Exotic properties of strongly interacting matter under acceleration and rotation

Maxim Chernodub

Institut Denis Poisson, CNRS, Tours, France
Department of Physics, West University of Timișoara, Romania

first-principle numerical (Monte Carlo) simulations

Plan(-maximum) of the talk

1) **A pre-puzzle:** Phase diagram of rotating gluon plasma: simulations vs. theory show no agreement

1) **A puzzle:** Negative moment of inertia of gluon plasma

V. V. Braguta, A. A. Roenko, D. A. Sychev, M.Ch, Phys. Lett. B 852 (2024) 138604 [ArXiv 2303.03147]

2) **A hint to resolve the puzzle? -** Negative Barnett effect and evaporation of the gluon condensate

V. V. Braguta, I. E. Kudrov, A. A. Roenko, D. A. Sychev, M. Ch., Phys. Rev. D 110 (2024) 1, 014511 [ArXiv:2310.16036]

3) **A mystery:** New mixed inhomogeneous phase in vortical gluon plasma in equilibrium: ~~defying~~ turning inside out the Tolman-Ehrenfest picture

V. V. Braguta, A. A. Roenko, M. Ch, Phys. Lett. B 855 (2024) 138783 [ArXiv:2312.13994]

4) **A hint to resolve the mystery? -** The importance of being magnetovortical: Is the gluon angular momentum important for thermodynamics of rotating plasmas?

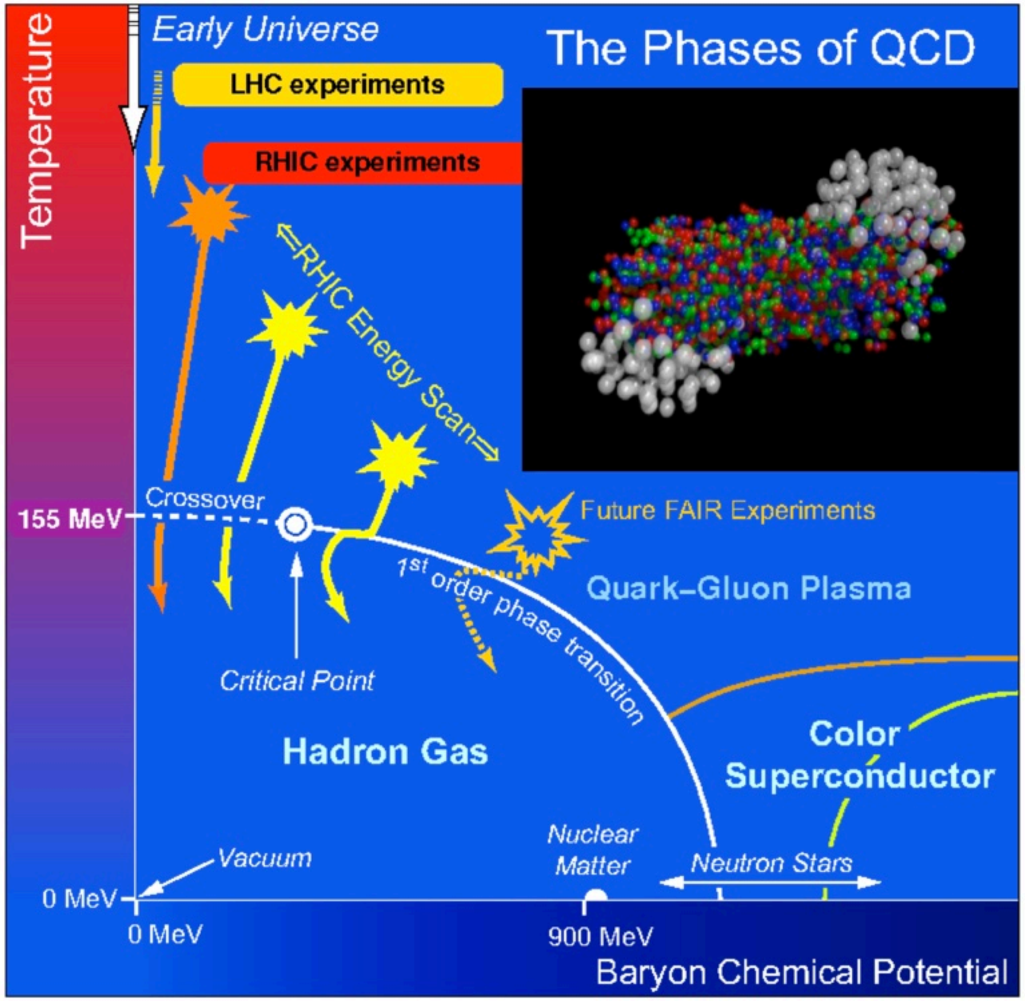
V. V. Braguta, A. A. Roenko, M. Ch., ArXiv:2411.15085

5) **Surprising results:** Acceleration has a softening effect on gluon plasma

V.A. Goy, A.V. Molochkov, D.V. Stepanov, A.S. Pochinok and M.Ch., Phys.Rev.Lett. 134 (2025) 11, 111904 [ArXiv:2409.01847]

Phase diagram of QCD

1) Hot quark-gluon plasma phase and cold hadron phase constitute, basically, one single phase because they are separated by a nonsingular transition ("crossover").



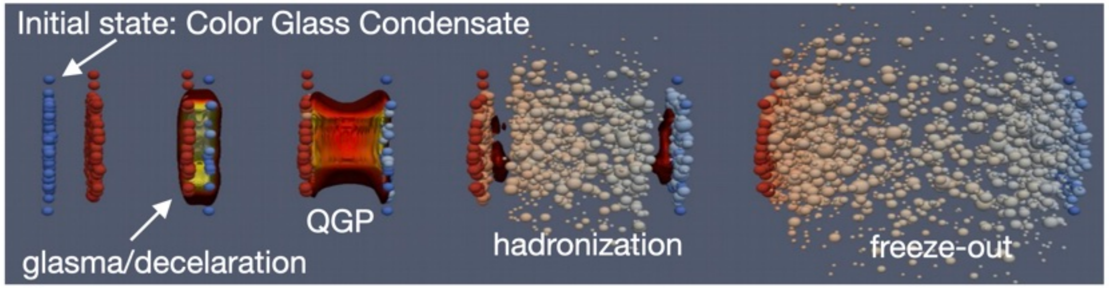
From a BNL webpage

2) The color superconducting phases at high baryonic chemical potential μ were extensively studied theoretically [they are out of reach of both lattice simulations and Earth-based experiments yet]

3) The LHC and RHIC experiments probe low baryon density physics. One can safely take $\mu = 0$ in further discussions.

Non-inertial regimes in stages of the collisions

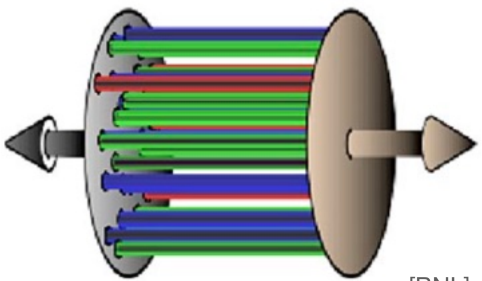
[adapted after MADAI collaboration, Hannah Petersen, Jonah Bernhard]



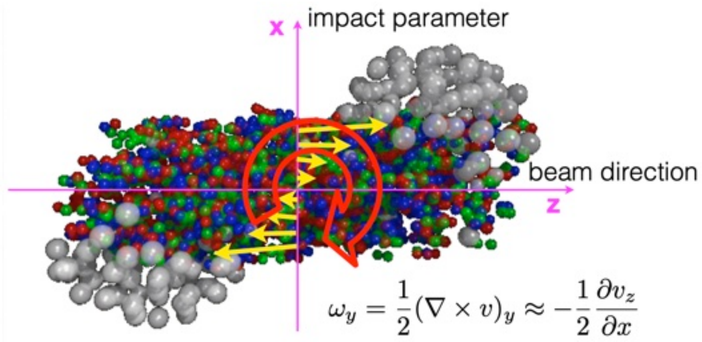
non-inertial regimes

(1) acceleration (deceleration)

(2) rotation in non-central collisions



[BNL]



thermodynamic equilibrium

a **gluon-rich medium**

ω **quark-gluon plasma regime**

strong gluon force at short distances cause deceleration

Glasma state, strong longitudinal chromo-electric and chromo-magnetic fields

The QGP from the earliest phase of ultrarelativistic heavy-ion collisions is expected to be far from equilibrium.

deceleration = acceleration taken with a minus sign

Thermal equilibrium in non-inertial frames

Systems under uniform acceleration$^{(*)}$ and/or solid rotation$^{(**)}$ admit the global thermal equilibrium state.

(1) we do not discuss here how the global thermalization is achieved

(2) take proper acceleration and the angular velocity to be time-independent

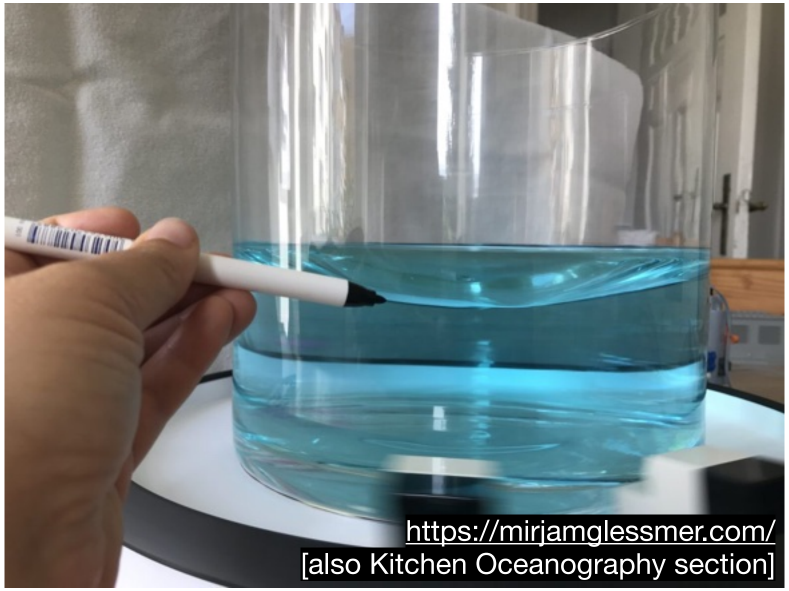
Examples:

acceleration

a

and

rotation



ω

For both examples, global thermal equilibrium is achieved in a non-uniform state

(*) non-uniform acceleration produces entropy/heat

(**) the same is true for non-solid rotation

(2) The most vortical fluid ever observed

The experimental result for the vorticity:

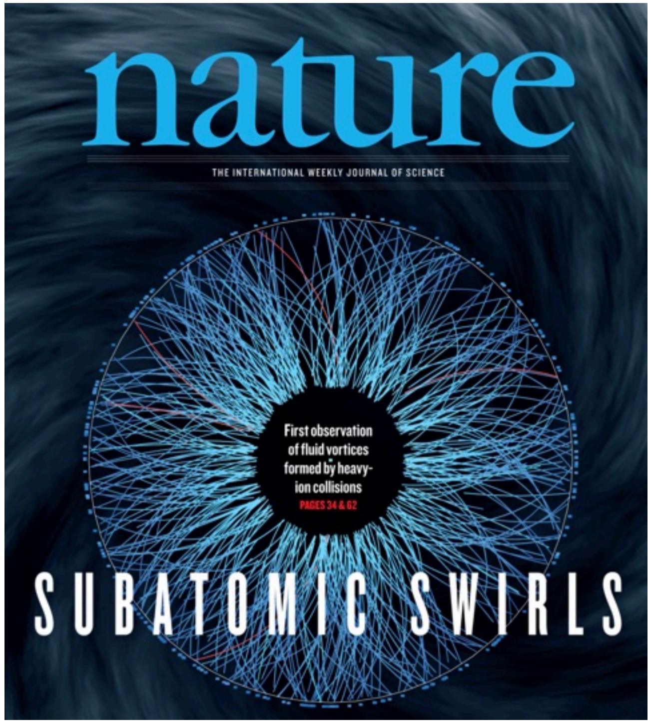
$$\omega \approx (9 \pm 1) \times 10^{21}\,\mathrm{s}^{-1}$$

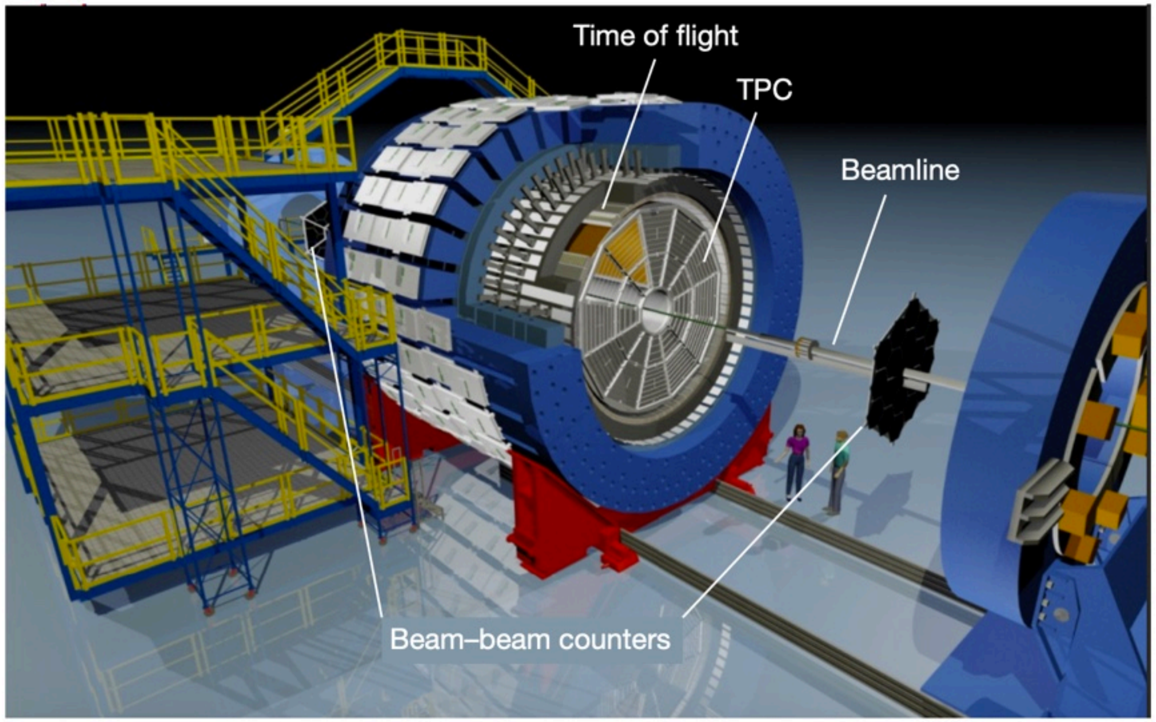
A "non-relativistic rotation" in a relativistic system:

1) $\omega \sim 6\,\mathrm{MeV};\ T_c \simeq 150\,\mathrm{MeV} \ \rightarrow\ \omega/T_c \simeq 0.04 \ll 1$

