Semiclassical aspects of two-dimensional black holes: singularity resolution via a negative central charge

José Navarro-Salas
Departamento de Física Teórica-IFIC
Universitat de València-CSIC, Spain

Partially based on
A. del Río, J. Marañón-González and J. N.-S., PRD 2025
C. García-Pérez, J. Marañón-Gonzalez, J. N.-S. and S. Pla, work in progress

Black Holes and their Symmetries, Tours, July 2-4, 2025

July 2, 2025, Tours

Why two-dimensional black holes ?

- Black hole singularities represent a fundamental breakdown of classical general relativity.
- To understand their resolution, we need a quantum theory of gravity.
- Full quantum gravity is notoriously hard
- Lower-dimensional toy models serve as *tractable laboratories*.
- In particular, 2D dilaton gravity models (e.g., Callan-Giddings-Harvey-Strominger (CGHS) model) capture key features:
  - Event horizons
  - Hawking radiation
  - Information paradox analogs
- Goal: Use semiclassical methods to explore singularity resolution in these models.

The role of quantum effects and central charge

- In 2D gravity, quantum effects are essentially modeled through the **conformal anomaly** of matter fields
$$\langle T \rangle = C \frac{\hbar}{24\pi} R$$
- The conformal anomaly is proportional to the central charge C of the underlying CFT.
- Standard scenarios use $C > 0$, describing ordinary matter fields.
- What happens if we consider **negative central charge** $(C < 0)$?
  - Emerges in ghost systems (typically, $C = -26$ for Fadeev -Popov (reparametrization) ghosts)
  - May act as a **semiclassical 2D analog** mimicking 4D gravity effects
- We explore whether such a modification can **smooth out the classical singularity**

Review of the CGHS Model: Classical Setup

- Proposed by Callan, Giddings, Harvey, and Strominger in 1991
- Action for 2D dilaton gravity coupled to N massless scalar fields:

$$S(g, \phi, matter) = \frac{1}{2}\int d^2x\sqrt{-g}\left[e^{-2\phi}\left(R + 4(\nabla\phi)^2 + 4\lambda^2\right) - \sum_{i=1}^{N}(\nabla f_i)^2\right]$$

- ϕ: dilaton field, f_i: scalar matter fields, λ: cosmological constant.
- $e^{-\phi} \sim r$ radius of 2-spheres in 4D gravity
- For a pulse of infalling matter, black hole formation occurs
- Event horizon and spacelike singularity naturally arise
- Remark: The CGHS action differs from that of the spherical reduction of general relativity in a seemingly minor way [$r_0 e^{-\phi} = r$]

$$S_{GR2d} = \frac{1}{2G_2}\int d^2x\sqrt{-g}e^{-2\phi}\left[\frac{R}{2} + (\nabla\phi)^2 + \frac{e^{2\phi}}{r_0^2}\right] + S_{matter},$$

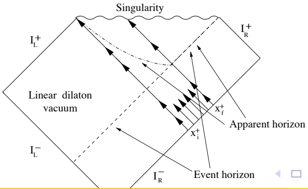
- Conformal gauge $ds^2 = -e^{2\rho}dx^+dx^-$
- We can completely fix coordinates (Kruskal gauge) by imposing: $e^{-2\phi} = e^{-2\rho}$
- This is possible due to an underlying global symmetry:

$$\partial_+\partial_-(\rho - \phi) = 0$$

- For $x^+ > x_f^+$ the solution take the form of a static (CGHS) black hole

$$ds^2 = -\frac{dx^+dx^-}{\frac{m}{\lambda} - \lambda^2 x^+ x^-}$$

- Apparent horizon $\equiv \partial_+\phi = 0$
- Curvature becomes singular as $x^+x^- \to m/\lambda^3$.



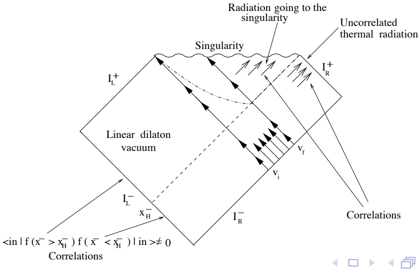
Hawking radiation

- Relation between the IN outgoing coordinate σ^-_{in} and the OUT outgoing coordinate σ^-_{out}

$$\sigma^-_{in} = -\frac{1}{\lambda}\log(e^{-\lambda\sigma^-_{out}} + P(x^+_f)/\lambda) \qquad P(x^+_f) \equiv \int_{x^+_i}^{x^+_f} dy^+ T_{++}(y^+)$$

