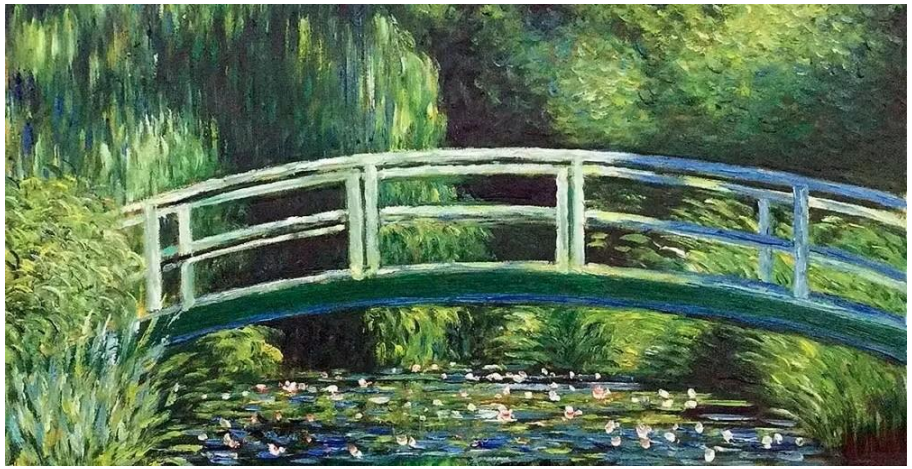
Gauge anomalies on shell and collinear factorization

Edoardo Alviani, IJCLab – Université Paris-Saclay
alviani@ijclab.in2p3.fr

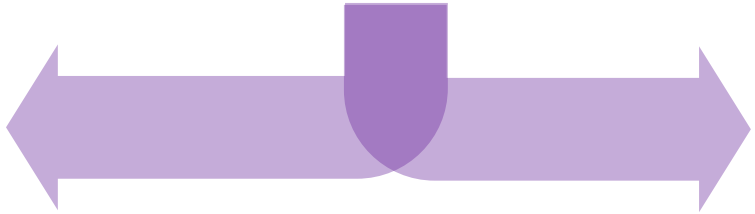
Based on work 2509.03368 in collaboration with Adam Falkowski

A river can be crossed with a bridge or a boat.



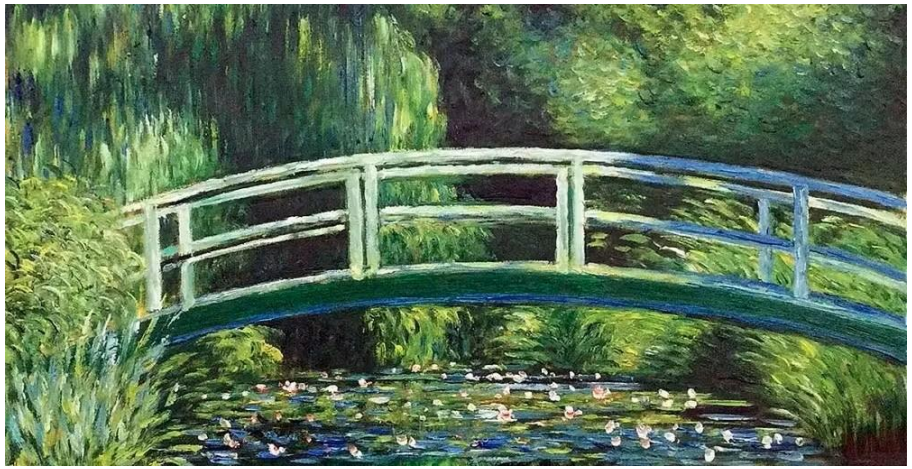
A storm calls for

Resistance to wind fluctuations



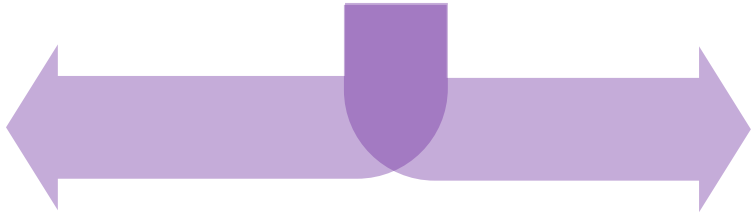
Resistance to water waves fluctuations

A river can be crossed with a bridge or a boat.



A storm calls for

Resistance to wind fluctuations



Resistance to water waves fluctuations

In the same way any theory has to be **resistant to quantum fluctuations** regardless of the formalism we use to navigate it.

In this talk

1. Introducing the question: anomalies and on-shell methods
2. Gravity as a probe for anomalies
3. Collinear factorization and unitarity
4. Gauge anomalies on shell
5. Outlook

In this talk

1. Introducing the question: anomalies and on-shell methods
2. Gravity as a probe for anomalies
3. Collinear factorization and unitarity
4. Gauge anomalies on shell
5. Outlook

Why???

In this talk

1. Introducing the question: anomalies and on-shell methods
2. Gravity as a probe for anomalies
3. Collinear factorization and unitarity
4. Gauge anomalies on shell
5. Outlook

[*Dark Matter from Anomaly Cancellation at the LHC*, Butterwort, Debnath, Pérez, Yeh]

[*On Anomaly-Free Dark Matter Models*, Perez, Golias, Li, Murgui, Plasencia]

[*Anomaly-free dark matter models with one-loop neutrino masses and a gauged U(1) symmetry*, de Boer, Klasen, Zeinstra]

Why???

- Any quantum field theory, visible or dark, must be anomaly-free
- DM models with new U(1) symmetry are potentially anomalous: DM candidates can be predicted from gauge anomay cancellation
- Anomaly-induced effective operators can mediate dark-visible interactions (ALPs)

1. Introducing the question: quantum fields

Lagrangians for spin-1 and -2 particles have **gauge symmetry**.

Unphysical: gauge dependence cancel in observables

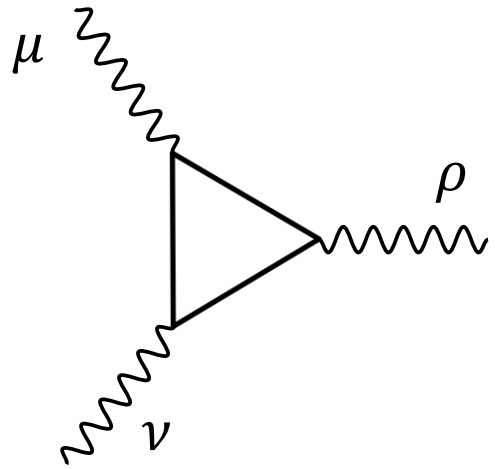
Necessary: prevents the existence of negative norm states

1. Introducing the question: quantum fields

Lagrangians for spin-1 and -2 particles have **gauge symmetry**.

Unphysical: gauge dependence cancel in observables

Necessary: prevents the existence of negative norm states



Anomaly: breaking of **a** symmetry upon computing loop correlators.

Not problematic in principle.

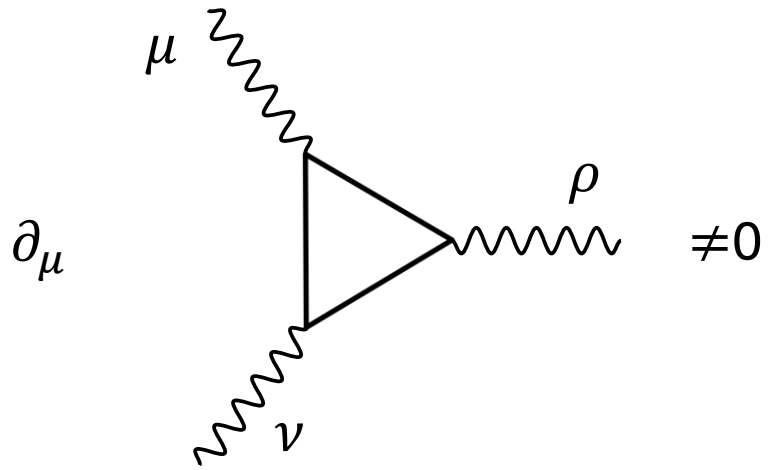
1. Introducing the question: quantum fields

Lagrangians for spin-1 and -2 particles have **gauge symmetry**.

