Black holes with electroweak hair: the non-perturbative analysis

Romain Gervalle

Black holes and their symmetries

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Reference paper: R. G. and M. S. Volkov, *Phys. Rev. Lett.* 133, 171402 **[arXiv:2406.14357]**
+ the detailed derivation submitted to *Phys. Rev. D* **[arXiv:2504.09304]**

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Hairy black holes: the qualitative picture

The RN instability should grow at the *non-perturbative* level to form **a hair.**

Same magnetic charge $P = n/(2e)$ as for RN black holes.

The hairy solutions emerge from the RN branch when $\mathrm{r_H} = \mathrm{r_H^0}(n) \approx \sqrt{|n|}/g$.

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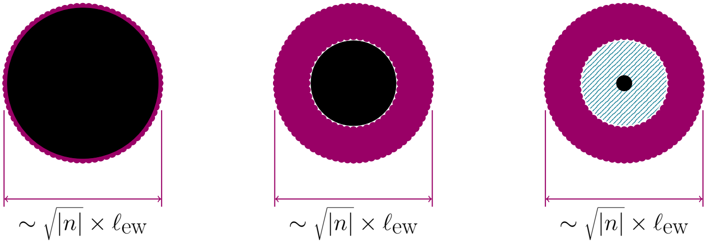
1) **The electroweak hair appears**, 2) **gets longer** as the horizon shrinks and 3) eventually **a bubble of symmetric phase appears.**

1) $|B(\mathrm{r_H})| \approx m_W^2$, 2) $m_W^2 < |B(\mathrm{r_H})| < m_H^2$, 3) $|B(\mathrm{r_H})| > m_H^2$



Axially symmetric ansatz

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$$ds^2 = -e^{2\mathrm{U}}\mathrm{N}(\mathrm{r})dt^2 + e^{2\mathrm{K}}\left(\frac{d\mathrm{r}^2}{\mathrm{N}(\mathrm{r})} + \mathrm{r}^2 d\vartheta^2\right) + e^{2\mathrm{S}}\,\mathrm{r}^2\sin^2\vartheta\, d\varphi^2,$$

where $\mathrm{N}(\mathrm{r})$ is a *gauge-fixing* function *s.t.* $\mathrm{N}(\mathrm{r}_\mathrm{H}) = 0$.

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→ **Purely magnetic** electroweak fields,

$$W_\mu dx^\mu = \frac{\tau_2}{2}\left(-\frac{1}{\mathrm{r}}H_1\, d\mathrm{r} + H_2\, d\vartheta\right) + \frac{n}{2}\left[(\cos\vartheta + H_3\,\sin\vartheta)\frac{\tau_3}{2} - H_4\,\sin\vartheta\,\frac{\tau_1}{2}\right] d\varphi,$$

$$B_\mu dx^\mu = \frac{n}{2}(\cos\vartheta + y\,\sin\vartheta)\,d\varphi \qquad \Phi = (\phi_1, \phi_2)^T \text{ with } \phi_1, \phi_2 \in \mathbb{R}.$$

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→ **10 nonlinear elliptic** field equations to be solved (**FreeFem** solver).

The hair charge

In the **Nambu description**, a part of the charge P is stored **in the hair.**

$$P_{\rm h} = \int_{\rm r>r_H} \tilde{\mathcal{J}}^0 \sqrt{-g}\, d^3x, \quad P_{\rm H} = P - P_{\rm h}.$$

$P_{\rm h}$: charge *inside the hair*, $P_{\rm H}$: *horizon* charge.

<u>RN black holes:</u> $P_{\rm h} = 0$, $P_{\rm H} = P$, all charge is **inside the horizon.**

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RN black holes: $P_{\mathrm{h}} = 0$, $P_{\mathrm{H}} = P$, all charge is **inside the horizon.**

Hairy black holes: P_{h} **grows when the horizon shrinks.** A maximum is reached in the extremal limit,

$$P_{\mathrm{h}} = g'^2 \times P = 0.22 \times P.$$

$\rightarrow$ **22% of the charge** moves to the hair.