2) $R \sim (3\ldots10)\,\mathrm{fm} \rightarrow\ \mathrm{v} = \omega R \sim (0.1\ldots0.3)c \rightarrow \gamma = (1.004\ \ldots\ 1.05)$

system's size — boundary velocity — the Lorentz factor

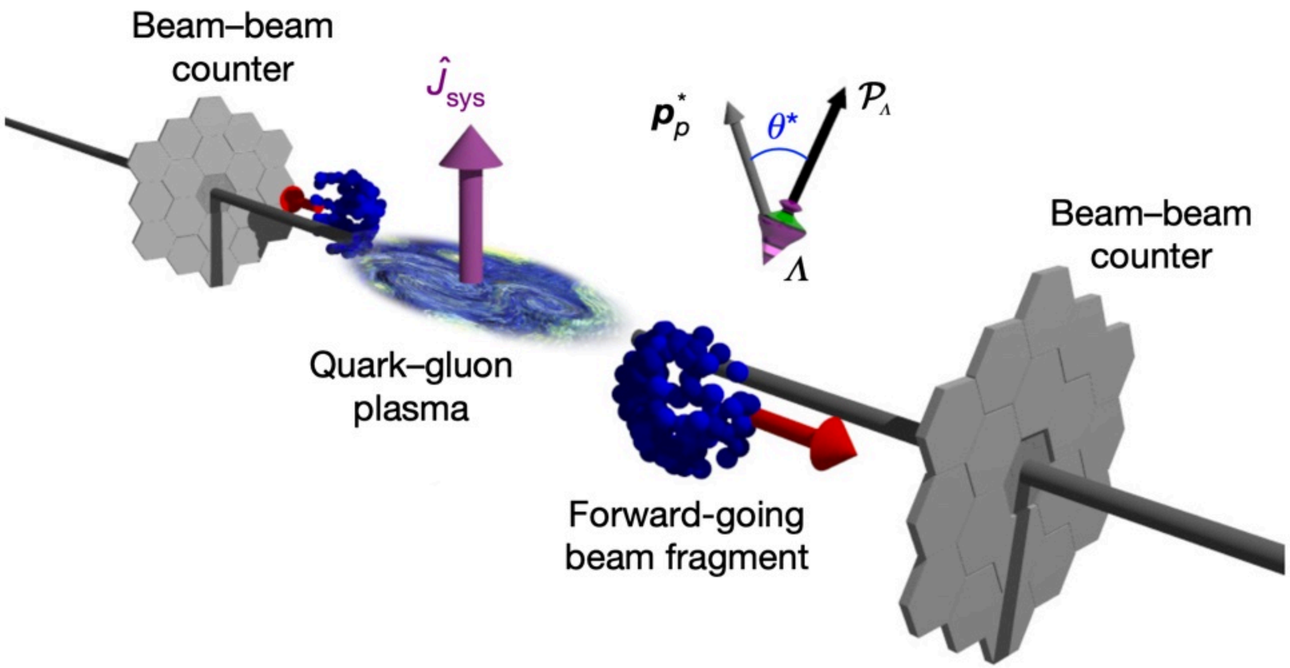


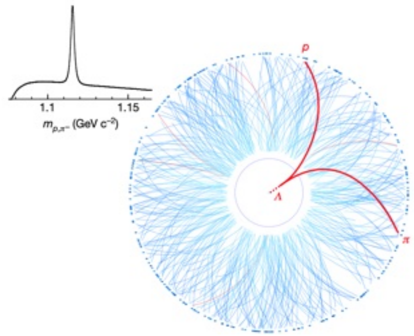


The STAR Collaboration, Nature 62, 548 (2017)

How to measure the polarization?

The observed hyperon spin polarization ignited much interest.





$$\Lambda \to p + \pi^-$$

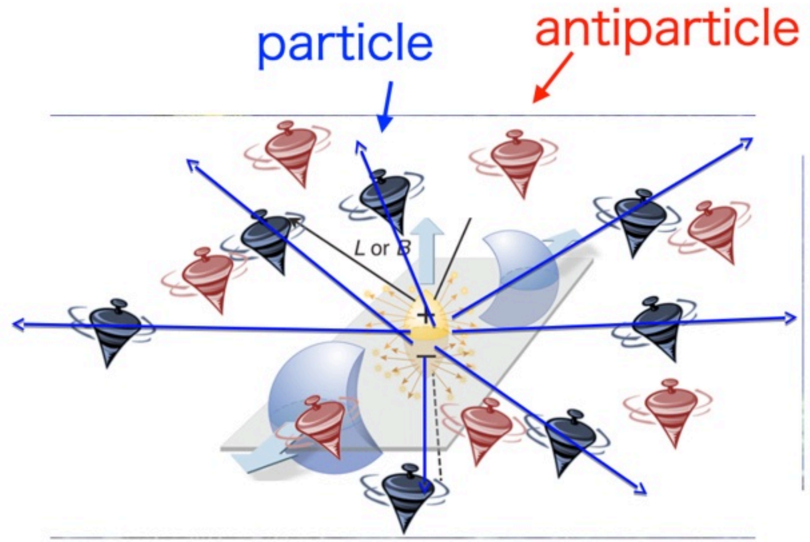
(BR: 63.9%, cτ ~7.9 cm)

Overview (including experimental status): “Polarization and Vorticity in the Quark–Gluon Plasma”, F. Becattini, M. A. Lisa, Ann.Rev.Nucl.Part.Sci. 70, 395 (2020)

Overview of the theoretical models: “Vorticity and Spin Polarization in Heavy Ion Collisions: Transport Models”, X.-G. Huang, J. Liao, Q. Wang, X.-L. Xia, Lect.Notes Phys. 987, 281 (2021)

How to measure the vorticity?

the vorticity could be probed via quark's spin polarization



The mechanism:

1) **orbital angular momentum of the rotating quark-gluon plasma is transferred to the particle spin**

2) **both particles and anti-particles are polarized in the same way (spin polarization is not sensitive to the particle charge)**

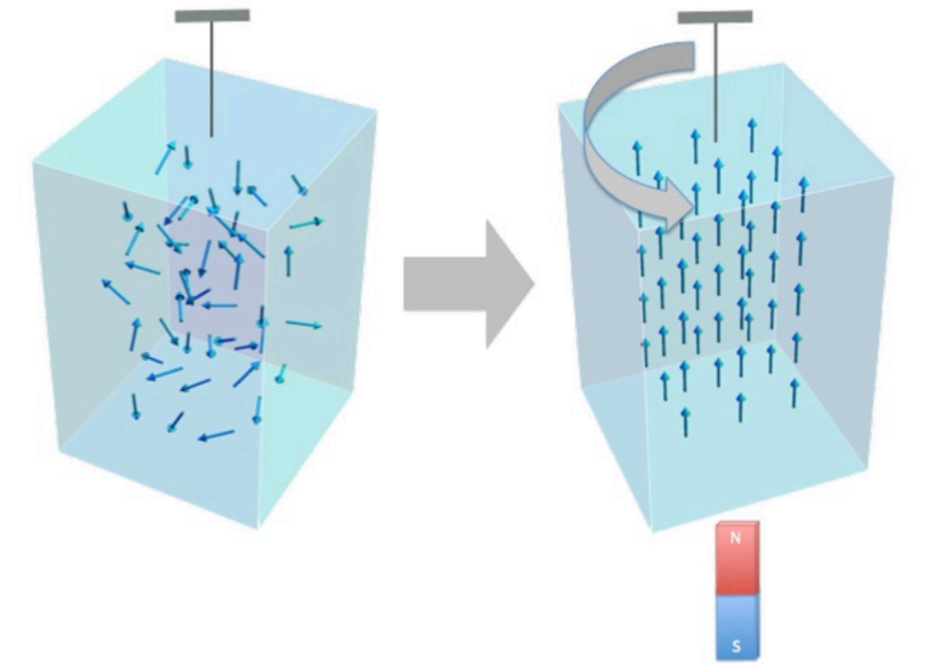
3) **The vorticity may be measured via the polarization of the produced particles**

In this example: particles = hyperons (+ vector mesons, etc)

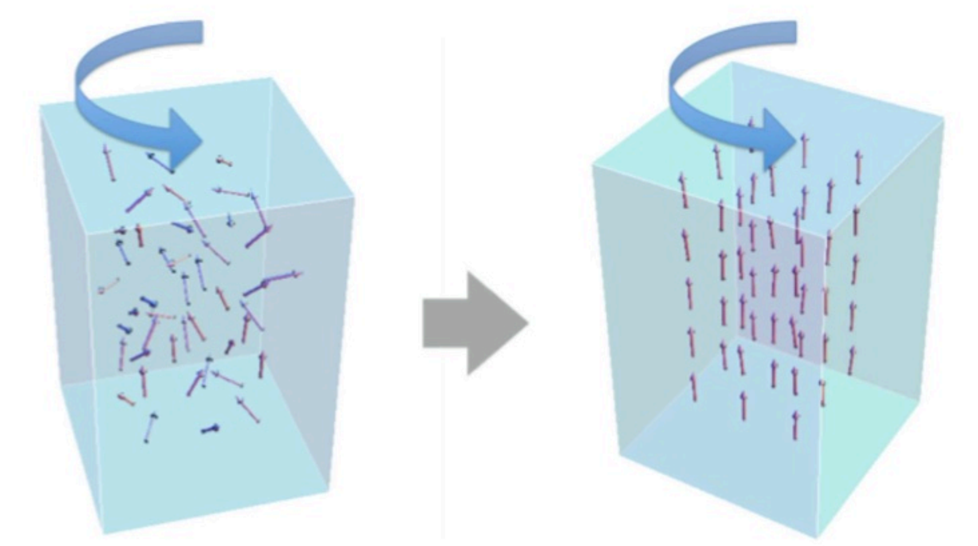
The mechanism is the quark/hadronic Barnett effect

Magnetization by rotation: The Barnett effect

Coupling between mechanical rotation and spin orientation



Einstein-de Haas effect. Modulation of magnetization of iron by applying the external magnetic filed, magnetic angular momentum is changed. As a result, the mechanical angular momentum is induced for compensating the modulation of the annular momentum.



Barnett effect. Magnetization is induced by applying mechanical rotation since an effective magnetic field, emerges in a rotating body.

Magnetization due to rotation: $M = \chi \Omega / \gamma$

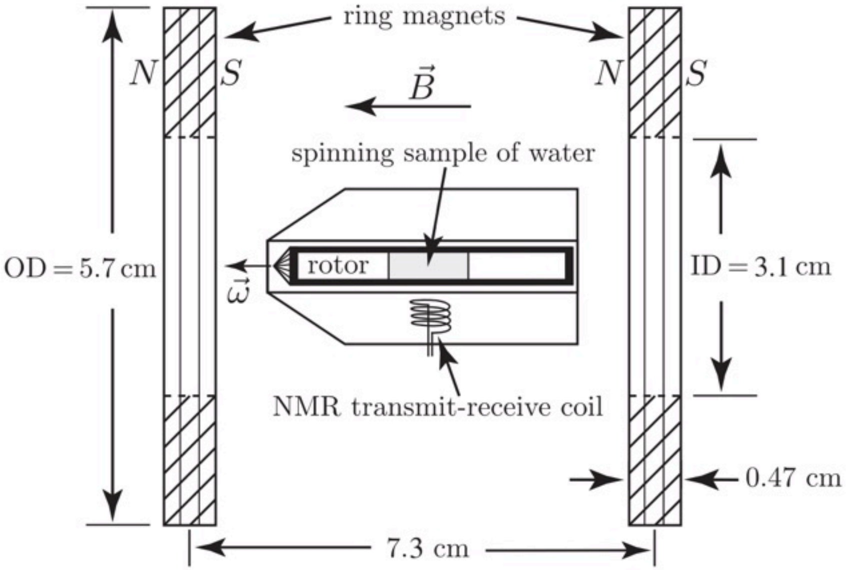
Effective magnetic field: $B_\Omega = \Omega / \gamma$

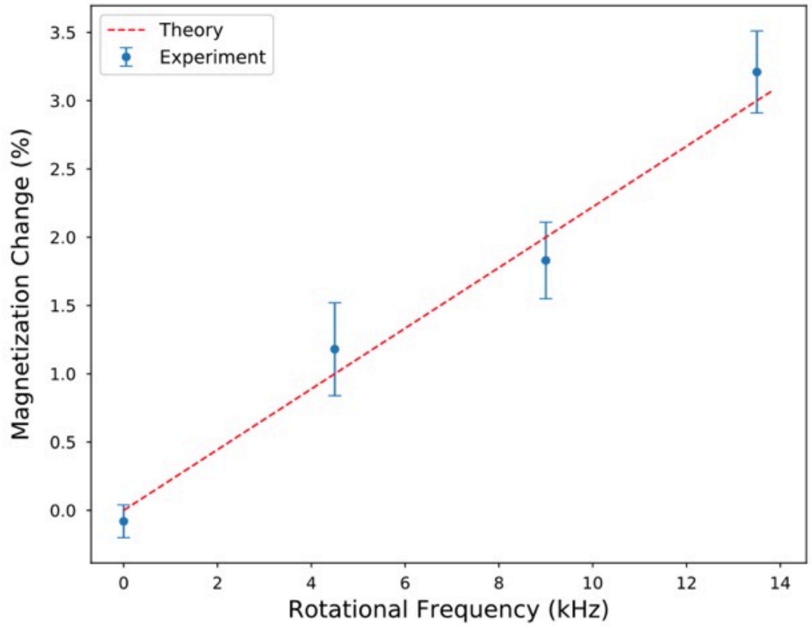
χ is the magnetization susceptibility of the medium

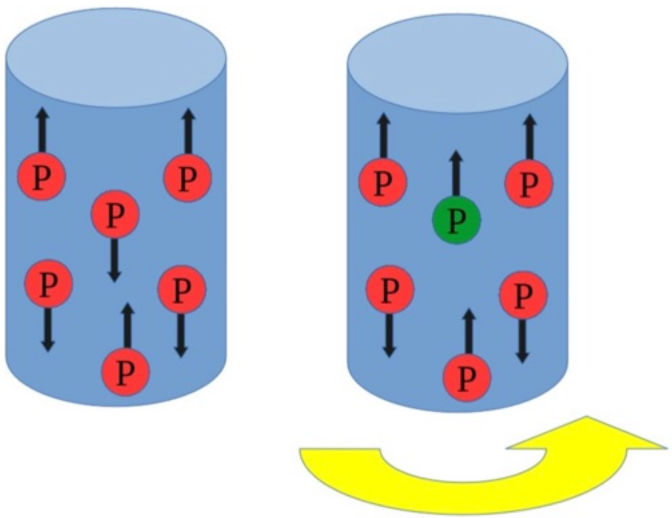
γ is the gyromagnetic ratio

The Barnett effect is a reciprocal phehomenon to the Einstein-de Haas effect

Nuclear Barnett Effect found in water





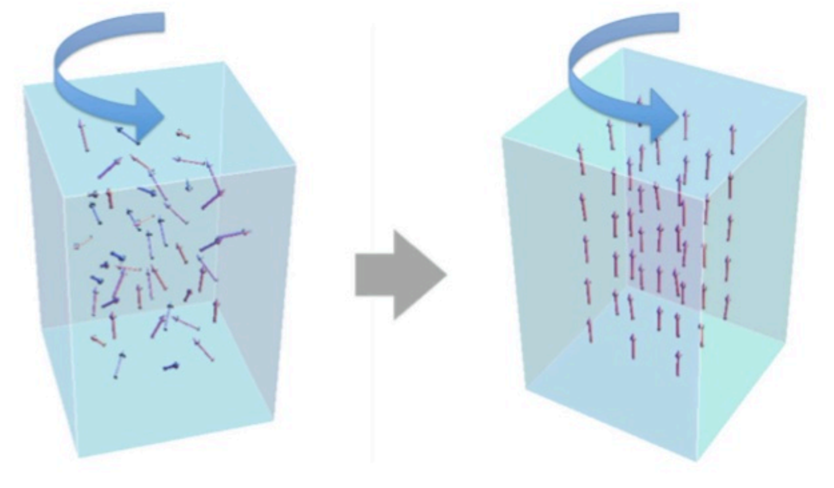


Measured the nuclear Barnett effect by rotating a sample of water at rotational speeds up to 13.5 kHz in a weak magnetic field and observed a change in the polarization of the protons in the sample that is proportional to the frequency of rotation.

