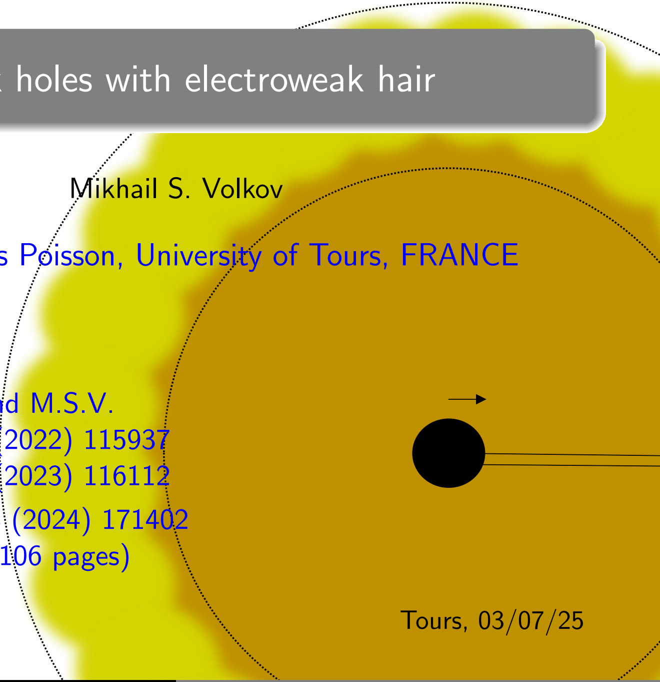
Black holes with electroweak hair

Mikhail S. Volkov

Institut Denis Poisson, University of Tours, FRANCE



Romain Gervalle and M.S.V.
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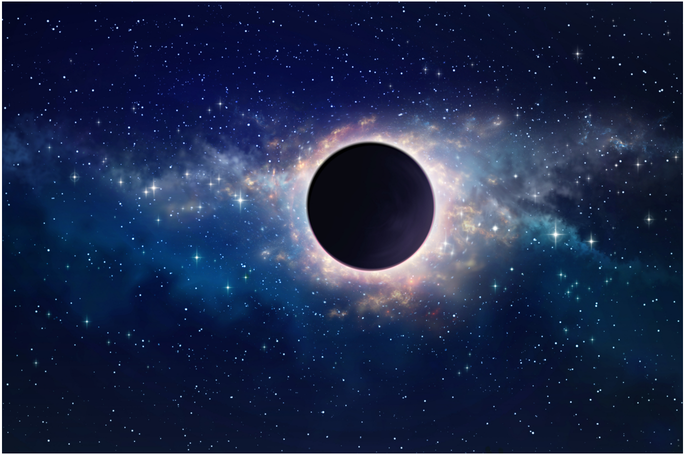
Isolated black hole – Kerr-Newman geometry



3 parameters: mass M, charge Q, angular momentum J.

Brief history: no-hair conjecture

No-hair conjecture /Ruffini and Wheeler, 1971/: black holes formed by gravitational collapse are characterized by their mass, angular momentum, and electric/magnetic charge. These are the only parameters that can survive during the gravitational collapse, all other information is lost. Black holes have no memory.

No-hair conjecture

From January 1971, pages 30–41

Introducing the black hole

Remo Ruffini and John A. Wheeler

According to present cosmology, certain stars end their careers in a total gravitational collapse that transcends the ordinary laws of physics.

At the time of this article, **Remo Ruffini** and **John Wheeler** were both at Princeton University; Wheeler, on leave from Princeton, was spending a year at the California Institute of Technology and Moscow State University.

The quasi-stellar object, the pulsar, the neutron star have all come onto the scene of physics within the space of a few years. Is the next entrant destined to be the black hole? If so, it is difficult to think of any development that could be of greater significance. A black hole, whether of "ordinary size" (approximately one solar mass, 1 $M_\odot$) or much larger (around $10^6\,M_\odot$ to $10^{10}\,M_\odot$, as proposed in the nuclei of some galaxies), provides our "laboratory model" for the gravitational collapse, predicted by Einstein's theory, of the universe itself.