The ADM mass

The black hole mass M is computed from the **asymptotic behavior** $g_{00} = -1 + 2M/r + \dots$ or from the **Komar integral**,

$$M = \frac{k_{\rm H} A_{\rm H}}{4\pi} + \frac{\kappa}{8\pi} \int_{\rm r>r_H} \left(-T_0^0 + T_k^k\right) \sqrt{-g}\, d^3x,$$

<u>surface gravity:</u> $k_{\rm H} = (1/2)\, N'(\rm r)\, e^{\rm U-K}|_{\rm r=r_H}$,

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Like for the charge, the mass split into,

$$M = M_{\mathrm{H}} + M_{\mathrm{h}}, \qquad M_{\mathrm{H}} \neq \frac{k_{\mathrm{H}} A_{\mathrm{H}}}{4\pi}.$$

The horizon mass M_{H} is defined as the mass of a RN black hole of size $\mathrm{r_H}$ and charge $P_{\mathrm{H}} \leq P$.

Hairy black holes: M_{h}/M **grows when the horizon shrinks.**

The horizon oblateness

The configurations are **not spherical.** We define,

$$\textit{equatorial}\text{ horizon radius:}\quad r_{\rm eq} = \sqrt{g_{\varphi\varphi}(\mathrm{r_H}, \pi/2)},$$

$$\textit{polar}\text{ horizon radius:}\quad r_{\rm pl} = \frac{1}{\pi}\int_0^\pi \sqrt{g_{\vartheta\vartheta}(\mathrm{r_H}, \vartheta)}\, d\vartheta,$$

$$\textit{average}\text{ radius:}\quad r_h = \sqrt{A_{\rm H}/(4\pi)},$$

$$\text{horizon }\textit{oblateness}\text{:}\quad \delta = r_{\rm eq}/r_{\rm pl} - 1.$$

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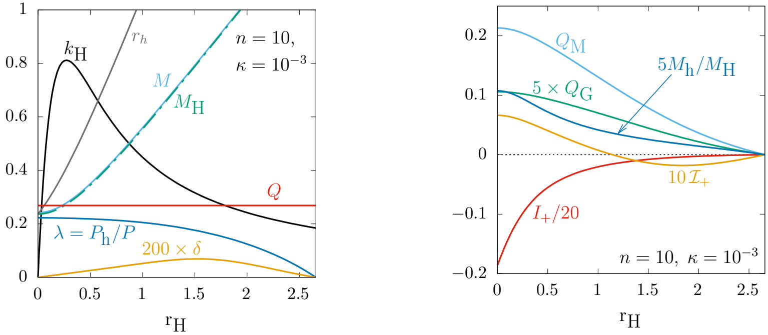
Hairy black holes: as the horizon shrinks, δ increases, then reaches a maximum, and eventually **vanishes in the extremal limit.**

→ Extremal hairy black holes have **a perfectly spherical horizon.**

Output parameters for $n = 10$, $\kappa = 10^{-3}$:



Additional quantities:

I_+ (*'t Hooft*), $\mathcal{I}_+$ (*Nambu*) : **electric currents** in upper half-space,

Q_G, Q_M : gravitational and magnetic **quadrupole moments** [Fodor *et al.*, 2021].

Magnetic field profile

We define the **'t Hooft** electromagnetic field ΔF of the condensate/hair as,

$$\boxed{\Delta F = F - \underbrace{\frac{n}{2e} \sin\vartheta \, d\vartheta \wedge d\varphi}_{\text{charge inside the BH}} .}$$

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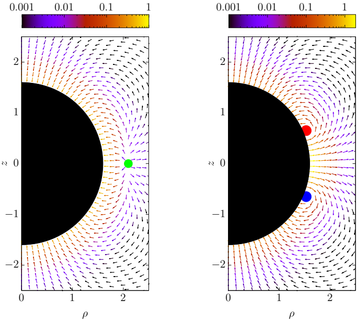
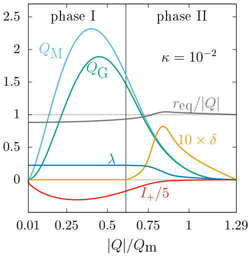


Nambu (left): condensate field $\Delta\vec{\mathcal{B}}$ is sourced by the **smooth distribution of magnetic charge** P_{h} outside the black hole.