Unphysical: gauge dependence cancel in observables

Necessary: prevents the existence of negative norm states

At one-loop, gauge symmetry (as Ward identities) is lost



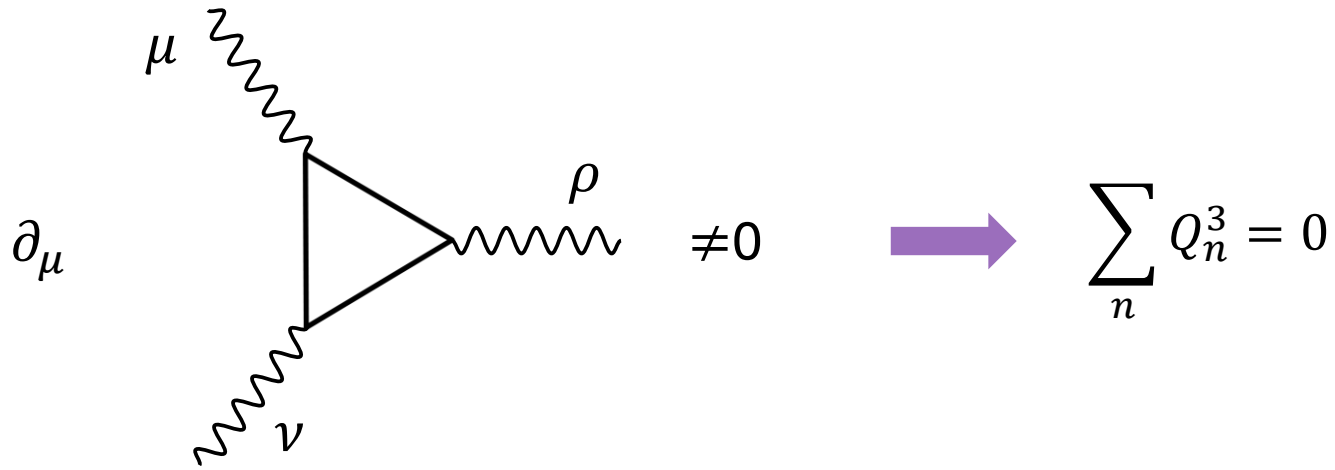
1. Introducing the question: quantum fields

Lagrangians for spin-1 and -2 particles have **gauge symmetry**.

Unphysical: gauge dependence cancel in observables

Necessary: prevents the existence of negative norm states

At one-loop, gauge symmetry (as Ward identities) is lost unless **gauge anomaly cancellation conditions** are satisfied

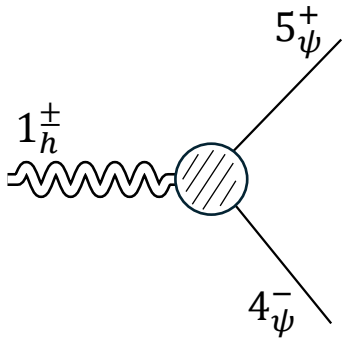


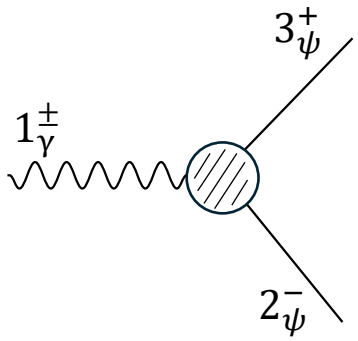
$$\sum_n Q_n^3 = 0$$

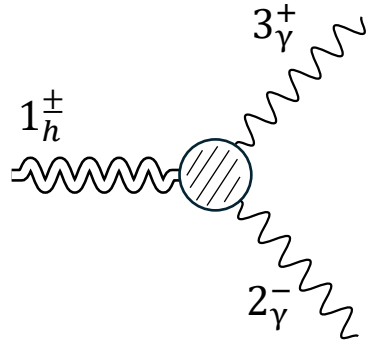
Physical constraints on the spectrum of the theory

1. Introducing the question: on-shell methods

We can construct our theory using as building blocks not quantum fields, but three-point amplitudes.

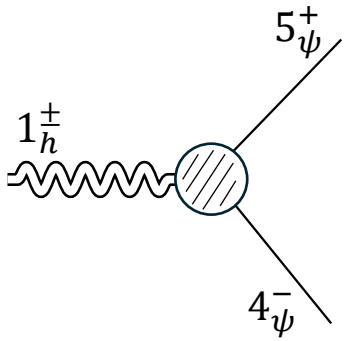


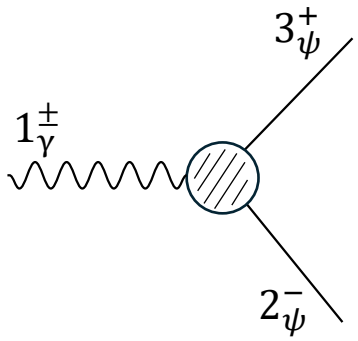


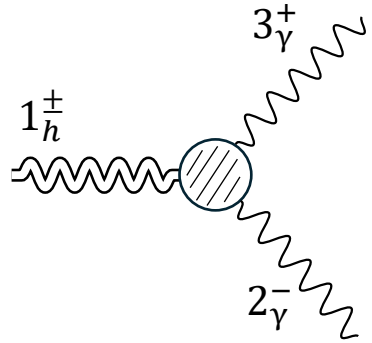


1. Introducing the question: on-shell methods

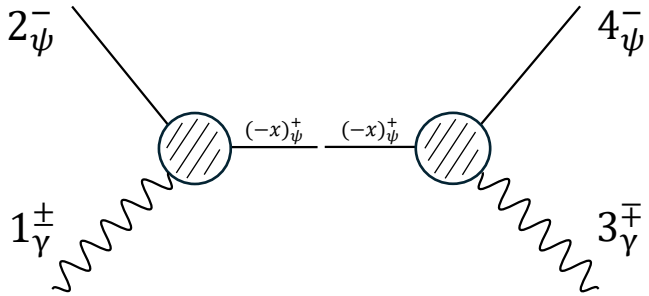
We can construct our theory using as building blocks not quantum fields, but three-point amplitudes.

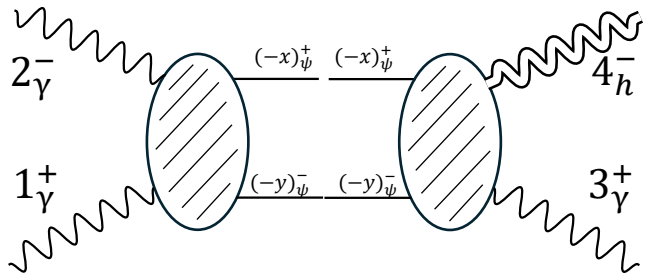






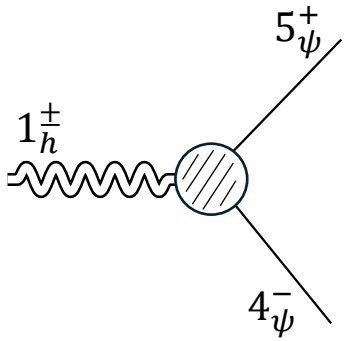
Higher-point and loop-amplitudes can be constructed from a set of three-point ones by unitarity

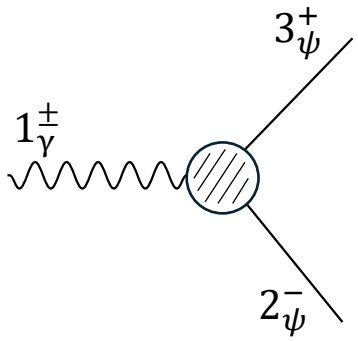


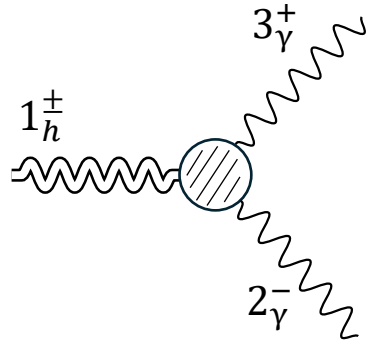


1. Introducing the question: on-shell methods

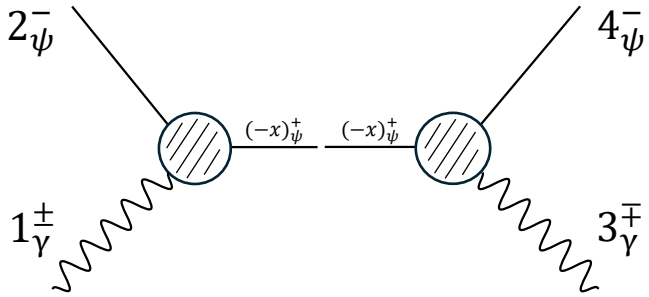
We can construct our theory using as building blocks not quantum fields, but three-point amplitudes.

