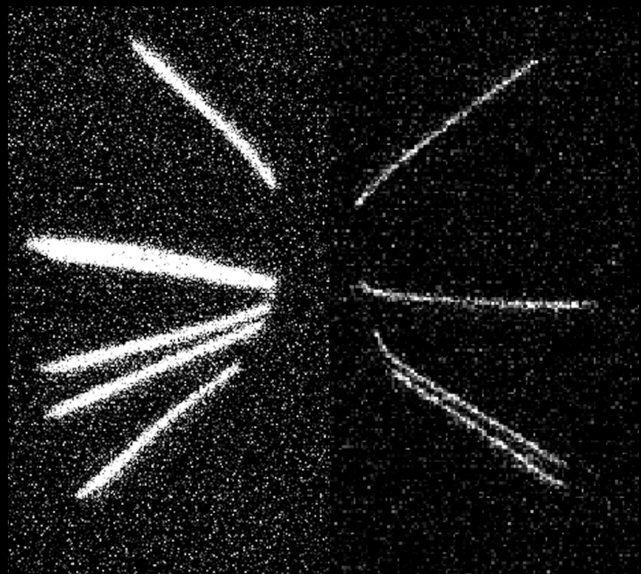
Elisabetta Baracchini

Gran Sasso Science Institute & Istituto Nazionale Fisica Nucleare

Negative ion drift operation:
a new window on high precision particle detection and tracking



Fernando Domingues Amaro [1], Elisabetta Baracchini [2,3], Luigi Benussi [4], Stefano Bianco [4], Cesidio Capoccia [4], Michele Caponero [4,5], Danilo Santos Cardoso [6], Gianluca Cavoto [7,8], André Cortez [2,3], Igor Abritta Costa [9], Rita Joanna da Cruz Roque [1], Emiliano Dané [4], Giorgio Dho [2,3], Flaminia Di Giambattista [2,3], Emanuele Di Marco [7], Giovanni Grilli di Cortona [4], Giulia D'Imperio [7], Francesco Iacoangeli [7], Herman Pessoa Lima Júnior [6], Guilherme Sebastiao Pinheiro Lopes [9], Amaro da Silva Lopes Júnior [9], Giovanni Maccarrone [4], Rui Daniel Passos Mano [1], Michela Marafini [10], Robert Renz Marcelo Gregorio [11], David José Gaspar Marques [2,3], Giovanni Mazzitelli [4], Alasdair Gregor McLean [11], Andrea Messina [7,8], Cristina Maria Bernardes Monteiro [1], Rafael Antunes Nobrega [9], Igor Fonseca Pains [9], Emiliano Paoletti [4], Luciano Passamonti [4], Sandro Pelosi [7], Fabrizio Petrucci [12,13], Stefano Piacentini [7,8], Davide Piccolo [4], Daniele Pierluigi [4], Davide Pinci [7,*], Atul Prajapati [2,3], Francesco Renga [7], Filippo Rosatelli [4], Alessandro Russo [4], Joaquim Marques Ferreira dos Santos [1], Giovanna Saviano [4,14], Neil John Curwen Spooner [11], Roberto Tesauro [4], Sandro Tomassini [4] and Samuele Torelli [2,3]

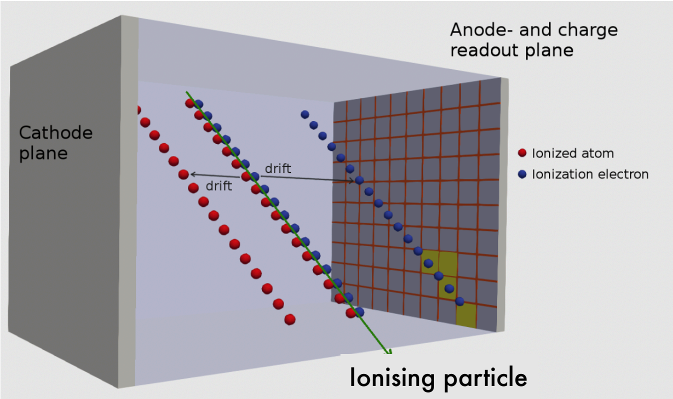
Bi-national Conference on Detector R&D - LPNHE Paris - 20th November 2025

This project has received fundings under the European Union's Horizon 2020 research and innovation programme from the European Research Council (ERC) grant agreement No 818744

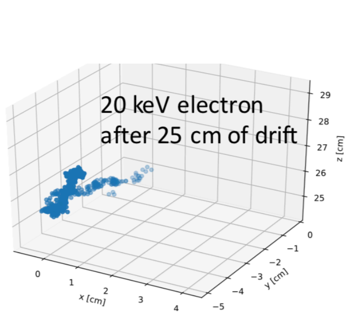
Outline

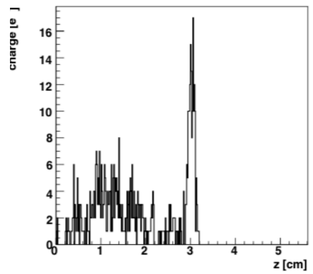
- Time Projection Chambers advantages and the limitations
- The CYGNO high precision optical 3D TPC experimental approach
- Negative ion drift operation with optical sCMOS + PMT readout
  - Demonstration of feasibility at LNGS atmospheric pressure (900 mbar)
  - Observation of minority carriers and negative ions mobilities evaluation at 650 mbar
- Spoiler on diffusion measurements ;)
- Conclusions & outlooks

Time Projection Chamber advantages & drawbacks

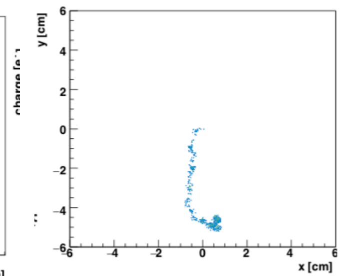


Depending on the anode segmentation (x-y) and time sampling (z), tracks can be reconstructed in 1D, 2D or 3D

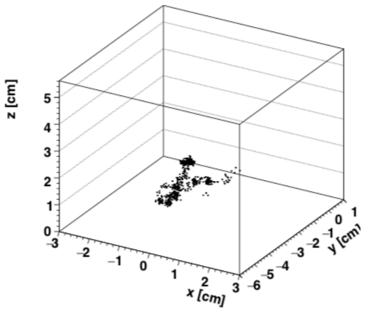




1D GEM



2D optical



3D pixels

Advantages:

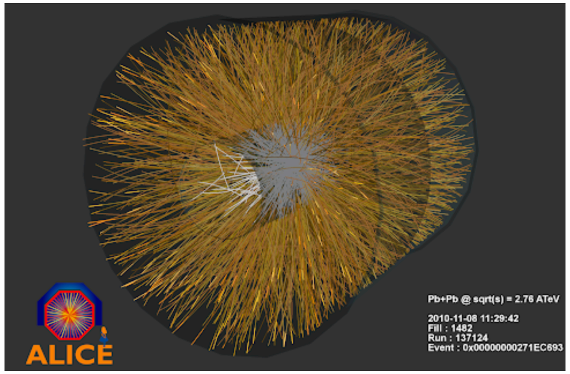
- Reconstruct track topology, i.e. "imaging"
- Measure track sense and direction through energy loss profile
- Infer particle identity through energy loss characteristics
- Gaseous TPCs down to O(keV), liquid TPCs down to O(MeV)

Drawbacks:

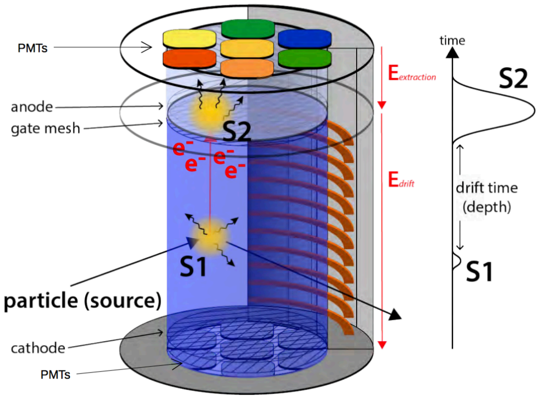
- TPCs need a "proxy" to define the absolute coordinate along the drift direction
  - Either a t_0 from another detector or a way to extract from data
- **All TPCs advantages result spoiled by diffusion during drift**

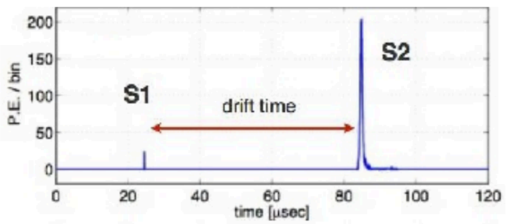
Classical TPC “fiducialization” techniques (i.e. absolute coordinate along drift direction)

By using t_0 from some other detector/beam (i.e. at collider/ beam experiment)



By exploiting primary scintillation (noble liquid TPCs)

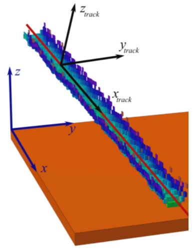


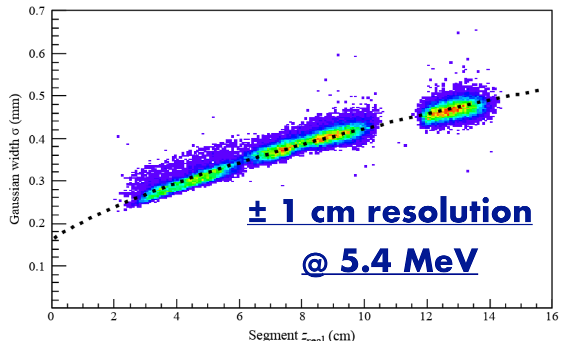


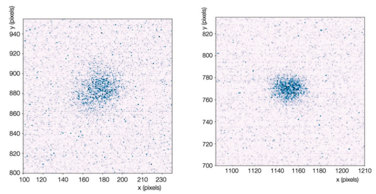
By fitting for the diffusion (gaseous high granularity TPCs)

$$\sigma = \sqrt{\sigma_0^2 + D_T^2 \cdot z}$$

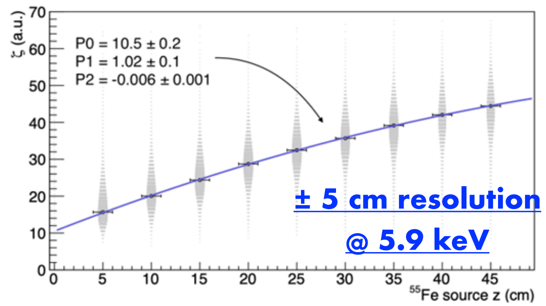
P. Lewis et al, Nucl.Instrum.Meth. A 789 (2015) 81-85





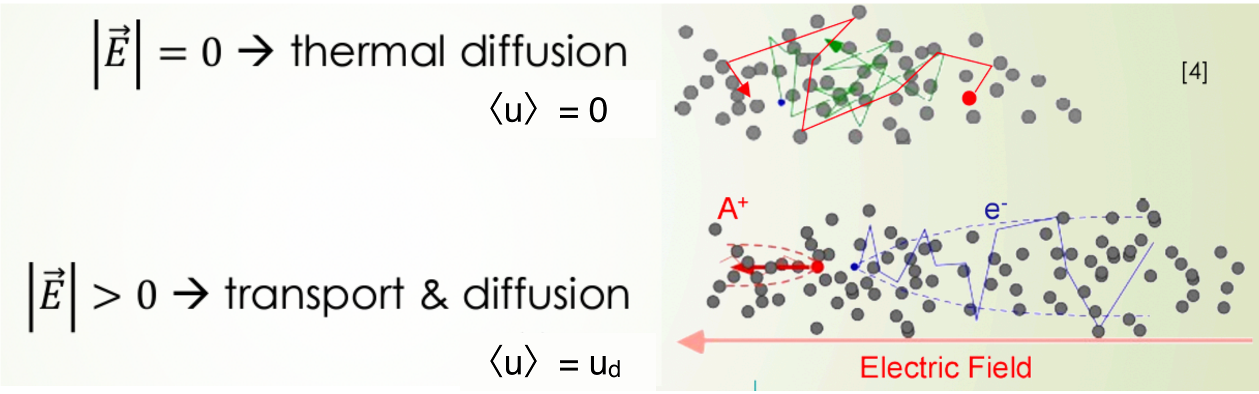


F. Amaro et al. *Eur.Phys.J.C* 83 (2023) 10, 946



Transport and diffusion during drift in TPCs

- Electrons and ions drifting through a gas under the influence of an *E* field are scattered on the gas molecules so that their direction of motion is randomised in each collision
- On average, they assume a constant drift velocity *u* in the direction of the *E* field, which is much smaller than the instantaneous velocity *c* between collisions
  - Electrons are accelerated very rapidly by the *E* field, and they lose very little energy when colliding elastically with the gas atoms due to their small mass. Electrons reach random energies far in excess of the energy of the thermal motion
  - Ions are accelerated much slower than electrons by the same *E* field due to their larger mass. They lose a large fraction of it in collisions, since their mass is comparable to the gas atoms and therefore ions random energy is mostly thermal



$$\sigma^2_{tot} = 2Dt$$

$$\frac{1}{\tau} = N\sigma c \quad l_0 = c\tau \quad D = \frac{c^2\tau}{3} = \frac{2}{3}\frac{\varepsilon}{m}\tau.$$

$$\Rightarrow \quad \epsilon_{\min} = \frac{3}{2}kT \quad \sigma^2_{therm} = \frac{2kTL}{eE}$$

