Towards unveiling the properties of the massive black hole binary population with LISA

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Doctorante
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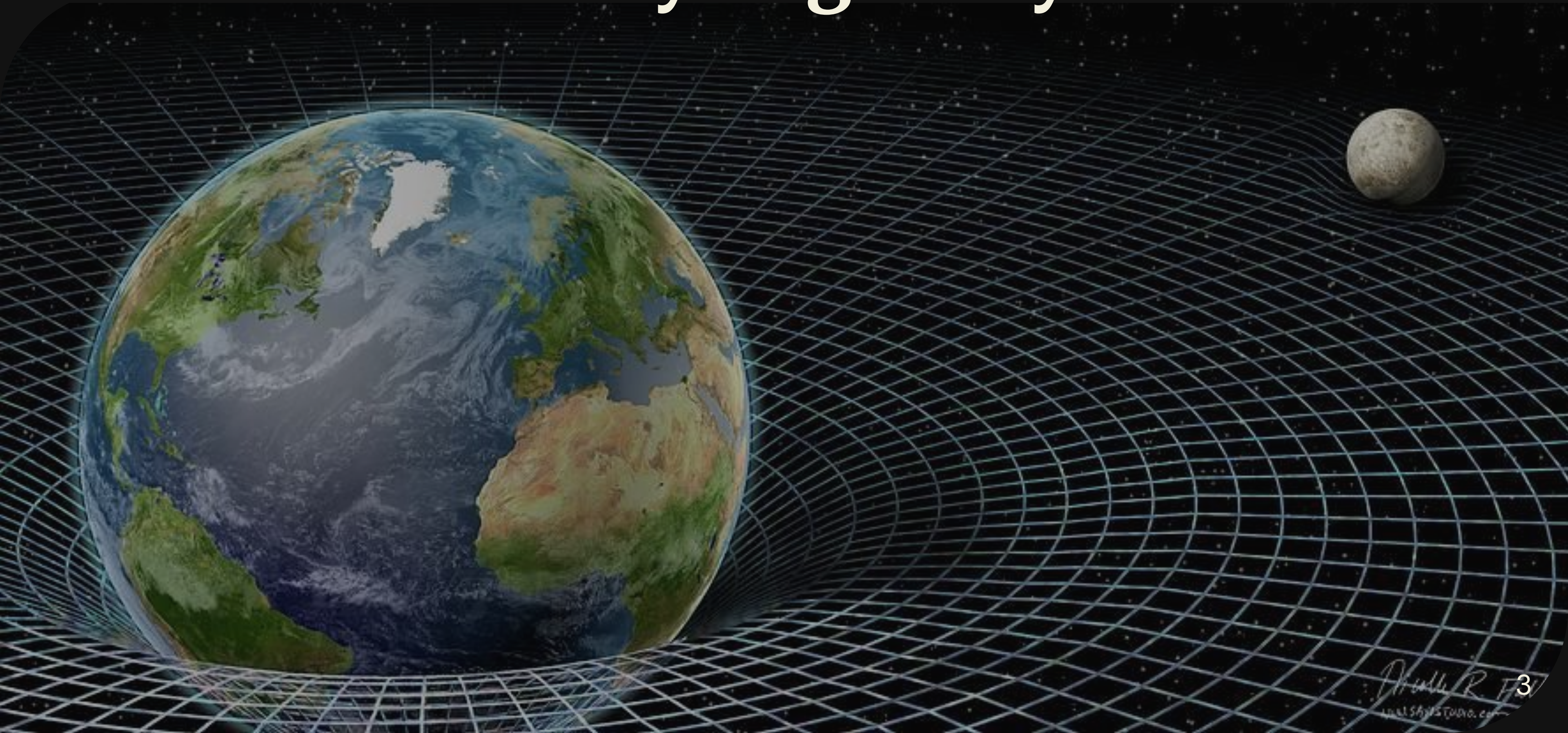
Amphi II @ Toulouse
4. Avril, 2025

Agenda

- Einstein’s Universe
- The Laser Interferometer Space Antenna (LISA)
- Massive Black Hole Binaries (MBHBs)
- The Analytical Model
- Hierarchical Bayesian Inference
- Large scale catalog comparison
- Conclusions

Einstein’s theory of gravity



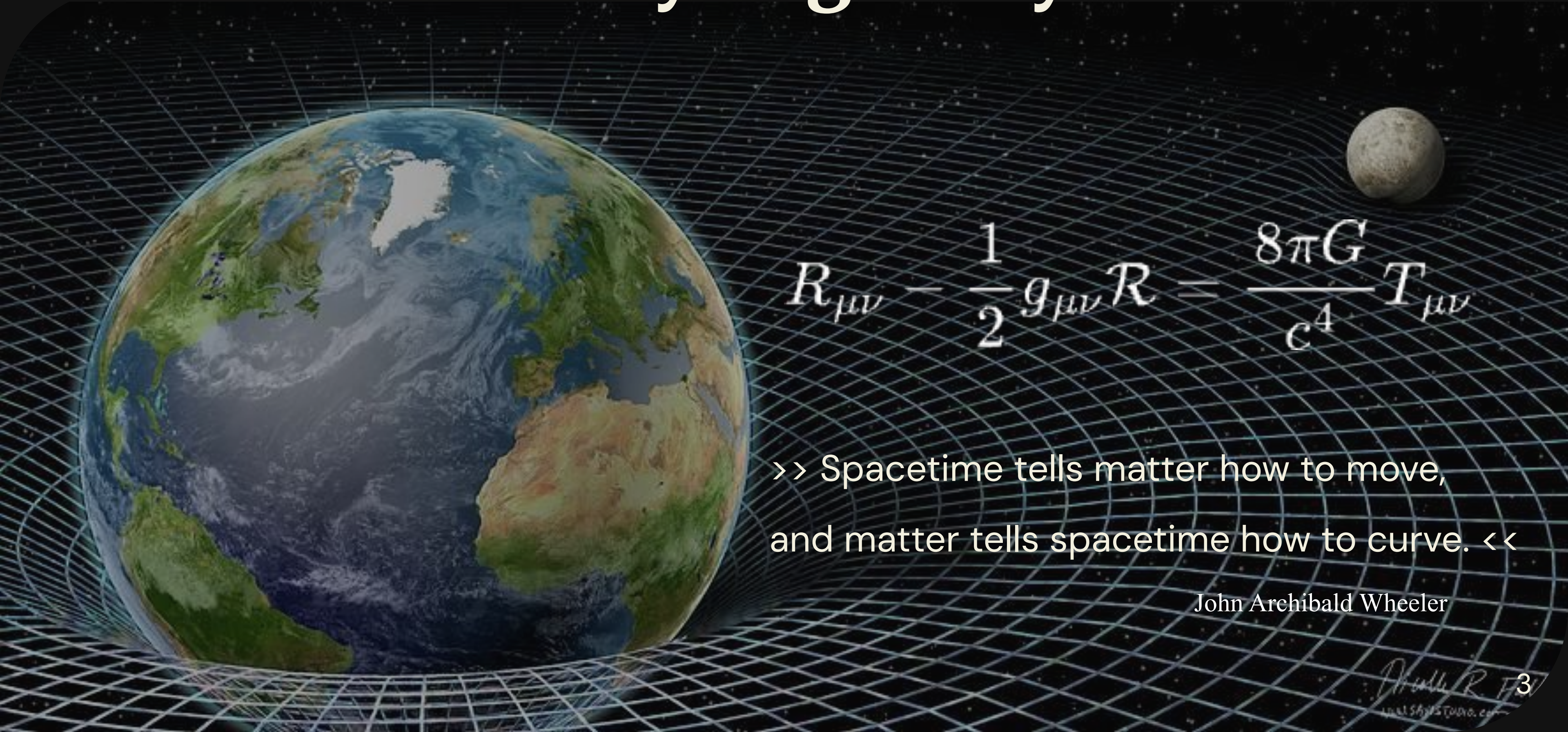
Einstein's theory of gravity

Gravity

=

Spacetime curvature

Einstein’s theory of gravity



>> Spacetime tells matter how to move,

and matter tells spacetime how to curve. <<

John Archibald Wheeler

Einstein’s theory of gravity

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

>> Spacetime tells matter how to move,

and matter tells spacetime how to curve. <<

John Archibald Wheeler

Gravitational wave (GW) equation

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad |h_{\mu\nu}| \ll 1$$

Flat spacetime + perturbation

Gravitational wave (GW) equation

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$$\partial^\nu \bar{h}_{\mu\nu} = 0 \qquad \wedge \qquad \bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h \qquad \wedge \qquad T_{\mu\nu} = 0$$

Lorentz gauge

Vacuum solution

Gravitational wave (GW) equation

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Lorentz gauge

Vacuum solution

$$\Longrightarrow \qquad \left(-\frac{\partial}{\partial t^2} + c^2\nabla^2\right)\bar{h}_{\mu\nu} = 0$$

Wave equation !

The plane wave solution

Ripples in spacetime

$$\bar{h}_{\mu\nu} = A_{\mu\nu} e^{i k_\alpha x^\alpha}$$

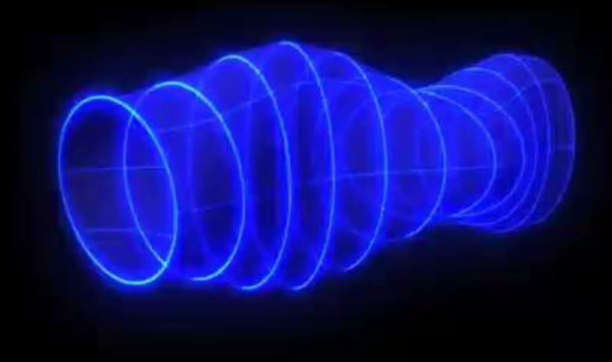
Plane wave solution

$$h^{\mathrm{TT}} = h_+ e^+_{ab} + h_\times e^\times_{ab}$$

Transverse traceless gauge

The plane wave solution

Ripples in spacetime



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Plane wave solution

Credit: ESA

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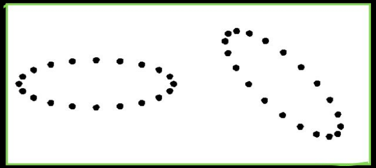
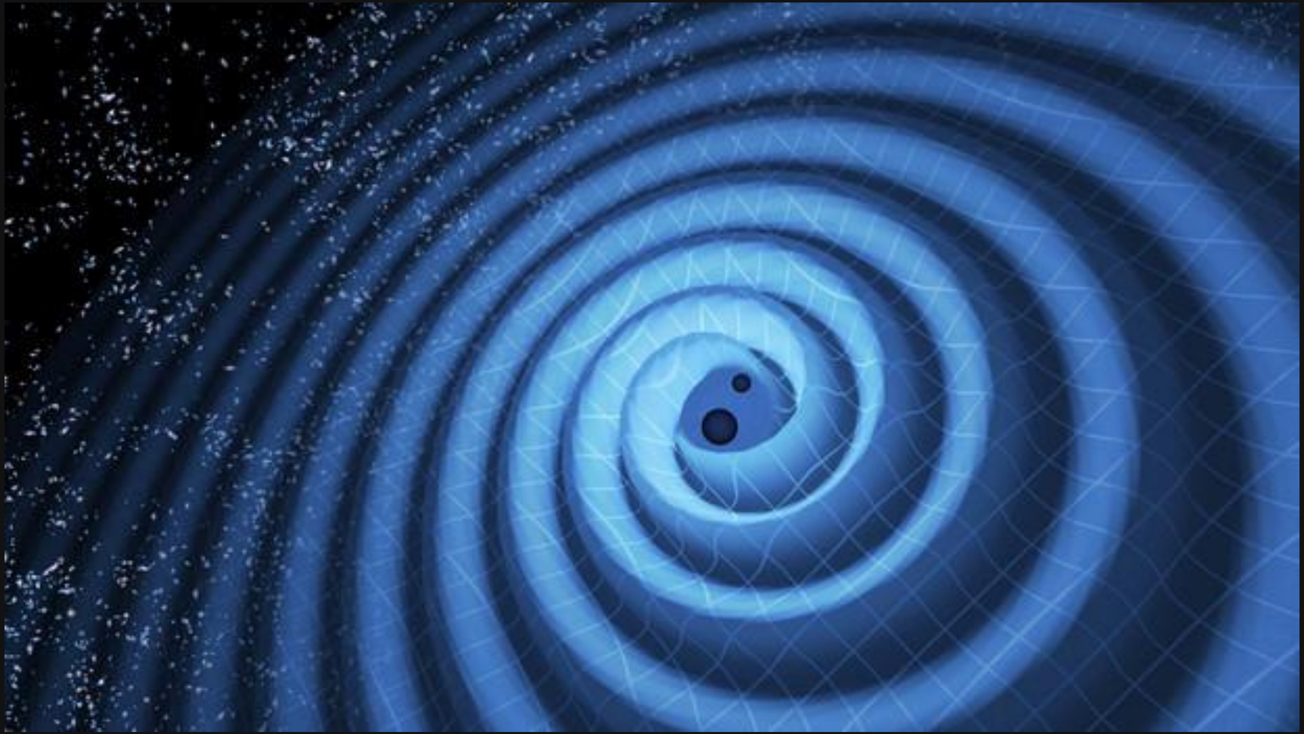
Transverse traceless gauge



h_+ $h_\times$

GWs from a binary

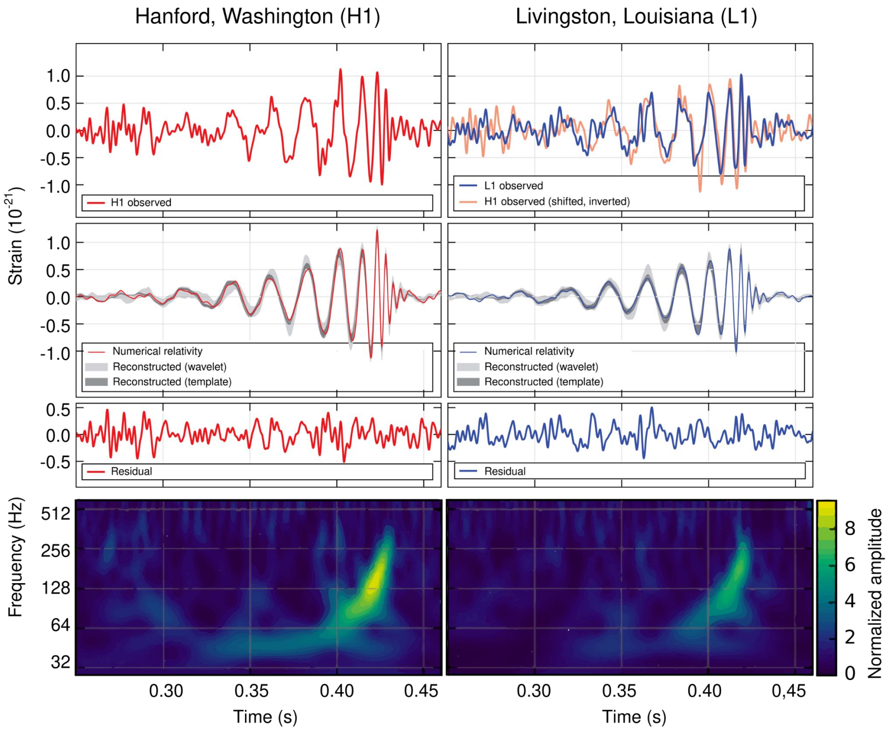
Ripples in spacetime



Credit:
LVK-collaboration

The first GW signal

GW140915 (September 15; 2014)



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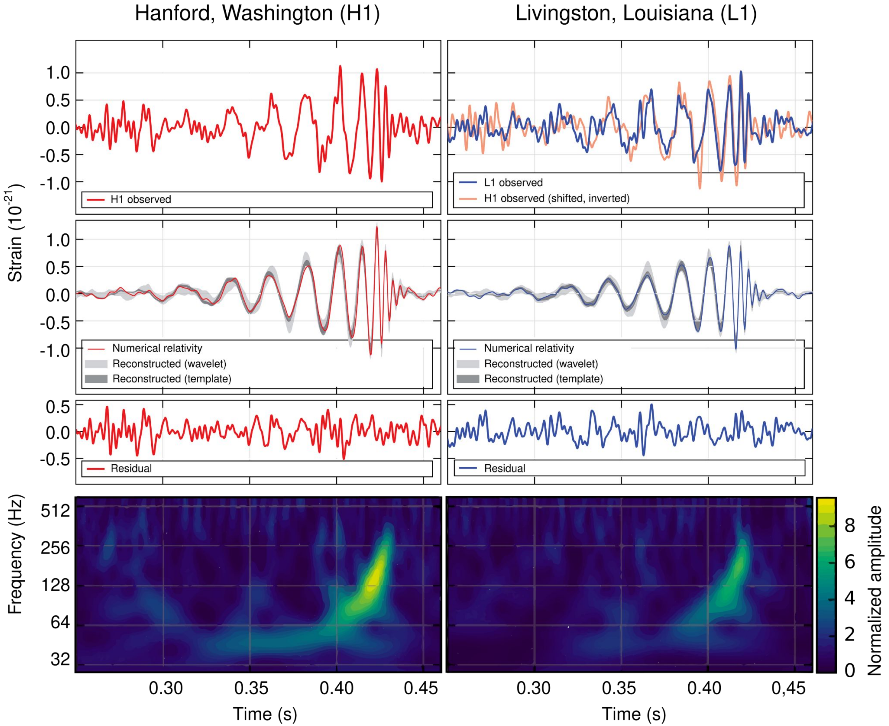
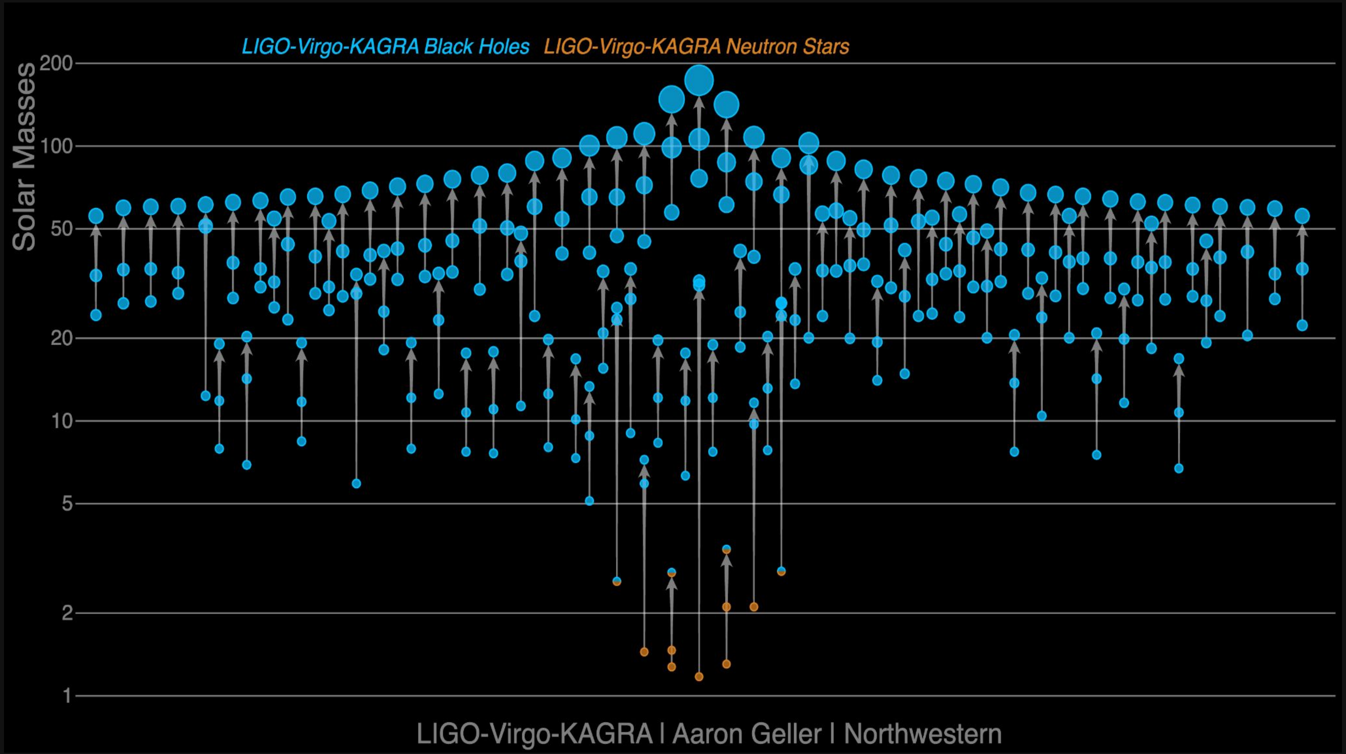


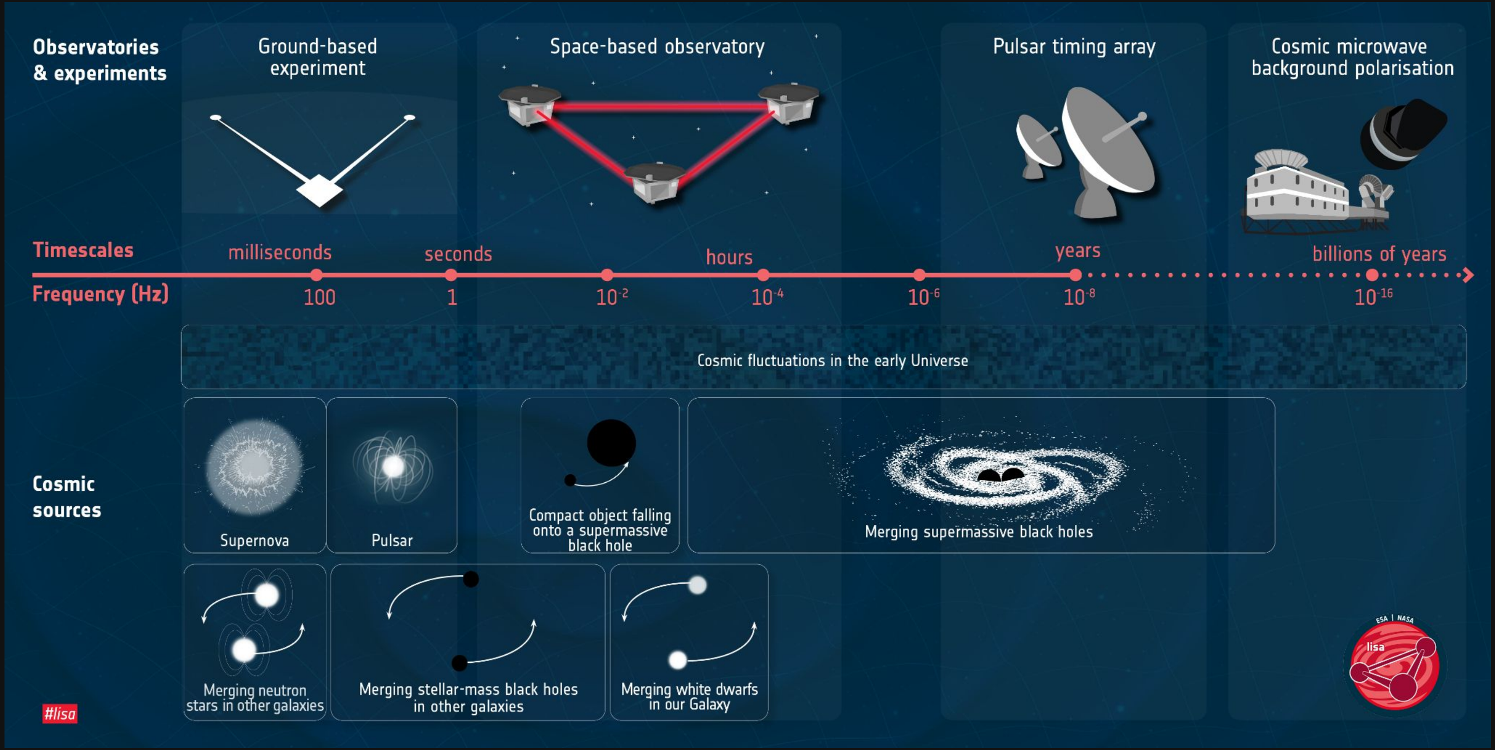
- Stellar origin black hole binary
- M_1 = ~ 35 $M_\odot$, M_2 = ~ 30 $M_\odot$ (equal mass)
- 35 Hz – 150 Hz
- ~ 400 Mpc (local universe)

Status of current observations



- > 90 events seen by the ground-based network (LIGO-Virgo-KAGRA)
- O1 - O3; O4 on-going
- Binary masses from 2 - 10^2 solar masses
- Local universe

GW spectrum


Observatories & experiments
Ground-based experiment
Space-based observatory
Pulsar timing array
Cosmic microwave background polarisation
Timescales
milliseconds
seconds
hours
years
billions of years
Frequency (Hz)
100
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10-2
10-4
10-6
10-8
10-16
Cosmic fluctuations in the early Universe
Cosmic sources
Supernova
Pulsar
Compact object falling onto a supermassive black hole
Merging supermassive black holes
Merging neutron stars in other galaxies
Merging stellar-mass black holes in other galaxies
Merging white dwarfs in our Galaxy
#lisa
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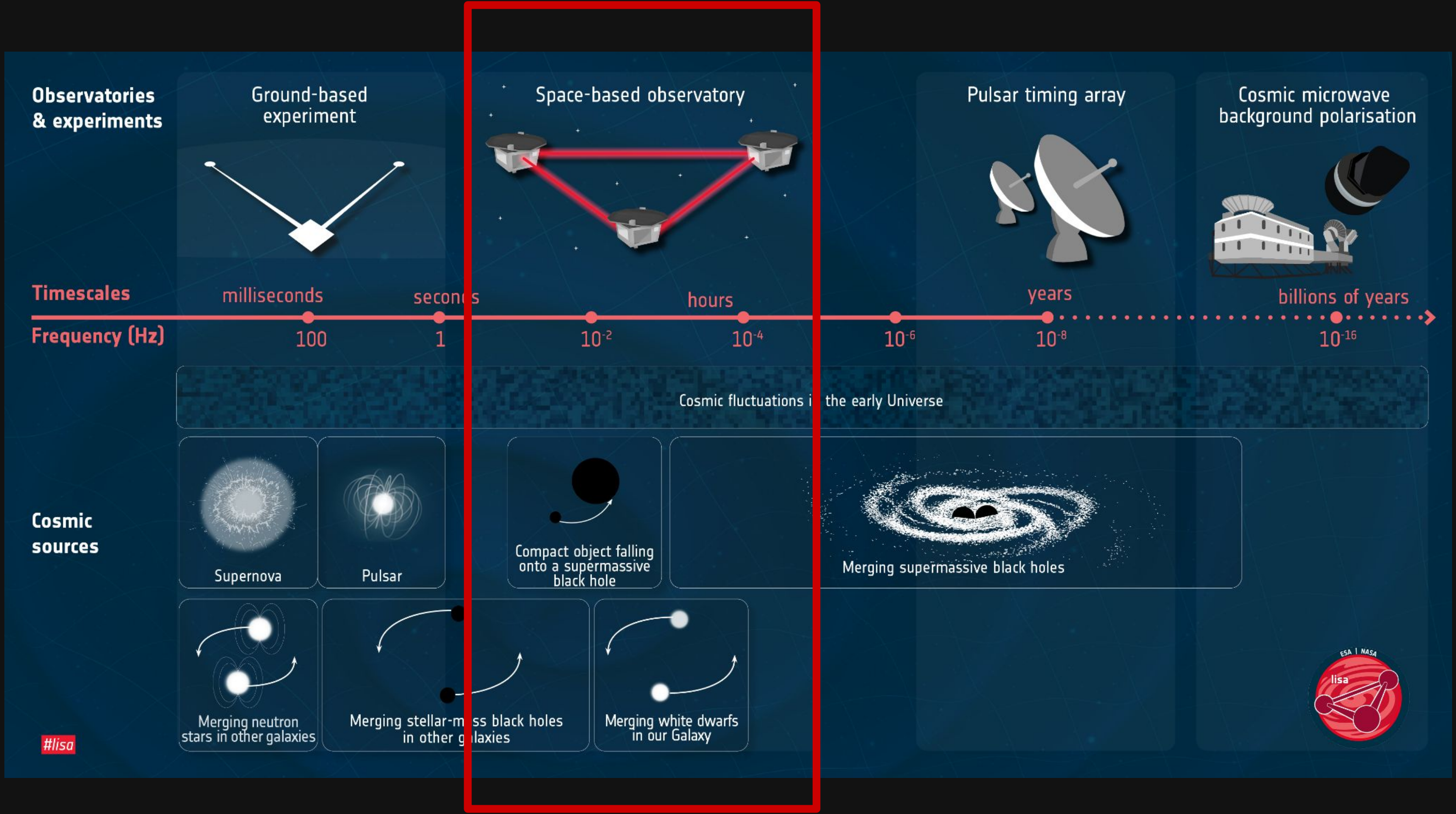

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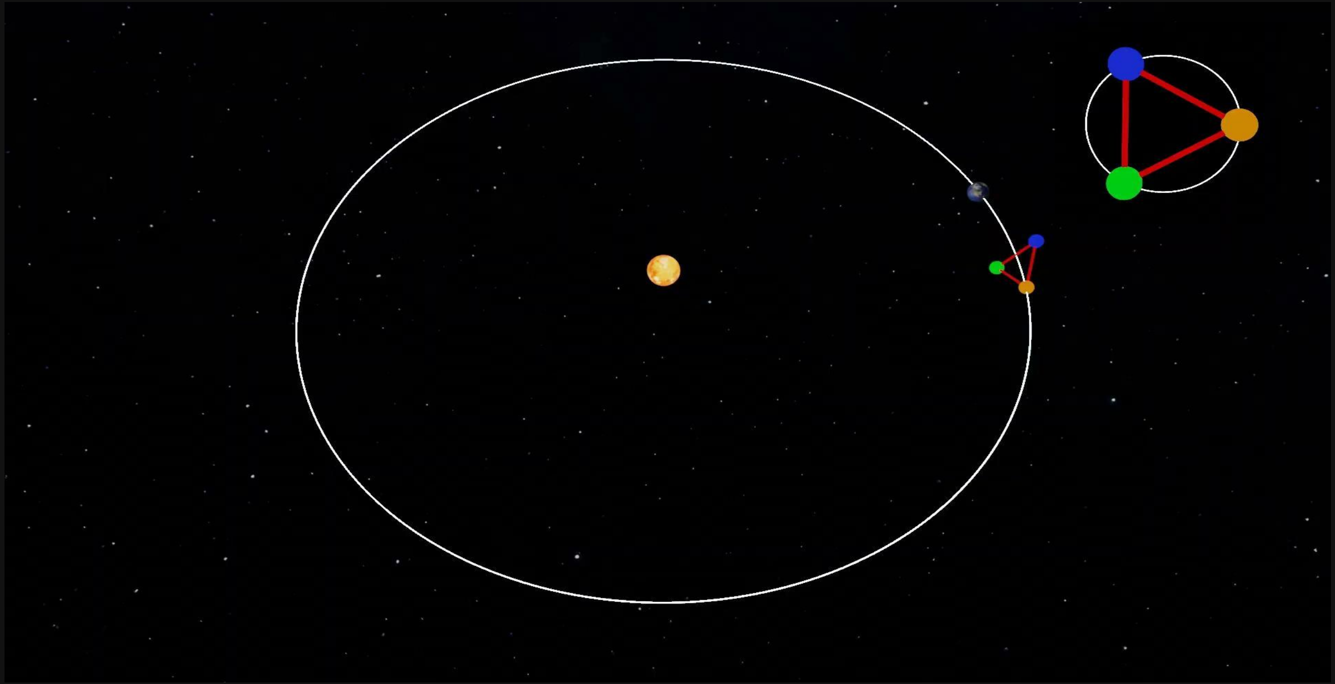
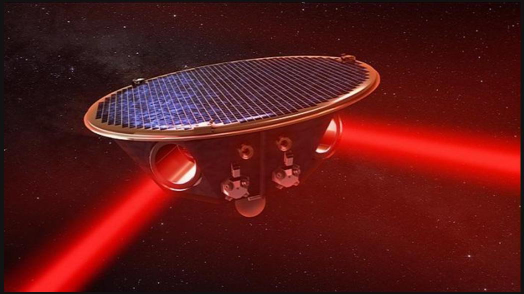
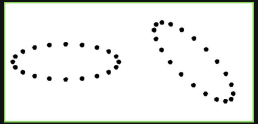
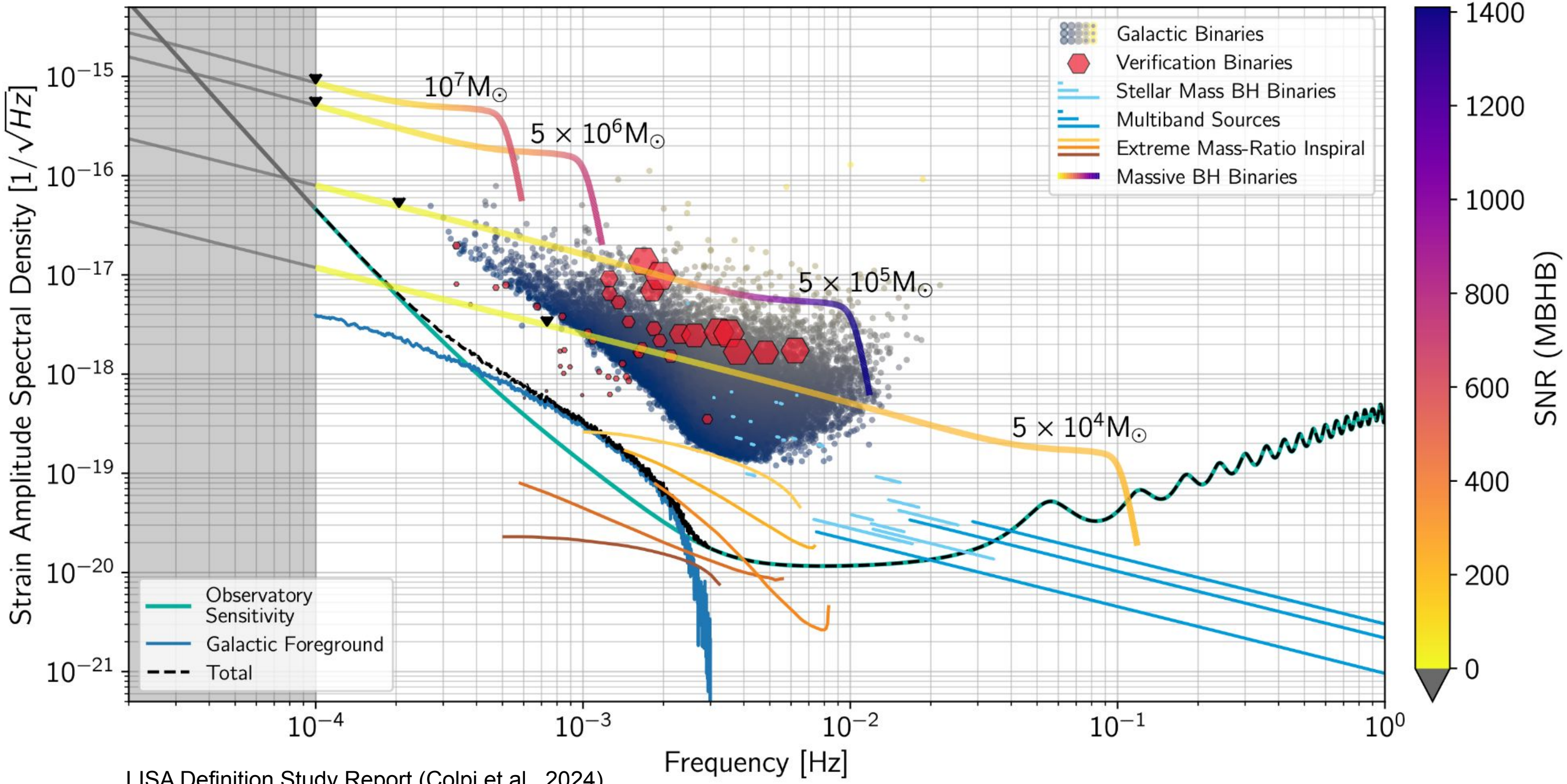






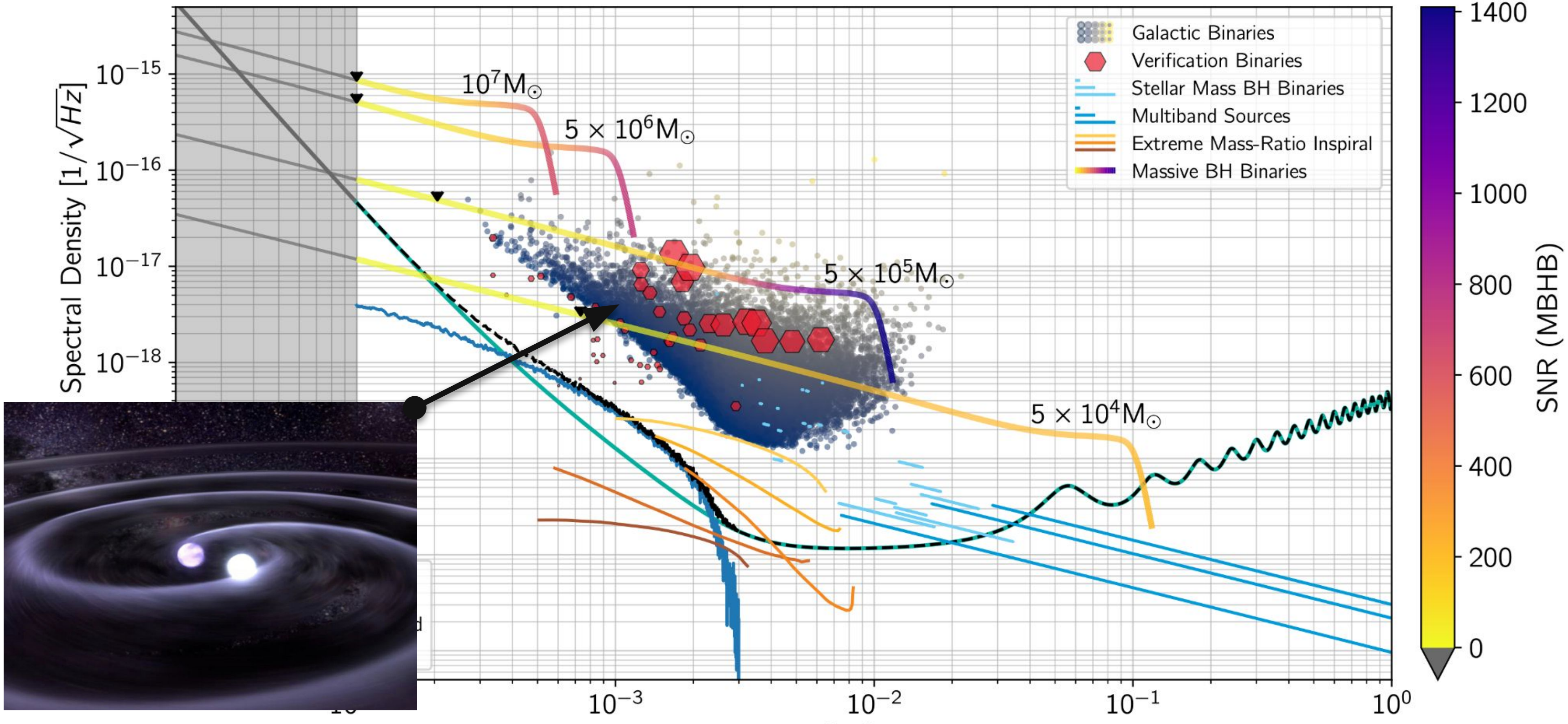
Credit: Stefan Strub

LISA sources



LISA Definition Study Report (Colpi et al., 2024)

LISA sources



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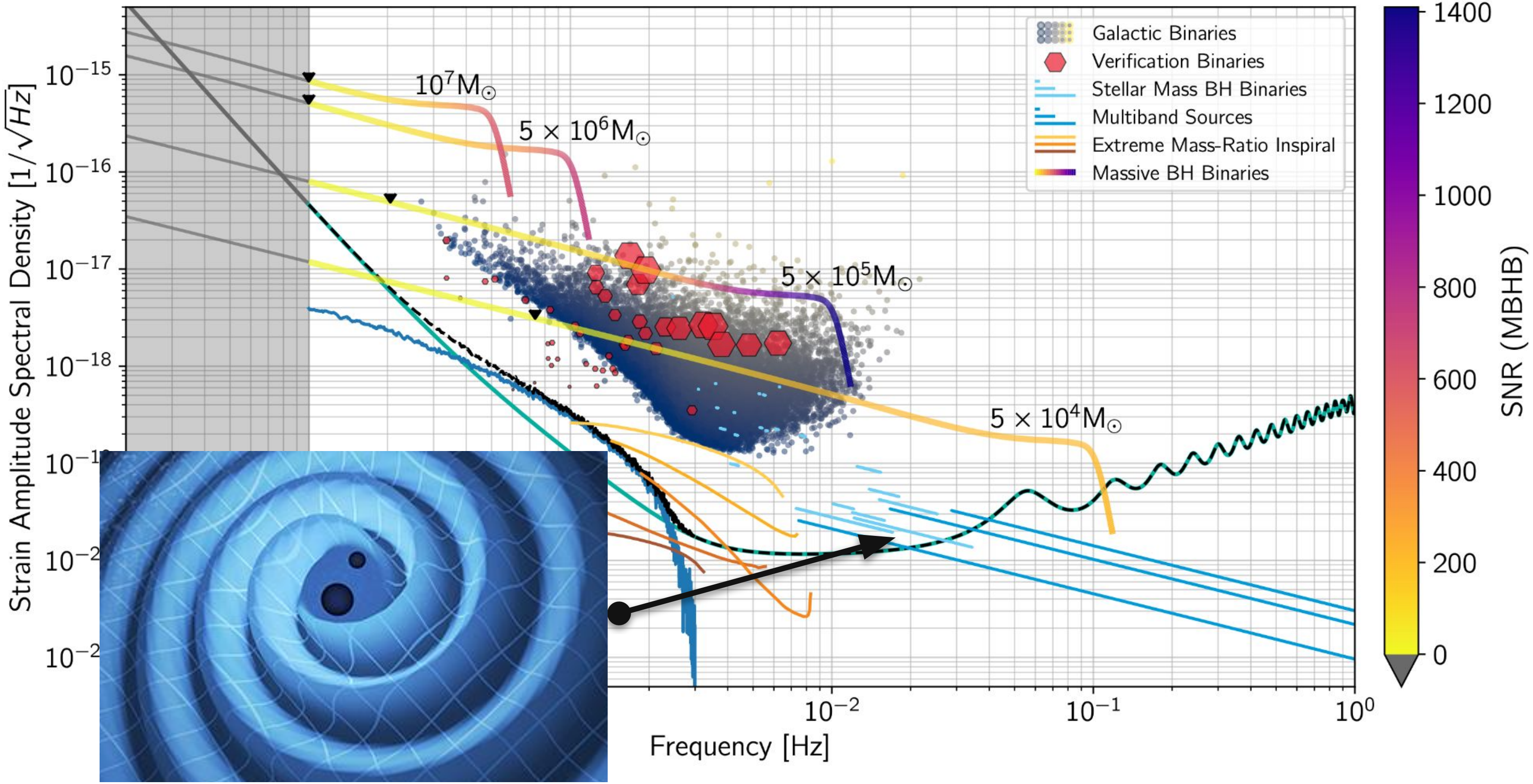

