Black holes and their symmetries
Tours, 2025

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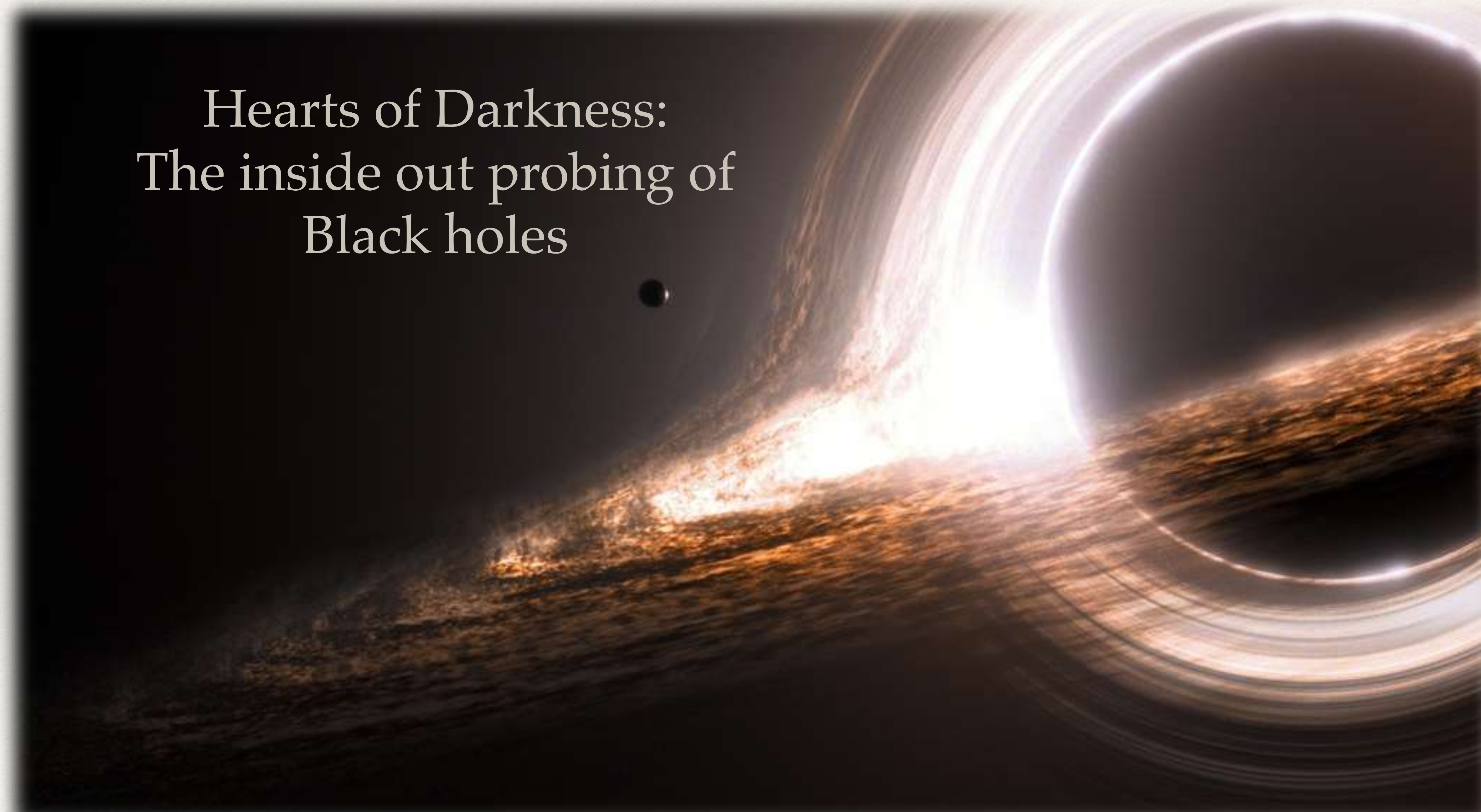


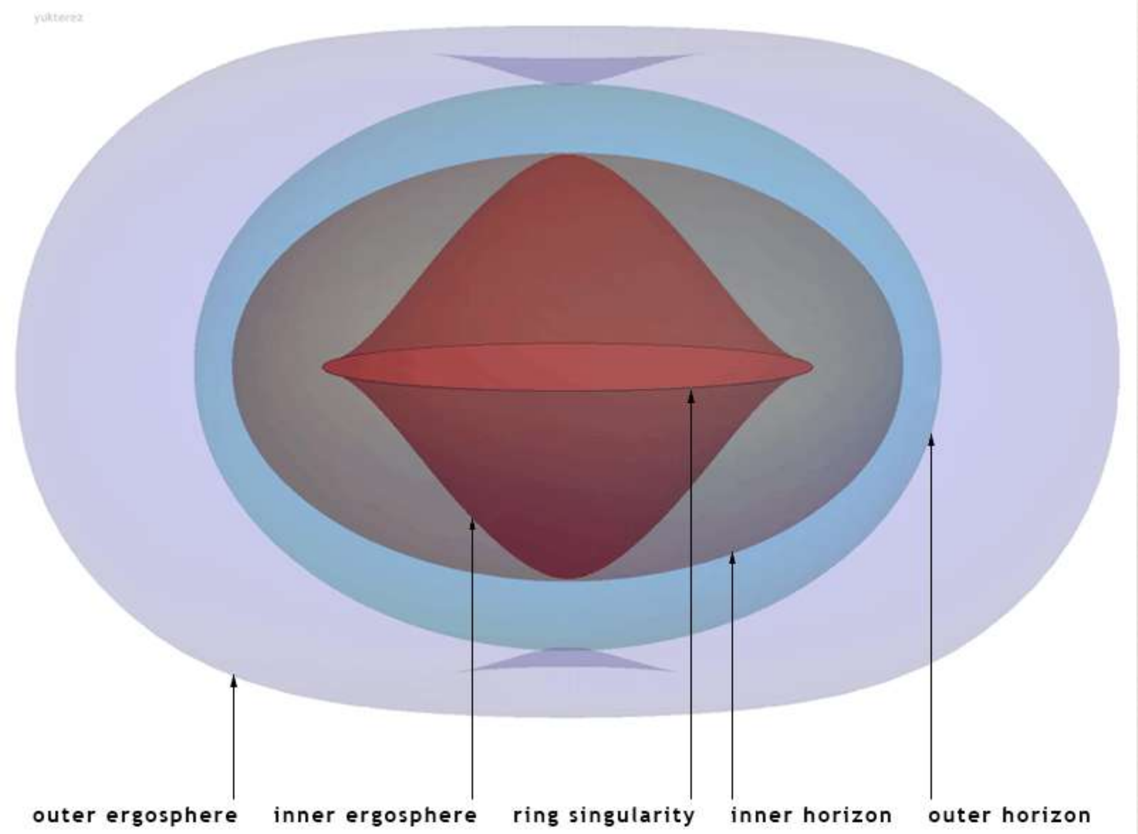
Hearts of Darkness: The inside out probing of Black holes

Work in collaboration with:
J. Arrechea, R. Carballo-Rubio, F. Di Filippo, E. Franzin , H. Neshat, J. Mazza, C. Pacilio, V. Vellucci, M. Visser

Black Holes:
the Rosetta stone of gravity

"The **black holes** of nature are the most perfect macroscopic objects there are in the universe: the only elements in their construction are our concepts of space and time."
Subrahmanyan Chandrasekhar

- Albeit we are nowadays familiar with the concept of Black Holes their acceptance as a physical solution of General relativity has been far from obvious.
- Even once was understood the nature of the event horizon, BH are still characterised by "hard to digest" structures
  - Singularities: infinite curvature
  - Cauchy horizons (associated to timelike singularities and time machines): end of predictability



QG is supposed to "cure" these features:
If it does so just in a hidden QG core of Planck scale then BH will be exactly as in GR.
But what if the "cure" requires long range (in time and/or space) effects?
Then maybe we could test QG using BH… could we?

Singularity

- A singularity is where General relativity is no more predictive: we cannot describe spacetime there —> missing points.
- Penrose's theorem is what makes very confident that singularities must form inside black holes generically

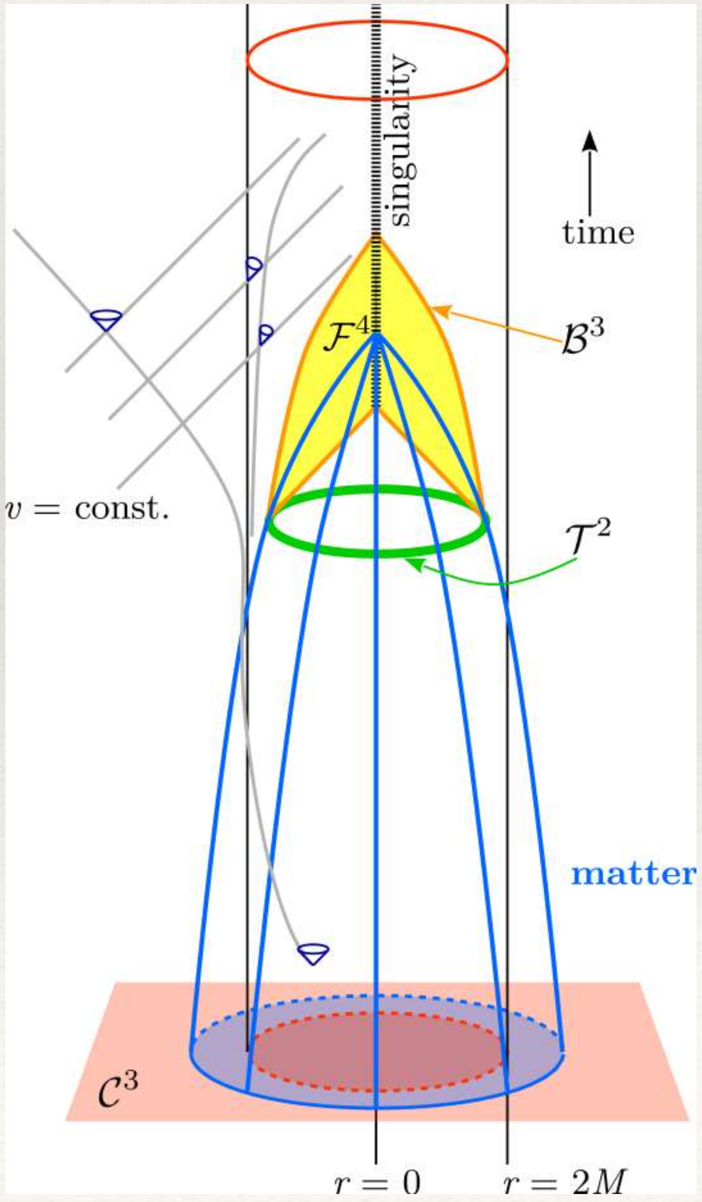
Penrose's singularity theorem

Assumptions

- The theory of gravity is GR
- The gravitational collapse becomes enough strong to have convergent light cones (trapped region)
- Matter gravitates in the standard way (no exotic/quantum matter: if p=wϱ w>-1)

