Gravitational wave tests of EFT theories of gravity

Laura Bernard

Black holes and their symmetries

July 2-4 2025, Tours

collaboration with S. Giri, L. Lehner & R. Sturani

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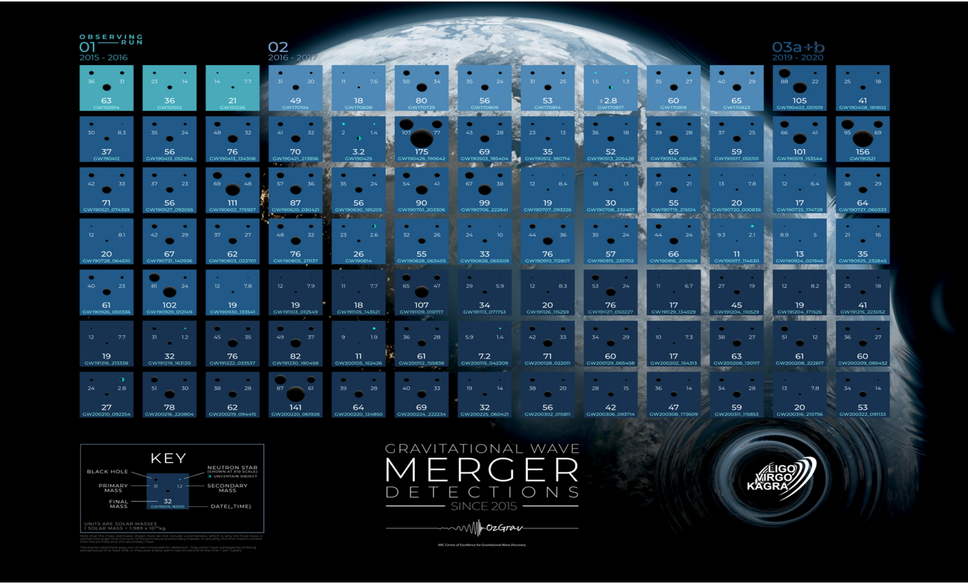
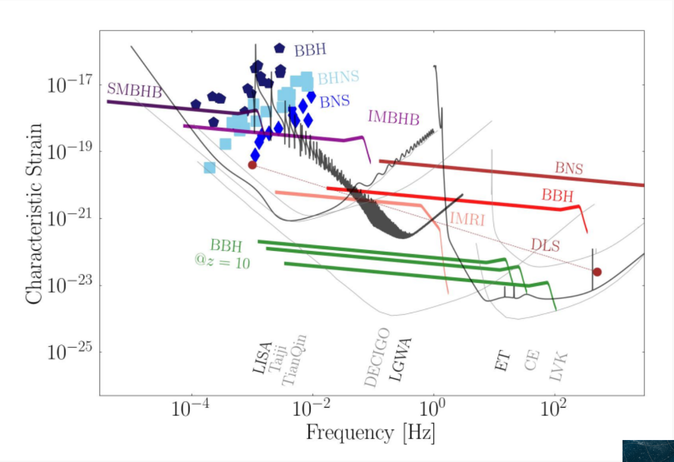
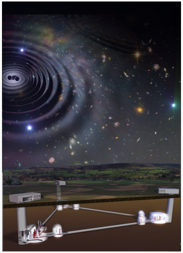
Gravitational wave detections

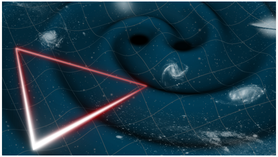


The future gravitational wave universe



Einstein Telescope blue book (2025)

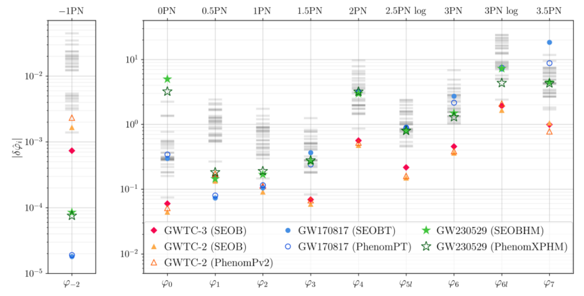




GR: a beautiful and successful theory

GR is highly non linear but:

- well-posed in harmonic coordinates [Choquet-Bruhat, 1952]
- not been challenged until now