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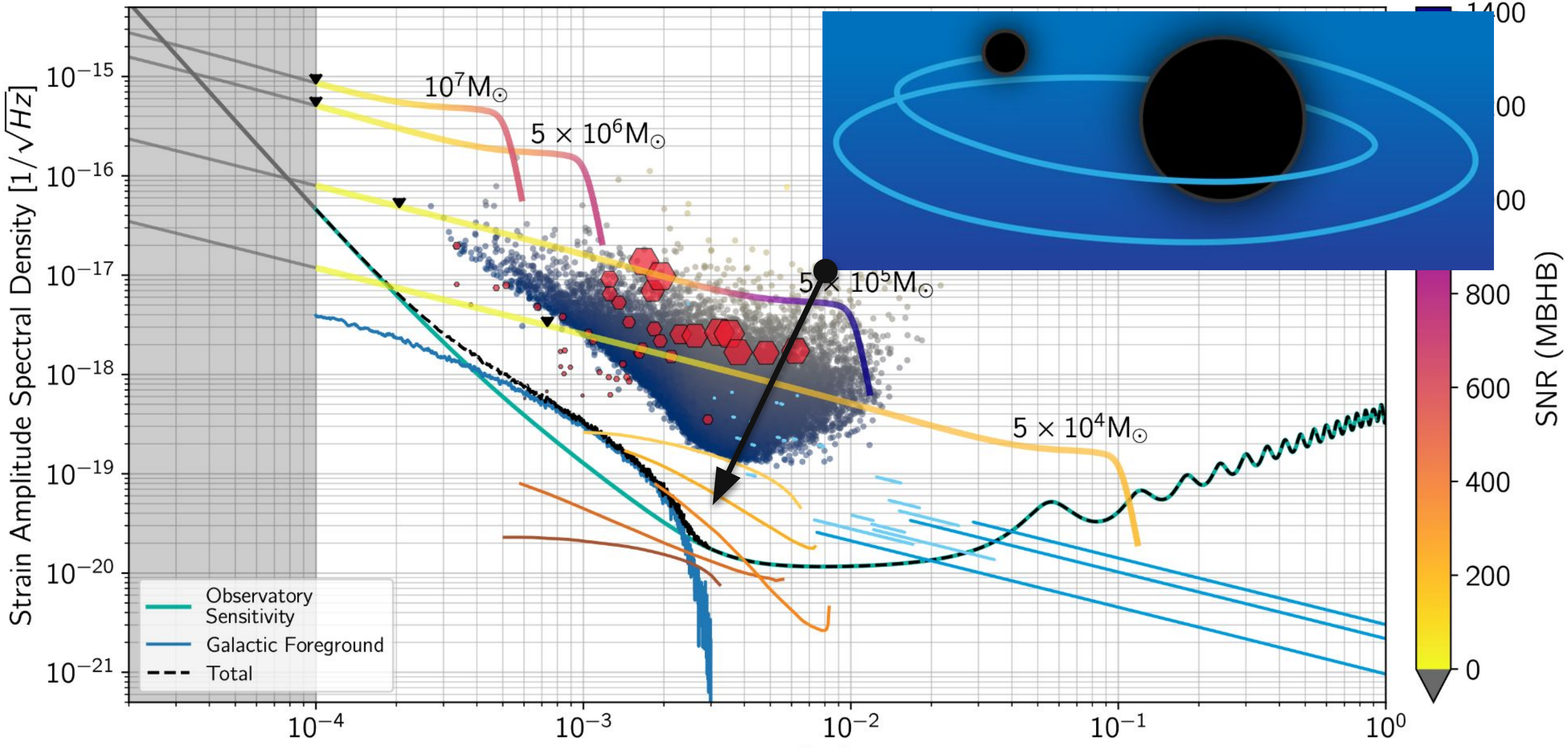

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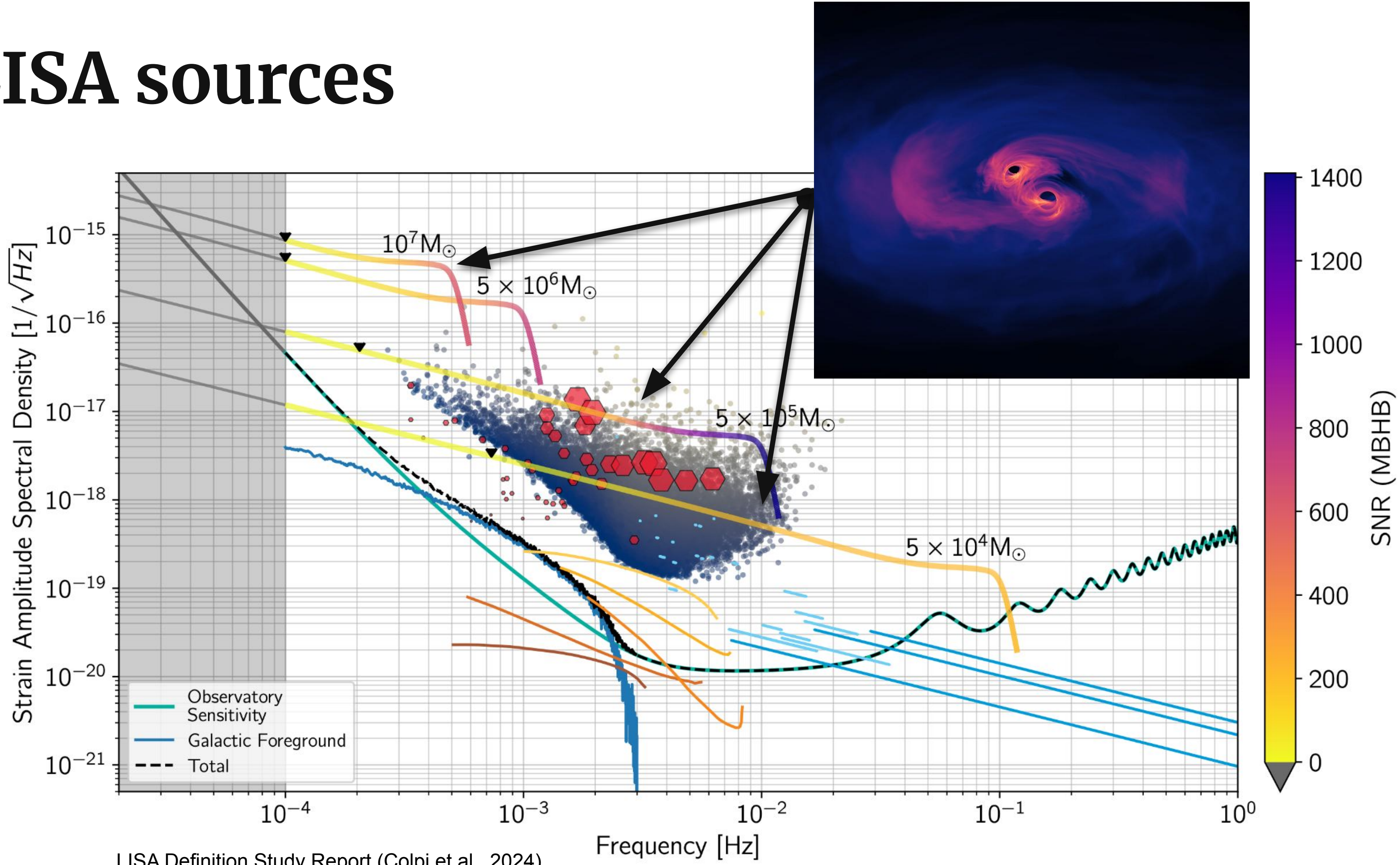

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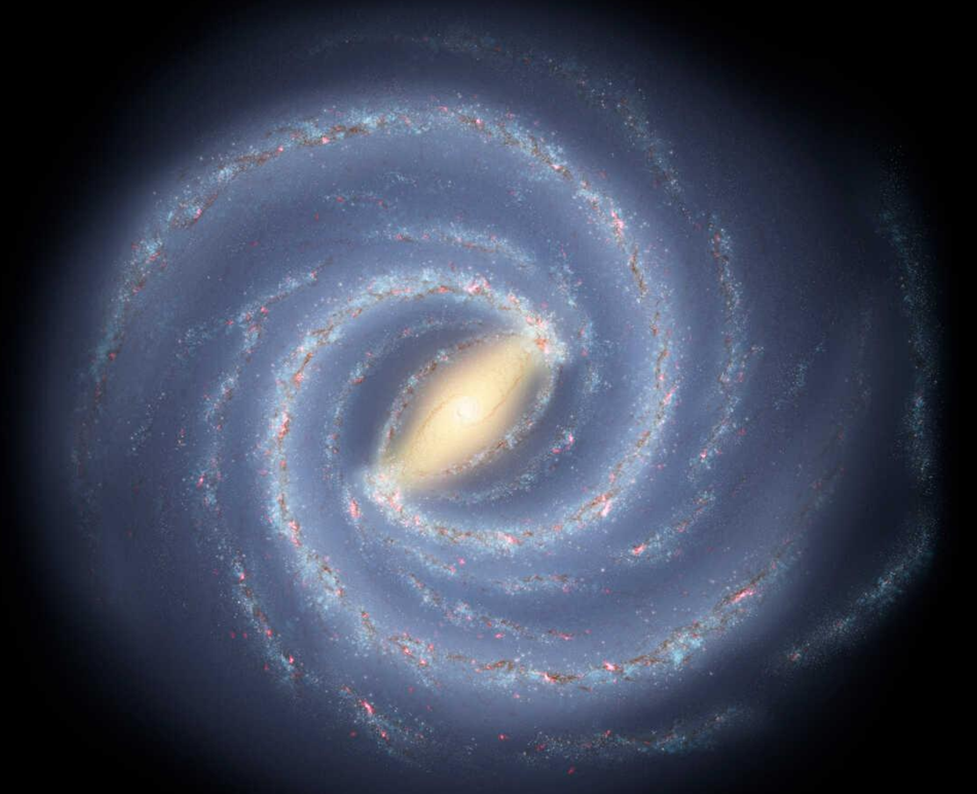
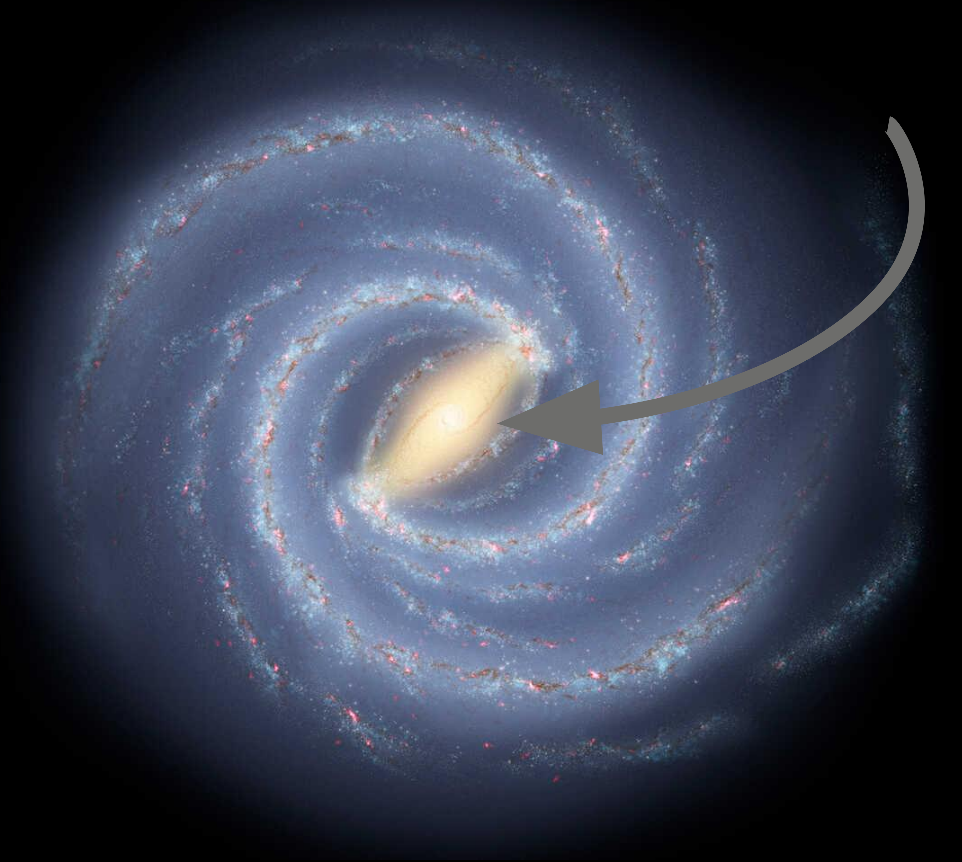
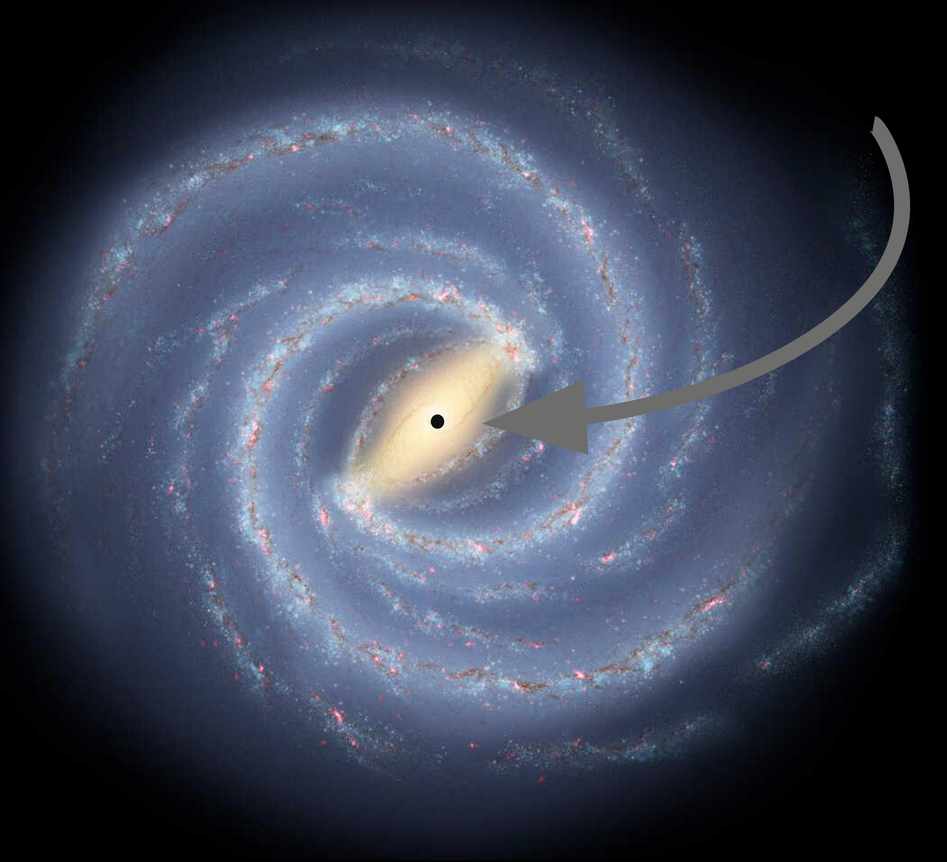
Massive black holes: the CORE of galaxies



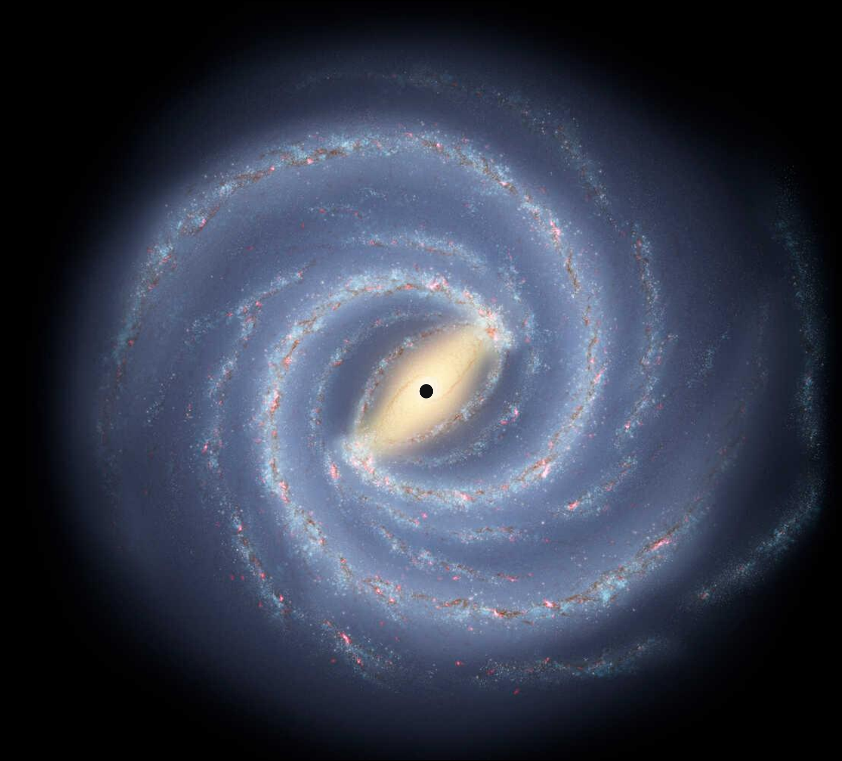
Massive black holes: the CORE of galaxies

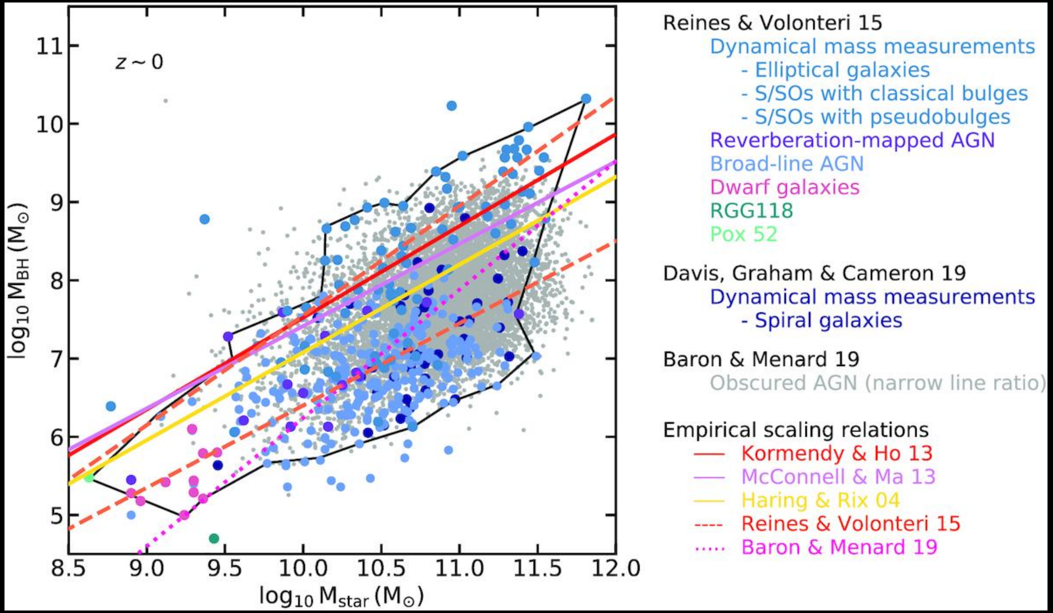


Massive black holes in the center of galaxies



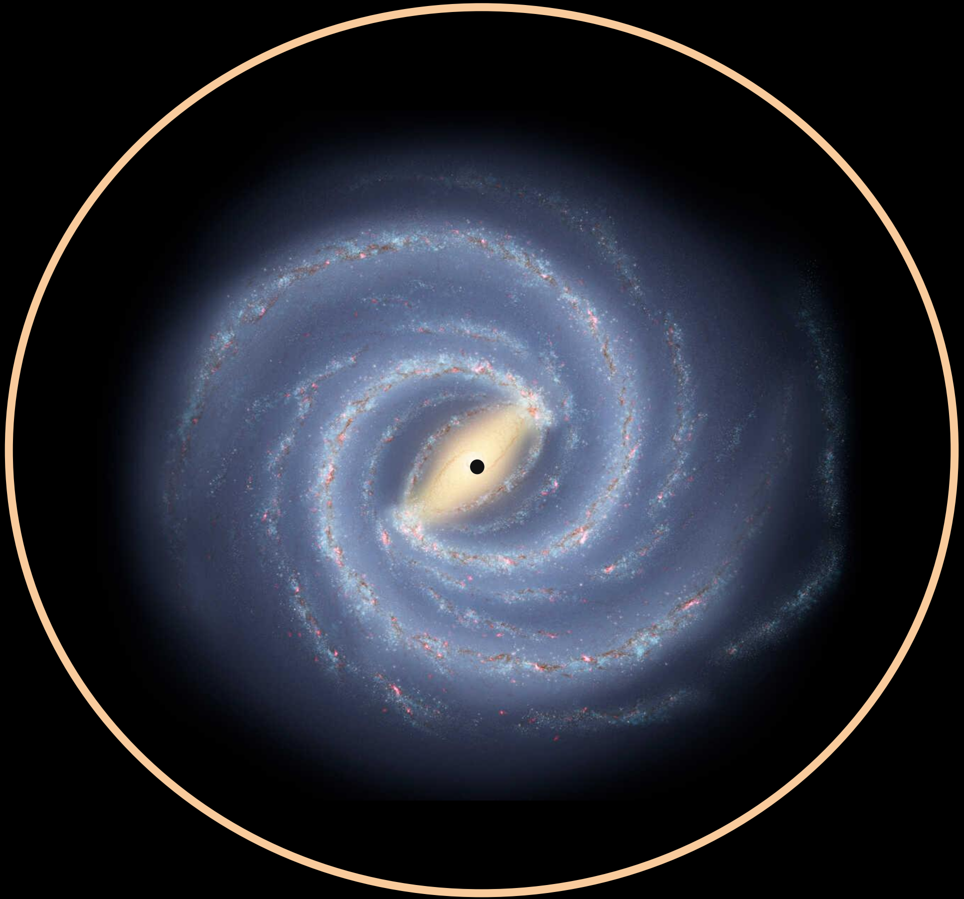
Black hole - galaxy co-evolution



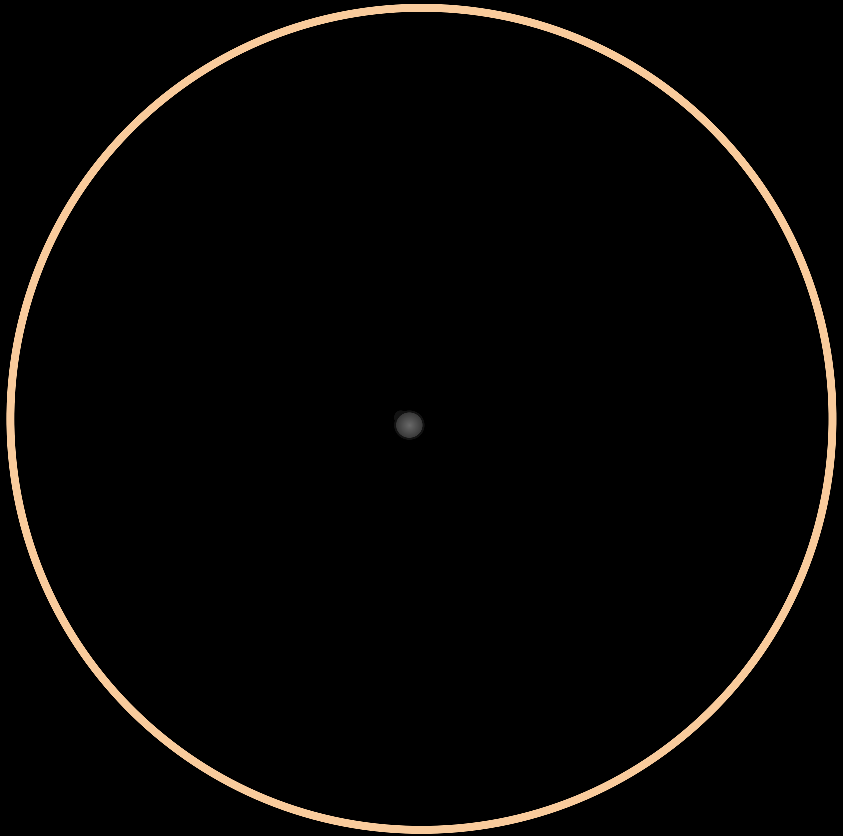


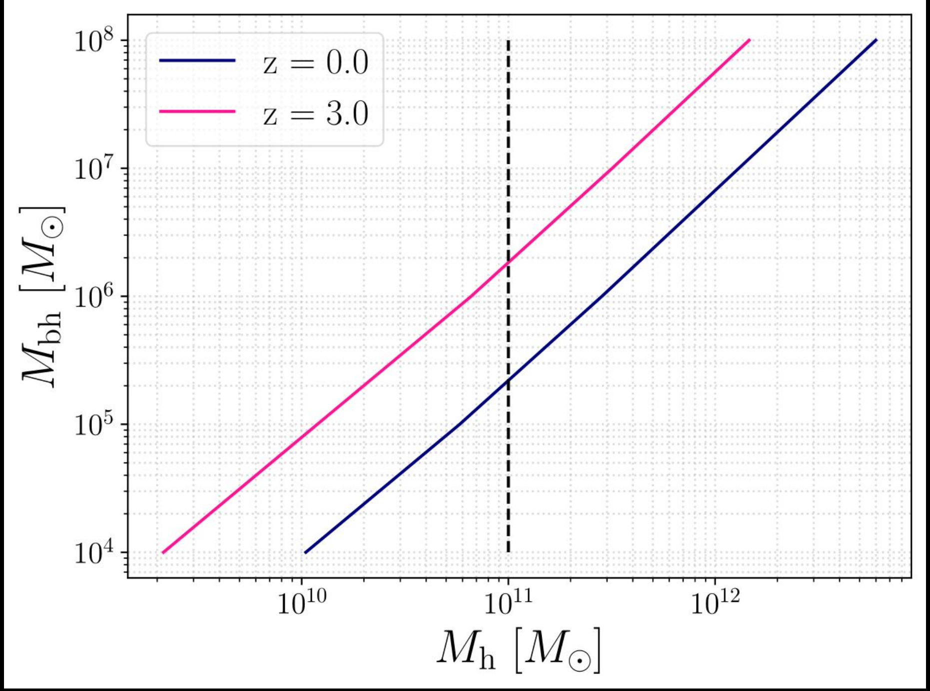
Habouzit, M., et al 2021; MNRAS 503(2), 1940-1975

Galaxies in the center of dark matter halos



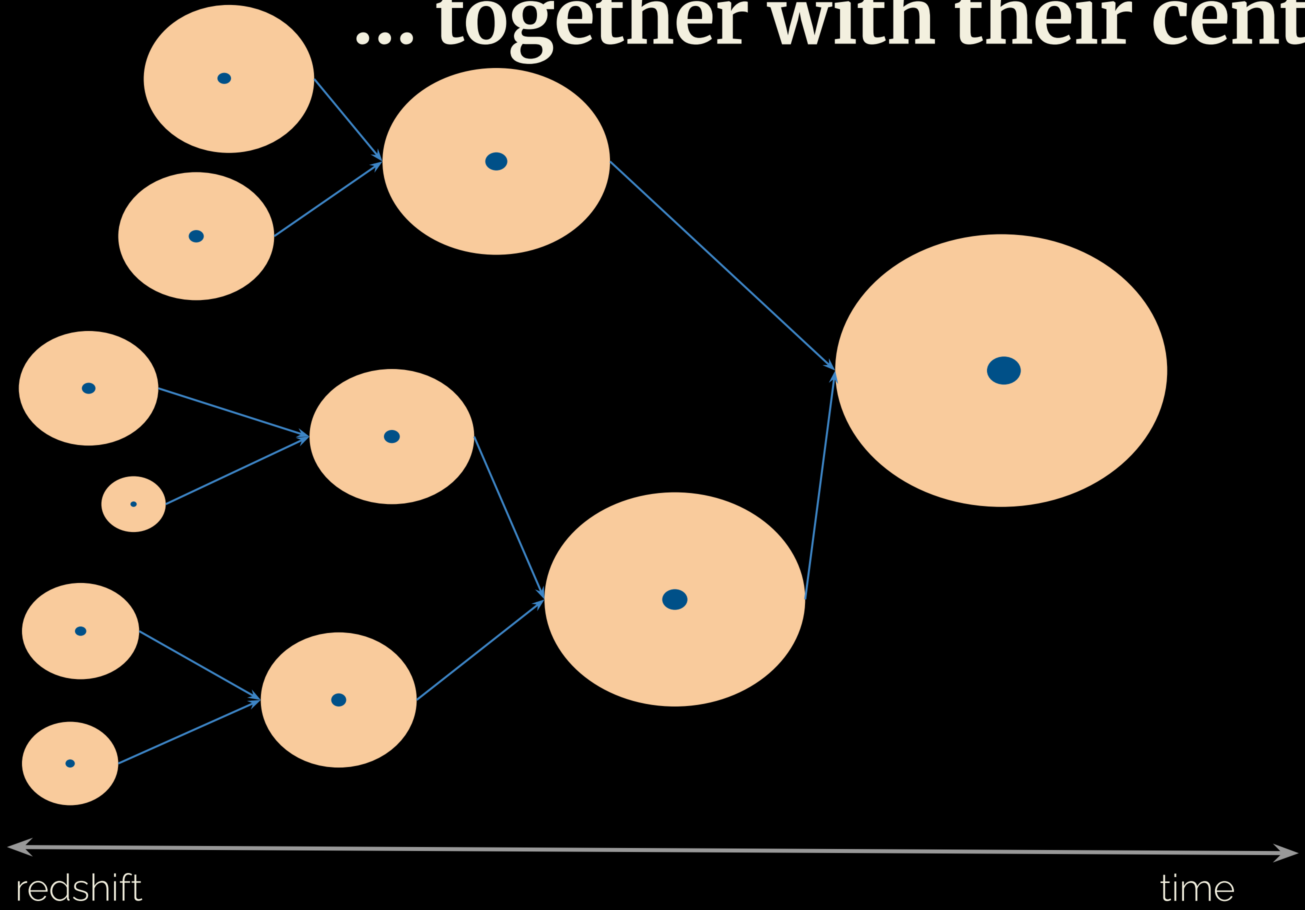
Halo - galaxy - BH co-evolution



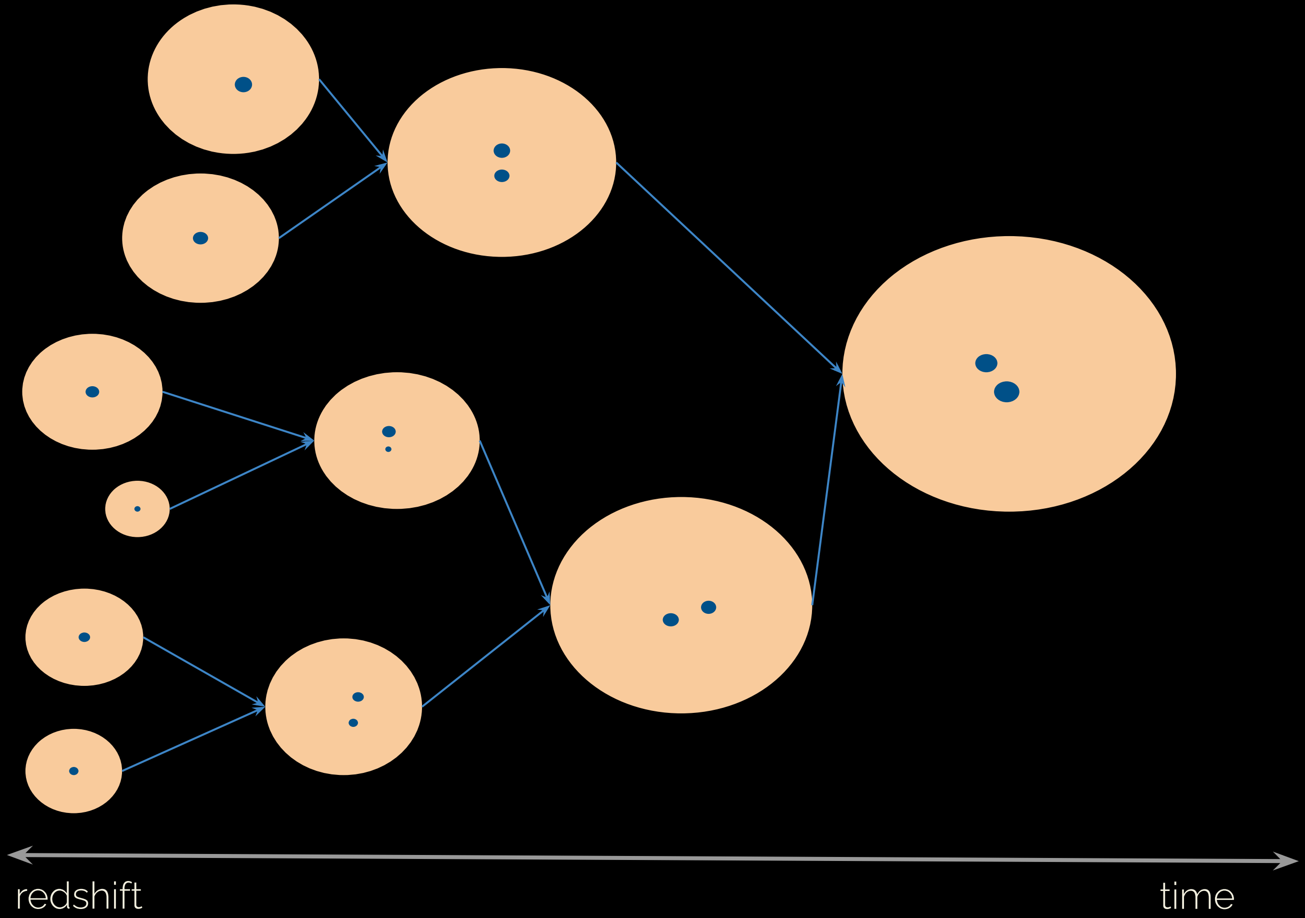


J. Stuart B. Wyithe et al., 2002; Astrophys.J. 581 (2002) 886

Halos & galaxies aggregate hierarchically
… together with their central BH

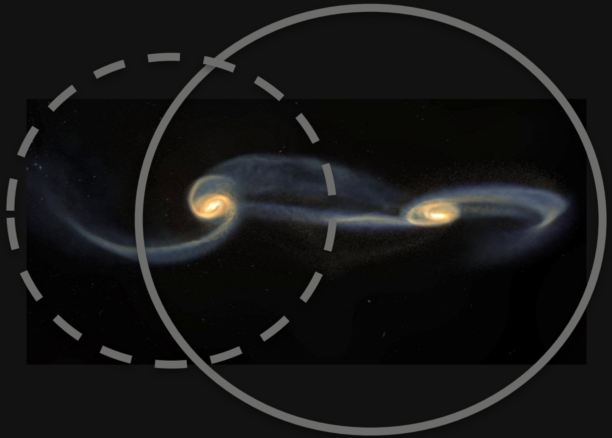


… forming many MBHB across cosmic time



Massive black hole binaries (MBHB)

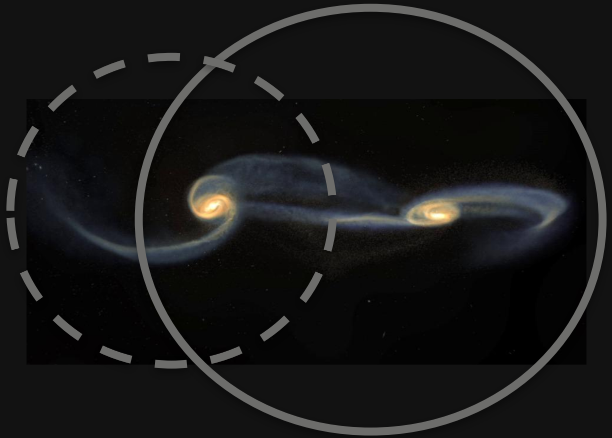
Path to coalescence



Dynamical friction phase

Massive black hole binaries (MBHB)

Path to coalescence



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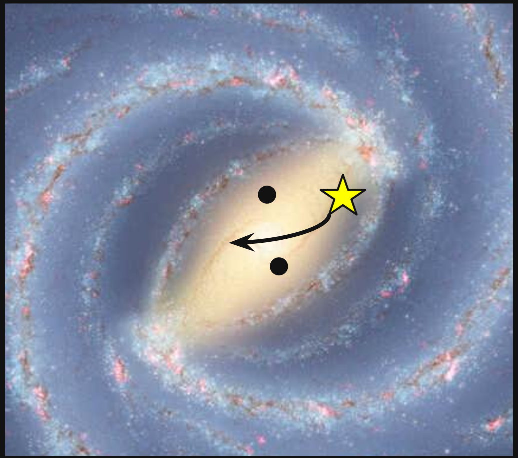

Stellar hardening

Massive black hole binaries (MBHB)

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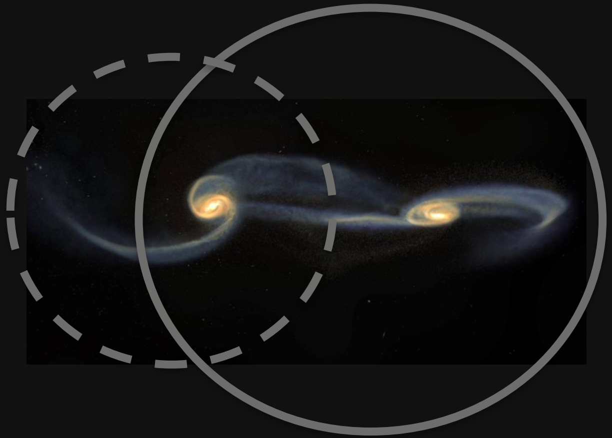

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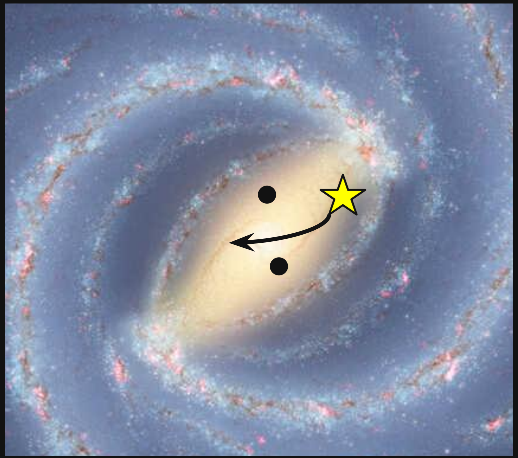

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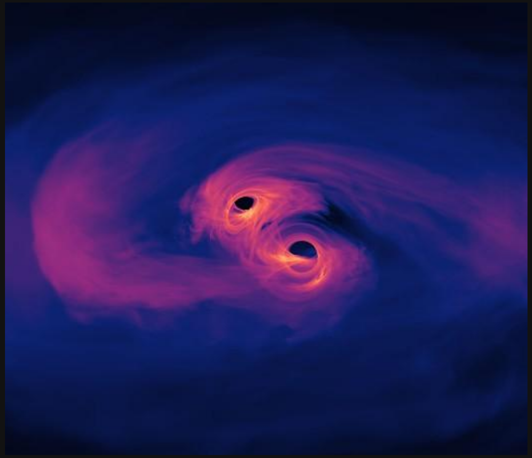
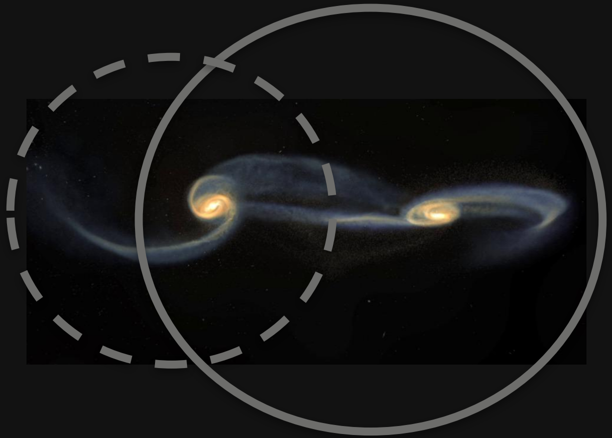