Implication

Once a trapped region forms the collapse would be unstoppable and has to lead to a singularity



Avoidance of this conclusion requires at least one of the following

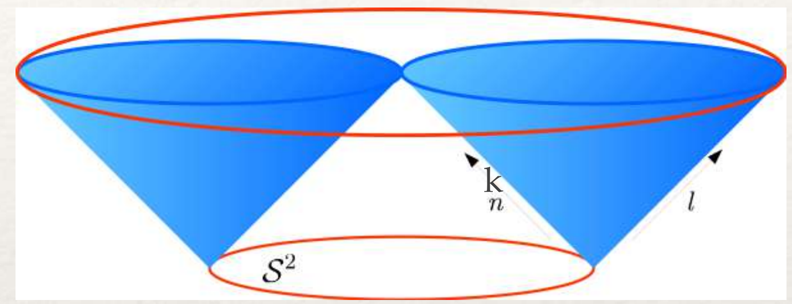
- The weak energy condition is violated.
- The Einstein field equations do not hold.
- Lorentzian geometry does not provide an adequate description of spacetime inside BHs.
- Global hyperbolicity (Cauchy evolution) breaks down.

We shall be ready to give up the first two and hold the last two…

Focussing on the focussing point

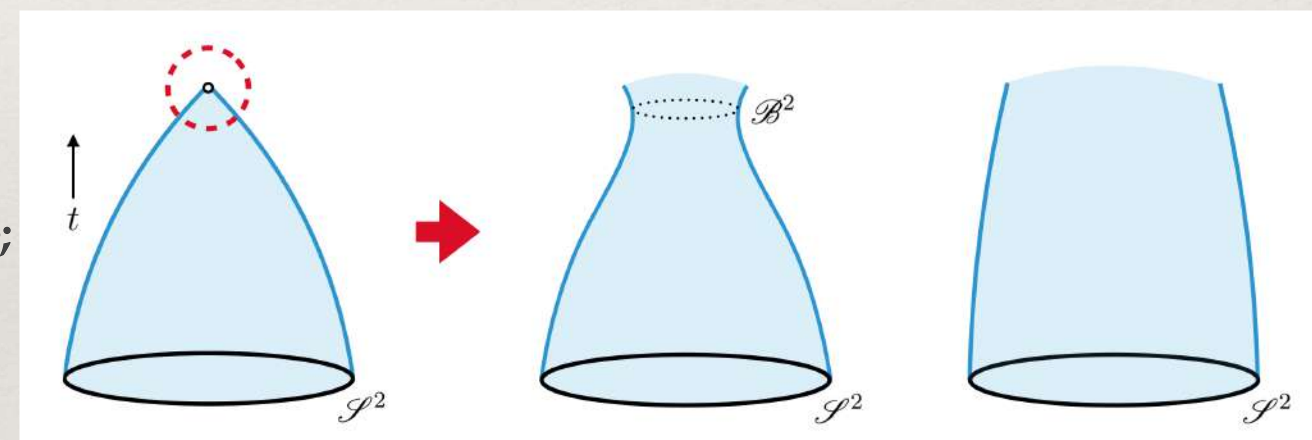
- **Let's assume that QG produces a <u>space-time which is regular and entirely predictable in the sense of a Cauchy problem</u>.**
  - **No singularities both in the sense of incomplete geodesic as well as curvature singularities (metric is at least C²).**

Penrose' theorem works by proving first that in a collapse a focussing point for outgoing light rays is reached and then by showing that this point (or sets of points) cannot be part of the spacetime. If QG removes such a focussing point what can happen? We can have



$$\theta^{(i)}_{\text{null}} = \frac{1}{\delta A}\frac{\delta A}{\delta \lambda} .$$

- **Defocusing point at a finite affine distance, $\lambda_{DEF}=\lambda_0$;**
- **Defocusing point at an infinite affine distance, $\lambda_{DEF}=\infty$;**
- **Focusing point at infinity, $\lambda_{DEF}=\varnothing$;** 
  - **still singular at finite affine parameter for ingoing congruence**



Apart from the above behaviour of the outgoing light rays
we can catalogue all the possible cases by considering the radius R at which defocussing happens and the behaviour of the ingoing light rays there.
We then get only

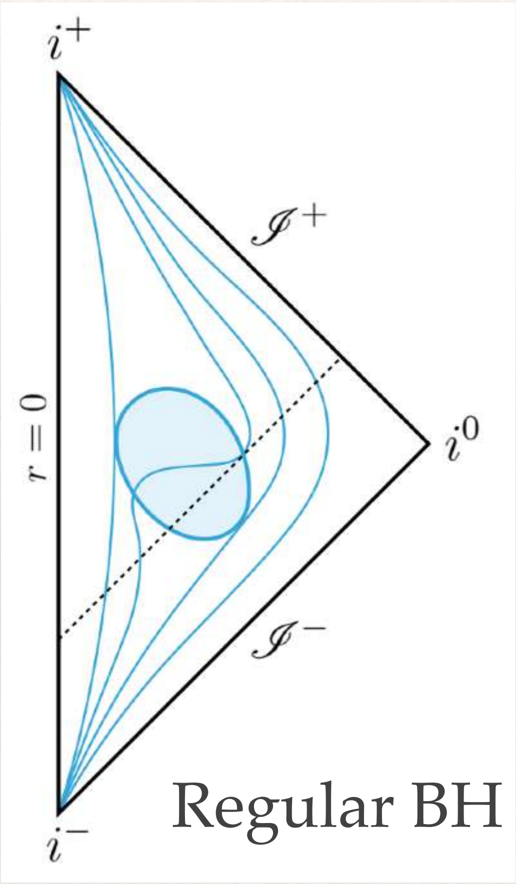
4 viable classes:

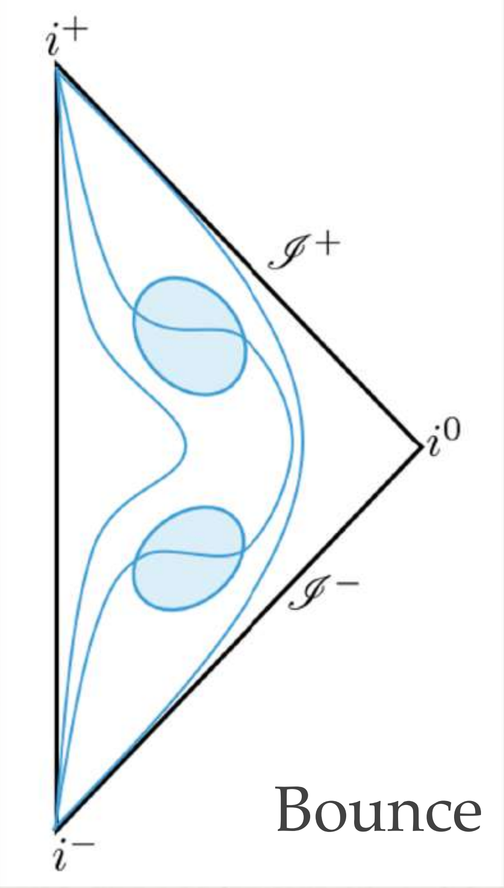
1. $(\lambda_0, R_0, \bar{\theta}^{(k)} < 0)$
2. $(\lambda_0, R_0, \bar{\theta}^{(k)} \geq 0)$
3. $(\infty, R_\infty, \bar{\theta}^{(k)} < 0)$
4. $(\infty, R_\infty, \bar{\theta}^{(k)} \geq 0)$

Class 1: Evanescent horizons

- The expansion relative to the outgoing null vector vanish and changes sign.
- The expansion of the intersecting ingoing radial null geodesics remains negative.

