Some unconventional enhanced black hole symmetries with physical implications

The tune of Love, and null infinity as an inverted extremal horizon

Based on: {[2209.02091], [2502.02694]}; [2506.15526].

In collaboration with: Sergei Dubovsky, Mikhail Ivanov; Laura Donnay, Shreyansh Agrawal.

Black holes and their symmetries

Tours; July 02-04, 2025

Panagiotis (Panos) Charalambous

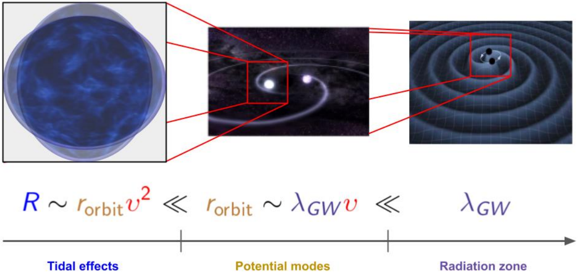
SISSA

European Research Council
Established by the European Commission

Table of Contents

Love numbers in GR - Worldline EFT

Goldberger & Rothstein [hep-th/0409156]
Porto [gr-qc/0511061]
Laura's talk



$$S^{\text{1-body}}_{\text{EFT}}\left[x_{\text{cm}}, h, A, \Phi\right] = S_{\text{bulk}}\left[\eta + h, A, \Phi\right] - M \int d\tau + S_{\text{finite-size}}\left[x_{\text{cm}}, h, A, \Phi\right]$$

Relativistic Love numbers := Wilson coefficients in worldline EFT

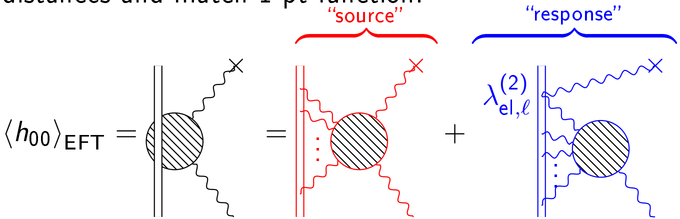
$$S_{\text{finite-size}} \supset \sum_j \frac{\lambda^{(j)}_{\text{el},\ell}}{2\ell!} \int d\tau \, \mathcal{E}^{(j)}_L\left(x_{\text{cm}}(\tau)\right) \mathcal{E}^{(j)\,L}\left(x_{\text{cm}}(\tau)\right) + \left(\mathcal{E} \leftrightarrow \mathcal{B}, \mathcal{T}\right) ,$$

$$\mathcal{E}^{(0)}_L = \partial_{\langle i_\ell} \ldots \partial_{i_1\rangle} \Phi , \quad \mathcal{E}^{(1)}_L = \partial_{\langle i_\ell} \ldots \partial_{i_2} F_{i_1\rangle 0} , \quad \mathcal{E}^{(2)}_L = \partial_{\langle i_\ell} \ldots \partial_{i_3} R_{i_2|0|i_1\rangle 0} .$$

Newtonian matching

Kol & Smolkin [1110.3764]
Hui & Joyce & Penco & Santoni & Solomon [2010.00593]
PC & Dubovsky & Ivanov [2102.08917]

- Put a pure 2^ℓ-pole background with source moments $\bar{\mathcal{E}}_L$ at large distances and match 1-pt function:



$$= \bar{\mathcal{E}}_L x^L \left[\left(1 + a_1 \frac{GM}{r} + \dots \right) + \frac{\lambda^{(2)}_{\text{el},\ell}}{r^{2\ell+1}} \left(b_0 + b_1 \frac{GM}{r} + \dots \right) \right]$$

$$\boxed{\lambda^{(j)}_\ell \propto k^{(j)\text{Love}}_\ell (\omega = 0)\, \mathcal{R}^{2\ell+1}}$$

- $j = 0$: Scalar response
- $j = 1$: Electric/Magnetic polarization
- $j = 2$: Electric/Magnetic tidal response

The tune of Love

- Totalitarian principle: "*Everything not forbidden is compulsory!*"
- 't Hooft naturalness (1980): "*At any energy scale, a physical parameter is allowed to be small if setting it to zero enhances the symmetry of the system. Otherwise, its natural value is an* $\mathcal{O}(1)$ *number*".