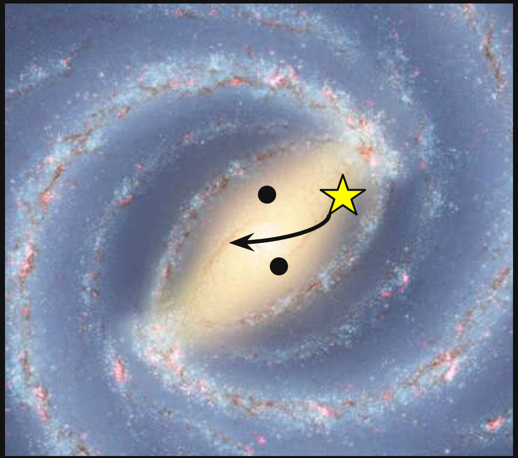
GW emission phase

Massive black hole binaries (MBHB)
Path to coalescence



Dynamical friction phase

- Mpc → kpc scale
- Timescale Gyr
- DM vs gaseous / stellar medium



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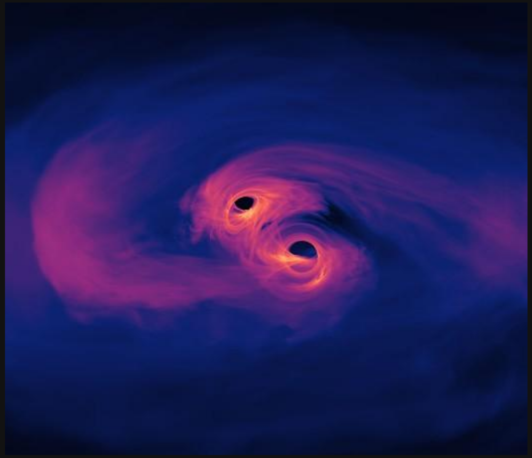

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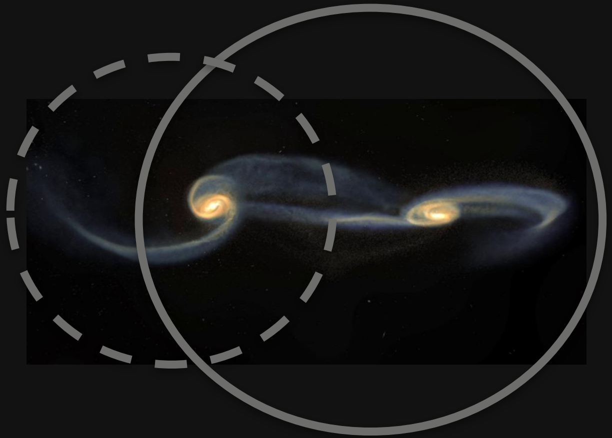

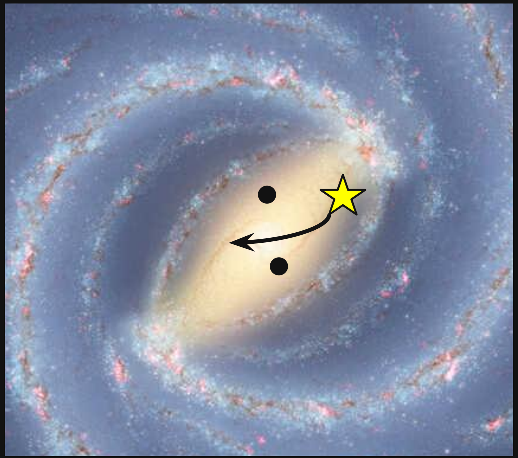

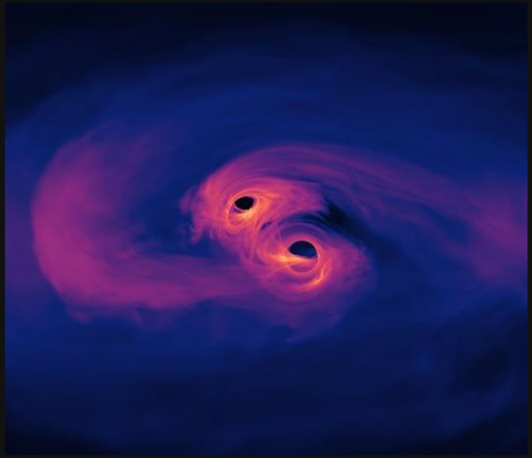

Dynamical friction phase

- Mpc → ~kpc scale
- Timescale ~Gyr
- DM vs gaseous / stellar medium

Stellar hardening

- Merged galaxy
- ~kpc – pc scale at galaxy core
- Timescale ~100 Myr
- 3 body stellar encounters

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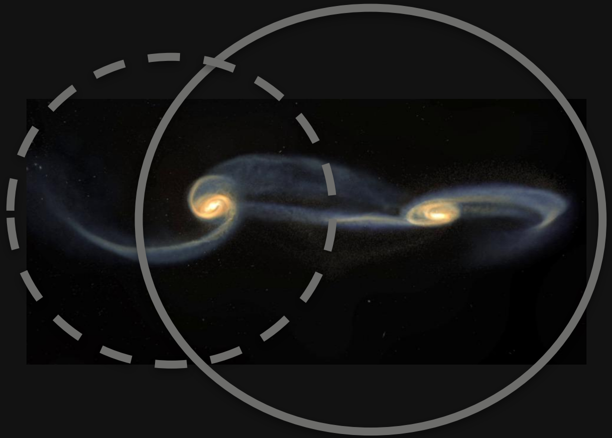

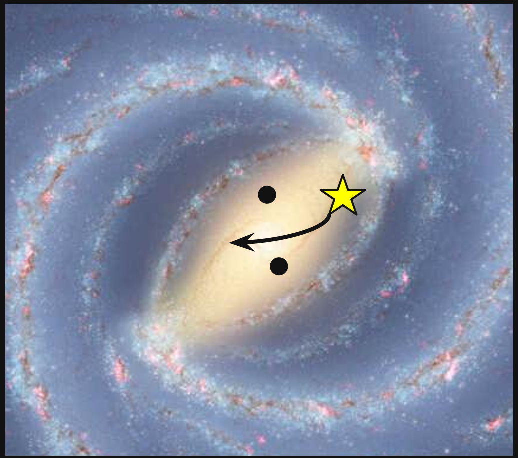

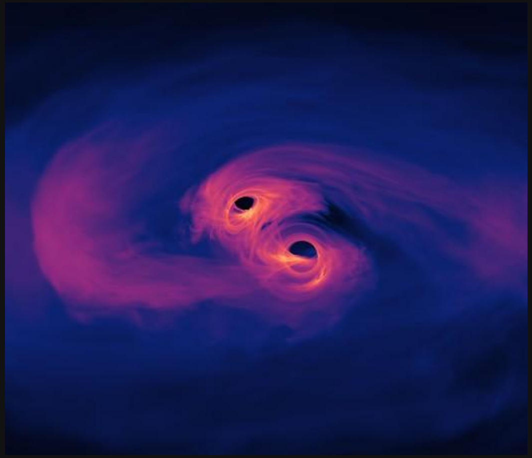

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Gas dynamics neglected !

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- Final stage
- < pc scale
- Shortest phase
- That's what LISA sees!

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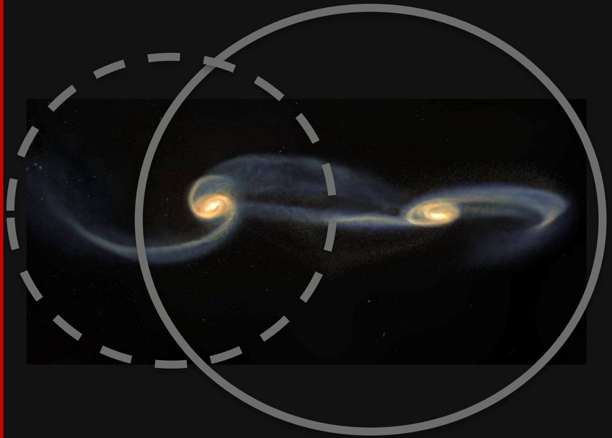

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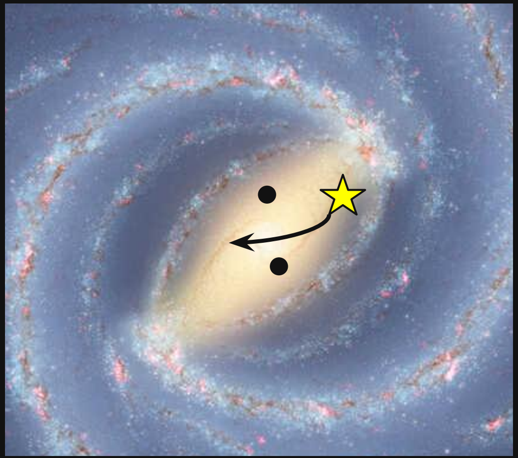

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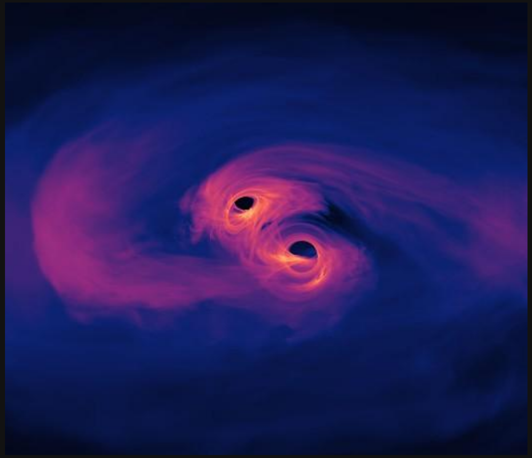

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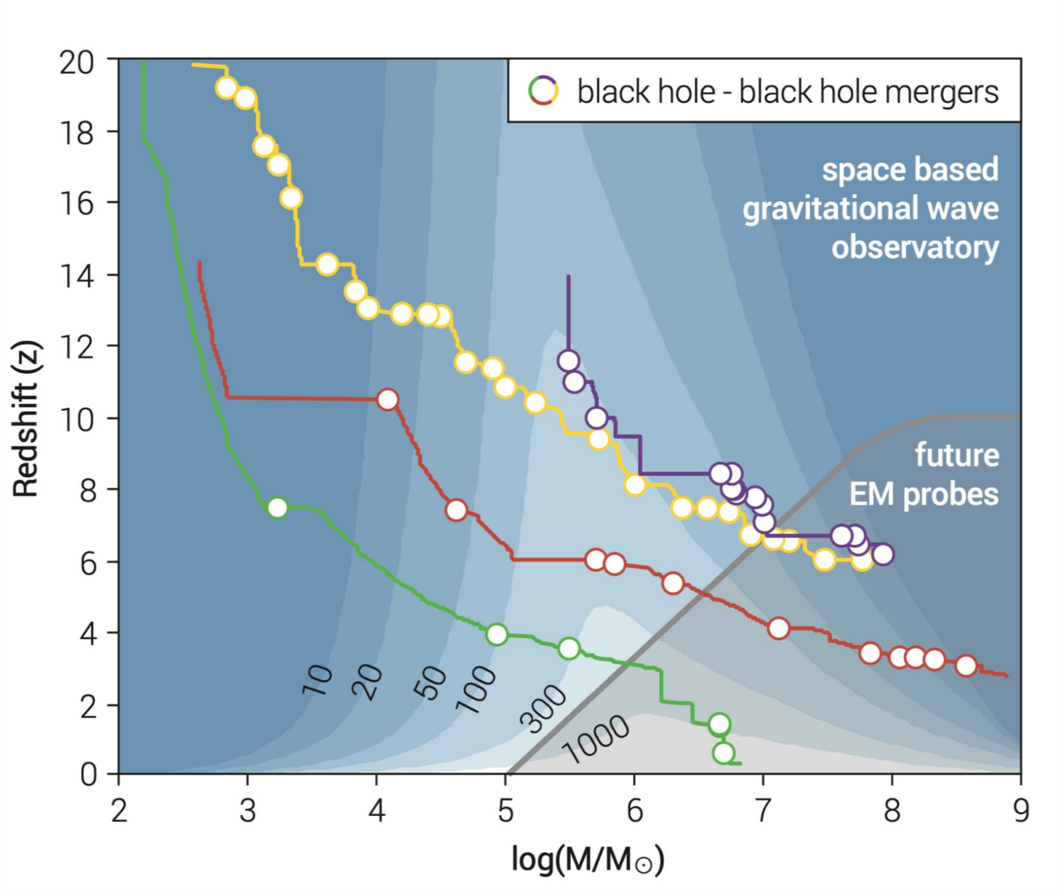
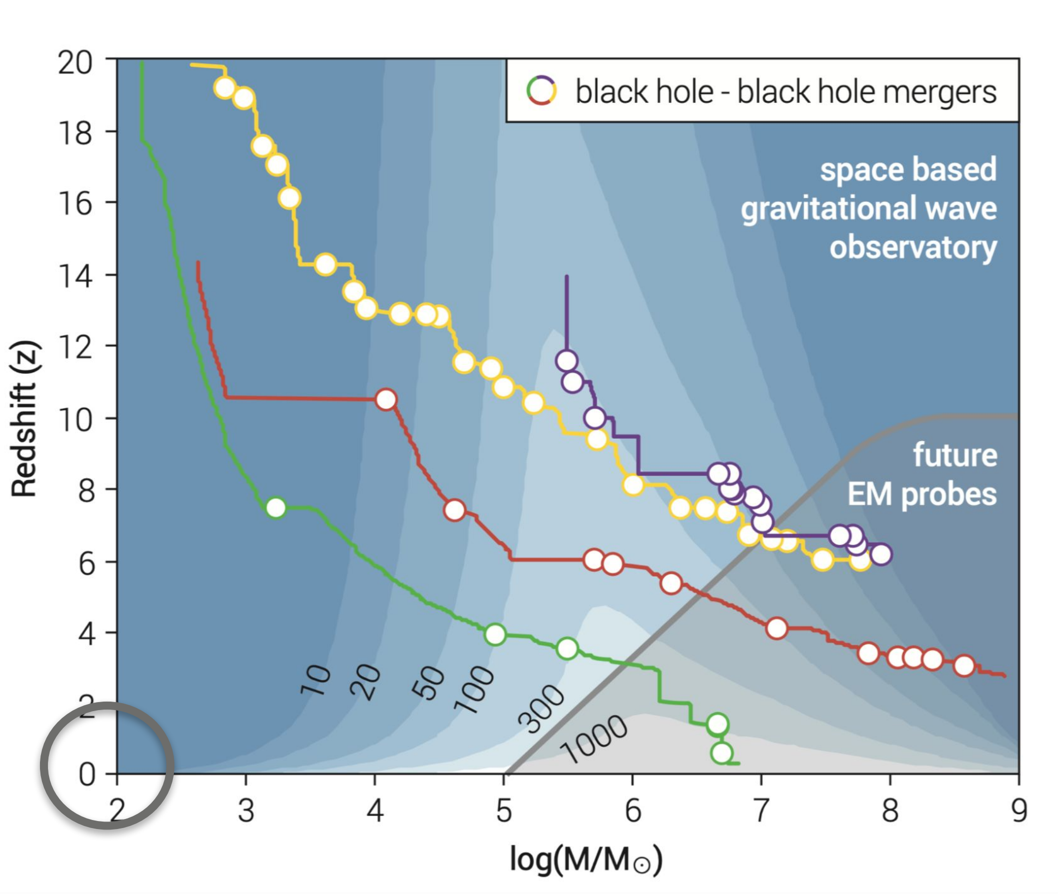
LISA horizon



Danzman, K., 2012. The Gravitational Universe

LISA horizon



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- Current GW observations

  ⟶ local universe & low masses

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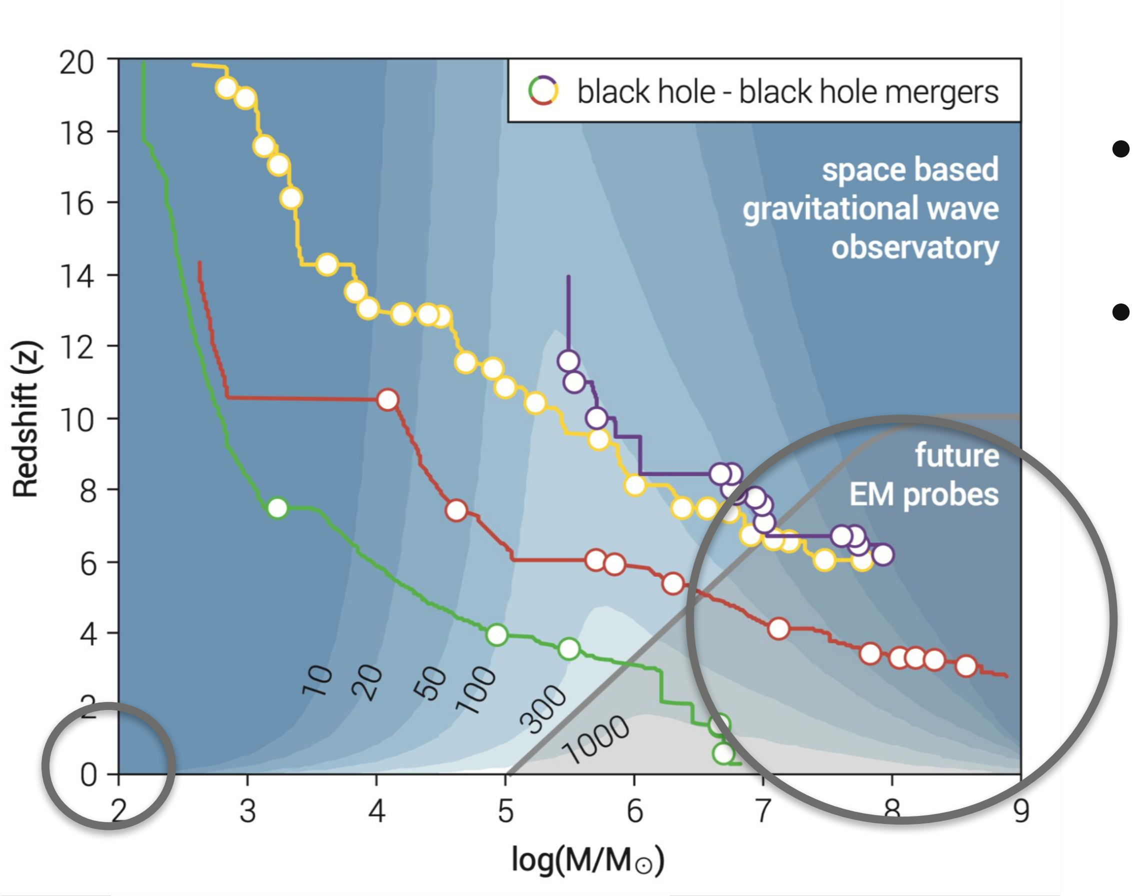

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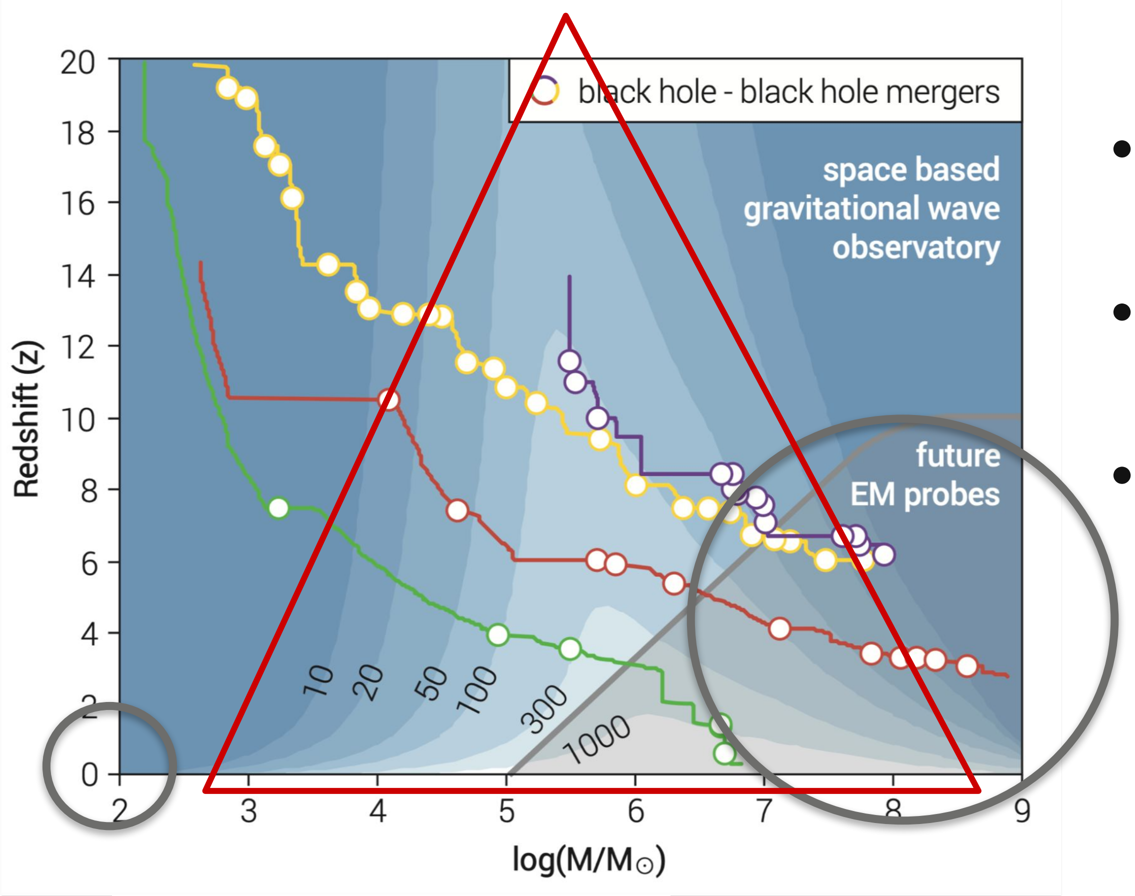

Danzman, K., 2012. The Gravitational Universe

- Current GW observations
  → local universe & low masses
- EM facilities
  → limited to high masses
- The spectrum of growing MBHs missing !
  → space-based GW facilities
  → *need LISA !*

MBHB modelling

Large scale simulations

Analytical models

Hydrodynamical,
N-body simulations

Semi-analytical
models

MBHB modelling

Large scale simulations

Analytical models

Hydrodynamical, N-body simulations

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- Very efficient
- Fast data generation
  → ideal for inference
- Very flexible
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- Limited sub-grid physics possible
- Limited by their resolution

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- Massive Black Hole Binaries (MBHBs)
- The Analytical Model
- Hierarchical Bayesian Inference
- Large scale catalog comparison
- Conclusions

The Analytical Model

H. Padmanabhan et al. 2020

$$\left(\frac{\mathrm{d}n_{\mathrm{bh}}}{\mathrm{dlog}M_{\mathrm{bh}}\mathrm{d}z\mathrm{d}q}\right) = \quad \times \quad \times$$

The Analytical Model

H. Padmanabhan et al. 2020

Number density of MBHBs !

$$\left(\frac{\mathrm{d}n_{\mathrm{bh}}}{\mathrm{dlog}M_{\mathrm{bh}}\mathrm{d}z\mathrm{d}q}\right) =$$

$\times$

$\times$

The Analytical Model

H. Padmanabhan et al. 2020

<u>Number density of MBHBs</u>

$$\left(\frac{\mathrm{d}n_{\mathrm{bh}}}{\mathrm{dlog}M_{\mathrm{bh}}\mathrm{d}z\mathrm{d}q}\right) = \ldots \times \ldots$$

[redacted]

$$\ldots \times \frac{\mathrm{d}n_{\mathrm{h}}}{\mathrm{dlog}M_{\mathrm{h}}}(z)$$

DM halo mass function
→ *Numerical package (S. Murrey et al. 2013)*

The Analytical Model

H. Padmanabhan et al. 2020

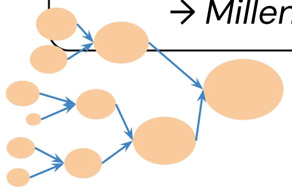
<u>Number density of MBHBs</u>

$$\left(\frac{\mathrm{d}n_{\mathrm{bh}}}{\mathrm{dlog}M_{\mathrm{bh}}\mathrm{d}z\mathrm{d}q}\right) = \qquad \times$$

[redacted]

$$\times q^{\frac{3}{\gamma}-1+\frac{3\beta}{\gamma}}(1+z)^{\eta} \times \exp\left[\left(\frac{q}{\bar{q}}\right)^{\frac{3\gamma_1}{\gamma}}\right] \times \frac{\mathrm{d}n_{\mathrm{h}}}{\mathrm{dlog}M_{\mathrm{h}}}(z)$$

Hale merger rate
→ *Millenium simulation (O. Fakhouri et al., 2010)*



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The Analytical Model

H. Padmanabhan et al. 2020

MBH – hale mass relation

<u>Number density of MBHBs</u>

$$\left(\frac{dn_{\rm bh}}{{\rm dlog}M_{\rm bh}{\rm d}z{\rm d}q}\right) =$$ [redacted] $$\times A_1\left(\frac{M_{\rm bh}}{10^{12}{\rm M}_\odot \times {\rm K}(z,\gamma,\epsilon)}\right)^{\frac{3\alpha}{\gamma}}$$

[redacted]

$$\times q^{\frac{3}{\gamma}-1+\frac{3\beta}{\gamma}}(1+z)^\eta \times \exp\left[\left(\frac{q}{\bar{q}}\right)^{\frac{3\gamma_1}{\gamma}}\right] \times \frac{dn_{\rm h}}{{\rm dlog}M_{\rm h}}(z)$$

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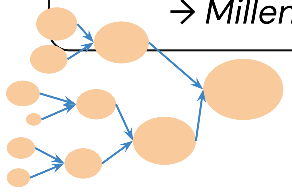

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The BH - halo mass scaling relation

J. Stuart B. Wyithe et al., 2002; Astrophys.J. 581 (2002) 886

$$M_{\rm bh} = \epsilon \cdot M_{\rm h} \left(\frac{M_{\rm h}}{10^{12} M_{\odot}} \right)^{\frac{\gamma}{3}-1} \times \left(\frac{\Omega_{\rm m} \cdot \Delta_c h^2}{\Omega_{\rm m}^{\rm z} \cdot 18\pi^2} \right)^{\frac{\gamma}{6}} (1+z)^{\frac{\gamma}{2}}$$

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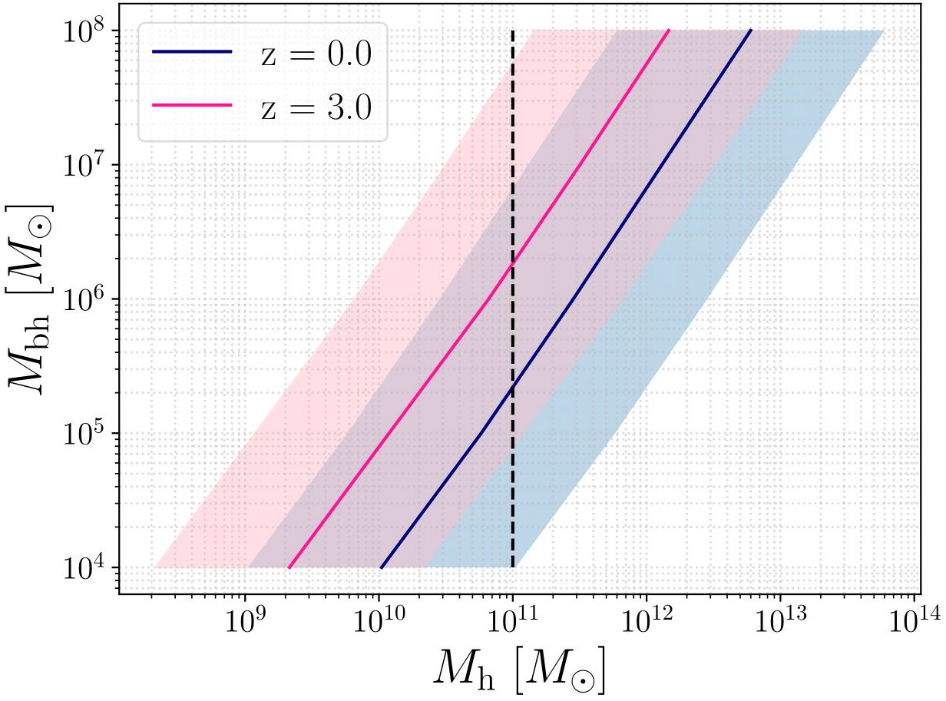

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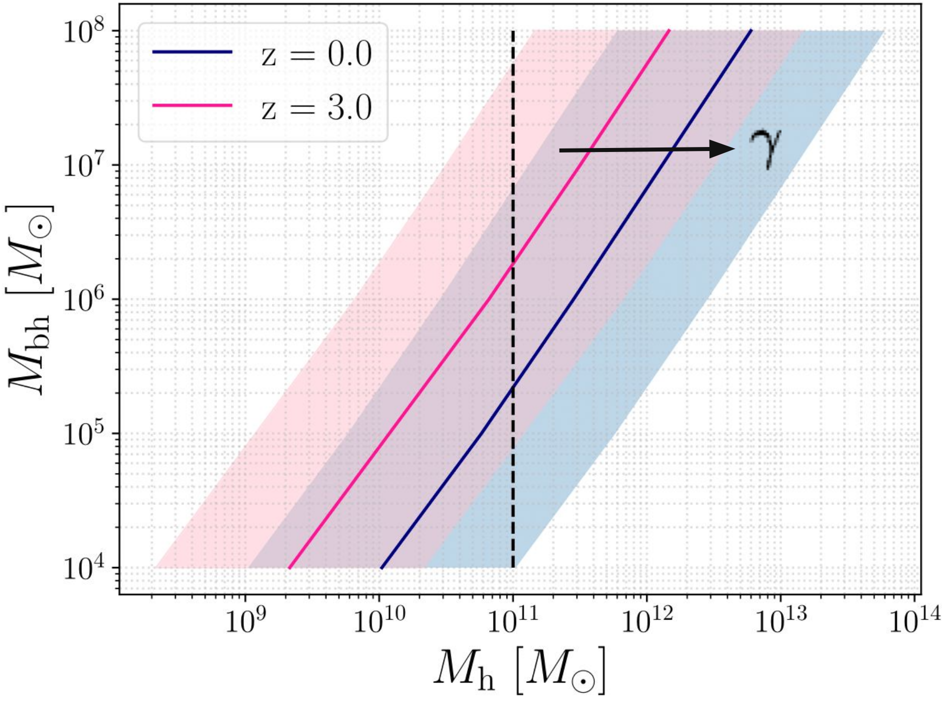

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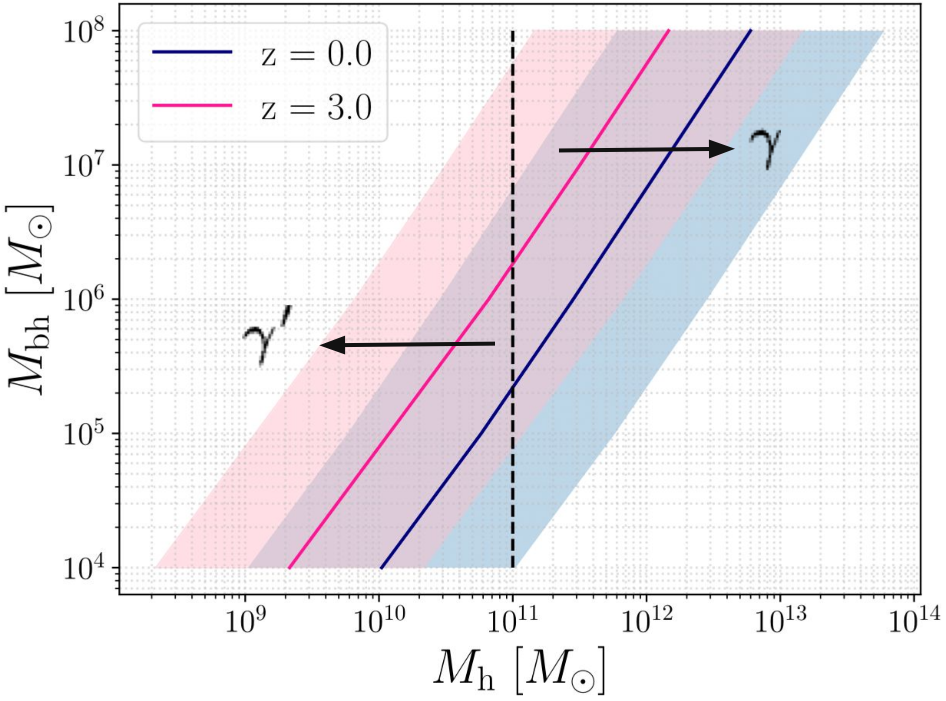

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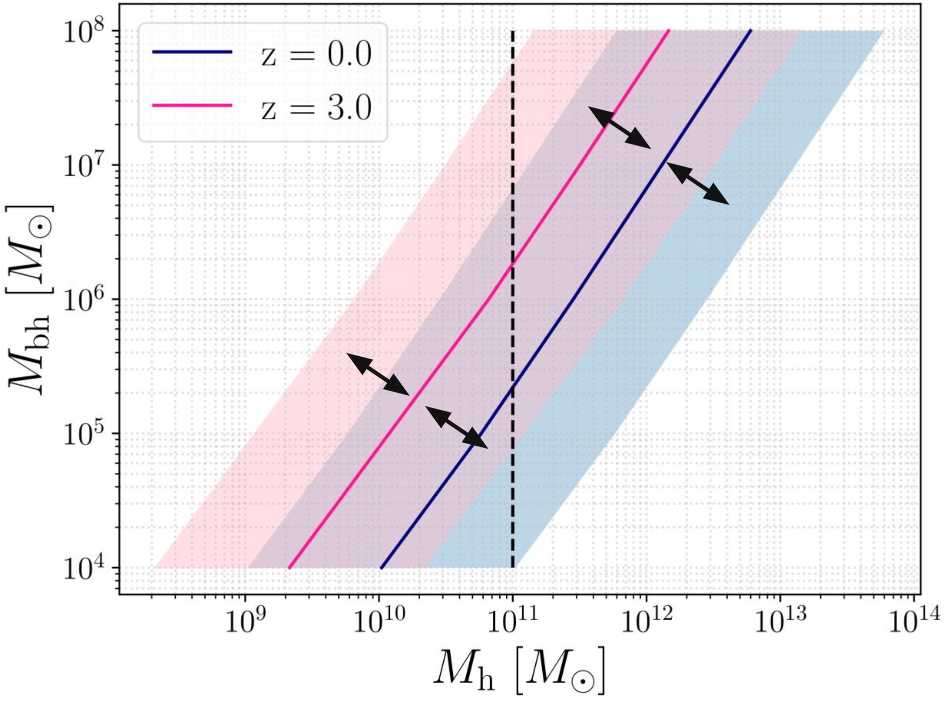

The BH - halo mass scaling relation

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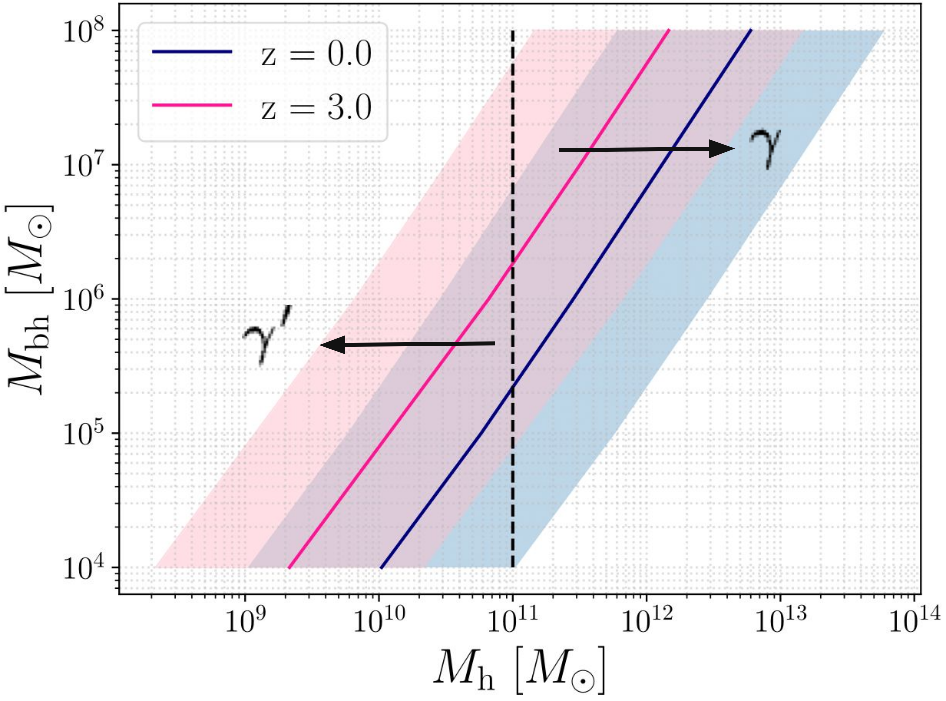
$$M_{\rm bh} = \epsilon \cdot M_{\rm h} \left(\frac{M_{\rm h}}{10^{12} M_\odot} \right)^{\frac{\gamma}{3} - 1} \times \left(\frac{\Omega_{\rm m} \cdot \Delta_c h^2}{\Omega_{\rm m}^{\rm z} \cdot 18\pi^2} \right)^{\frac{\gamma}{6}} (1+z)^{\frac{\gamma}{2}}$$

$$\gamma = 4.53$$

$$\gamma' = 4.03$$

$$\epsilon = 10^{-5}$$



The Analytical Model

H. Padmanabhan et al. 2020

MBH – hale mass relation

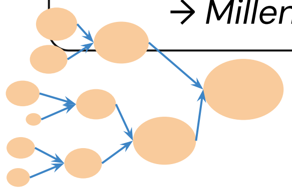
Number density of MBHBs

$$\left(\frac{dn_{\rm bh}}{d\log M_{\rm bh} dz dq}\right) =$$ [redacted] $$\times A_1 \left(\frac{M_{\rm bh}}{10^{12} M_{\odot} \times K(z, \gamma, \epsilon)}\right)^{\frac{3\alpha}{\gamma}}$$

[redacted]

$$\times q^{\frac{3}{\gamma} - 1 + \frac{3\beta}{\gamma}} (1+z)^{\eta} \times \exp\left[\left(\frac{q}{\bar{q}}\right)^{\frac{3\gamma_1}{\gamma}}\right] \times \frac{dn_{\rm h}}{d\log M_{\rm h}}(z)$$

Hale merger rate
→ *Millenium simulation (O. Fakhouri et al., 2010)*



DM halo mass function
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The Analytical Model

H. Padmanabhan et al. 2020

Occupation fraction

MBH – hale mass relation

Number density of MBHBs

$$\left(\frac{\mathrm{d}n_{\mathrm{bh}}}{\mathrm{d}\log M_{\mathrm{bh}}\,\mathrm{d}z\,\mathrm{d}q}\right) = f_{\mathrm{bh}}(M^1_{\mathrm{bh}}, z, f_i, \epsilon_i)\, f_{\mathrm{bh}}(M^2_{\mathrm{bh}}, z, f_i, \epsilon_i) \times A_1 \left(\frac{M_{\mathrm{bh}}}{10^{12}\mathrm{M}_\odot \times \mathrm{K}(z,\gamma,\epsilon)}\right)^{\frac{3\alpha}{\gamma}}$$

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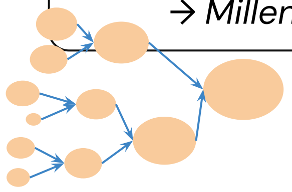

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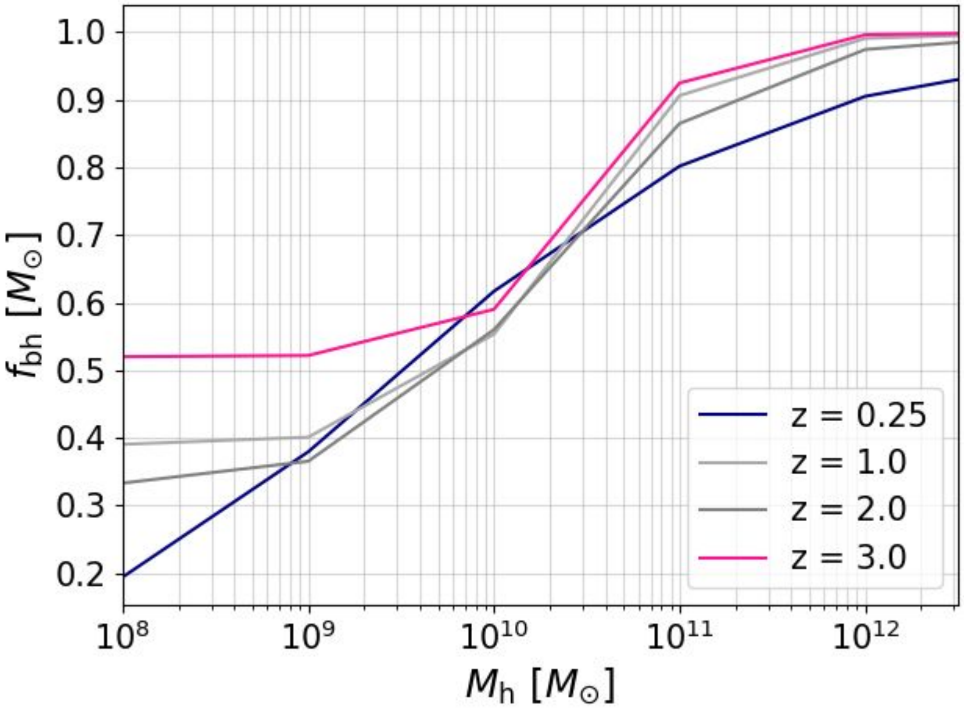
The occupation fraction