Sanger et al. 2024

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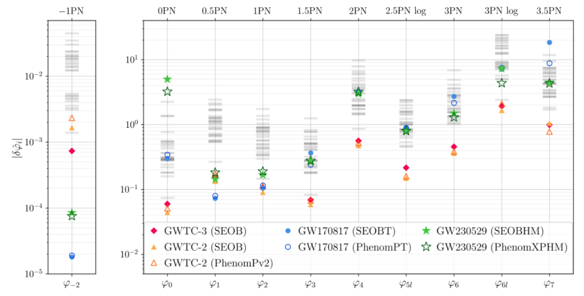
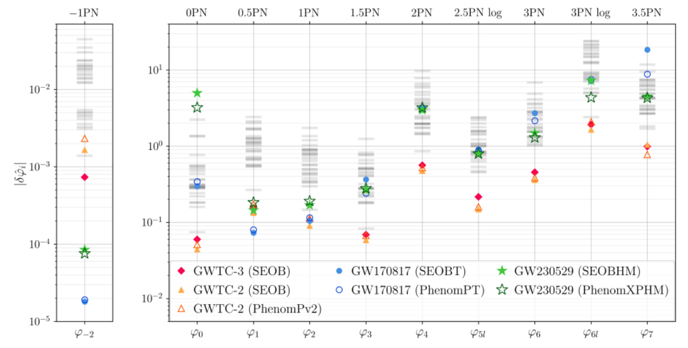


Sanger et al. 2024

But we still try to go beyond… **why**?

- to better understand gravity & compact objects
- to answer fundamental physics questions (quantum gravity, beyond SM)
- to explain cosmology inconsistencies

Limitation of current tests



LIGO-Virgo '21

- agnostic searches: $h(f) = \mathcal{A}(f)\, e^{\psi_n(f)+\delta\psi_n(f)}$, $\delta\psi_n = \{\delta\varphi_{-2,0,..7}\}$
  - parametrized deviations from GR PN waveforms
  - no deviations from GR

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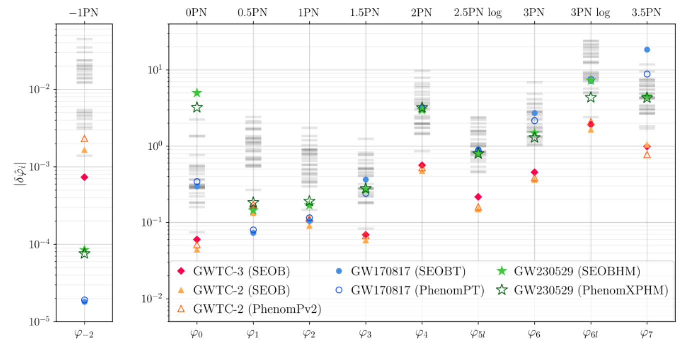

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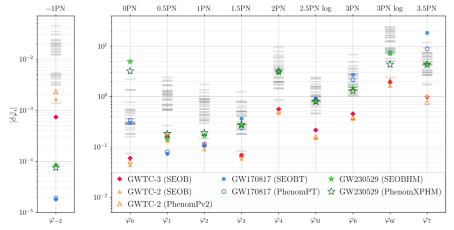
  - parametrized deviations from GR PN waveforms
  - no deviations from GR
- limited theory-specific tests, ex:

  ST theories: 3PN dynamics, 2.5PN radiation, NNLO tides

  EsGB, Chern-Simmons: LO correcion, including spin

Question



But:

- ★ we can miss some deviations
- ★ no systematic way to connect deviations to new physics

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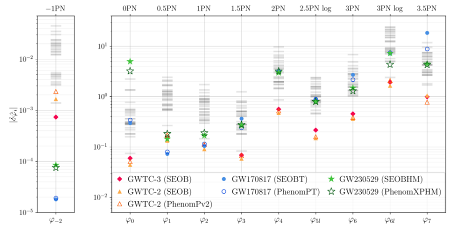

But:

- ★ we can miss some deviations
- ★ no systematic way to connect deviations to new physics

What if we had a dictionnary?

Should we detect a small deviation from GR in a gravitational waveform, what can we infer from it?

(or... towards building more realistic GW tests of GR)

The effective field theory of gravity

$$S = 2\Lambda^2 \int \mathrm{d}^4x \, \sqrt{-g} \left[R + \sum_{p\geq 3} \alpha_{(p)} \, M_\Lambda^{2p-2} \, \mathcal{R}^{(p)} \right]$$

- M_Λ: typical scale of new physics
- $\mathcal{R}^{(p)} \sim (R_{....})^p$: products of p Riemann tensors
  - ex. cubic gravity

$$\mathcal{R}^{(3)} = R_{\mu\rho\nu\sigma} \, R^{\rho\alpha\sigma\beta} \, R_{\alpha\ \beta}^{\ \mu\ \nu} + R_{\mu\nu\rho\sigma} \, R^{\rho\sigma\alpha\beta} \, R_{\alpha\beta}^{\ \ \mu\nu} + \epsilon_{\mu\nu\delta\gamma} R_{\delta\gamma\rho\sigma} \, R^{\rho\sigma\alpha\beta} \, R_{\alpha\beta}^{\ \ \mu\nu}$$