A black hole is what is left behind after an object has undergone complete gravitational collapse. Spacetime is so strongly curved that no light can come out, no matter can be ejected, and no measuring rod can ever survive being put in. Any kind of object that falls into the black hole loses its separate identity, preserving only its mass, charge, angular momentum, and linear momentum (see figure 1). No one has yet found a way to distinguish between two black holes constructed out of the most different kinds of matter if they have the same mass, charge, and angular momentum. Measurement of these three determinants is permitted by their effect on the Kepler orbits of test objects, charged and uncharged, in revolution about the black hole.

How the physics of a black hole looks depends more upon an act of choice by the observer himself than on anything else. Suppose he decides to follow the collapsing matter through its collapse down into the black hole. Then he will see it crushed to indefinitely high density, and he himself will be torn apart eventually by indefinitely increasing tidal forces. No restraining force whatsoever has the power to hold him away from this catastrophe, once he crossed a certain critical surface known as the "horizon." The final collapse occurs a finite time after the passage of this surface, but it is inevitable. Time and space are interchanged inside a black hole in

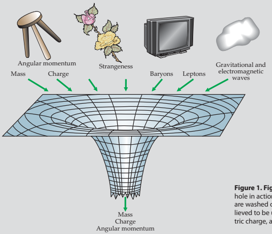


Figure 1. Figurative representation of a black hole in action. All details of the infalling matter are washed out. The final configuration is believed to be uniquely determined by mass, electric charge, and angular momentum.

Uniqueness and no-hair theorems

- Uniqueness theorems /Israel, Robinson, Mazur/: All electrovacuum holes are described by the Kerr-Newman metrics. This confirms the conjecture.

- Are there other black holes, not described by Kerr-Newman metrics ?

- No-hair theorems /Bekenstein, 1972,.../ confirm the conjecture for a number of special cases. Considering

$$G_{\mu\nu} = T_{\mu\nu}(\Phi), \qquad \Box\Phi = U(\Phi),$$

  where $\Phi =$ scalar, spinor, massive vector field, etc., field, one can show that the only black hole solutions are of the Kerr-Newman type.

- However, if $\Phi = A^a_\mu$ is a pure Yang-Mills field then there are new black holes without new charges:

First counterexample – black holes with Yang-Mills field

Non-Abelian Einstein-Yang–Mills black holes

M. S. Volkov and D. V. Gal'tsov
M. V. Lomonosov Moscow State University

(Submitted 7 September 1989)
Pis'ma Zh. Eksp. Teor. Fiz. **50**, No. 7, 312–315 (10 October 1989)

Solutions of the self-consistent system of Einstein-Yang–Mills equations with the SU(2) group are derived to describe black holes with a non-Abelian structure of gauge fields in the external region.

In the case of the electrovacuum, the most general family of solutions describing spherically symmetric black holes is the two-parameter Reissner–Nordström family, which is characterized by a mass M and an electric charge Q. It was recently shown for the Einstein-Yang–Mills systems of equations with the SU(2) group that a corresponding assertion holds when the hold has a nonvanishing color-magnetic charge. In this case the structure of the Yang–Mills hair is effectively Abelian.[1] In the present letter we numerically construct a family of definitely non-Abelian solutions for Einstein-Yang–Mills black holes in the case of zero magnetic charge. These solutions are characterized by metrics which asymptotically approach the Schwarzschild metric far from the horizon but are otherwise distinct from metrics of the Reissner–Nordström family. In addition to the complete Schwarzschild metric, the family of solutions is parametrized by a discrete value of n: the number of nodes of the gauge function. For a

Zoo of hairy black holes

- <u>before 2000</u>: Einstein-Yang-Mills black holes and their generalizations – higher gauge groups, additional fields (Higgs, dilaton), non-spherical solutions, stationary generalizations, Skyrme black holes, Gauss-Bonnet, . . . /M.S.V.+Gal'tsov, Phys.Rep. 319 (1999) 1/