New Horizon Simulation: R. S. Beckmann et al., 2022

$$f_{\mathrm{bh}}(M_{\mathrm{h}}; z_i, f_i, M_i', \varepsilon_i) = 1 - \frac{f_i}{1 + \left(\frac{\log_{10}(M_{\mathrm{h}})}{M_i'}\right)^{\varepsilon_i}}$$



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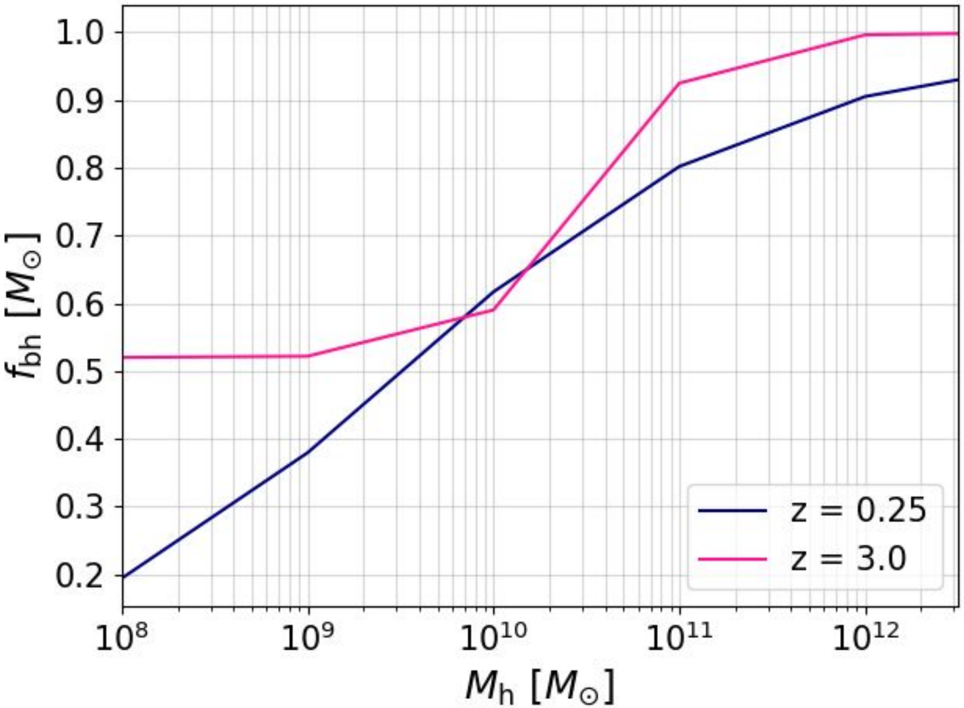

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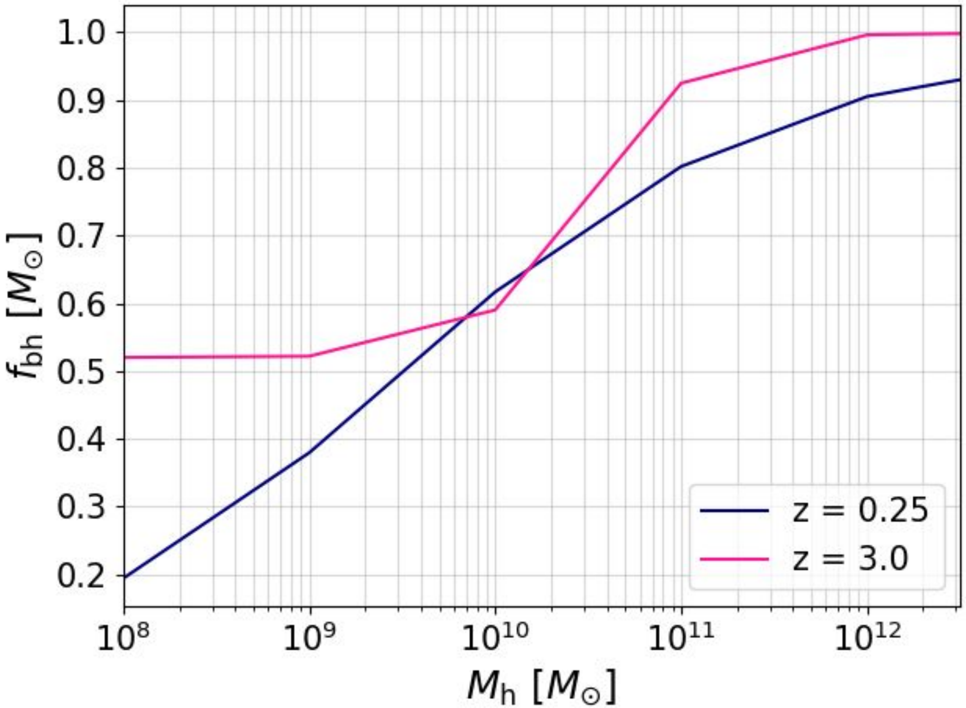

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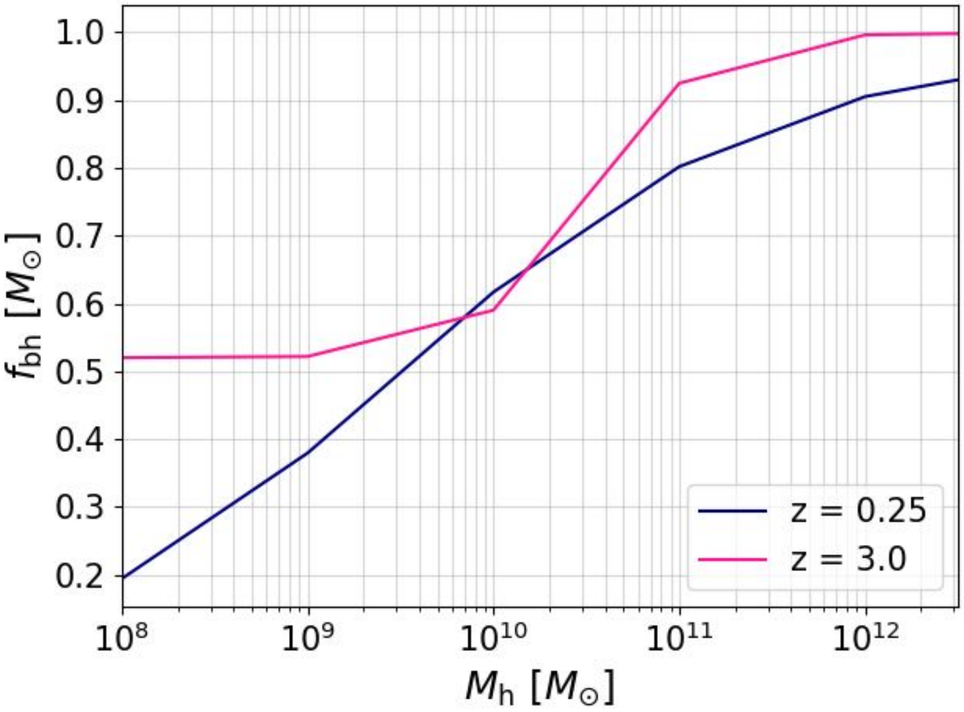
$z = 0.25 : f_0 = 0.98, M_0' = 9.65, \epsilon_0 = 9.98$

$z = 3 : f_3 = 0.48, M_3' = 10.5, \epsilon_3 = 36.2$



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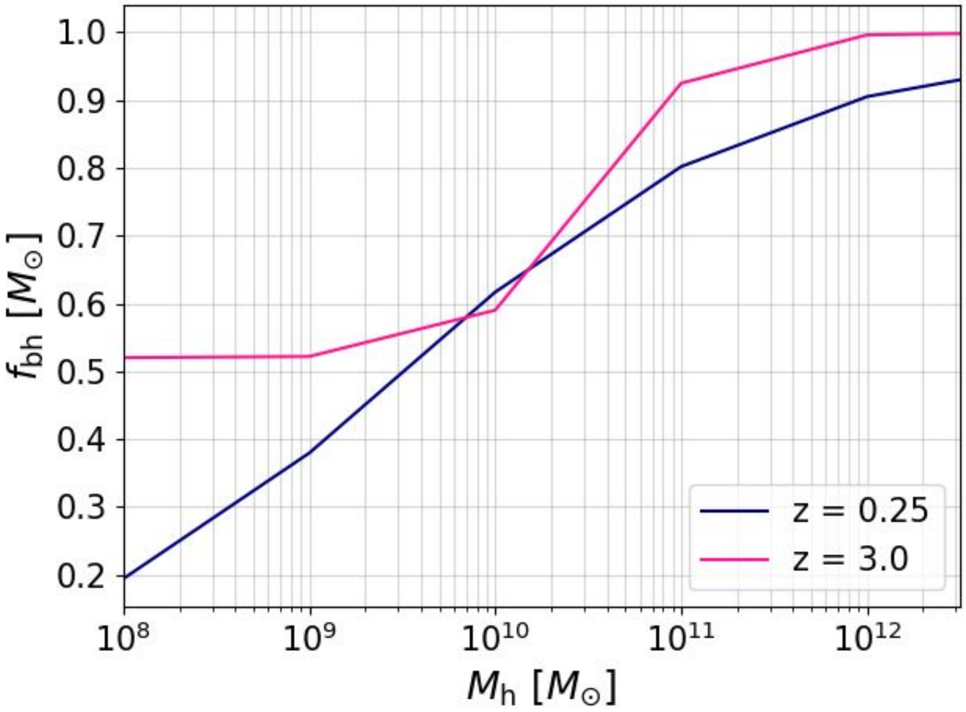


The Analytical Model

H. Padmanabhan et al. 2020

Occupation fraction

MBH – hale mass relation

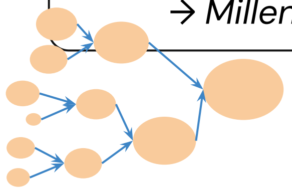
Number density of MBHBs

$$\left(\frac{\mathrm{d}n_{\mathrm{bh}}}{\mathrm{d}\log M_{\mathrm{bh}}\mathrm{d}z\mathrm{d}q}\right) = f_{\mathrm{bh}}(M^1_{\mathrm{bh}}, z, f_i, \epsilon_i) f_{\mathrm{bh}}(M^2_{\mathrm{bh}}, z, f_i, \epsilon_i) \times A_1 \left(\frac{M_{\mathrm{bh}}}{10^{12}\mathrm{M}_\odot \times \mathrm{K}(z,\gamma,\epsilon)}\right)^{\frac{3\alpha}{\gamma}}$$

$$\times q^{\frac{3}{\gamma}-1+\frac{3\beta}{\gamma}}(1+z)^{\eta} \times \exp\left[\left(\frac{q}{\bar{q}}\right)^{\frac{3\gamma_1}{\gamma}}\right] \times \frac{\mathrm{d}n_{\mathrm{h}}}{\mathrm{d}\log M_{\mathrm{h}}}(z)$$

[redacted]

Hale merger rate
→ *Millenium simulation (O. Fakhouri et al., 2010)*



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The Analytical Model

H. Padmanabhan et al. 2020

Occupation fraction

MBH – hale mass relation

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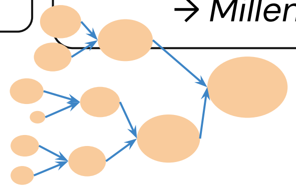
$$t_{\rm delay} = t_{\rm DF} + t_{\rm hard} + t_{\rm GW}$$

Time delays

Hale merger rate
→ *Millenium simulation (O. Fakhouri et al., 2010)*



DM halo mass function
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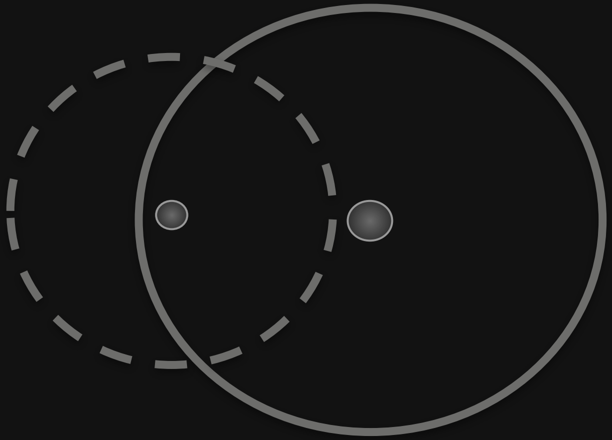
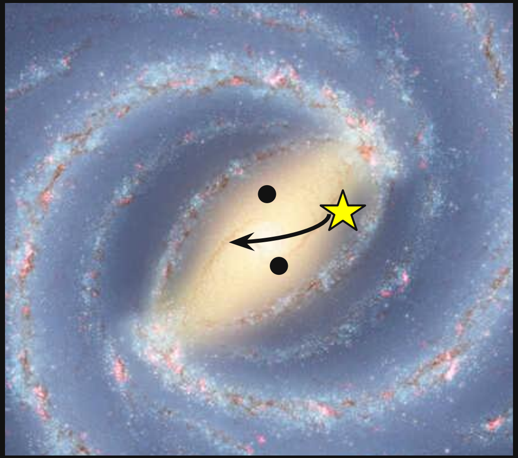
Path to coalescence

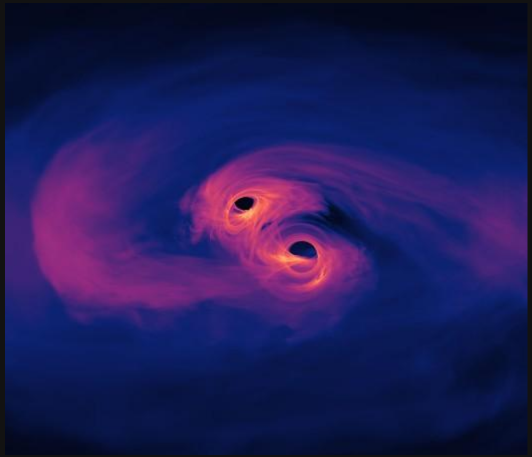
In collaboration with E. Bortolas (Univ. Milano Bicocca)



Dynamical friction phase



Stellar hardening



GW emission phase

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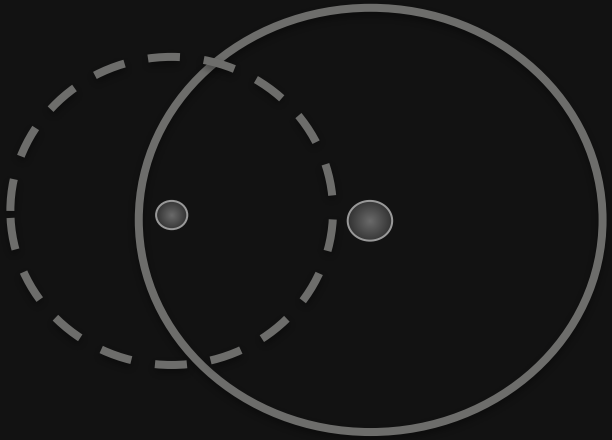

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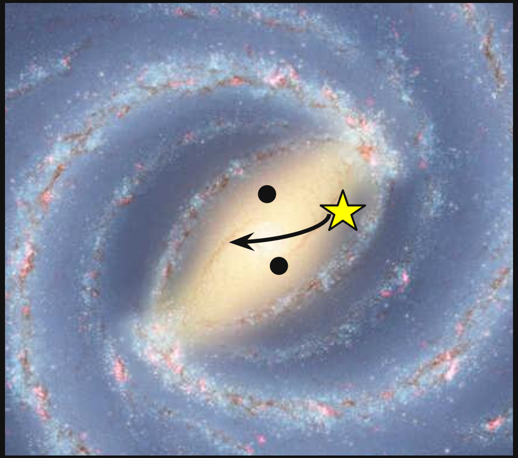

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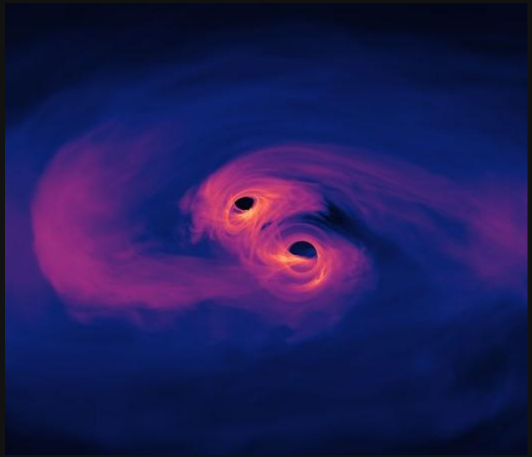

GW emission phase

$$t_{\mathrm{DF}} = \alpha_{\mathrm{fric}} \times \frac{V_{\mathrm{virial}} \cdot R^2_{\mathrm{virial}}}{G \cdot m_{\mathrm{virial}} \cdot \ln\left(1 + \frac{M_{\mathrm{virial}}}{m_{\mathrm{virial}}}\right)}$$

Guo et al., 2011

Path to coalescence

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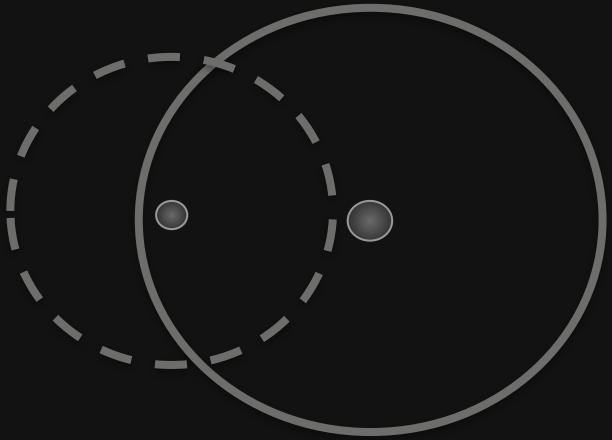

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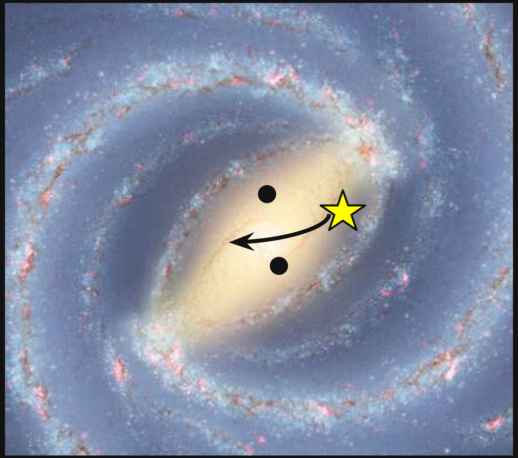

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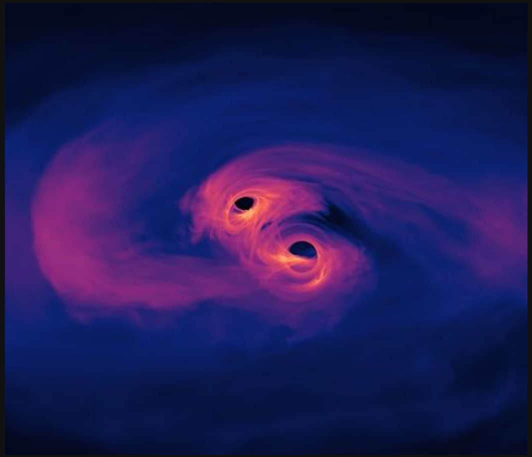

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$$t_{\rm hardening} = \frac{\sigma_\infty}{G H \rho_\infty a_{GW}}$$

Sesana & Khan, 2015

Path to coalescence

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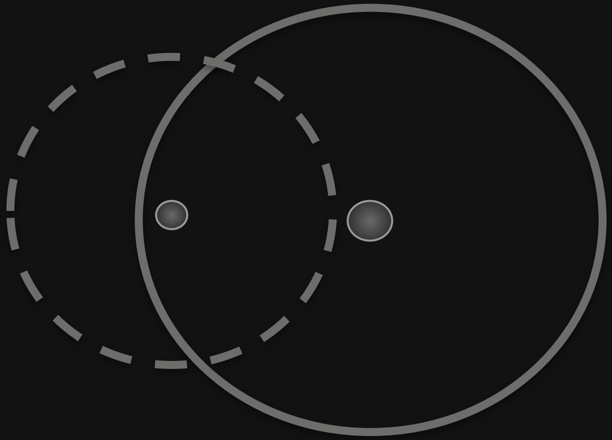

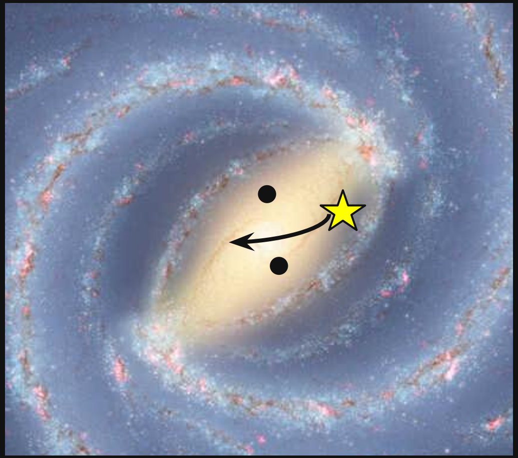

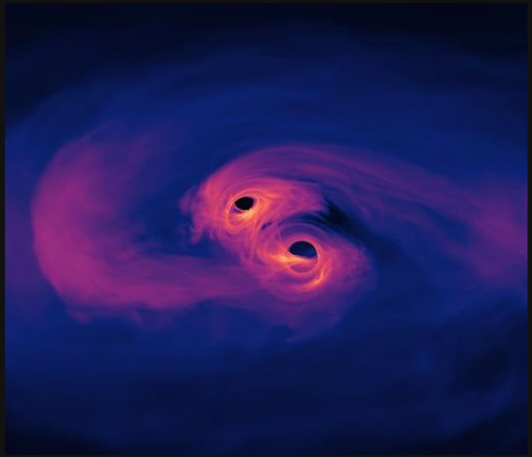

Dynamical friction phase

Stellar hardening

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Guo et al., 2011

$$t_{\rm hardening} = \frac{\sigma_\infty}{G H \rho_\infty a_{GW}}$$

Sesana & Khan, 2015

$$t_{\rm GW} = 5.81 \times 10^6 \,{\rm yr} \left(\frac{a_{\rm GW}}{0.01}\right)^4 \left(\frac{10^8}{m_1}\right)^3 \frac{m_1^2}{m_2(m_1 + m_2)}$$

Maggiore, 2018

The Analytical Model

H. Padmanabhan et al. 2020; arXiv:2007.12710v2

Occupation fraction

MBH – hale mass relation

$$\left(\frac{\mathrm{d}n_{\mathrm{bh}}}{\mathrm{dlog}M_{\mathrm{bh}}\mathrm{d}z\mathrm{d}q}\right) = f_{\mathrm{bh}}(M^1_{\mathrm{bh}}, z, f_i, \epsilon_i) f_{\mathrm{bh}}(M^2_{\mathrm{bh}}, z, f_i, \epsilon_i) \times A_1 \left(\frac{M_{\mathrm{bh}}}{10^{12}\mathrm{M}_\odot \times \mathrm{K}(z,\gamma,\epsilon)}\right)^{\frac{3\alpha}{\gamma}}$$

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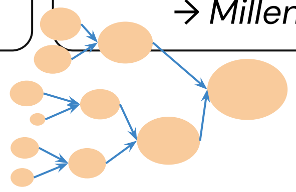
Time delays

Hale merger rate
→ *Millenium simulation (O. Fakhouri et al., 2010)*



DM halo mass function
→ *Numerical package (S. Murrey et al. 2013)*

Generating a population of MBHB

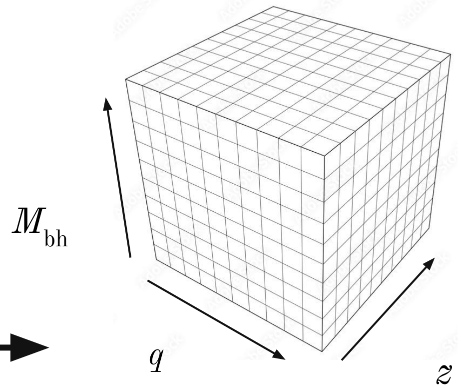
Mock LISA data

$$\left(\frac{\mathrm{d}n_{\mathrm{bh}}}{\mathrm{dlog}M_{\mathrm{bh}}\mathrm{d}z\mathrm{d}q}\right)$$

Generating a population of MBHB

Mock LISA data

$$\left(\frac{\mathrm{d}n_{\mathrm{bh}}}{\mathrm{dlog}M_{\mathrm{bh}}\mathrm{d}z\mathrm{d}q}\right) \longrightarrow$$



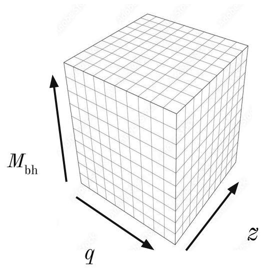
Evaluation
on a 3D grid

$M_2 > 10^4\ M_\odot$
$q = 1 - 10$
$z = 0 - 20$

Generating a population of MBHB

Mock LISA data

$$\left(\frac{\mathrm{d}n_{\mathrm{bh}}}{\mathrm{dlog}M_{\mathrm{bh}}\mathrm{d}z\mathrm{d}q}\right)$$



Poisson draw

Evaluation on a 3D grid

$M_2 > 10^4\ M_\odot$
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Generating a population of MBHB

Mock LISA data

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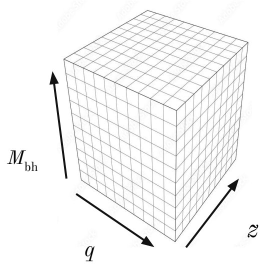

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Poisson draw →

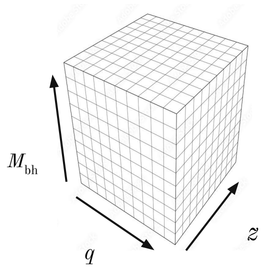
Mbh_desc	z	q	snr_merger
45067.34663848167	0.4601581301143115	0.6175212755240307	717.6741155480061
106763.66825689132	0.4891246352921214	0.1816927875976829	966.0925116586465
114479.99112655567	0.5604190733782815	0.6966205806485715	1250.8661886034865
329685.6130418605	0.5808666359461838	0.2963160492717632	2127.3399089549343
787278.3397360283	0.5762229392915306	0.4002448584596854	4561.126684382561
305921.9088253438	0.6712454254020717	0.1496229244081406	1320.3450490374712
338683.9060766871	0.6183427909925768	0.218959508506081	1802.9666574132323
65177.75154886915	0.7873802553841631	0.3499873929022703	526.6212492646217
434294.70816228125	0.8862611899375896	0.245575474461648	1591.1707438102412
271021.7900681352	0.9425961196991082	0.6077905342141617	1450.4526560171087
1115073.022574774	0.9661786294116	0.3277342554932912	3585.9456548926873
86824.08728314856	1.0491167594271302	0.8633681053025086	588.7843609892852
488919.5376022535	1.011570965121033	0.102535286501045	935.4422270566105
475190.6468197063	1.039180083126099	0.9159803422984928	2192.095532211184
8442682.285731195	1.1644533579103826	0.3371133367488694	1707.7967607265728
96789.3081604071	1.2747434801747128	0.3306392076225322	461.05931909750205
645246.2625567305	1.25396466460293	0.1255972450172499	1040.5666729500942
1771600.4097073562	1.2473896646099587	0.1769236188538361	2212.32550997808

Generating a population of MBHB

Mock LISA data

lisabeta package
Marsat et al., 2021

$$\left(\frac{\mathrm{d}n_{\mathrm{bh}}}{\mathrm{dlog}M_{\mathrm{bh}}\mathrm{d}z\mathrm{d}q}\right) \longrightarrow$$



Evaluation on a 3D grid

$M_2 > 10^4\ M_\odot$
$q = 1 - 10$
$z = 0 - 20$

Poisson draw →

Mbh_desc	z	q	snr_merger
45067.34663848167	0.4601581301143115	0.61752127552403[illegible]7	717.6741155480061
106763.66825689132	0.4891246352921214	0.1816927875976829	966.0925116586465
114479.99112655567	0.5604190733782815	0.6966205806485715	1250.8661886034865
329685.6130418605	0.5808666359461838	0.2963160492717632	2127.3399089549343
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305921.9088253438	0.6712454254020717	0.1496229244081406	1320.3450490374712
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65177.75154886915	0.7873802553841631	0.3499873929022703	526.6212492646217
434294.70816228125	0.8862611899375896	0.245575474461648	1591.1707438102412
271021.7900681352	0.9425961196991082	0.6077905342141617	1450.4526560171087
1115073.022574774	0.9661786294116	0.3277342554932912	3585.9456548926873
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488919.5376022535	1.011570965121033	0.102535286501045	935.4422270566105
475190.6468197063	1.039180083126099	0.9159803422984928	2192.095532211184
8442682.285731195	1.1644533579103826	0.3371133367488694	1707.7967607265728
96789.3081604071	1.2747434801747128	0.3306392076225322	461.05931909750205
645246.2625567305	1.25396466460293	0.1255972450172499	1040.5666729500942
1771600.4097073562	1.2473896646099587	0.1769236188538361	2212.32550997808

Population statistics

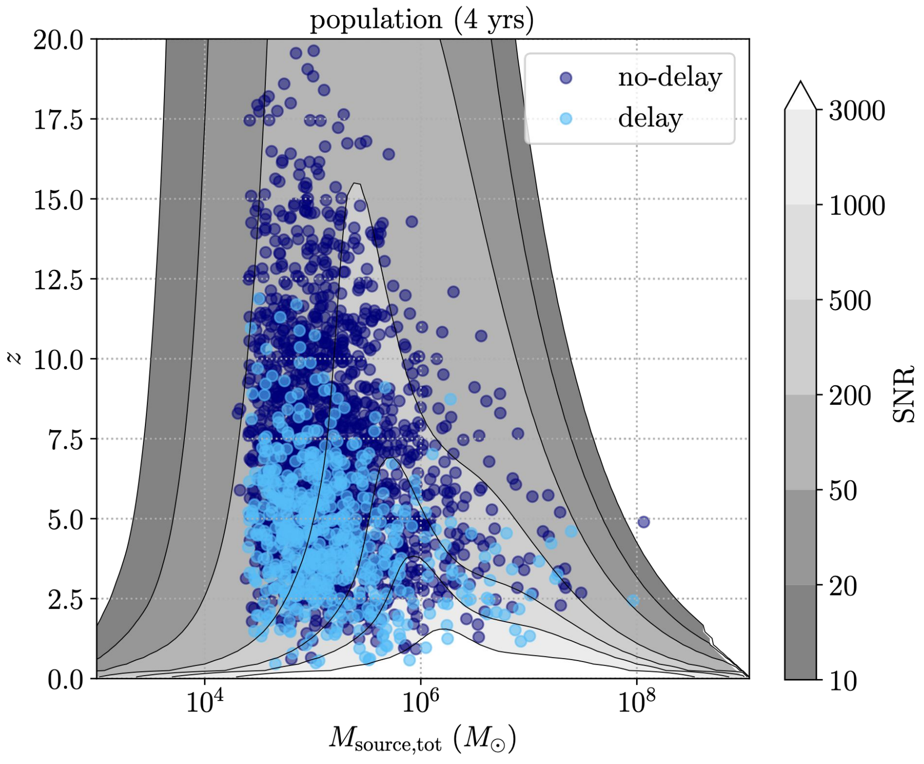
Langen et al., 2025. MNRAS 536(4), 3366-3385.

Detection rates [/yr] (*Stochastic sc.r.*)	No - delay model	Delay model
Fiducial rates	385.7	144.5
Reduced rates	38.5	14.5

Detection rates [/yr] (*Deterministic sc.r.*)	No - delay model	Delay model
Fiducial rates	216	98.0
Reduced rates	21.6	9.8

Population statistics

Langen et al., 2025. MNRAS 536(4), 3366-3385.



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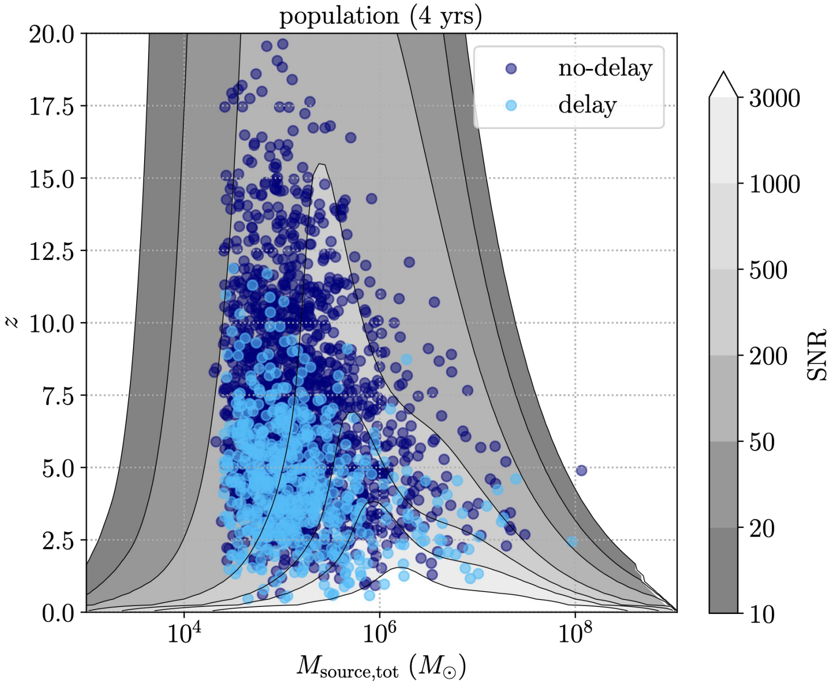

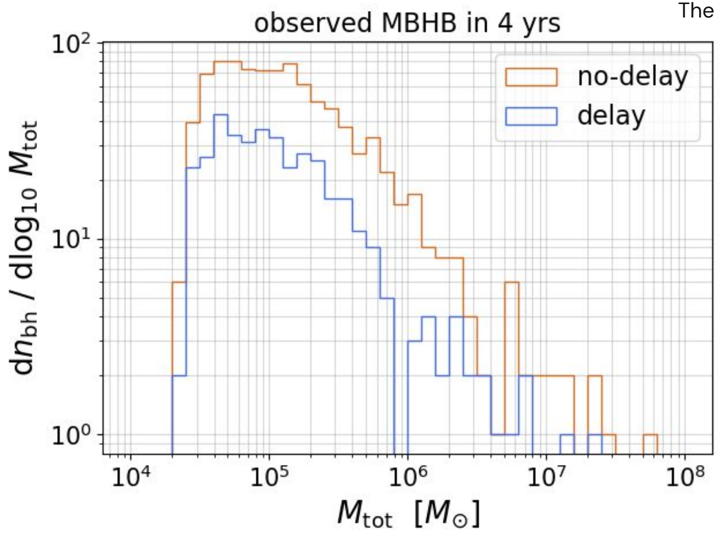

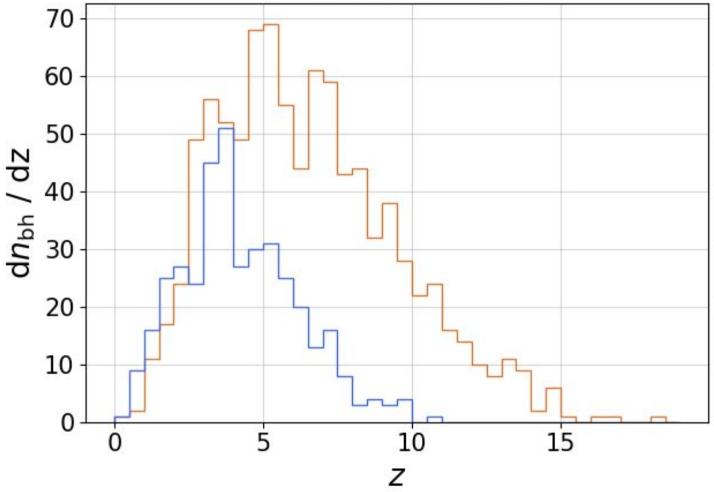

That’s it for the *astrophysics*,

Now it’s about ***data analysis*** **!**

That's it for the *astrophysics*,

Now it's about ***population inference*** !

Our chosen hyper-parameters

- BH - halo mass scaling relation : $\gamma, \ \gamma', \epsilon$
- Occupation fraction : f_3
- DF time delay efficiency : α_{fric}

- Einstein's Universe
- The Laser Interferometer Space Antenna (LISA)
- Massive Black Hole Binaries (MBHBs)
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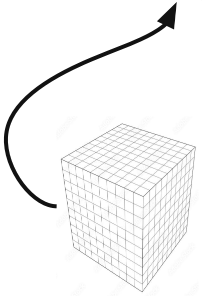
MCMC approach

- 4 yrs of LISA data
- Simple Poisson likelihood
- No selection effects
- Zero Poisson-noise
- Simplistic LISA-noise

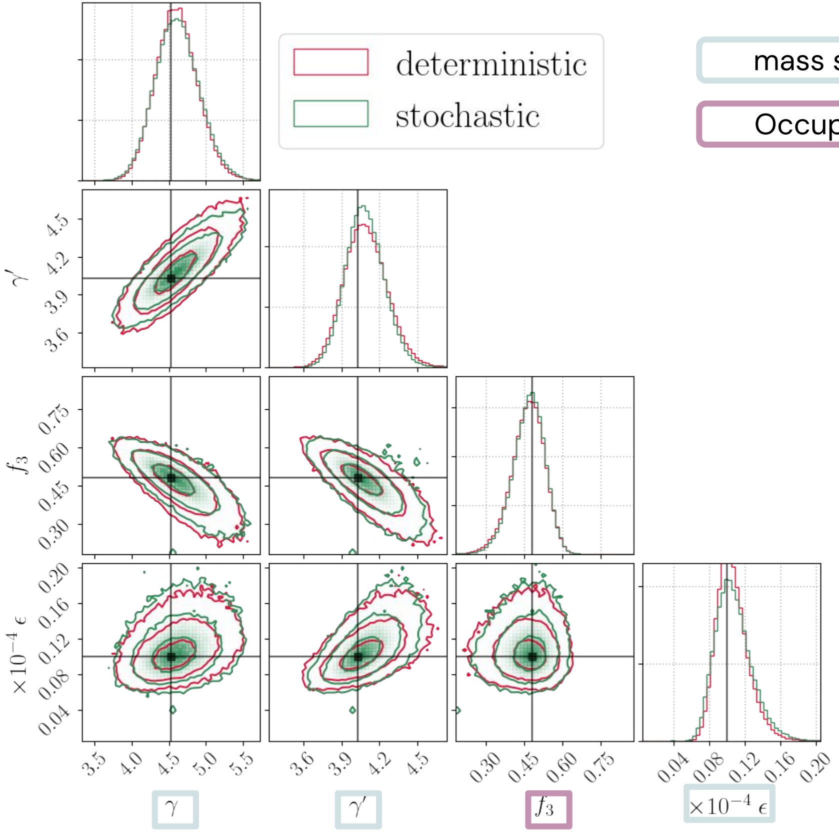
➔ Simplified scenario

$$\mathcal{L}_{\rm tot}\left(\text{data}|\text{model}\right) = \prod_{i}^{N_m \cdot N_q \cdot N_z} \frac{\left(\lambda_i^{\rm exp}\right)^{n_i^{\rm obs}} e^{-\lambda_i^{\rm exp}}}{n_i^{\rm obs}!}$$



No-delay model: deterministic VS stochastic sc. r.

Langen et al., 2025. MNRAS 536(4), 3366-3385.

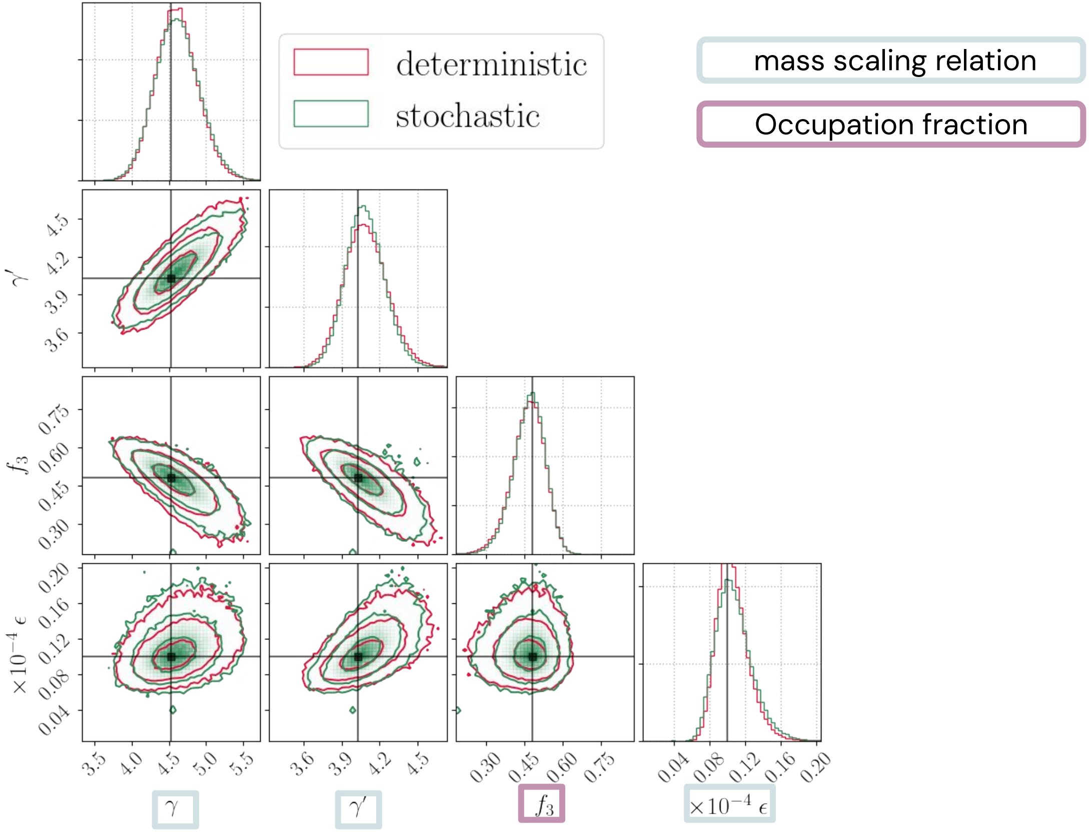


mass scaling relation

Occupation fraction

No-delay model: deterministic VS stochastic sc. r.