Arabgol and Sleator, PRL 122, 177202 (2019)

Gluons are important!

Coupling between mechanical rotation and spin orientation



Barnett effect. Magnetization is induced by applying mechanical rotation since an effective magnetic field, emerges in a rotating body.
[Matsuo, Ieda, Maekawa, Frontiers in Physics 3, 54 (2015)]

The Barnett effect:

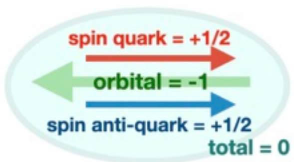
Effective magnetic field: $B_\Omega = \Omega / \gamma$

γ is the gyromagnetic ratio

Spin and orbital momenta are getting polarized by rotation

Mechanism, graphically, for fermions:

$$\langle \bar{\psi}\psi \rangle = -\frac{\sigma}{2G}$$

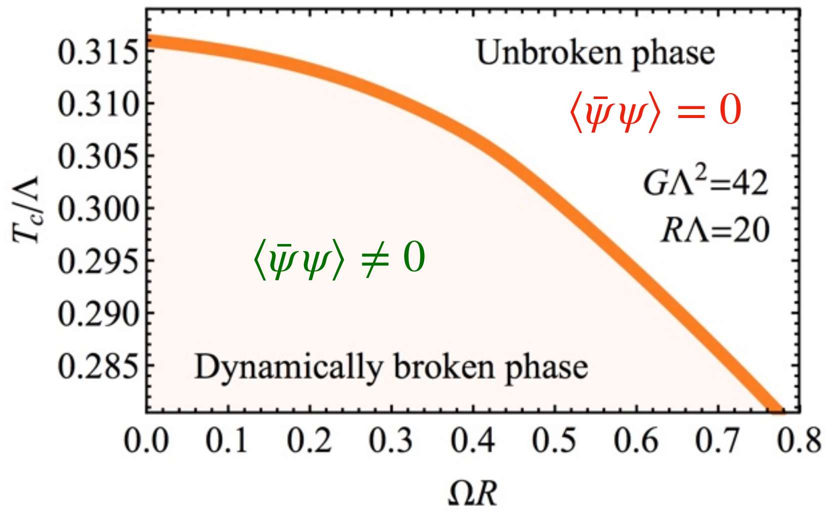


the chiral condensate (a quark-anti-quark pairing state with $L = S = 1$ but $J = 0$)

$\langle \bar{\psi}\psi \rangle \neq 0$ $\quad$ $\langle \bar{\psi}\psi \rangle = 0$

A typical prediction from theory:

(a result from the NJL model)

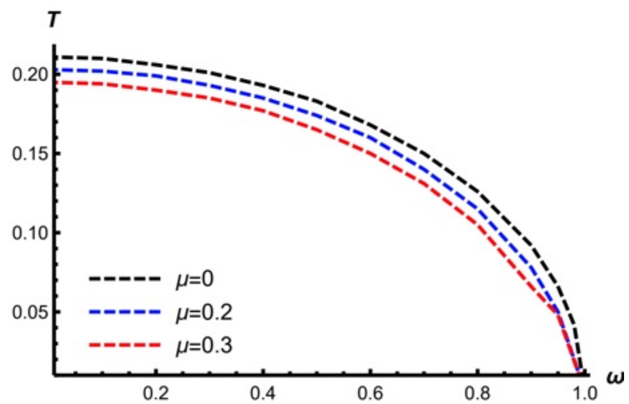


[S. Gongyo, M.Ch, JHEP 01 (2017) 136]

Head-on collision of analytics and numerics ?

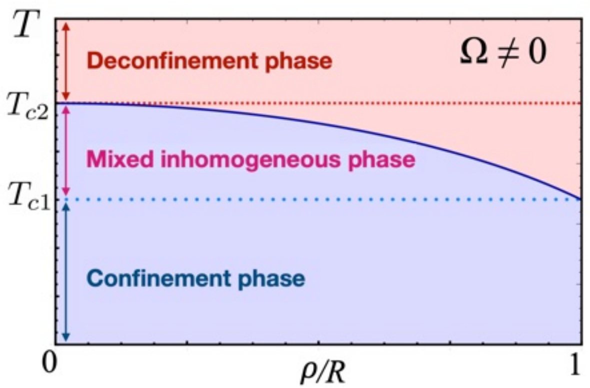
Analytical results: rotation destroys hadron phase (similar to baryonic density)

holography



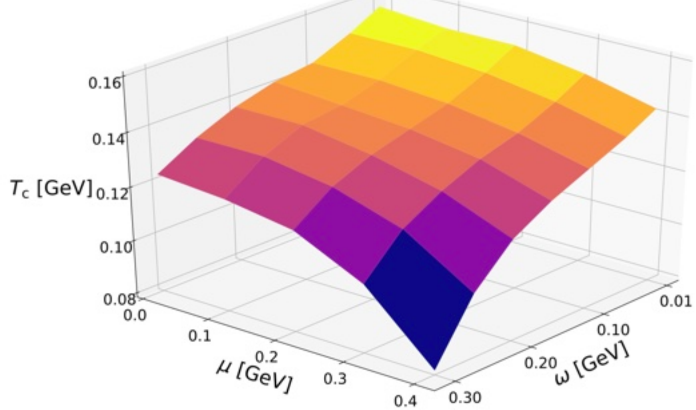
Chen, Zhang, Li, Hou, Huang (arxiv:2010.14478)

Tolman-Ehrenfest



M. Ch. (arxiv: 2012.04924)

hadron resonance gas



Fujimoto, Fukushima, Hidaka (arxiv:2101.09173)

+ many other works that do not match lattice data, M.Ch.&Co included

First-principle numerical results in lattice Yang-Mills theory (imaginary rotation in Euclidean + analytical continuation to Minkowski):

Rotation increases hadronic phase by increasing the deconfinement temperature (!):

Why?

$$T_c(\Omega)/T_c(0) = 1 + C_2\Omega^2 \text{ with } C_2 > 0$$

Braguta, Kotov, Kuznedelev, Roenko (arxiv:2102.05084)

Thermodynamical and mechanical properties of a rotating system.

The free energy in the co-rotating frame:

Let's spin the gluons!

$$F = -T \ln \int DA e^{iS} \equiv -T \ln \mathcal{Z}$$

where S is the Yang-Mills action in the co-rotating frame:

$$S = -\frac{1}{2g_{\rm YM}^2} \int d^4x \sqrt{-g}\, g^{\mu\nu} g^{\alpha\beta} F^a_{\mu\alpha} F^a_{\nu\beta}$$

with the metric

$$g_{\mu\nu} = \begin{pmatrix} 1 - r_\perp^2\Omega^2 & \Omega y & -\Omega x & 0 \\ \Omega y & -1 & 0 & 0 \\ -\Omega x & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

curved metric (with $R_{\mu\nu\alpha\beta} = 0$)

For example, the moment of inertia can be obtained as

$$F(T,R,\Omega) = F_0(T,R) - \frac{1}{2} I(T,R)\Omega^2 + \dots$$

The expectation values of order parameters (the Polyakov loop, etc) are computed with the partition function $\mathcal{Z}$ defined above in the co-rotational reference frame.

Calculation from first principles

PHYSICAL REVIEW D **103,** 094515 (2021)

Influence of relativistic rotation on the confinement-deconfinement transition in gluodynamics

V. V. Braguta,[1,2,3,*] A. Yu. Kotov,[4,†] D. D. Kuznedelev,[3,‡] and A. A. Roenko[1,§]

Lattice Yang-Mills in curved Euclidean spacetime

$$S_G = \frac{1}{4g^2}\int d^4x \sqrt{g_E} g_E^{\mu\nu} g_E^{\alpha\beta} F^a_{\mu\alpha} F^a_{\nu\beta}$$

Going to Euclidean via a Wick transform ($t \to i\tau$)

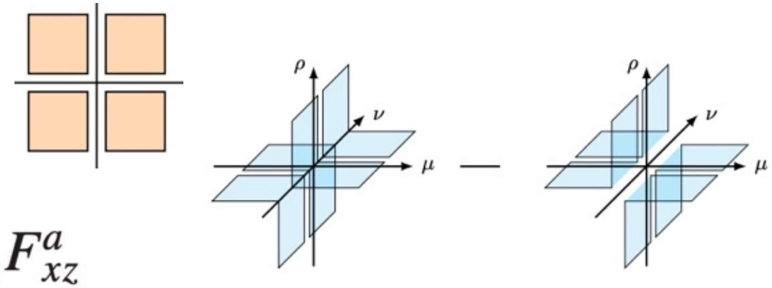
$$\begin{aligned} S_G = \frac{1}{2g^2}\int d^4x \,[&(1 - r^2\Omega^2) F^a_{xy}F^a_{xy} + (1 - y^2\Omega^2) F^a_{xz}F^a_{xz} \\ &+ (1 - x^2\Omega^2) F^a_{yz}F^a_{yz} + F^a_{x\tau}F^a_{x\tau} + F^a_{y\tau}F^a_{y\tau} \\ &+ F^a_{z\tau}F^a_{z\tau} - 2iy\Omega(F^a_{xy}F^a_{y\tau} + F^a_{xz}F^a_{z\tau}) \\ &+ 2ix\Omega(F^a_{yx}F^a_{x\tau} + F^a_{yz}F^a_{z\tau}) - 2xy\Omega^2 F^a_{xz}F^a_{zy}]. \end{aligned}$$

A need for imaginary rotation:

$$\Omega = i\Omega_I$$

Metric in Minkowski

$$g_{\mu\nu} = \begin{pmatrix} 1 - r^2\Omega^2 & \Omega y & -\Omega x & 0 \\ \Omega y & -1 & 0 & 0 \\ -\Omega x & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$



First work on the subject:

Lattice QCD in Rotating Frames

Arata Yamamoto and Yuji Hirono

Phys. Rev. Lett. **111**, 081601 – Published 22 August 2013

Analytic continuation $\Omega_I \to -i\Omega$

Lattice result for critical deconfining temperature:

$$\frac{T_c(\Omega_I)}{T_c(0)} = 1 - C_2\Omega_I^2$$

imaginary rotation

$$\frac{T_c(v_I)}{T_c(0)} = 1 - B_2\frac{v_I^2}{c^2}$$

$\Omega_i = -i\Omega$

$v_i = -iv$

$$\frac{T_c(\Omega)}{T_c(0)} = 1 + C_2\Omega^2$$

real rotation

$$\frac{T_c(v)}{T_c(0)} = 1 + B_2\frac{v^2}{c^2}$$

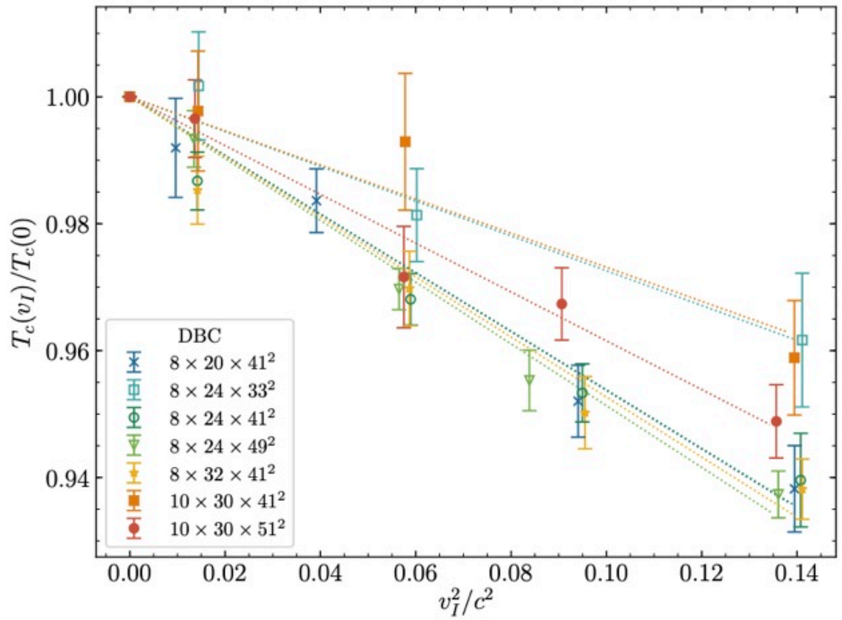
as a function of … angular frequency

… linear velocity on the boundary $v = \Omega R$

Contrary to all theoretical expectations, the critical temperature of deconfining phase transition raises with increase of the angular velocity Ω:

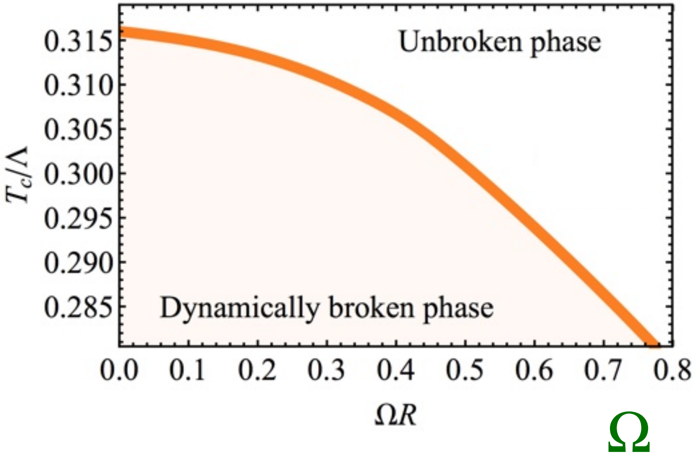
$$T_c(\Omega)/T_c(0) = 1 + C_2\Omega^2 \text{ with } C_2 > 0$$

V.V. Braguta, A.Yu. Kotov, D.D. Kuznedelev, A.A. Roenko, arXiv:2102.05084



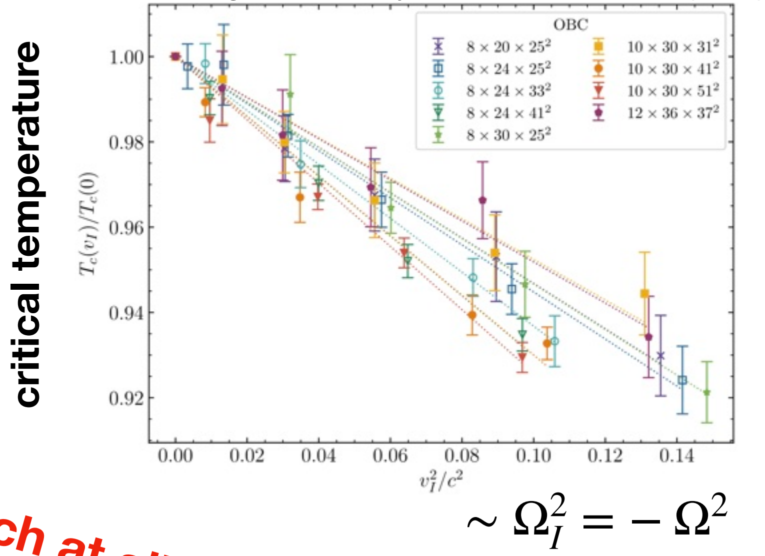
Evidence of the failure of our understanding

theory at real-valued rotation



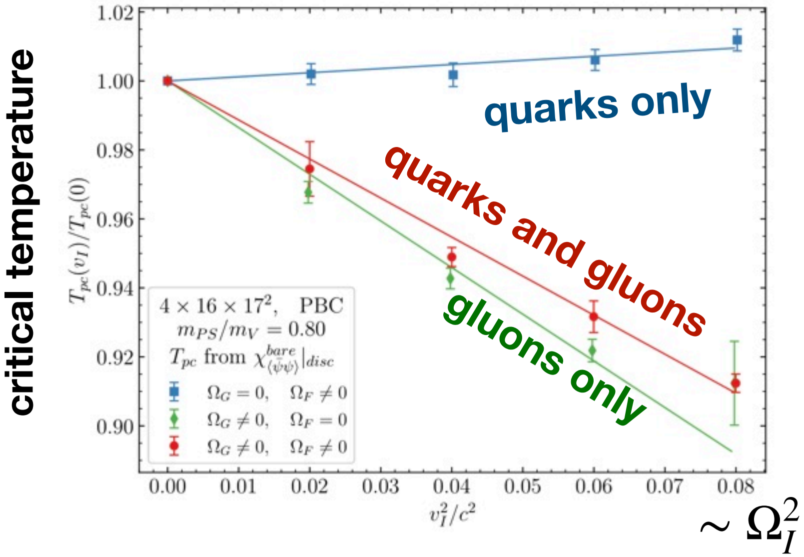
lattice at imaginary rotation

V.V. Braguta et al, Phys.Rev.D 103, 094515 (2021)