- <u>after 2000</u>: black holes via engineering the scalar field potential, Horndeski black holes, spontaneously scalarized black holes, black holes supporting spinning clouds of ultralight bosons /Herdeiro-Radu/, hairy black holes in higher dimensions, with stringy corrections, with massive gravitons /Gervalle+M.S.V., 2020/, etc, . . . /M.S.V., 1601.0823/

- Which of these solutions are physically relevant ? Unfortunately, one cannot be too optimistic in this respect.

Present status of hairy black holes

- Almost all known hairy solutions have been obtained either within too much simplified models, or within exotic models relying on a new physics = yet undiscovered particles and fields. They are nice theoretically but their physical relevance is not obvious.

- New physics (stringy effects, SUSY, GUT fields, Horndeski fields, ultralight Dark Matter, massive gravitons, etc) may exist. However, its existence has not been confirmed yet.

- We therefore adopt the following viewpoint: a physically relevant solutions should be obtained within General Relativity (GR) + Standard Model (SM) of fundamental interactions.

- The SM contains the QCD sector with pure Yang-Mills (gluons). Therefore, hairy black holes with Yang-Mills field may have some relevance. However, classical configurations in QCD are destroyed by large quantum corrections.

Electroweak black holes ?

- The Standard Model contains also the electroweak (EW) sector where the quantum corrections are not very large. Therefore, it makes sense to study classical solutions of the Einstein-Weinberg-Salam theory. This theory contains the Einstein-Maxwell sector and hence describes the Kerr-Newman black holes.
- Does it describe something else ?
- Only unphysical limits of the electroweak theory (vanishing Weinberg angle) have been analyzed in the black hole context, since in the full theory the spherical symmetry is lost due to the electroweak condensation.

Electroweak condensation /Ambjorn-Olesen 1989/

- Constant homogeneous magnetic field $\vec{\mathcal{B}} = (0, 0, \mathcal{B})$ may exist if only $\mathcal{B} < m_{\rm w}^2/e \approx 10^{20}$ Tesla.
- For $m_{\rm w}^2/e < \mathcal{B} < m_{\rm h}^2/e$ the vacuum structure changes leading to the appearance of a condensate of massive W, Z, Φ fields forming a lattice of vortices (flux tubes). Anti-Lenz: the magnetic field is maximal where the condensate is maximal.

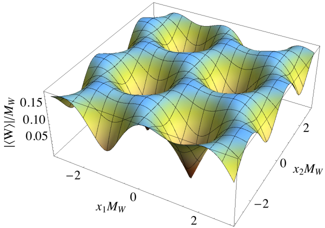
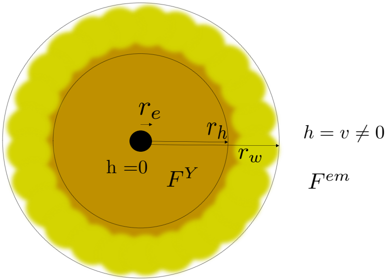


Figure: /Chernonub et al 2013/

- For $B > m_{\rm h}^2/e$ the vortices disappear and the Higgs field approaches zero – the full electroweak symmetry is restored.

Magnetic electroweak black hole /Maldacena 2020/



Radial magnetic field near the horizon where Higgs=0, followed by electroweak "corona" made of vortex pieces, followed by radial magnetic field in the far field where Higgs is constant = magnetic Reissner-Nordstrom.

Nobody tried to confirm this

Preliminary analyzis in flat space

The electroweak corona should exist already in flat space around a magnetic charge.

The best known magnetic monopole of 't Hooft-Polyakov is not described by the Standard Model.