- we can allow for additional derivatives or new fields: $\left\{\psi^s, \, \nabla^{2r}, \, \mathcal{R}^{(p-r)}\right\}$
  - ex. EsGB gravity

$$\psi \, \mathcal{G}^{(2)} = \psi \, \left[R^2 - 4R_{\mu\nu} \, R^{\mu\nu} + R_{\mu\nu\rho\sigma} \, R^{\mu\nu\rho\sigma}\right]$$

The goal: building a dictionary

We want to determine:

- the scaling with respect to the curvature correction: $q(p)$
- the leading PN contribution of new physics: $n(p)$

$$h \sim h_{\mathrm{GR}} \cdot \delta \cdot \left(\frac{M_\Lambda}{m}\right)^q \cdot \left(\frac{v^2}{c^2}\right)^n \cdot \alpha_{(p)}$$

- m characteristic mass of the system $\sim$ lightest component

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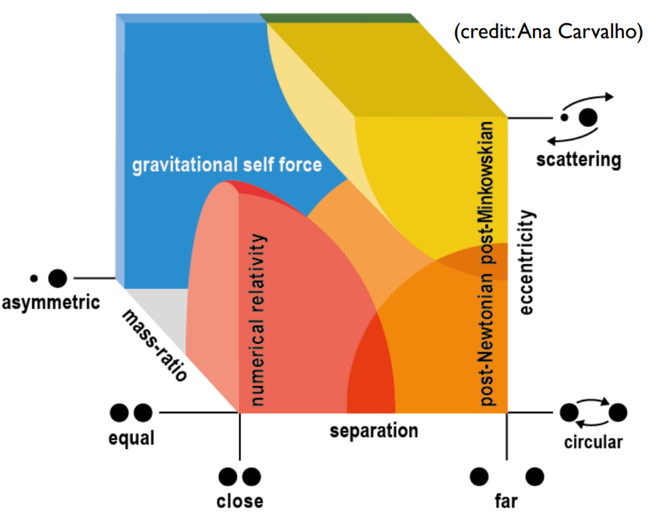
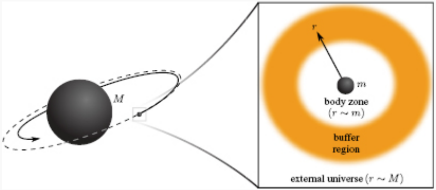
We will use a **perturbative expansion** valid for $v \ll c$ and $G \ll 1$ together with a **diagrammatic** approach

The different methods



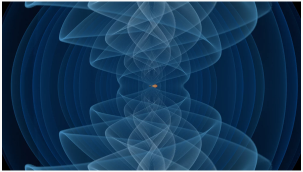
- Inspiral-Merger-Ringdown (IMR): *effective-one-body, phenomenological & surrogate models*

The different methods: gravitational self-force

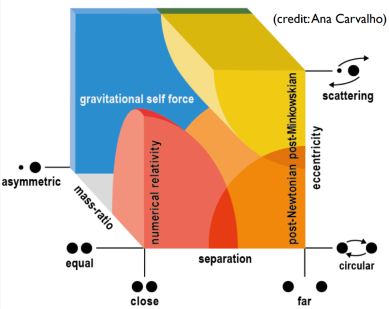


Self force:

- expansion in $q = \frac{m_1}{m_2} \ll 1$
- resonances, par ex. 2:3



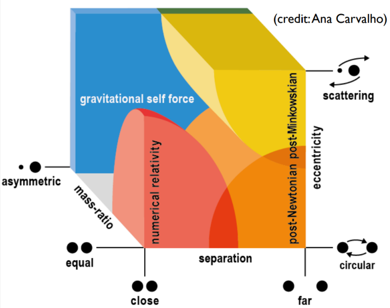
I. Markin, T. Dietrich, H. Pfeiffer, A. Buonanno (Potsdam University and Max Planck Institute for Gravitational Physics)

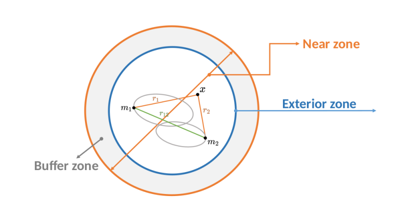


Numerical relativity:

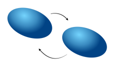
- solving the full Einstein equations
- computationally expensive
- add spins, eccentricity, *etc.*

The different methods: post-Newtonian

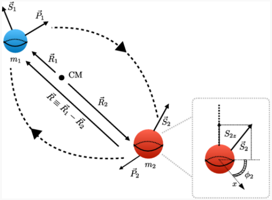




- expansion in $\epsilon = \frac{v_{12}^2}{c^2} \sim \frac{G\,m_{1,2}}{r_{12}c^2} \ll 1$
- point-particle approximation



- add spins, tides, *etc.*



Tanay et al. '23

Current state-of-the-art:

- **4.5PN for p.p.**
- NNLO for tides
- $\sim$ 3PN for spins

Post-Newtonian vs post-Minkowskian formalisms

Traditional PN: direct iteration and integration in the direct space

Effective Field Theory: diagramatic and integration in Fourier space

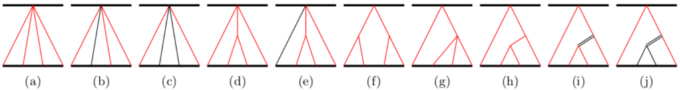


Figure 4: Diagrams contributing at order $G^4 m_1^4 \lambda_2^{(0)}$

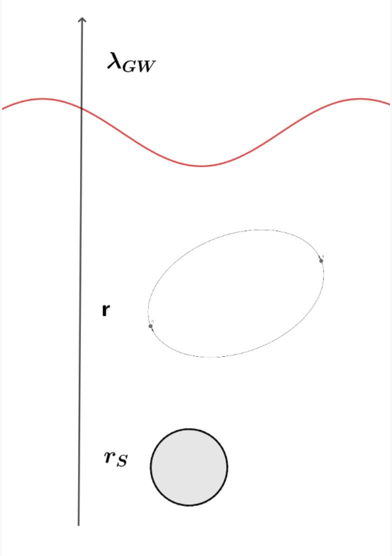
Scattering amplitudes: PM expansion, diagrammatic and integration in Fourier space

	0PN		1PN		2PN		3PN		4PN		5PN		6PN		7PN			
1PM	(1	+	v^2	+	v^4	+	v^6	+	v^8	+	v^{10}	+	v^{12}	+	v^{14}	+	...)	G^1
2PM			(1	+	v^2	+	v^4	+	v^6	+	v^8	+	v^{10}	+	v^{12}	+	...)	G^2
3PM					(1	+	v^2	+	v^4	+	v^6	+	v^8	+	v^{10}	+	...)	G^3
4PM							(1	+	v^2	+	v^4	+	v^6	+	v^8	+	...)	G^4
5PM									(1	+	v^2	+	v^4	+	v^6	+	...)	G^5
6PM											(1	+	v^2	+	v^4	+	...)	G^6

Comparison table of powers used for PN and PM approximations in the case of two non-rotating bodies.

The PN-EFT algorithm

It relies on a **hierarchy of scales**



$$r_S \ll r \sim \frac{r_S}{v} \ll \lambda_{GW} \sim \frac{r}{v}$$

Kol-Smolkin decomposition

$$g_{\mu\nu} = e^{\frac{2\phi}{\Lambda}} \begin{pmatrix} -1 & \frac{A_j}{\Lambda} \\ \frac{A_i}{\Lambda} & e^{-\frac{4\phi}{\Lambda}} \left(\delta_{ij} + \frac{\sigma_{ij}}{\Lambda}\right) + \frac{A_i A_j}{\Lambda^2} \end{pmatrix}$$

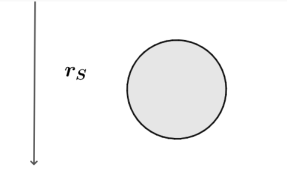
$$\equiv \eta_{\mu\nu} + h_{\mu\nu}$$

The inner region

Integrating out the internal dofs: **one-particle EFT**

Point particles

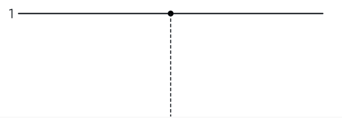
$$S_{\rm pp} = -\sum_{a=1,2} m_a \int \mathrm{d}\tau_a$$



- In presence of scalar field:

$$m_a(\psi) = \overline{m}_a \sum_{n\geq 0} \frac{d_n^{(a)}}{n!}\left(\frac{\psi}{\Lambda}\right)^n .$$

$$S_{\rm pp}^{\rm (quad)} = -\sum_a m_a \int \mathrm{d}t \left\{1 - \frac{1}{2}v_a^2 - \frac{1}{8}v_a^4 + \frac{1}{\Lambda}\left[\left(1+\frac{3}{2}v_a^2\right)\phi - A_i v_a^i - \frac{1}{2}v_a^i v_a^j \sigma_{ij}\right] + \cdots\right\}$$



$$\propto -i\frac{m_a}{\Lambda}\int \mathrm{d}t \int_{\mathbf{k}} \mathrm{e}^{i\,\mathbf{k}\cdot\mathbf{x}} \sim \frac{t}{r^3}\frac{m}{\Lambda},$$

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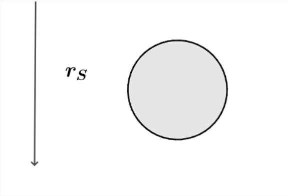
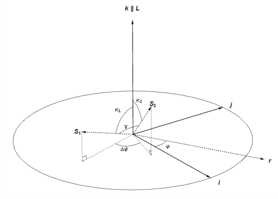