Magic zeroes in the black hole response problem

- For *all* isolated asymptotically flat GR (Kerr-Newman) black holes:

Fang & Lovelace [0505156]
Hinderer [0711.2420]
Damour & Nagar [0906.0096]
Binnington & Poisson [0906.1366]

Poisson [1411.4711]
Le Tiec & Casals [2007.00214]
Chia [2010.07300]
Le Tiec et. al [2010.15795]
PC & Dubovsky & Ivanov [2102.08917]

Gürlebeck [1503.03240]
De Luca et al. [2305.14444]
Riva et al. [2312.05065]

$$k_{\ell m}^{(s)\text{Love}}(\omega = 0) = 0 \quad \Rightarrow \quad \boxed{\text{Fine-tuning or enhanced symmetry?}}$$

Porto [1606.08895]

Scalar perturbations of Schwarzschild black hole ($c = 1$)

- Massless scalar field in Schwarzschild background ($r_{\text{h}} = r_s = 2GM$):

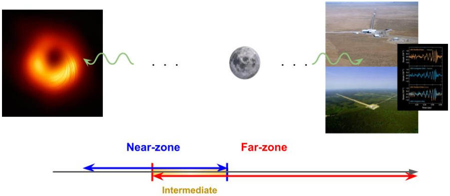
$$\Box\Phi = 0\,, \quad ds^2 = -\frac{\Delta(r)}{r^2}dt^2 + \frac{r^2}{\Delta(r)}dr^2 + r^2 d\Omega_2^2\,, \quad \Delta(r) = r(r - r_s)$$

- Full e.o.m.: $\mathbb{O}_{\text{full}}\,\Phi_{\omega\ell m} = \left[\partial_r\,\Delta\,\partial_r - \frac{r^4}{\Delta}\partial_t^2\right]\Phi_{\omega\ell m} = \ell(\ell+1)\,\Phi_{\omega\ell m}$

Response as low-energy scattering problem

Starobinsky (1973,1974)

- Near-zone region: $\omega(r - r_{\text{h}}) \ll 1$
- Far-zone region: $\omega r \gg 1$
- Intermediate region: $r_{\text{h}} \ll r \ll \frac{1}{\omega}$



Chia [2010.07300]
PC & Dubovsky & Ivanov [2102.08917]
Ivanov & Zhou [2209.14324]
PC & Ivanov [2303.16036]
Saketh & Zhou & Ivanov [2307.10391]

Scalar perturbations of Schwarzschild black hole ($c = 1$)

- Massless scalar field in Schwarzschild background ($r_{\rm h} = r_s = 2GM$):

$$\Box\Phi = 0\,, \quad ds^2 = -\frac{\Delta(r)}{r^2}dt^2 + \frac{r^2}{\Delta(r)}dr^2 + r^2 d\Omega_2^2\,, \quad \Delta(r) = r(r - r_s)$$

- Full e.o.m.: $\mathbb{O}_{\rm full}\,\Phi_{\omega\ell m} = \left[\partial_r\,\Delta\,\partial_r - \frac{r^4}{\Delta}\partial_t^2\right]\Phi_{\omega\ell m} = \ell(\ell+1)\,\Phi_{\omega\ell m}$
- NZ e.o.m.: $\mathbb{O}_{\rm NZ}\,\Phi_{\omega\ell m} = \left[\partial_r\,\Delta\,\partial_r - \frac{r_s^4}{\Delta}\partial_t^2\right]\Phi_{\omega\ell m} = \ell(\ell+1)\,\Phi_{\omega\ell m}$

$SL(2,\mathbb{R})$ symmetry of $\mathbb{O}_{\rm NZ}$:

Bertini & Cacciatori & Klemm [1106.0999]
Kim & Myung & Park [1205.3701]

$$L_0 = -\kappa^{-1}\partial_t\,, \quad L_{\pm1} = e^{\pm\kappa t}\left[\mp\sqrt{\Delta}\,\partial_r + \left(\sqrt{\Delta}\right)'\kappa^{-1}\partial_t\right], \quad \kappa = 2r_s\,.$$

$$[L_m, L_n] = (m-n)\,L_{m+n}\,,\ \mathcal{C}_2 = \mathbb{O}_{\rm NZ}$$

$$L_0\Phi_{\omega\ell m} = i\kappa^{-1}\omega\Phi_{\omega\ell m}$$

$$\mathcal{C}_2^{SL(2,\mathbb{R})}\Phi_{\omega\ell m} = \ell(\ell+1)\,\Phi_{\omega\ell m}$$

Scalar perturbations of Schwarzschild black hole ($c=1$)

- Massless scalar field in Schwarzschild background ($r_h = r_s = 2GM$):

$$\Box\Phi = 0\,, \quad ds^2 = -\frac{\Delta(r)}{r^2}dt^2 + \frac{r^2}{\Delta(r)}dr^2 + r^2 d\Omega_2^2\,, \quad \Delta(r) = r(r-r_s)$$