- Hawking flux at I^+_R [at late times the flux goes to a constant, with $T_H = \frac{\lambda}{2\pi}$]

$$\langle 0_{in}|T_{--}\left(\sigma^-_{out}\right)|0_{in}\rangle|_{I^+_R} = \frac{N\lambda^2}{48\pi}\left[1 - \frac{1}{(1+\frac{P(x^+_f)}{\lambda}e^{\lambda\sigma^-})^2}\right],$$



Quantum corrections: semiclassical CGHS model

- Quantum effects from $N = C$ scalar fields included via the conformal anomaly
- Effective action includes the nonlocal Polyakov term:

$$S_P = -\frac{N}{96\pi} \int d^2x \sqrt{-g} R \frac{1}{\Box} R$$

- In general, the semiclassical theory cannot be solved in full closed form
- Reason: the quantum Polyakov term breaks the classical global symmetry

$$\delta g_{ab} = 2e^{2\phi} g_{ab} \qquad \delta\phi = e^{2\phi}$$

  and hence the conservation law $\partial_+ \partial_- (\rho - \phi) = 0$
- Russo-Susskind-Thorlacius (1992) proposed to slightly modify the semiclassical effective action by adding a local counterterm

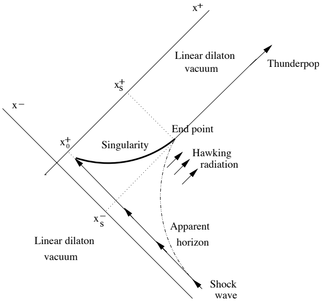
$$S_P \to S_P - \frac{N}{48\pi} \int d^2x \sqrt{-g} \phi R$$

- The main features of the RST model are:
  i) $\partial_+ \partial_- (\rho - \phi) = 0$ is preserved
  ii) The 2d Minkowski space is a semiclassical solution
- It is possible to find an analytical solution describing an evaporating black hole

Black Hole evaporation in the semiclassical RST model

- The energy radiated by the black hole E_{rad} plus the (negative) energy of the thunderpop E_{th} equals the energy of the classical collapsing shell of matter m
- Energy conservation:

$$E_{rad} + E_{thp} = m$$



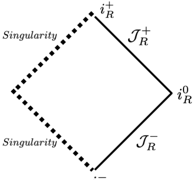
- Singularity and information loss persist in the semiclassical theory
- A thunderbolt is also generated at the end-point (Anderson-DeWitt mechanism), breaking energy conservation [Fabbri and N.-S., ICP-World Scientific '05]

Static semiclassical solutions in the RST model

- Let us go back to static configurations
- The semiclassical, static, asymptotically flat, radiationless solution (Boulware vacuum state) in the RST model can be written as

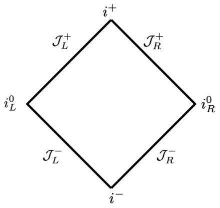
$$\rho = \phi \qquad\qquad e^{-2\phi} + \frac{C}{24\pi}\phi = -\lambda^2 x^+ x^- - \frac{C}{48\pi}\ln(-\lambda^2 x^+ x^-) + \frac{m}{\lambda}$$

- Remark 1: Boulware state is singular at the classical event horizon
- Remark 2: Boulware state required for unphysical fields ($C < 0$). No Hawking radiation, only vacuum polarization
- The event horizons are destroyed, and the null lines $x^+ = 0$ and $x^- = 0$ are converted into curvature singularities
- The geometry has the form of a non-symmetric wormhole with throat at $x^+ x^- = -\lambda^2/48$



Negative central charge

- Remarkably, reversing the sign of the central charge from $C > 0$ to $C < 0$ removes the curvature singularity !! [Potaux, Sarkar and Solodukhin, 2022]



Question: Is this an accident resulting from the specific features of the RST model?

1. What would happen if the RST counterterm were modified while maintaining solvability?
2. What would happen if the RST counterterm were removed ?
3. What about a different classical theory, such as spherically reduced Einstein's gravity ?