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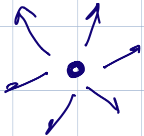
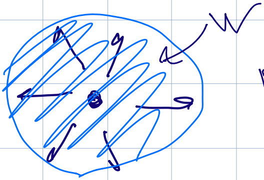
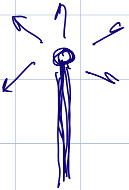
Electroweak monopoles

1)  $P = \frac{n}{2e}$, $n \in \mathbb{Z}$ Dirac

2) 

$P = \frac{1}{e}$ Cho - Maison

3)  $P = \frac{\sin^2\theta_W}{e}$ Nambu

Static monopoles

Monopoles of Dirac and Cho-Maison are static and spherically symmetric, both with infinite energy:

- Pointlike Dirac monopole for any value of the magnetic charge $P = n/(2e)$, $n \in \mathbb{Z}$. It turns out that all Dirac monopoles with $|n| > 1$ are unstable with respect to condensation.

- Non-Abelian monopole of Cho-Maison for $n = \pm 2 \Rightarrow$ superposition of a pointlike hypermagnetic U(1) monopole and a regular SU(2) condensate. It turns out that this solution is stable $\Rightarrow$ it can be viewed as the Dirac monopole dressed by the condensate.

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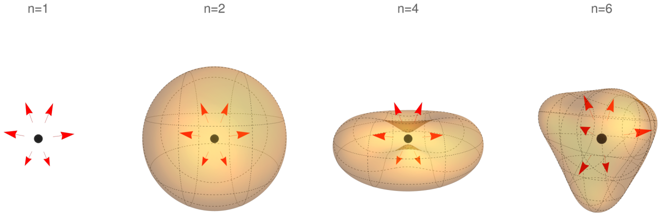
High charge generalizations of Cho-Maison



We constructed numerically monopoles with axial symmetry up to $|n| = 200$. They contain a pointlike magnetic charge surrounded by a condensate. The energy is infinite due to the central singularity.

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When gravity is taken into account, the singularity should be shielded by a horizon and the energy will become finite.

Including gravity

/PRL 133 (2024) 171402;
arXiv:2504.09304 /

Einstein-Weinberg-Salam theory

$$\mathcal{L} = \frac{1}{2\kappa} R + \mathcal{L}_{\rm WS}$$

$$\mathcal{L}_{\rm WS} = -\frac{1}{4g^2}\, \mathrm{W}^a_{\mu\nu} \mathrm{W}^{a\mu\nu} - \frac{1}{4g'^2}\, B_{\mu\nu} B^{\mu\nu} - (D_\mu \Phi)^\dagger D^\mu \Phi - \frac{\beta}{8} \left(\Phi^\dagger \Phi - 1 \right)^2$$

where Higgs is a complex doublet, $\Phi = (\phi_1, \phi_2)^{\rm T}$,

$$\begin{aligned} \mathrm{W}^a_{\mu\nu} &= \partial_\mu \mathrm{W}^a_\nu - \partial_\nu \mathrm{W}^a_\mu + \epsilon_{abc} \mathrm{W}^b_\mu \mathrm{W}^c_\nu \,, \qquad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \,, \\ D_\mu \Phi &= \left(\partial_\mu - \frac{i}{2}\, B_\mu - \frac{i}{2}\, \tau^a \mathrm{W}^a_\mu \right) \Phi \,. \end{aligned}$$

The length scale and mass scale are $\boldsymbol{l}_0 = 1.5 \times 10^{-16}$ cm and $\boldsymbol{m}_0 = 128.6$ GeV. The couplings $g' = \sin\theta_{\rm W}$, $\quad g = \cos\theta_{\rm W}$,

$$g^2 = 0.78, \quad g'^2 = 0.22, \quad \beta = 1.88, \quad \kappa = \frac{4e^2}{\alpha} \frac{m_{\rm z}^2}{M_{\rm pl}^2} = 5.30 \times 10^{-33}.$$

Electron charge $e = gg'$, $\alpha = 1/137$. The Z, W, Higgs masses in unites of $\boldsymbol{m}_0$ are $m_{\rm z} = 1/\sqrt{2}$, $m_{\rm w} = g m_{\rm z}$, $m_{\rm h} = \sqrt{\beta} m_{\rm z}$.