- Full e.o.m.: $\mathbb{O}_{\text{full}}\,\Phi_{\omega\ell m} = \left[\partial_r\,\Delta\,\partial_r - \frac{r^4}{\Delta}\partial_t^2\right]\Phi_{\omega\ell m} = \ell(\ell+1)\,\Phi_{\omega\ell m}$
- NZ e.o.m.: $\mathbb{O}_{\text{NZ}}\,\Phi_{\omega\ell m} = \left[\partial_r\,\Delta\,\partial_r - \frac{r_s^4}{\Delta}\partial_t^2\right]\Phi_{\omega\ell m} = \ell(\ell+1)\,\Phi_{\omega\ell m}$

$SL(2,\mathbb{R})$ symmetry of $\mathbb{O}_{\text{NZ}}$:

Bertini & Cacciatori & Klemm [1106.0999]
Kim & Myung & Park [1205.3701]

$$L_0 = -\kappa^{-1}\partial_t\,, \quad L_{\pm1} = e^{\pm\kappa t}\left[\mp\sqrt{\Delta}\,\partial_r + \left(\sqrt{\Delta}\right)'\kappa^{-1}\partial_t\right]\,, \quad \kappa = 2r_s\,.$$

$[L_m, L_n] = (m-n)\,L_{m+n}\,,\ \mathcal{C}_2 = \mathbb{O}_{\text{NZ}}$

$$L_0\Phi_{\omega\ell m} = i\kappa^{-1}\omega\Phi_{\omega\ell m}$$
$$\mathcal{C}_2^{SL(2,\mathbb{R})}\Phi_{\omega\ell m} = \ell(\ell+1)\,\Phi_{\omega\ell m}$$

***Selection rule*:**

$$L_{+1}^{\ell+1}\Phi_{\omega=0,\ell m} = 0$$

PC&Dubovsky&Ivanov [2103.01234] [2209.02091]

$\rightsquigarrow \partial_r^{\ell+1}\Phi_{\omega=0,\ell m} = 0 \Rightarrow$ No $r^{-\ell-1}$ term!

Love symmetry for Kerr-Newman black holes

PC & Dubovsky & Ivanov [2103.01234,2209.02091]

$$L_0^{(s)} = -\kappa^{-1}\partial_t + s$$

$$L_{\pm 1}^{(s)} = e^{\pm\kappa t}\left[\mp\sqrt{\Delta}\,\partial_r + \left(\sqrt{\Delta}\right)'\kappa^{-1}\partial_t + \frac{a}{\sqrt{\Delta}}\partial_\phi - s\,(1\pm 1)\left(\sqrt{\Delta}\right)'\right]$$

Teukolsky (1973)

$$\phi_0 = F_{\mu\nu} l^\mu m^\nu\,, \qquad \phi_1 = \tfrac{1}{2}F_{\mu\nu}(l^\mu n^\nu + m^{*\mu} m^\nu)\,, \qquad \phi_2 = F_{\mu\nu} m^{*\mu} n^\nu\,, \tag{1.1}$$

$$\psi_0 = -C_{\alpha\beta\gamma\delta} l^\alpha m^\beta l^\gamma m^\delta\,, \qquad \psi_4 = -C_{\alpha\beta\gamma\delta} n^\alpha m^{*\beta} n^\gamma m^{*\delta}\,, \tag{1.3}$$

$$ds^2 = (1 - 2Mr/\Sigma)dt^2 + (4Mar\sin^2(\theta)/\Sigma)dt\,d\varphi - (\Sigma/\Delta)dr^2 - \Sigma d\theta^2 - \sin^2(\theta)(r^2 + a^2 + 2Ma^2 r\sin^2(\theta)/\Sigma)d\varphi^2\,. \tag{4.1}$$

Here M is the mass of the black hole, aM its angular momentum, $\Sigma = r^2 + a^2\cos^2\theta$, and[1] $\Delta = r^2 - 2Mr + a^2$. When $a = 0$, the metric reduces to the Schwarzschild metric, a nonrotating black hole.