't Hooft (right): condensate field $\Delta\vec{\mathcal{B}}$ is sourced by **two oppositely directed electric currents** $I_{\pm}$ (all charge inside the horizon).

Extremal hairy black holes



If $|Q| = \sqrt{\kappa/2}\, P \ll Q_{\rm m} = \sqrt{2/(\kappa\beta)}/g$:

electroweak fields at $r_{\rm H}$ are in the **false vacuum** ($|\Phi| = W^a_\mu = 0$, B_μ is Coulombian).

$\rightarrow$ The horizon geometry is **spherical** ($\delta = 0$).

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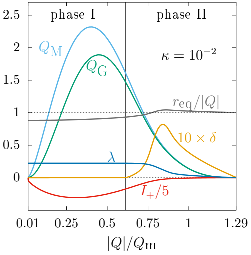

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  $|Q| \approx 0.62\, Q_{\rm m} \quad \Rightarrow \quad |B(r_{\rm H})| \approx m_H^2$.
  $\rightarrow$ The black hole starts *absorbing* the hair.

- Extremal hairy BHs merge with extremal RN at
  $|Q| \approx 1.29\, Q_{\rm m} \quad \Rightarrow \quad |B(r_{\rm H})| \approx m_W^2$.
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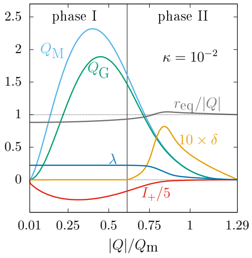

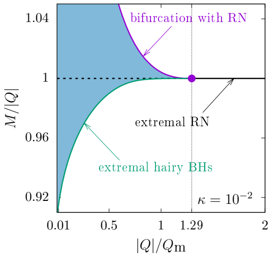

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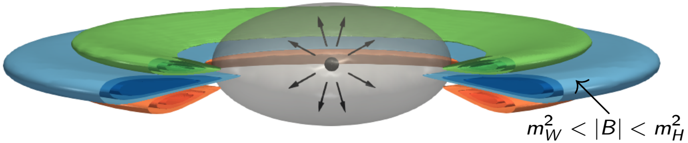
- *For all* extremal hairy solutions: $\boxed{M \leq |Q|}$.

An extremal hairy black hole in phase I **(Nambu description):**



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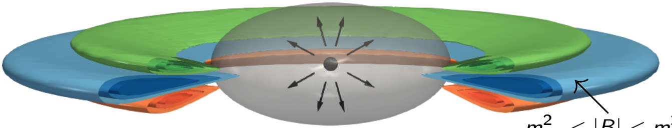

$\rightarrow$ The strong magnetic field produces a *bubble* of **Higgs false vacuum**: restoration of the *full* electroweak gauge symmetry near the horizon.

$\rightarrow$ Inside ($|B| > m_H^2$): $|\Phi| = W_\mu^a = 0$, B_μ is **Coulombian** and

$$g_{\mu\nu} \sim \text{ RN-de Sitter with } r_{\text{eq}} = r_{\text{ext.}}^{\text{RNdS}} \approx g|Q|.$$

$\rightarrow$ Far field ($|B| < m_W^2$): $|\Phi| = 1$, B_μ, W_μ^a are **Coulombian** and

$$g_{\mu\nu} \sim \text{ RN with } \boxed{M \approx g|Q| < |Q|}.$$

The weakness of gravity

The hair mass $M_{\mathrm{h}} \ll M$ is very small because of the **negative Zeeman energy** of the condensate interacting with the background magnetic field,

$$m_W^2 \to m_W^2 - |B| \approx 0 \quad \Rightarrow \quad \frac{M_{\mathrm{h}}}{Q_{\mathrm{h}}} \ll 1, \qquad (Q_{\mathrm{h}} = 0.22 \times Q).$$