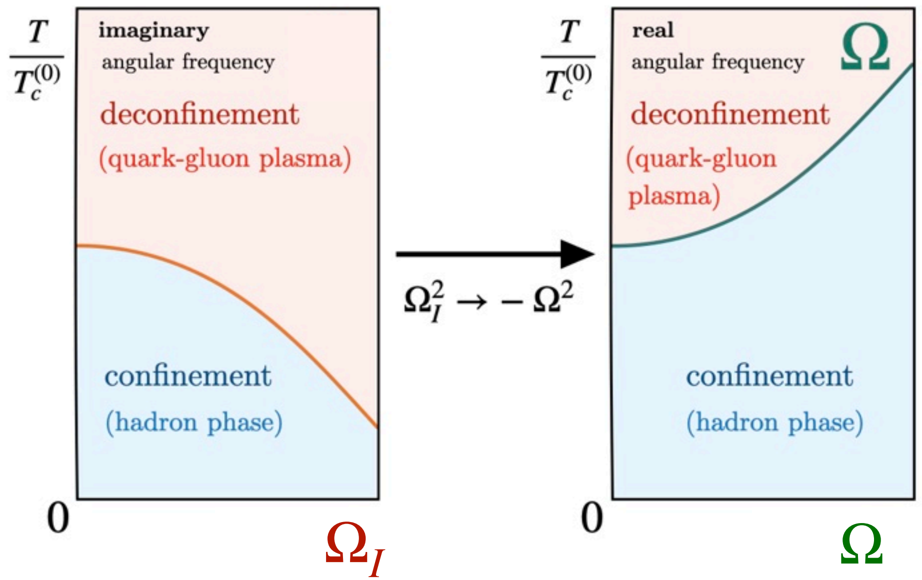
do not match at all

who rotates?



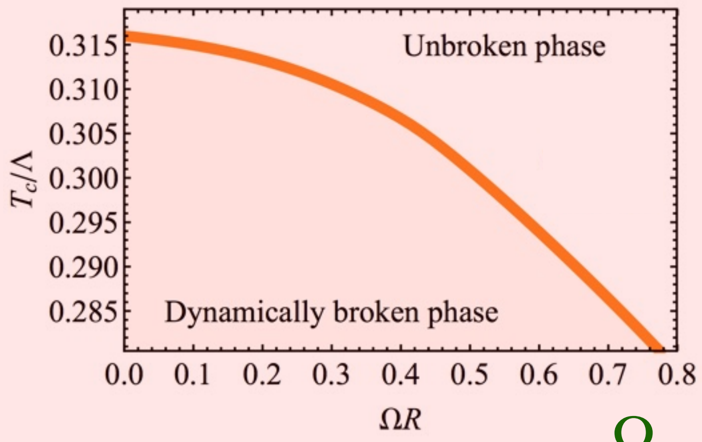
[V.V. Braguta, A. Kotov, A. Roenko, D. Sychev, ArXiv:2212.03224, also J.-C. Yang, X.-G. Huang, ArXiv: 2307.05755]

imaginary rotation | real rotation



Evidence of the failure of our understanding

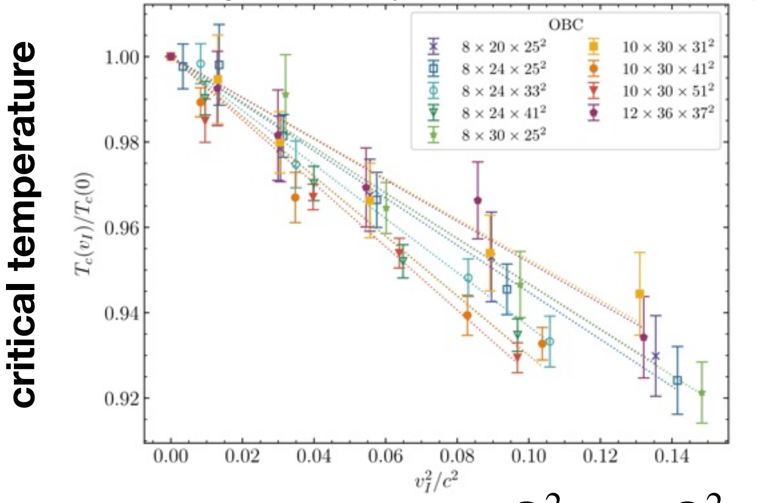
theory at real-valued rotation



Theory

lattice at imaginary rotation

V.V. Braguta et al, Phys.Rev.D 103, 094515 (2021)

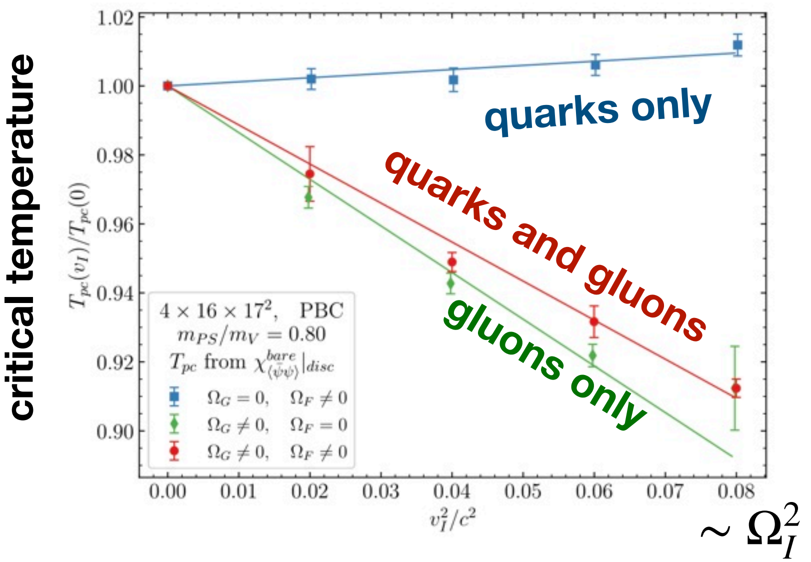


$\sim \Omega_I^2 = -\Omega^2$

do not match at all

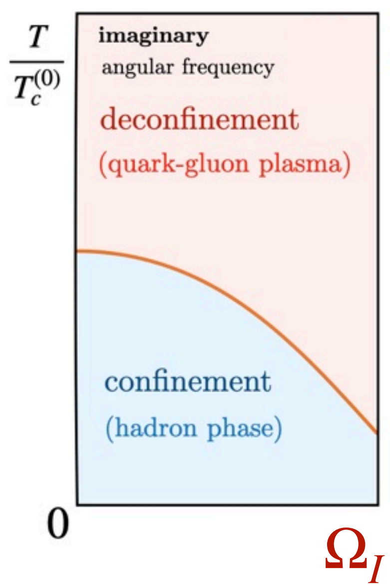
Lattice

who rotates?



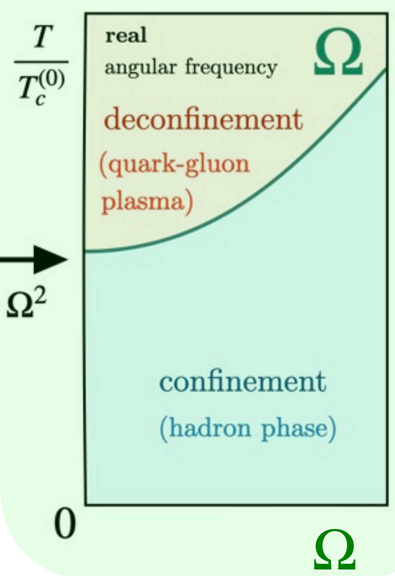
[V.V. Braguta, A. Kotov, A. Roenko, D. Sychev, ArXiv:2212.03224,
also J.-C. Yang, X.-G. Huang, ArXiv: 2307.05755]

imaginary rotation



$\Omega_I^2 \to -\Omega^2$

real rotation



Negative moment of inertia and rotational instability (?) of gluon plasma

Standard thermodynamics

Angular momentum:

$$\boldsymbol{J} = -\left(\frac{\partial E}{\partial \boldsymbol{\Omega}}\right)_S = -\left(\frac{\partial F}{\partial \boldsymbol{\Omega}}\right)_T$$

(Isothermal) moment of inertia:

$$I(T,\Omega) = \frac{J(T,\Omega)}{\Omega} = -\frac{1}{\Omega}\left(\frac{\partial F}{\partial \Omega}\right)_T$$

Free energy in co-rotating frame

$$F(T,R,\Omega) = F_0(T,R) - \frac{1}{2} I(T,R)\Omega^2$$
$$= F_0(T,R)\Big(1 + \frac{1}{2} K_2 v_R^2 + O\big(v_R^4\big)\Big)$$

Dimensionless moment of inertia

For a rigidly rotating cylinder

Moment of inertia:

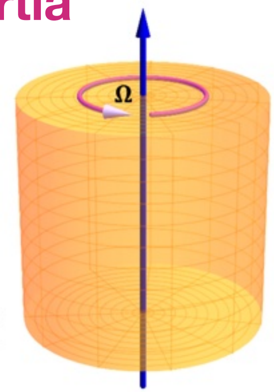
$$I(T,R,\Omega) = \int_V d^3x\, x_\perp^2 \rho(T, x_\perp, \Omega)$$
$$= \frac{\pi}{2} L_z R^4 \rho_0(T)$$
$$= -K_2(T) F_0(T) R^2$$

Dimensionless moment of inertia

(notice that $F_0 < 0$)

velocity at the boundary

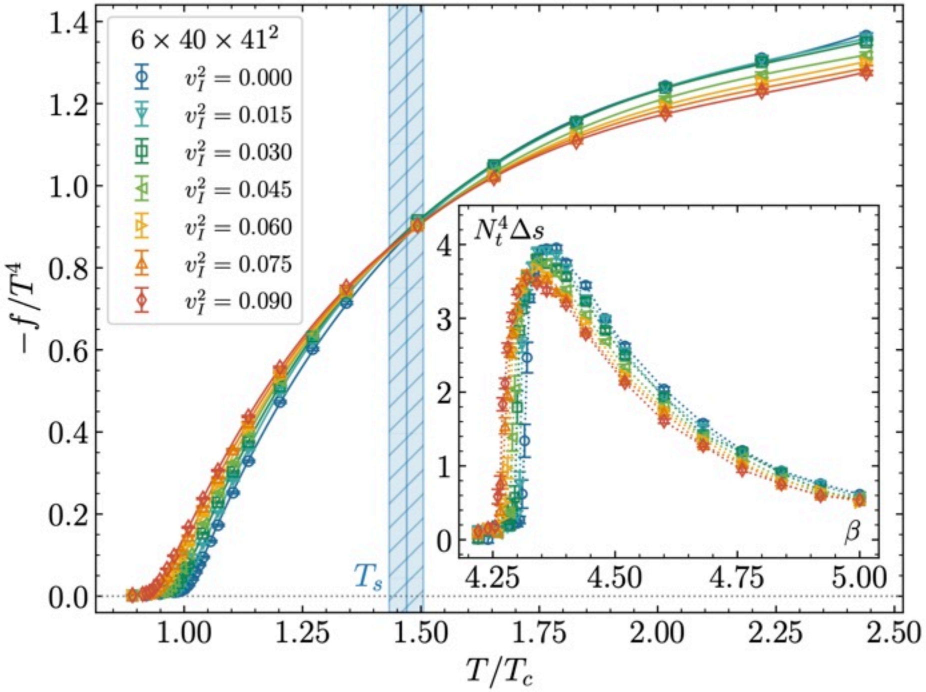
$$v_R = \Omega R$$



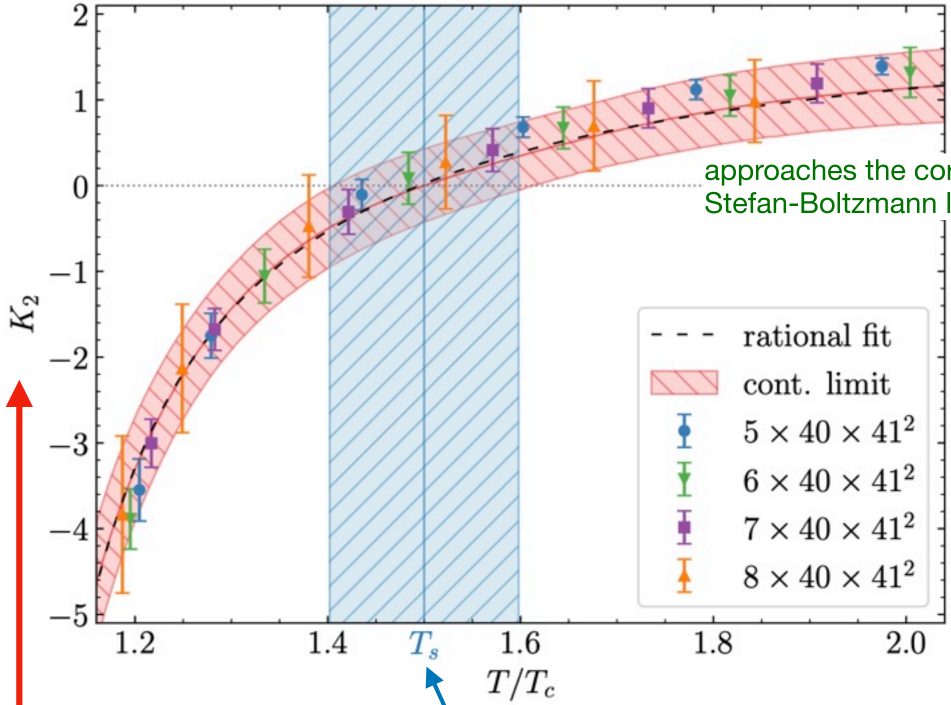
V. V. Braguta, A. A. Roenko, D. A. Sychev, M. Ch., arXiv:2303.03147

Exotic behavior of critical temperature: a result of an exotic Barnett effect for gluons?