Spinning point-particles

$$S_{\rm spin} = \int \mathrm{d}\tau \left[p_\mu u^\mu + \frac{1}{2} S_{\mu\nu}\Omega^{\mu\nu}\right],$$

$$S^{\mu\nu} p_\nu = 0 \quad \text{(spin sup. cond.)}$$



Beyond point particles: finite-size effects

Gravito electric and magnetic tidal Love numbers

$$S_{\text{tid.}} = \int \mathrm{d}\tau \left[c_E^{(l)} E_L E^L + c_B^{(l)} B_L B^L \right], \quad E_L = -\nabla_{L-2} C_{\mu_{l-1}\rho\nu_l\sigma} u^\rho u^\sigma$$

- ⋆ effacement principle: starts at $5PN \sim c_{E,B} \cdot \left(\frac{v}{c}\right)^{10}$
- ⋆ BHs in GR: $c_E = c_B = 0$

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Scalar tidal Love numbers

- ○ scalar dipole moment $\mathcal{E}_i \propto \partial_i \psi \Rightarrow$ scalar-induced tidal deformability

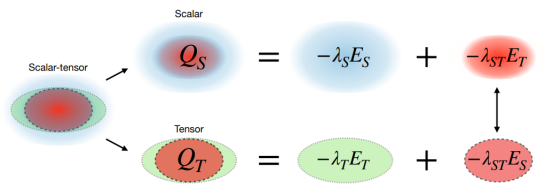
$$S_{\text{tidal}} = -\frac{c}{2} \sum_{a=1,2} \int \mathrm{d}\tau_a \left\{\lambda_a(\psi)\left(\nabla_L^{\perp}\psi\right)_a \left(\nabla_{\perp}^{L}\psi\right)_a + \mu_a(\psi)\left(\nabla_L^{\perp}\psi\right)_a (E^L)_a\right\}$$

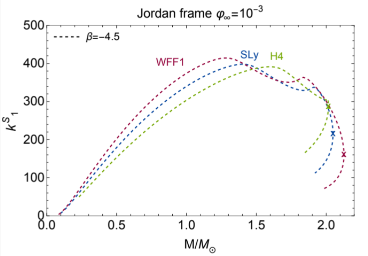
- ○ new PN scaling: starts at $3PN$, computed up to NNLO [E.Dones+ '24, '25]
- ○ dimensionless scalar tidal deformability: $k_s \equiv \frac{\lambda_s}{R^3} \longrightarrow$ enhanced effect

$$\delta V_{(\mathrm{fs})} \propto \sum_{a\neq b} V_{(\mathrm{N})} \cdot d_b^{(s)} \cdot k_a^{(s)} \cdot \frac{R_a^3}{r^3}$$

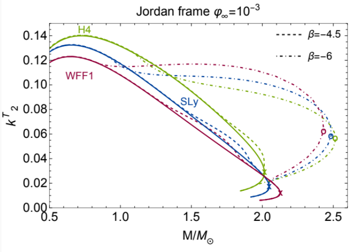
- ★ more important at low frequency or highly scalarized objects

Different types of tidal effects





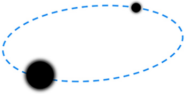
scalar



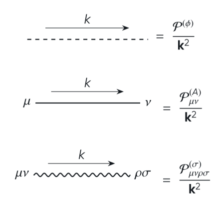
tensor

Creci et al. '23

The near zone



The **potential** modes are off-shell: $k = (\omega, \boldsymbol{k}) \sim (v/r, 1/r)$, $\omega \ll |\boldsymbol{k}|$



$$S^{(\mathrm{quad})}_{\mathrm{bulk}} = \int \mathrm{d}^4 x \Big\{ -4\partial_\mu\phi\partial^\mu\phi + \partial_\mu A_\nu \partial^\mu A^\nu + \frac{1}{4}\left(\partial_\mu \sigma^\alpha_\alpha \partial^\mu \sigma^\beta_\beta - 2\partial_\mu \sigma_{\alpha\beta}\partial^\mu\sigma^{\alpha\beta}\right) \Big\}$$

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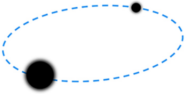

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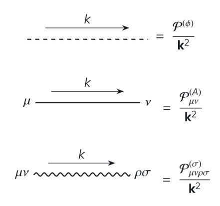

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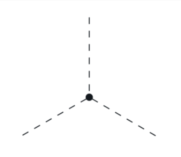
$$S^{(\mathrm{cub})}_{\mathrm{bulk}} \supset \frac{1}{\Lambda}\int \mathrm{d}^4 x \left\{-16\phi\partial_t\phi\partial_t\phi\right\}$$