How to minimise electrons diffusion during drift

By using polyatomic gases

$$\frac{1}{\tau} = N\sigma c \qquad \frac{1}{2}mc^2 = \varepsilon = \varepsilon_E + \frac{3}{2}kT \qquad u = E\frac{e}{m}\tau \qquad u^2 = \frac{eE}{mN\sigma}\sqrt{\frac{\lambda}{2}}, \qquad c^2 = \frac{eE}{mN\sigma}\sqrt{\frac{2}{\lambda}}$$

$$\varepsilon_E \gg (3/2)kT$$

τ average time between collisions = $1/l_0$

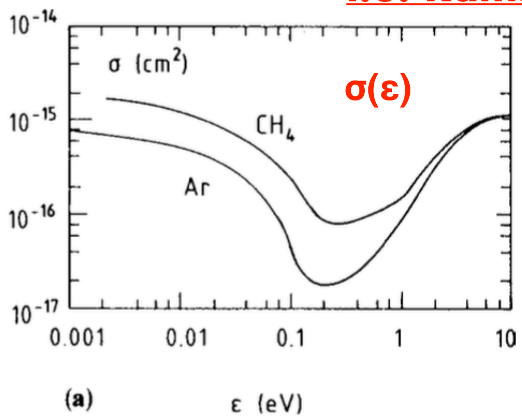
N number density

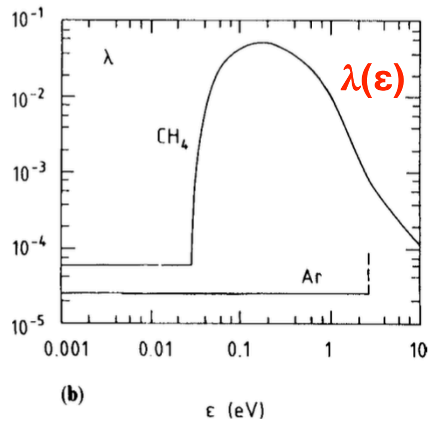
$\sigma = \sigma(\varepsilon)$ momentum transfer cross section

$\lambda = \lambda(\varepsilon)$ fractional energy loss per collisions

hence $c = c(\varepsilon)$, $u = u(\varepsilon)$ and $D = D(\varepsilon)$

i.e. Ramsauer minima





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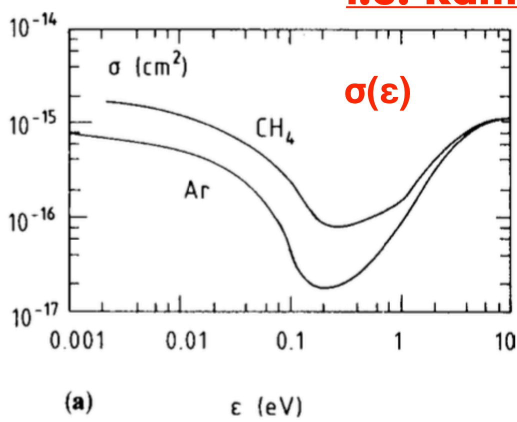
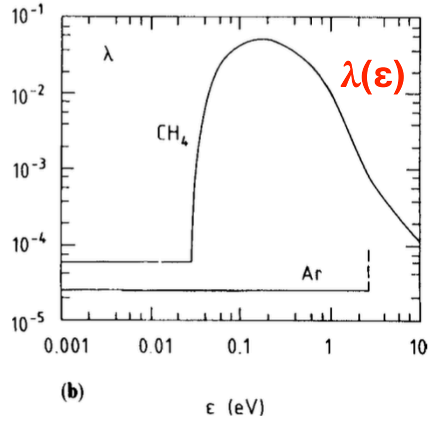
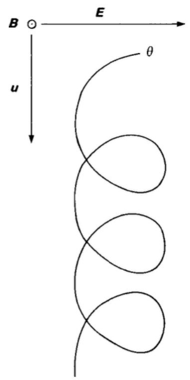




By properly introducing a magnetic field



$$\omega\tau = B\mu \qquad \omega\tau \gg 1$$

$$c^2 = \left(\frac{eE}{m\omega}\right)^2 \frac{2}{\lambda},$$

$$u^2 = \left(\frac{eE}{m\omega}\right)^2, \qquad u = \frac{E}{B},$$

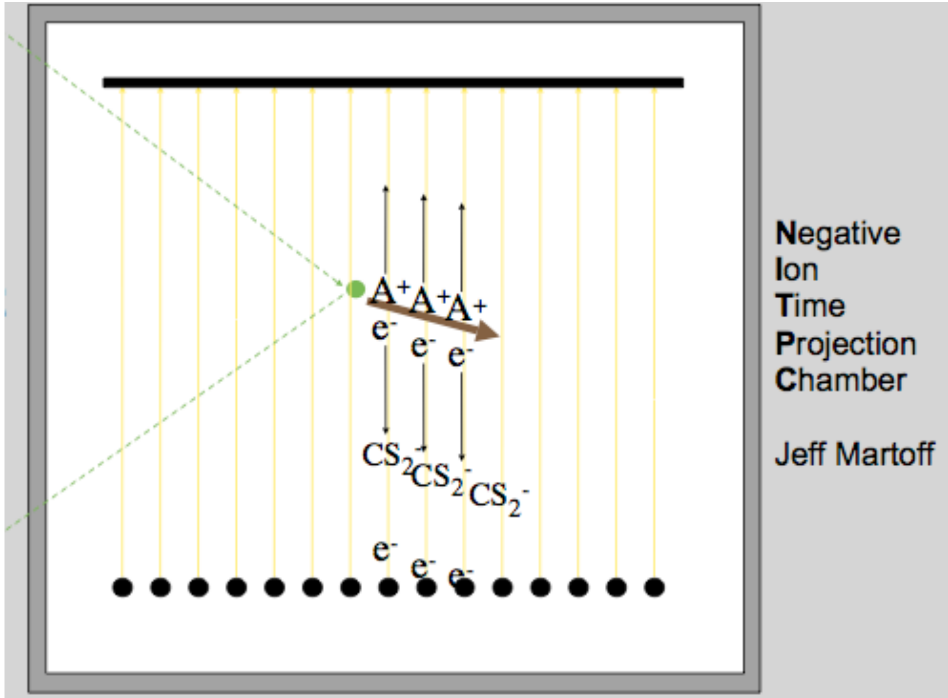
independent of gases!

NOTE: only transverse diffusion can be reduced with magnetic field

$$\sigma_{\parallel} = \sqrt{2dt}$$

$$\sigma_{\perp} = \frac{\sigma_{\parallel}^2}{1 + \omega^2\tau^2}$$

How to minimise diffusion: exploit negative ions as charge carriers instead of electrons



T. Ohnuki et al., NIM A 463

J. Martoff et al., NIM A 440 355

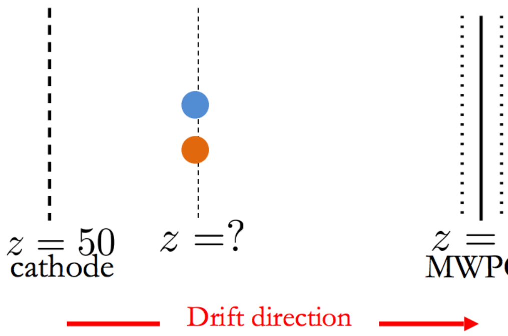
$$\sigma^2_{ions} = \sigma^2_{therm} = \frac{2kTL}{eE}$$

± 100 um/sqrt(cm)

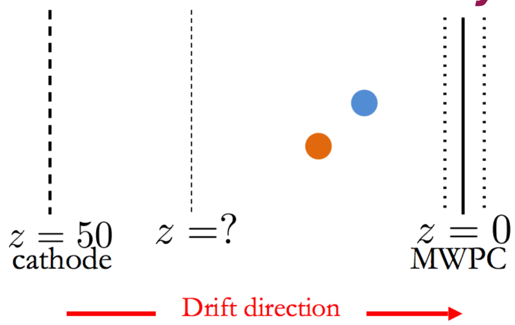
- Electronegative dopant in the gas mixture (SF_6, CS_2, CH_3NO_2, ...)
- Primary ionization electrons captured by electronegative gas molecules at O(100) um
- Anions drift to the anode acting as the effective image carrier instead of the electrons
- Longitudinal and transverse diffusion reduced thanks to the large mass of the charge carrier
  - Allow for realisation of larger TPC volume with same (or improved) tracking performance
- Negative ion drift velocity is O(cm/ms), compared to O(cm/us) electon drift velocity because of larger mass
  - Significant improvement of resolution along drift direction thanks to slower image carriers for low rate applications

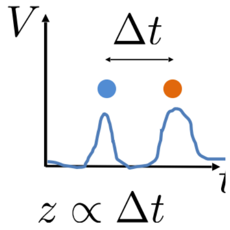
Additional feature: minority carriers in negative ions

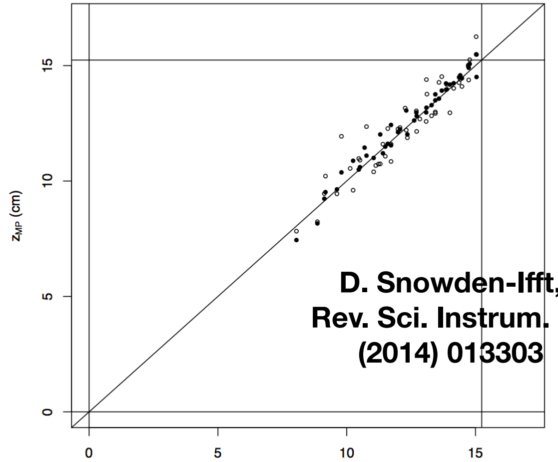
start together



…but drift at different velocity



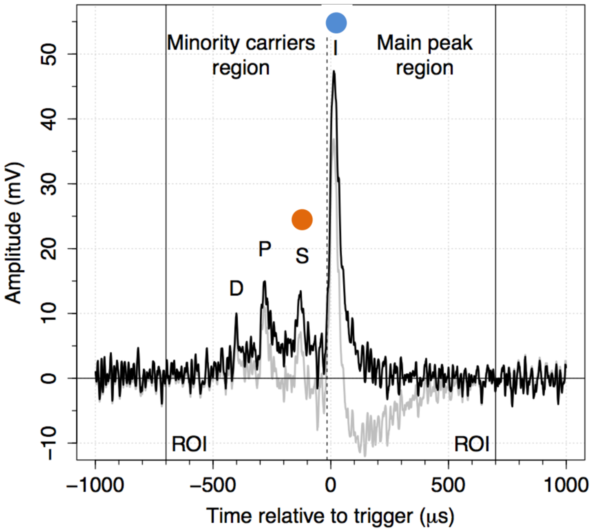




D. Snowden-Ifft, Rev. Sci. Instrum. 85 (2014) 013303

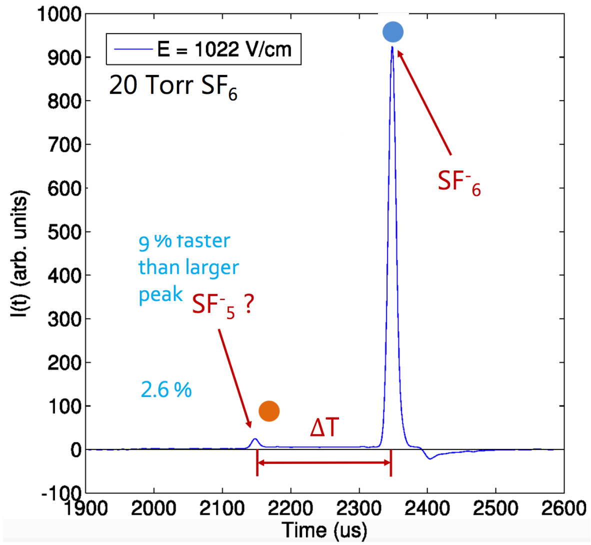
$$z = (t_m - t_p)\frac{v^m_{drift} v^p_{drift}}{v^m_{drift} - v^p_{drift}}$$

CS_2:CF_4:O_2 30:10:1 Torr



J. Battat et al *Phys.Dark Univ.* 9-10 (2015) 1-7

SF_6 20 Torr



N. S. Phan et al., *JINST* 12 (2017) 02, 02

Multiple charge carriers = event coordinate along the drift direction!!