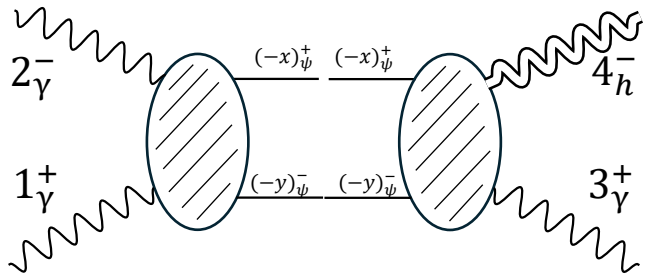






Higher-point and loop-amplitudes can be constructed from a set of three-point ones by unitarity





Gauge invariance is bypassed from the get-go

1. Introducing the question: goal of this work

In standard formulation of QFT gauge invariance gives constraints on the charges of the spectrum.

On-shell formulation of QFT bypasses gauge invariance.

Where are such constraints from in the on-shell formulation?

1. Introducing the question: goal of this work

In standard formulation of QFT gauge invariance gives constraints on the charges of the spectrum.

On-shell formulation of QFT bypasses gauge invariance.

Where are such constraints from in the on-shell formulation?

Earlier work [Y. Huang, D. McGady] suggests that they are hidden in a tension between unitarity and locality

The question was discussed from people in the audience.

[Q. Bonnefoy, S. De Angelis, E. Gendy, C. Grojean, J. Roosmale Nepveu]

1. Introducing the question: goal of this work

In standard formulation of QFT gauge invariance gives constraints on the charges of the spectrum

On-shell formulation of QFT bypasses gauge invariance

Where are such constraints from in the on-shell formulation?

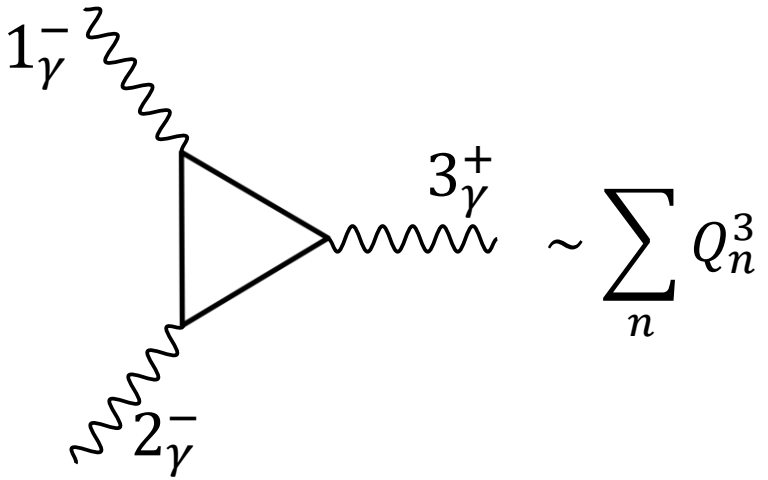
Earlier work [Y. Huang, D. McGady] suggests that they are hidden in a tension between unitarity and locality

The question was discussed from people in the audience. [Q. Bonnefoy, S. De Angelis, E. Gendy, C. Grojean, J. Roosmale Nepveu]

Answer: **anomalies lead to a breakdown of collinear factorization**

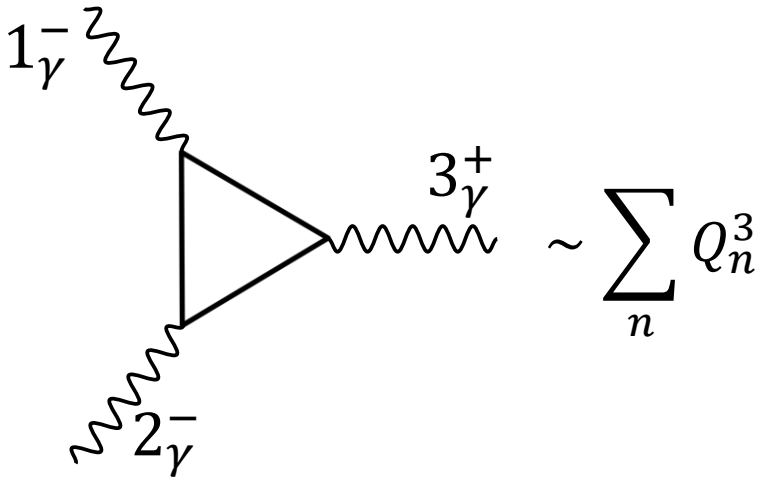
2. Gravity as a probe for anomalies

Anomalies appear in triangle one-loop diagrams, but for technical reasons these are hard to study on-shell.

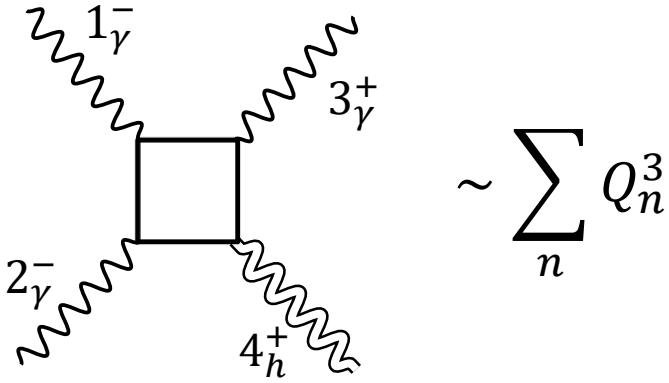


2. Gravity as a probe for anomalies

Anomalies appear in triangle one-loop diagrams, but for technical reasons these are hard to study on-shell.

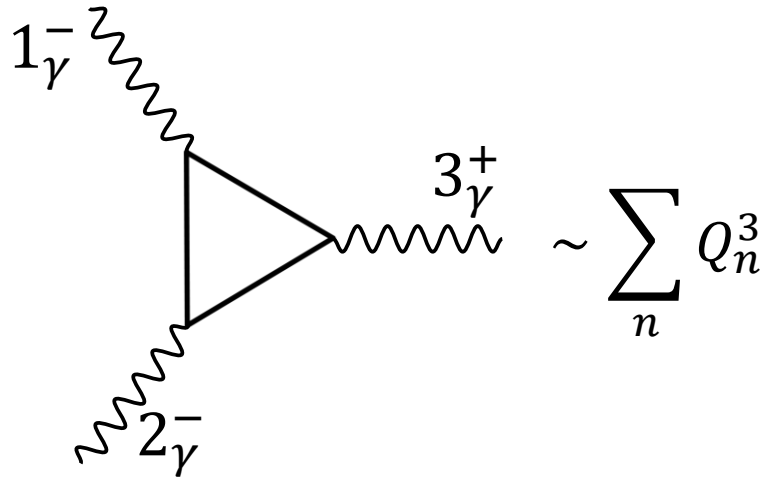


Certain four-point amplitudes probe the anomaly, thanks to gravity. These are computable on-shell, up to rational terms.



2. Gravity as a probe for anomalies

Anomalies appear in triangle one-loop diagrams, but for technical reasons these are hard to study on-shell.