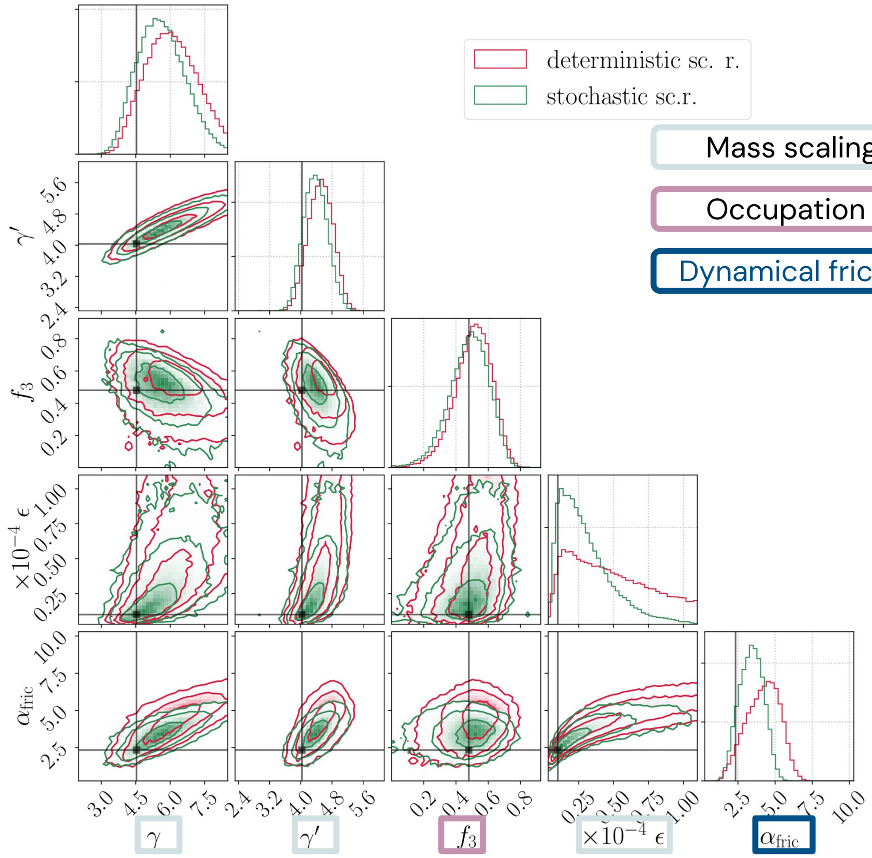
Langen et al., 2025. MNRAS 536(4), 3366-3385.



- Zero delays
- Both scenarios consistent with each other
- All parameters constrained within < 10% for the 90% C.I.

Delay model: deterministic VS stochastic sc. r.

Langen et al., 2025. MNRAS 536(4), 3366-3385.



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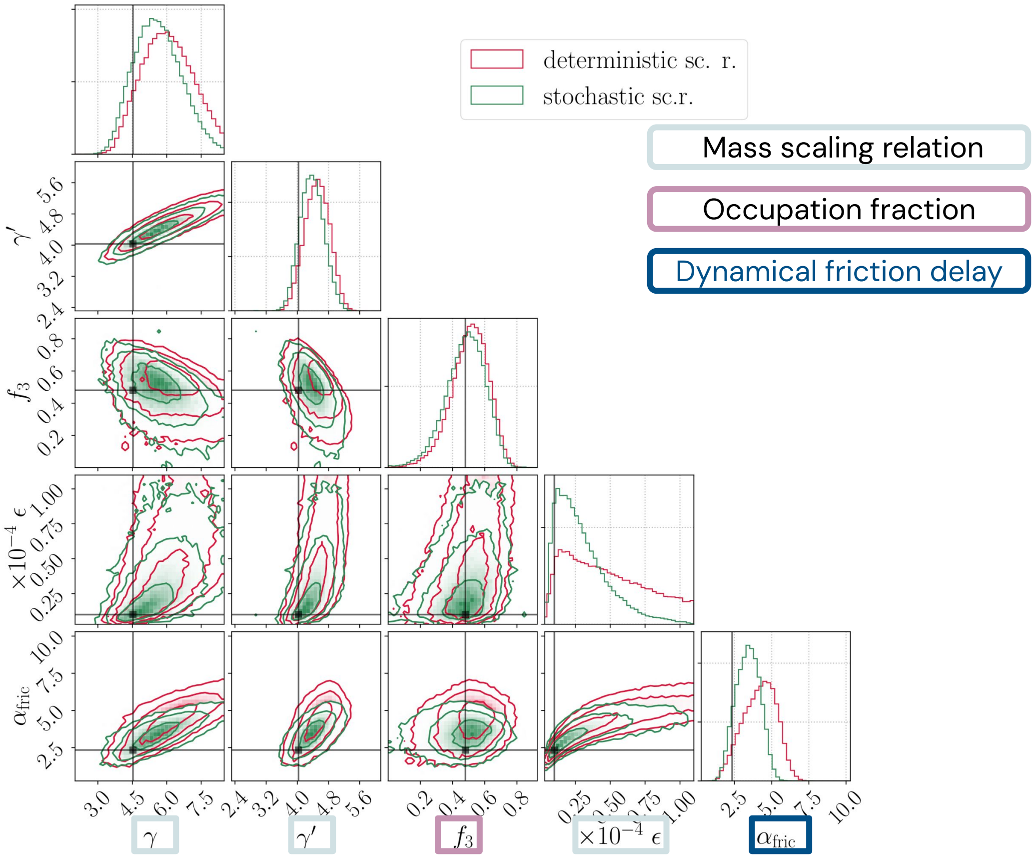

- *Higher* dimensions & *smaller* rate →worse constraints
- Errors enlarged to < 20% for 90% C.I.
- Except γ' remains within 10% →better constraints at low masses!
- Degeneracy between ε and $\alpha_{\rm fri}$
- Degeneracies for deterministic sc. r. *Less events at high masses?*

Predictive posterior distributions

Langen et al., 2025. MNRAS 536(4), 3366-3385.

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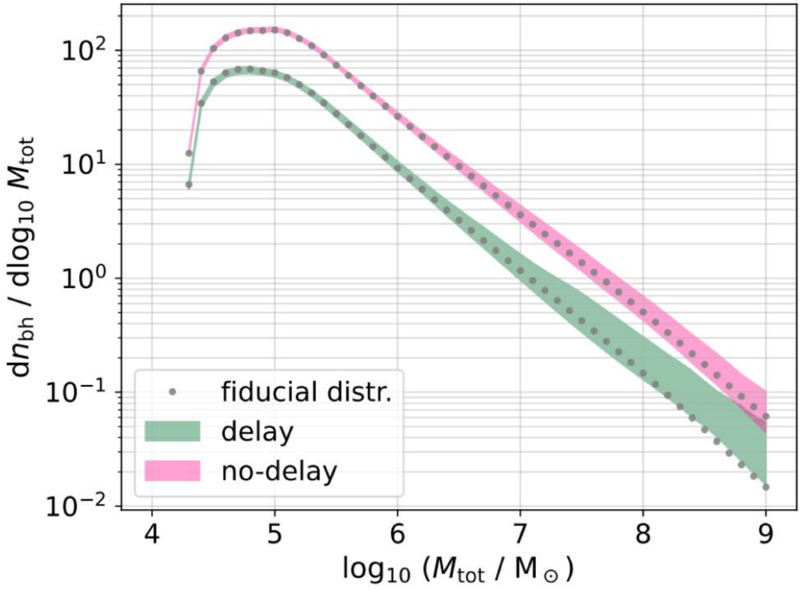
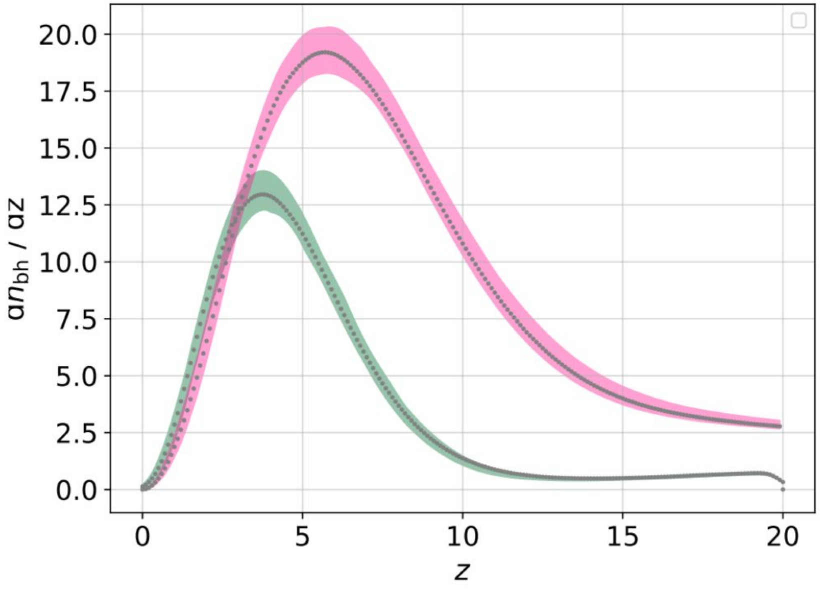
Langen et al., 2025. MNRAS 536(4), 3366-3385.





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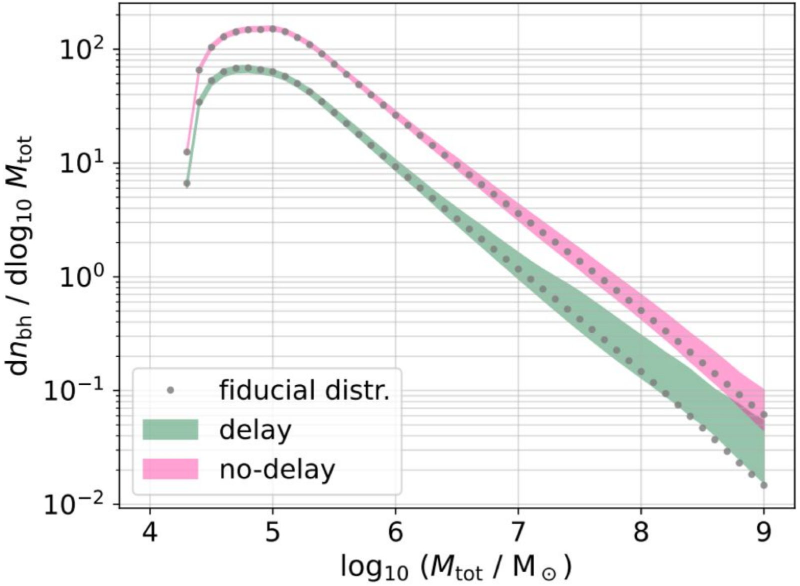

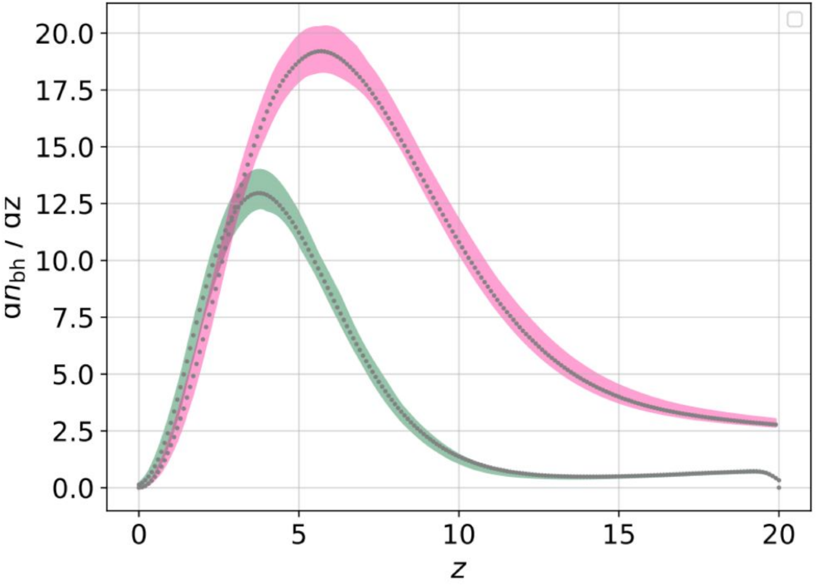

- Better constraints at lower BH masses
- Smaller errors for the no-delay model

- Less evident difference between delay and no-delay model.

BH - halo mass scaling relation

Langen et al., 2025. MNRAS 536(4), 3366-3385.

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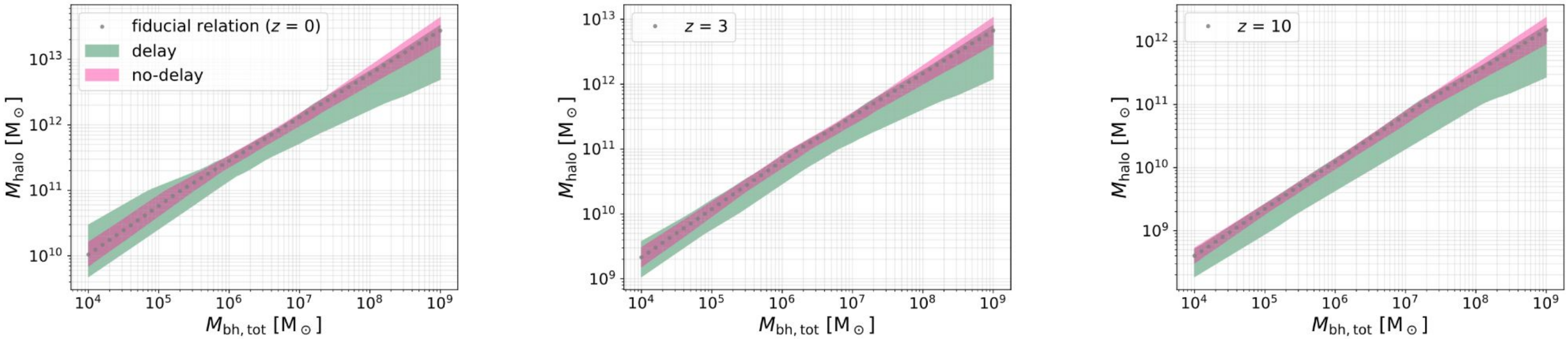

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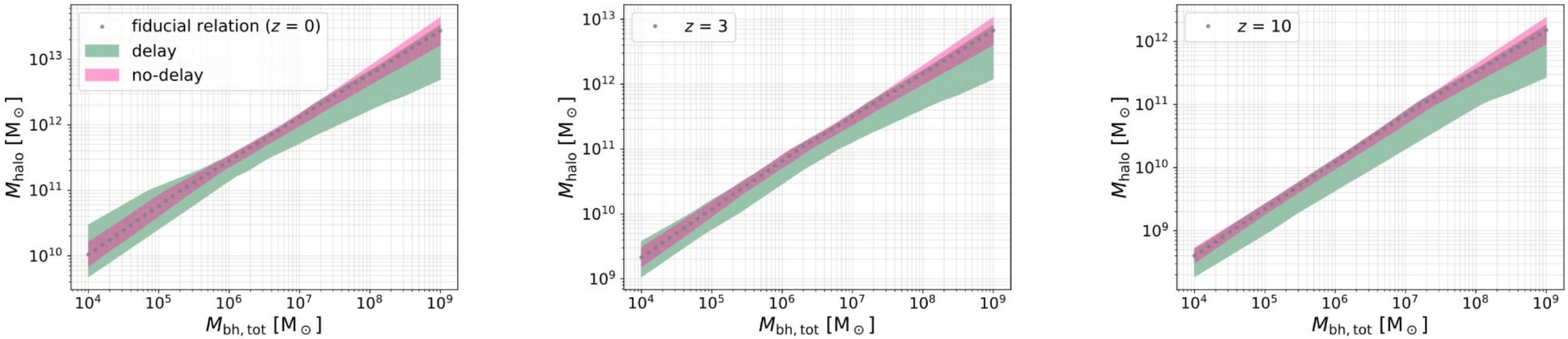
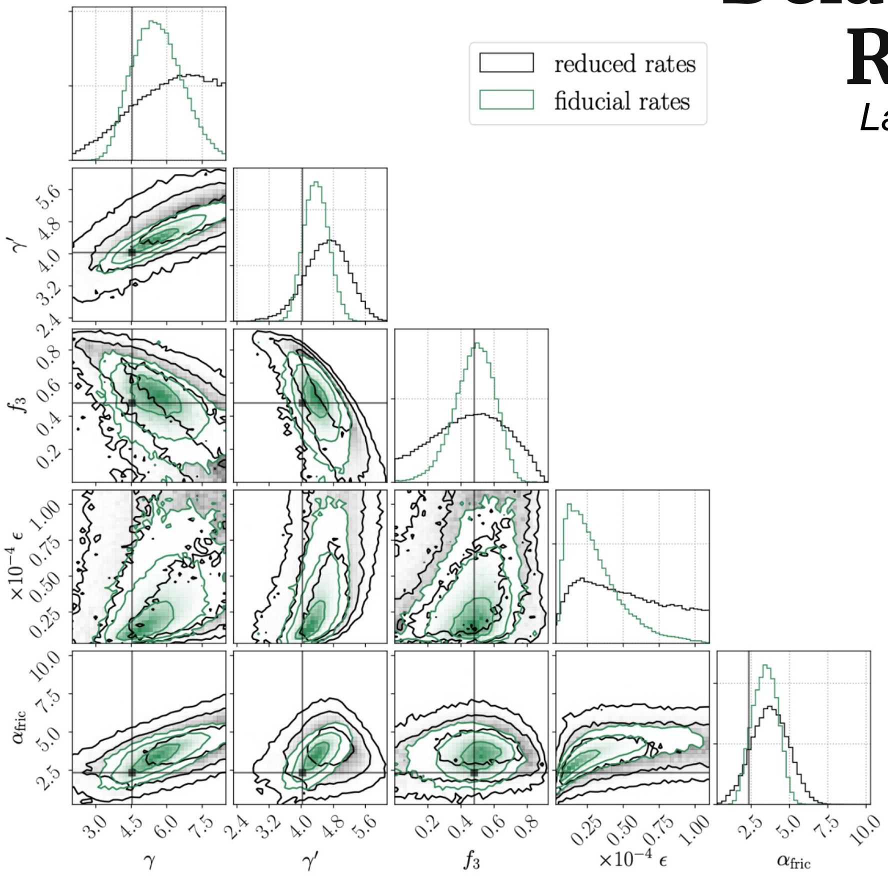


- Good constraints on the scaling relations up to high redshift
- Especially low masses hardly accessible by EM observations

Delay model: Reduced VS fiducial rates

Langen et al., 2025. MNRAS 536(4), 3366-3385.



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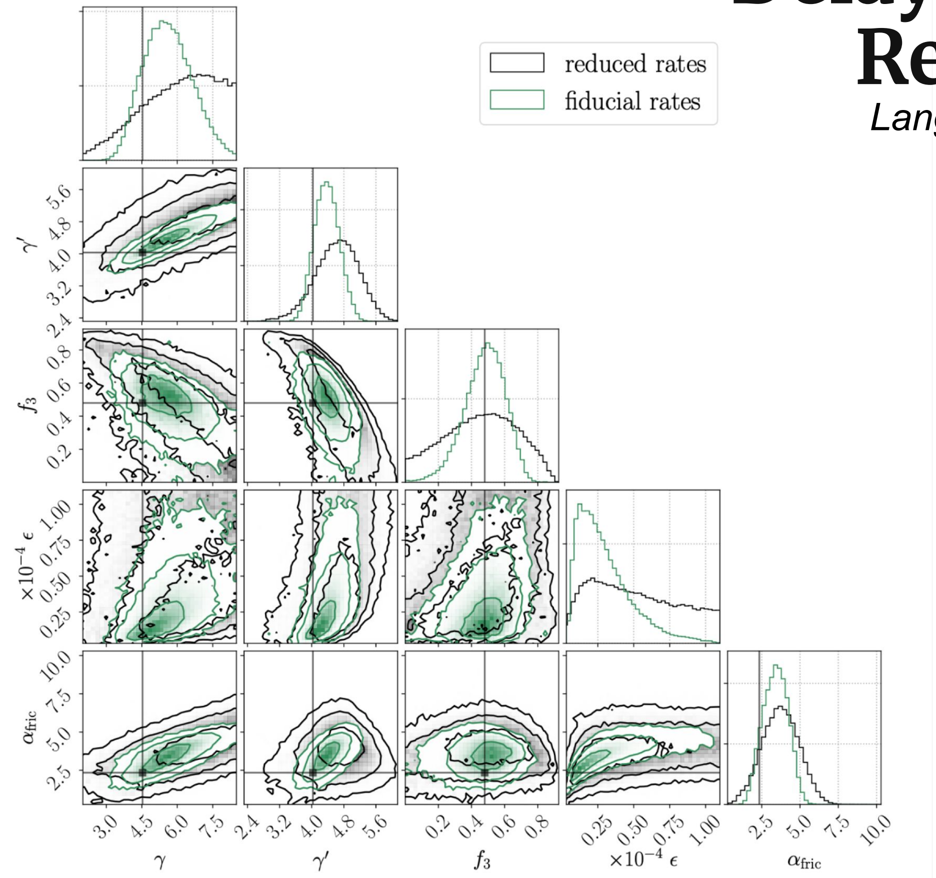
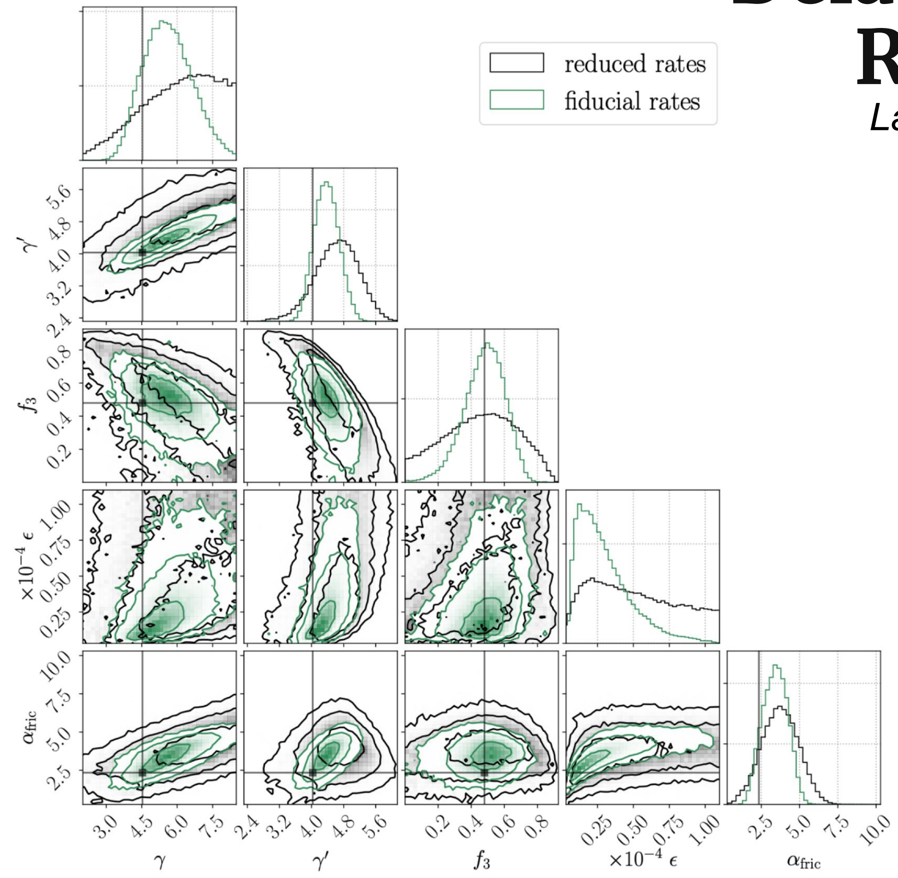
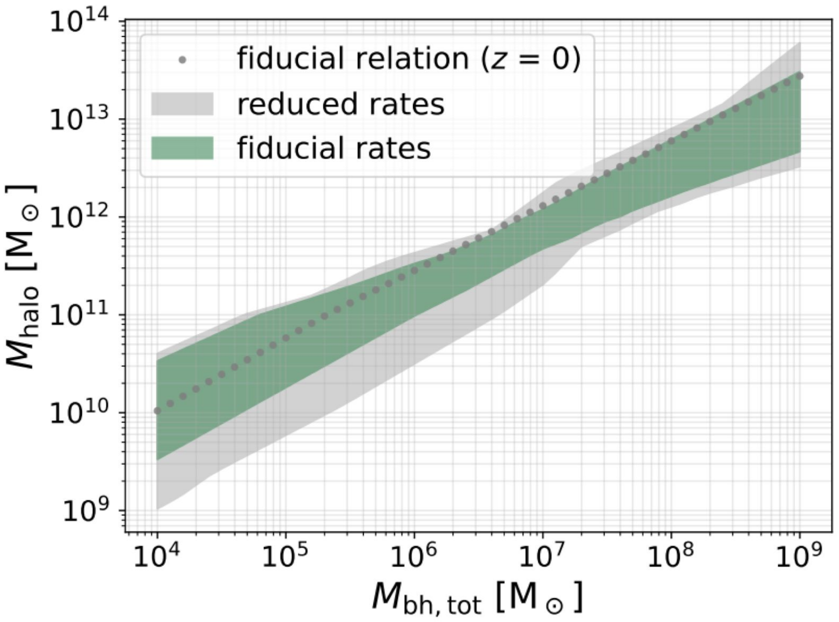


- Stochastic sc. r.
- fiducial: 144 /yr
  vs
  reduced = 14.4 /yr
- Smaller rates
  → larger errors
- Meaningful constraints on γ’ and f_3

Delay model: Reduced VS fiducial rates

Langen et al., 2025. MNRAS 536(4), 3366-3385.





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LISA Astrophysics Working Group, 2025 in prep.

Overview

- Project started in September 2022 during the annual AstroGW meeting
- Comparison of 18 state-of-the-art semi-analytical and hydrodynamical / N-body simulations

My contribution

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Results preliminary !

→ paper writing in progress

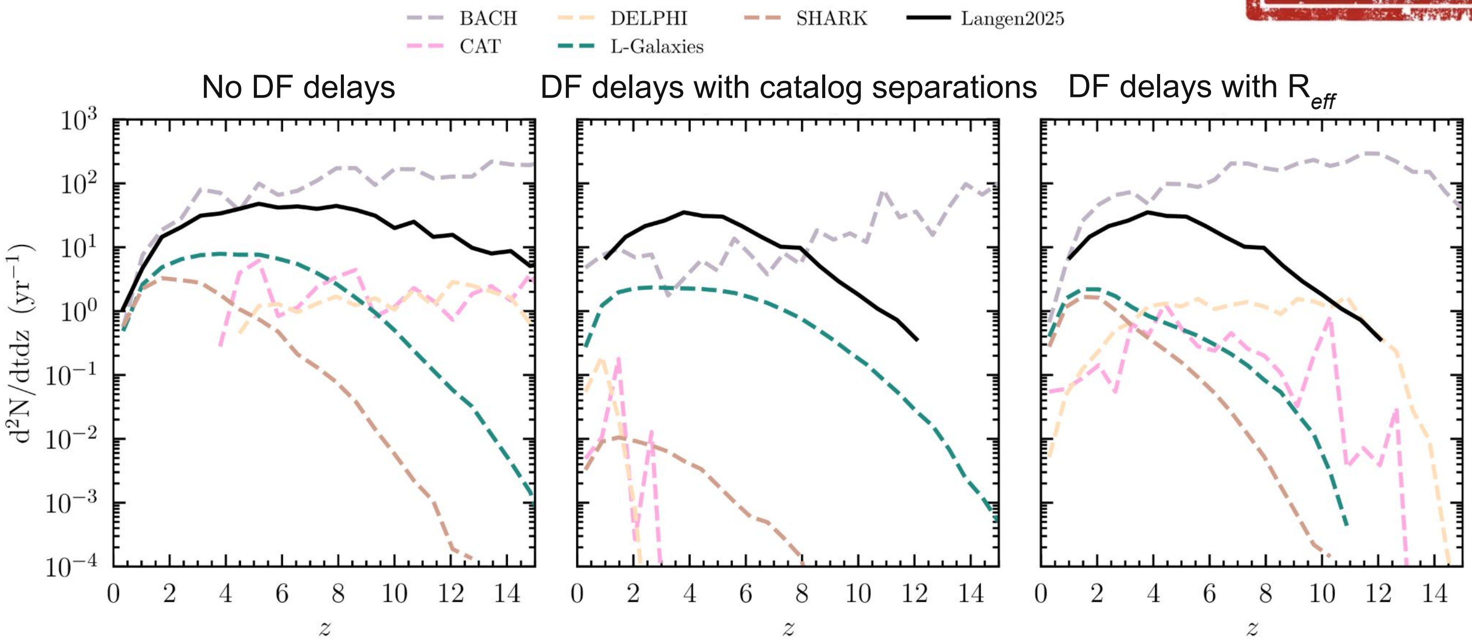
My contribution

- Participation since the beginning for the last 2.5 years of my PhD
- Computation of merger rates
- Implementation of time delays
- Calculation of signal-to-noise ratios
- Section writing and interpretation (to lesser extent)

Semi-analytical models

LISA Astrophysics Working Group, 2025 in prep.

PRELIMINARY

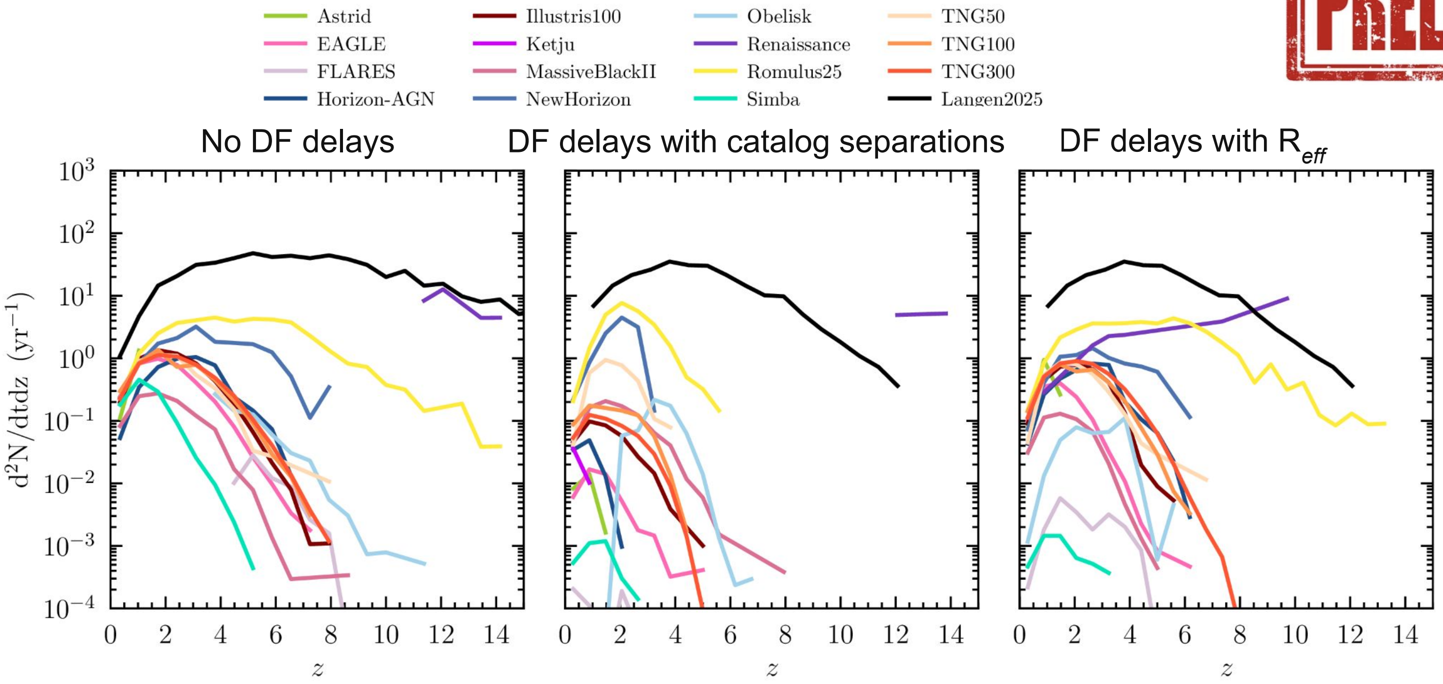


- Large spread between merger rates → analytical model consistent among predictions
- Convergence at low redshift for no-DF
- Comparable DF time delays
- No clear difference between DF delay methods

Hydrodynamical, N-body simulations

LISA Astrophysics Working Group, 2025 in prep.

PRELIMINARY



- Lower rates compared to the semi-analytical and our analytical model (especially at high z)
- Lower spatial & lower mass resolution
  → longer DF delays and missing low mass mergers

Take-home messages & conclusions

- ***Improved*** analytical model of the MBHB population
  → In ***agreement*** with state-of-the-art models
  → ***Delays reduce*** the MBHB merger ***rates***
  → Minor impact of stochastic scaling relation

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  → ***Inference*** up to ***5 parameters;*** despite degeneracy between ε and α_{fric}
  → Slope of the ***scaling relation better*** measured at ***low masses (for all z)***
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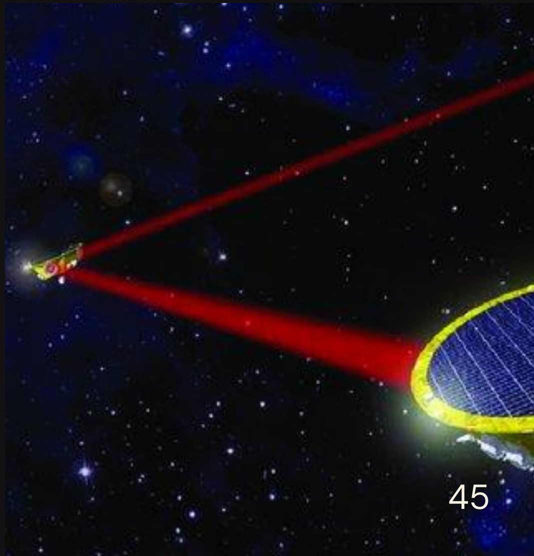
⇒ ***LISA will complement EM observations at high*** z ***low masses***

Future prospects

- A lot of potential to improve the model
  → implementation of *light-seeds* & *heavy-seeds* populations
  → introduction of *spins*
  → simple binary *accretion prescription*
  → *inference of the mixing fraction* between spin populations / seed populations ?



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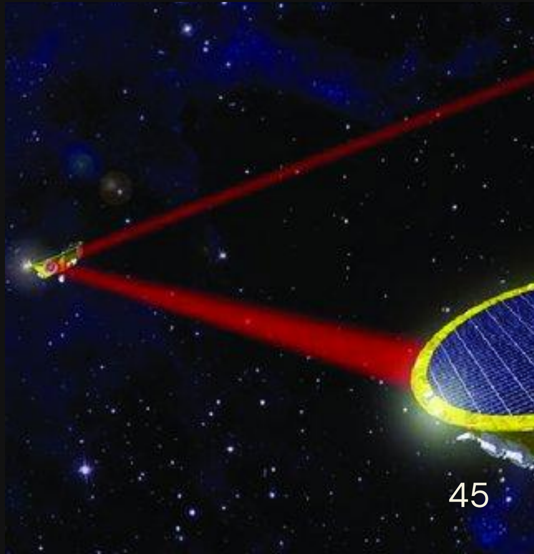

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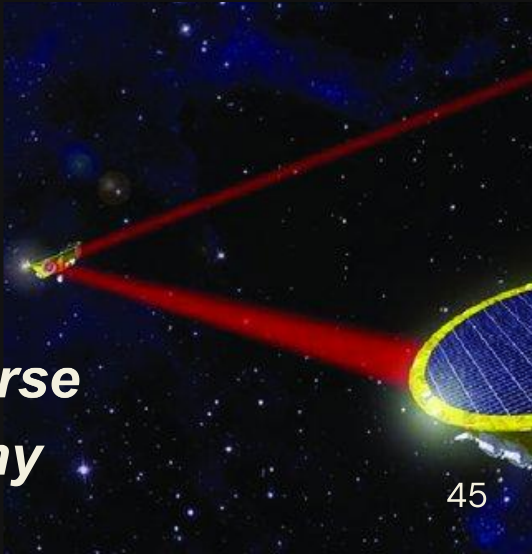
- ***LISA: an unique window into the high-z universe***

  → *a new milestone in observational astronomy*

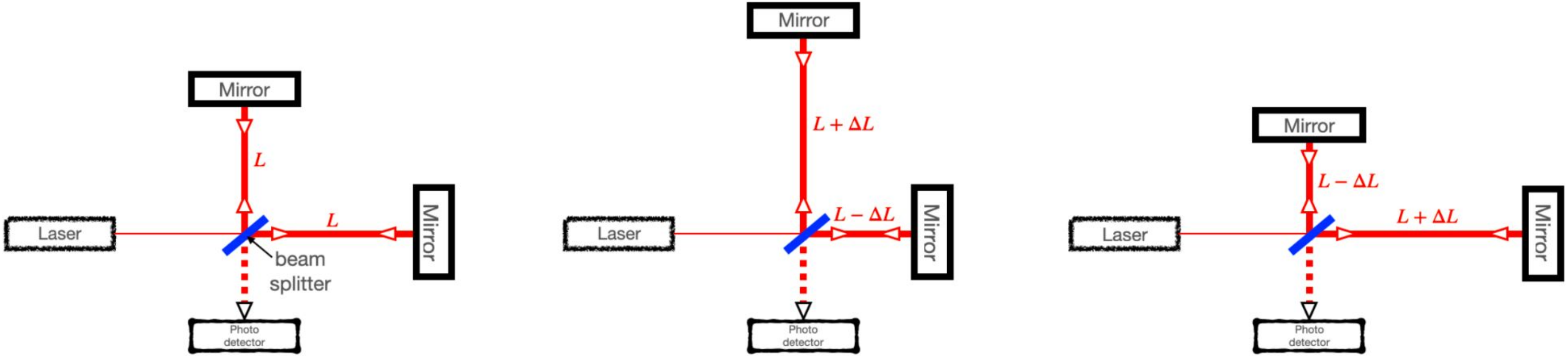


Thank you for your attention!