Free energy density in SU(3) Yang-Mills theory



The dimensionless moment of inertia



negative moment of inertia

supervortical temperature

$$T_s = 1.50\,(10)\,T_c$$

$$f(T, v_I) = f_0(T)\Big(1 - \frac{1}{2} K_2(T) v_I^2\Big)$$

linear imaginary velocity at the boundary $v_I = \Omega_I R$

$$I(T) \equiv \lim_{\Omega \to 0} I(T, \Omega) = -K_2(T) F_0(T) R^2$$

notice that $F_0(T) < 0$

$$K_2^{(\text{fit})}(T) = K_2^{(\infty)} - \frac{c}{T/T_c - 1}$$

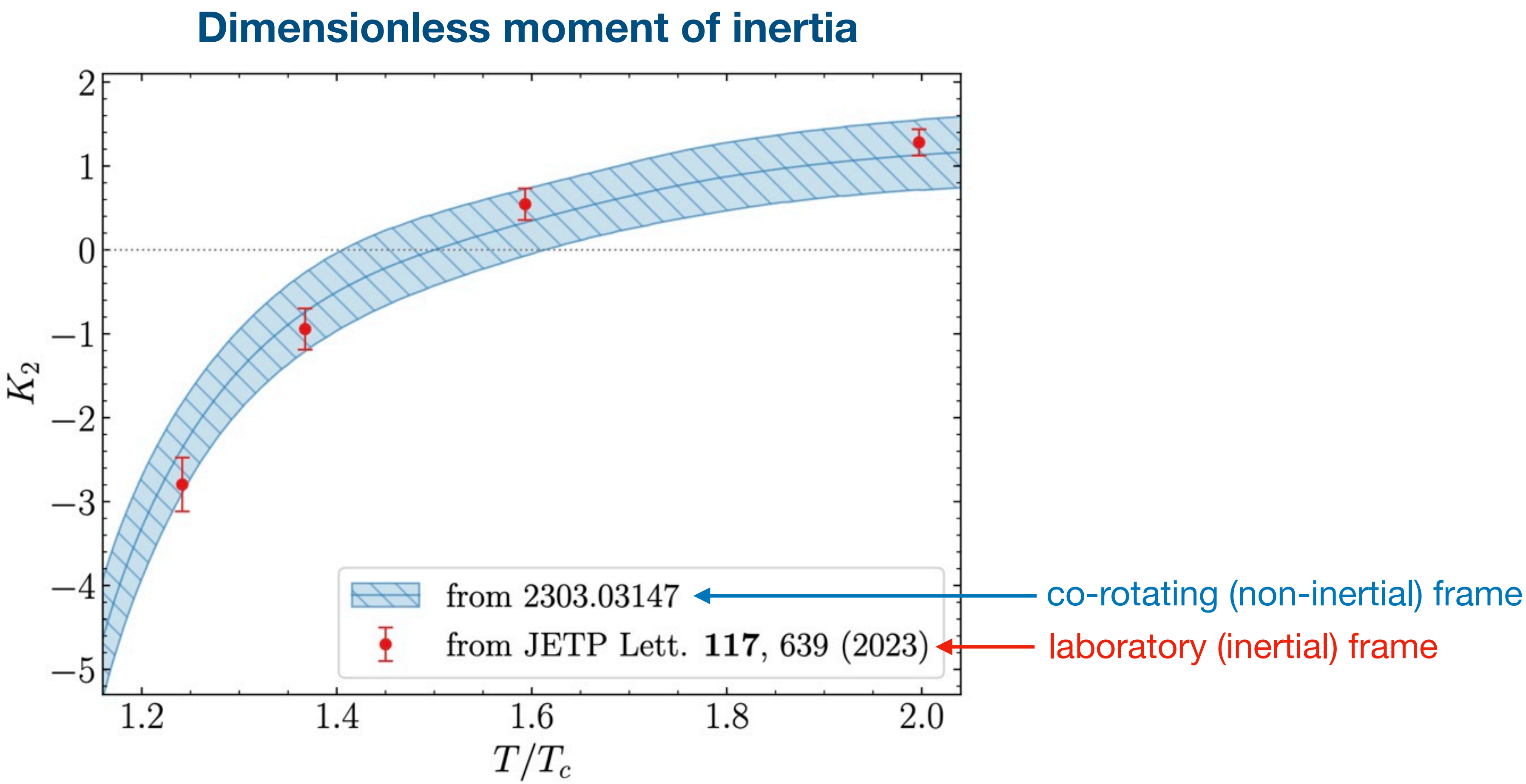
$$K_2^{(\infty)} = 2.23(39)$$

for a free particle: $K_2 = 2$

[Braguta et al, Phys. Lett. B 852, 138604 (2024)]

[V. E. Ambruş et al., Phys.Rev.D 108,085016 (2023)]

Co-Rotating vs Laboratory frames



[Braguta et al, PoS LATTICE2023 (2024) 181; ArXiv: 2311.03947]

Fermions vs vector bosons in co-rotating frame

Fermions:

$$\mathcal{L}_\psi^{(n)} \propto \Omega^n \ (n = 0, 1)$$

$$\mathcal{L}_\psi = \mathcal{L}_\psi^{(0)} + \mathcal{L}_\psi^{(1)}$$

Lagrangian in co-rotating frame

$$\mathcal{L}_\psi^{(0)} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$$

Lagrangian in laboratory frame

Mechanical part

$$\mathcal{L}_\psi^{(1)} = \boldsymbol{\Omega} \cdot \boldsymbol{J}_\psi$$

Linear in angular momentum

$$\hat{J}_{\psi,z} = -i\left(-y\partial_x + x\partial_y\right) + \frac{1}{2}\Sigma^{xy}$$

Vector bosons:

$$\mathcal{L}_G^{(n)} \propto \Omega^n \text{ for } n = 0, 1, 2$$

$$\mathcal{L}_G = \mathcal{L}_G^{(0)} + \mathcal{L}_G^{(1)} + \mathcal{L}_G^{(2)}$$

$$\mathcal{L}_G^{(0)} = \frac{1}{4g_{YM}^2}\eta^{\mu\nu}\eta^{\alpha\beta}F_{\mu\alpha}^a F_{\nu\beta}^a$$

Lagrangian in laboratory frame

$$\mathcal{L}_G^{(1)} = \boldsymbol{\Omega} \cdot \boldsymbol{J}_G$$

mechanical part

$$\mathcal{L}_G^{(2)} = \frac{1}{2g_{YM}^2}\left[\Omega^2(\boldsymbol{B}^a \cdot \boldsymbol{r})^2 + r^2(\boldsymbol{B}^a \cdot \boldsymbol{\Omega})^2\right]$$

magneto-vortical coupling

angular momentum

$$\boldsymbol{J}_G = \frac{1}{g_{YM}^2}\boldsymbol{r} \times (\boldsymbol{E}^a \times \boldsymbol{B}^a)$$

Gluons in co-rotating frame

The action in the co-rotating frame is quadratic in the angular frequency Ω:

$$S = S_0 + S_1\Omega + \frac{S_2}{2}\Omega^2$$

$$S = -\frac{1}{2g_{\mathrm{YM}}^2}\int d^4x\,\sqrt{-g}\,g^{\mu\nu}g^{\alpha\beta}F^a_{\mu\alpha}F^a_{\nu\beta}$$

$$g_{\mu\nu} = \begin{pmatrix} 1-r_\perp^2\Omega^2 & \Omega y & -\Omega x & 0 \\ \Omega y & -1 & 0 & 0 \\ -\Omega x & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

with

chromoelectric fields

$$S_1 = \frac{1}{g_{YM}^2}\int d^4x\left[xF^a_{yx}F^a_{xt} + xF^a_{yz}F^a_{zt} - yF^a_{xy}F^a_{yt} - yF^a_{xz}F^a_{zt}\right]$$

standard "mechanical" contribution

chromomagnetic fields

$$S_2 = -\frac{1}{g_{YM}^2}\int d^4x\left[r_\perp^2(F^a_{xy})^2 + y^2(F^a_{xz})^2 + x^2(F^a_{yz})^2 + 2xyF^a_{xz}F^a_{zy}\right]$$

chromo-magnetic contribution

chromomagnetic contribution only

Moment of inertia:

$$I^{\mathrm{gl}} = \lim_{\Omega\to 0}\left[-\frac{1}{\Omega}\left(\frac{\partial F}{\partial\Omega}\right)_T\right] = \lim_{\Omega\to 0}\left[-\left(\frac{\partial^2 F}{\partial\Omega^2}\right)_T\right]$$

where

$$F = -T\ln\int DA\,e^{iS}$$

for a good smooth $F = F(\Omega)$ $\Longrightarrow$ S_2 **will contribute!!!**

Decomposition of the moment of inertia: the mechanical part

Moment of inertia of the gluon plasma can be decomposed into two parts:

$$I^{\rm gl} = I^{\rm gl}_{\rm mech} + I^{\rm gl}_{\rm magn}$$

(nonlocal) ↑ **standard mechanical** (exists for quark and gluons)

↑ **(local)** **non-trivial chromomagnetic** (gluons are special! No such term for quarks)

The standard mechanical part:

$$I^{\rm gl}_{\rm mech} = T \left\langle\!\left\langle S_1^2 \right\rangle\!\right\rangle_T = \frac{1}{T} \left\langle\!\left\langle \left(\boldsymbol{n} \cdot \boldsymbol{J}^{\rm gl}\right)^2 \right\rangle\!\right\rangle_T$$

total angular momentum of gluons

$$J^{\rm gl}_i = \frac{T}{2} \int_V d^4x\, \epsilon_{ijk} M^{jk}_{\rm gl}(x) \qquad i,j = 1,2,3$$

or: $\vec{\rm J}_g = \int d^3x [\vec{\rm x} \times (\vec{\rm E} \times \vec{\rm B})]$

the local angular momentum of gluons

$$M^{ij}_{\rm gl}(\boldsymbol{x}) = x^i T^{j0}_{\rm gl}(\boldsymbol{x}) - x^j T^{i0}_{\rm gl}(\boldsymbol{x})$$

gluonic stress-energy tensor

$$T^{\mu\nu}_{\rm gl} = F^{a,\mu\alpha} F^{a,\nu}{}_{\alpha} - (1/4)\eta^{\mu\nu} F^{a,\alpha\beta} F^a_{\alpha\beta}$$

a Belinfante-improved form
(symmetric, gauge invariant, and conserved)

thermal expectation value

$$\langle\!\langle \mathcal{O} \rangle\!\rangle_T = \langle \mathcal{O} \rangle_T - \langle \mathcal{O} \rangle_{T=0}$$

"cold vacuum cannot be set into rotation"

"Mechanical part of the moment of inertia with respect to an axis n is the susceptibility of the projection of total angular momentum on the axis n."

Decomposition of the moment of inertia: the chromomagnetic part

$$I^{\mathrm{gl}}_{\mathrm{magn}} = T\,\langle\!\langle S_2 \rangle\!\rangle_T = \int_V d^3x \Big[\langle\!\langle (\boldsymbol{B}^a \cdot \boldsymbol{x}_\perp)^2 \rangle\!\rangle_T + \langle\!\langle (\boldsymbol{B}^a \cdot \boldsymbol{n})^2 \rangle\!\rangle_T\, \boldsymbol{x}_\perp^2 \Big]$$

chromomagnetic field:

$$B^a_i = \frac{1}{2}\epsilon^{ijk} F^a_{jk}$$

distance to the axis of rotation:

$$\boldsymbol{x}_\perp = \boldsymbol{x} - \boldsymbol{n}(\boldsymbol{n}\cdot\boldsymbol{x})$$

In the static limit, $\Omega \to 0$, the space is $O(3)$ isotropic:

$$\langle\!\langle B^a_i B^a_j \rangle\!\rangle_T = \frac{1}{3}\delta_{ij}\, \langle\!\langle (\boldsymbol{B}^a)^2 \rangle\!\rangle_T$$

The chromomagnetic contribution to the moment of inertia is proportional to the thermal part of the chromomagnetic condensate:

$$I^{\mathrm{gl}}_{\mathrm{magn}} = \frac{2}{3}\int_V d^3x\, \boldsymbol{x}_\perp^2\, \langle\!\langle (\boldsymbol{B}^a)^2 \rangle\!\rangle_T$$

Compare to the formula from classical mechanics:

$$I_{\mathrm{class}} = \int_V d^3x\, \boldsymbol{x}_\perp^2\, \rho(\boldsymbol{x})$$

"classical" mass density

$$\rho(\boldsymbol{x}) \to \tfrac{2}{3}\langle\!\langle (\boldsymbol{B}^a)^2 \rangle\!\rangle_T$$

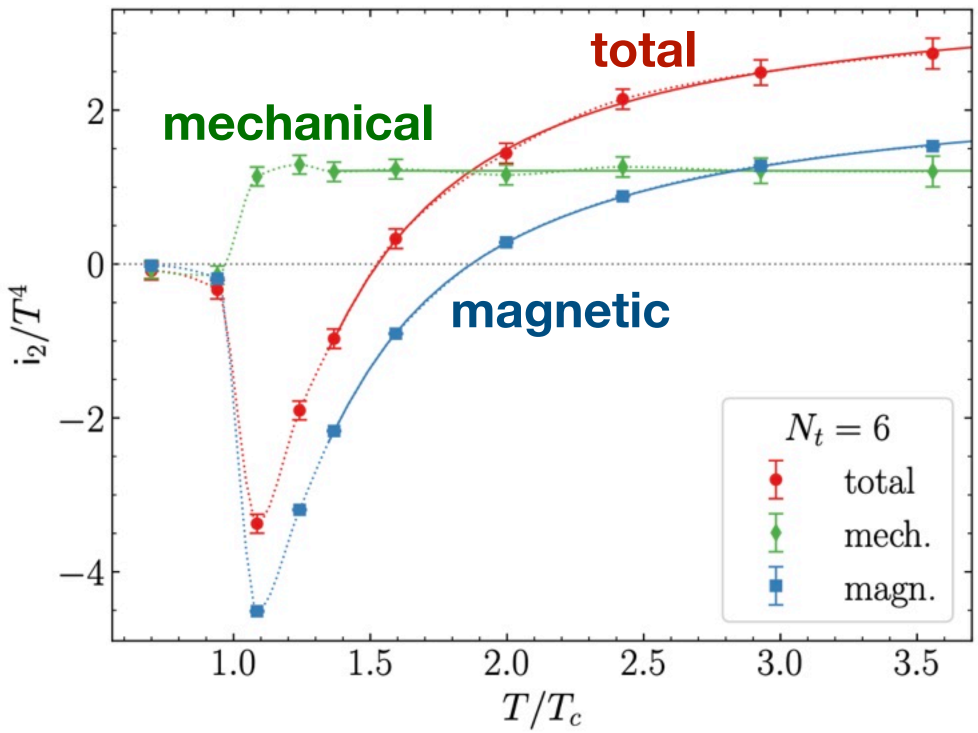
Mechanism behind the negativity of gluonic moment of inertia?

Melting of the gluon condensate, $\langle\langle (\boldsymbol{B}^a)^2 \rangle\rangle_T < 0$!