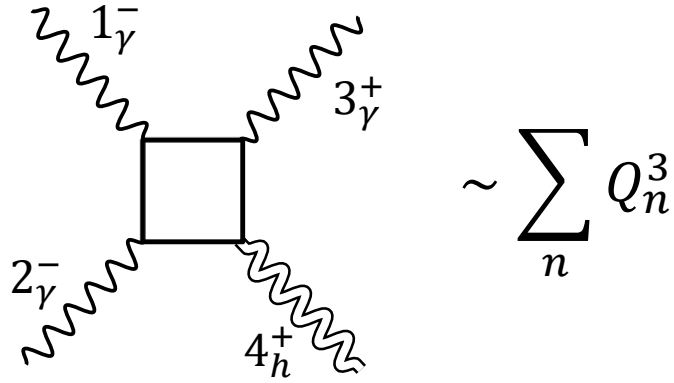


Certain four-point amplitudes probe the anomaly, thanks to gravity. These are computable on-shell, up to rational terms.

$$\mathcal{M}^{(1)}\left[1_\gamma^- 2_\gamma^- 3_\gamma^+ 4_h^+\right] = M_{3\gamma h} + R_{3\gamma h},$$

$$M_{3\gamma h} \equiv -\left[\sum_{n=1}^{N} s_n Q_n^3\right] \frac{\sqrt{2}e^3}{16\pi^2 M_{\rm Pl}} \langle 12\rangle^2 [34]^2 [4|p_1 p_2|4] F_{3\gamma h}(s,t),$$

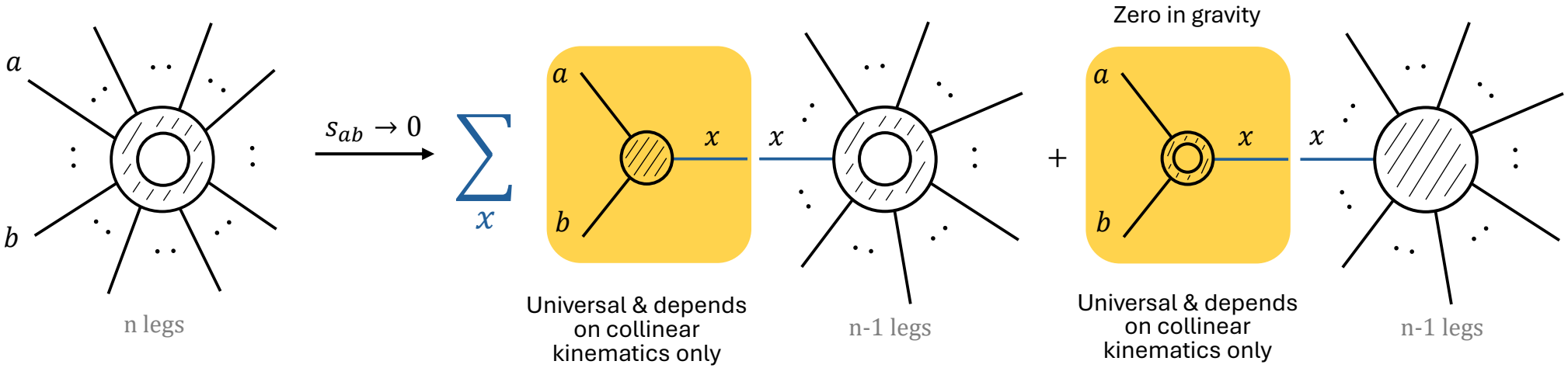
$$F_{3\gamma h}(s,t) \equiv \frac{1}{s^3}\left\{4\log\left(\frac{t}{u}\right) + \frac{t-u}{s}\left[\log\left(\frac{t}{u}\right)^2 + \pi^2\right]\right\}|_{u=-s-t}.$$



3. Collinear factorization

Our statement is that anomalies manifest on-shell as the breakdown of collinear factorization, formulated as

[Bern, Chalmers, Dixon, Kosower, Perelstein, Rowowsky]

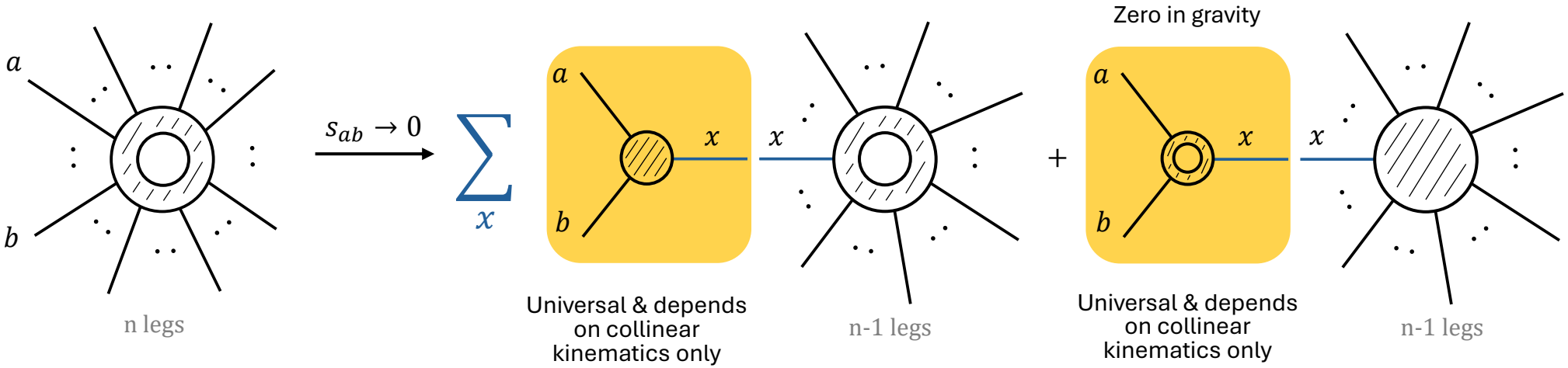


(1)

3. Collinear factorization

Our statement is that anomalies manifest on-shell as the breakdown of collinear factorization, formulated as

[Bern, Chalmers, Dixon, Kosower, Perelstein, Rowowsky]



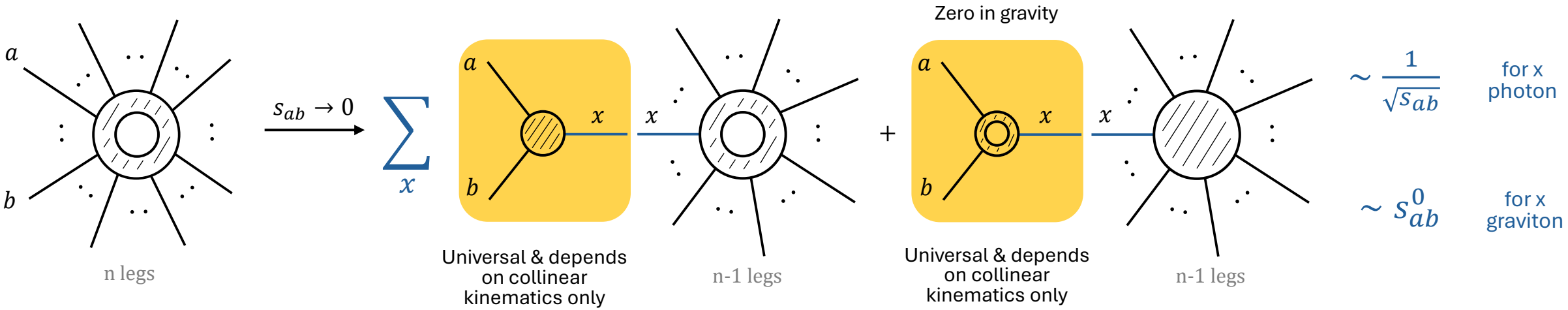
(1)

Crucially, this factorization holds for $n \geq 5$

3. Collinear factorization

Our statement is that anomalies manifest on-shell as the breakdown of collinear factorization, formulated as

[Bern, Chalmers, Dixon, Kosower, Perelstein, Rowowsky]