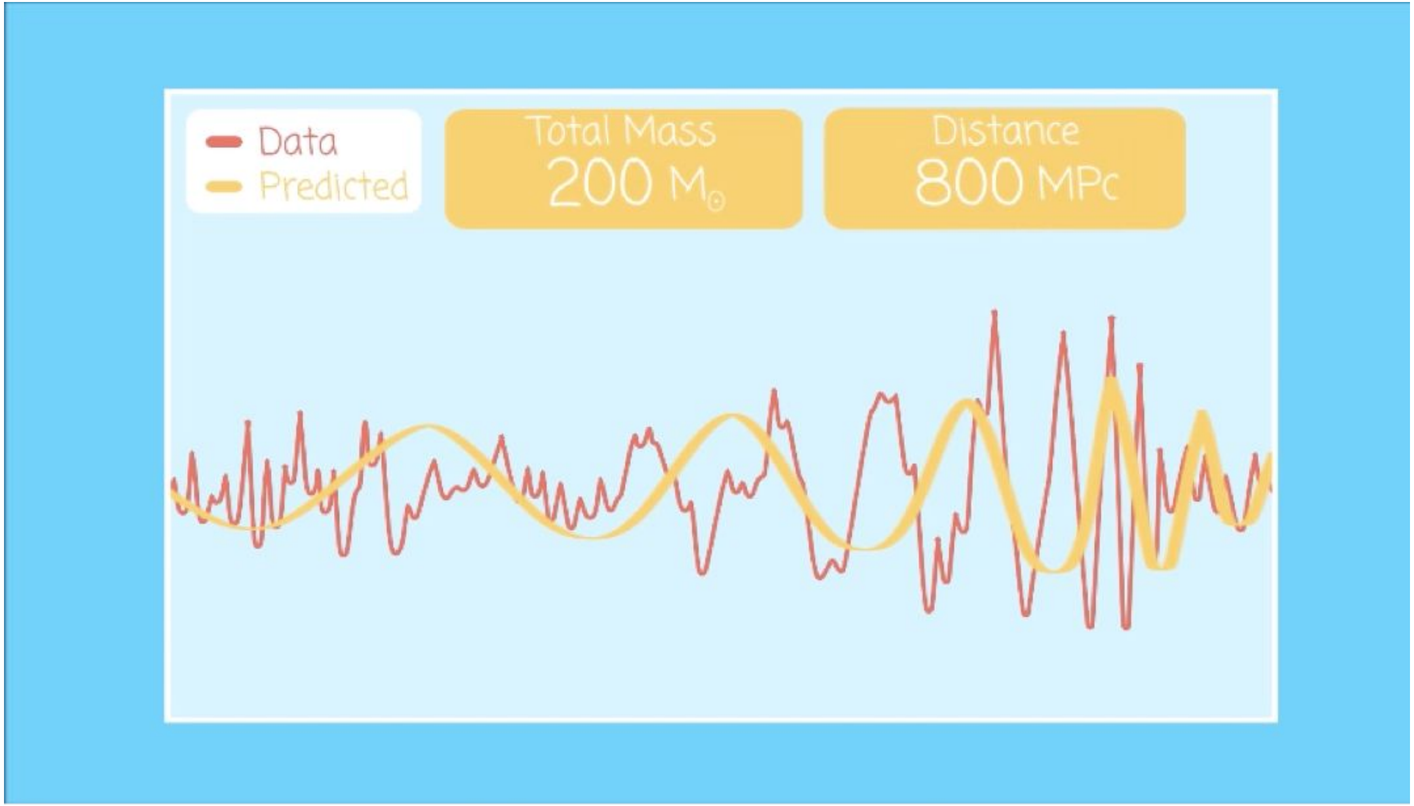
Back-up slides

- Introduction and MBHs
- Full occupation fraction

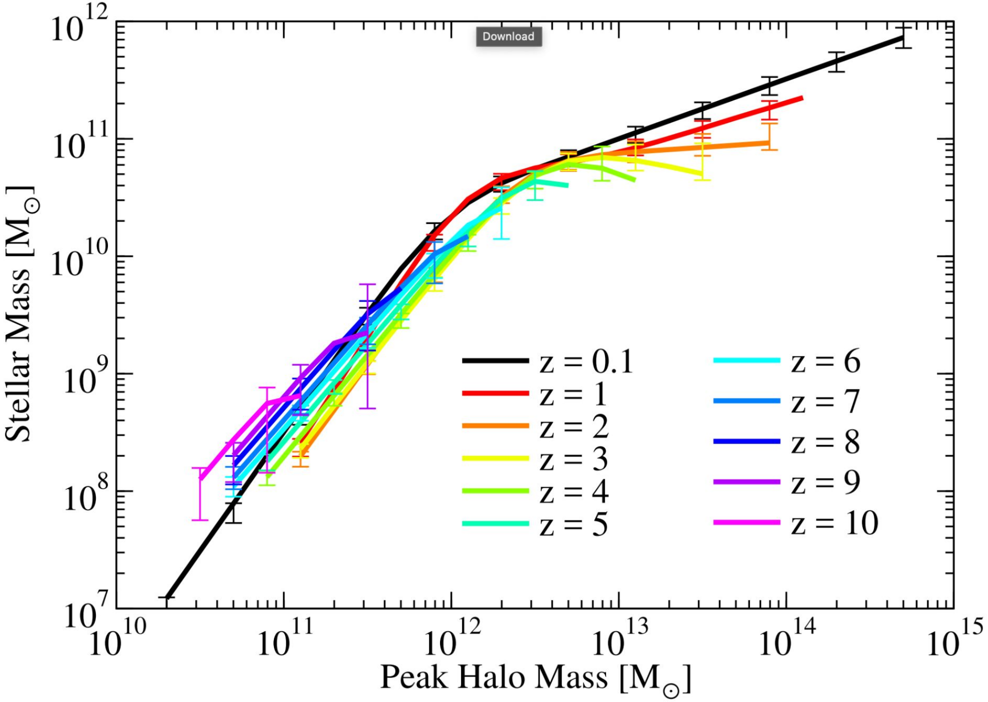
Michelson - Morley interferometer



Measuring mass VS redshift



Galaxy - halo mass scaling relation



Behroozi, P., Wechsler, R. H., et al, 2019; MNRAS, 488, 3143

Darf galaxy – BH mass scaling relation

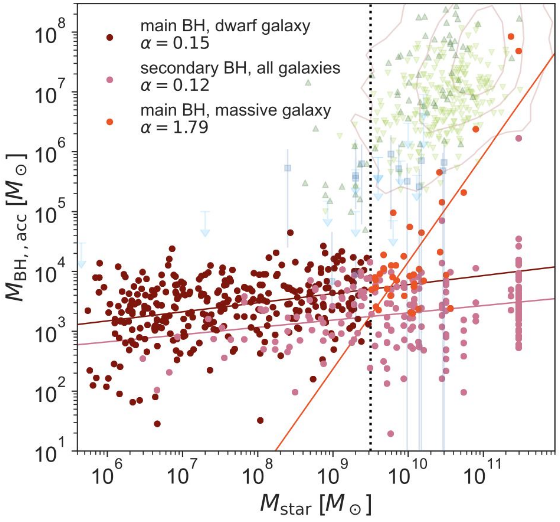


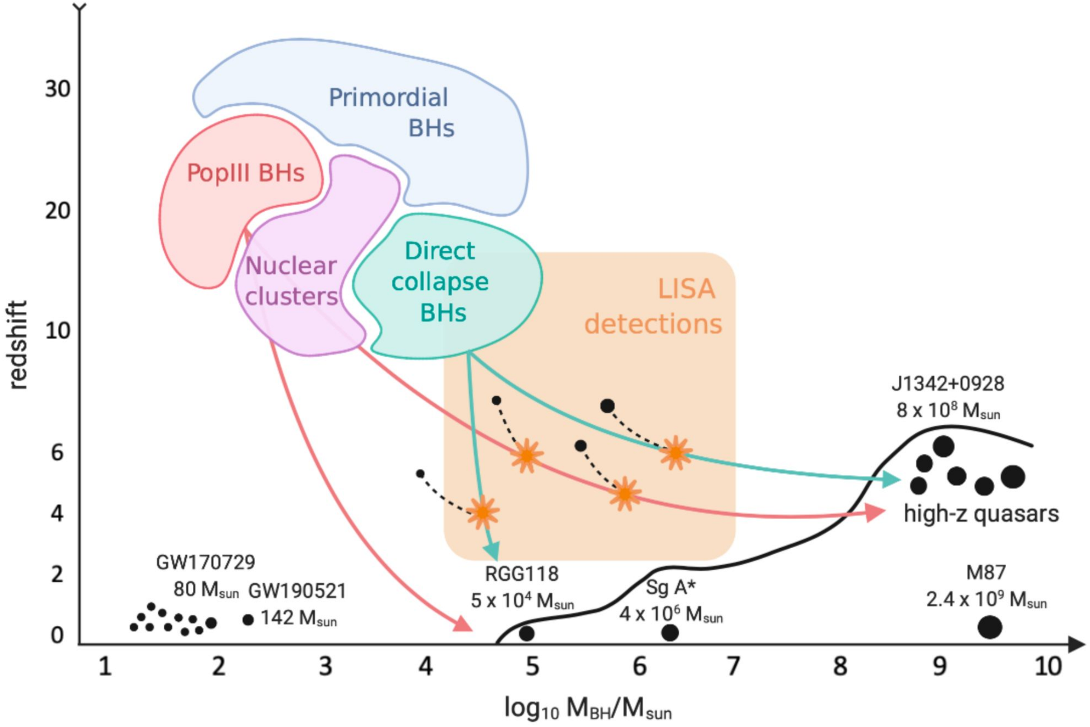
Figure 2. Correlations between mass and host galaxy properties at $z = 0.25$: [Top] Galaxy stellar mass versus BH mass $M_{\rm BH}$ **for all main BHs**. The grey distribution shows that stacked sample at all redshifts. [Bottom] Galaxy stellar mass versus accereted BH mass $M_{\rm BH,acc}$ **for all BHs**. Shown on both plots for comparison are observational data from Reines & Volonteri (2015) (RV15, green triangles), Baron & Ménard (2019) (BM19, brown contours) and Greene et al. (2019) (Greene20, blue markers and limits).The same observations are shown on both panels. **α denotes the slope of the fits for each population of BHs.** Errorbars for RV15 are omitted for clarity. Galaxies left of the dotted black line are considered dwarf galaxies.

New Horizon Simulation:

R. S. Beckmann et al., 2022

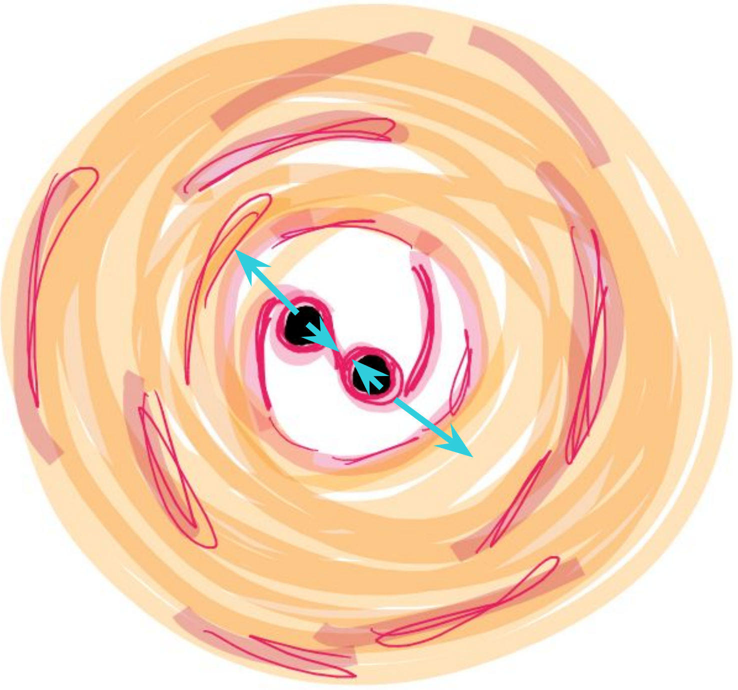
Behroozi, P., Wechsler, R. H., et al, 2019; MNRAS, 488, 3143

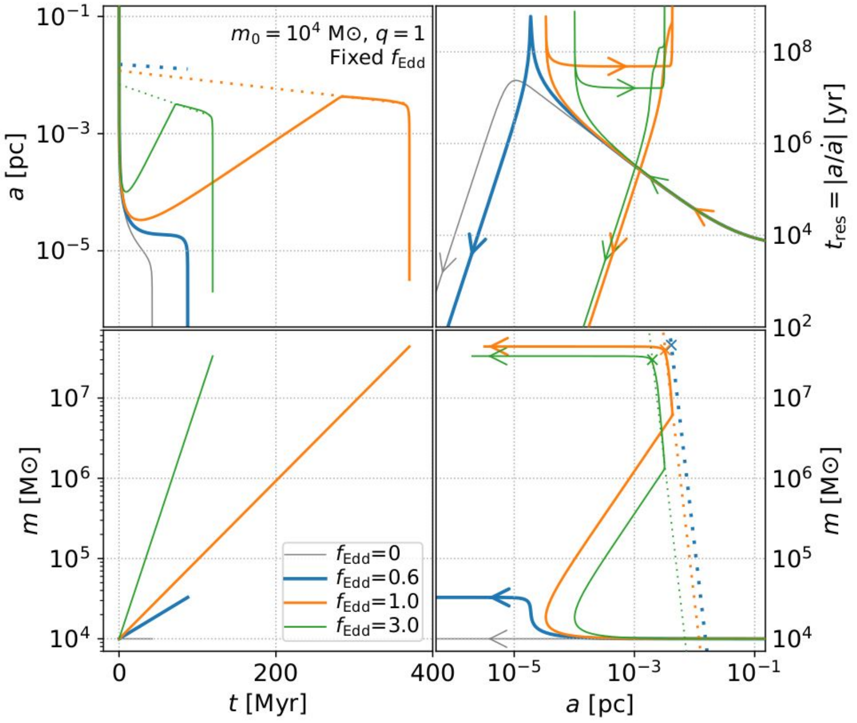
MBH seeds



Pau Amaro-Seoane et al., 2023; Living reviews in Relaitiviy

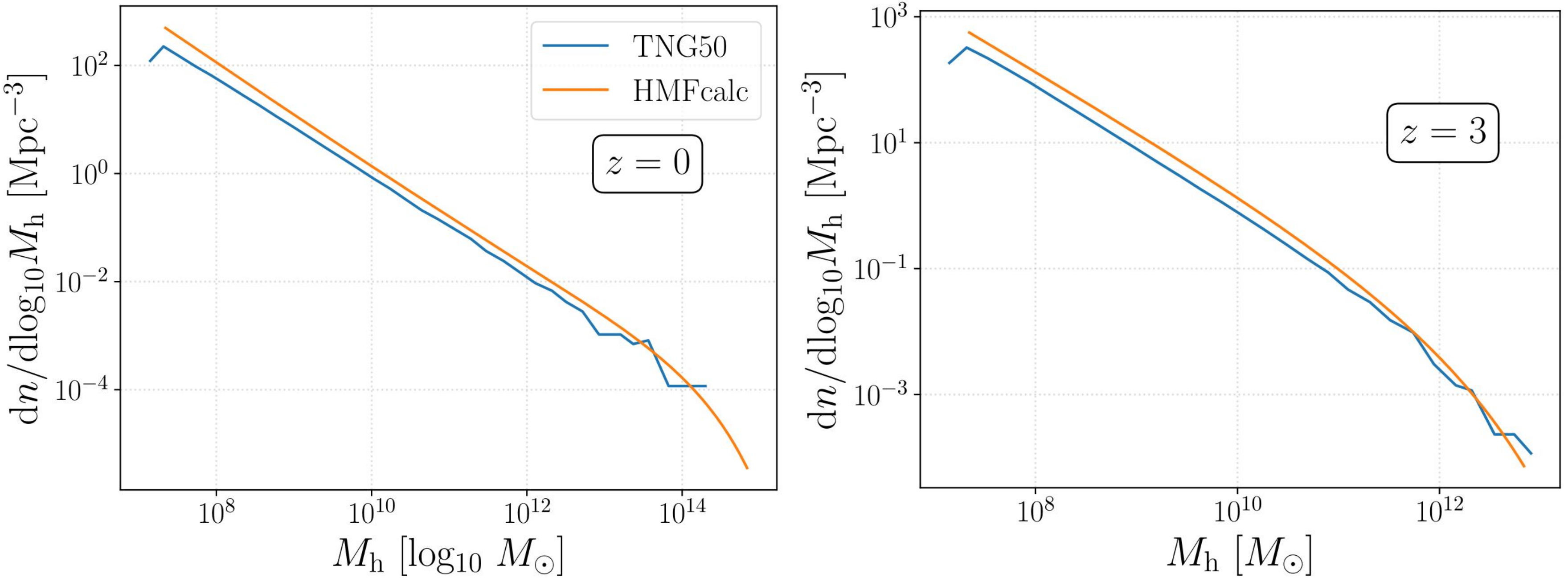
The role of gas in the hardening phase



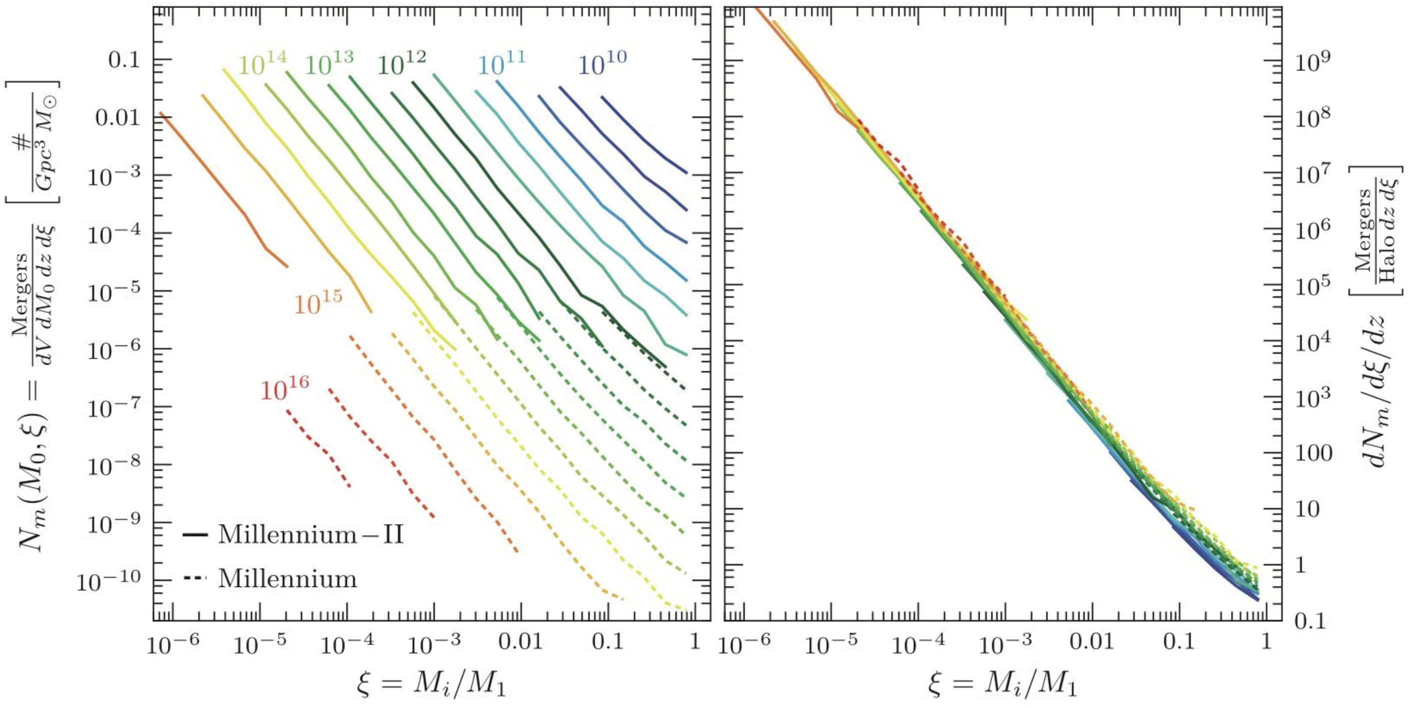


Bortolas+2021, ApjL, 918 L15

HMFcalc compared to TNG50

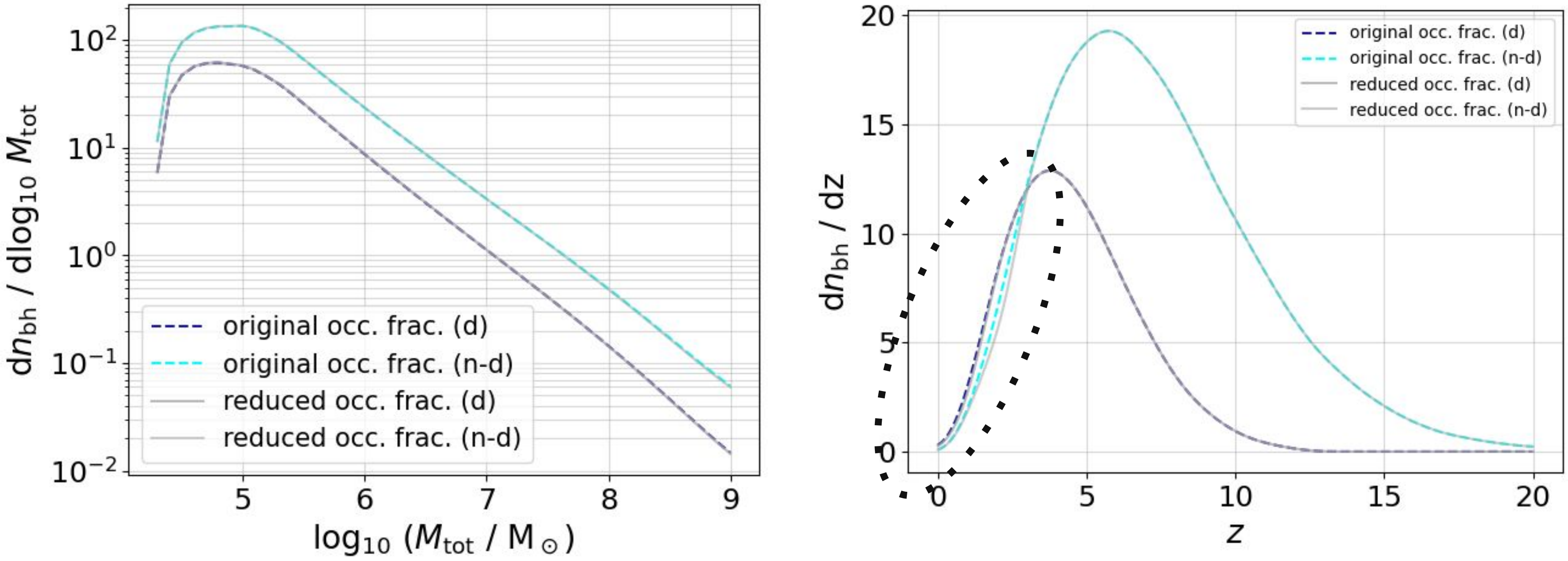


Halo merger rate per halo

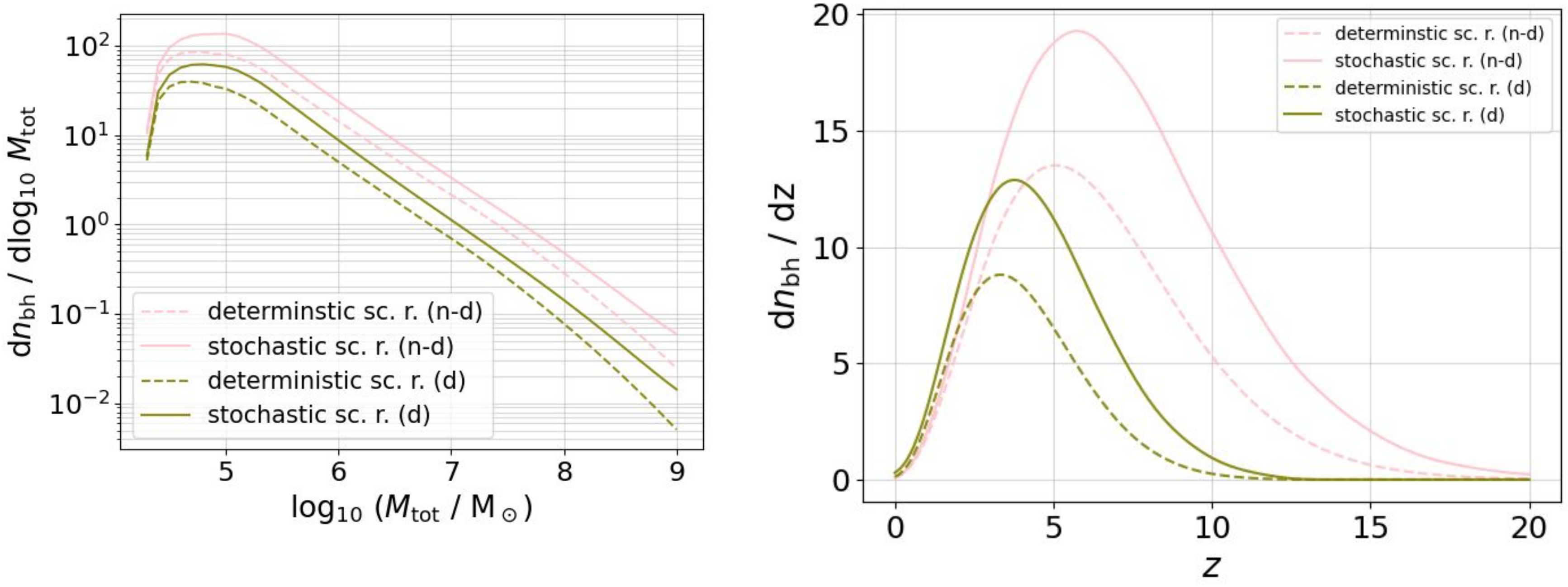


O. Fakhouri et al., 2010

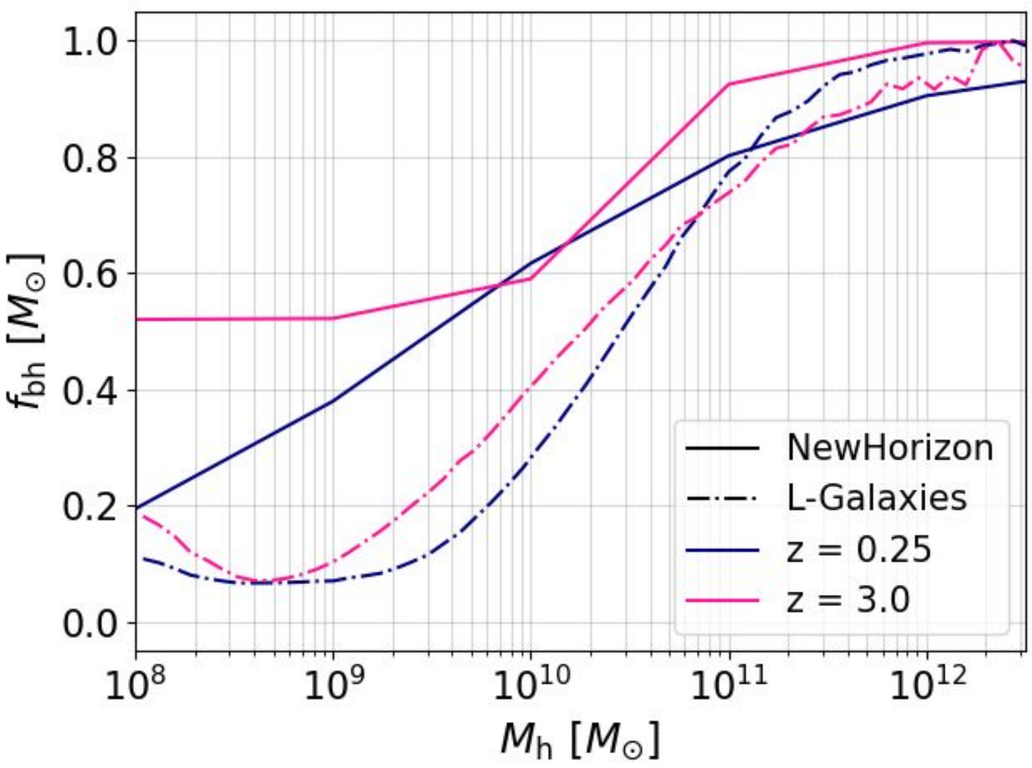
Original versus reduced occupation fraction



Deterministic VS stochastic relation: population distributions



Occupation fraction compared to *L-galaxies* model

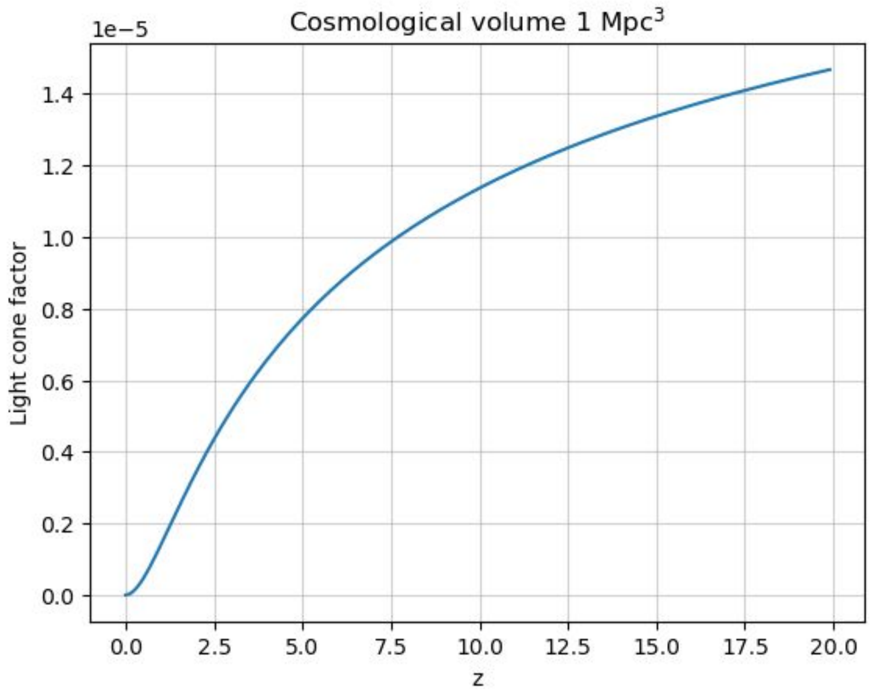


Observed merger rate (equation)

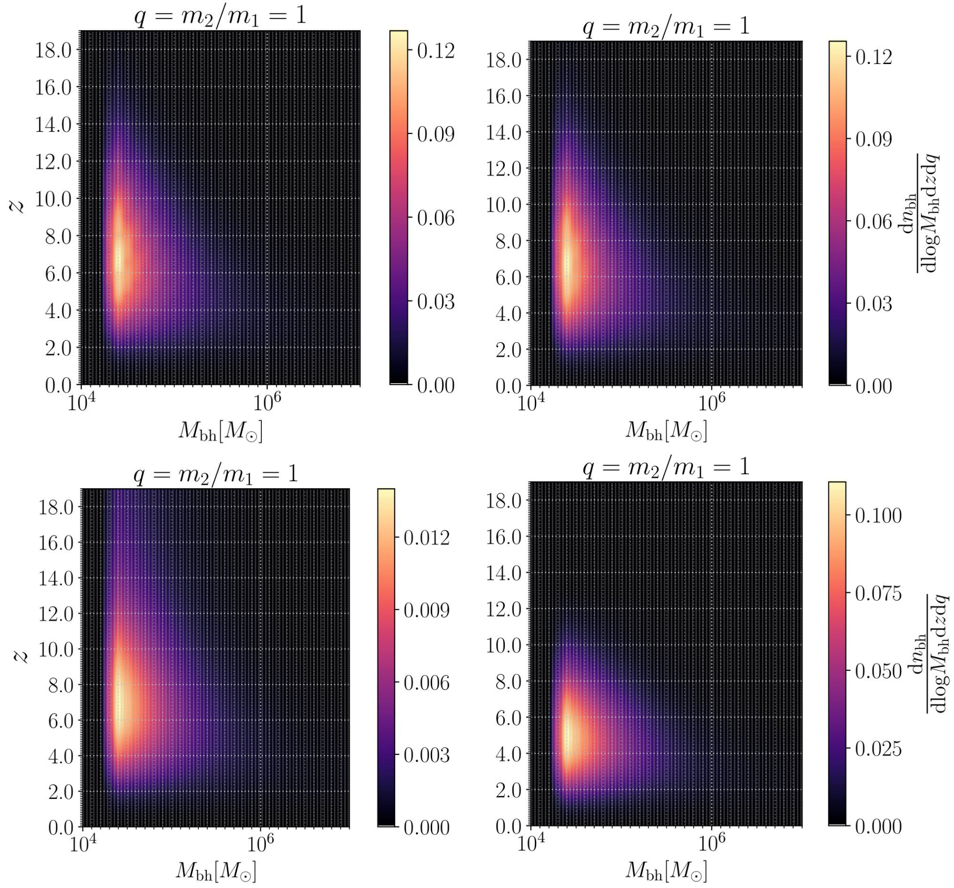
$$\frac{dN}{dt} = \int_0^{z_{\max}} dz \frac{dn}{dz} \times \frac{4\pi c\, d_L(z)^2}{(1+z)^2} \,.$$

$$(n_{\text{obs}})_{ijk} = n_{ijk} \times \frac{4\pi c\, d_L(z)^2}{(1+z)^2} \,.$$

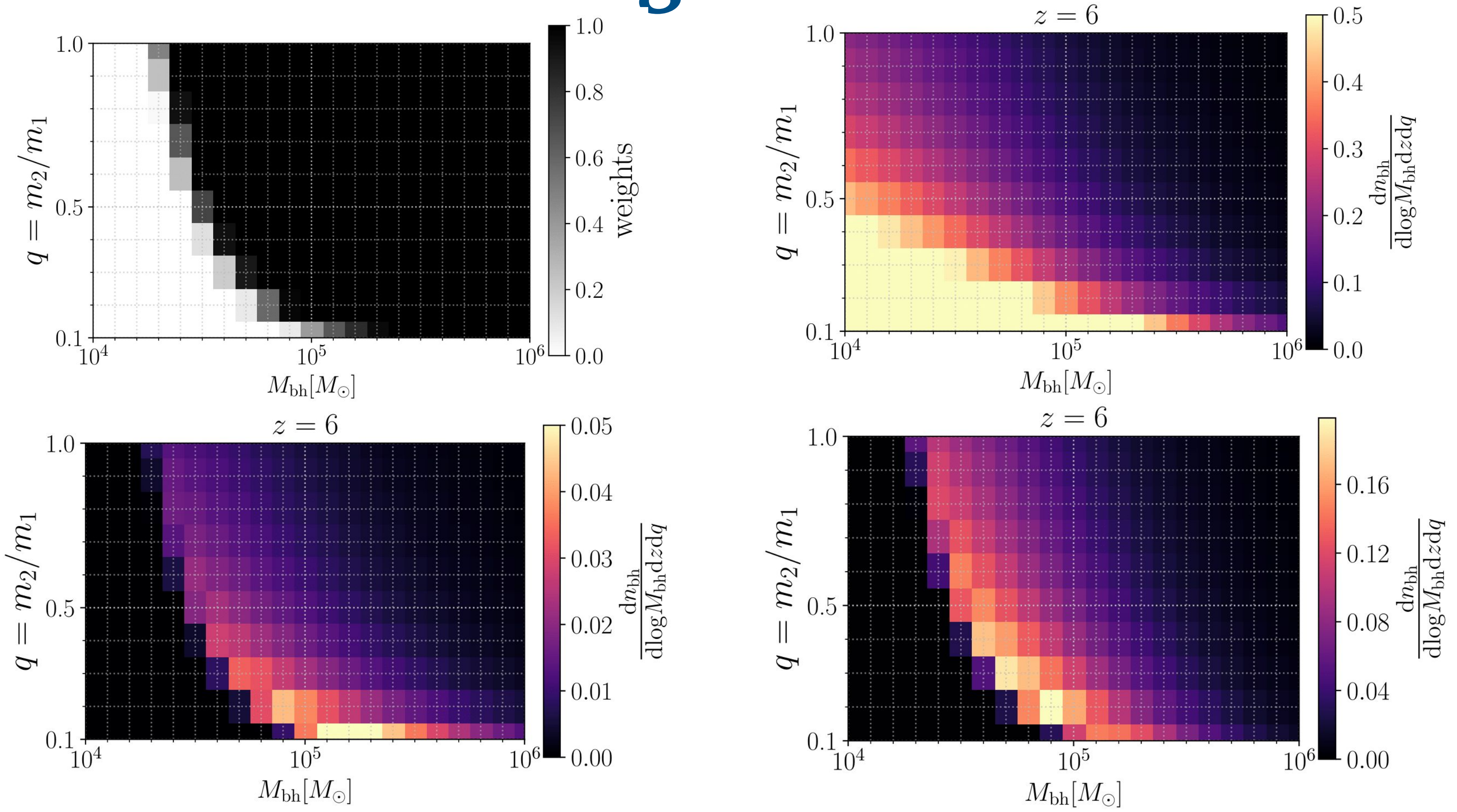
$$\frac{dN}{dt} = \sum_{i,j,k=0}^{n_z, n_q, n_m} (n_{\text{obs}})_{ijk} .$$