- We recover the geometry of an evanescent regular black hole.
- The geometry possesses an outer and an inner horizon that merge in finite time.
- This situation corresponds to a regular BH with no singularity
- Or to a bounce from a Trapped Region to an Anti-Trapped Region (where the two region are separated by an un-trapped, standard region)





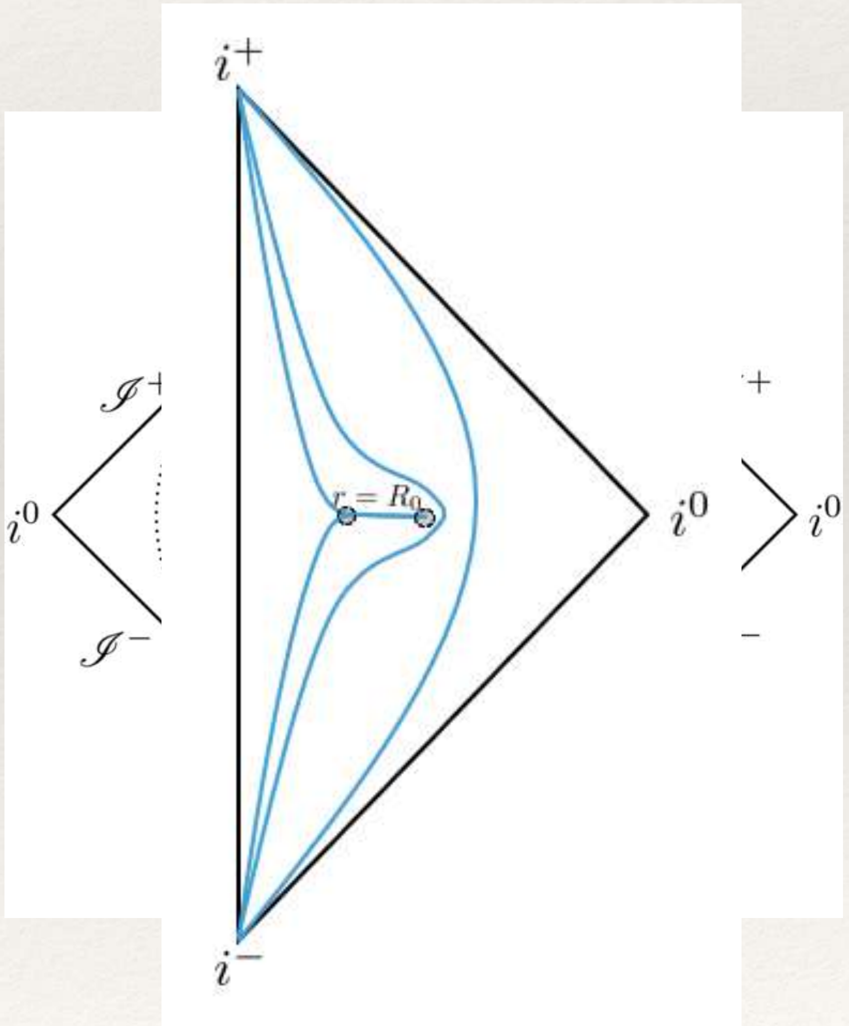
Note: bounce behaviour explicitly found in 2D dilation gravity semiclassical collapse
J. Barenboim, A. V. Frolov, G. Kunstatter
Phys.Rev.D 111 (2025) 10, 104068 • e-Print: 2503.03191 [gr-qc]

Class 2: One way hidden wormholes

In this case

- The expansion relative to the outgoing null rays vanish and changes sign.
- The expansion of the intersecting ingoing radial null rays changes sign as well.

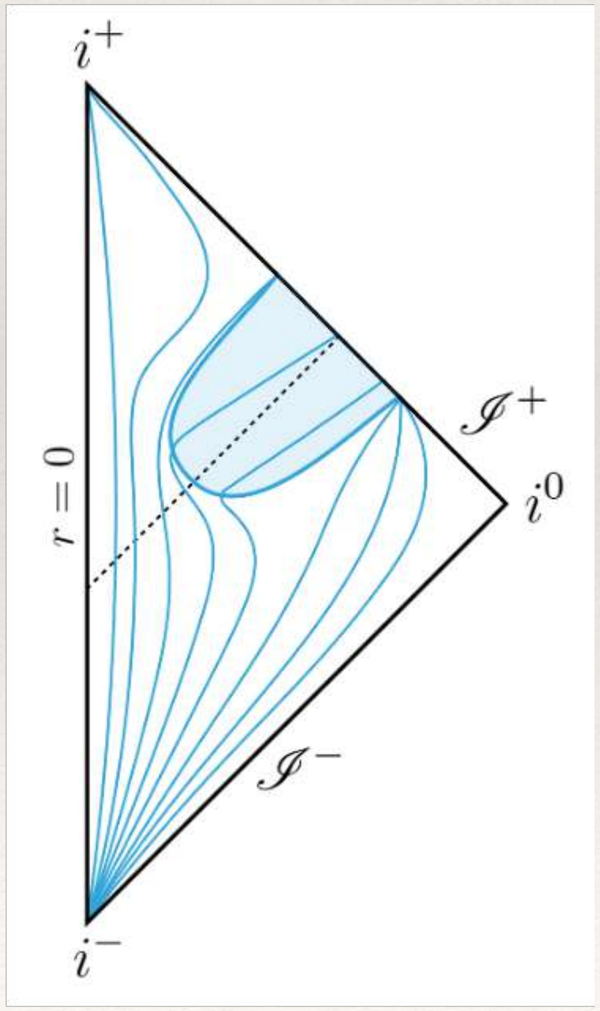
- The geometry possesses a minimum radius throat that resembles the one of a wormhole;
- The throat is inside a trapping horizon and can be traversed only in one direction.
- Problematic creation from gravitational collapse as topology change is incompatible with global hyperbolicity. However, if one gives up (at least in two points) metric analyticity requirement then possible to conceive a geometry with minimum finite radius locally.



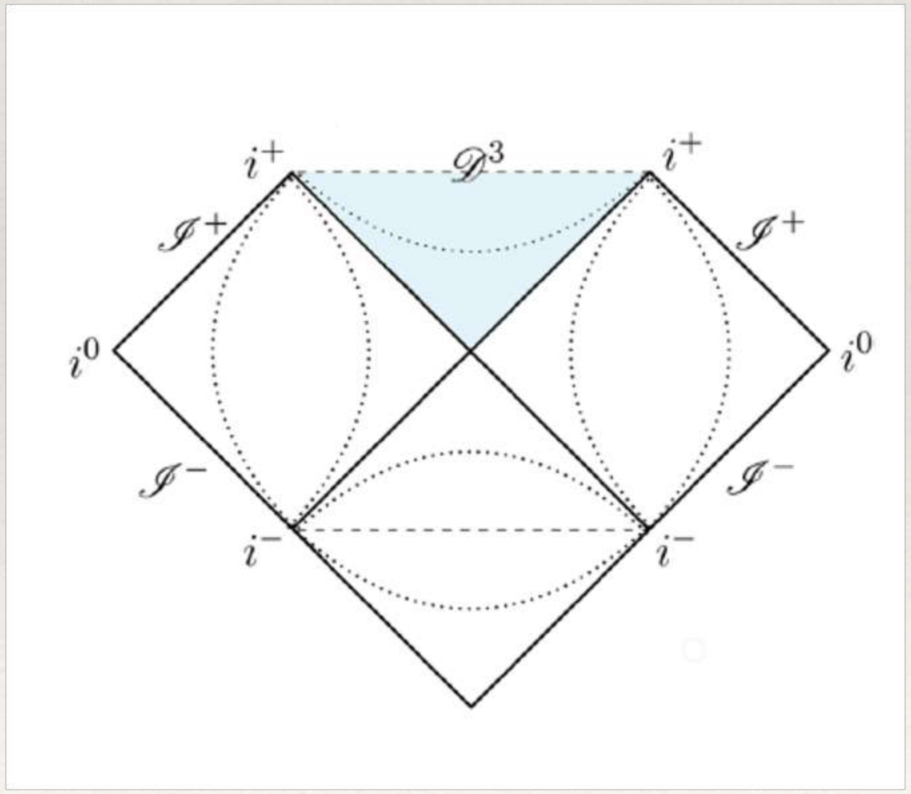
Asymptotic resolutions: Cases 3,4

- These are (idealised?) cases in which the defocussing point is pushed at infinity.

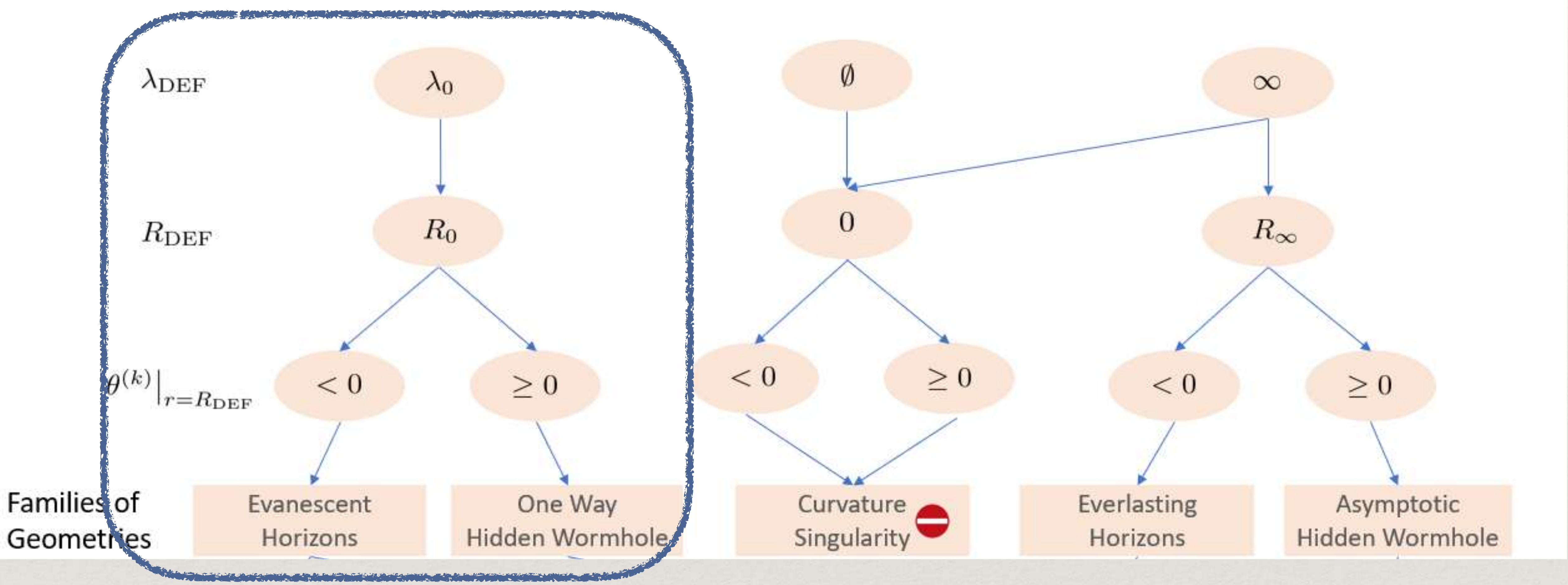
Everlasting horizons



Asymptotic hidden wormholes



Viability summary

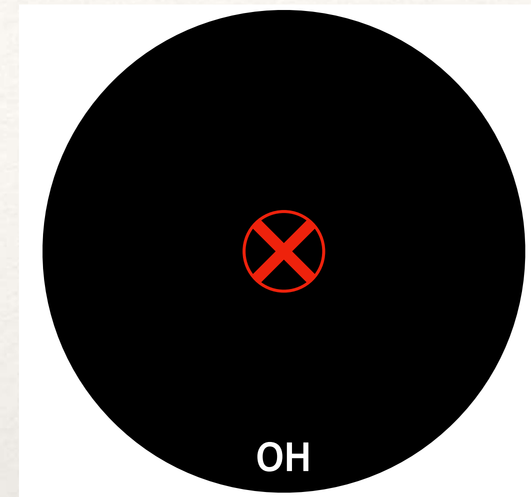


- Asymptotic resolutions of the singularity are rather unphysical. So, we shall not deal with these asymptotic cases (Case 3 and 4) any further…
- We shall focus instead on the finite time resolutions