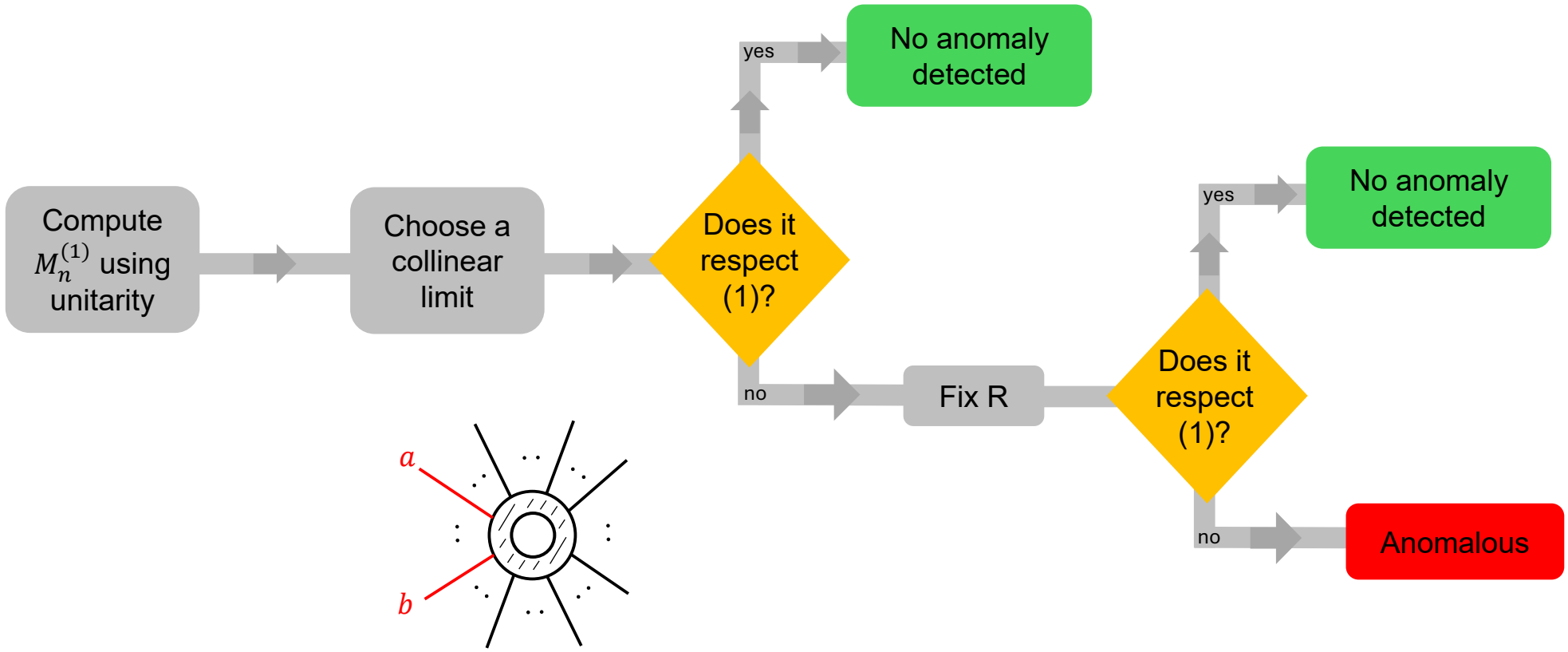


Crucially, this factorization holds for $n \geq 5$

Any deviation from this pattern leads to an inconsistency.

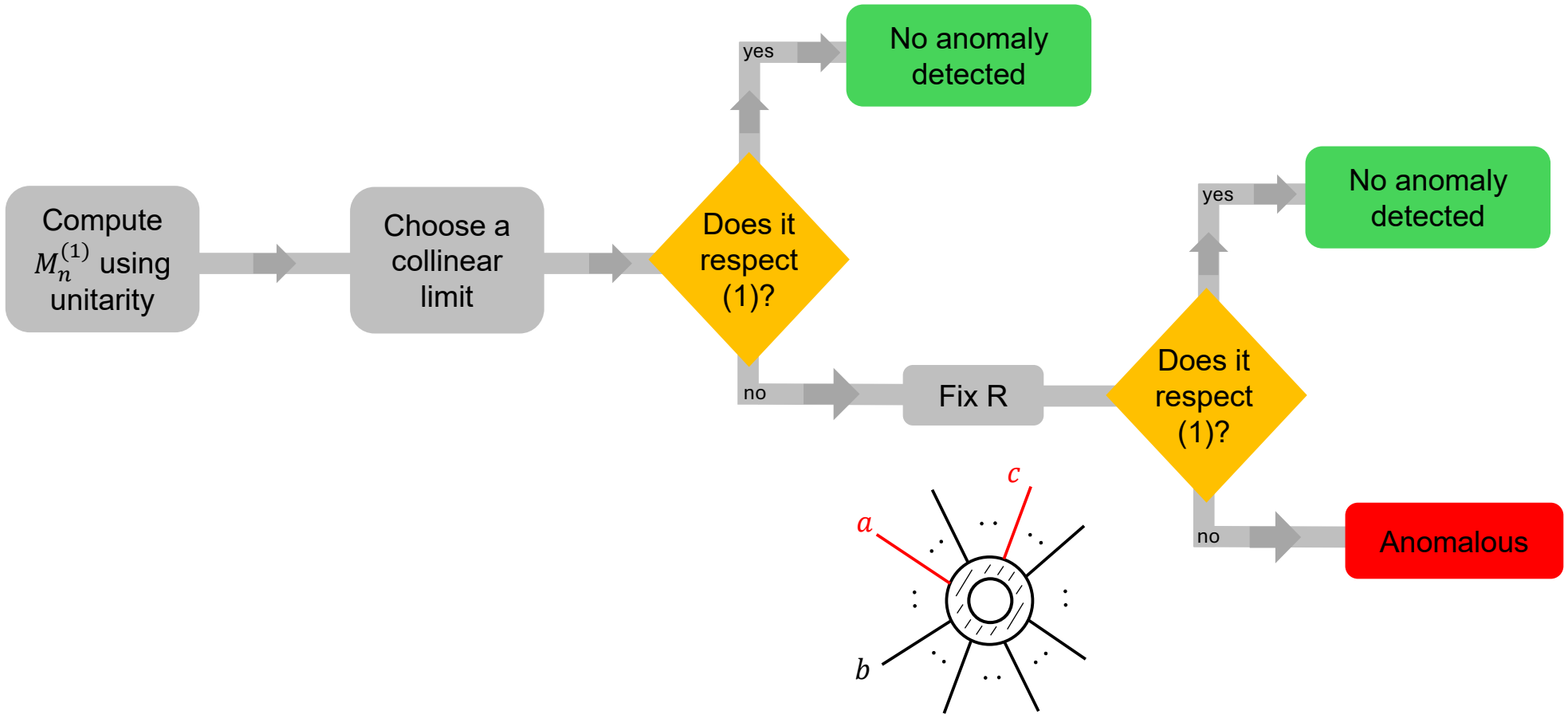
3. Collinear factorization

We constructed the following algorithm to detect anomalous theories fully on-shell