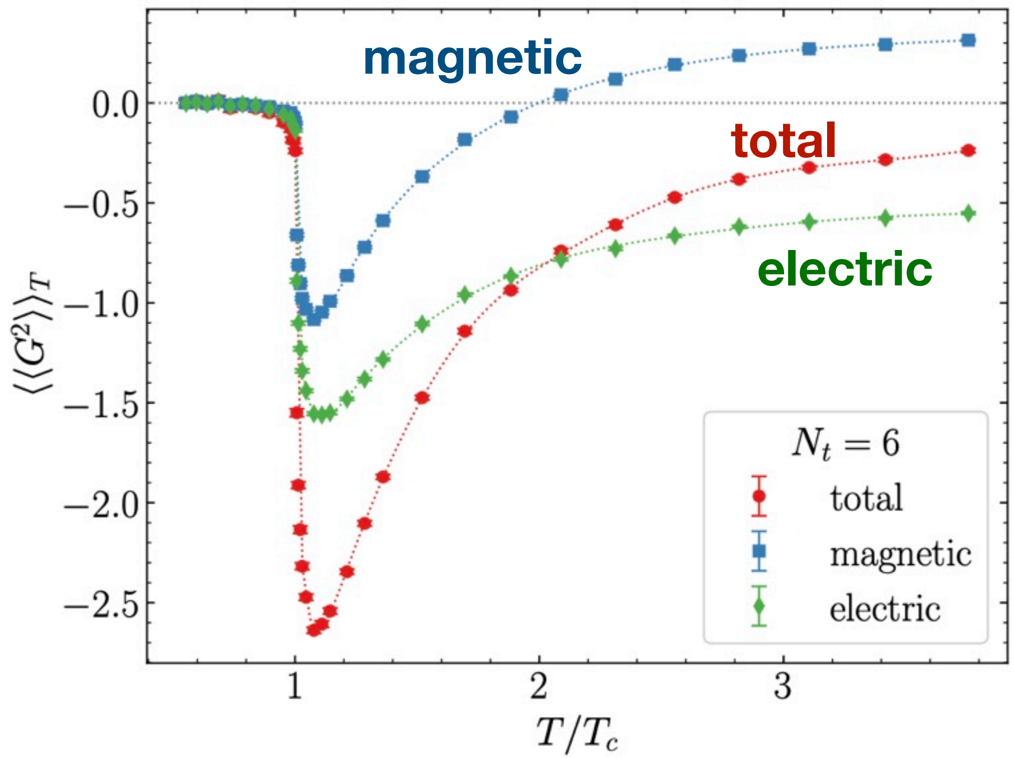
Gluon condensate melts at $T \gtrsim T_c$, and the moment of inertia receives a negative contribution

$$F(T, R_\perp, \Omega) = F_0(T, R_\perp) - V \sum_{k=1}^{\infty} \frac{\mathrm{i}_{2k}(T)}{(2k)!} R_\perp^{2k} \Omega^{2k} \equiv F_0 - \frac{I}{2}\Omega^2 + O(\Omega^4)$$

specific (normalized) moment of inertia



(normalized) gluon condensates



$$I^{\mathrm{gl}} = I^{\mathrm{gl}}_{\mathrm{mech}} + I^{\mathrm{gl}}_{\mathrm{magn}}$$

$$I^{\mathrm{gl}}_{\mathrm{magn}} = \frac{2}{3} \int_V d^3x\, \boldsymbol{x}_\perp^2 \left\langle\!\left\langle (\boldsymbol{B}^a)^2 \right\rangle\!\right\rangle_T$$

Negative moment of inertia: instability of rigid rotation?

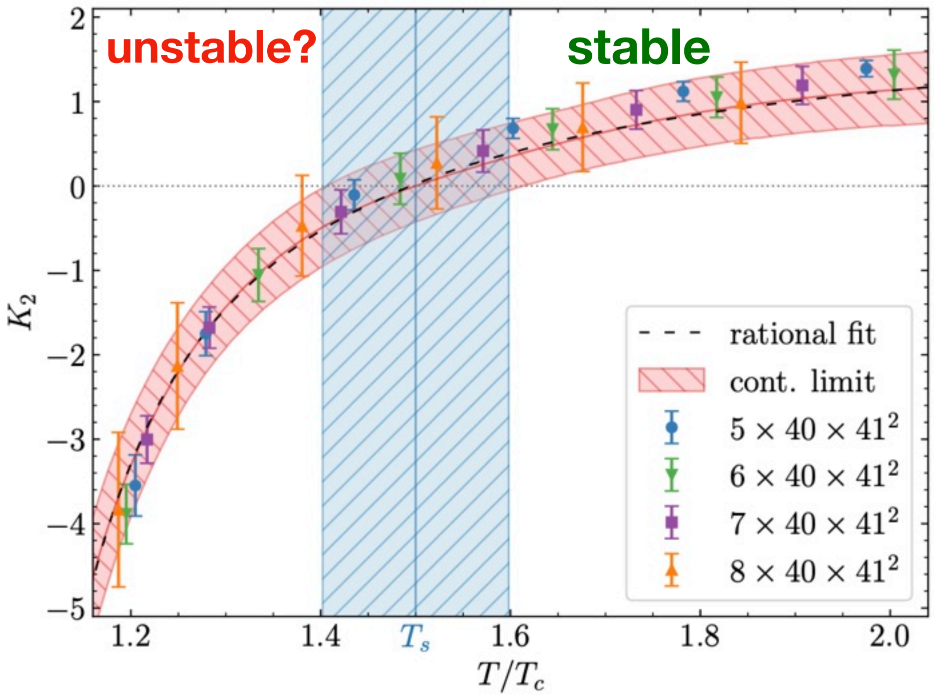
Thermodynamic equilibrium:

$$\delta E - T\delta S - \boldsymbol{\Omega}\delta \boldsymbol{J} > 0$$

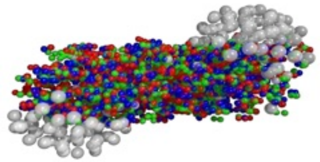
For rotating system: all eigenvalues of the inverse Weinhold metric

$$g^{(W),\mu\nu} = -\frac{\partial^2 f(T,\boldsymbol{\Omega})}{\partial X_\mu \partial X_\nu}\,, \qquad X_\mu = (T, \Omega_i)$$

should be positively defined:



$$C_J > 0\,, \qquad C_J = T\left(\frac{\partial S}{\partial T}\right)_J$$ ← **specific heat** $C_V = \frac{\langle E^2\rangle - \langle E\rangle^2}{k_B T^2}$



$$\mathrm{spec}(I^{ij}) > 0\,, \qquad I^{ij} = \left(\frac{\partial J^i}{\partial \Omega_j}\right)_T$$ ← **tensor of moments of inertia**

In our notations: $K_2(T) > 0$ ← **condition of thermodynamic stability**

Emerges also in spinning black holes

[B.F. Whiting, J. W. York Jr, PRL 61, 1336 (1988); T. Prestidge, PRD 61 (2000), 084002;
H. S. Reall, PRD 64 (2001), 044005; R. Monteiro, M. J. Perry, H. E. Santos, PRD 80 (2009), 024041]

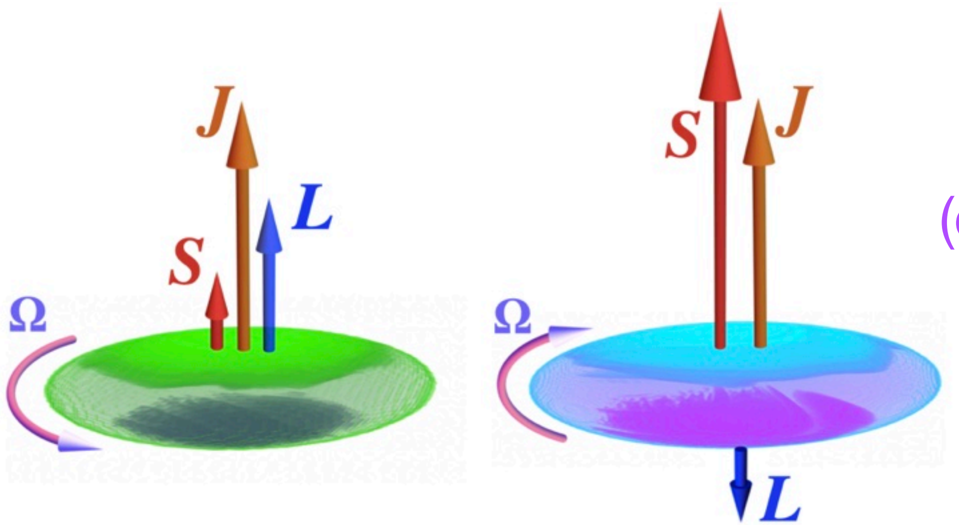
Physical picture: a negative Barnett effect for gluons?

$$\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$$

total angular momentum = orbital part + spin part

ordinary fluid (gas)

$$\boldsymbol{S} = \kappa\,\boldsymbol{\Omega}$$
$$\kappa > 0$$



(quark) gluon plasma

$$\boldsymbol{S} = \kappa\,\boldsymbol{\Omega}$$
$$\kappa < 0$$

Barnett **negative Barnett**

1) gluon spins $\boldsymbol{S}$ are over-polarized by rotation leading to $\boldsymbol{S} \| \boldsymbol{J}$ with $S > J$

2) since $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$, the L must take a negative value, $L < 0$, so do $\Omega < 0$

3) one arrives to $S > 0$ and $\Omega < 0$, leading to the negative Barnett effect

$$\boldsymbol{S} = \kappa\,\boldsymbol{\Omega} \text{ with } \kappa < 0$$

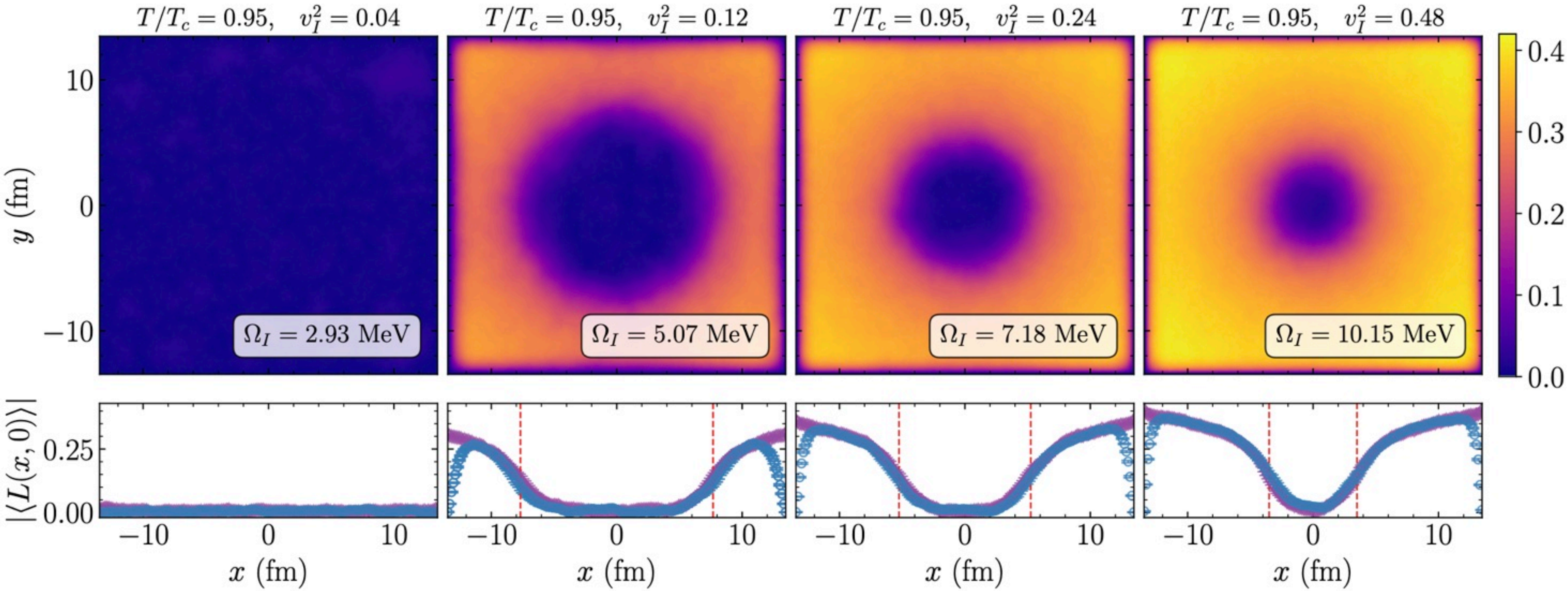
open question: any link to the proton spin crisis?

[Braguta et al, ArXiv: 2310.16036]

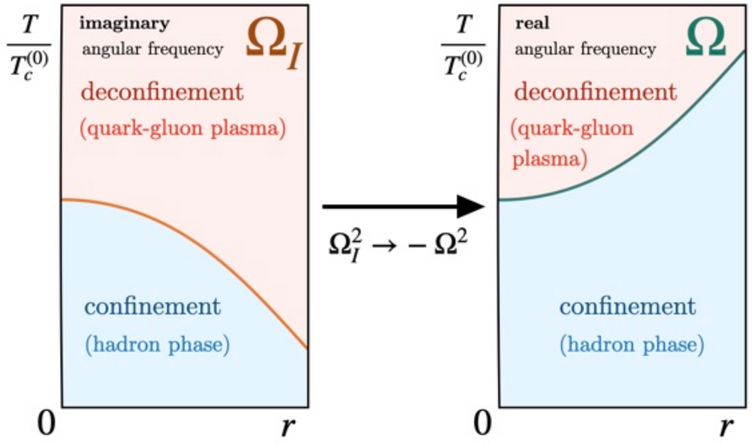
New mixed phase in QCD

rotation with imaginary angular velocity Ω_I (then we should make the analytical continuation, $\Omega_I^2 = -\Omega^2$)

Increase in rotation →



analytical continuation:



a local expectation value of the Polyakov loop

$$L(\boldsymbol{x}) = \mathrm{Tr}\,\mathcal{P}\exp\left(\oint_0^{1/T} dx_4 A_4(x_4, \boldsymbol{x})\right)$$

the order parameter of confinement:
$\langle L \rangle = 0$: confinement; $\langle L \rangle \neq 0$: deconfinement

[New mixed inhomogeneous phase in vortical gluon plasma: First-principle results from rotating SU(3) lattice gauge theory, V. V. Braguta, A. A. Roenko, M. Ch, Phys. Lett. B 855 (2024) 138783, ArXiv:2312.13994]

Violation of the Tolman-Ehrenfest (TE) law in gluon plasma

The TE law: $\sqrt{g_{00}(x)}\,T(x) = T_0 = \text{const}$ in a static inhomogeneous gravitational field

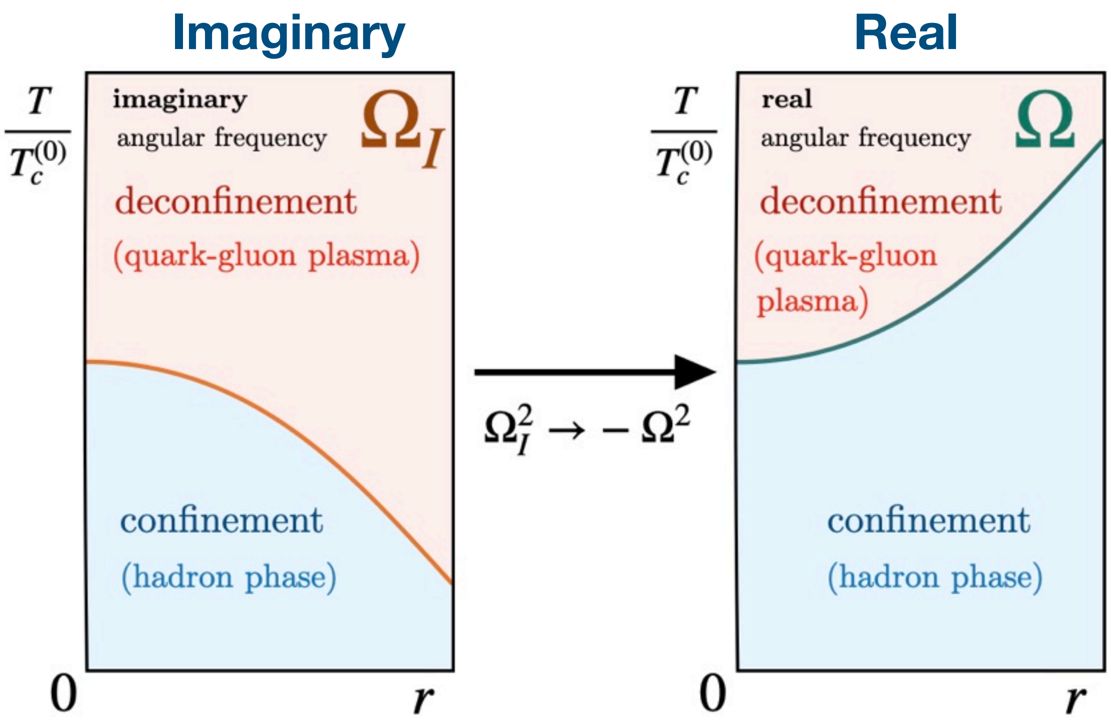
For rotation,

$$g_{\mu\nu} = \begin{pmatrix} 1 - r^2\Omega^2 & \Omega y & -\Omega x & 0 \\ \Omega y & -1 & 0 & 0 \\ -\Omega x & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

the equilibrium TE temperature is

$$T(r) = \frac{T_0}{\sqrt{1+\Omega_I^2 r^2}} = \frac{T_0}{\sqrt{1-\Omega^2 r^2}}$$

We observe:



Close to the rotation axis: Confinement ("hot")

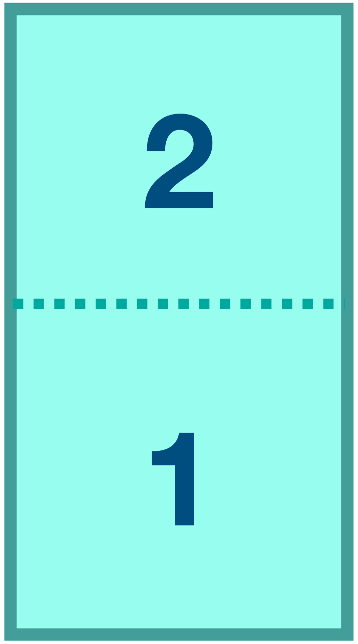
Far from the rotation axis: Deconfinement ("cold")

(contradicts our intuition)

The TE law is not satisfied in the rotating gluon plasma.