Negative ion drift: history and status

Charge Readout

Low pressure

- Concept demonstratred in 2000 at 40 Torr CS_2 with MWPC [1]
- Pioneered in a actual experiment by DRIFT with CS_2:CF_4:O_2 at 40 Torr with MWPC [2]
- 20-40 Torr pure SF_6 in 2017 with THGEM [3]
- 20 Torr pure SF_6 with THGEM-multiwire [4] and muPIC in 2020 [5]

(nearly) Atm pressure

- Demonstrated in 2010's in He:CS_2[6] and CO_2:Ne:CH_3NO_2[7] with GEMs and MWPC
- In 2017 at 610 Torr of He:CF_4:SF_6 with GEMs and TimePix2 [8]
- In 2021 in Ar:iC_4H_{10}:CS_2 with GridPix (Ingrid + Timepix3) [9]

Optical Readout

- 50-150 Torr CF_4:CS_2 with glass GEM and CMOS [D. Loomba, talk at RD51 June 2022 meeting]

THIS TALK

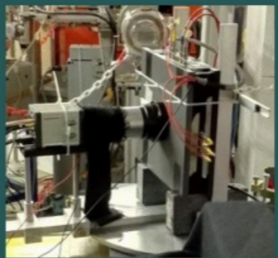
[1] C. J. Martoff et al. NIM A 440 335
[2] G. J. Alner et al., NIM A 535
[3] N. S. Phan et al, *JINST* 12 (2017) 02, 02
[4] A. C. Ezeribe NIM A 987
[5] T. Ikeda et al, *JINST* 15 07, P07015
[6] C. J. Martoff et al, NIM A 555
[7] C. J. Martoff et al, NIM A 598
[8] E. Baracchini et al, *JINST* 13 04, P04022
[9] C. Ligtenberg et al, NIM A 1014 165706

CYGNO experiment

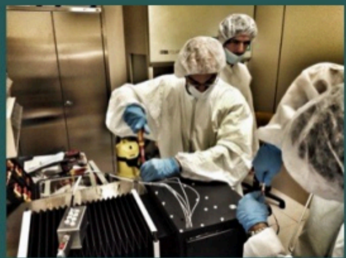
High precision 3D optical TPC for directional Dark Matter searches and solar neutrino spectroscopy

PHASE 0:
R&D and prototypes

2015/16 ROMA1	2017/18 LNF	2019/22 LNF/LNGS
ORANGE	LEMON	LIME



- 1 cm drift



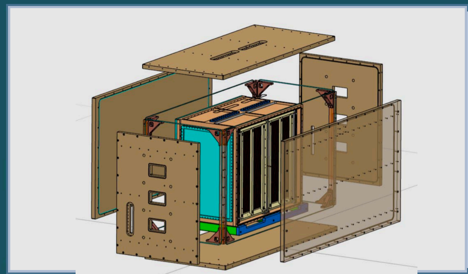
- 3D printing
- 20 cm drift

- 50 cm drift
- underground tests
- MC validation

PHASE 1:
O(1) m³ Demonstrator

2023/26 LNF/LNGS

CYGNO_04

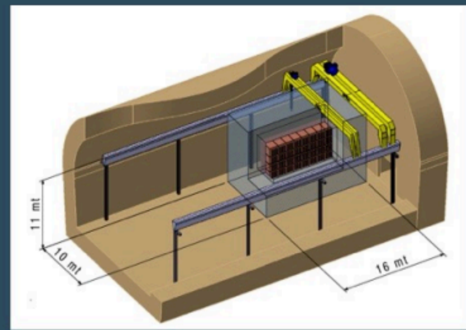


- background
- materials test, gas purification
- scalability

PHASE 2:
30 m³ Experiment

2026.. LNGS

CYGNO_30



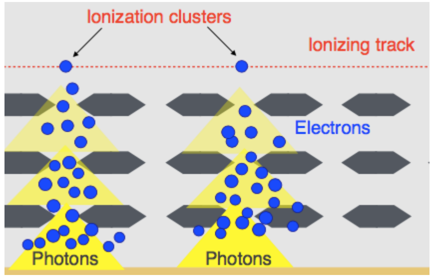
- Physics research

Instruments 6 (2022) 1, 6
JINST 15 (2020) 12, T12003
JINST 15 (2020) P08018
Measur.Sci.Tech. 32 (2021) 2, 025902
JINST 15 (2020) P10001
2019 JINST 14 P07011
NIM A 999 (2021) 165209
arXiv:2305.06168

https://web.infn.it/cygnus/

CYGNO :3D TPC with optical readout via PMT + sCMOS

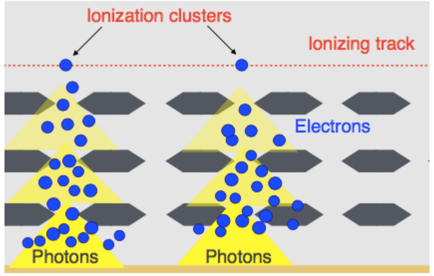
JINST 13 (2018) no.05, P05001



$He:CF_4$ @ 1 atm

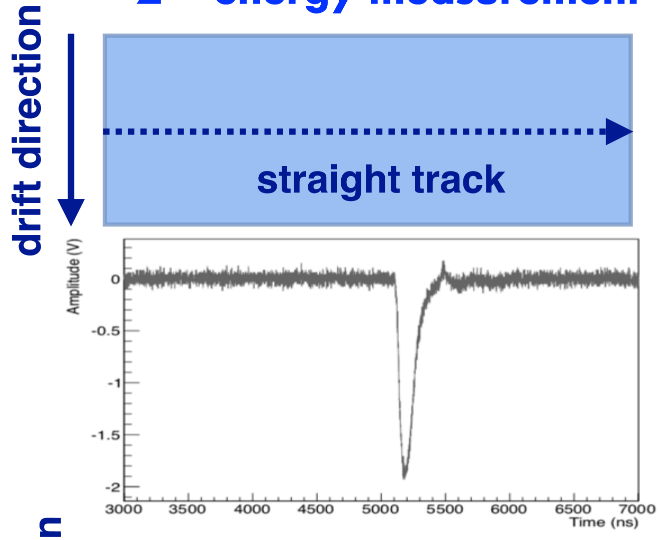
CYGNO :3D TPC with optical readout via PMT + sCMOS

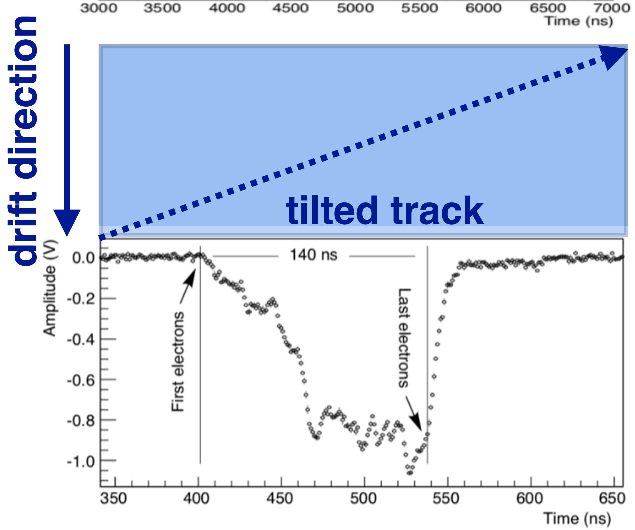
JINST 13 (2018) no.05, P05001



PMT:

integrated
Z + energy measurement



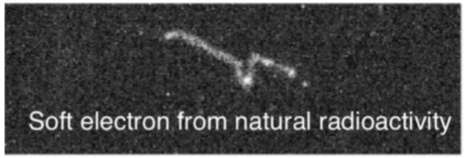


CYGNO :3D TPC with optical readout via PMT + sCMOS

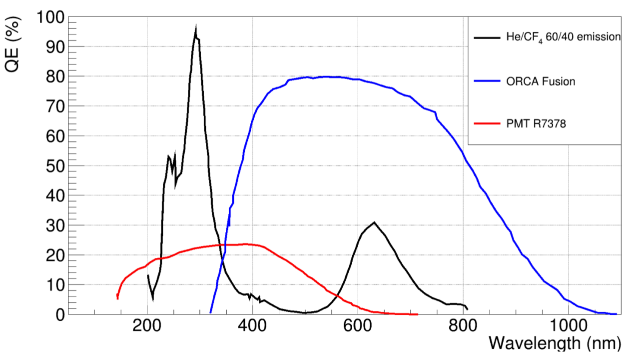
JINST 13 (2018) no.05, P05001

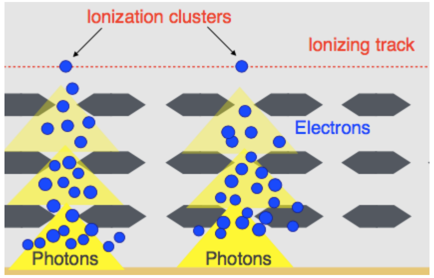
sCMOS:

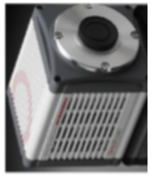
high granularity
X-Y + energy measurements



- **1/3 noise w.r.t. CCDs**
- **Market pulled**
- **Single photon sensitivity**
- **Decoupled from target**
- **Large areas with proper optics**







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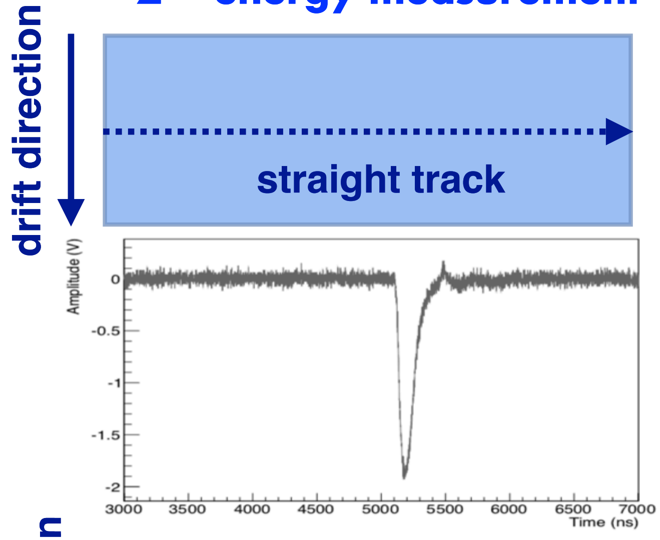
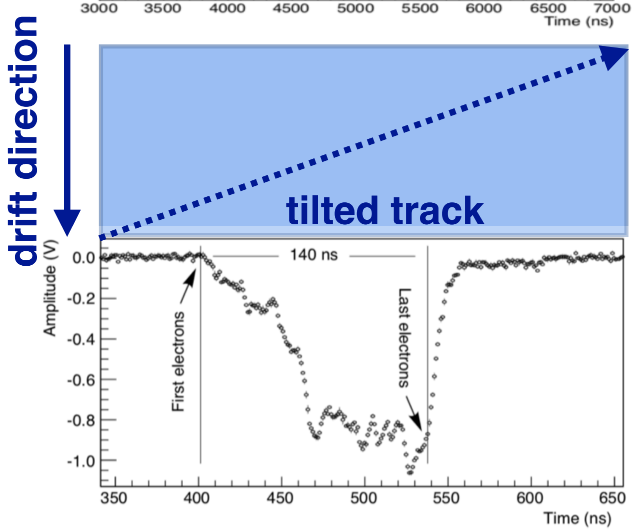


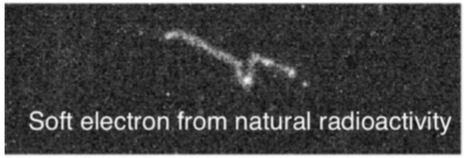


CYGNO :3D TPC with optical readout via PMT + sCMOS

JINST 13 (2018) no.05, P05001

sCMOS:

high granularity
X-Y + energy measurements



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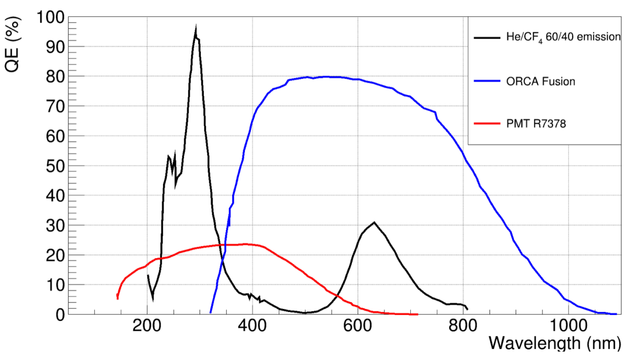
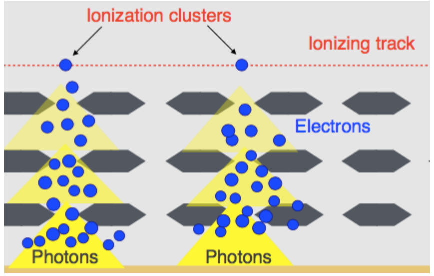
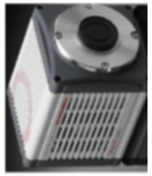
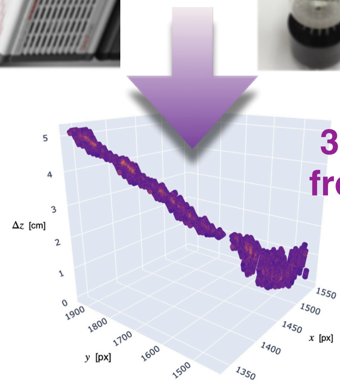