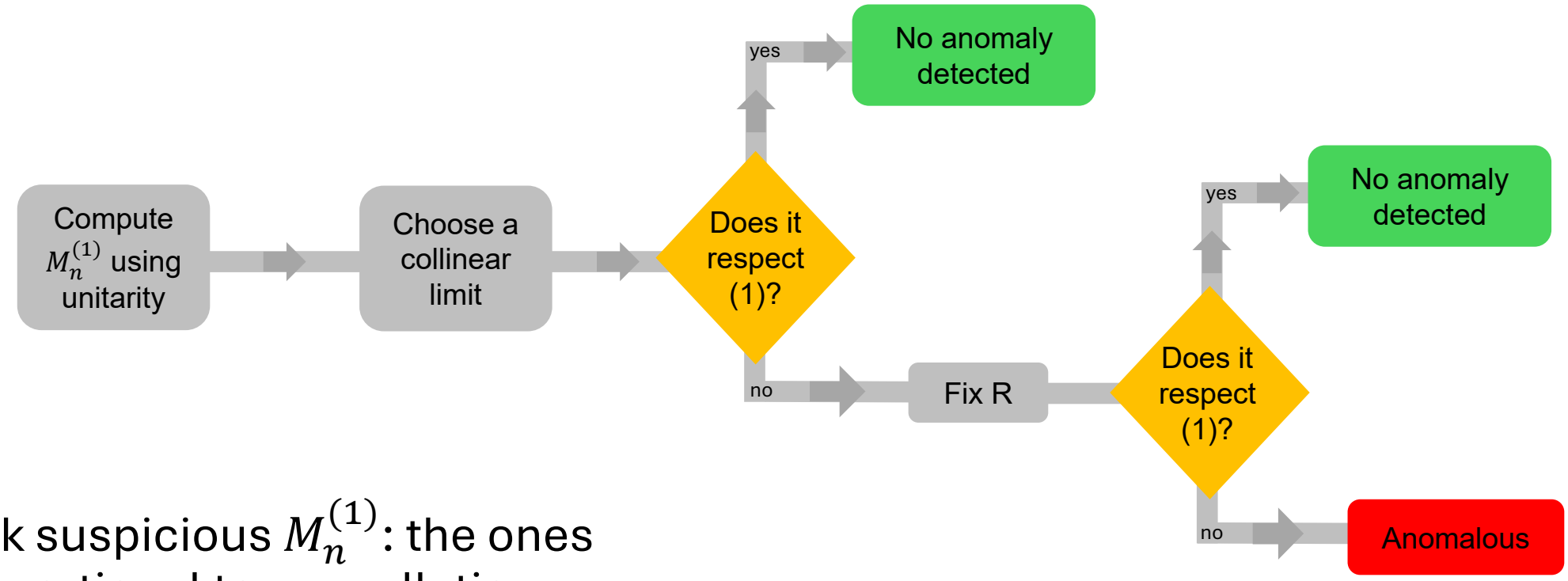
3. Collinear factorization

We constructed the following algorithm to detect anomalous theories fully on-shell



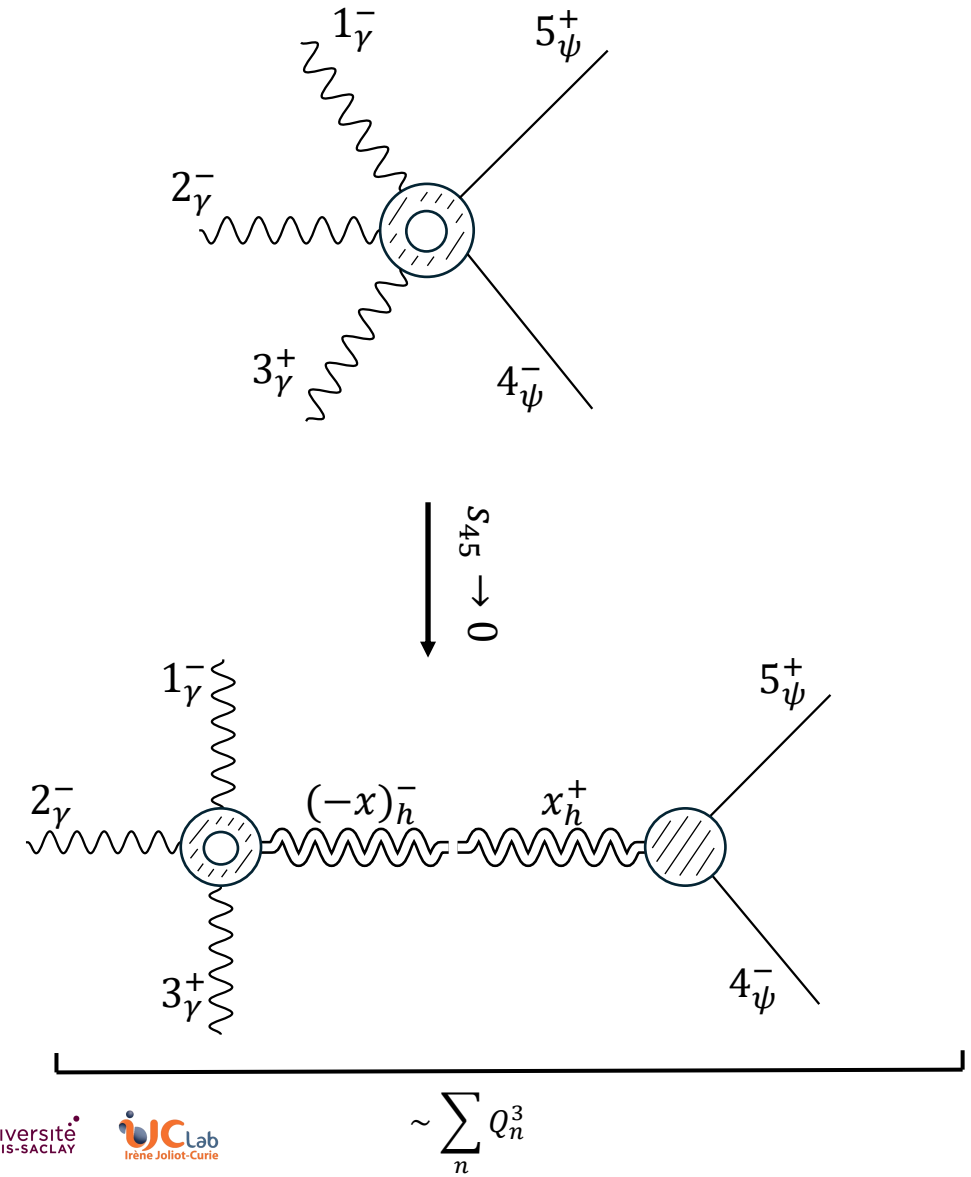
3. Collinear factorization

We constructed the following algorithm to detect anomalous theories fully on-shell



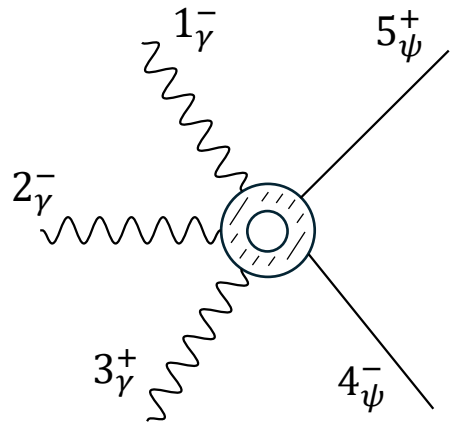
Pick suspicious $M_n^{(1)}$: the ones proportional to cancellation conditions (e.g. $\sum_n Q_n^3$)

4. Gauge anomalies on-shell: $U(1)^3$

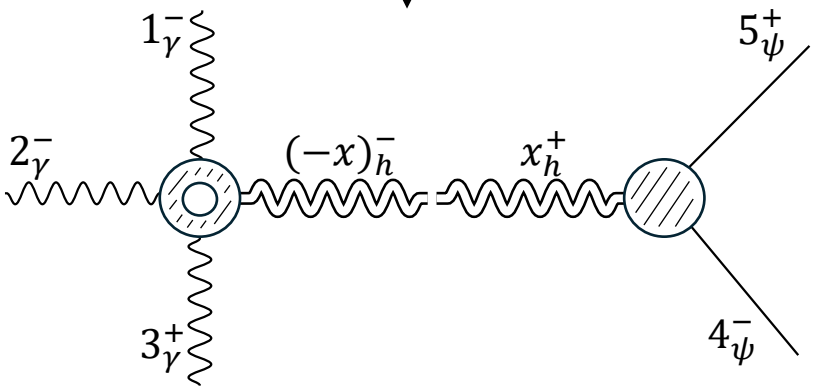


Start from a five point amplitude and take a collinear limit to expose the structure of interest.