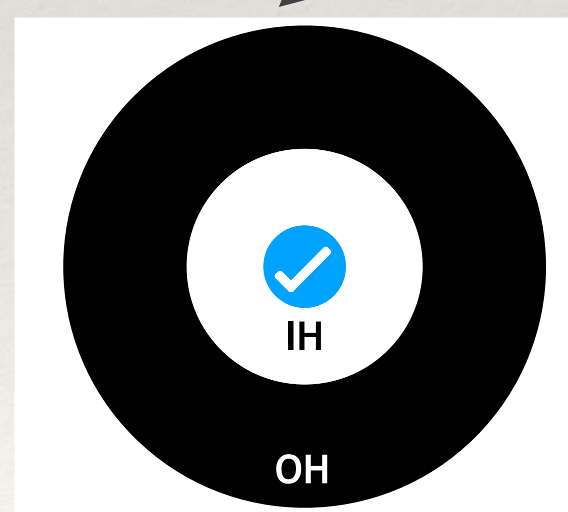
P.S.: same analysis can be done to regularise the Big Bang in FRW…
Raúl Carballo-Rubio, Stefano Liberati, Vania Vellucci (Apr 19, 2024)
Published in: Phys.Rev.D 110 (2024) 4, 044055 • e-Print: 2404.13112 [gr-qc]
Result: a bouncing universe, an emergent universe, and an asymptotically emergent universe

First “take-home” message

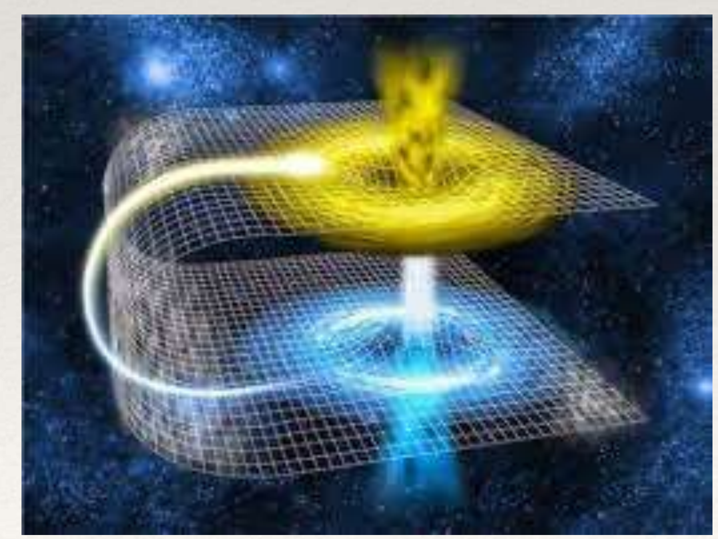
- The analysis of the singularity resolutions tells us that substantially, once a trapping horizon forms, there are two classes of singularity free solutions (local in space and time) available:
    - Simply connected topology: Regular black holes (and bounces) with inner horizons.
    - Non-simply connected topology: Hidden Wormholes (wormholes shielded by a trapping horizons)



GR



Regular BH



Hidden WH

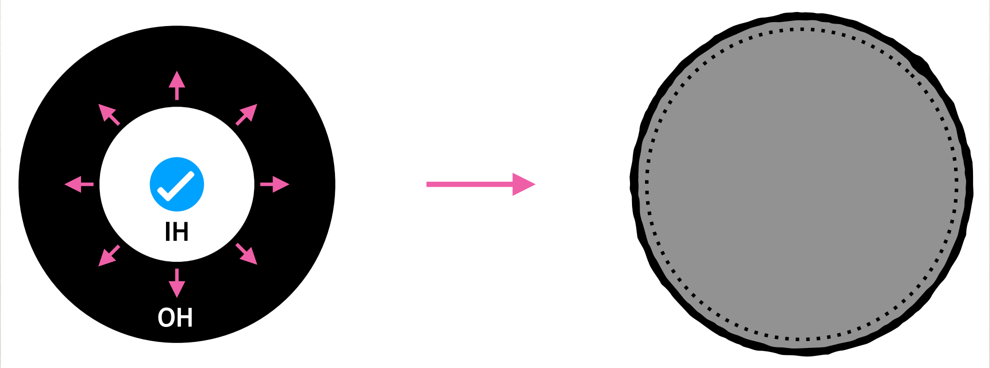
Figures by courtesy of R. Carballo-Rubio

Limiting cases - BH Mimickers

- In both these cases one can ask what happens if the defocussing happens before an horizon is formed $R_0 > r_{horizon}$. Answer: one gets two corresponding new classes of objects of horizonless objects.
  - Horizonless BH Mimickers
  - Naked wormholes

BH Mimickers

Let us define a static and spherically symmetric quasi-black hole as a spacetime satisfying:
(i) there are no event or trapping horizons.
(ii) Still sufficiently compact to have a light ring (as seen by EHT)

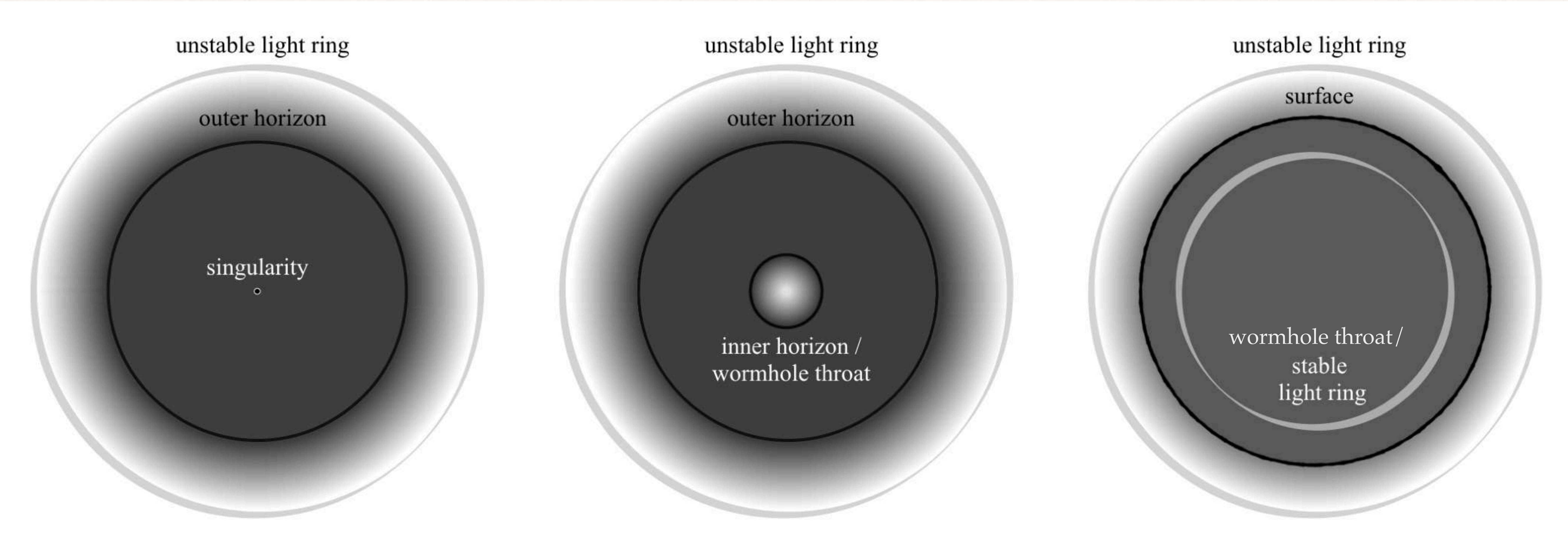


Naked Wormhole

The hidden wormhole spacelike trout becomes so large to engulf the trapped region and leave a traversable wormhole (timelike throat)

Theorem: such horizonless configurations are characterised by an pair of light rings (or none).
See e.g. F. Di Filippo. Phys.Rev.D 110 (2024) 8, 084026
The usual unstable one plus an inner stable one (for the naked wormhole on the WH throat) which tends to accumulate massless excitations (an instability source?)

So in summary…



GR BH

Regular BH/
Hidden wormhole

BH Mimicker/
Traversable wormhole

This is the general structure of the regularised BH.
But so far our analysis is local.
How about the global metric associated to these categories?