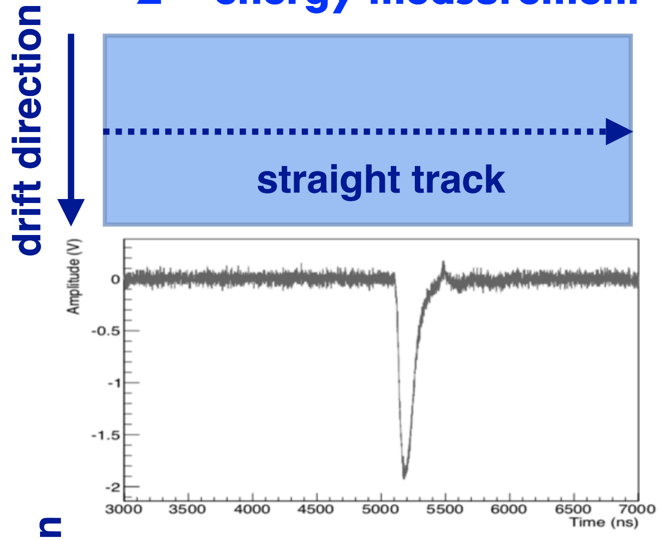


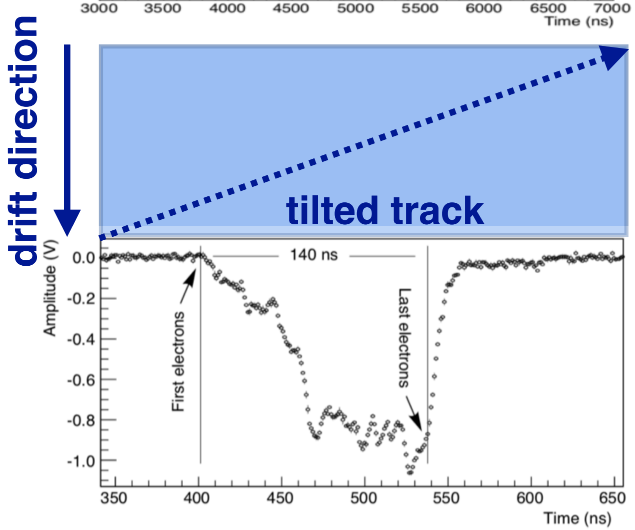


3D track from data

PMT:

integrated
Z + energy measurement





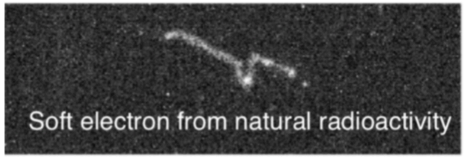
He:CF_4 @ 1 atm

CYGNO :3D TPC with optical readout via PMT + sCMOS

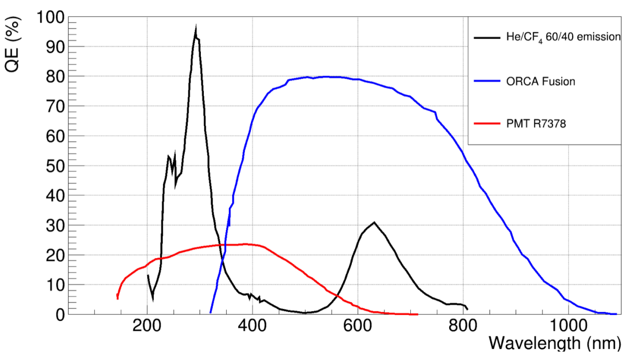
JINST 13 (2018) no.05, P05001

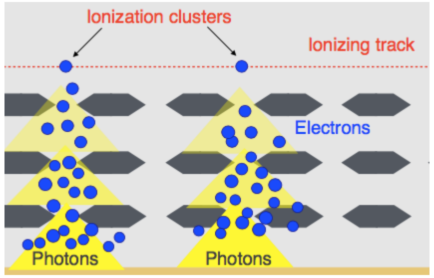
sCMOS:

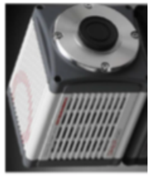
high granularity
X-Y + energy measurements

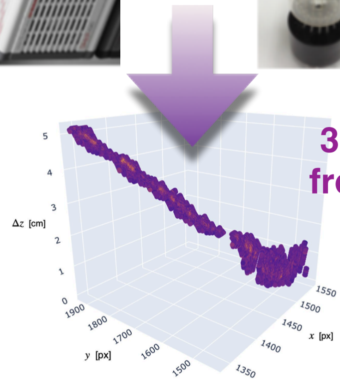


- **1/3 noise w.r.t. CCDs**
- **Market pulled**
- **Single photon sensitivity**
- **Decoupled from target**
- **Large areas with proper optics**









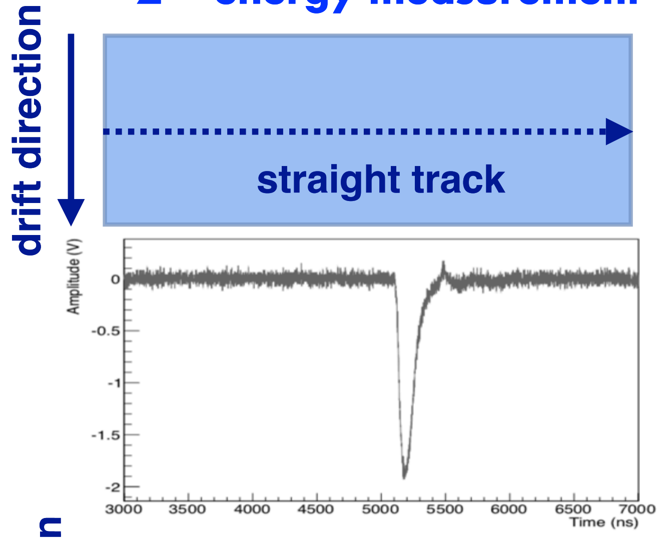
3D track from data

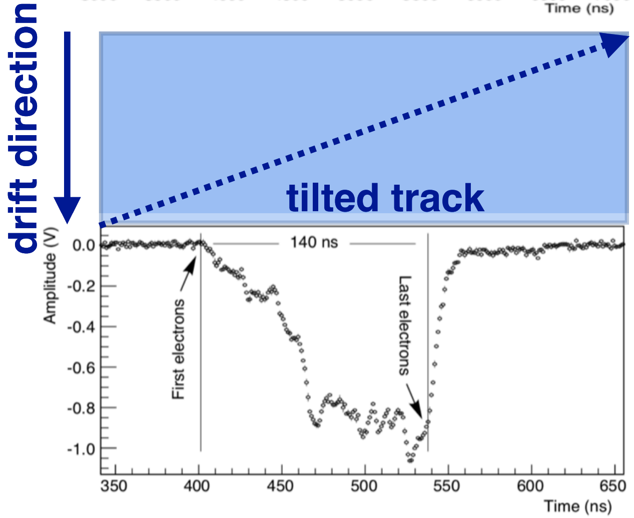
+ SF_6 for negative ion drift

European Research Council
INITIUM

PMT:

integrated
Z + energy measurement





sCMOS camera readout features

- **Camera focused on last amplification stage**

Lens de-magnification

$$\delta = \frac{f}{d - f}$$

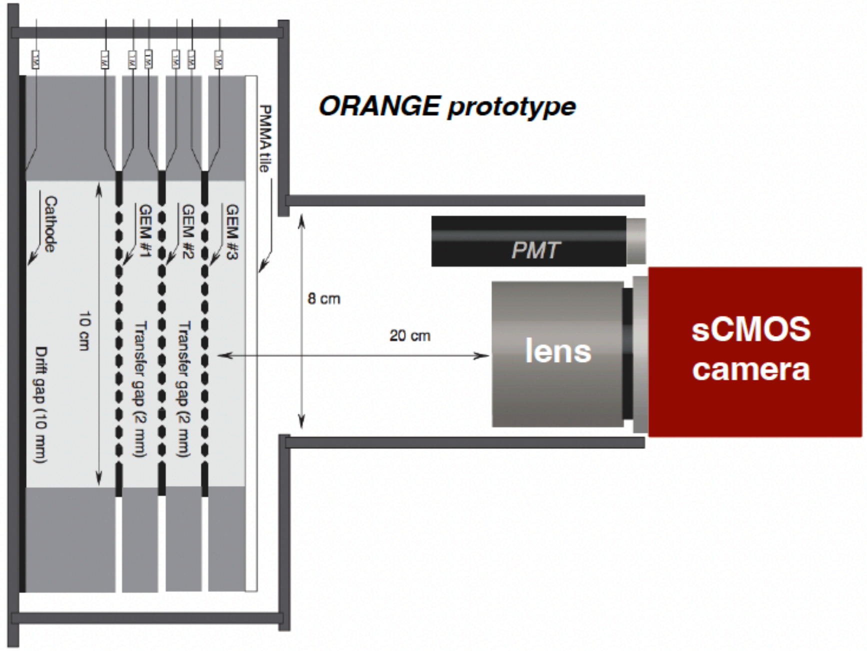
sCMOS-GEM distance (d) — **Focal lenght F.L.** (f)

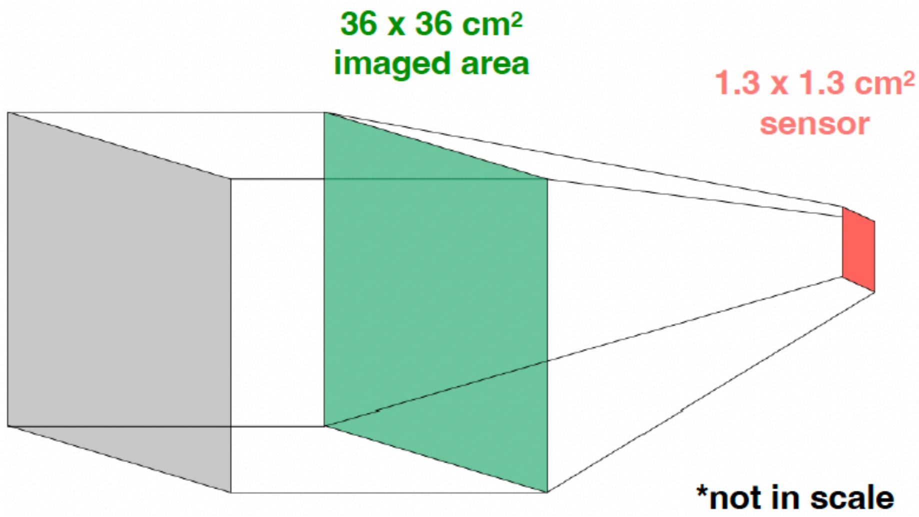
sCMOS sensor geometrical acceptance

$$\Omega = \frac{1}{(4(1/\delta + 1) \times a)^2}$$

- **The further the camera, the larger the area it can image**
  - a 36 x 36 cm² area with an effective granularity of 155 x 155 um² (large volume application)
  - a 10 x 10 cm² area with an effective granularity of 43 x 43 um² (small volume application)
- **The further the camera, the lower the light yield detectable**
  - ± 1 x 10^{-4} coverage for large volume application
  - ± 1 x 10^{-3} coverage for large volume application