4. Gauge anomalies on-shell: $U(1)^3$



$s_{45} \to 0$



$\sim \sum_n Q_n^3$

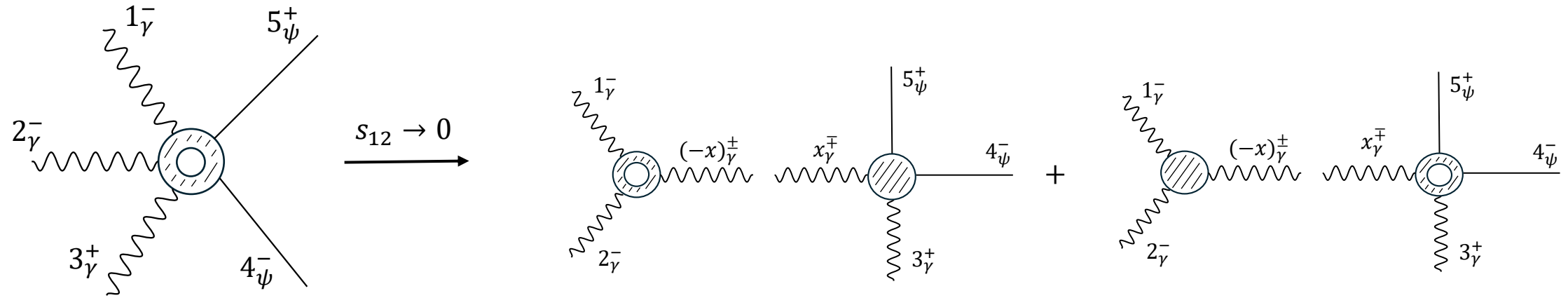
Take a second collinear limit to test the pole structure. Does it scale as $\frac{1}{\sqrt{s_{12}}}$ or s_{12}^0?

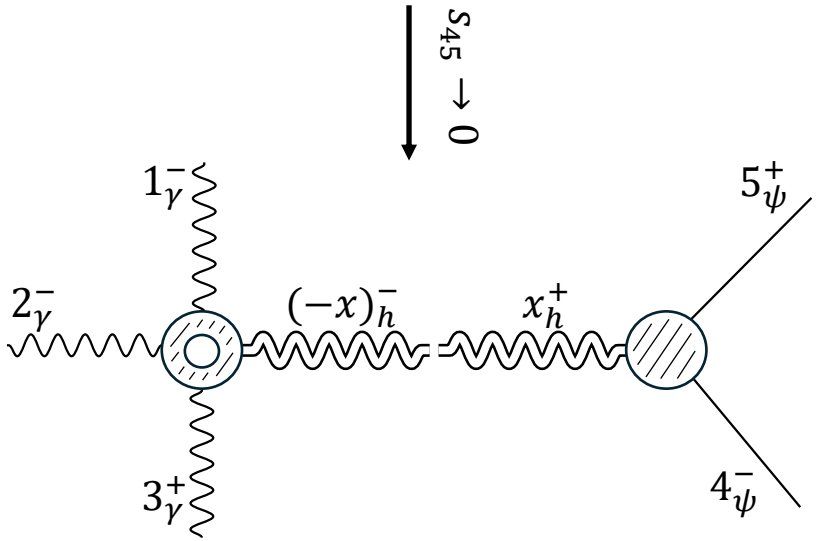
If yes, can we **interpret this as a collinear factorization** of the 5-point amplitude?

$$\xrightarrow{s_{12} \to 0} \quad A\, S_{QED} \left(\frac{\sum_n Q_n^3}{s_{12} s_{13}} + O({s_{12}}^0, {s_{45}}^1) \right) + R$$

$\sim \frac{1}{\sqrt{s_{12}}}$

4. Gauge anomalies on-shell: $U(1)^3$





$\sim \sum_n Q_n^3$

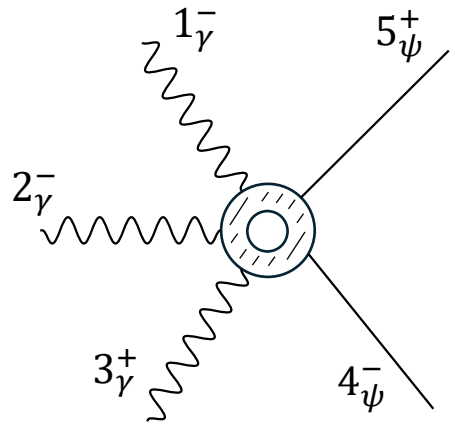
$s_{12} \to 0$

$\sim \frac{1}{\sqrt{s_{12}}}$

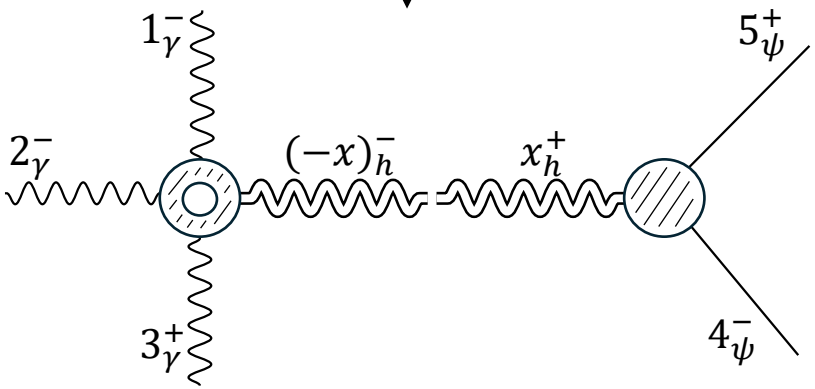
$$A\, S_{QED}\left(\frac{\sum_n Q_n^3}{s_{12}s_{13}} + O({s_{12}}^0, {s_{45}}^1)\right) + R$$

The **collinear interpretation fails**: non universal splitting function, mismatching charge structure.
We can still save the amplitude by **adjusting** R.

4. Gauge anomalies on-shell: $U(1)^3$



$s_{45} \to 0$



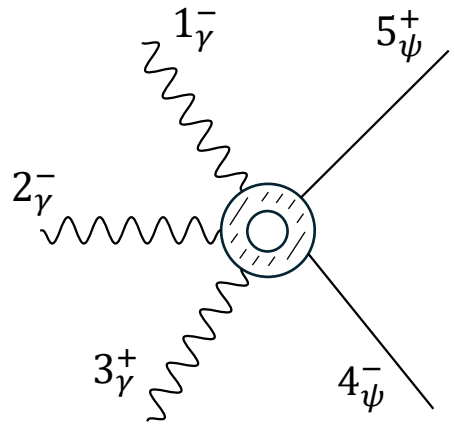
$\sim \sum_n Q_n^3$

R **cancels** the s_{12} singularity

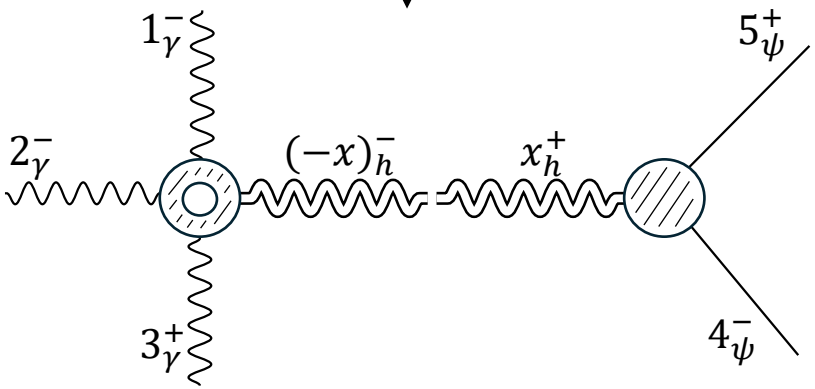
$$R = A \sum_n Q_n^3 \; S_{QED} \frac{1}{{s_{12}}^2 s_{13}}$$

$$\xrightarrow{s_{12} \to 0} \; A \, S_{QED} \left(\frac{\sum_n Q_n^3}{s_{12} s_{13}} + O({s_{12}}^0, {s_{45}}^1) \right) + R \; \sim O(\sqrt{s_{12}})$$

4. Gauge anomalies on-shell: $U(1)^3$



$s_{45} \to 0$



$\sim \sum_n Q_n^3$

R cancels the s_{12} singularity, but **it introduces another one** in s_{13}.
This one cannot be interpreted as a collinear factorization, and there is no other way to cancel it.