Class 1: Stationary Metrics

Let starts with the **Evanescent Horizons** class.
A one parameter family of static metrics embodying this category is

$$ds^2 = -\left(1 - \frac{2m(r)}{r}\right) \mathrm{d}t^2 + \frac{\mathrm{d}r^2}{\left(1 - \frac{2m(r)}{r}\right)} + r^2 \left[\mathrm{d}\theta^2 + \sin^2\theta \, \mathrm{d}\phi^2\right] .$$

m(r)=Misner-Sharp Mass

Model	$m(r)$
Bardeen [44]	$M \frac{r^3}{(r^2+\ell^2)^{3/2}}$
Hayward [45]	$M \frac{r^3}{r^3+2M\ell^2}$
Dymnikova [46]	$M \left[1 - \exp\left(\frac{r^3}{\ell^3}\right)\right]$
Fan–Wang [47]	$M \frac{r^3}{(r+\ell)^3}$

- Requirements for the mass function
  $m(r) \to M$ as $r \to \infty$ and $m(r) = O(r^3)$ as $r \to 0$ (at least)
- Asympt. flatness+Regularity at the core+Outer Horizon imply also Inner Horizon (actually Cauchy). The position of the inner and outer horizons and their surface gravity depend on m(r)
- Within GR, RBHs are non-vacuum solutions, the effective stress-energy tensor can be read off from the Einstein tensor; several interpretations in terms of non-linear electrodynamics.
- In general, they imply violations of the strong energy condition (not of NEC).
- Remember Hawking-Penrose theorem
- Rotating regular black holes (Kerr-like) can be constructed e.g. using generalised Janis–Newman procedure (albeit care is required…)

Class 1: Regular-BH limit

- Let us take Hayward RBH for concreteness: $m(r) = \dfrac{Mr^3}{r^3 + 2\ell^2 M}$, $\qquad \phi(r) = 0.$
- The effective stress energy tensor takes the form associated with an anisotropic perfect fluid

$$\rho(r) = \frac{3\ell^2}{2\pi}\left(\frac{m(r)}{r^3}\right)^2 = -p_r(r), \qquad p_t(r) = \frac{3\ell^2}{\pi}\frac{r^3 - \ell^2 M}{r^3 + 2\ell^2 M}\left(\frac{m(r)}{r^3}\right)^2 = \frac{2r^3 - 2\ell^2 M}{r^3 + 2\ell^2 M}\rho(r)\,.$$

- $2m(r) = r$ has 2 roots for $M/\ell > 3\sqrt{3}/4$ a degenerate/double root for $M/\ell = 3\sqrt{3}/4$ (at $r_* = \sqrt{3}\ell$) and no roots for $M/\ell < 3\sqrt{3}/4$

Assuming $M/\ell > 3\sqrt{3}/4$ and $M \gg \ell$ one has a RBH a ultra compact object with 4 “zones”

- The (approximately isotropic) dS core [$r \sim \ell < 2M$]:

$$\rho(\ell) \equiv -p_r(\ell) = \frac{3}{8\pi\ell^2}\left[1 - \mathcal{O}\left(\ell/M\right)\right] = -p_t(\ell)\,.$$

- The (mildly anisotropic) crust [$r \sim L_+ \equiv \sqrt[3]{2\ell^2 M}$]:

$$\rho(L_+) \equiv -p_r(L_+) = \frac{\Lambda_0}{4}\left[1 + \mathcal{O}\left(\ell/M\right)\right], \qquad p_t(L_+) = \frac{\Lambda_0}{8}\left[1 + \mathcal{O}\left(\ell/M\right)\right].$$

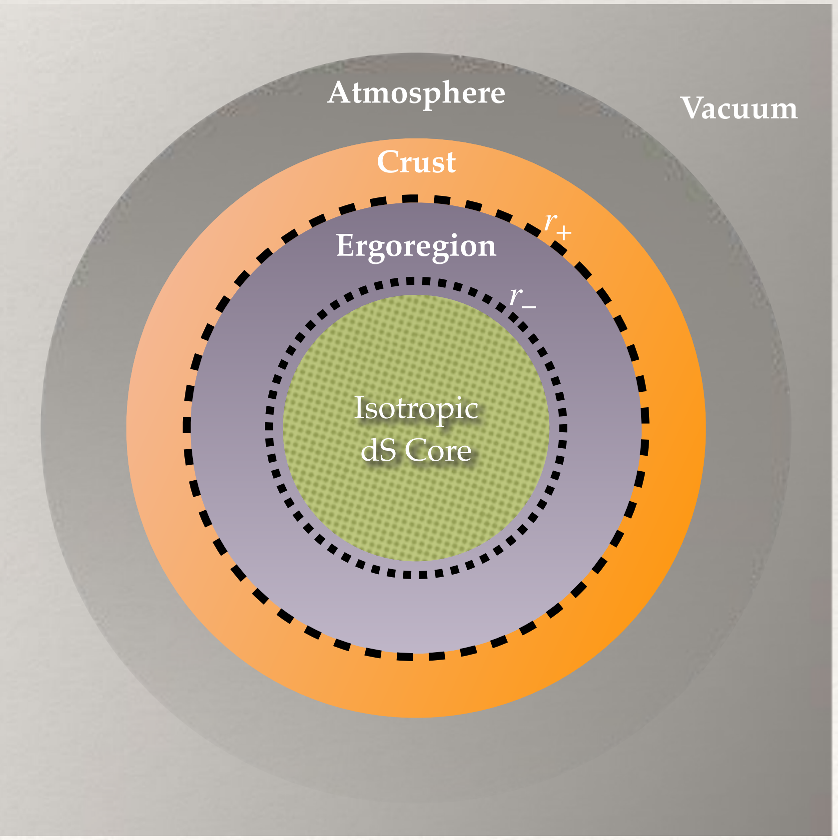
- The (grossly anisotropic) atmosphere [$r \sim 2M$]:

$$\rho(M) \equiv -p_r(M) = \Lambda_0\left(\frac{\ell}{2M}\right)^4\left[1 + \mathcal{O}\left(\ell^2/M^2\right)\right], \qquad p_t(M) = 2\rho(M)\left[1 + \mathcal{O}\left(\ell^2/M^2\right)\right].$$

- The (approximately vacuum) asymptotic region [$r \sim R \gg M$]:

$$\rho(R) \equiv -p_r(R) = \Lambda_0\left(\frac{\ell}{2M}\right)^4\left(\frac{2M}{R}\right)^6\left[1 + \mathcal{O}\left(\ell^2 M/R^3\right)\right], \quad p_t(R) = 2\rho(R)\left[1 + \mathcal{O}\left(\ell^2 M/R^3\right)\right].$$

Typical structure of a RBH with dS core



Note: RBH with AdS cores are also possible. Arrechea, Neshat , SL, Vellucci. To Appear soon.

Class 1: BH Mimicker limit

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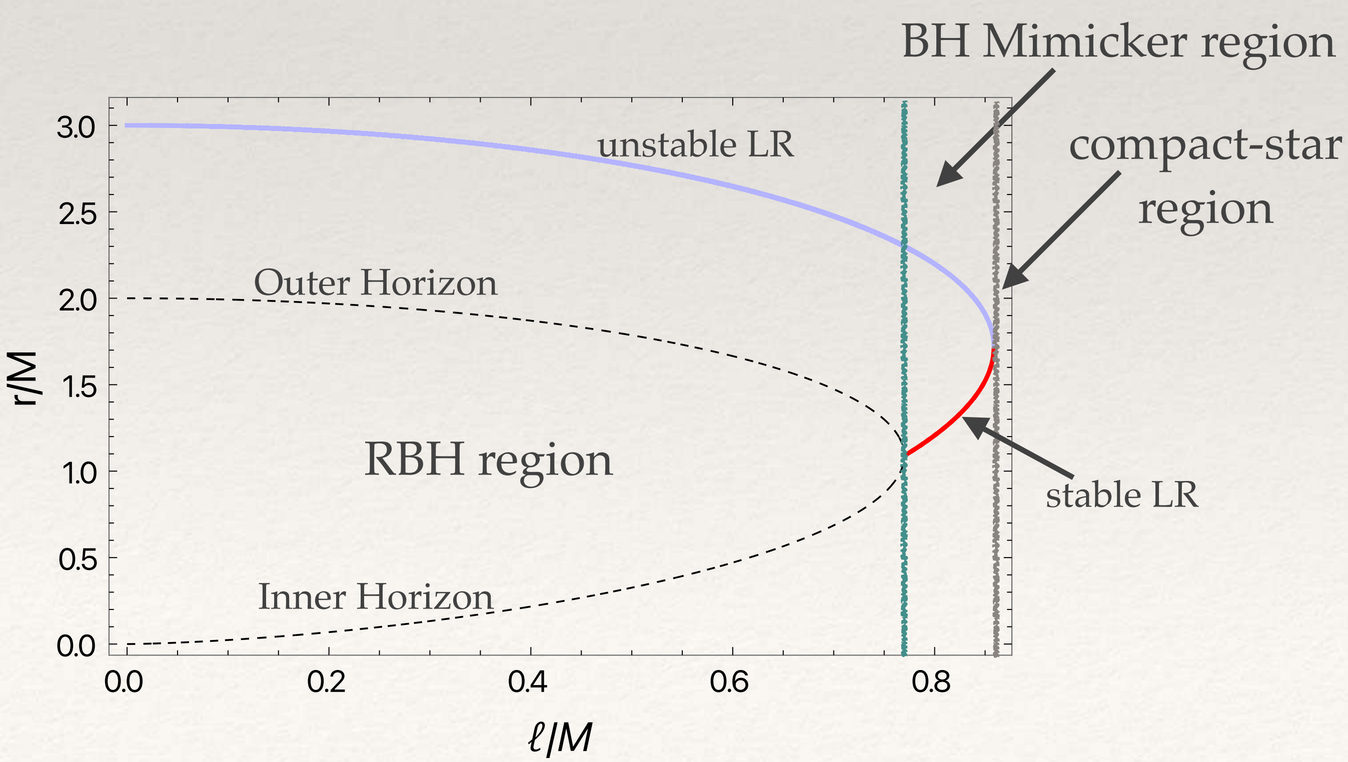
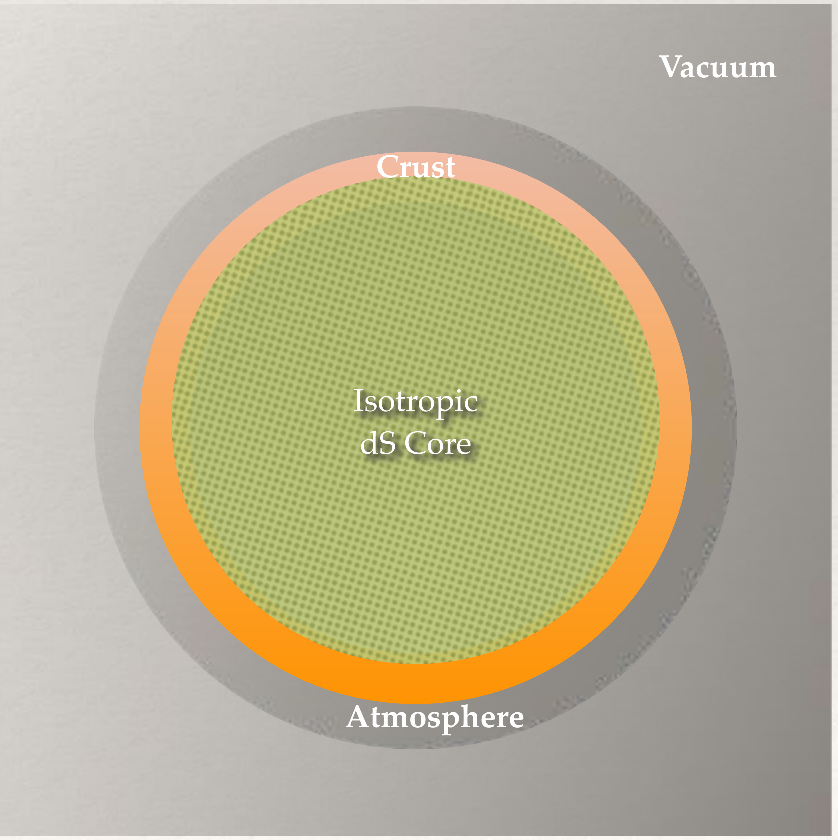
Assuming $M/\ell \lesssim 3\sqrt{3}/4$. In this case, the different scales ℓ and M coalesce, the horizons disappear, with the dS-like core growing in size and the crust and atmosphere shrinking.