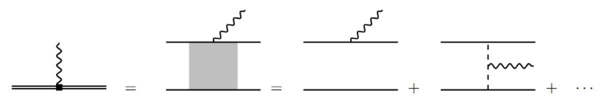
$$\sim \delta(t)\, r^{-1}$$

The wave zone

The **radiation** modes are on-shell: $k = (\omega, \boldsymbol{k}) \sim (v/r, v/r)$, $|\boldsymbol{k}|_{\text{ext.}} \ll |\boldsymbol{k}|_{\text{int.}}$

$$S_{\text{eff}} = \frac{1}{2\Lambda} \int \mathrm{d}^4 x \, T^{ab} \, \bar{h}_{ab}$$

$$\underbrace{=}_{\lambda_{\text{gw}} \gg r} \frac{1}{2\Lambda} \sum_{l=0}^{+\infty} \frac{1}{l!} \underbrace{\int \mathrm{d}^3 x \, T^{ab}(t, \mathbf{x}) x^{i_1} \cdots x^{i_l}}_{\text{source moments}} \, \bar{h}_{\mu\nu, i_1 \ldots i_l}$$



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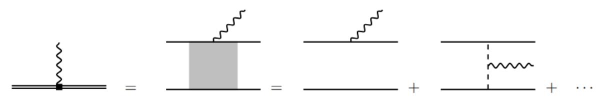

$$S^{(\text{cub})}_{\text{eff, rad}} = \frac{1}{\Lambda}\int \mathrm{d}t\left(\frac{m_1}{2}v_1^i v_1^j - \frac{G m_1 m_2}{4r}n^i n^j\right)\sigma_{ij}$$

$$= \frac{1}{2\Lambda}\int \mathrm{d}t\,\frac{\mathrm{d}^2}{\mathrm{d}t^2}\underbrace{\left[\frac{\mu}{2}r^i r^j\right]}_{\text{mass quadrupole: } Q_{ij}}\sigma_{ij}$$

An example: Einstein-cubic gravity

$$S_{\text{bulk}} = 2\Lambda \int \mathrm{d}^4x \sqrt{-g} \Big[R + \alpha_{(3)}\, M_\Lambda^4\, \mathcal{R}^{(3)} + \beta_{(3)}\, M_\Lambda^4\, \widetilde{\mathcal{R}}^{(3)} + \gamma_{(3)}\, M_\Lambda^4\, {}^*\mathcal{R}^{(3)} \Big] \,,$$

with

$$\begin{aligned} \mathcal{R}^{(3)} &= R_{abcd}\, R^{cdef}\, R_{ef}{}^{ab} \\ \widetilde{\mathcal{R}}^{(3)} &= R_{abcd}\, R^{bgde}\, R_{g}{}^{a}{}_{e}{}^{c} \\ {}^*\mathcal{R}^{(3)} &= \epsilon_{abcd}\, R^{cd}{}_{ef}\, R^{efgh}\, R_{gh}{}^{ab} \end{aligned}$$

An example: Einstein-cubic gravity

$$S_{\text{bulk}} = 2\Lambda \int \mathrm{d}^4x\,\sqrt{-g}\left[R + \alpha_{(3)}\,M_\Lambda^4\,\mathcal{R}^{(3)} + \beta_{(3)}\,M_\Lambda^4\,\widetilde{\mathcal{R}}^{(3)} + \gamma_{(3)}\,M_\Lambda^4\,{}^*\mathcal{R}^{(3)}\right],$$

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Some remarks

- ★ the dual term ${}^*\mathcal{R}^{(3)}$ adds a factor $v \sim$ 0.5PN order

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Some remarks

- ★ the dual term ${}^*\mathcal{R}^{(3)}$ adds a factor $v \sim 0.5$PN order
- ○ some algebra

$$\begin{aligned} G_3 &= \mathcal{R}^{(3)} - 2\widetilde{\mathcal{R}}^{(3)} \\ &= \frac{5}{4} R^3 - 9RR^{ab}R_{ab} + \frac{3}{4} RR^{abcd} R_{abcd} + 8R^{ab} R_a{}^c R_{bc} + 6R^{ab} R_{acbd} R^{cd} \end{aligned}$$

- ○ each term $\propto R, R_{ab}$ can be removed by field redefinitions
- ▹ only one dynamical term (in vacuum)

A note on Ricci-Kretschmann Lagrangian

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★ we get $c_{E\;|RK}^{(a)} = -4 m_a \lambda M_\Lambda^4$

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★ we get $c_E^{(a)}{}_{|RK} = -4 m_a \lambda M_\Lambda^4$

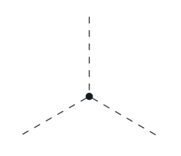
- we expect from GR (in vacuum): $\delta V = V_{RK} + V_{\rm tid.} = 0$