Negative central charge

- What would happen if the RST counterterm were modified while maintaining solvability?
- Consider the one-parameter $a \geq 0$ family of semiclassical models preserving $\partial_+\partial_-(\rho - \phi) = 0$

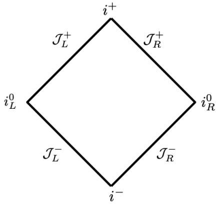
$$S_{RST} = -\frac{C}{48\pi}\int d^2x\sqrt{-g}\phi R \to \frac{C}{24\pi}\int d^2x\sqrt{-g}[(1-2a)(\nabla\phi)^2 + (a-1)\phi R]]$$

[J. Cruz and J. N.-S. PLB '96] $a = 1/2$ RST-model $\qquad a = 0$ BPP-model

- Semiclassical solutions with $C < 0$

$$\frac{aC}{12\pi}\phi + e^{-2\phi} = -\lambda^2 x^+ x^- - \frac{C}{48\pi}\ln(-\lambda^2 x^+ x^-) + \frac{M}{\lambda} \ .$$

- Curvature singularity removed $a > 0$ [Case $a = 0$: geodesically complete]



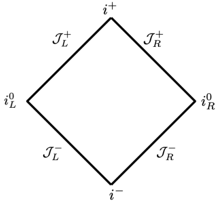
Negative central charge

- What would happen if the RST counterterm were removed ?

$$S_{one-loop} = -\frac{C}{96\pi}\int d^2x\sqrt{-g}R\frac{1}{\Box}R + S_{local}$$

$$S_{local} = 0$$

- Assuming that the solutions are static, asymptotically flat and radiationless, we can reduce the problem to a nonlinear differential equation with the appropriate boundary conditions
- Numerical integration confirms that, when $C < 0$, the semiclassical, backreacted (conformal) geometry obtained with solvable counterterms is reproduced

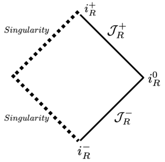


- What about spherically reduced Einstein's gravity ?

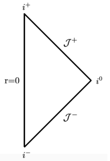
$$\frac{1}{2G_2}\int d^2x\sqrt{-g}e^{-2\phi}\left[\frac{R}{2}+(\nabla\phi)^2+\frac{e^{2\phi}}{r_0^2}\right]+S_{matter}$$
$$-\quad\frac{C}{96\pi}\int d^2x\sqrt{-g}R\frac{1}{\Box}R$$

Assuming static, asymptotically flat, and radiationless solutions, we get

- For $C > 0$ [Fabbri et al. PRD '06; Ho and Matsuo, CQG '18; Arrechea et al. PRD '20]



- When $C < 0$ [A. del Rio, J. Marañón-González and J. N.-S., PRD '25]



Dynamical solutions

- A proposal: [inspired by Potaux, Sarkar and Solodukhin PRL '23]

  Hybrid quantum states → Hybrid effective action

- Assume conventional matter fields with total $C_+ > 0$ and unconventional fields with $C_- < 0$ [$\hat{g}_{\mu\nu} \equiv g_{\mu\nu} e^{-2\phi}$ is a flat metric]

$$\begin{aligned} S(g,\phi,matter) &= \frac{1}{2}\int d^2x\sqrt{-g}\left[e^{-2\phi}\left(R+4(\nabla\phi)^2+4\lambda^2\right)-\sum_{i=1}^{N}(\nabla f_i)^2\right] \\ &- \frac{C_-}{96\pi}\int d^2x\sqrt{-\hat{g}}R(\hat{g})\frac{1}{\Box}R(\hat{g}) \\ &- \frac{C_+}{96\pi}\int d^2x\sqrt{-g}R\frac{1}{\Box}R \\ &+ \frac{C_+}{24\pi}\int d^2x\sqrt{-g}[(1-2a)(\nabla\phi)^2+(a-1)\phi R]] \end{aligned}$$

- The non-local Polyakov term for unconventional fields is constructed with respect the (on-shell) flat metric

$$\hat{g}_{\mu\nu} \equiv g_{\mu\nu} e^{-2\phi}$$

  This ensures absence of Hawking radiation for ghost fields [Strominger '93]

- The local counterterm for conventional matter ensures $\partial_+\partial_-(\rho-\phi)=0$

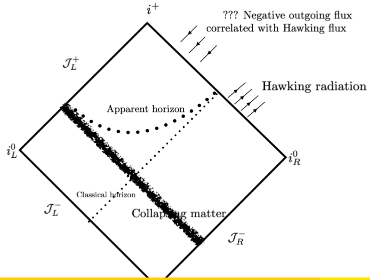
- Choose the parameter a such that 2D Minkowski is a semiclassical solution

$$a = \frac{C_+ + C_-}{2C_+}$$

- We want to start with 2D Minkowski space, all the fields in the Boulware state, and throw a shockwave of the classical ordinary matter. The semiclassical solution is

$$\begin{aligned} \frac{aC_+}{12\pi}\phi + e^{-2\phi} &= -\lambda^2 x^+ x^- - \frac{C_+ + C_-}{48\pi}\ln(-\lambda^2 x^+ x^-) \\ &- \frac{m}{\pi\lambda x_0^+}(x^+ - x_0^+)\theta(x^+ - x_0^+) \ . \end{aligned}$$