Thermal equilibrium (classical thermodynamics)

— Consider a closed system divided arbitrarily into two subsystems



— Thermal equilibrium happens when the total entropy reaches its maximum

$$S = S_1 + S_2 \qquad dS_1 + dS_2 = 0$$

— Assume that we have no gravitational field

— If some quantity of heat leaves the first subsystem, it always enters the second subsystem:

$$dE_1 = -dE \rightarrow dE_2 = dE \rightarrow dS_1/dE_1 = dS_2/dE_2 \rightarrow T_1 = T_2$$

the definition of temperature: $1/T = \partial S/\partial E$

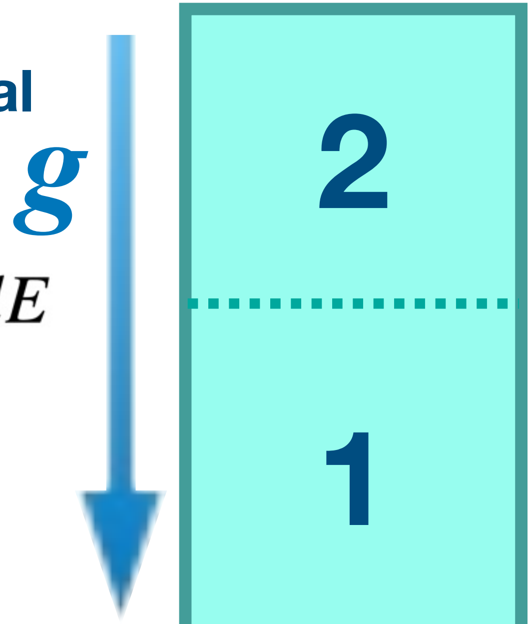
In the absence of gravitational field, temperature is constant

How to understand the Tolman-Ehrenfest law?

— In a static gravitational field Φ, the heat quantity dE possesses an inertial mass

$$dm = dE/c^2$$

— the equivalence between inertial and gravitational masses: a quantity of heat has a weight



— When heat leaves the first subsystem, $dE_1 = -dE$ it enters the second subsystem, and performs work agains the gravity (heat = mass):

$$dE_2 = dE + (\Phi_2 - \Phi_1)dm = dE_2(1 + \Delta\Phi/c^2)$$

$$\Delta\Phi = \Phi_2 - \Phi_1$$

a change in the gravitational potential

$$T_1 = T_2$$

— Entropy maximum

$$dS_1 + dS_2 = 0$$

— Local temperature

$$T_2 = T_1(1 + \Delta\Phi/c^2)$$

$$g_{00} = 1 + \frac{2\Phi}{c^2}$$

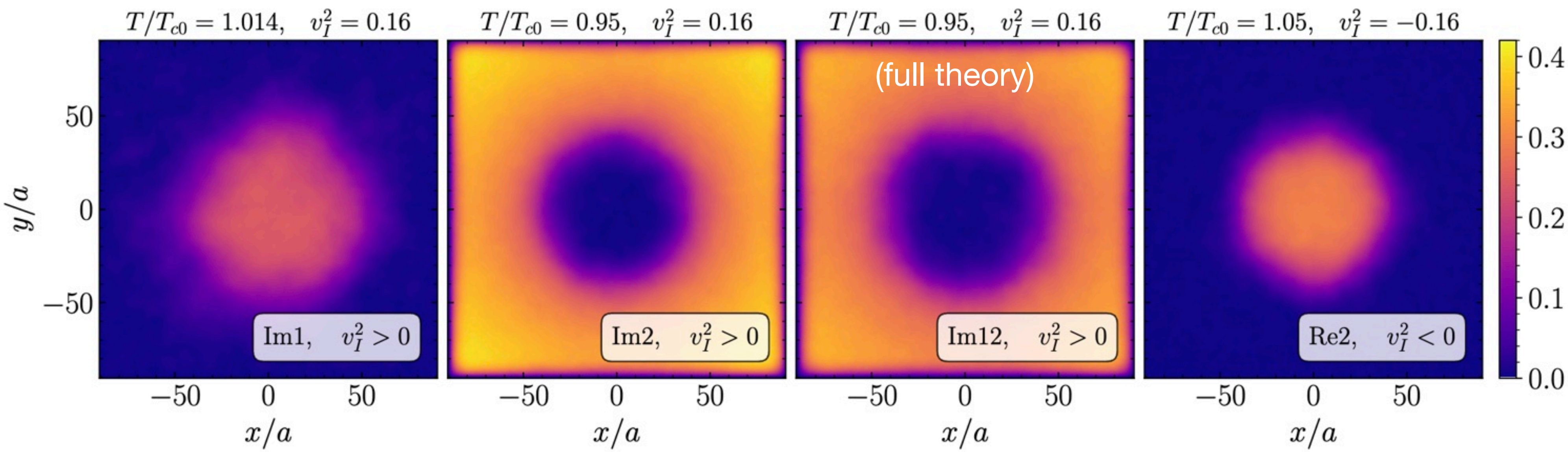
Tolman-Ehrenfest law

$$T(x) = T_0/\sqrt{g_{00}(x)}$$

[e.g. C. Rovelli and M. Smerlak, "Thermal time and Tolman-Ehrenfest effect: temperature as the speed of time" Class. Quant. Grav. 28, 4 (2010)]

Importance of the (quadratic) magnetovortical coupling

Non-Importance of (linear) mechanical coupling to angular momentum



$\mathcal{L}_G^{(0)} + \mathcal{L}_G^{(1)}$ $\quad$ $\mathcal{L}_G^{(0)} + \mathcal{L}_G^{(2)}$ $\quad$ $\mathcal{L}_G^{(0)} + \mathcal{L}_G^{(1)} + \mathcal{L}_G^{(2)}$ $\quad$ $\mathcal{L}_G^{(0)} + \mathcal{L}_G^{(2)}$

Imaginary rotation — **real rotation**

Tolman-Ehrenfest is satisfied — **Tolman-Ehrenfest is broken**

$$\mathcal{L}_G = \mathcal{L}_G^{(0)} + \mathcal{L}_G^{(1)} + \mathcal{L}_G^{(2)}$$

$$\mathcal{L}_G^{(0)} = \frac{1}{4g_{YM}^2}\eta^{\mu\nu}\eta^{\alpha\beta}F^a_{\mu\alpha}F^a_{\nu\beta}$$

Lagrangian in laboratory frame
(no sign problem)

$$\mathcal{L}_G^{(1)} = \boldsymbol{\Omega}\cdot\boldsymbol{J}_G$$

mechanical coupling
(sign problem)

$$\mathcal{L}_G^{(2)} = \frac{1}{2g_{YM}^2}\left[\Omega^2(\boldsymbol{B}^a\cdot\boldsymbol{r})^2 + r^2(\boldsymbol{B}^a\cdot\boldsymbol{\Omega})^2\right]$$

magnetovortical coupling
(no sign problem)

(1) Gluon plasma under acceleration

Several reasons to study

1. Quark-gluon plasma in a new environment:

Effects of high temperature, high density, strong (electro)magnetic fields, vorticity on quark-gluon plasma have been intensively studied. Here we ask the question: what is the effect of acceleration on the phase diagram of QCD?

2. A practical angle:

→ Emerges an early stage of the collisions (no thermal equilibrium though)

→ Suggested to lead to a rapid thermalization of gluon matter due to very high deceleration set by the gluon saturation scale, $a \sim 1 \text{ GeV}$

→ Related to tunneling through the Rindler horizon of accelerating gluons

[D. Kharzeev, K. Tuchin, From Color Glass Condensate to Quark Gluon Plasma through the event horizon, Nucl. Phys. A753, 316 (2005)]

→ Extreme astrophysical environments (the Very Early Universe, black hole mergers?)

3. Academic (deeper?) questions:

→ Uniformly accelerating system possesses an event horizon, similar to black holes.

→ The Unruh temperature, the Hawking radiation, the Tolman-Ehrenfest law, …

→ Accelerating detector/observer vs. thermal state of accelerating system?

A uniform acceleration of a relativistic fluid

[for example: C. Cercignani and G. M. Kremer, The Relativistic Boltzmann Equation: Theory and Applications (Springer, 2002)]

(1) Under a uniform acceleration a, a generic particle system resides in **global thermal equilibrium** characterized by the inhomogeneous temperature $T(x)$, which is a function of the spacetime coordinate x.

(2) The fluid is described by the inverse temperature four-vector $\beta^\mu(x) = u^\mu(x)/T(x)$ associated with the local fluid four-velocity $u^\mu(x)$ normalized as $u^\mu u_\mu = -1$.

(3) The vector β^μ satisfies the Killing equation: $\partial^\mu \beta^\nu + \partial^\nu \beta^\mu = 0$.

(4) The solution that corresponds to acceleration a directed along the z axis:

$$\beta^\mu(x)\partial_\mu = \frac{1}{T_0}[(1 + a_0 z)\partial_t + a_0 t \partial_z].$$

A shorthand notation, equivalent to $\beta^t = \frac{1}{T_0}(1 + a_0 z)$ and $\beta^z = \frac{a_0 t}{T_0}$

→ local physical quantities:

temperature

$$T(x) = \frac{T_0}{\sqrt{(1+az)^2 - (at)^2}}$$

four-velocity

$$u^\mu(x)\partial_\mu = \frac{T(x)}{T_0}\left[(1+az)\partial_t + at\partial_z\right]$$

acceleration $a^\mu = u^\nu \partial_\nu u^\mu$

$$a^\mu(x)\partial_\mu = a\frac{T^2(x)}{T_0^2}\left[at\partial_t + (1+az)\partial_z\right]$$

Killing vector fields are associated with isometries, representing transformations that preserve distances on the manifold.

The Killing vector β^μ guarantees no heat flow between adjacent volume elements of the fluid.

Thermodynamic equilibrium at the fixed time $t = 0$:

We associate this particular temperature profile with the acceleration "a_0" of hot matter with the central temperature T_0.

temperature

$$T(z) = \frac{T_0}{1 + a_0 z}$$

four-velocity

$$u^\mu(z) = \delta^{\mu,t}$$

acceleration

$$a^\mu(z) = a(z)\delta^{\mu,z} = \frac{a_0 \delta^{\mu,z}}{1 + a_0 z}$$

from the previous page:

$$T(x) = \frac{T_0}{\sqrt{(1+az)^2 - (at)^2}}$$

$$u^\mu(x)\partial_\mu = \frac{T(x)}{T_0}\left[(1+az)\partial_t + at\partial_z\right]$$

$$a^\mu(x)\partial_\mu = a\frac{T^2(x)}{T_0^2}\left[at\partial_t + (1+az)\partial_z\right]$$

(1) temperature is inhomogeneous

(2) the fluid is static everywhere ($\boldsymbol{u} = 0$)

(3) proper thermal acceleration $\alpha^\mu = a^\mu / T$
has a constant length $\alpha = -\alpha^\mu \alpha_\mu = a_0 / T_0$

(4) The Rindler event horizon ($t = 0$):

$$z > z^{\mathrm{R}} = -\frac{1}{a_0}$$

(5) the Tolman-Ehrenfest (Luttinger) relation:

$$a(z) = -\frac{1}{T(z)}\frac{\partial T(z)}{\partial z}$$

→ fluid acceleration leads to a temperature gradient
→ a temperature gradient generates fluid acceleration

[F. Becattini and D. Rindori, "Extensivity, entropy current, area law and Unruh effect," Phys. Rev. D 99, 125011 (2019)]
Maik Selch, Ruslan A. Abramchuk, and M. A. Zubkov, "Effective Lagrangian for the macroscopic motion of fermionic matter," Phys. Rev. D 109, 016003 (2024)

General remarks - intermediate summary

(1) Equivalence principle: effects of **gravity** are locally indistinguishable from those of **acceleration**

(2) Tolman-Ehrenfest: in the background static **gravitational** field, the local equilibrium is achieved at inhomogeneous **temperature**

[Richard C. Tolman, “On the weight of heat and thermal equilibrium in general relativity,” Phys. Rev. 35, 904–924 (1930).
Richard C. Tolman and Paul Ehrenfest, “Temperature equilibrium in a static gravitational field,” Phys. Rev. 36, 1791– 1798 (1930).]

(3) Luttinger: **temperature** gradient can be treated as a **gravitational** force

[J. M. Luttinger, “Theory of thermal transport coefficients,” Phys. Rev. 135, A1505–A1514 (1964)]

“In fact, if the gravitational field didn’t exist, one could invent one for the purposes of this paper.”