TABLE 1
Field Quantities ψ, Spin-Weight s, and Source Terms T for Equation (4.7)

ψ	s	T
Φ	0	$\Box\Phi = 4\pi T$
χ_0 $\rho^{-1}\chi_1$	$\frac{1}{2}$ $-\frac{1}{2}$	See references in Appendix B
ϕ_0 $\rho^{-2}\phi_2$	1 -1	J_0 (eq. [3.6]) $\rho^{-2}J_2$ (eq. [3.8])
$\psi_0{}^B$ $\rho^{-4}\psi_4{}^B$	2 -2	$2T_0$ (eq. [2.13]) $2\rho^{-4}T_4$ (eq. [2.15])

$$\left[\frac{(r^2+a^2)^2}{\Delta} - a^2\sin^2\theta\right]\frac{\partial^2\psi}{\partial t^2} + \frac{4Mar}{\Delta}\frac{\partial^2\psi}{\partial t\partial\varphi} + \left[\frac{a^2}{\Delta} - \frac{1}{\sin^2\theta}\right]\frac{\partial^2\psi}{\partial\varphi^2} - \Delta^{-s}\frac{\partial}{\partial r}\left(\Delta^{s+1}\frac{\partial\psi}{\partial r}\right) - \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) - 2s\left[\frac{a(r-M)}{\Delta} + \frac{i\cos\theta}{\sin^2\theta}\right]\frac{\partial\psi}{\partial\varphi} - 2s\left[\frac{M(r^2-a^2)}{\Delta} - r - ia\cos\theta\right]\frac{\partial\psi}{\partial t} + (s^2\cot^2\theta - s)\psi = 4\pi\Sigma T\,. \tag{4.7}$$

- Selection rule: $(L_{+1}^{(s)})^{\ell+s+1}\psi_{s,\omega=0,\ell m} = 0 \quad \rightsquigarrow k_{\ell m}^{(s)\mathrm{Love}}(\omega = 0) = 0$
- Love $SL(2,\mathbb{R}) \times \overline{SL(2,\mathbb{R})}$, with $\overline{SL(2,\mathbb{R})}$ not globally defined, $\bar{L}_m^{(s)} \xrightarrow{\phi\to\phi+2\pi} e^{2\pi m\kappa/\Omega}\bar{L}_m^{(s)}$, and without smooth spinless limit.

Hui & Joyce & Penco & Santoni & Solomon [2010.00593]
Pereñiguez & Cardoso [2112.08400]
PC & Dubovsky & Ivanov [2103.01234,2209.02091]
PC & Ivanov [2303.16036]
Rodriguez & Santoni & Solomon & Temoche [2304.03743]
PC [2402.07574]
Gray & Keeler & Kubiznak & Martin [2409.05964]
PC & Dubovsky & Ivanov [2502.02694]
Kol & Smolkin [1110.3764]
Gürlebeck [1503.03240]
De Luca & Khoury & Wong [2305.14444]
Riva & Santoni & Savić & Vernizzi [2312.05065]
Iteanu & Riva & Santoni & Savić & Vernizzi [2410.03542]
Combaluzier-Szteinsznaider et al. [2410.10952]
Kehagias & Riotto [2410.11014]
Gounis & Kehagias & Riotto [2412.08249]

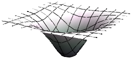
Black configuration	Type of perturbation	Magic zeroes?	Love symmetry?
Schwarzschild-Tangherlini ($d \geq 4$)	p-form "el."	✓($\frac{\ell \pm p}{d-3} \in \mathbb{N}$)	✓
	p-form "mag."	✓($\frac{\ell \pm (d-2-p)}{d-3} \in \mathbb{N}$)	✓
	Gr. el. (Z)	✓($\frac{\ell \pm (d-3)}{d-3} \in \mathbb{N}$)	(? for $d > 4$)
	Gr. mag. (RW)	✓($\frac{\ell \pm (d-2)}{d-3} \in \mathbb{N}$)	✓
	Gr. tensor ($d > 4$)	✓($\frac{\ell}{d-3} \in \mathbb{N}$)	✓
Reissner-Nordström ($d \geq 4$)	Scalar	✓($\frac{\ell}{d-3} \in \mathbb{N}$)	✓
	p-form ($p \geq 2$)	×	×
	Gr. tensor ($d > 4$)	✓($\frac{\ell}{d-3} \in \mathbb{N}$)	✓
Kerr ($d = 4$)	Scalar, Em., Gr.	✓($\ell \in \mathbb{N}$)	✓
Myers-Perry ($d = 5$)	Scalar	✓($\frac{\ell}{2} \in \mathbb{N}$)	✓
Einstein-Maxwell Lense-Thirring ($d \geq 4$)	Scalar	✓($\frac{\ell}{d-3} \in \mathbb{N}$)	✓
Schw.+Riemann$^{n \geq 2}$ ($d \geq 4$)	Scalar	×	×
Static black p-branes (non-dilatonic)	Scalar	✓($\frac{\ell}{d-p-3} \in \mathbb{N}$)	✓
Schwarzschild, Kerr ($d = 4$)	*Non-linear gravitational, static & axisymmetric*	✓($\ell \in \mathbb{N}$)	?