An example: Einstein-cubic gravity

$$S_{\text{bulk}} = 2\Lambda \int \mathrm{d}^4 x \sqrt{-g} \left[R + \alpha_{(3)} M_\Lambda^4 \mathcal{R}^{(3)} + \beta_{(3)} M_\Lambda^4 \widetilde{\mathcal{R}}^{(3)} \right] ,$$

$$\Big\downarrow \quad R_{\mu\nu\rho\sigma} \sim \partial^2 \phi$$

$$S^{(3)}_{\text{cub}} \supset \frac{4 M_\Lambda^4}{\Lambda} \int \mathrm{d}^4 x \Big[3 \left(4\alpha_{(3)} + \beta_{(3)}\right) \nabla^2 \phi \, \partial_{cd} \phi \partial^{cd} \phi + \beta_{(3)} (\nabla^2 \phi)^3 - 4 \left(2\alpha_{(3)} + \beta_{(3)}\right) \partial_b \partial^c \phi \partial_{ac} \phi \partial^{ab} \phi \Big]$$



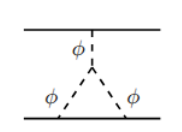
$$\sim \frac{M_\Lambda^4}{\Lambda} \frac{r^3}{t^2} \left(\alpha_{(3)}, \beta_{(3)} \right)$$

Binding potential and radiation

$$S_{\text{bulk}} = 2\Lambda \int \mathrm{d}^4x\,\sqrt{-g}\left[R + \alpha_{(3)}\,M_\Lambda^4\,\mathcal{R}^{(3)} + \beta_{(3)}\,M_\Lambda^4\,\widetilde{\mathcal{R}}^{(3)}\right],$$

The binding potential

$$\delta V_{\text{cub.}} = 9\frac{G\,m_1 m_2}{r}\;\frac{G(m_2+m_1)}{r\,c^2}\left(\frac{G\,M_\Lambda}{r\,c^2}\right)^4\beta_{(3)}$$



This is a **5PN** effect

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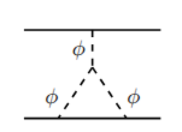
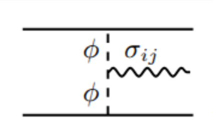


The radiation

$$S^{\text{(cub)}}_{\text{eff, rad}} = \frac{1}{\Lambda}\int \mathrm{d}t \left\{ \frac{\mu}{4}\frac{\mathrm{d}^2}{\mathrm{d}t^2}\left[r^i r^j \underbrace{-36\beta_{(3)}\frac{GM}{r^3} n^i n^j}_{\delta Q_{ij}} \right]\sigma_{ij} - 27\beta_{(3)}\frac{G^2 M^2 \mu}{r^6} n^i n^j \sigma_{ij} \right\}$$



[Cannella et al. '09, Lins & Sturani '21]

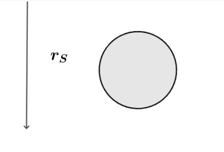
This is a **5PN** effect

But... tidal effects

Unlike in GR, BH Love numbers are not necessary zero in higher curvature gravity

$$S_{\text{tidal}} = -\sum_{a=1,2} \int \mathrm{d}\tau_a \, c_E^{(a)} \, E_{ab} E^{ab}$$

$$\text{with} \qquad E_{ab} = C_{abcd} \, u^c \, u^d$$



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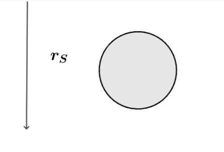

- Love numbers are sourced by the higher curvature interaction

$$c_E \propto \bar{\beta}_{(3)} \left(\frac{M_\Lambda}{m}\right)^4 \left(\frac{m}{r_{\rm H}}\right)^5$$

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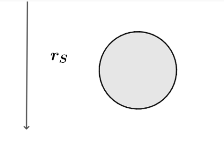

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★ as in GR, tidal effects give a **5PN correction**

$$V_{\text{tid}} = -c_E^{(1)} \frac{6G^2 m_2^2}{r^6} + (1 \leftrightarrow 2)$$

▹ exactly cancel V_{cub} for $c_E^{(1)} = \frac{3m_1}{2} \beta_{(3)} \, M_\Lambda^4$

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★ binding potential

$$V_{\text{cub.}} = -\frac{3}{128} \frac{m_1 m_2^2}{r^6} \frac{M_\Lambda^4}{\pi^2 \Lambda^4}$$

$$V_{\text{tid.}} = -\frac{3}{512} \frac{m_2^2}{r^6} \frac{c_{\mathrm{E}}}{\pi^2 \Lambda^4}\,.$$

▹ as expected (in vacuum): $\delta V = V_{\text{cub.}} + V_{\text{tid.}}$ [Wilson-Gerow '25]

The dictionary - generic case

$$S = 2\Lambda^2 \int \mathrm{d}^4x \, \sqrt{-g} \left\{ R + \sum_{p\geq 3} \alpha_{(p-2)} \, M_\Lambda^{2p-2} \, f\left[\nabla^{2r}, \mathcal{R}^{(p-r)}\right] \right\}$$