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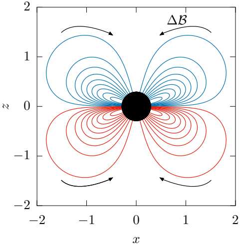
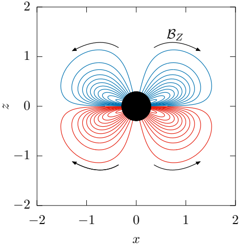
$$M = M_{\rm H} + M_{\rm h} \approx M_{\rm H} \approx g|Q| < |Q|.$$

$\rightarrow$ Hairy black holes are **energetically favored** as compared to RN.

The black hole "corona"





- The magnetic field lines form **loops** starting and ending at the horizon **('t Hooft description).**

  $\rightarrow$ The black hole **"corona".**

- The *massless* and *massive* magnetic fields $\Delta\mathcal{B}$ and $\mathcal{B}_Z$ flow in opposite directions.

  $\rightarrow$ They are sourced by **opposite currents** of gauge bosons.

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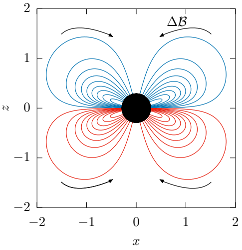

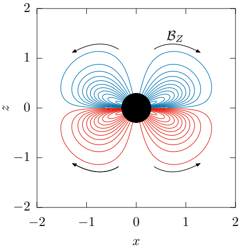

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Order of magnitudes: (for *maximally* hairy BHs, $|n| \sim 10^{32}$)

$$\boxed{\begin{array}{l} \boldsymbol{r}_{\text{hor}} \approx 1.4\,\text{cm}, \quad \boldsymbol{r}_{\text{hair}} \approx 2.2\,\text{cm}, \quad |\boldsymbol{\mathcal{B}}(\boldsymbol{R}_\odot)| \approx 0.1\,\text{T}, \\ \boldsymbol{M} \approx 2 \times 10^{25}\,\text{kg}, \quad \boldsymbol{M}_{\text{hair}} \approx 11\% \times \boldsymbol{M}. \end{array}}$$

Summary

- Black holes with an **axially symmetric** electroweak hair are constructed numerically.

  $\rightarrow$ Very compelling candidates for both **hairy black holes** and **magnetic monopoles.**

  No physics beyond the Standard Model + GR is required!

- The hair features an electroweak *symmetric phase* where $|\Phi| \approx 0$ and contains **W/Z-boson** currents generating the black hole **"corona"**.

- They *cannot* decay into RN because they are **less energetic:** $M \leq |Q|$.

  $\rightarrow$ They can still *reduce* their energy by splitting their hair into a hedgehog of vortices: the **"fully spread" corona** (never been described at the non-perturbative level).

- They could have been created in the fluctuating magnetic field of the **early Universe** $\rightarrow$ *primordial black holes*.

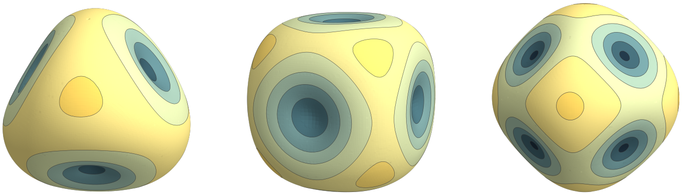
Thank you for your attention.

Beyond axial symmetry

Perturbative analysis reveals that the axially symmetric condensate **do not achieve the energy minimum** (except for $|n| = 4$).

$\rightarrow$ Decay to configurations with only **discrete symmetries:**

$|n| = 6$ $\quad$ $|n| = 8$ $\quad$ $|n| = 10$



Similar situation as in flat space, with an *external and uniform* magnetic field [Ambjorn, Olesen, 1988] - [Chernodub *et al.*, 2023].