Relation to enhanced NHE isometry

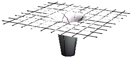
PC & Dubovsky & Ivanov [2209.02091]
PC & Ivanov [2303.16036]
PC & Dubovsky & Ivanov [2502.02694]

- At extremality, near-horizon develops an infinite throat that *decouples* from far-horizon geometry and acquires an AdS structure.
- RN_d: $\text{SL}(2,\mathbb{R})_{\text{NHE}} \times SO(d-1)$
- MP_d: $\text{SL}(2,\mathbb{R})_{\text{NHE}} \times U(1)^{\lfloor \frac{d-1}{2} \rfloor}$
- p-branes: $SO(p+1,2)_{\text{NHE}} \times SO(d-p-1)$

Carter (1973)
Kunduri & Lucietti & Reall [0705.4214]
Galajinsky [1209.5034]
Bardeen & Horowitz [hep-th/9905099]
Figueras et. al. [0803.2998]
Galajinsky et. al [1303.4901]



$Q^2 < M^2$



$Q^2 = M^2$

$$\text{RN}_d: \lim_{Q^2 \to M^2} \text{SL}(2,\mathbb{R})_{\text{Love}} = \text{SL}(2,\mathbb{R})_{\text{NHE}}$$

$$\text{MP}_{4,5}: \lim_{a^2 \to M^2} \left(\text{SL}(2,\mathbb{R})_{\text{BH}} \subset \text{SL}(2,\mathbb{R})_{\text{Love}} \ltimes \hat{U}(1)^{\lfloor \frac{d-1}{2} \rfloor}\right) = \text{SL}(2,\mathbb{R})_{\text{NHE}}$$

$$\text{Black strings: } \lim_{Q^2 \to M^2} SO(2,2)_{\text{Love}} = SO(2,2)_{\text{NHE}}$$

Summary and extensions

- Love numbers are Wilson coefficients in worldline EFT that capture the conservative response of compact bodies to external "tidal" fields.
- Black holes in $d = 4$ GR have vanishing static Love numbers.
- Enhanced $SL(2,\mathbb{R})$ symmetry in near-zone $\rightsquigarrow$ Highest-weight banishes Love
- For $d > 4$ black holes and black strings, Love symmetries still exist and are in accordance with the more intricate vanishing of Love numbers.
- For modified GR, Love symmetry in general does not exist and Love numbers have their natural non-zero and RG-flowing values.
- For scalar perturbations of RN and p-form perturbations of Schwarzschild, Love $SL(2,\mathbb{R})$ admits a full centerless Virasoro (Witt) algebra extension.
- Approximate near-zone symmetries are isometries of near-NHE geometries.
- **Closely related to enhanced isometry of NHE throat.**

Table of Contents

Null infinity as a non-expanding horizon

Ashtekar & Speziale {[2401.15618] [2402.17977]

Null infinity as a non-expanding horizon (NEH)

Simone's talk

$\mathscr{I}^+$ is a NEH of the conformal completion of an asymptotically flat spacetime (AFS),

$$ds^2_{\mathscr{I}^+} = \Omega^2 d\tilde{s}^2_{\mathscr{I}^+} \quad \text{with} \quad \Omega = \frac{\alpha}{r} \,.$$

- Even in the presence of radiation, if $T_{\mu\nu} = \mathcal{O}\left(\Omega^2\right)$.
- $\exists$ divergence-free conformal frames $\rightsquigarrow$ $\mathscr{I}$ is an extremal NEH.
- $g_{uA} = \mathcal{O}\left(r^0\right) \rightsquigarrow \mathscr{I}$ is a non-twisting NEH.