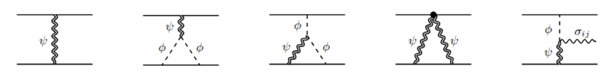
$$\delta V \sim V_\mathrm{N} \cdot \left(\frac{v^2}{c^2}\right)^{3p-r-4} \cdot \left(\frac{M_\Lambda}{m}\right)^{2p-2} \alpha_{(p)}$$

$$\delta h_{ij}^{\mathrm{TT}} \sim h_{ij}^{\mathrm{TT}}\Big|_\mathrm{N} \cdot \left(\frac{v^2}{c^2}\right)^{3p-r-4} \cdot \left(\frac{M_\Lambda}{m}\right)^{2p-2} \alpha_{(p)}$$

The dictionary - with an additional scalar field

$$S = 2\Lambda^2 \int \mathrm{d}^4x \sqrt{-g} \left\{ R - \frac{1}{4}\nabla_a\psi\nabla^a\psi + \beta_{(p)}\, M_\Lambda^{2p-2}\, f\left[\left(\nabla^a, \psi^b\right); \left(\nabla^c, \mathrm{Riem}^d\right)\right]\right\}$$

$$\delta V \sim V_\mathrm{N} \cdot \left(\frac{v^2}{c^2}\right)^{2p-4+d+b} \cdot \left(\frac{M_\Lambda}{m}\right)^{2(p-1)} \cdot \bar{\beta}_{(p)} \cdot \left(d_a^{(1)}\right)^b$$



- linear coupling in the scalar field (a=0,b=1)
  - $d_a^{(1)} \equiv (M_\Lambda/m_a)^{2p-2}\,\bar{\beta}_{(p)}\bar{d}_a^{(1)}$ for a linear coupling in ψ:
- no derivative $n \geq 2$: different dependency in the coupling
  - for $n = 2$, $\psi \propto \beta_{(p)}^{1/2}$
  - for $n \geq$, $\psi \propto \frac{1}{\beta_{(p)}^q} \Rightarrow$ breaking of the EFT
- with derivatives, no simple scaling in $\beta_{(p)}$

The dictionary - some examples

$$S = 2\Lambda^2 \int \mathrm{d}^4x \, \sqrt{-g} \left\{ R - \frac{1}{4} \nabla_a \psi \nabla^a \psi + \alpha(p) \, M_\Lambda^{2p-2} \, f\left[\left(\nabla^a, \psi^b\right); \left(\nabla^c, \mathrm{Riem}^d\right) \right] \right\}$$

quantity / $\alpha_{(p)}$	scalar+quadratic	cubic	quartic
$\delta V / V\|_N \propto$	$\left(\frac{v^2}{c^2}\right)^3 \cdot \left(\frac{M_\Lambda}{m}\right)^2 \cdot d_a$	$\left(\frac{v^2}{c^2}\right)^5 \cdot \left(\frac{M_\Lambda}{m}\right)^4$	$\left(\frac{v^2}{c^2}\right)^8 \cdot \left(\frac{M_\Lambda}{m}\right)^6$
$\delta h_{\{+,\times\}} / h\|_{\mathrm{GR}} \propto$	$\left(\frac{v^2}{c^2}\right)^3 \cdot \left(\frac{M_\Lambda}{m}\right)^2 \cdot d_a$	$\left(\frac{v^2}{c^2}\right)^5 \cdot \left(\frac{M_\Lambda}{m}\right)^4$	$\left(\frac{v^2}{c^2}\right)^8 \cdot \left(\frac{M_\Lambda}{m}\right)^6$
$\delta V_{\mathrm{tid}} / V\|_N \propto$	$\left(\frac{v^2}{c^2}\right)^3 \cdot \left(\frac{M_\Lambda}{m}\right)^2 \cdot d_a$	$\left(\frac{v^2}{c^2}\right)^5 \cdot \left(\frac{M_\Lambda}{m}\right)^4$	$\left(\frac{v^2}{c^2}\right)^5 \cdot \left(\frac{M_\Lambda}{m}\right)^6$
$h_b \propto$	$\left(\frac{v^2}{c^2}\right)^3 \cdot d_a$	–	–

Final comments

$$h \sim h_{\mathrm{GR}} \cdot \delta \cdot \left(\frac{v^2}{c^2}\right)^{5,\,3p-4} \cdot \left(\frac{M_\Lambda}{m}\right)^{2p-2} \cdot \alpha_{(p)}$$

- **leading correction at 5PN** if no extra degree of freedom
- there can be additional cancelations increasing the PN order
- if extra degrees of freedom:
    - **scalar tidal effects** start at 3PN
    - PN and new physics orders: $n(p) = 3(p-3)$, $q(p) = 2(p-1)$
    - dependence of the scalar charge w.r.t. the new physics coupling more involved

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★ systematic way to **distinguish new physics from systematics**

★ can be **directly integrated** into existing data analysis methods

Thank you!