Electromagnetic field (no unique definition if $\Phi \neq const$):

Nambu: $e\mathcal{F}_{\mu\nu} = g^2 B_{\mu\nu} - g'^2 n_a \mathrm{W}^a_{\mu\nu}, \quad n_a = (\Phi^\dagger \tau_a \Phi)/(\Phi^\dagger \Phi)$

defines conserved electric and magnetic currents

$$4\pi \mathcal{J}^\mu = \nabla_\nu \mathcal{F}^{\mu\nu}, \qquad 4\pi \tilde{\mathcal{J}}^\mu = \nabla_\nu \tilde{\mathcal{F}}^{\mu\nu},$$

magnetic charge

$$\boxed{P = \int \tilde{\mathcal{J}}^0 \sqrt{-\mathrm{g}} d^3 x.}$$

't Hooft: $F_{\mu\nu} = \mathcal{F}_{\mu\nu} + \epsilon_{abc} n^a \mathcal{D}_\mu n^b \mathcal{D}_\nu n^c = \partial_\mu A_\nu - \partial_\nu A_\mu,$

electric current

$$\boxed{4\pi J^\mu = \nabla_\nu F^{\mu\nu}}.$$

Using Nambu for magnetic charge and t'Hooft for electric current.

30 coupled equations to solve:

Weinberg-Salam:

$$\nabla^\mu B_{\mu\nu} = g'^2 \, \frac{i}{2} \, (\Phi^\dagger D_\nu \Phi - (D_\nu \Phi)^\dagger \Phi),$$

$$\mathcal{D}^\mu \mathrm{W}^a_{\mu\nu} = g^2 \, \frac{i}{2} \, (\Phi^\dagger \tau^a D_\nu \Phi - (D_\nu \Phi)^\dagger \tau^a \Phi),$$

$$D_\mu D^\mu \Phi - \frac{\beta}{4} \, (\Phi^\dagger \Phi - 1) \Phi = 0,$$

Einstein:

$$G_{\mu\nu} = \kappa T_{\mu\nu}$$ where $\kappa \sim 10^{-33}$ is very small and

$$T_{\mu\nu} = \frac{1}{g^2} \mathrm{W}^a{}_{\mu\sigma} \mathrm{W}^a{}_\nu{}^\sigma + \frac{1}{g'^2} B_{\mu\sigma} B_\nu{}^\sigma + 2 D_{(\mu} \Phi^\dagger D_{\nu)} \Phi + \mathrm{g}_{\mu\nu} \mathcal{L}_{\mathrm{WS}}$$

=30 coupled equations. Vacuum solution:

$$\mathrm{g}_{\mu\nu} = \eta_{\mu\nu}, \quad \mathrm{B} = \mathrm{W} = 0, \;\; \Phi = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Simplest solutions: Reissner-Nordstrom, RN-de Sitter

Same electroweak fields as for Dirac monopole, *Higgs in vacuum:*

$$B = W^3 = -\frac{n}{2}(\cos\vartheta \mp 1)\, d\varphi, \quad W^1 = W^2 = 0, \quad \Phi = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

$$A = \frac{g}{g'} B + \frac{g'}{g} W^3 = \frac{B}{e}, \quad \vec{\nabla} \wedge \vec{A} = \frac{P\vec{r}}{r^3}, \quad P = \frac{n}{2e}, \quad \boxed{n \in \mathbb{Z}}$$

n, P=magnetic charge; RN geometry:

$$ds^2 = -N(r)\, dt^2 + \frac{dr^2}{N(r)} + r^2 \left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right),$$

$$N(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}, \quad Q^2 = \frac{\kappa n^2}{8e^2}, \quad r_h = M + \sqrt{M^2 - Q^2}$$

Describes the $r \to \infty$ limit of the hairy black holes.
RN-de Sitter: $W^1 = W^2 = W^3 = \Phi = 0$, same B. Its extremal limit describes the horizon geometry of extremal hairy black holes.