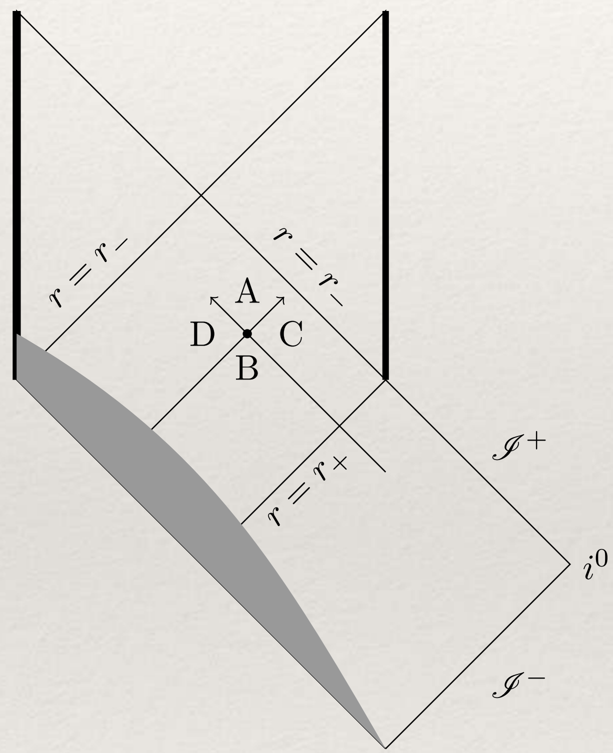




Similar structure to gravastars…

Problem: Mass inflation instability

Problem: The inner horizon is generically classically unstable



$$m_A(r_0(v)\,|_{u=u_o}) \propto v^{-\gamma} e^{|\kappa_-|v}$$

Without fine tuning there is an instability at inner horizon (mass inflation) in QG time scale, while evaporation time is generically infinite.

Note also that possible cosmological constant relevant only after a time $v \sim 1/\sqrt{\Lambda}$.

Similarly, ingoing Hawking flux can become relevant (see Buonanno et al. 2022) but too late for astrophysical black holes

R.Carballo-Rubio, F.Di Filippo, SL, C.Pacilio and M.Visser,

JHEP 1807, 023 (2018). [arXiv:1805.02675 [gr-qc]].

JHEP 05 (2021) 132 • e-Print: 2101.05006 [gr-qc]

***Phys.Rev.D* 108 (2023) 12, 12 • e-Print: 2212.07458**

An open issue was if this instability is characteristic of any trapping (dynamical) inner horizon or just of Cauchy horizons. The latter, as event horizons, are just abstractions associated to eternal black holes.
Trapping horizons are instead physically relevant.

Mass inflation without Cauchy horizons

It can be shown that finite (but often large) exponential buildups of energy are generically present for dynamical geometries endowed with slowly-evolving inner trapping horizons, even in the absence of Cauchy horizons, if the following conditions are satisfied

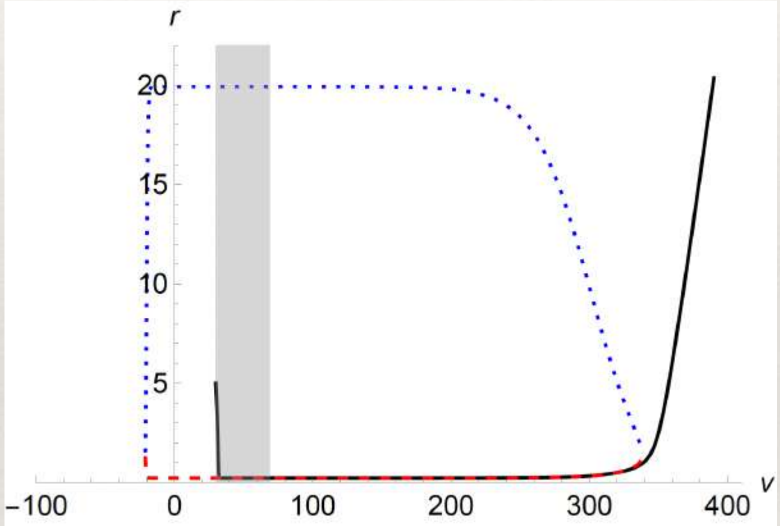
- *Adiabatic condition for radius of the inner horizon*:

$$\left|\frac{\mathrm{d}r_{\mathrm{in}}(v)}{\mathrm{d}v}\right| \ll |\kappa_{\mathrm{in}}(v)|\,|r(v) - r_{\mathrm{in}}(v)|\,. \tag{7}$$

- *Adiabatic condition for surface gravity of the inner horizon*:

$$\left|\frac{\mathrm{d}\kappa_{\mathrm{in}}(v)}{\mathrm{d}v}\right| \ll |\kappa_{\mathrm{in}}(v)|^2. \tag{8}$$

These are conditions on the first and second derivatives of $r_{in}(v)$, and thus can be violated or satisfied independently of each other.



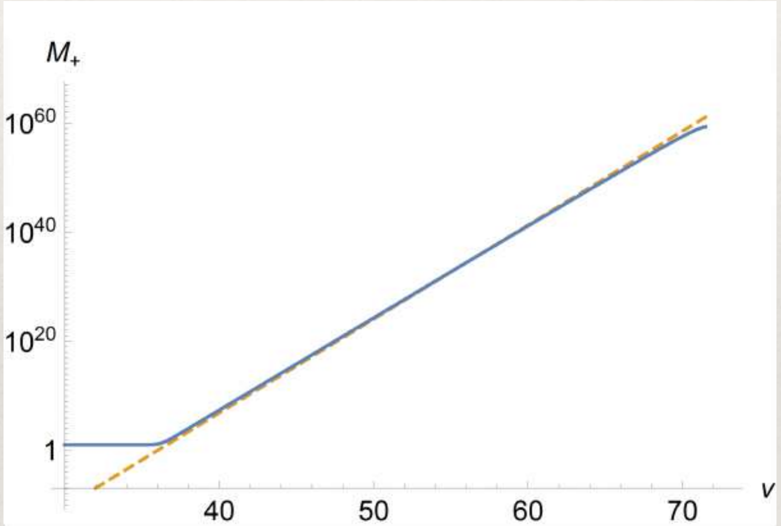


Figure 2. *Left:* Specific realization of Fig. 1 for the Misner-Sharp mass in Eq. (18) and parameters $M_\infty = 10$, $\ell = 1$, $\beta = 1$, $\gamma = 2$, $v_i = -20$, $v_f = 300$, $s_1 = 1$, $s_2 = 1/40$. Outer and inner horizons are indicated by the blue dotted line and the red dashed line, respectively, and the outgoing null thin shell by the solid black line. The radius of the outgoing shell is set initially at $R(v = 30) = 5$. The shaded region marks the interval of time for which $M_+(v)$ is plotted in the right-hand panel. *Right:* Misner-Sharp mass $M_+(v)$ in the region interior to the outgoing shell for the the initial condition $m_-(v = 30) = M_\infty + 1 = 11$. The dashed line corresponds to the exponential mass inflation for a stationary regular black hole with the same parameters. The exponential buildup of $M_+(v)$ pushes the linear approximation beyond its regime of validity in a short interval of time, as in the stationary case.

This imply that black hole geometries with non-extremal inner horizons, including the Kerr geometry in general relativity, and non-extremal regular black holes in theories beyond general relativity, can describe dynamical transients but not the long-lived endpoint of gravitational collapse! (at least in their interior)

Stable regular black holes?

- Basic idea: a possible stable endpoint is a Regular BH with zero surface gravity at the IH but non zero one at the outer horizon given that mass inflation is exponential in κ_-

R. Carballo-Rubio, F. Di Filippo, S. Liberati, C. Pacilio and M. Visser, ``Regular black holes without mass inflation instability,'' JHEP 09 (2022), 118.