Camera electronics is integrated, the output is an USB plug

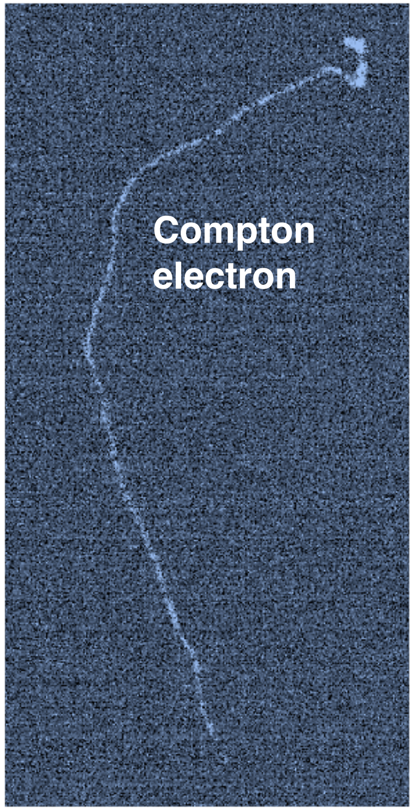


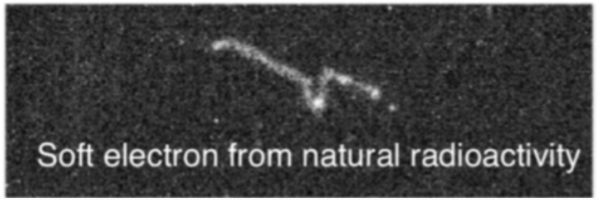


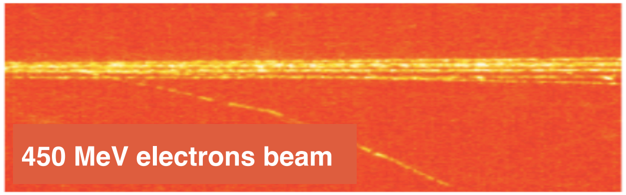
He:CF$_4$ @ 1 atm

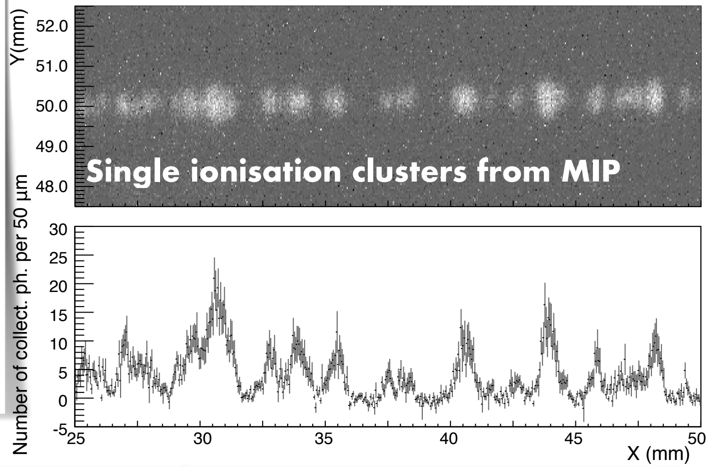
Imaging tracks with CYGNO

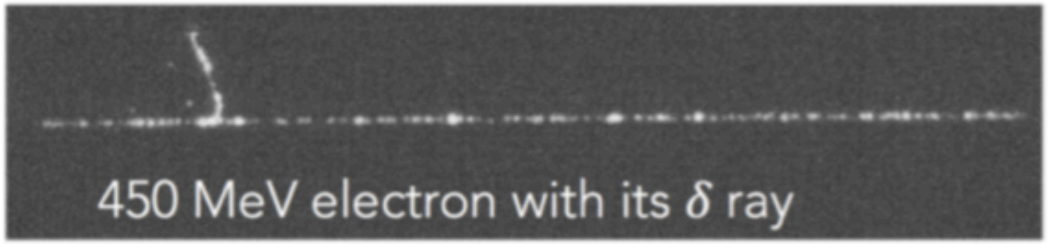
….with classical electron drift

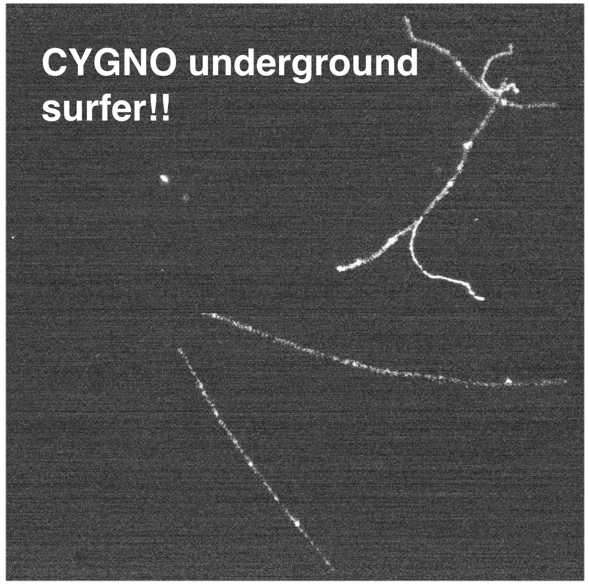


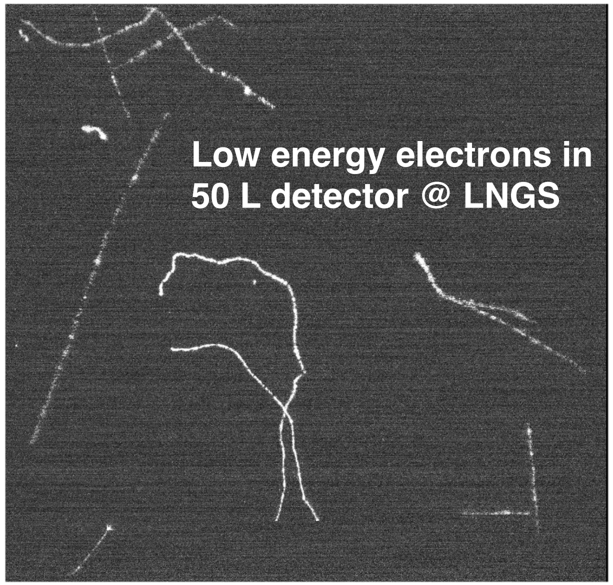












*Detector operated at LNGS (1100 m): atm pressure is 900 mbar

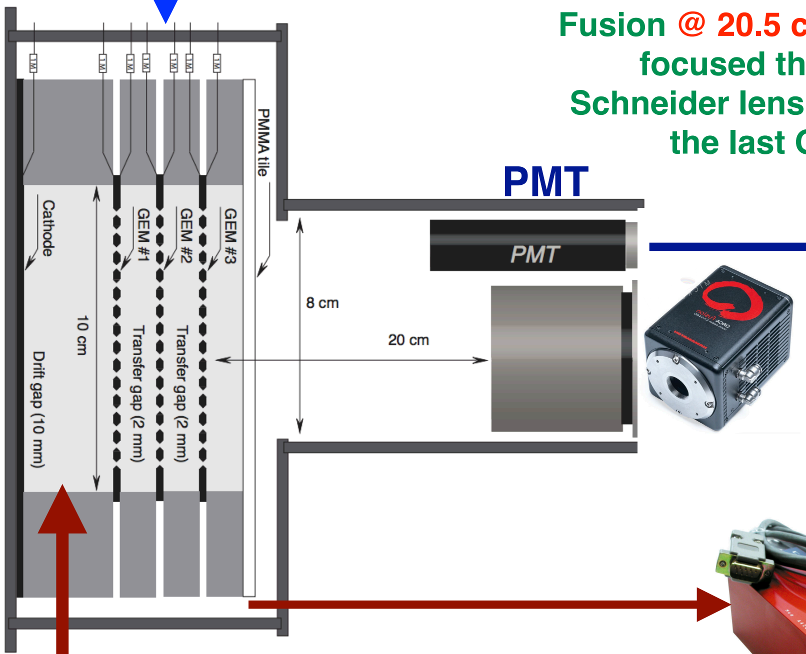
Negative ion R&D setup: the MANGO detector

Triple thin 50 um GEM stack 10 x 10 cm^2

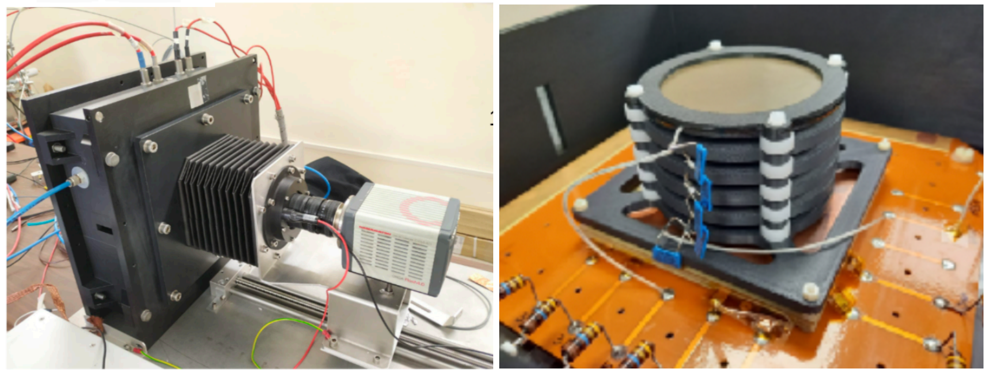
Acquired with Hokawo 3.0 software

sCMOS camera Orca Fusion @ 20.5 cm distance focused through Schneider lens (f/0.95) on the last GEM

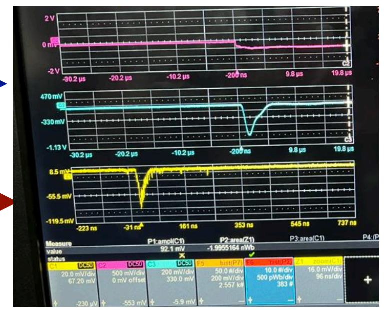
PMT



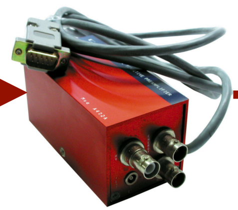
MANGO short 5 cm field cage setup



Acquire sCMOS images, PMT & GEM waveforms with ^{241}Am source



Drift gap (variable from 5 to 15 cm)



Charge sensitive preamplifier

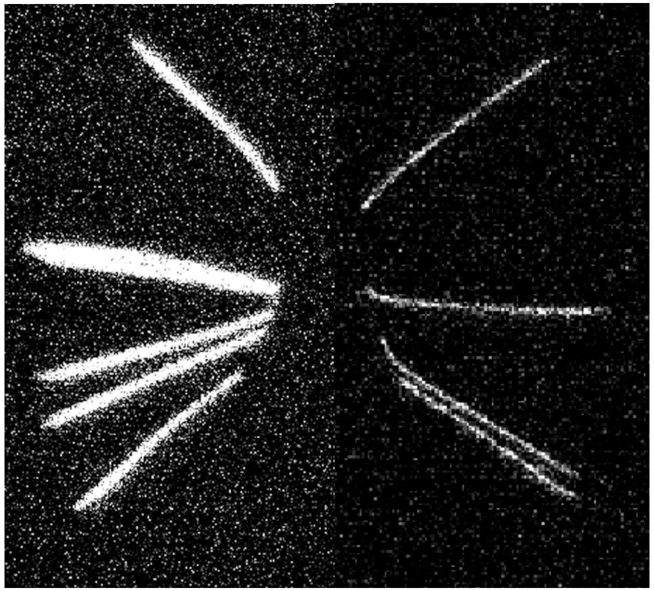
300 us decay time

0.113 mV/fC

Oscilloscope

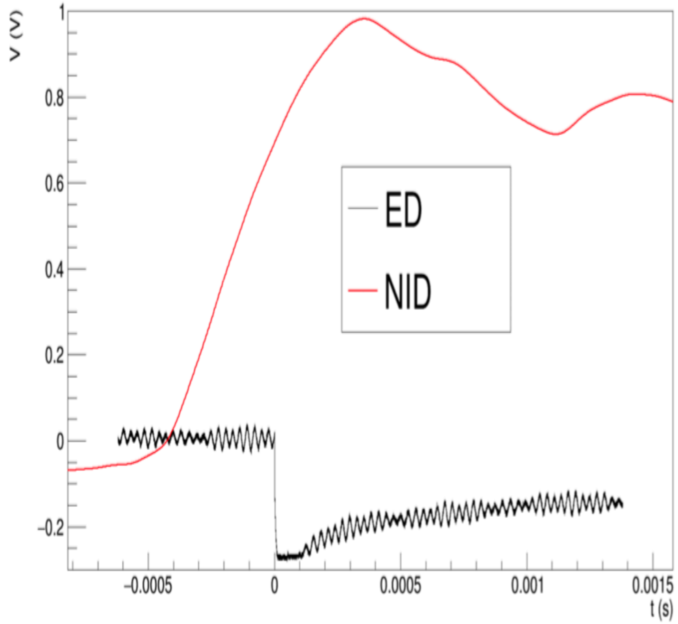
Eyes (and waveforms) can't lie

sCMOS image



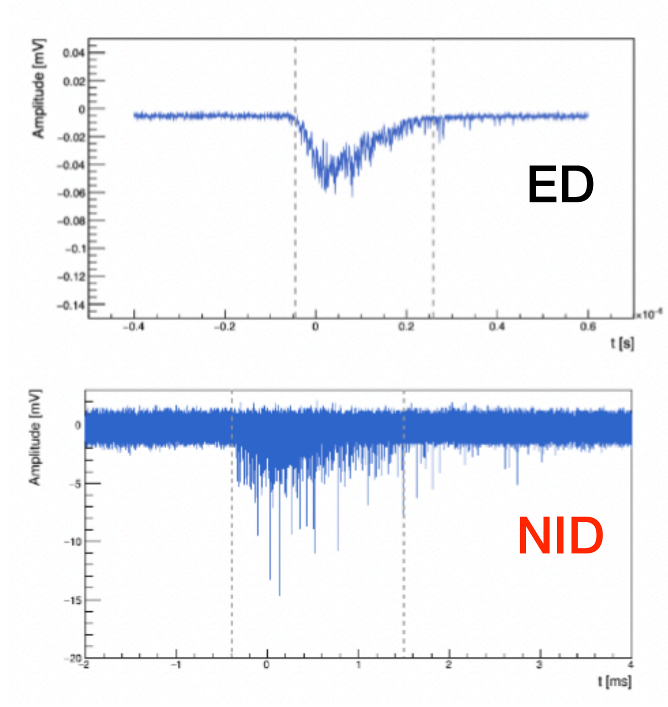
He:CF_4	He:CF_4:SF_6
60:40	59:39.4:1.6
1 kV/cm	0.4 kV/cm
(ED)	(NID)

GEM preamp output



O(us) rise for ED
O(ms) rise for NID

PMT waveforms

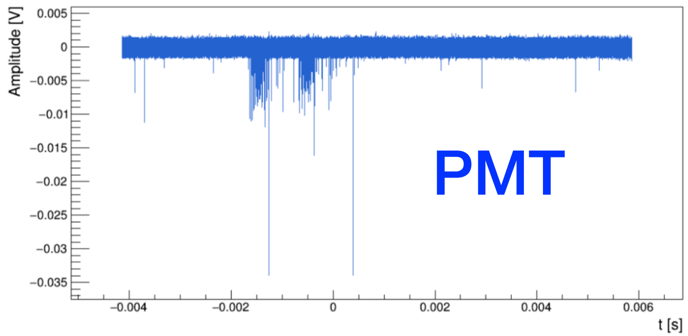


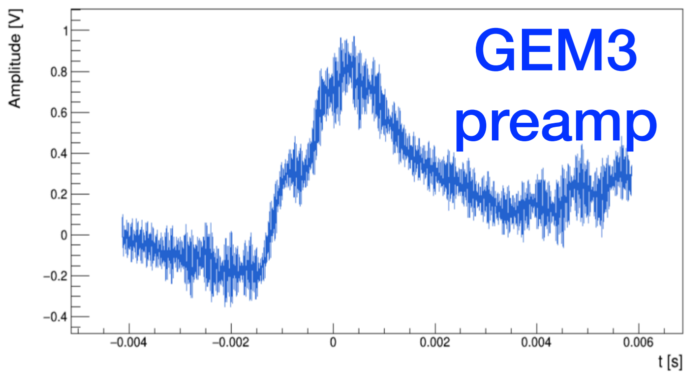
O(0.1 us) time extent for ED
O(10 ms) time extent for NID

0.90 atm

(LNGS atmospheric pressure)

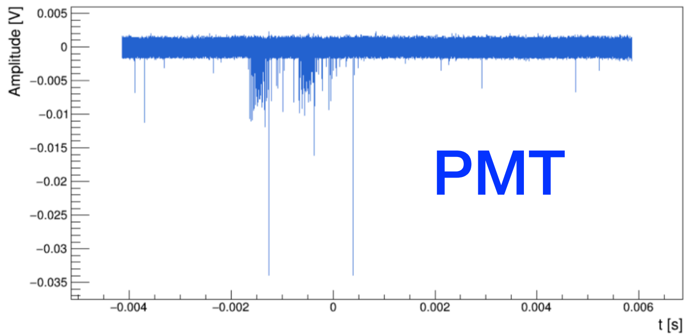
NID PMT waveforms: how peculiar!