$\mathscr{H}$ / $\mathscr{I}$ correspondence

Godazgar & Godazgar & Pope [1707.09804]
Agrawal & PC & Donnay [2506.15526]

Conformally completed AFS $\simeq$ Geometry near extremal horizon

Under the spatial inversion

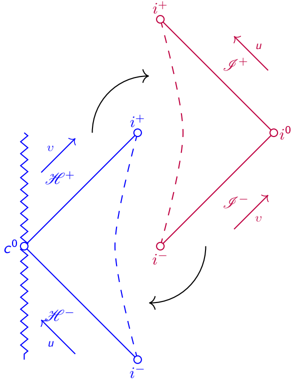
$$\boxed{r \mapsto \frac{\alpha^2}{\rho}\,, \quad u \mapsto v \quad \Rightarrow \quad d\tilde{s}^2_{\mathscr{I}^+} \mapsto ds^2_{\mathscr{H}^+}}\,,$$

with:

$$\mathcal{F}(v,\rho,x^A) = \alpha^{-2} F\left(u \mapsto v, r \mapsto \frac{\alpha^2}{\rho}, x^A\right)\,,$$

$$\theta^A(v,\rho,x^B) = -\alpha^{-4}\rho U^A\left(u \mapsto v, r \mapsto \frac{\alpha^2}{\rho}, x^B\right)\,,$$

$$g_{AB}(v,\rho,x^C) = \alpha^2 \mathcal{H}_{AB}\left(u \mapsto v, r \mapsto \frac{\alpha^2}{\rho}, x^C\right)\,.$$



$$ds^2_{\mathscr{H}^+} = -\rho^2 \mathcal{F} dv^2 + 2 dv d\rho + g_{AB}(dx^A + \rho\,\theta^A dv)(dx^B + \rho\,\theta^B dv)\,,$$

$$ds^2_{\mathscr{I}^+} = -F du^2 - 2 du dr + r^2 \mathcal{H}_{AB}\left(dx^A - \frac{U^A}{r^2}du\right)\left(dx^B - \frac{U^B}{r^2}du\right) = \left(\frac{\alpha}{r}\right)^2 d\tilde{s}^2_{\mathscr{I}^+}\,.$$

$\mathscr{H}/\mathscr{I}$ correspondence

Agrawal & PC & Donnay [2506.15526]

$\mathscr{H}/\mathscr{I}$ dictionary

$\mathscr{H}$	Name	Evolution equation	$\mathscr{I}$
κ	"surface gravity"	-	0
Θ	longitudinal expansion	Null Raychaudhuri eq.	0
ω_A	twist 1-form	Damour eq.	0
σ_{AB}	longitudinal shear	Tidal force eq.	0
Ω_{AB}	horizon metric	(free data)	q_{AB} (fixed)
λ_{AB}	transversal shear	Trans. deform. rate ev. eq.	C_{AB} (free data)

$$ds^2_{\mathscr{H}^+} = -2\rho\,\kappa\,dv^2 + 2dv d\rho + 2\rho\,\vartheta_A dv dx^A + (\Omega_{AB} + \rho\lambda_{AB})\,dx^A dx^B + \dots\,,$$
$$ds^2_{\mathscr{I}^+} = -F_0\,du^2 - 2du dr - 2u_A du dx^A + \left(r^2 q_{AB} + rC_{AB}\right) dx^A dx^B + \dots\,.$$

⚠ *The AFS and its "dual" extremal, non-rotating horizon do <u>not</u> in general live in the same spacetime.*

The Couch-Torrence inversion symmetry of ERN

Eric's talk

Extremal Reissner-Nordström (ERN) black hole ($d=4$, $G=c=1$):

$$ds^2_{\rm ERN} = -\frac{\Delta(r)}{r^2}dt^2 + \frac{r^2 dr^2}{\Delta(r)} + r^2 d\Omega_2^2\,, \quad \Delta(r) = (r-M)^2\,.$$

Couch-Torrence (CT) inversion

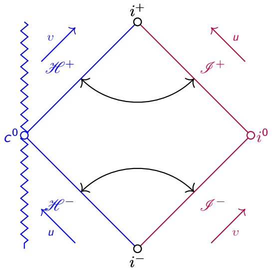
Couch & Torrence (1984)
Borthwick & Gourgoulhon & Nicolas [2303.14574]

$$\boxed{r \xrightarrow{\rm CT} \tilde{r} = \frac{Mr}{r-M} : \textbf{Isometry of } r^{-2}ds^2_{\rm ERN}}\,,$$

CT inversion = Reflection of tortoise coord.
that preserves $r_{\rm ph} = 2M$:

$$r_* = r - M - \frac{M^2}{r-M} + 2M\ln\left|\frac{r-M}{M}\right| \xrightarrow{\rm CT} -r_*$$

$$\Rightarrow (v, r, x^A) \xleftrightarrow{\rm CT} \left(u, \frac{Mr}{r-M}, x^A\right) \Leftrightarrow \boxed{\mathscr{H}^\pm \xleftrightarrow{\rm CT} \mathscr{I}^\pm}\,.$$



Physical implications: Matching of near-$\mathscr{H}$ and near-$\mathscr{I}$ charges