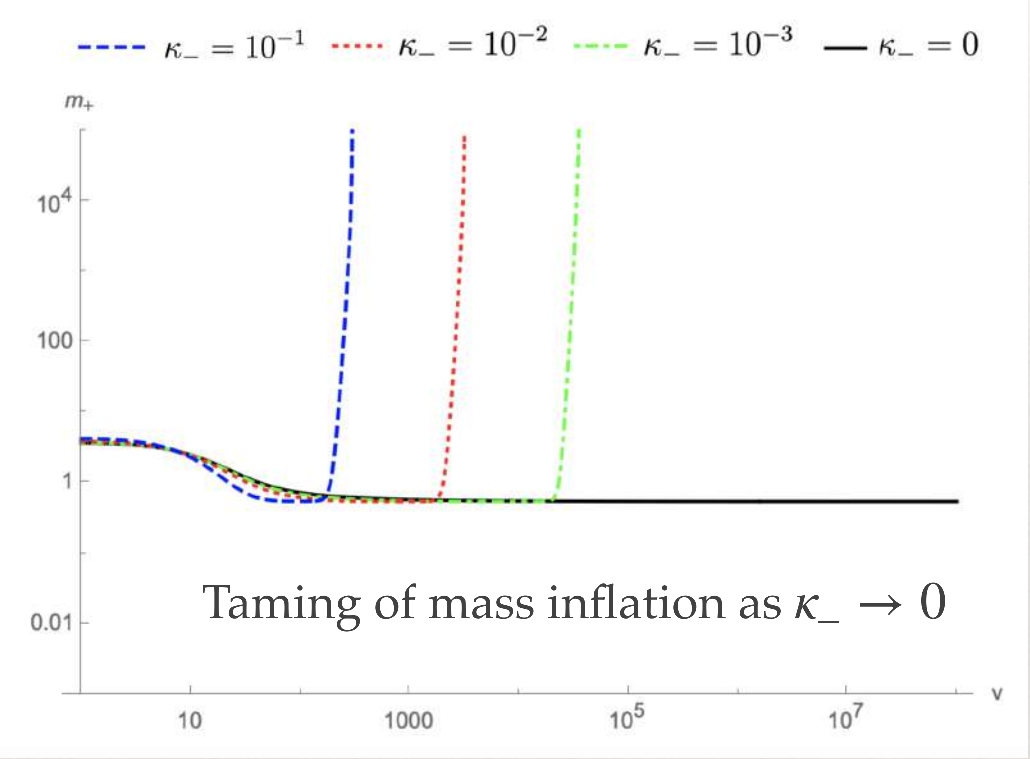
$$\mathrm{d}s^2 = -e^{-2\phi(r)}F(r)\mathrm{d}v^2 + 2e^{-\phi(r)}\mathrm{d}v\mathrm{d}r + r^2\mathrm{d}\Omega^2,$$

Misner-Sharp quasi-local mass m $\quad F(r) = 1 - \frac{2m(r)}{r}.$

$$F(r) = \frac{(r-r_-)^3(r-r_+)}{(r-r_-)^3(r-r_+) + 2Mr^3 + [a_2 - 3r_-(r_+ + r_-)]r^2}, \qquad \phi(r) = 0,$$

subject to

$$r_- \ll r_+ \sim 2M; \qquad r_- \sim |r_+ - 2M|; \qquad a_2 \gtrsim \frac{9}{4}r_+r_-.$$



- Generalisation to rotating black holes.

E. Franzin, S.Liberati, J. Mazza and V. Vellucci, ``Stable Rotating Regular Black Holes,''. [arXiv:2207.08864 [gr-qc]].

$$ds^2 = \frac{\Psi}{\Sigma}\left[-\left(1 - \frac{2m(r)r}{\Sigma}\right)dt^2 - \frac{4a\,m(r)r\sin^2\theta}{\Sigma}\,dt\,d\phi + \frac{\Sigma}{\Delta}\,dr^2 + \Sigma\,d\theta^2 + \frac{A\sin^2\theta}{\Sigma}\,d\phi^2\right],$$

$$\Psi = \Sigma + \frac{b}{r^3}, \quad \Sigma = r^2 + a^2\cos^2\theta, \quad \Delta = r^2 - 2m(r)r + a^2, \quad A = (r^2+a^2)^2 - \Delta a^2\sin^2\theta\,,$$

$$m(r) = M\,\frac{r^2 + \alpha r + \beta}{r^2 + \gamma r + \mu}.$$

$$\alpha = \frac{a^4 + r_-^3 r_+ - 3a^2 r_-(r_- + r_+)}{2a^2 M},$$
$$\beta = \frac{a^2(2M - 3r_- - r_+) + r_-^2(r_- + 3r_+)}{2M},$$
$$\gamma = 2M - 3r_- - r_+\,,$$
$$\mu = \frac{r_-^3 r_+}{a^2}\,.$$

$$r_+ = M + \sqrt{M^2 - a^2},$$
$$r_- = a^2\left[M + (1-e)\sqrt{M^2-a^2}\right]^{-1}$$

Still unclear if evolution can lead to this (apparently) fined tuned solutions…

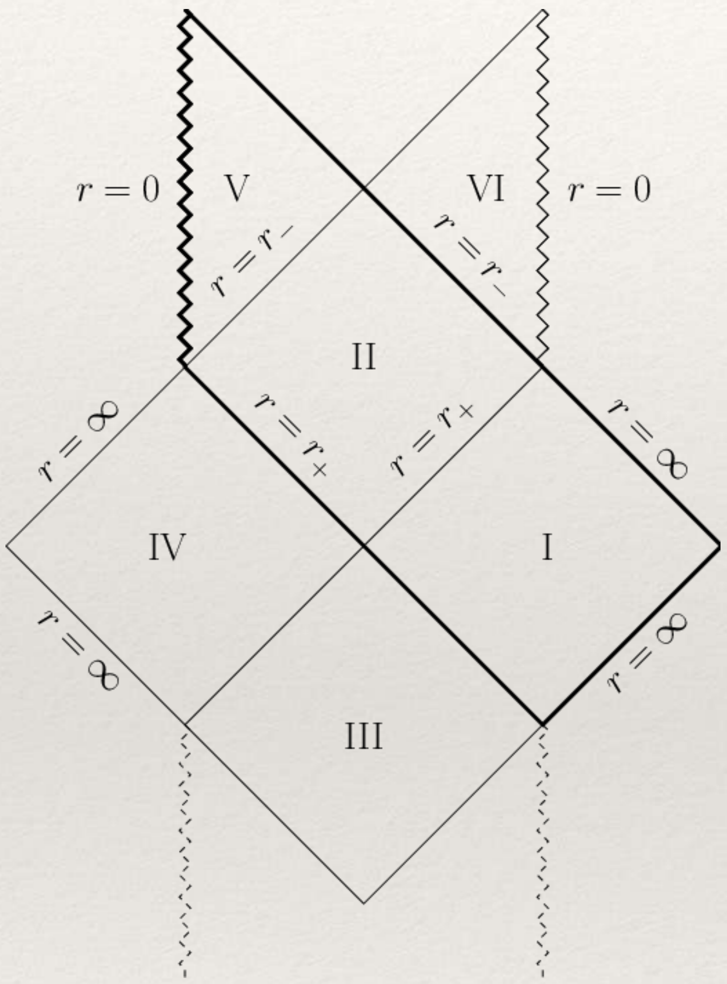
However… semiclassical instability

S. Hollands, R.M. Wald and J. Zahn,
Class. Quant. Grav. 37 (2020) no.11, 115009

T. McMaken,
Phys. Rev. D107 (2023) no.12, 125023

C. Barcelò, V. Boyanov, R. Carballo-Rubio and L.J. Garay,
Phys. Rev. D106 (2022) no.12, 124006

R. *Balbino, A Fabbri.*
Universe 10 (2024) 18 • e-Print: 2311.09943



- Zero surface gravity at the inner horizon might not be enough to stabilise a regular black hole: there is a divergence in the SET at the Cauchy horizon ruled by

$$\lim_{r\to r_-} < U|T_{uu}|U > = -\frac{1}{48\pi}\left(\kappa_-^2 - \kappa_+^2\right)$$

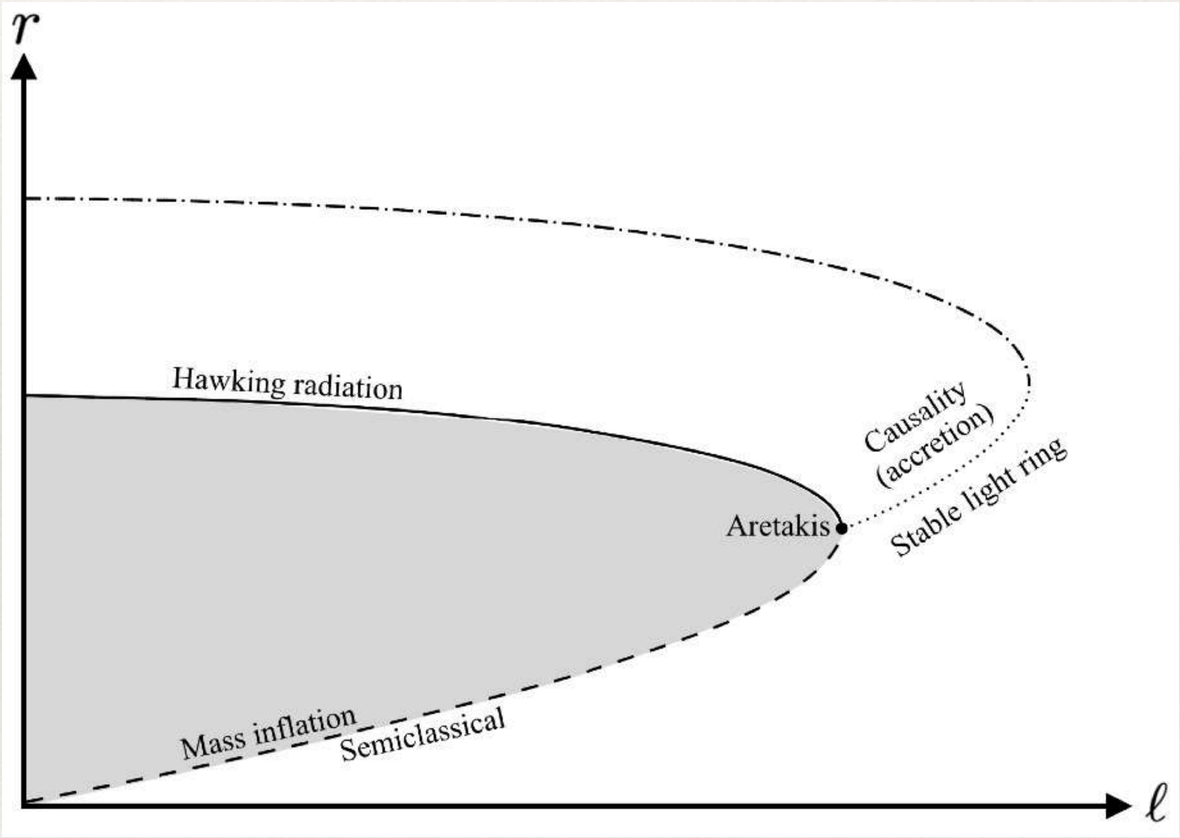
- Still this divergence might be absent in a dynamical geometry as it is strictly associated to Cauchy horizons. (Carballo-Rubio, Di Filippo, SL, Visser. In Preparation)

- In any case a large back reaction is expected also such from semiclassical instability.