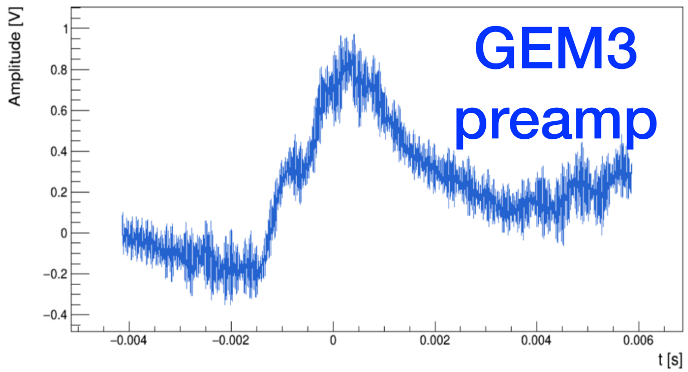


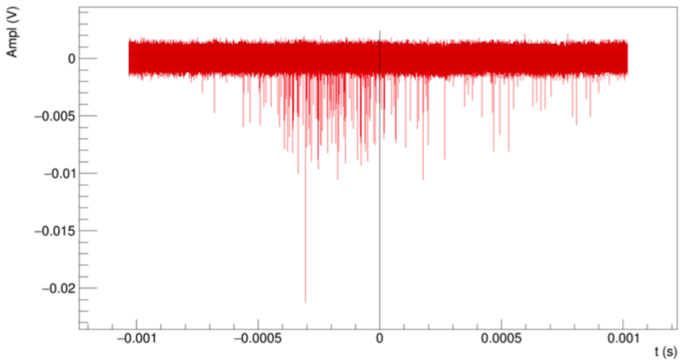


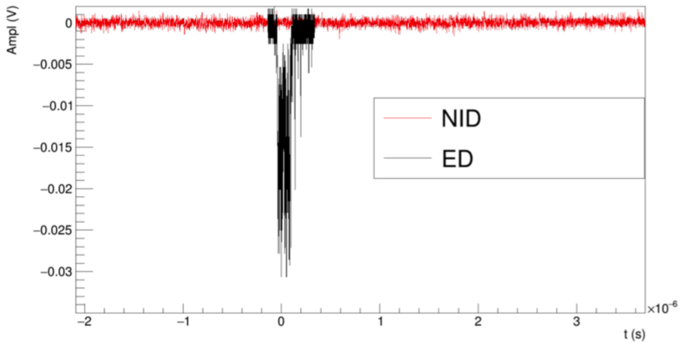
*First time ever NID are observed with PMTs!

NID PMT waveforms: how peculiar!









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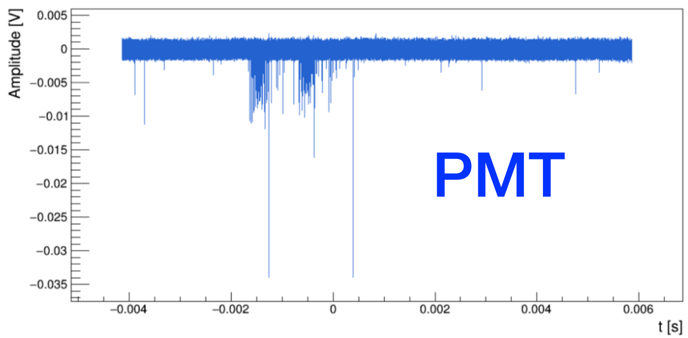
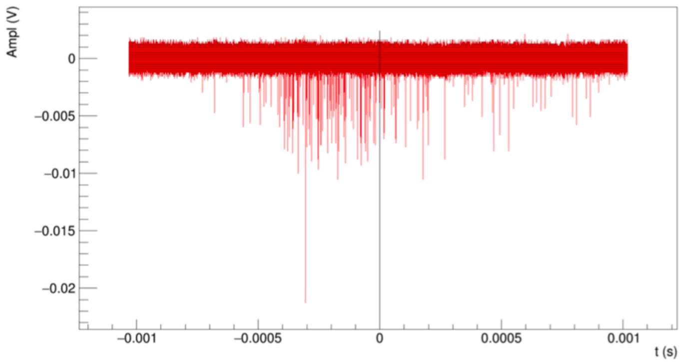
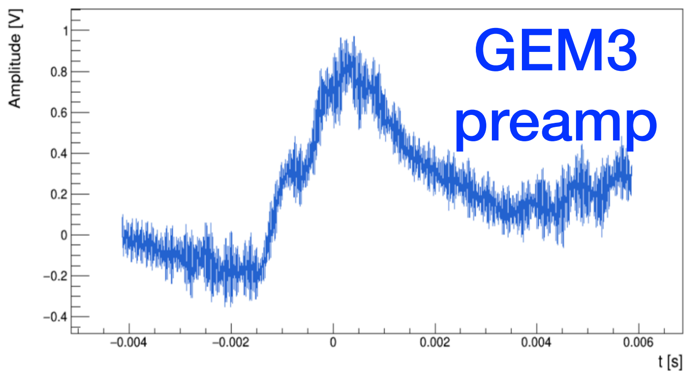
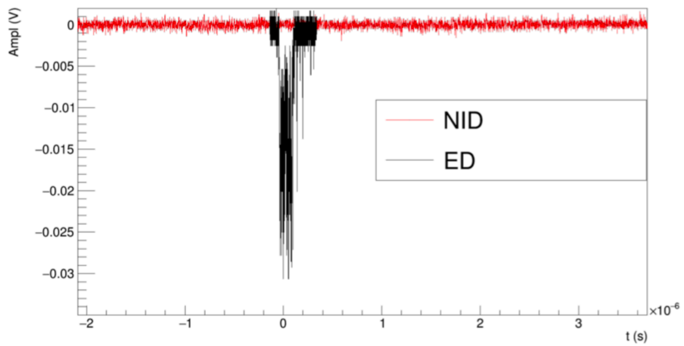
NID PMT waveforms: how peculiar!

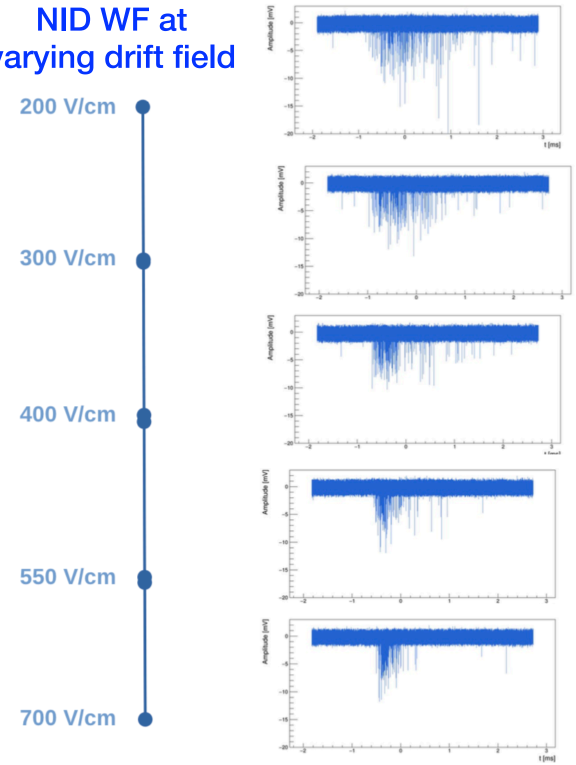








NID WF at varying drift field



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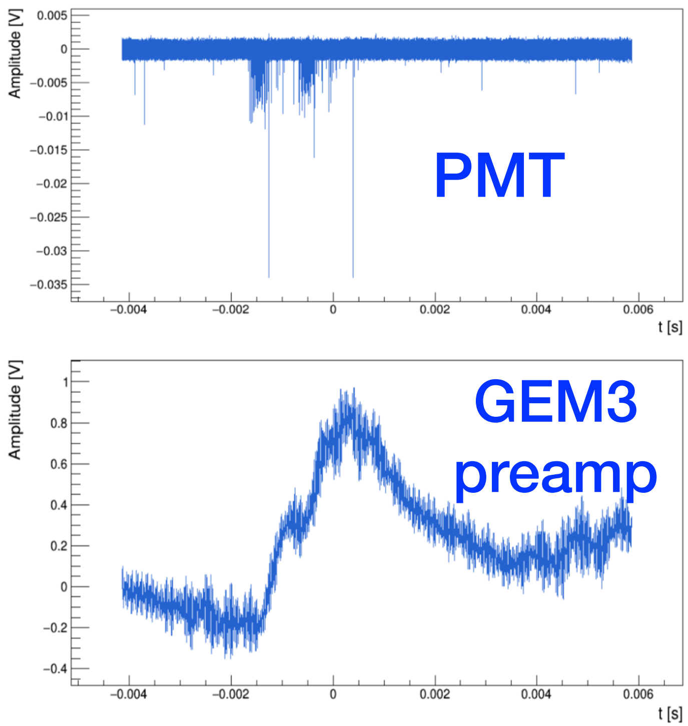
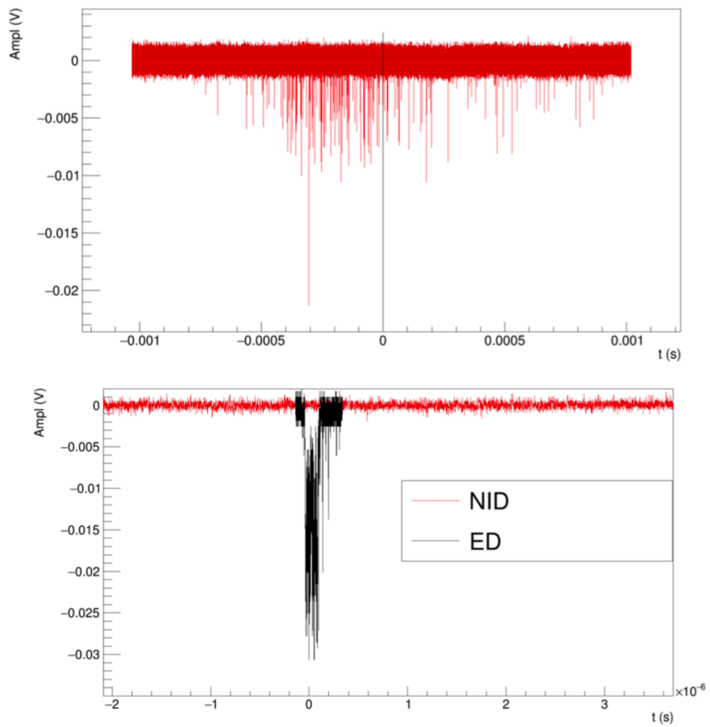
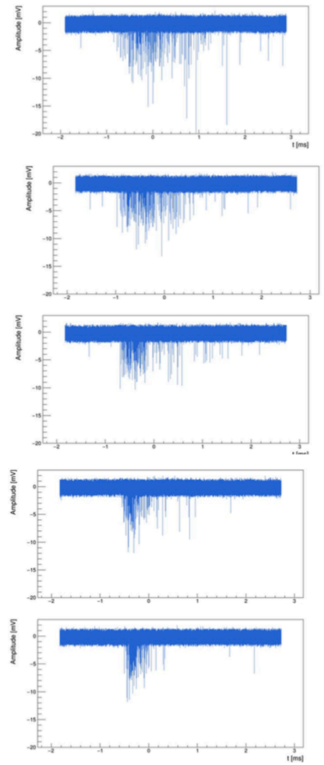
NID PMT waveforms: how peculiar!







Given the PMT bandwidth and the "slow" arrival of charge carriers, individual clusters are visible in the PMT signal --> ideally, this could allow cluster counting technique for superior energy resolution

H. Fischle et al., NIM A 301, 202 (1991)

G. Cataldi et al., NIM A 386, 458 (1997)

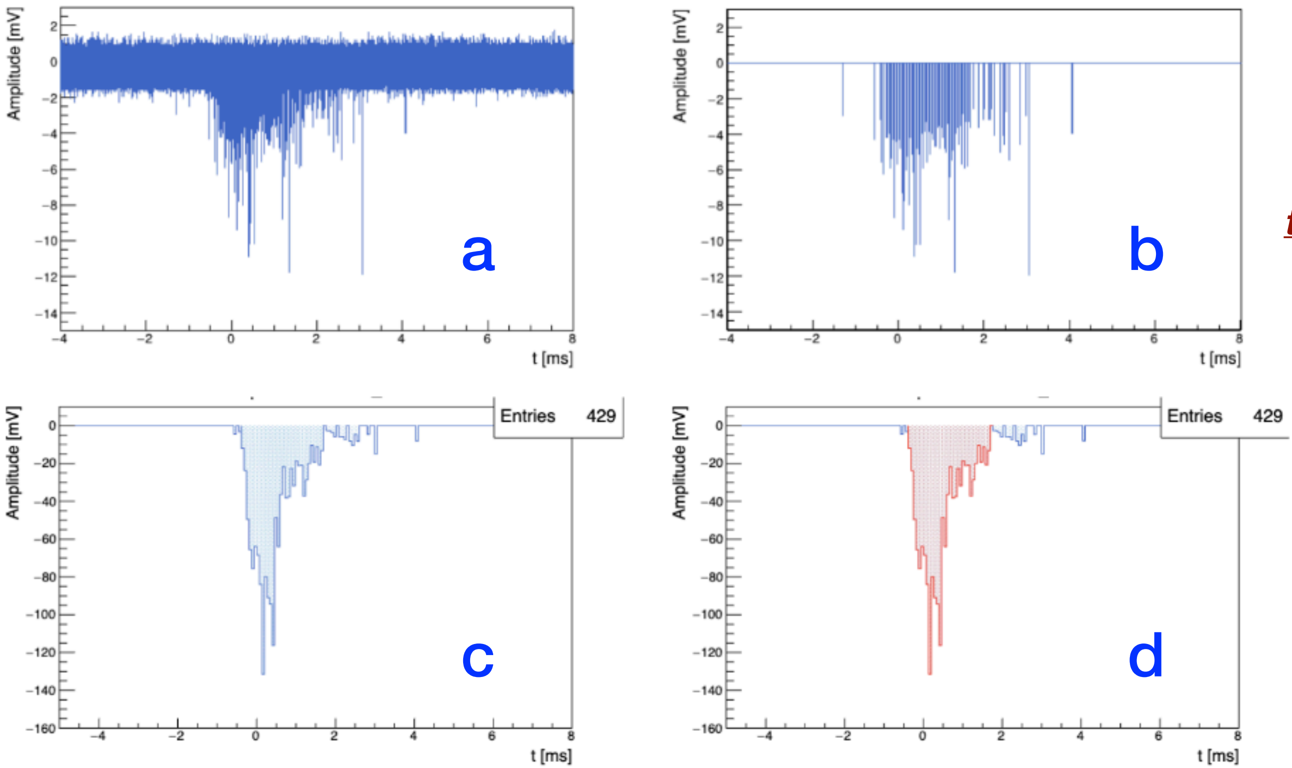
G. Chiarello et al., JINST 12 (07) C07021