Bizon & Friedrich [1212.0729], Lucietti & Murata & Reall & Tanahashi [1212.2557]
Bhattacharjee & Chakrabarty & Chow & Paul & Virmani [1805.10655]
Fernandes & Ghosh & Virmani [2008.04365], Borthwick & Gourgoulhon & Nicolas [2303.14574]
Agrawal & PC & Donnay [2506.15526]

Massless (minimally coupled) scalar perturbations of ERN black hole:

$$\Box^{(0)}_{\text{ERN}}\Phi = 0 \Rightarrow \begin{cases} \Box^{(0)}_{\mathscr{I}^+}\Phi(u,r,x^A) = 0\,, & \Phi(u,r,x^A) \sim \frac{1}{r}\sum_{n=0}^{\infty}\Phi^{(n)}(u,x^A)\frac{1}{r^n}\,; \\ \Box^{(0)}_{\mathscr{H}^+}\hat{\Phi}(v,r,x^A) = 0\,, & \hat{\Phi}(v,r,x^A) \sim \sum_{n=0}^{\infty}\hat{\Phi}^{(n)}(v,x^A)\left(\frac{r-M}{M}\right)^n\,; \end{cases}.$$

$$r^2\Box^{(0)}_{\mathscr{I}^+} = \partial_r\left(r-M\right)^2\partial_r - 2r\partial_u\partial_r\,r + \Delta_{\mathbb{S}^2}\,,\quad r^2\Box^{(0)}_{\mathscr{H}^+} = \partial_r\left(r-M\right)^2\partial_r + 2r\partial_v\partial_r r + \Delta_{\mathbb{S}^2}\,.$$

Newman-Penrose conserved quantities

Newman & Penrose (1965, 1968)
Exton & Newman & Penrose (1969)

$$N^{(0)}_{\ell m} = \frac{(-1)^{\ell+1}}{(\ell+1)!}\lim_{r\to\infty}\int_{\mathbb{S}^2}Y^*_{\ell m}\left[\left(r-M\right)^2\partial_r\right]^{\ell}\left[r\left(r-M\right)\partial_r\left(r\Phi\right)\right] \rightsquigarrow \partial_u N^{(0)}_{\ell m} = 0\,.$$

Aretakis conserved quantities

Aretakis[1110.2007,1110.2009][1206.6598]

$$A^{(0)}_{\ell m} = \frac{M^{\ell-1}}{(\ell+1)!}\lim_{r\to M}\int_{\mathbb{S}^2}Y^*_{\ell m}\partial^{\ell}_r\left[r\partial_r\left(r\hat{\Phi}\right)\right] \rightsquigarrow \partial_v A^{(0)}_{\ell m} = 0\,.$$

CT inversion $\rightsquigarrow N^{(0)}_{\ell m} \simeq A^{(0)}_{\ell m}$

Bizon & Friedrich [1212.0729]
Lucietti & Murata & Reall & Tanahashi [1212.2557]
Bhattacharjee&Chakrabarty& Chow&Paul&Virmani [1805.10655]

- Scalar wave operator transforms homogeneously under CT inversion:

$$\square^{(0)}_{\mathscr{I}^+} \xrightarrow{\text{CT}} \tilde{\square}^{(0)}_{\mathscr{I}^+} = \Omega^{+1}\square^{(0)}_{\mathscr{H}^+}\Omega^{+1}\,, \quad \Omega = \tfrac{r-M}{M}\,.$$

- Action of CT inversion on scalar field:

$$\Phi(u,r,x^A) \xrightarrow{\text{CT}} \tilde{\Phi}(u,r,x^A) = \Omega^{-1}\hat{\Phi}\left(\upsilon \mapsto u, r \mapsto \tfrac{Mr}{r-M}, x^A\right).$$

$\rightsquigarrow$ If $\square^{(0)}_{\mathscr{H}^+}\hat{\Phi}(\upsilon,r,x^A) = 0$, then $\square^{(0)}_{\mathscr{I}^+}\tilde{\Phi}(u,r,x^A) = 0 \xRightarrow{b.c.'s} \Phi(u,r,x^A) = \tilde{\Phi}(u,r,x^A)$.

$$\therefore \boxed{\underline{\text{Matching condition:}} \quad \Phi(u,r,x^A) = \frac{M}{r-M}\hat{\Phi}\left(\upsilon \mapsto u, r \mapsto \frac{Mr}{r-M}, x^A\right)},$$

$$\Rightarrow \cdots \Rightarrow \boxed{\boxed{N^{(0)}_{\ell m} = M^{\ell+2}A^{(0)}_{\ell m}}}\,.$$