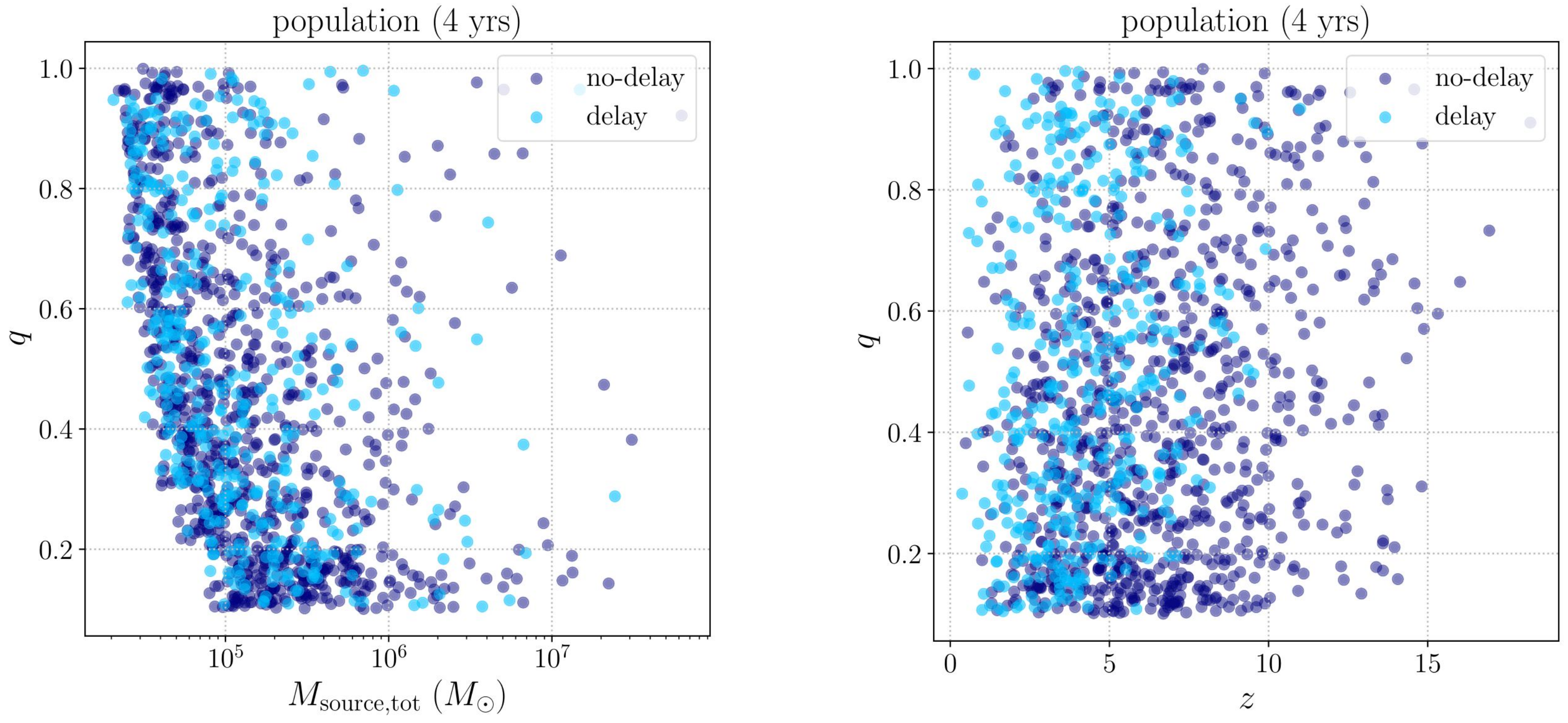
Population on a grid



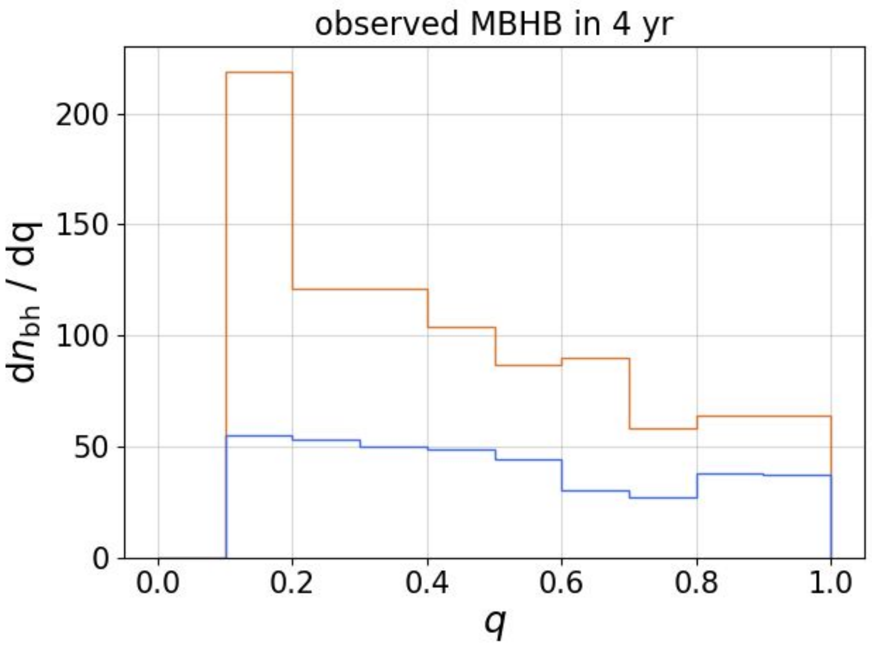
The mass-cut on a grid

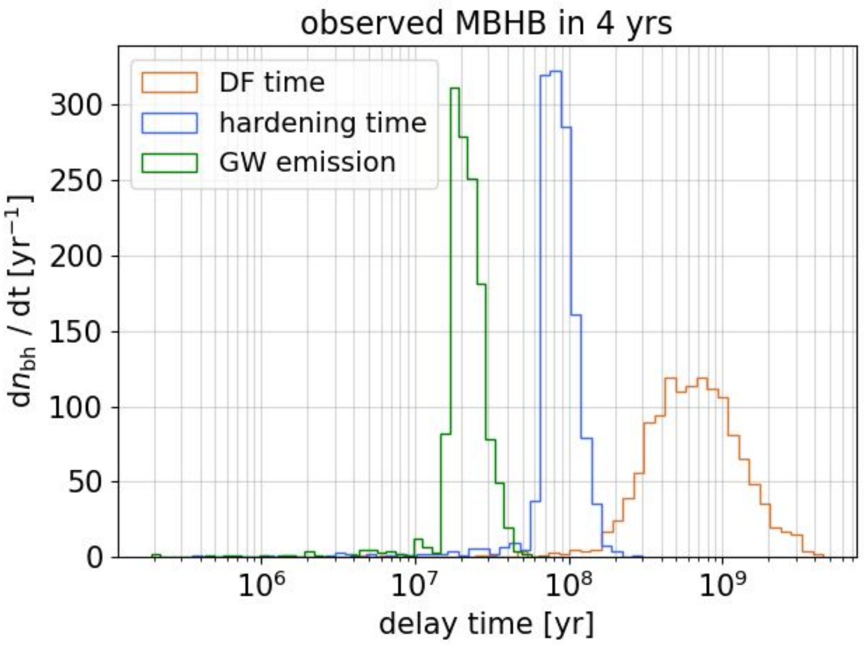


Scatter plots for *q*-*M* and *q*-*z*

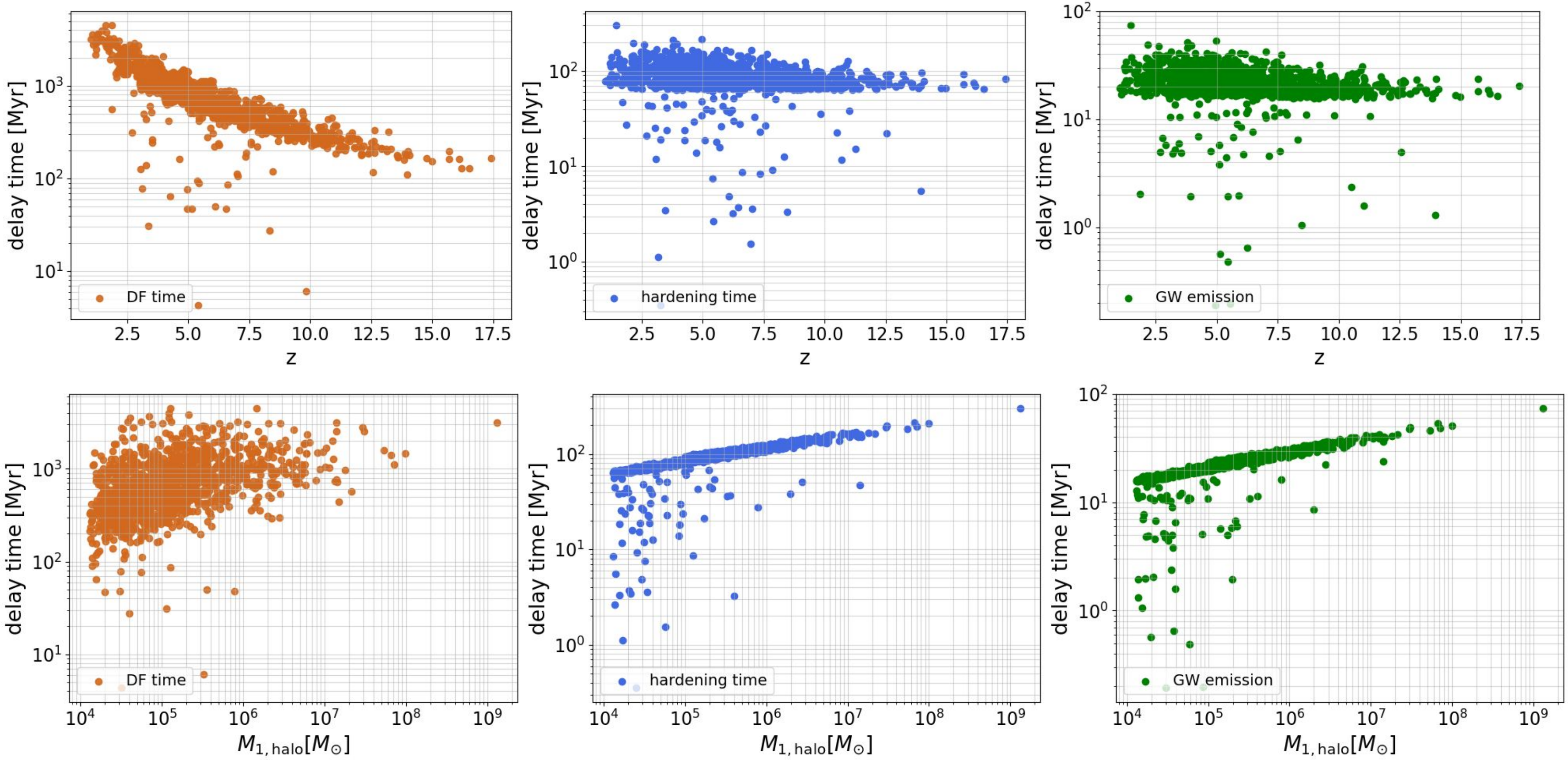


Histogram in q and time delays

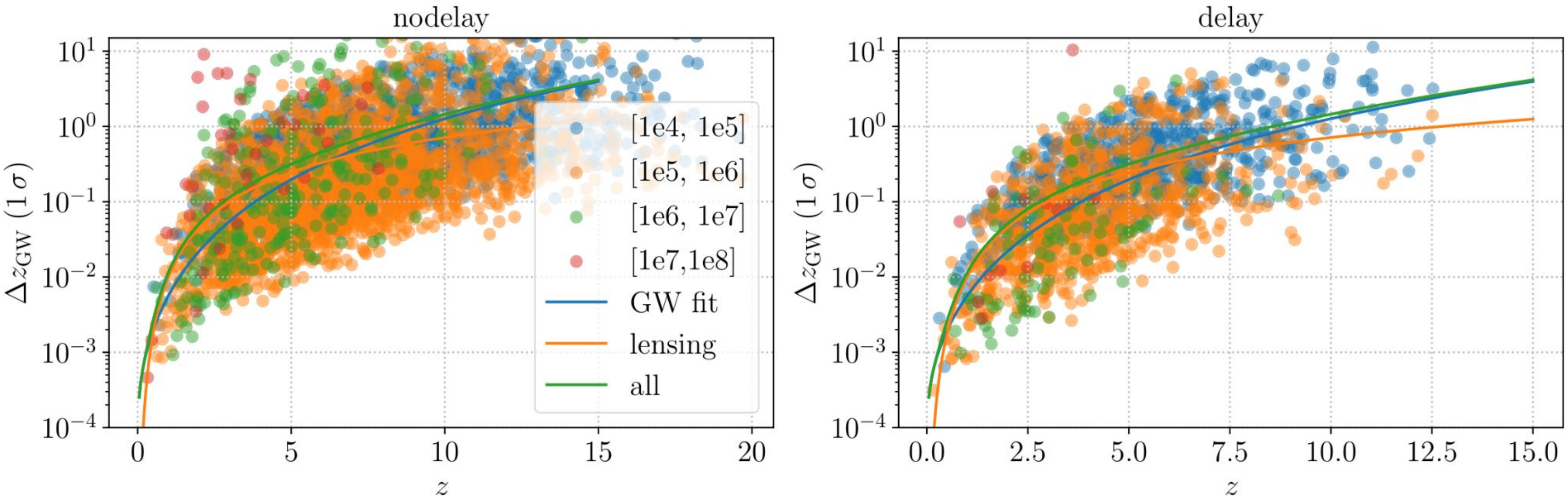




Dependency of time delays on M, q, z



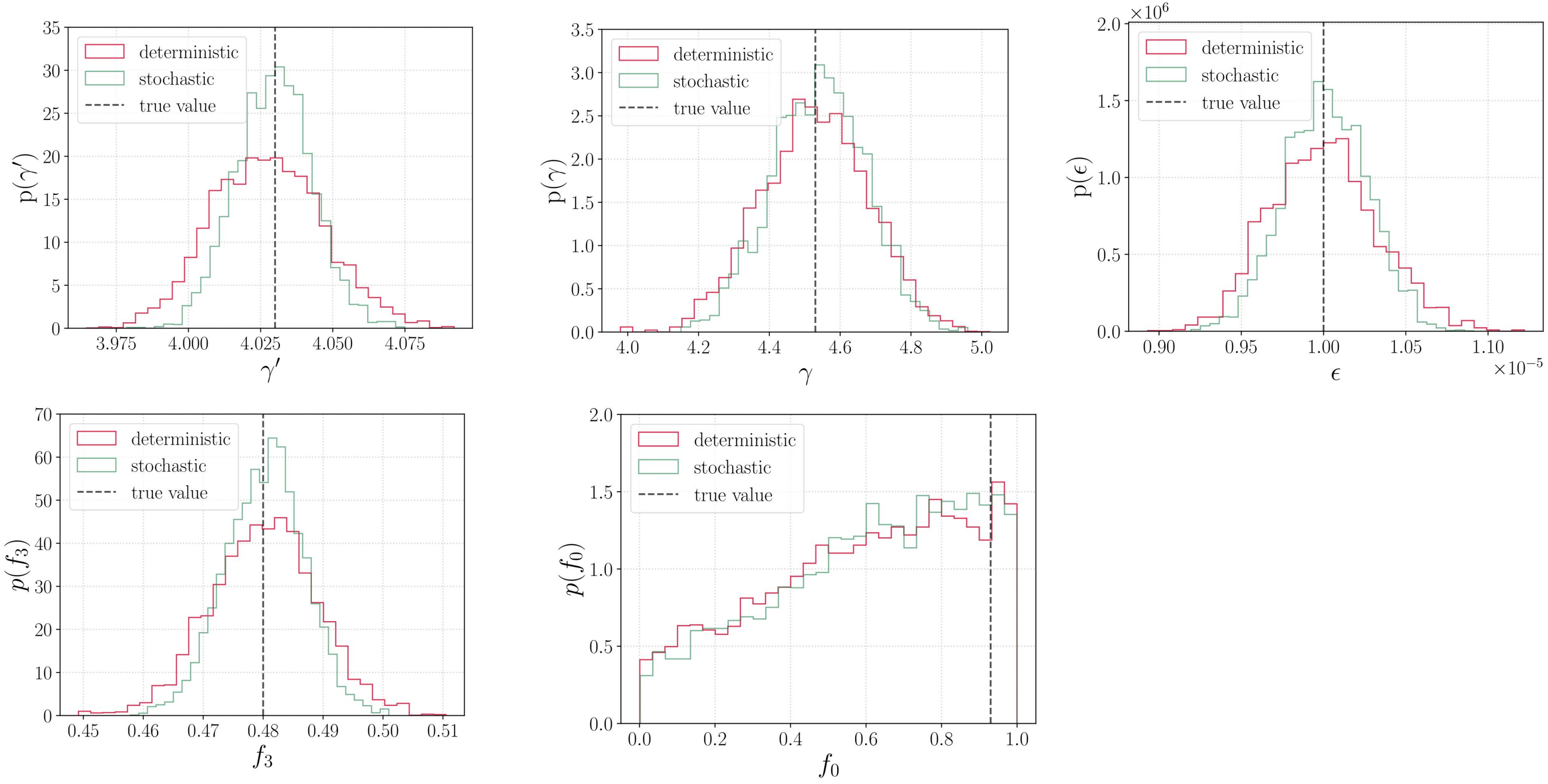
Intrinsic LISA measurement errors



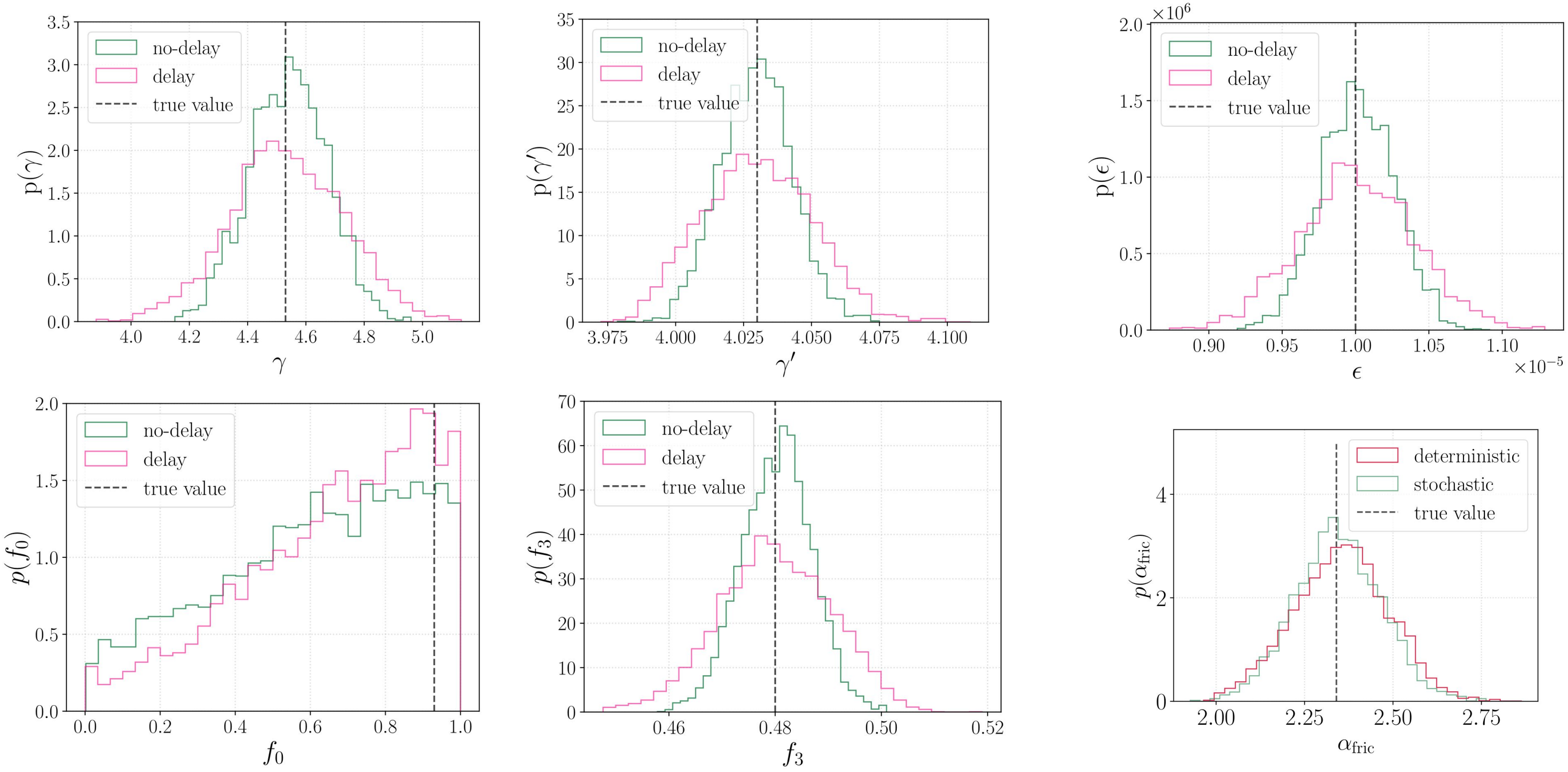
$$\sigma_{d_L}(z) = A \cdot z^{\alpha} \cdot (1+z)^{\beta},$$

$$\sigma_{\rm lensing}(z) = \frac{0.096}{2}\left(\frac{1-(1+z)^{-0.62}}{0.62}\right)^{2.36} \frac{\partial d_L}{\partial z}.$$

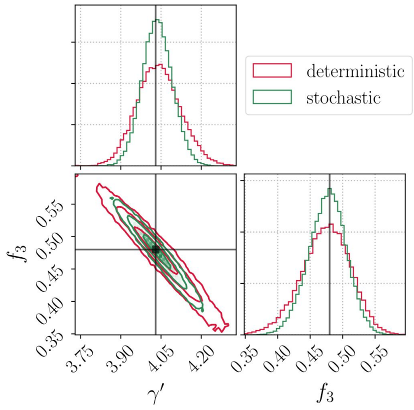
1D parameter estimation: no-delay

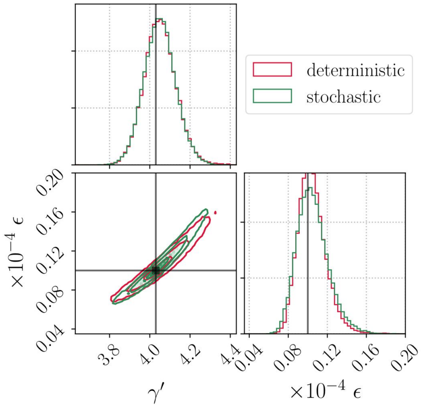


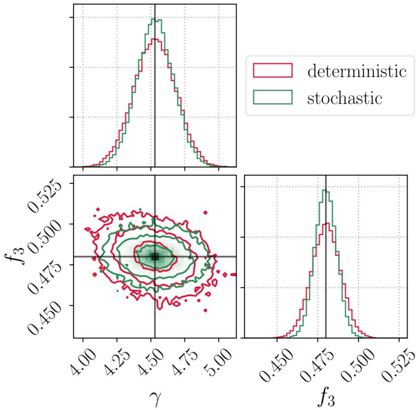
1D parameter estimation: delay VS no-delay

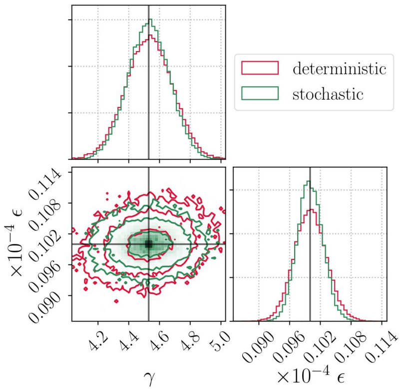


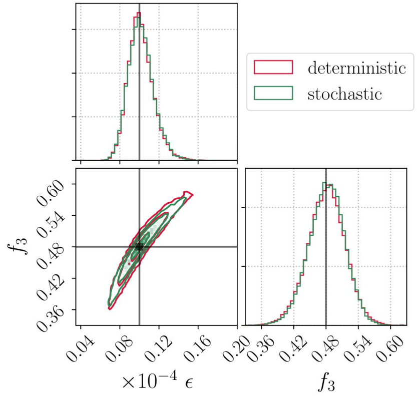
2D/3D posteriors: no-delay model

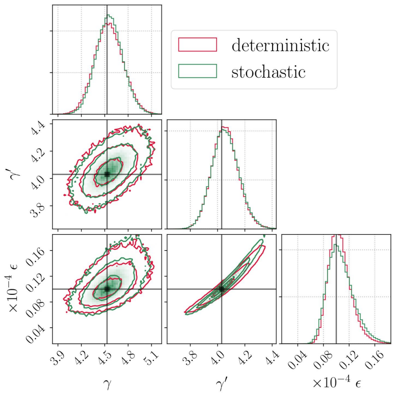




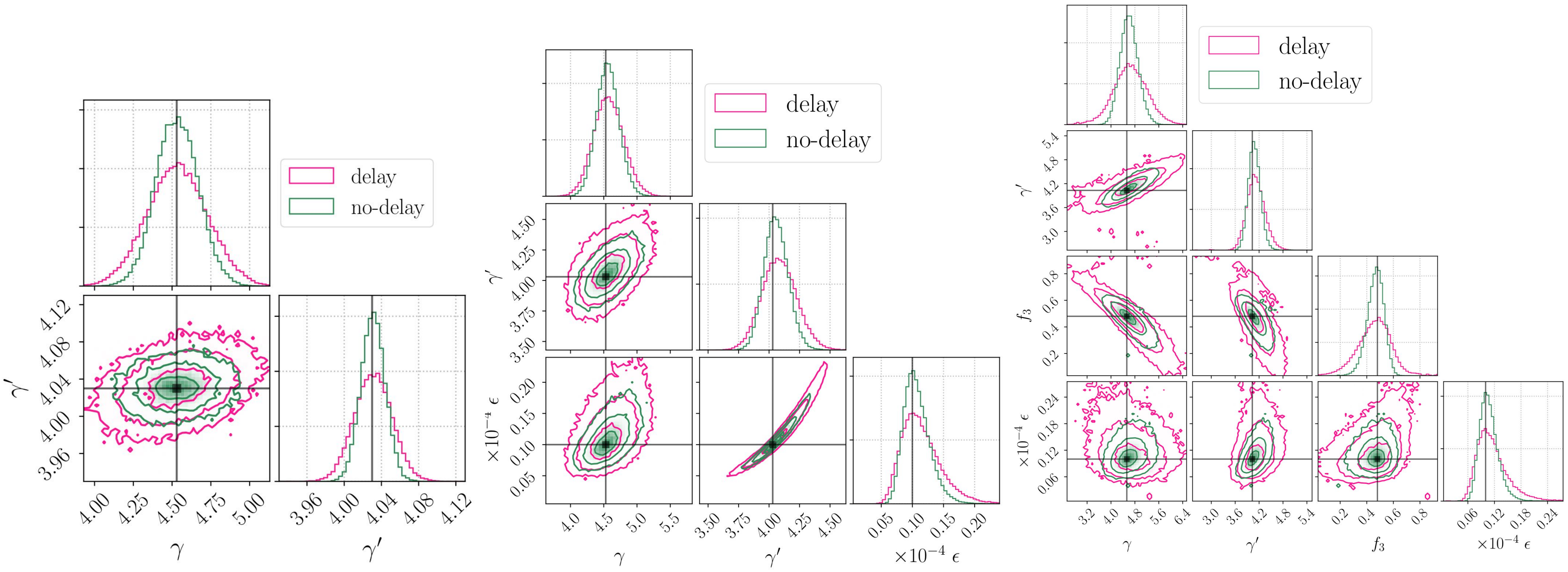




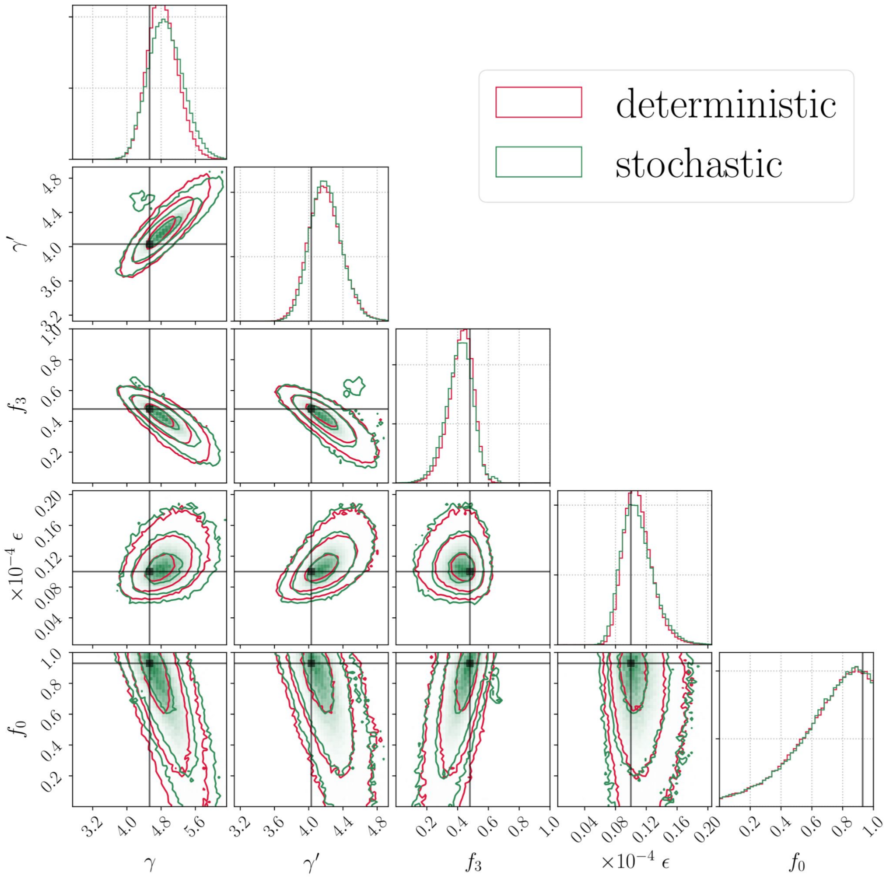




2D/3D/4D posteriors: delay VS no-delay

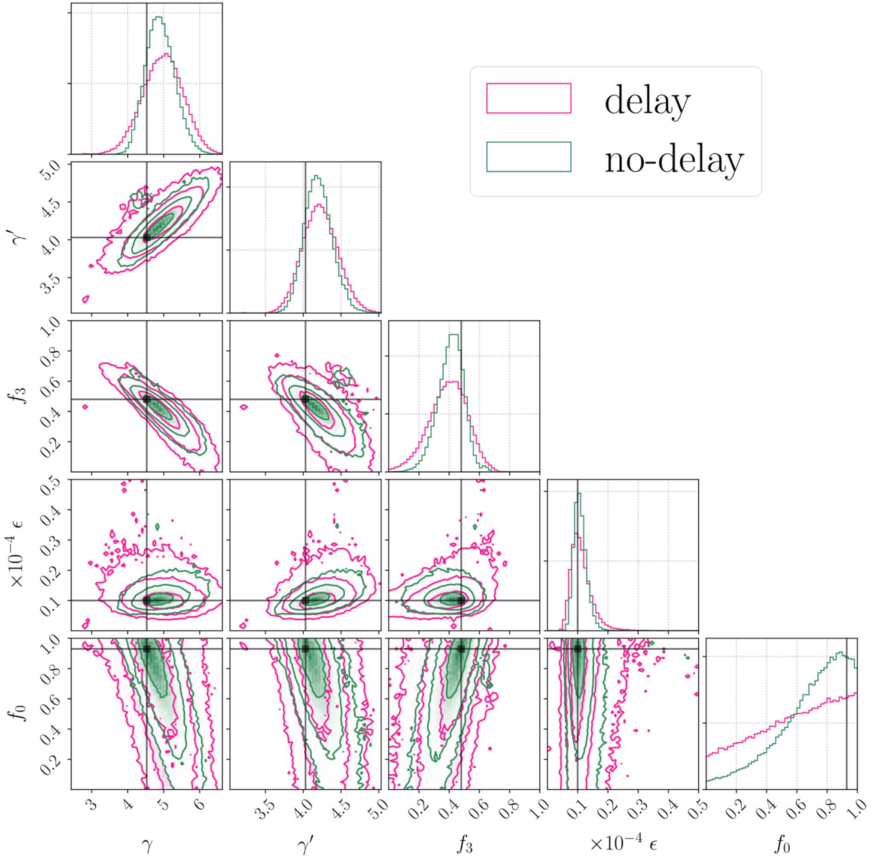


Alternative 5D case: degeneracy check



- No-delay model

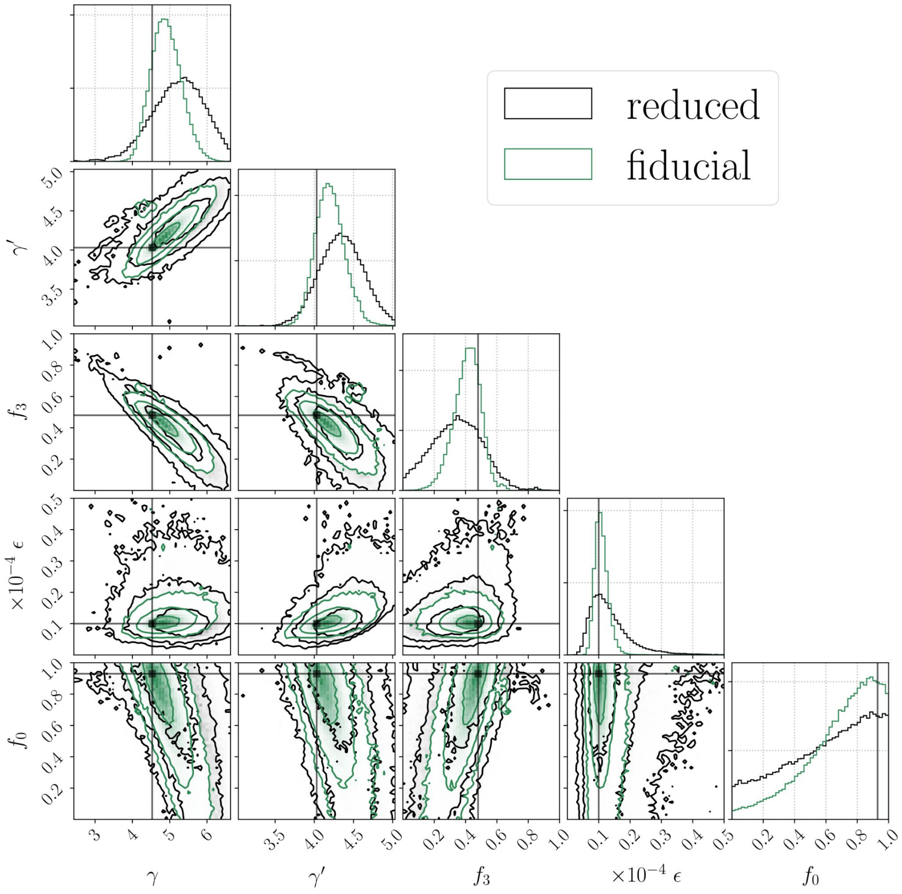
Alternative 5D case: degeneracy check



- Delay model
- Stochastic scaling relation

- Better constraints in all parameters than for 5D delay case with α_{fric}

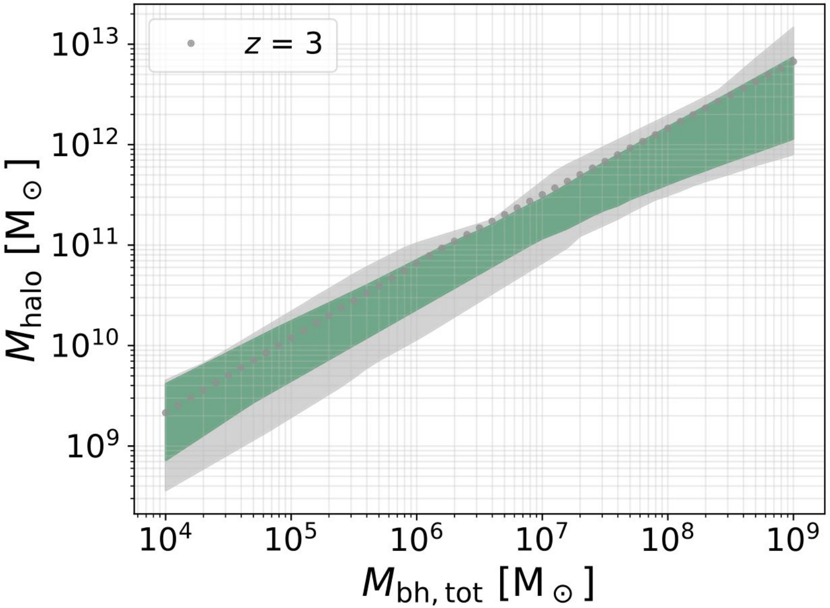
Alternative 5D case: degeneracy check

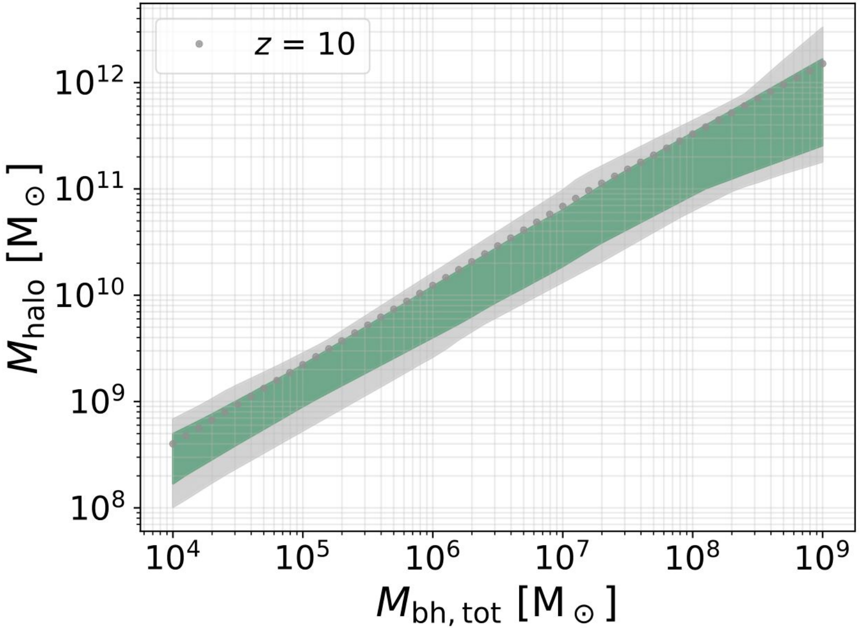


- Delay model
- Stochastic scaling relation

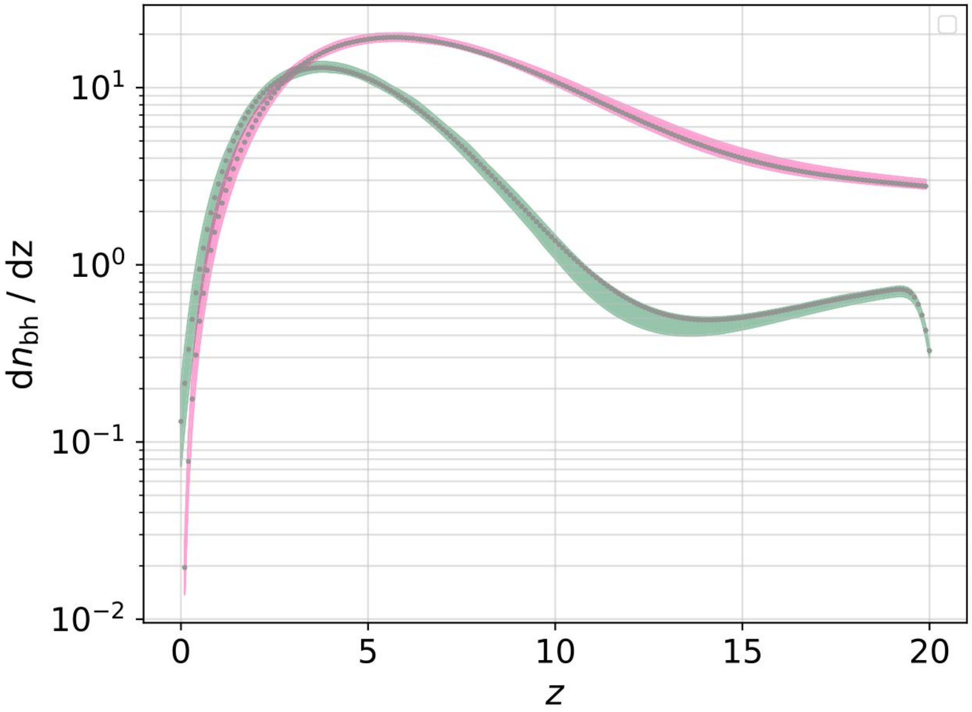
- No apparent degeneracy

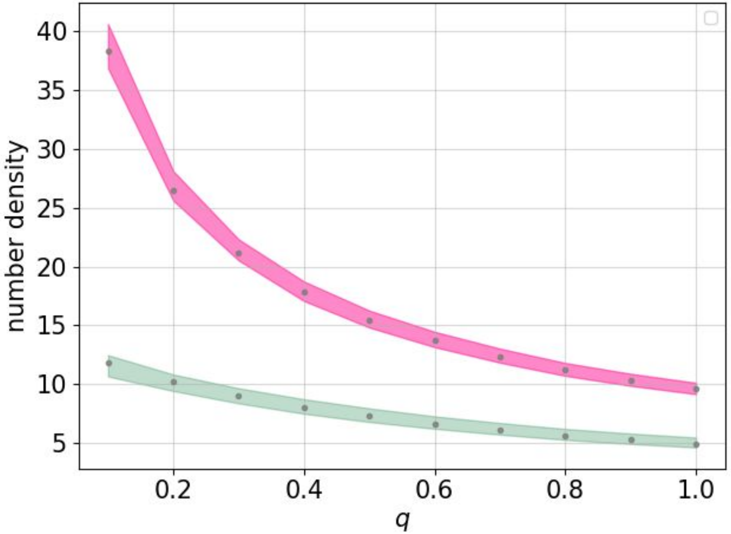
Scaling relation: reduced VS fiducial rates, high z