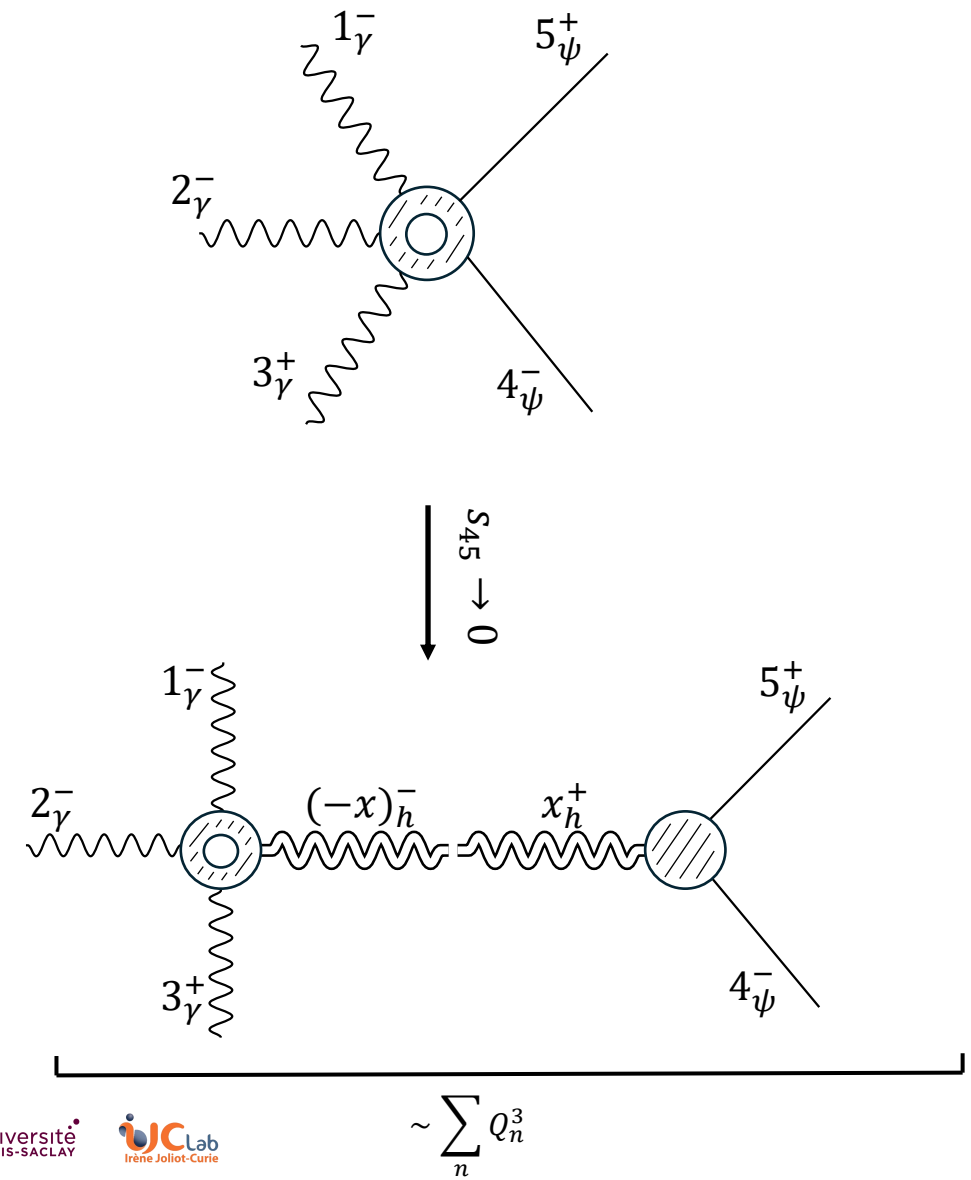
$s_{13} \to 0$

$$R = A \sum_n Q_n^3 \; S_{QED} \frac{1}{{s_{12}}^2 s_{13}} \sim \frac{1}{s_{13}}$$

$s_{12} \to 0$

$$A \; S_{QED} \left(\frac{\sum_n Q_n^3}{s_{12} s_{13}} + O({s_{12}}^0, {s_{45}}^1) \right) + R \sim O(\sqrt{s_{12}})$$

4. Gauge anomalies on-shell: $\mathrm{U}(1)^3$



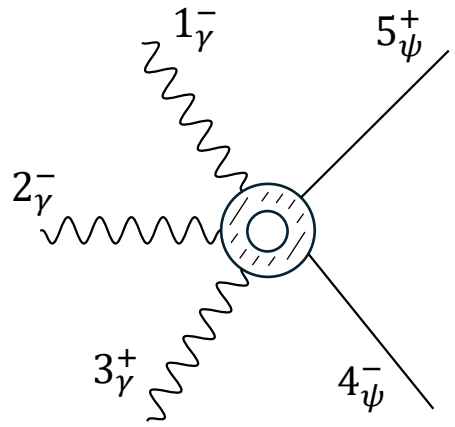
R cancels the s_{12} singularity, but **it introduces another one** in s_{13}.
This one cannot be interpreted as a collinear factorization, and there is no other way to cancel it.

The amplitude is inconsistent, unless this piece is identically zero. This means imposing

$$\sum_n Q_n^3 = 0$$

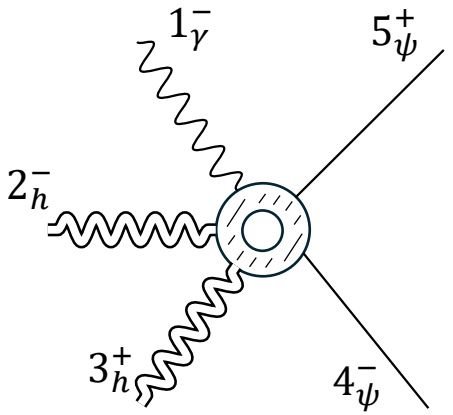
Anomaly cancellation conditions follow from restoring collinear factorization

4. Gauge anomalies on-shell: general



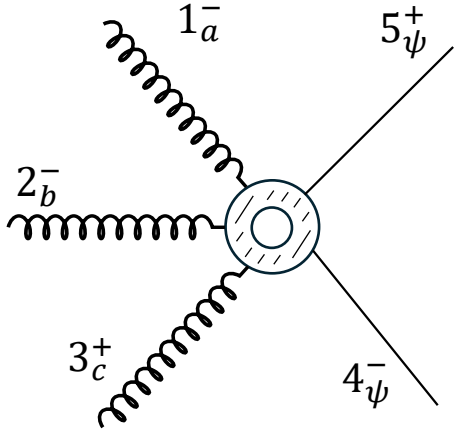
$\mathrm{U}(1)^3$

$$\sum_n Q_n^3 = 0$$



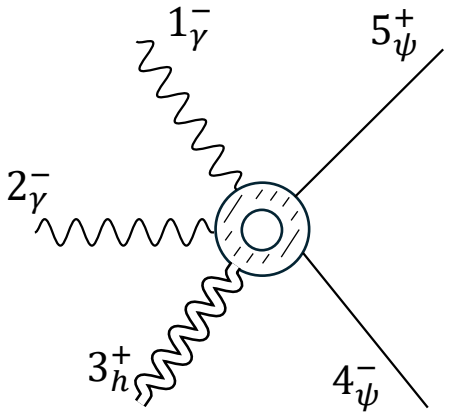
U(1)-gravitational

$$\sum_n Q_n = 0$$



$\mathrm{SU}(3)^3$

$$d^{abc} = 0$$



$$\sum_n Q_n^2 \neq 0$$

5. Outlook

In this work we showed how:

- Gauge anomalies appear on shell as breakdown of collinear factorization
- Restoring the factorization yields gauge anomaly cancellation conditions

For the future:

- Include Green-Schwarz formalism: how new DOF restore factorization
- Study global anomalies via a gravity probe
- Possible on-shell proof of the Adler-Bardeen theorem

5. Outlook

In this work we showed how:

- Gauge anomalies appear on shell as breakdown of collinear factorization
- Restoring the factorization yields gauge anomaly cancellation conditions

For the future:

- Include Green-Schwarz formalism: how new DOF restore factorization
- Study global anomalies via a gravity probe
- Possible on-shell proof of the Adler-Bardeen theorem

And I thank you!