CT inversion $\rightsquigarrow N^{(s)}_{\ell m} \simeq A^{(s)}_{\ell m}$

Fernandes & Ghosh & Virmani [2008.04365]
Agrawal & PC & Donnay [2506.15526]

$$\psi_s := \begin{cases} \Phi & \text{for } s=0 \text{ (Massless scalar perturbations)}\,; \\ \phi_0 & \text{for } s=+1 \text{ (EM perturbations (frozen gr.))}\,; \\ \Psi_0 & \text{for } s=+2 \text{ (Gr. perturbations (constrained em.))}\,; \end{cases} \quad \rightsquigarrow \Box^{(s)}_{\text{ERN}}\psi_s = 0\,.$$

NP constants: $N^{(s)}_{\ell m} = \frac{(-1)^{\ell-s+1}}{(\ell-s+1)!}\lim_{r\to\infty}\int_{\mathbb{S}^2} {}_sY^*_{\ell m}\left[(r-M)^2\partial_r\right]^{\ell-s}\left[\frac{(r-M)^{2s+1}}{r^{2s-1}}\partial_r\left(r^{2s+1}\psi_s\right)\right]$,

Aretakis constants: $A^{(s)}_{\ell m} = \frac{M^{\ell-s-1}}{(\ell-s+1)!}\lim_{r\to M}\int_{\mathbb{S}^2} {}_sY^*_{\ell m}\,\partial_r^{\ell-s}\left[\frac{1}{r^{2s-1}}\partial_r\left(r^{2s+1}\hat{\psi}_s\right)\right]$.

Matching condition: $\psi_s(u,r,x^A) = \left(\frac{M}{r-M}\right)^{2s+1}\hat{\psi}_s\left(v\mapsto u, r\mapsto \frac{Mr}{r-M}, x^A\right)$,

$$\Rightarrow \cdots \Rightarrow \boxed{N^{(s)}_{\ell m} = M^{\ell+s+2}A^{(s)}_{\ell m}}\,.$$

Summary and extensions

- $\mathscr{I}$ is an extremal and non-twisting NEH of the conformally completed AFS.

  Ashtekar & Speziale [2401.15618,2402.17977]

  $\rightsquigarrow$ The conformally completed AFS is a spatially inverted extremal and non-rotating near-horizon geometry. Godazgar & Godazgar & Pope [1707.09804] Agrawal & PC & Donnay [2506.15526]

- Self-inverted paradigm: ERN black hole ($d = 4$). Couch & Torrence (1984)

- Physical implications of existence of such conformal mappings:
  near-$\mathscr{I}$ Newman-Penrose constants $\simeq$ near-$\mathscr{H}$ Aretakis constants for scalar (Bizon & Friedrich [1212.0729], Lucietti & Murata & Reall & Tanahashi [1212.2557] Bhattacharjee & Chakrabarty & Chow & Paul & Virmani [1805.10655]), electromagnetic (Fernandes & Ghosh & Virmani [2008.04365]) and gravitational (Agrawal & PC & Donnay [2506.15526]) perturbations.

- Extremal Kerr-Newman black holes: Horizon is twisting and there is no (simple) CT inversion conformal isometry. E.o.m.'s *do* have a CT inversion conformal symmetry, which *acts non-locally in coordinate space*
  $\rightsquigarrow$ $N^{(s)}_{\ell,m=0} = M^{\ell+s+2} A^{(s)}_{\ell,m=0}$. Agrawal & PC & Donnay [2506.15526]

Outlook

- Matching of asymptotic symmetries — Donnay & Giribet & González & Pino {[1511.08687], [1607.05703]}; Mao & Wu & Zhang [1606.03226]
- Generalized Couch-Torrence inversion symmetry — Cvetic & Pope & Saha {[2008.04944], [2102.02826]}; Bianchi & Russo {[2110.09579], [2203.14900]}
- Full symmetry structure of AFS and $w_{1+\infty}$ for black holes — Strominger [2105.14346]; Freidel & Pranzetti & Raclariu [2112.15573]; Geiller [2403.05195]; Ruzziconi & Zwikel [2504.08027]
- $\mathscr{H}/\mathscr{I}$ correspondence for twisting horizons? — Godazgar & Godazgar & Pope [1707.09804]; Agrawal & PC & Donnay [2506.15526]
- Celestial holography Vs Kerr/CFT?

"Black holes are the hydrogen atom of the 21st century"

't Hooft (2016), EHT (April 10, 2019)

Thank you