Not a topic of this talk, but ongoing work along this line

Innovative NID PMT waveforms analysis

□□ **First ever PMT analysis for optical NID signals** □□

- **Initial approach ⇒ Time rebinning**
  a. **Threshold** ➡ Only peaks above 6*RMS are taken into account
  b. **Rebin** ➡ Selected points are put into histogram
  c. **Delimitation** ➡ Start (end) when 2 bins are above(below) 10 mV
  d. **Systematics** ➡ Varying #bins & threshold voltage **(reworked analysis)**

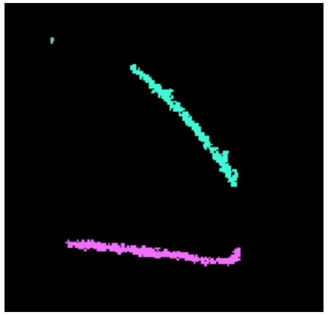


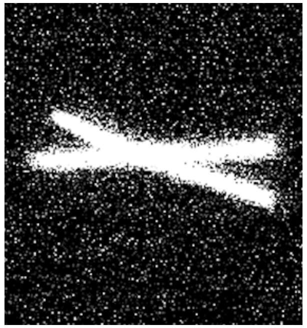
Dr. D. Marques PhD thesis, arXiv:2509.10890

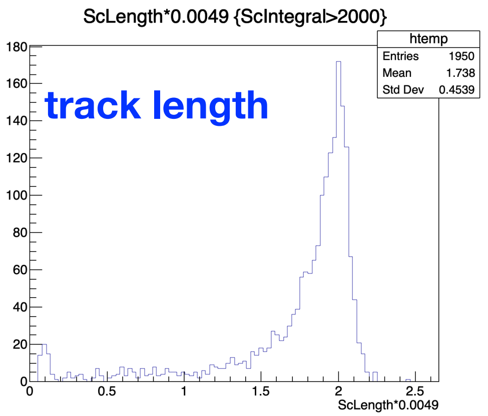
sCMOS camera images analysis

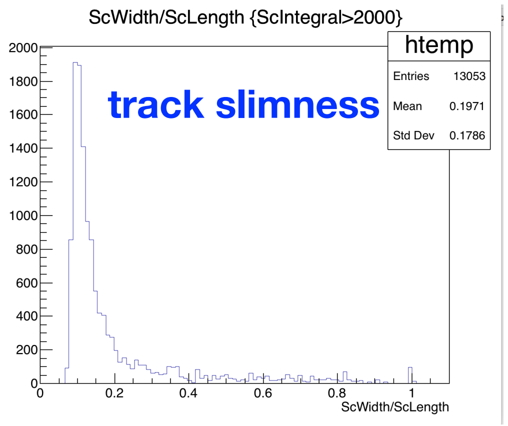
- Alphas selection:
  - tracks reconstructed with iterative DBSCAN algorithm [10]
  - track length > 1.47 cm
  - track slimness < 0.3
- Sum of pixel content is light integral

[10] E. Baracchini et al, JINST 15 (2020) 12 T12003

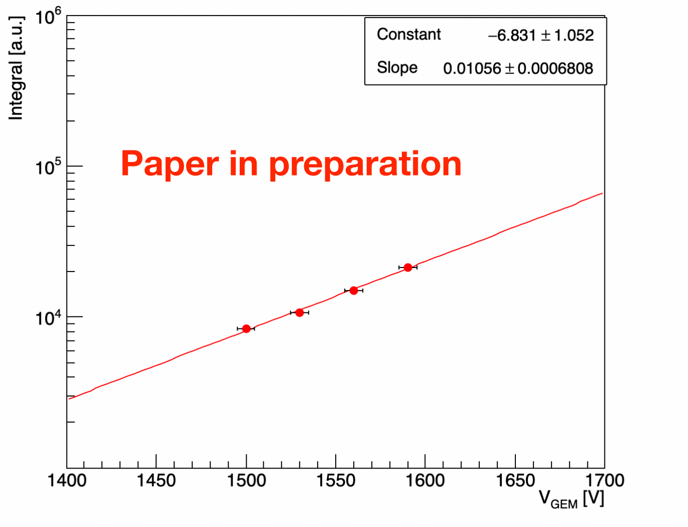




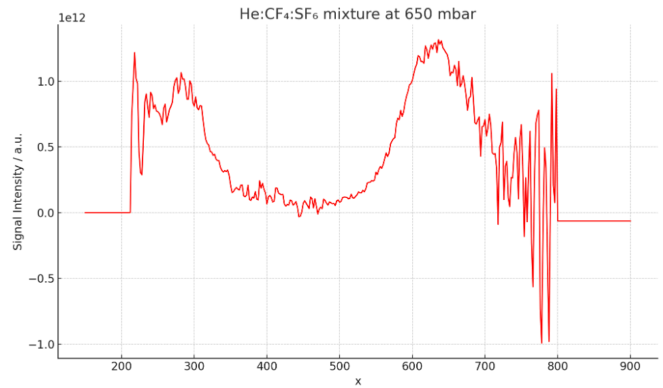




Absolute alpha track integral at 900 mbar



NID mixture scintillation spectrum

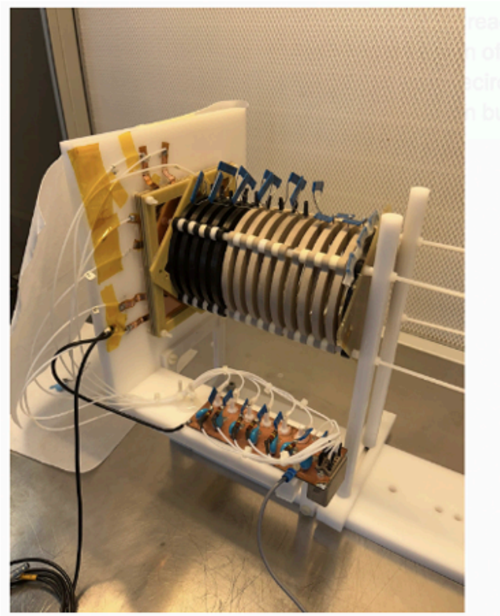


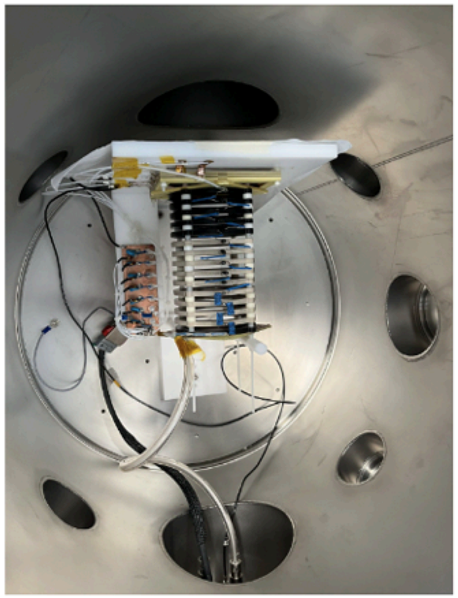
Preliminary charge gain evaluation ~ O(10³)

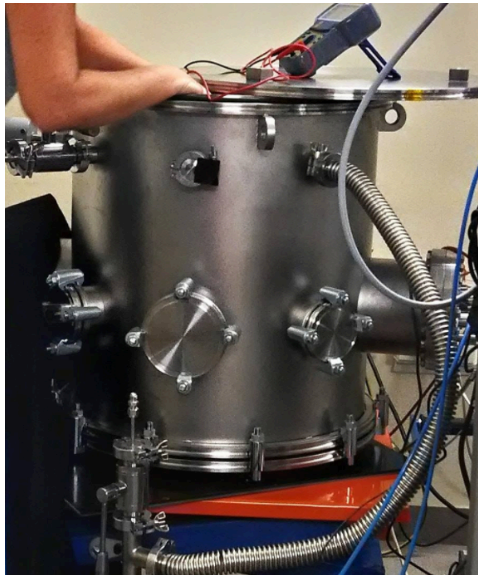
Implies O(keV) theshold at atmospheric pressure for charge readout!

A MANGO "in the keg"

Longer drift distance is necessary to evaluate diffusion and mobility: MANGO was installed in a vacuum vessel that could host a longer field cage







Because of geometry constraints, the camera is now at 26.6 cm distance (w.r.t. 20.5 cm of the previous setup): the light yield reaching the camera sensor is reduced of 2/3 with respect to previous configuration

For this reason and in order to be able to measure the diffusion at ~10 cm drift length and low ~250 V/cm drift fields, we reduced the pressure to 650 mbar in the diffusion measurements

NOTE: diffusion and (reduced) mobility are independent of pressure

Minority carriers demonstration and mobility evaluation

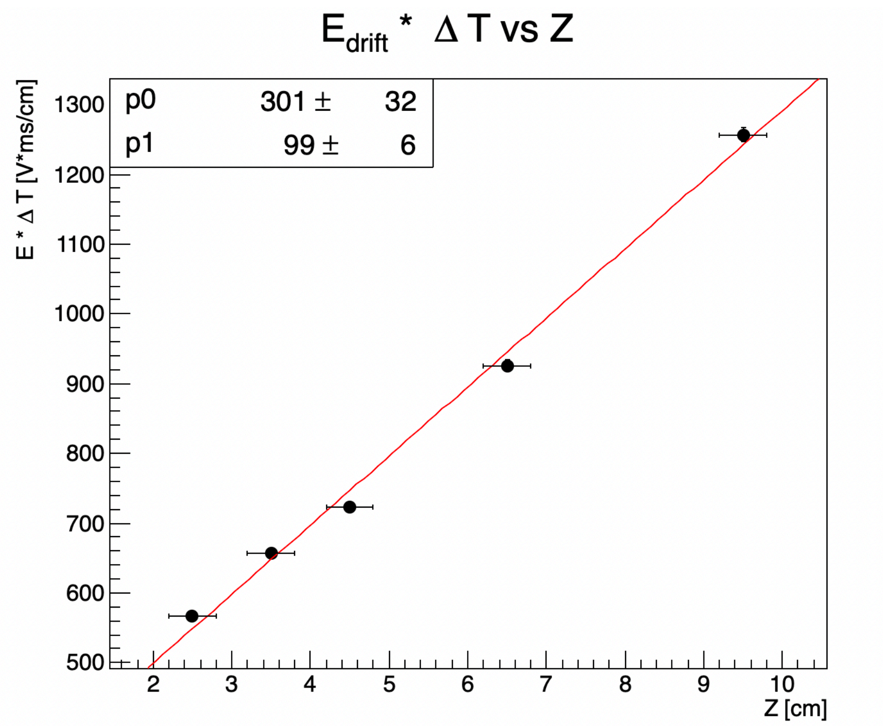
In presence of minority carriers, the measured average waveform time extension depends linearly on the drift distance Z.