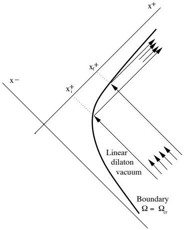
- Physical picture for $C_+ + C_- < 0$ [we are studying the flux inside the apparent horizon: consistency with energy conservation !!!. Work in progress]



An aside

- The regime $C_+ + C_- < 0$ may preserve unitarity in a manner somewhat similar to the subcritical regime in the conventional RST or BPP models

$$T_{++}^{classical} < \frac{N}{48\pi(x^+)^2} \qquad\qquad m < M_{critical}$$



- Bose-Parker-Peleg PRL '96: *... When a black hole almost forms, the radiation reaching infinity in advance of the original outgoing null matter has the properties of Hawking radiation. The radiation reaching infinity after the null matter consist of a brief burst of negative energy that preserves unitarity and restores correlations rapidly as a result of strong backreaction.*

Central charges in four dimensions

- The contribution of the known fields of the Standard Model to $\langle T_\mu^\mu \rangle$ (ignoring masses and interactions) is given by

$$\langle T_\mu^\mu \rangle = \hbar (c\, C^2 - a\, E)$$

with

$$a = \frac{1}{360(4\pi)^2}[N_0 + \frac{11}{2} N_{1/2} + 62 N_1] > 0 \,, \quad c = \frac{1}{120(4\pi)^2}[N_0 + 3 N_{1/2} + 12 N_1] > 0 \,,$$

- $N_1 = 12$ (electroweak bosons and gluons)
- $N_{1/2} = 3 \times 15$ (three generations of left-handed and right-handed leptons and quarks)
- $N_0 = 4$ (real components of the Higgs doubled)

[We have ignored contributions of the form $\Box R$ as they are intrinsically ambiguous and can be shifted by local counterterms]

Central charges in four dimensions

- However, in sharp contrast to two-dimensional conformal invariance, it is now possible to introduce a new field with negative contribution to c and a while preserving unitarity
- Spin $3/2$: $a < 0$ $c < 0$ [Christensen and Duff, '78]
- Spin 0: "dimensionless scalar field" ξ obeying a 4th order field equation [Bogolubov et al. textbook, 1987]

$$\Box^2 \xi = 0$$

In curved spacetime, it can be uniquely extended to a conformally invariant theory

$$S = -\frac{1}{2}\int d^4x\sqrt{-g}\,\xi \triangle_4\,\xi\ ,$$

where $\triangle_4$ is the unique conformally-invariant fourth order operator

$$a = -\frac{28}{360(4\pi)^2}, \qquad c = -\frac{8}{120(4\pi)^2}\ .$$

[Gusynin'89]

- An important property of this theory is that its physical content consists of a single quantum state: the vacuum [Bogolubov et al. 87]. Therefore, only the Boulware vacuum is allowed [No Hawking radiation, only vacuum polarization], as the Fadeed-Popov ghosts in 2D gravity

Central charges in four dimensions

- With
$$N^{\xi} \geq 36$$
the total central charges are negative [with matter content $=$ Standard Model]
$$a_{total} < 0 \quad c_{total} < 0$$
- An interesting coincidence:
The Standard Model (including right-handed neutrinos, 3 generations, excluding the Higgs)$+36$ dimension-zero scalars
$$a_{total} = 0 \quad c_{total} = 0$$
[Boyle and Turok, '21; J. Miller, G.E. Volovik, Zubkov '22]
It protects the smoothness of the big bang singularity (conformal regularity; Weyl Curvature Hypothesis)
- It is tempting to also speculate that the fate of gravitational collapse depends on the matter content of the universe, as seems to be the case in the CGHS model !!!

Conclusions

- We have provided strong evidence for singularity resolution in 2D black holes via a negative central charge
- This behaviour is very interesting for the information loss problem
- A very appealing picture could emerge, with the fate of gravitational collapse ultimately determined by the matter content of the universe

THANKS !!!