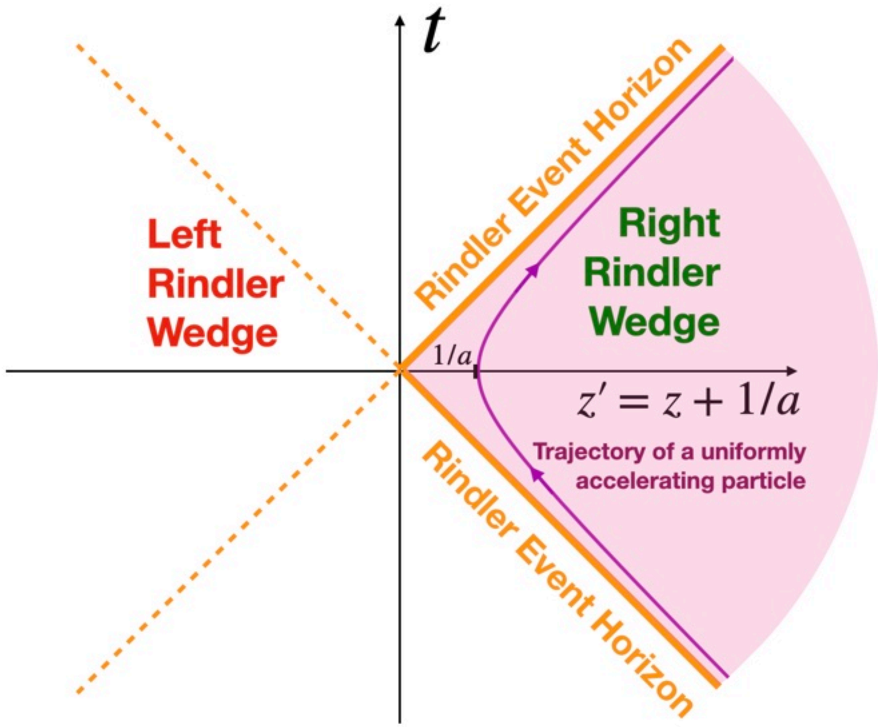
$$a(z) = -\frac{1}{T(z)}\frac{\partial T(z)}{\partial z}$$

(4) Temperature profile for a uniform acceleration:

$$T(x) = \frac{T_0}{\sqrt{(1+az)^2-(at)^2}}$$

(5) Singularity at the Rindler event horizon:

$$|\,1+az\,| \equiv |\,z'\,| = |\,t\,|$$



Study acceleration in lattice Monte Carlo?

Good news first: **No sign problem in imaginary time formalism!**

[acceleration] = meters per second^2

$$a = \frac{\partial^2 x}{\partial t^2}$$

for $t \to -i\tau$, one gets: $\frac{l}{t^2} \to -\frac{l}{\tau^2}$

$$a_{\mathrm{E}} = -a$$

Acceleration in imaginary time formalism in Euclidean spacetime

acceleration in Minkowski

just a sign flip

Thermal equilibrium under uniform acceleration can be accessed in Monte Carlo simulations on the lattice in imaginary time formalism!

Details of lattice simulations

Following [V.A. Goy, A.V. Molochkov, D.V. Stepanov, A.S. Pochinok, M.N.C., Extreme softening of QCD phase transition under weak acceleration: first principle Monte Carlo results for gluon plasma, PRL, ArXiv:2409.01847]

to keep finite volume effects under full control ↓

In order to simulate the lattice gauge theory at **fixed** spatial volume geometry, $L_x = L_y = L_z = \text{const}$, and **varying** temperature, $T = T(z)$, we need to work with an anisotropic lattice.

↑ to simulate acceleration at global thermodynamic equilibrium

$$L_\mu = a_\mu N_\mu\,, \quad \mu = x, y, z, \tau, \quad (\text{no sum over } \mu)$$

Simulation at: $N_\tau = 8;\ N_x = N_y = 42$

$N_z = 170, 148, 126, 104$

Two different lattice spacings in the spatial (a_σ) and imaginary-time (a_τ) directions.

[F. Karsch, SU(N) gauge theory couplings on asymmetric lattices, Nucl. Phys. B 205, 285 (1982).]

The Wilson action with anisotropic couplings:

$$S = \sum_x \sum_{i>j=1}^{3} \beta_\sigma(x_3)\left(1 - \mathcal{P}_{x,ij}\right) \quad \text{spatial plaquettes}, \quad z \equiv x_3$$

$$+ \sum_x \sum_{i=1}^{3} \beta_\tau(x_3)\left(1 - \mathcal{P}_{x,4i}\right) \quad \text{temporal plaquettes}$$

Physical temperature:

$$T(z) = \frac{1}{L_\tau(z)}$$

The length of the imaginary time:

$$L_\tau(z) = N_\tau a_\tau(z)$$

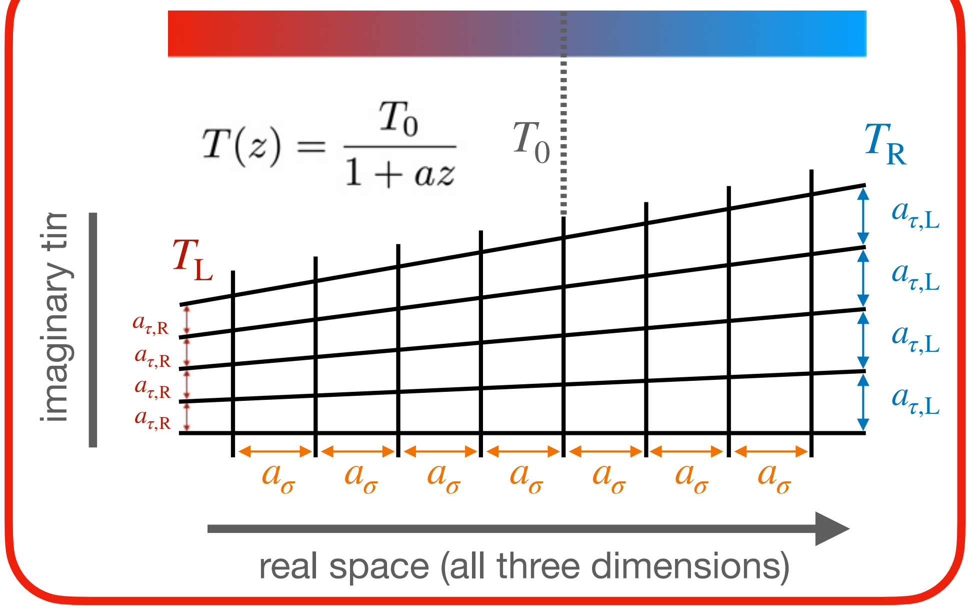
Acceleration:

$$T(z) = \frac{T_0}{1 - a_E z}$$

The spacing of the imaginary time:

$$a_\tau(z) = a_0(1 - a_E z)$$

$$a_E = -a < 0$$



a_σ, a_τ, a_0 : lattice spacings (very standard notation)

a : acceleration in Minkowski spacetime (also, confusingly, very standard notation)

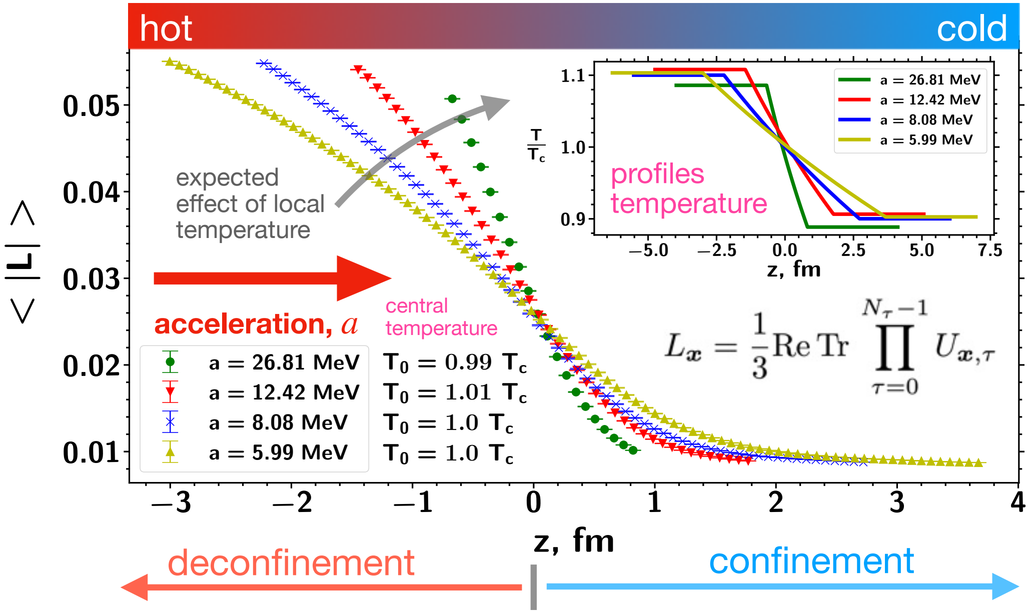
$a_E \equiv -a$: acceleration in the Euclidean space (after Wick rotation to imaginary time)

Lattice results on uniform acceleration

$$\langle L \rangle = e^{-F/T}$$

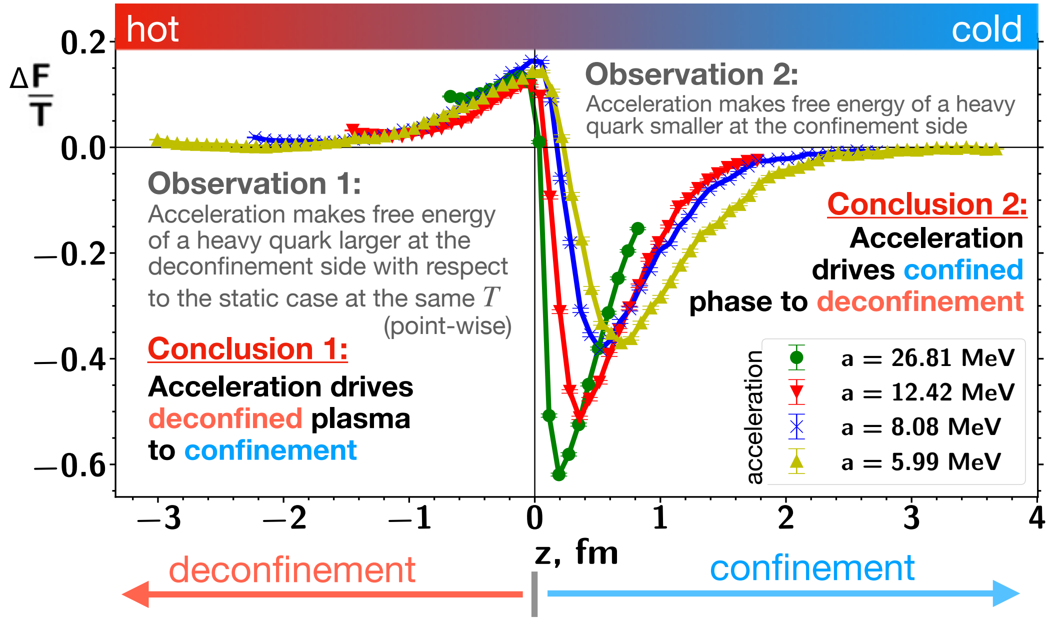
Polyakov loop (unrenormalized)

order parameter of confinement of color

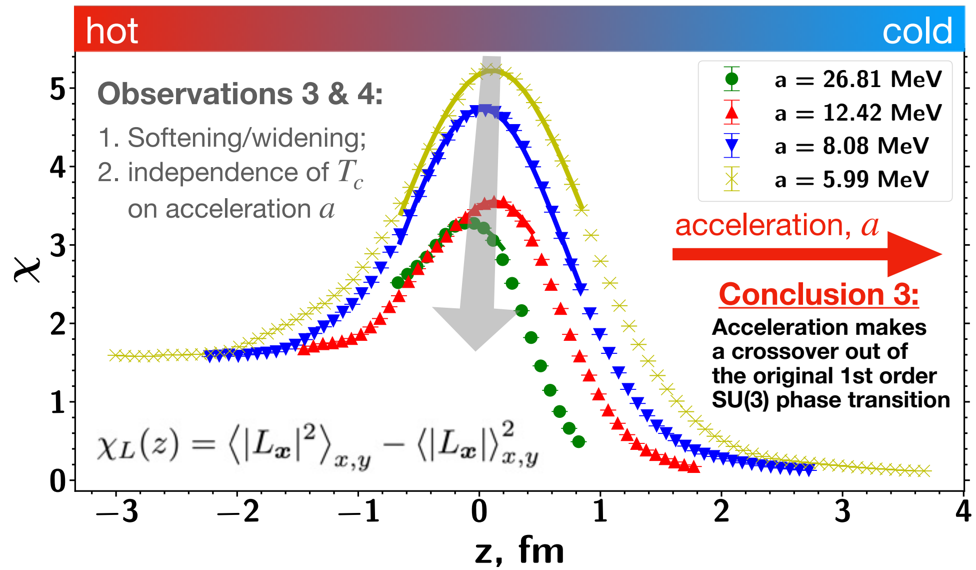


$$L_{\boldsymbol{x}} = \frac{1}{3} \mathrm{Re\,Tr} \prod_{\tau=0}^{N_\tau - 1} U_{\boldsymbol{x},\tau}$$

Change in free energy of a heavy quark due to acceleration (renormalized free energy)

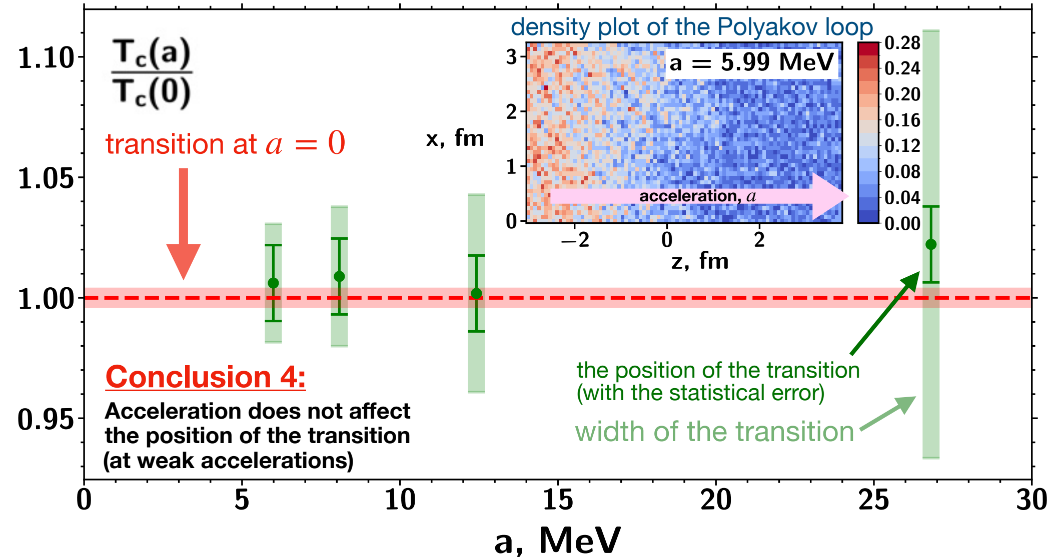


Polyakov loop susceptibility



$$\chi_L(z) = \langle |L_{\boldsymbol{x}}|^2 \rangle_{x,y} - \langle |L_{\boldsymbol{x}}| \rangle_{x,y}^2$$

Phase diagram of gluon plasma under acceleration



Conclusions

1. Effective models: rotation inhibits chiral condensate and reduces the chiral transition temperature (mechanism for quarks: fermionic Barnett effect)
2. Almost all (up to a few with fine-tunings) effective infrared models predict that the critical temperature decreases with rotation (contradicting the lattice data)
3. Possible mechanisms (related to each other):
   A) Thermal melting of the magnetic gluon condensate (magnetovortical coupling);
   B) Conformal anomaly in QCD (related, obviously, to “A”)
   C) Negative Barnett effect (negative spin-orbital coupling) for gluons
4. Globally rotating gluon plasma possesses a negative moment of inertia up to supervortical temperature $T_s = (1.50 \pm 0.1)T_c$
5. Fast rigid rotation of Yang-Mills plasma generates a new inhomogeneous phase
6. This inhomogeneous phase violates our intuition based on the Tolman-Ehrenfest Law the effect (due to the magnetovortical coupling?)
7. Acceleration does not affect the critical temperature of the deconfinement transition (at least, for the weak accelerations studied, $a \simeq (6\ldots27)\ \mathrm{MeV}$)
8. Even the weakest studied acceleration, $a \simeq 6\ \mathrm{MeV}$, makes a crossover out of the original 1st order thermal phase transition in SU(3) Yang-Mills theory