This dependence can not exist if only on charge carrier is present

$$\langle \Delta T \rangle (Z, E) = p_0(E) + p_1(E)\, Z, \qquad p_0(E) = \frac{C}{\mu_s E} \qquad p_1(E) = \frac{1}{E}\left(\frac{1}{\mu_s} - \frac{1}{\mu_f}\right).$$

$\mu_{s/f}$ = mobility of the slower/faster charge carrier

- Perform multiple ΔT measurements varying drift distances and drift fields
- Multiply for E to remove drift field dependence and average for each drift distance
- Linear fit returns p_0 and p_1
- From p_0 obtain slower carriers SF_6^- mobility consistent with [1]
- From p_1 estimate a $\mu_f \pm 3$ $cm^2\ V^{-1}\ s^{-1}$

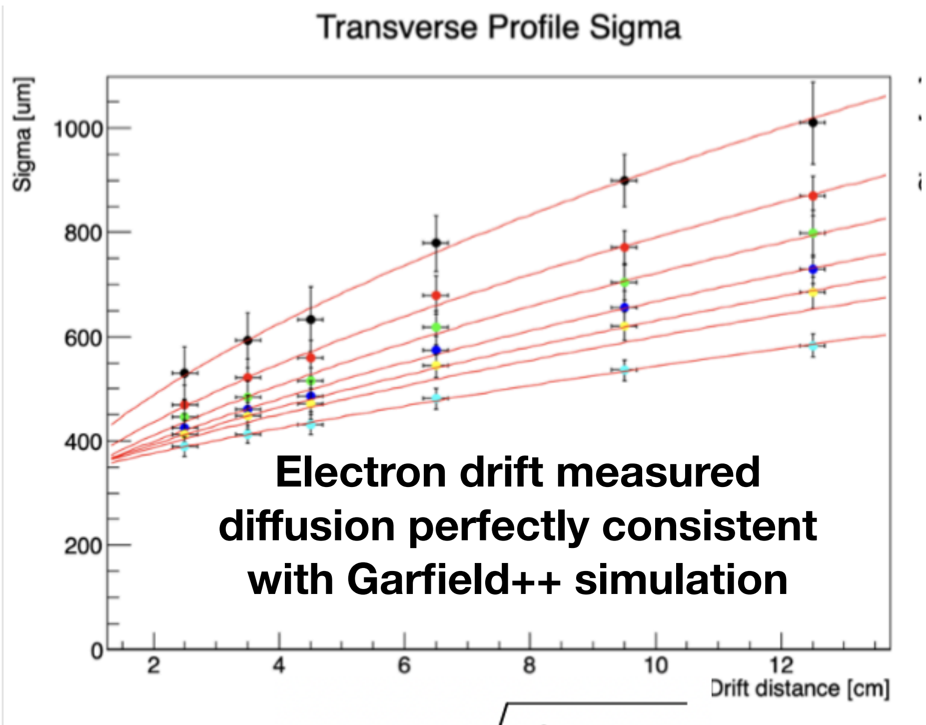


Paper under internal collaboration review

[1] E. Baracchini et al, JINST 13 04, P04022

Spoiler! ED & NID diffusion

He:CF_4 60:40 (ED)

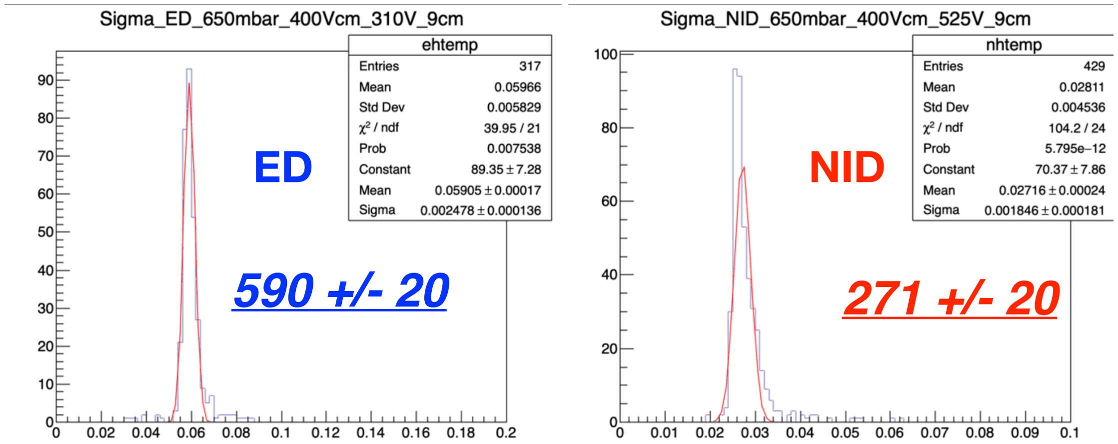


Electron drift measured diffusion perfectly consistent with Garfield++ simulation

$$\sigma_{meas} = \sqrt{\sigma_0^2 + \xi^2 L}$$

Drift field [V/cm]	σ_0^{ED} [μm]	ξ^{ED} [μm/$\sqrt{\text{cm}}$]
150	300 ± 100	280 ± 20
200	290 ± 60	230 ± 10
250	290 ± 60	210 ± 10
300	300 ± 40	190 ± 10
350	300 ± 40	170 ± 10
400	310 ± 30	160 ± 10
600	320 ± 20	140 ± 10

Transverse profile sigma at 9 cm distance and 400 V/cm



Note that He:CF_4 is already a very "cold" gas with one of the smallest diffusion coefficients

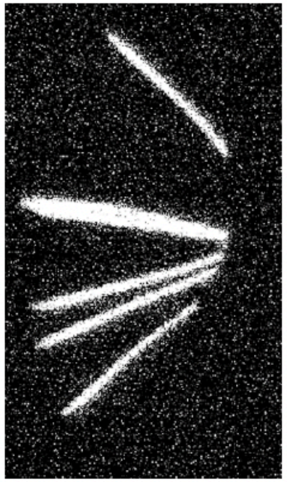
Analysis performed, results interpretation under finalisation and paper under preparation

Negative ion drift operation implies

From this….

….to this with same experimental layout

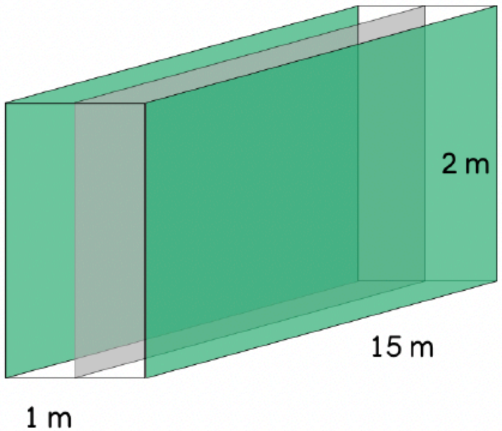
***He:CF$_4$*
*60:40***



***He:CF$_4$:SF$_6$*
*59:39.4:1.6***

with full fiducialization (i.e. 3D)

From this….

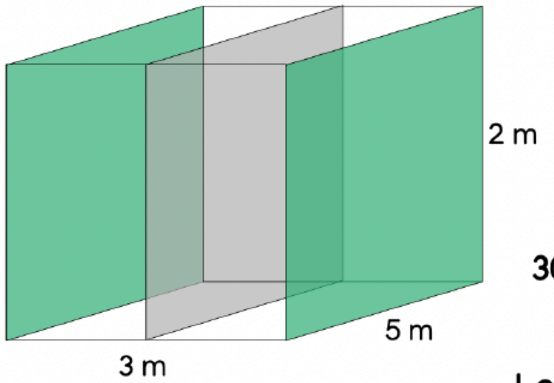


30 m^3

94 m^2

60 m^2
readout

….to this with same tracking performances



30 m^3

62 m^2

20 m^2
readout

30 m^2 less surface:
Less material
(background)
Less redout sensors

Conclusions & outlook

- **We obtained Negative Ion Drift operation at LNGS atmospheric pressure with optical readout with both PMT and sCMOS**
  - **First time NID are observed with a PMT!**
    - **We developed an innovative analysis for NID PMT waveforms**
    - **Possibility of cluster counting and improved energy resolution and PID?**
  - **O(10^3) charge gain achieved**

- **We performed measurements at 650 mbar (due to detector constraints)**
  - **We demonstrated for the first time the existence of minority carriers in He:CF_4:SF_6**
  - **We evaluated and SF_6^- mobility consistent with literature and estimated from this a mobility of the fast charge carriers about 30% faster than SF_6^-**

- **We measured diffusion and transport properties and analysis in ongoing**
- **Only the first step towards a systematic investigation of He:CF_4:SF_6 NID mixture potentialities at atmospheric pressure**


THANKS!

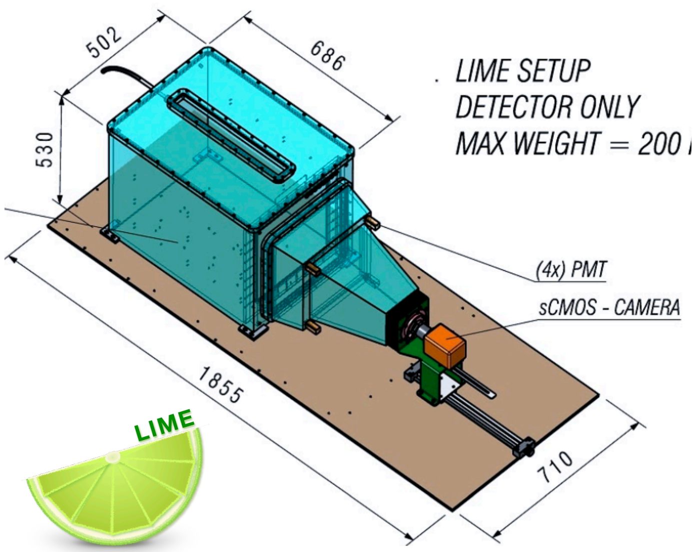
Backup slides

....with classical electron drift

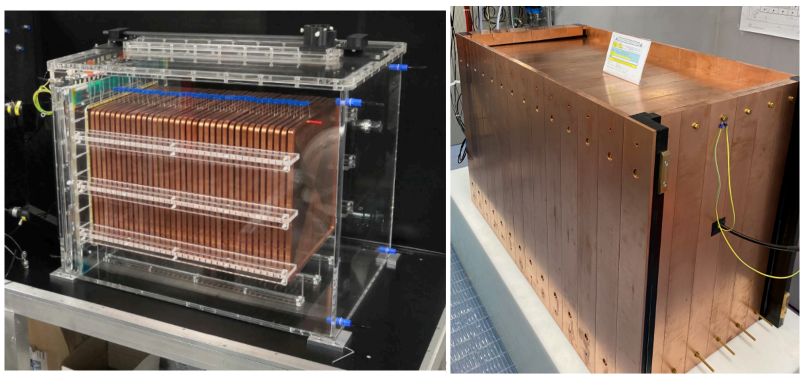
arXiv:2305.06168

Selected CYGNO results with LIME

50 L active volume

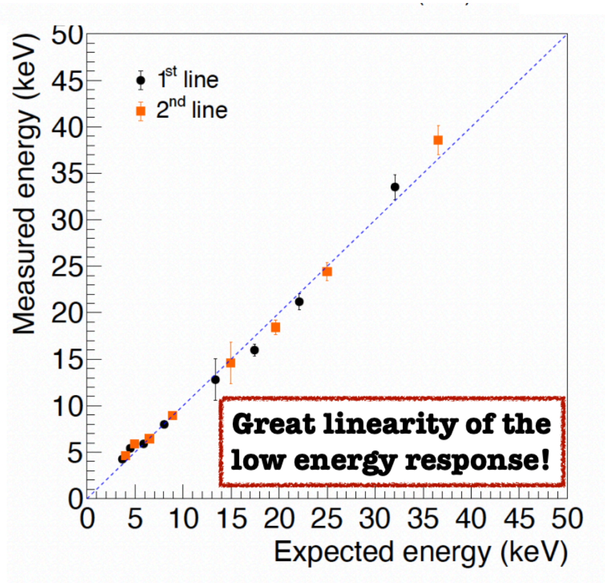


1 sCMOS + 4 PMT + 3 GEMs
33 x 33 cm^2 readout area
50 cm drift length

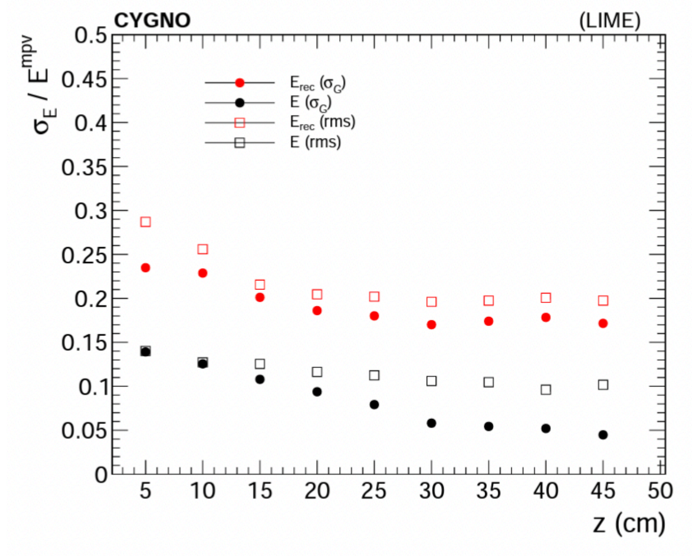


Underground shielded installation @ LNGS

Energy response linearity



Energy resolution @ 5.9 keV



Particle Identification

40% nuclear recoil efficiency for energies < 20 keV_{ee}, with 99% ^{55}Fe events rejected

Signal efficiency			Background efficiency		
ε_S^{presel}	ε_S^{δ}	ε_S^{total}	ε_B^{presel}	ε_B^{δ}	ε_B^{total}
0.98	0.51	0.50	0.70	0.050	0.035
0.98	0.41	0.40	0.70	0.012	0.008

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Improvement with ML over the entire 1-35 keV energy range

Models	Signal Efficiency $[\epsilon^S]$%	Bkg. Rej. Efficiency $[1-\epsilon^B]$%
RFC	40	99.1
	50	97.5
GBC	40	98.3
	50	96.5
DNN	40	96.6
	50	93.5

Preliminary

Diffusion during drift recap

Rolandi - Blum - Riegler book

$$\sigma_{tot}^2 = 2Dt$$

$$D = \frac{c l_0}{3}$$

Macroscopic quantities

- σ_{tot} r.m.s. normal distance at time t
- D diffusion coefficient
- u constant drift velocity <<< c
- μ mobility
- L = ut distance travelled after time t

Microscopic quantities

- c instantaneous velocity between collisions
- l_0 mean free path of the drifting particle
- τ average time between collisions
- N number density

If the drifting particle are electrons, the average time τ between collision can be approximated by:

$$\frac{1}{\tau} = N\sigma c \qquad l_0 = c\tau$$

$$D = \frac{l_0^2}{3\tau} = \frac{c l_0}{3} = \frac{c^2\tau}{3} = \frac{2}{3}\frac{\varepsilon}{m}\tau.$$

- m mass of the drifting electron
- ε energy of the drifting electron

For electrons with the lowest possible energy, i.e. due to thermal motion only:
by exploiting the definition of mobility μ, the thermal diffusion limit can be written as

$$u = \mu E \qquad \mu = \frac{e}{m}\tau \qquad \epsilon_{\min} = \frac{3}{2}kT$$

$$\sigma_{\text{therm}}^2 = \frac{2kTL}{eE}$$