Change of stability of Reissner-Nordström

- The solution is expected to change stability when r_h decreases, the horizon value of the magnetic field $\mathcal{B}_h = P/r_h^2$ then increases until the condensation starts ($\Leftrightarrow$ scalarization).
- When a tachyonic mode emerges, this indicates the appearance of new solution branch which bifurcates from the Reissner-Nordström family.

We wonsider generic perturbations around RN,

$$g_{\mu\nu} \to g_{\mu\nu} + \delta g_{\mu\nu}, \quad \mathrm{W}_\mu^a \to \mathrm{W}_\mu^a + \delta \mathrm{W}_\mu^a$$
$$B_\mu \to B_\mu + \delta B_\mu, \quad \Phi \to \Phi + \delta\Phi$$

Perturbations $w_\mu = \delta \mathrm{W}^1_\mu + i\delta \mathrm{W}^2_\mu$ fulfil charged Proca:

$$
\begin{aligned}
D^\mu w_{\mu\nu} + ieF_{\nu\sigma} w^\sigma &= m_{\mathrm{w}}^2 w_\nu \quad \Rightarrow \quad \text{(Newman-Penrose)} \\
w_\mu dx^\mu &= \sum_{\mathrm{m}\in[-j,j]} c_{\mathrm{m}} e^{i\omega t} \psi(r) w_{j\mathrm{m}}(\vartheta) e^{i\mathrm{m}\varphi}, \\
w_{j\mathrm{m}}(\vartheta) &= (\sin\vartheta)^j \left(\tan\frac{\vartheta}{2}\right)^{\mathrm{m}} (d\vartheta + i\sin\vartheta d\varphi), \\
j &= |n|/2 - 1 \ \Rightarrow \ \boxed{j = 0 \text{ only if } |n| = 2}.
\end{aligned}
$$

For $|n| > 2$ perturbations are not spherically symmetric.

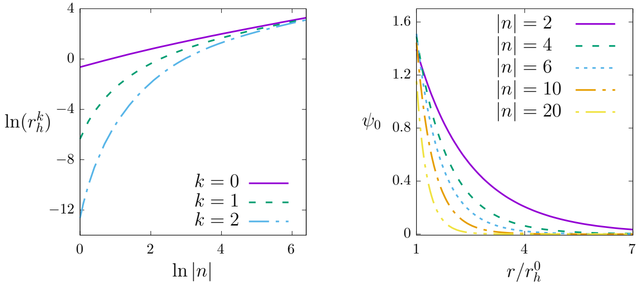
$$
\left(-\frac{d^2}{dr_\star^2} + N(r)\left[m_{\mathrm{w}}^2 - \frac{|n|}{2r^2}\right]\right)\psi(r) = \omega^2\psi(r),
$$

$\omega^2 > 0$ if r_h is large, $\omega^2 < 0$ if r_h is small $\Rightarrow \exists$ an intermediate value: a condensation threshold $r_h = r_h^0(n)$ for which there is a zero mode $\psi_0(r)$ describing a condensate that starts to appear.

Zero modes

n	2	4	6	10	20	40	100	200
r_H^0	0.89	1.47	1.93	2.69	4.12	6.19	10.33	15.03

One has $r_h^0(n) \approx \sqrt{|n|}/g$ for $n \gg 1$ hence $B(r_h^0) \approx m_{\mathrm{w}}^2$, which is the condition for the condensate to appear. The condensate is maximal at the horizon.