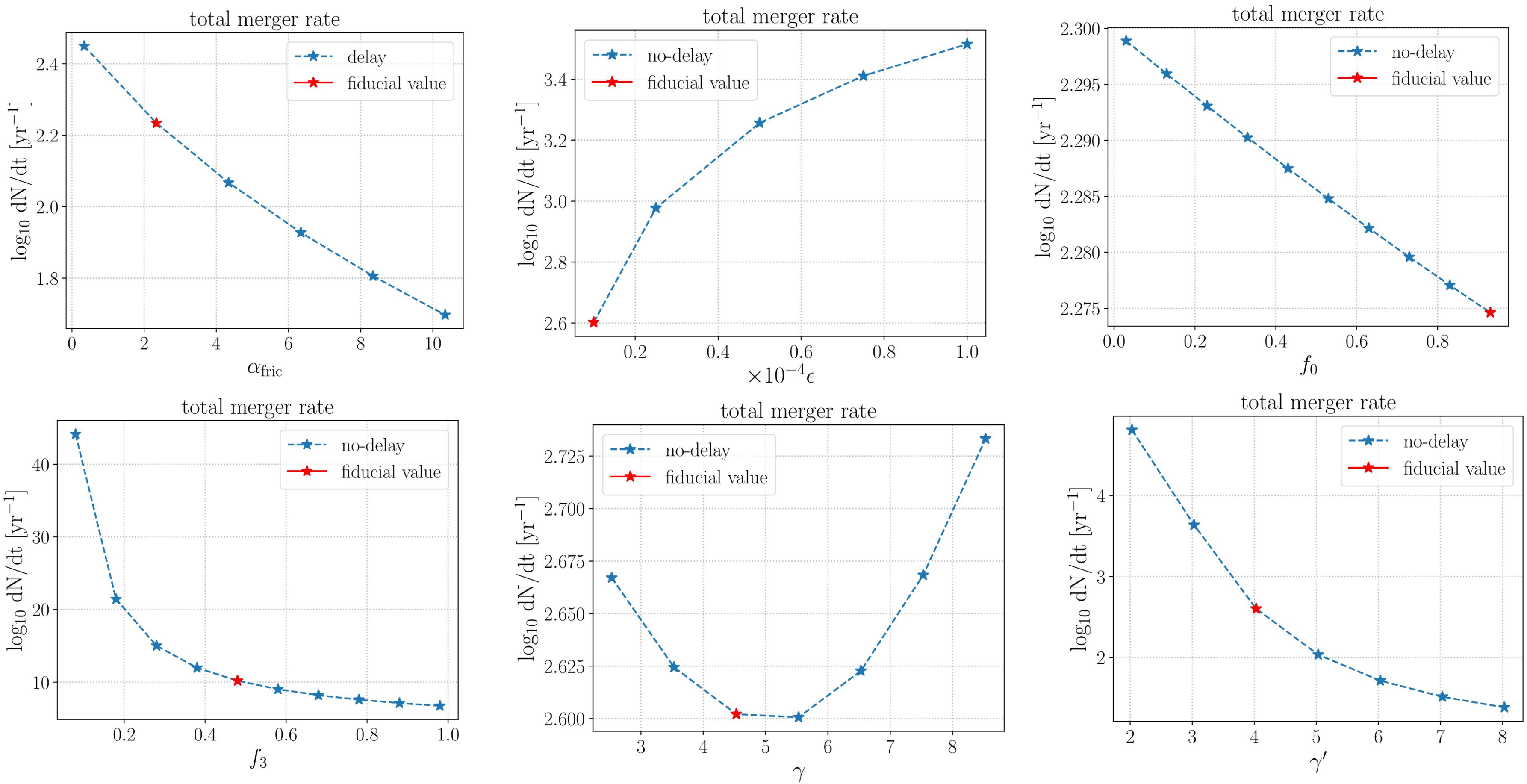


PPD in z log-scale and q

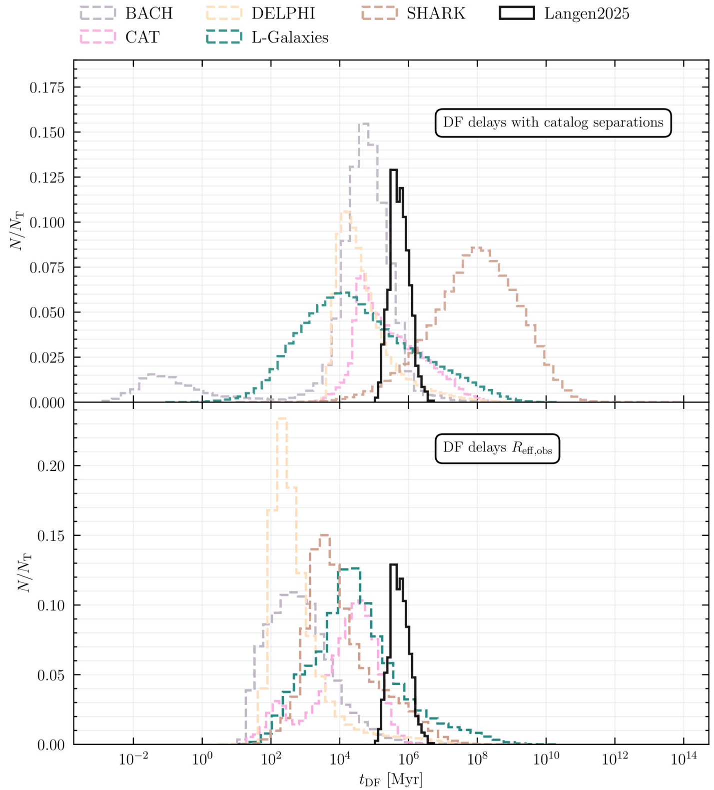


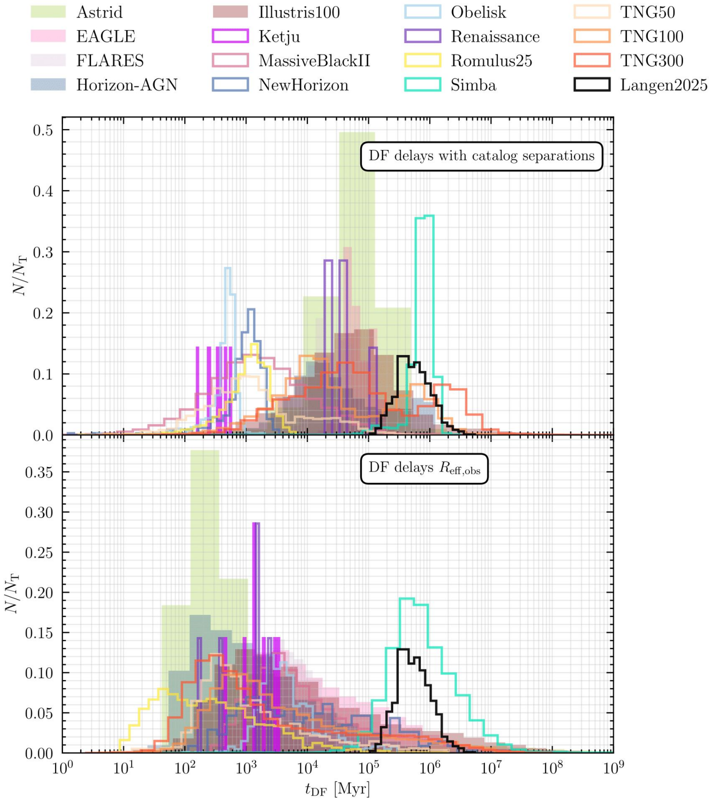


Parameter tests

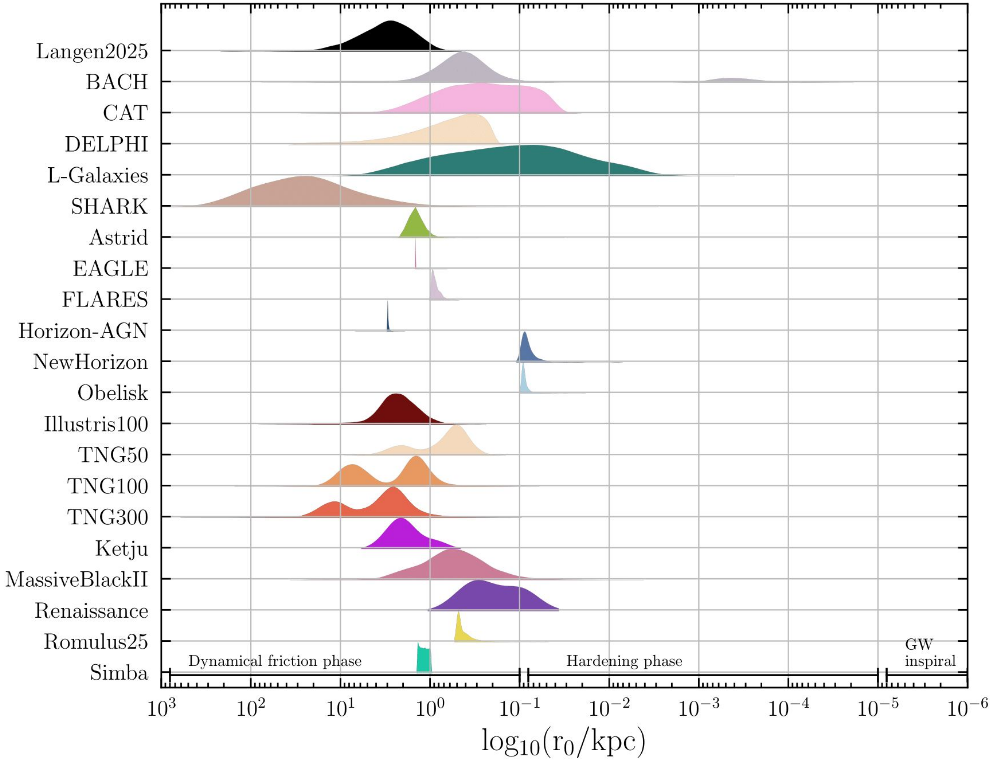


DF time delays in *MBHcat* project

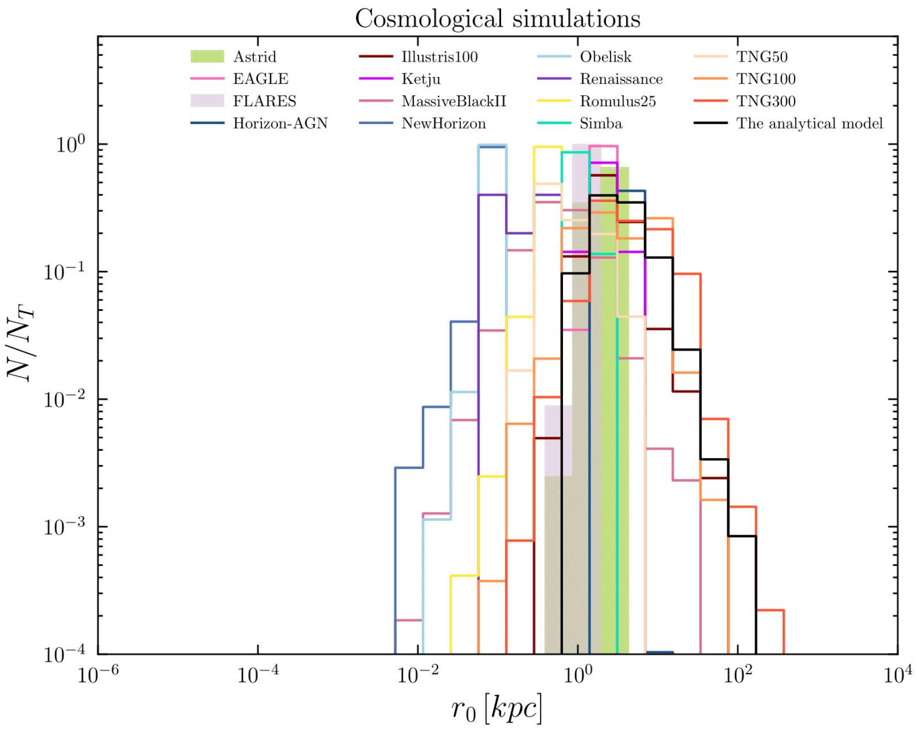


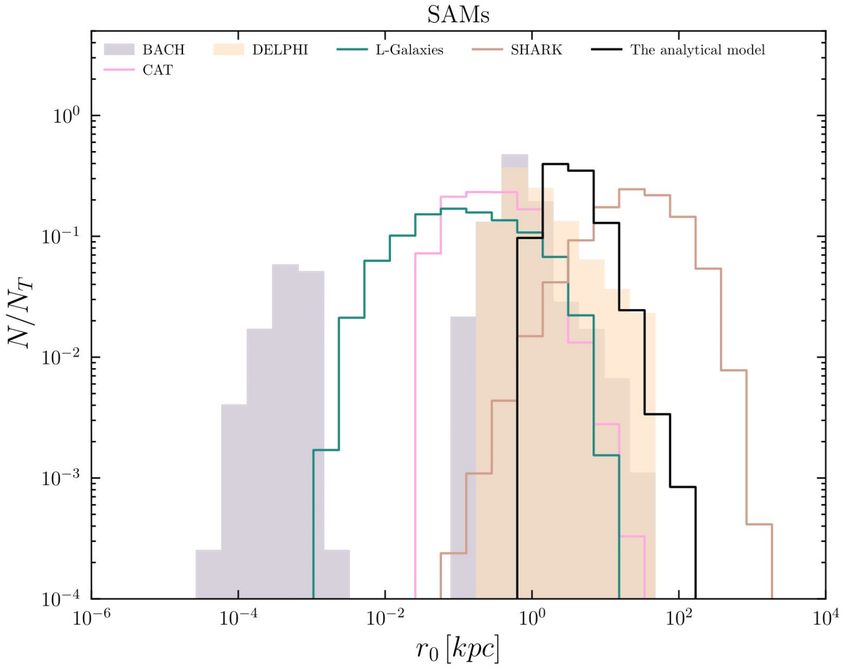


MBHcat project: resolutions and catalog separations

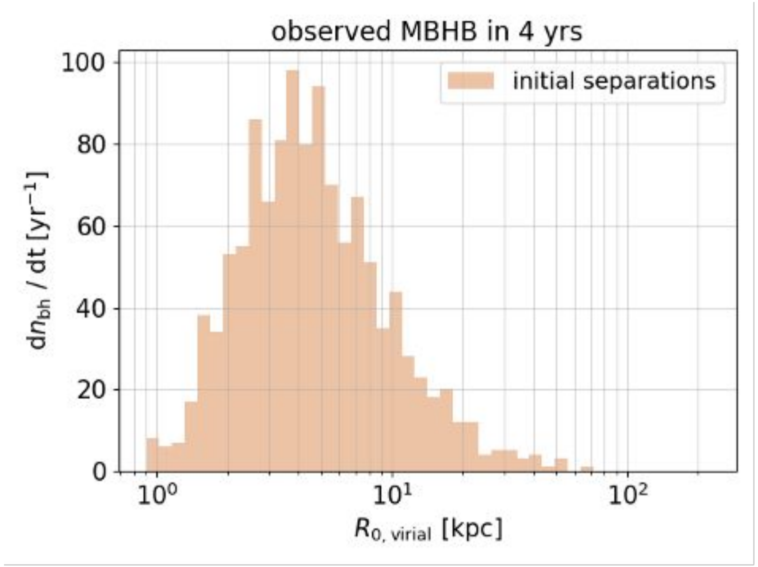


MBHcat project: resolutions and catalog separations (histogram)

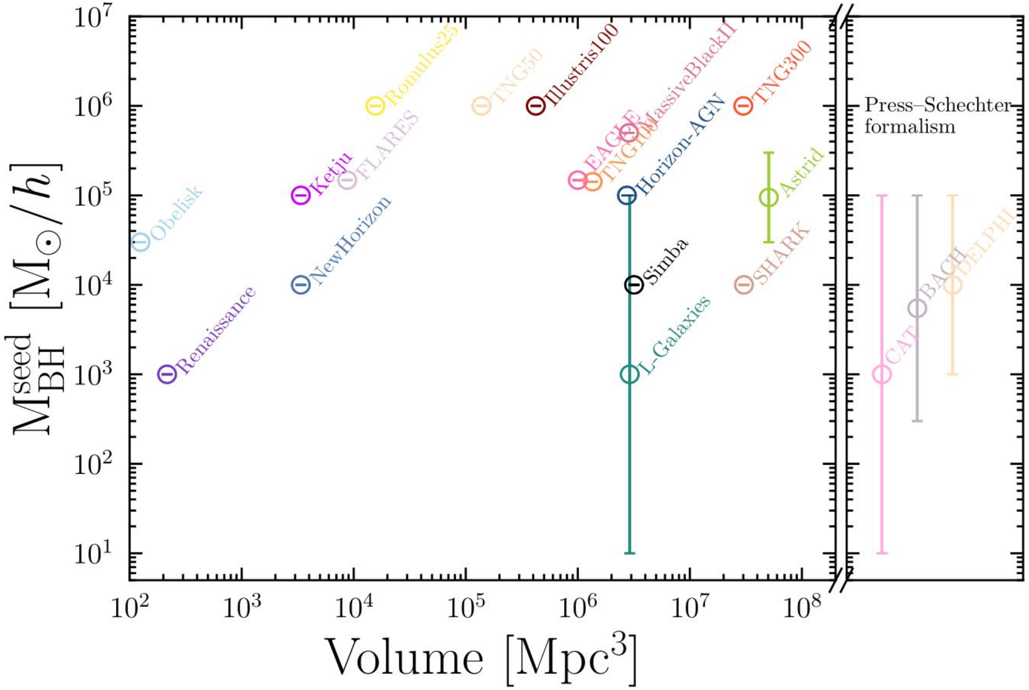


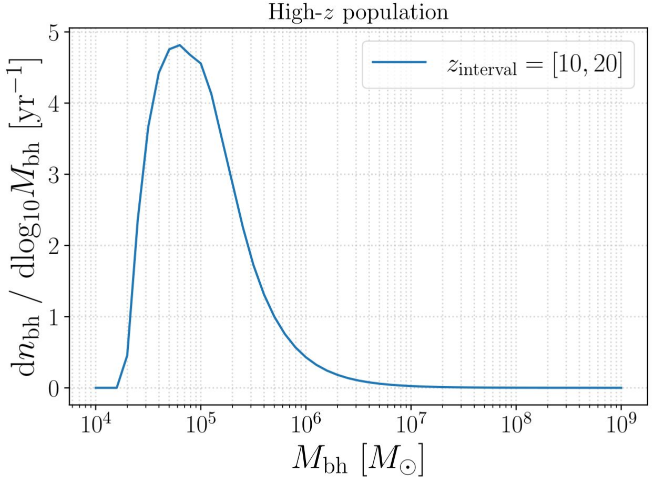


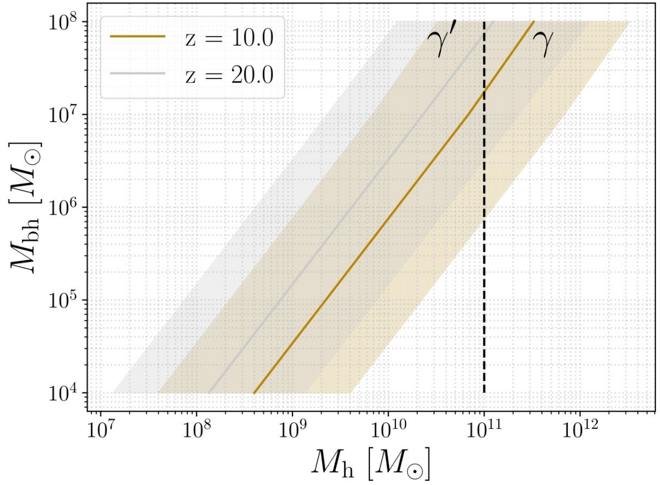
Initial separations for DF modelling



MBHcat project: volumes



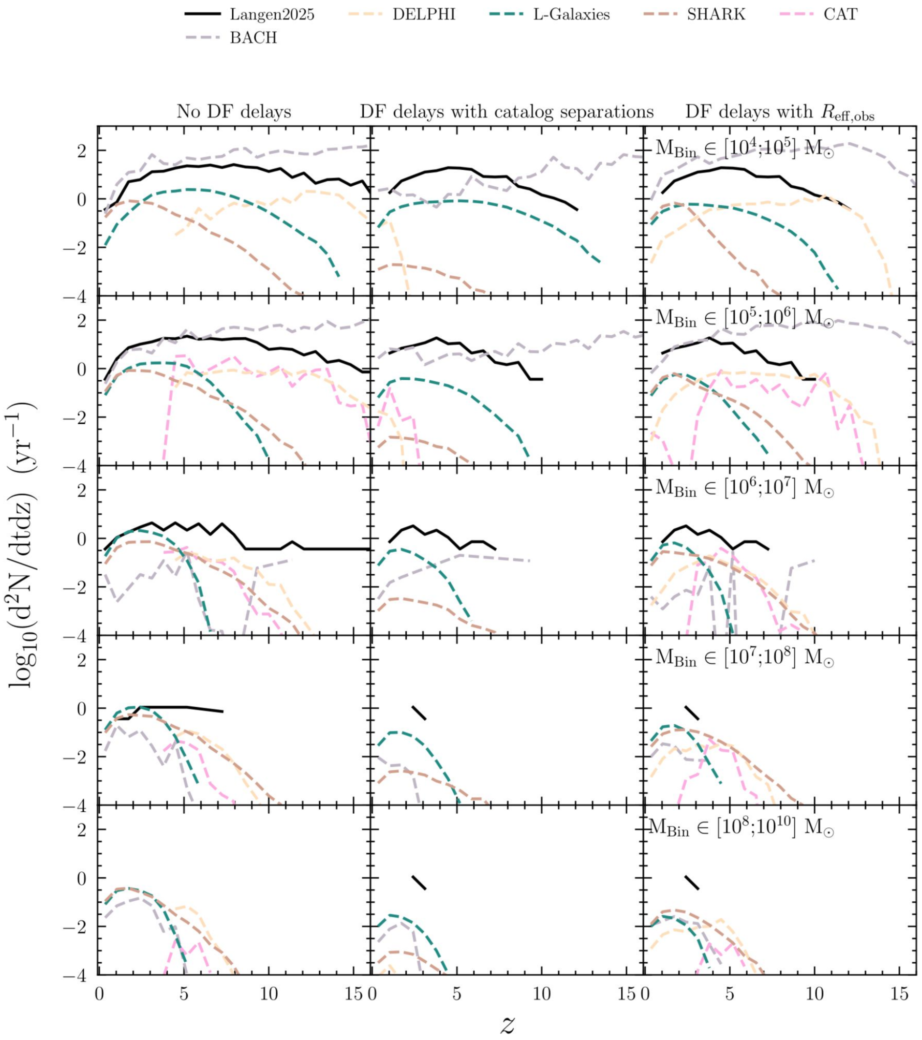




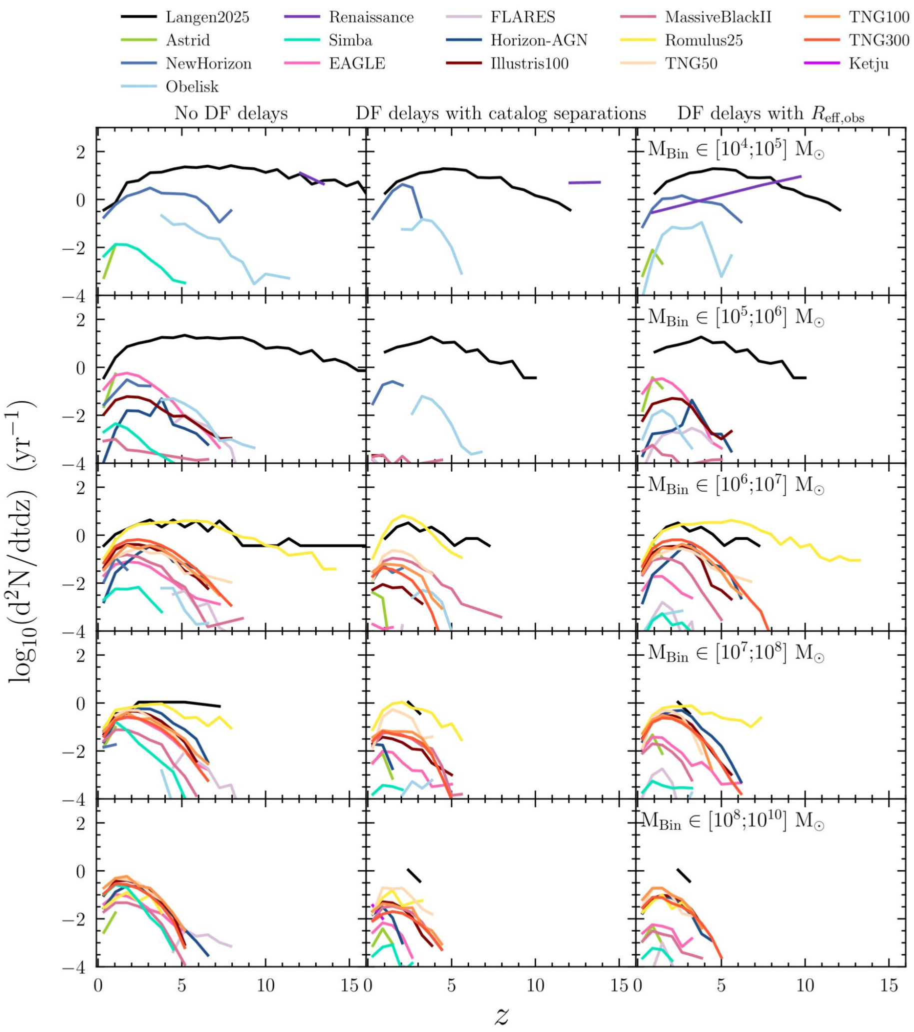
MBHcat project: absolute rates

	Total merger rate [yr^{-1}]		
	No-delay	DF delays with catalog sep.	DF delays with $R_{\mathrm{eff,obs}}$
	Cosmological simulations		
Astrid	0.991	0.0119	0.749
EAGLE	2.42	0.0257	0.7
FLARES	0.0437	0.000297	0.0112
Horizon-AGN	3.021	0.0476	2.0106
Illustris100	3.606	0.183	1.921
Ketju	0.0292	0.0245	0.00932
MassiveBlackII	0.6906	0.477	0.271
NewHorizon	10.721	6.639	4.3409
Obelisk	0.457	0.3508	0.2338
Renaissance	20.6109	5.955	11.4150
Romulus	26.468	15.2068	23.6387
Simba	0.694	0.00182	0.00276
TNG50	2.9313	1.735	1.9306
TNG100	3.2714	0.4258	2.0038
TNG300	3.359	0.2606	2.4882
	Semi-analytical models		
BACH	2373.587	568.969	2057.519
CAT	29.8195	0.1206	3.465
DELPHI	15.098	0.1472	11.6016
L-Galaxies	46.319	14.712	7.2424
SHARK	11.2608	0.0335	4.2379
	The analytical model		
Our model	379.5	160.5	-

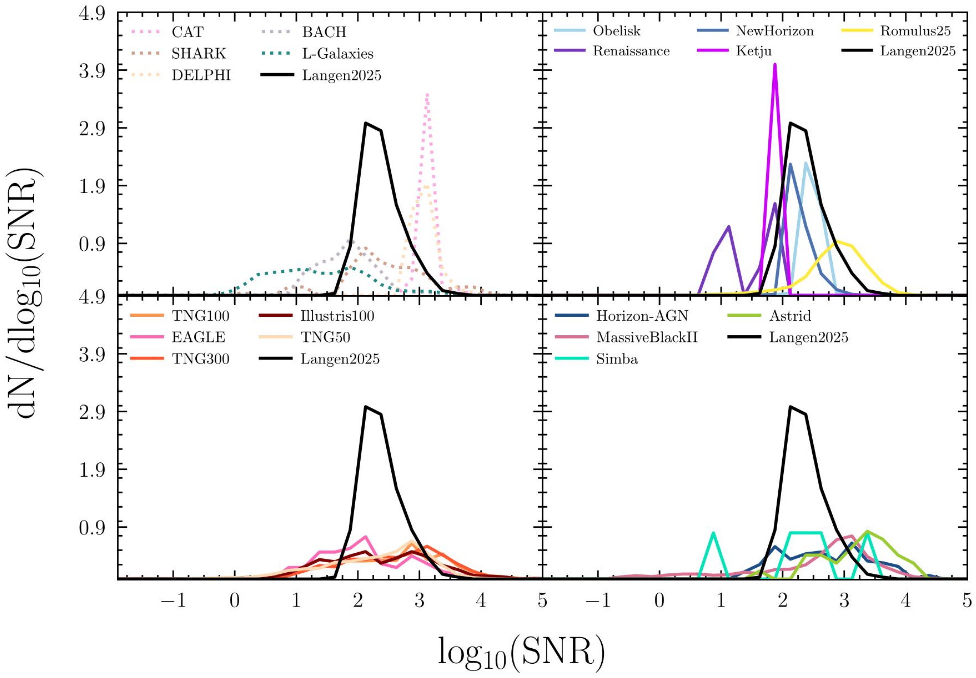
Merger rates for specific mass bins: SAMs



Merger rates for specific mass bins: numerical simulations

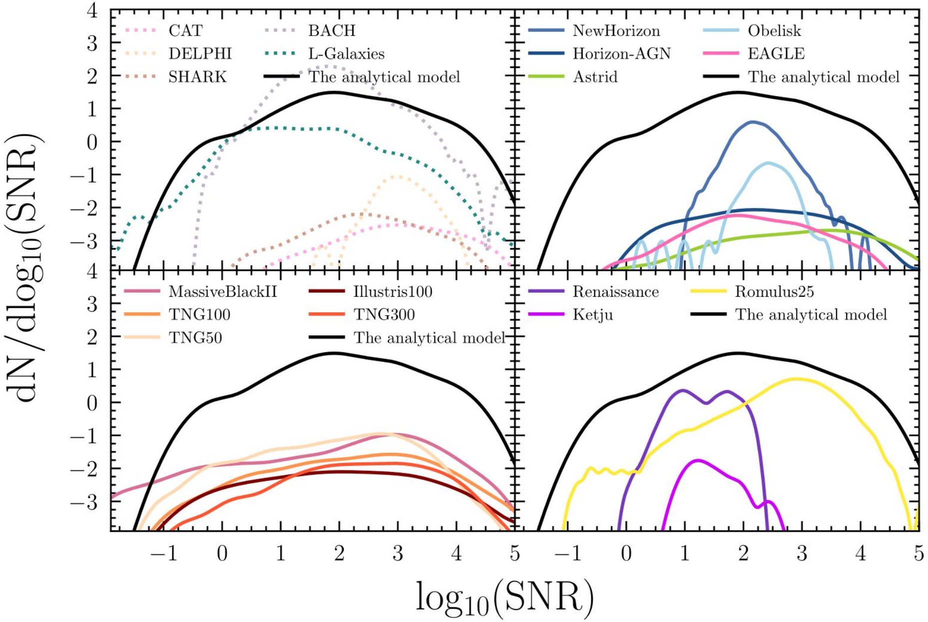


SNR distribution for all models

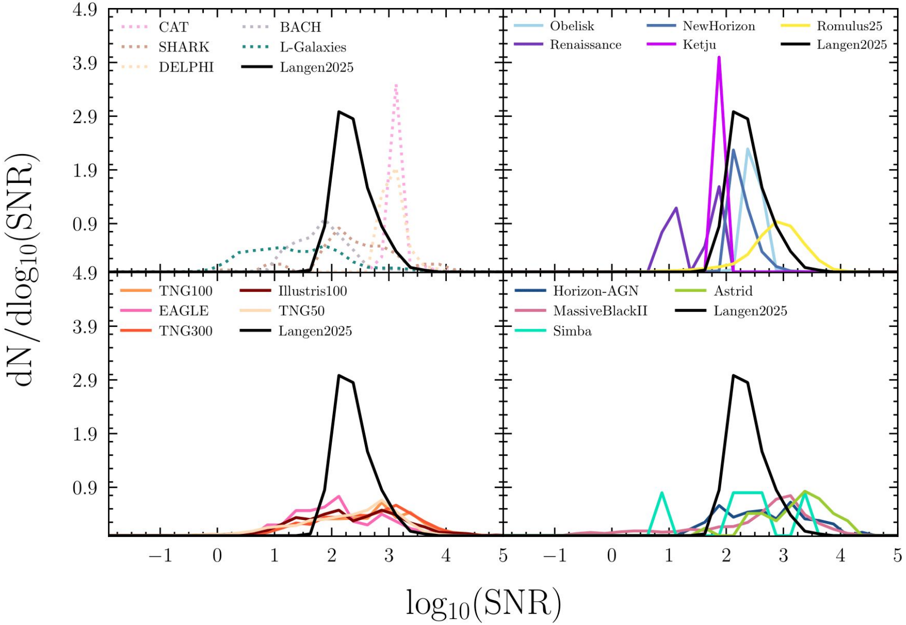


- models + DF delays using catalog separations
- no-smoothing

SNR distribution for all models



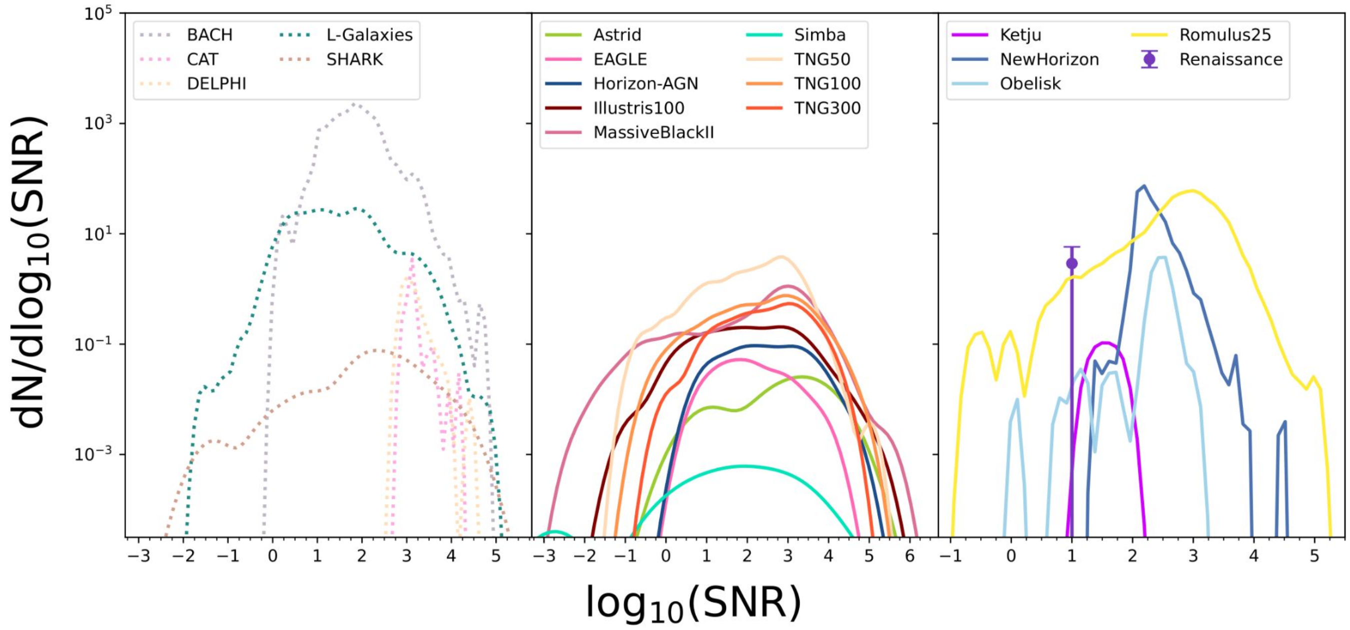
Smoothing / log-scale



No-smoothing

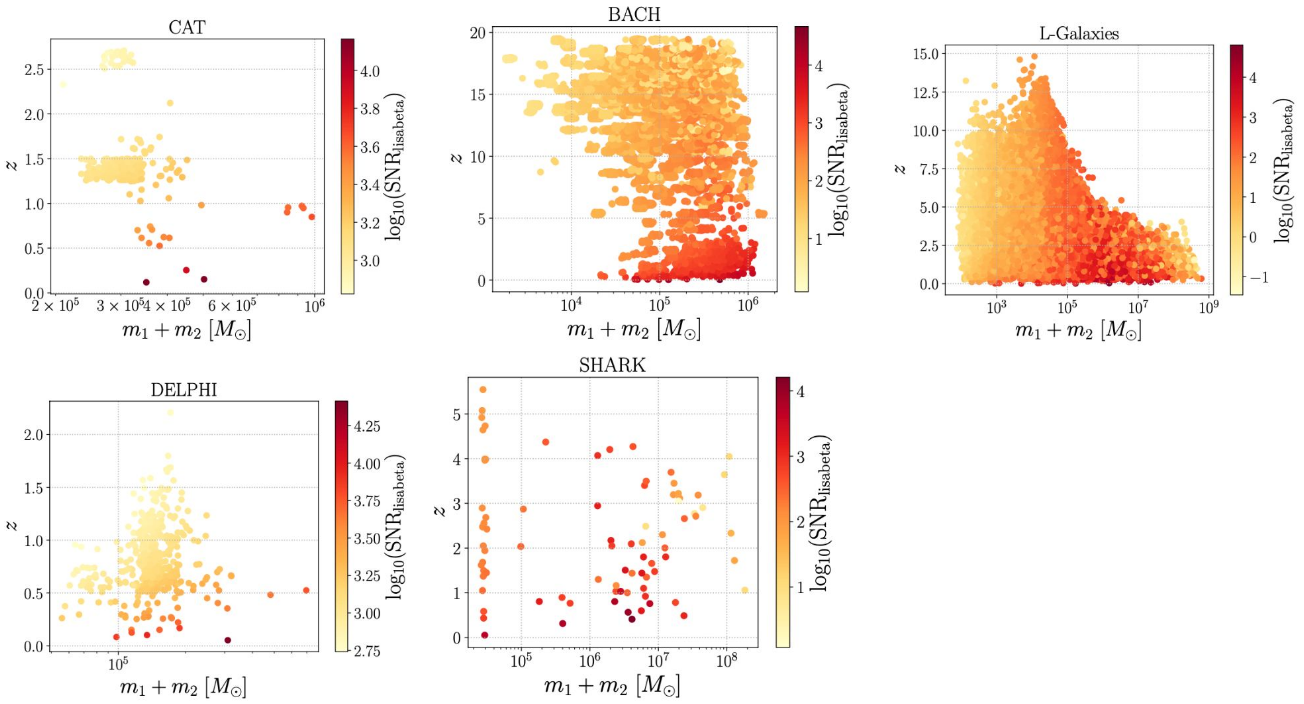
- models + DF delays using catalog separations

SNR distribution for all models (updated)

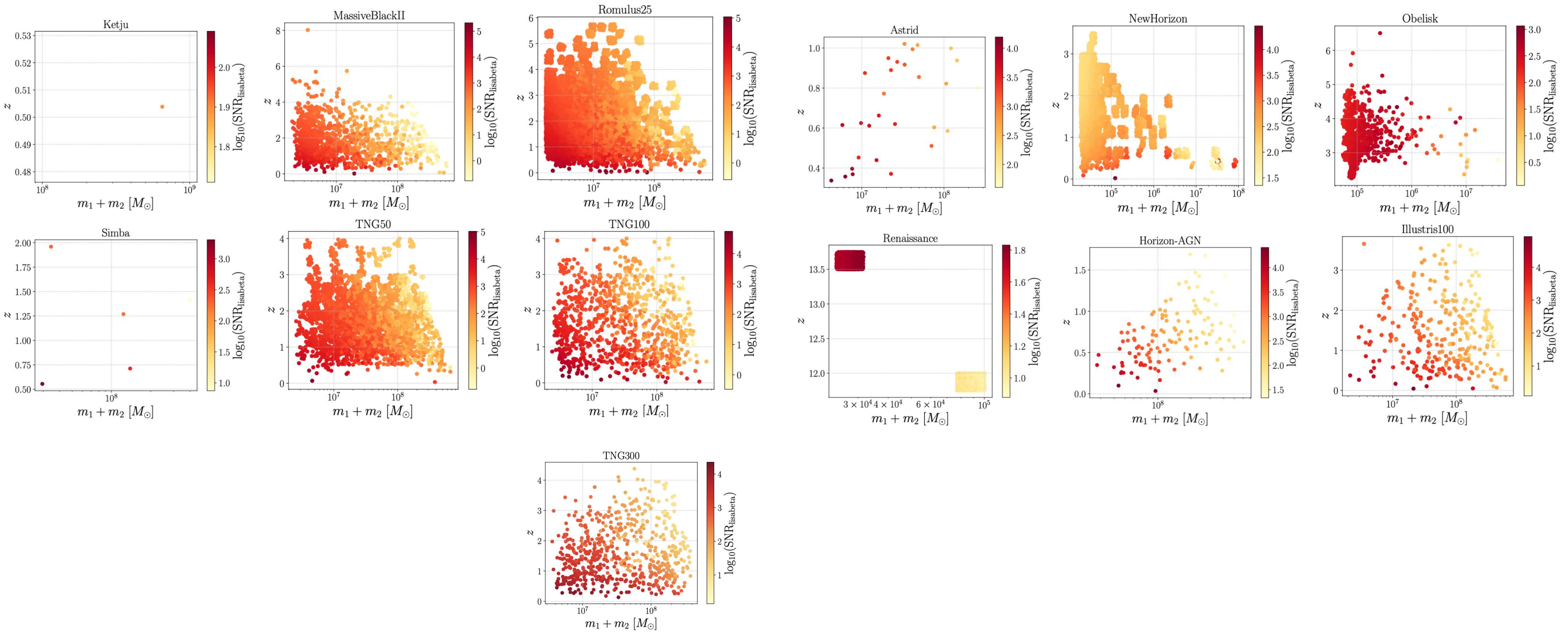


- models + DF delays using catalog separations

MBHcat scatter plots: SAMs (appendix)



MBHcat scatter plots: numerical simulations (appendix)



Additional paper material

POMPOCO LISA merger rate

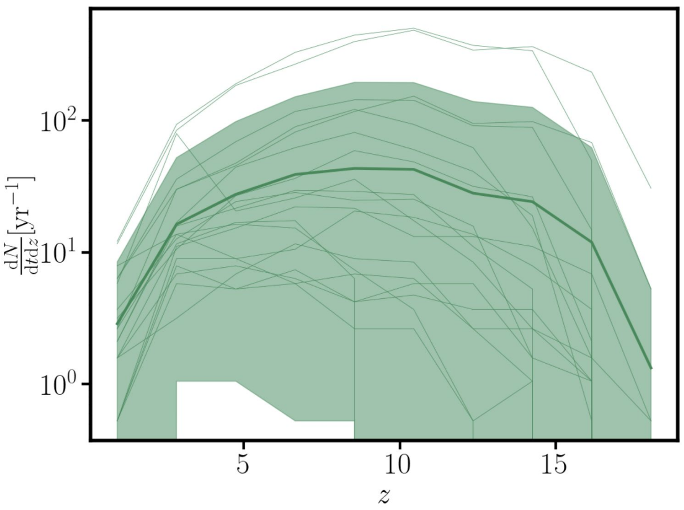
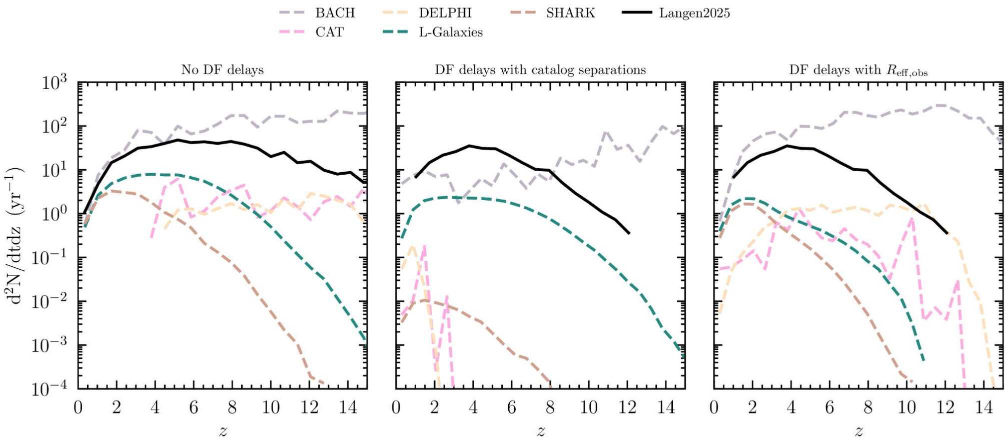


Fig. 8. Prediction of the rate of mergers detectable by LISA as a function of the redshift, assuming an SNR threshold of 8. See also Fig. 7.



Ref: *arXiv:2410.17916* (2024).

POMPOCO constraints on seeding and delays

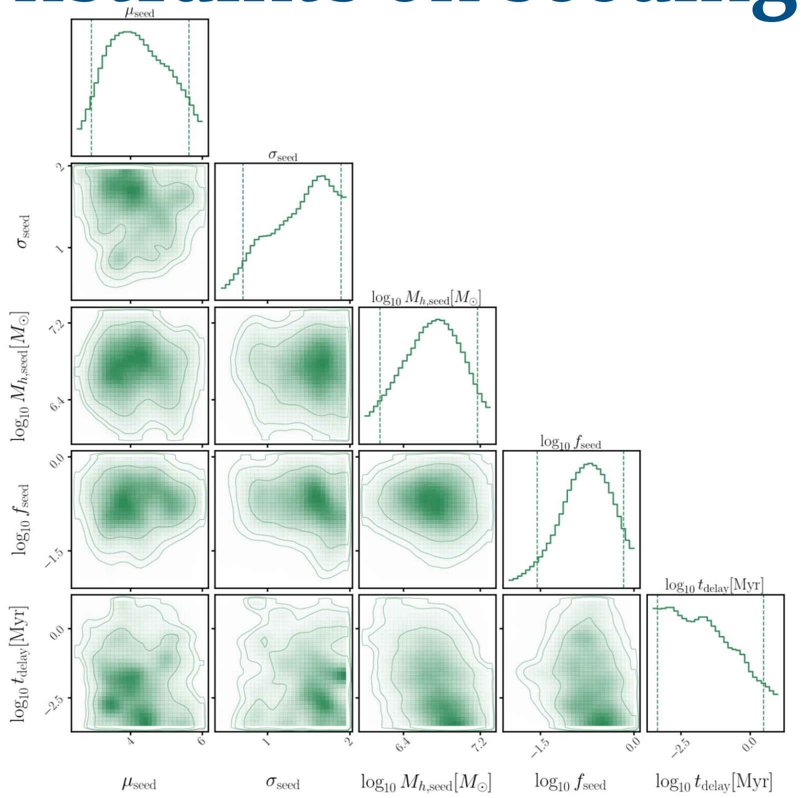


Fig. 4. Posterior on the model parameters related to seeding and BH mergers, when fitting for the LF and GW background. We show: the mean and standard deviation of the log-normal distribution of seed BH masses, $\mu_{\rm seed}$ and $\sigma_{\rm seed}$; the minimum mass of halos seeded $M_{h,\rm seed}$ and the seeding probability $f_{\rm seed}$; the delay of binary BH mergers (in addition to halo dynamical friction), $t_{\rm delay}$.

Ref: *arXiv:2410.17916* (2024).

Backup list

- Think of legit answer how to include spins and accretion in my model; possible combined? Aligned spins favor coherent accretion?
- Intuition why a_GW is in t_hard
- Backup slide for error sm matrices and error formulas
- Put all fucking thesis plots in the back up slides.
- Read abstract from each paper in model again
- Measure halo masses in observation
- Give values for relative uncertainty in mass and redshift.
- Read email conversation between massimo and S/N again
- Read the reports again; check comments