Static condensate and horizon currents

The static condensate

$$w = \sum_{\mathrm{m}\in[-j,j]} c_{\mathrm{m}} \psi_0(r) w_{j\mathrm{m}}(\vartheta) e^{i\mathrm{m}\varphi}$$

produces a current $J^\mu = \nabla_\sigma \Im(\bar{w}^\sigma w^\mu)$ tangent to the horizon. The current sources second order corrections for the F, Z, Φ fields forming vortices orthogonal to the horizon. The coefficients c_{m} are obtained via minimizing the condensate energy:

$$\langle |w_\mu|^4 \rangle \equiv \int |w_\mu|^4 \sqrt{-\mathrm{g}}\, d^3x \,,$$

by keeping fixed the norm

$$\langle |w_\mu|^2 \rangle \equiv \int |w_\mu|^2 \sqrt{-\mathrm{g}}\, d^3x = const.$$

This gives values of c_{m} determining positions of $|n| - 2$ vortices homogeneously distributed over the horizon:

Lattice of vortices – corona

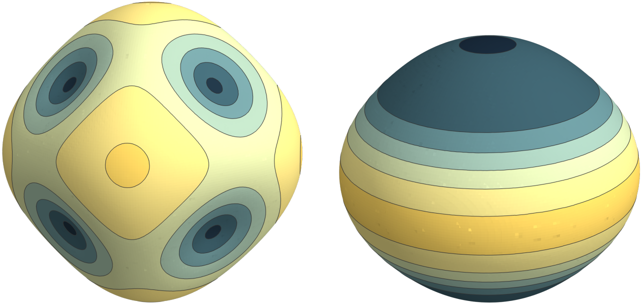
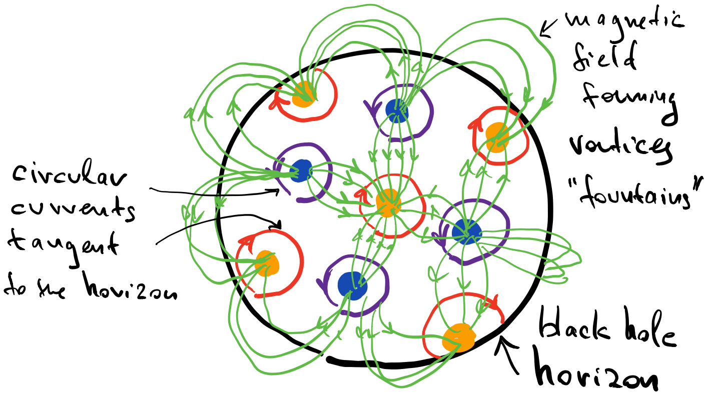


Figure: Left: the horizon distribution of the W-condensate $\bar{w}^{\mu} w_{\mu}$ corresponding to the global energy minimum for $n = 10$. The level lines coincide with the electric current flow forming loops around 8 radial vortices (dark spots) repelling each other and forming a lattice.
Right: the same when all vortices merge into two oppositely directed multi-vortices with axial symmetry, $c_{\rm m} \sim \delta_{0\rm m}$, also a stationary point.

Coronal loops in the black hole "atmosphere"



Non-perturbative axial symmetry

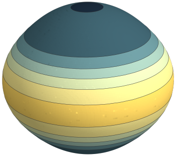
- Generic hairy black holes have no symmetries – full 3D simulations are needed.
- We study the axially symmetric case, which allows for any value of the magnetic charge and requires only 2D simulations.
- The axially symmetric solutions are unstable because the two multi-vortices tend to split into individual vortices.
- However, we find that far away from the horizon their geometry approaches the magnetic Reissner-Nordstrom whose mass is below the charge,

$$M < |Q|$$

This is explained by the presence of the charged condensate outside the horizon whose Zeeman energy is negative. Therefore, they cannot get rid of the hair and become RN, so the presence of the hair is a stable feature.

Stability

- According to Maldacena, the corona enhances the Hawking evaporation rate, hence non-extremal black holes should quickly relax to the extremal state where $M < |Q|$ and they cannot decay into RN black holes.
- However, axially symmetric black holes can further reduce their mass by splitting their hair into a hedgehog of vortices – "spreading the corona". Then the condensate energy achieves an absolute minimum and the hairy black holes seem to become absolutely stable. The corresponding solutions have not yet been obtained.



$\Rightarrow$