Take home message: RBH are most probably dynamical objects at most metastable.
Compatibility of this metastability with observations is an open issue.
Let's see more in detail what we know so far…

Class 1: a conundrum of Instabilities

- Preliminary investigations (see Barceló et al. *Phys.Rev.D* 106 (2022) 12, 124006) seems to suggest that
  - classical mass inflation would push the inner horizon inwards
  - the quantum instability would push the inner horizon outwards and actually dominate over mass inflation.
  - The position of the IH is basically set by ℓ, so the semiclassical instability suggests that one effectively gets $\ell \to \ell(v)$
  - So, this chain of instabilities may lead the RBH to end up extremal or a quasi-BH…
  - But extremal black holes seems to suffer from the Aretakis instability: higher radial derivatives of a field propagating on the horizon unboundedly grow in time. [Open question if it does apply also to inner extremal horizons… work in progress with L. Donnay group]
  - But quasi-BH have necessarily an inner stable light ring! $\longrightarrow$ possibly unstable again?



So what is going to be the end point?!

Class 2: The Simpson-Visser Black-Bounce

$$ds^2 = -\left(1 - \frac{2M}{\sqrt{r^2+\ell^2}}\right) dt^2 + \left(1 - \frac{2M}{\sqrt{r^2+\ell^2}}\right)^{-1} dr^2 + (r^2+\ell^2)\left[d\theta^2 + \sin^2\theta d\phi^2\right],$$

A.Simpson, M.Visser. JCAP 02 (2019) 042
e-Print: 1812.07114 [gr-qc]

- a two-way, traversable wormhole à la Morris-Thorne for $\ell > 2M$,
- a one-way wormhole with a null throat for $\ell = 2M$, and
- a regular black hole, in which the singularity is replaced by a bounce to a different universe, when $\ell < 2M$; the bounce happens through a spacelike throat shielded by an event horizon and is hence dubbed "black-bounce" in [6] or "hidden wormhole" as per [4].

RN extension:
E.Franzin,SL, J.Mazza, A.Simpson, M.Visser. JCAP 07 (2021) 036.
e-Print: 2104.11376 [gr-qc]

Rotating counterpart

J.Mazza, E.Franzin, SL. JCAP 04 (2021) 082 • e-Print: 2102.01105 [gr-qc]

$$ds^2 = -\left(1 - \frac{2M\sqrt{r^2+\ell^2}}{\Sigma}\right) dt^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 - \frac{4Ma\sin^2\theta\sqrt{r^2+\ell^2}}{\Sigma} dt d\phi + \frac{A\sin^2\theta}{\Sigma} d\phi^2 \quad (2.16)$$

with

$$\Sigma = r^2 + \ell^2 + a^2\cos^2\theta, \qquad \Delta = r^2 + \ell^2 + a^2 - 2M\sqrt{r^2+\ell^2},$$
$$A = (r^2+\ell^2+a^2)^2 - \Delta a^2 \sin^2\theta.$$

WoH traversable wormhole;

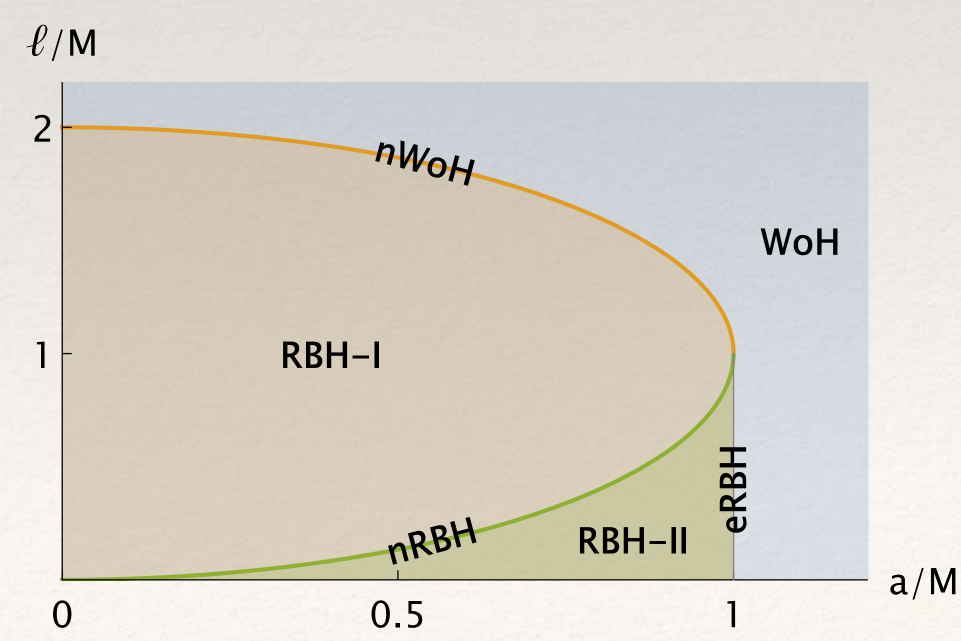
nWoH null WoH, i.e. one-way wormhole with null throat;

RBH-I regular black hole with one horizon (in the $r > 0$ side, plus its mirror image in the $r < 0$ side);

RBH-II regular black hole with an outer and an inner horizon (per side);

eRBH extremal regular black hole (one extremal horizon per side);

nRBH null RBH-I, i.e. a regular black hole with one horizon (per side) and a null throat.



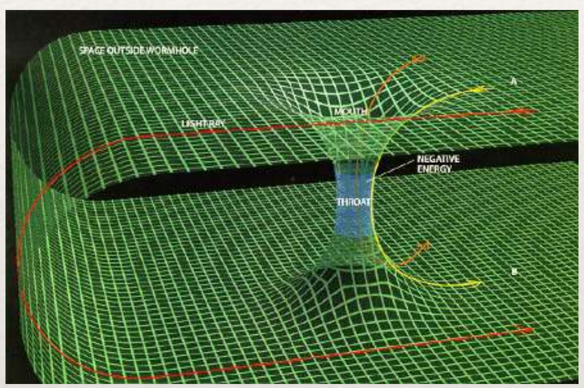
E. Franzin, S. Liberati, J. Mazza, R. Dey and S.Chakraborty,
Phys. Rev. D105, no.12, 124051 (2022)

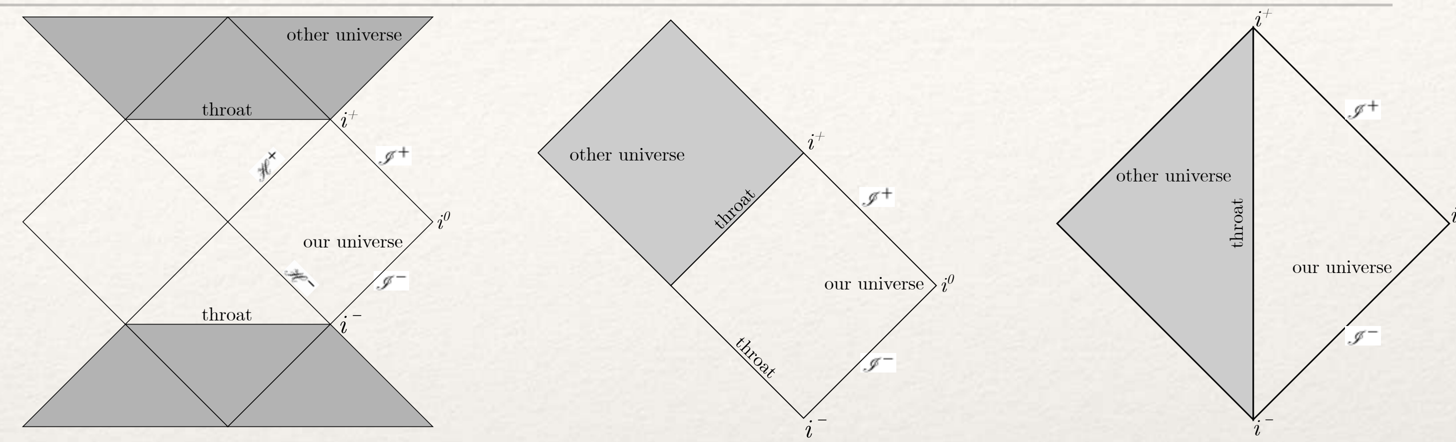
Class 2 limiting case: traversable wormholes

$$ds^2 = -\left(1 - \frac{2M}{\sqrt{r^2+\ell^2}}\right)dt^2 + \left(1 - \frac{2M}{\sqrt{r^2+\ell^2}}\right)^{-1}dr^2 + (r^2+\ell^2)\left[d\theta^2 + \sin^2\theta d\phi^2\right],$$

$$\boxed{\left(1 - 2M/(\sqrt{r^2+\ell^2}\right) = 0 \text{ has no roots for } \ell > 2M}$$

(similarly for the rotating case)





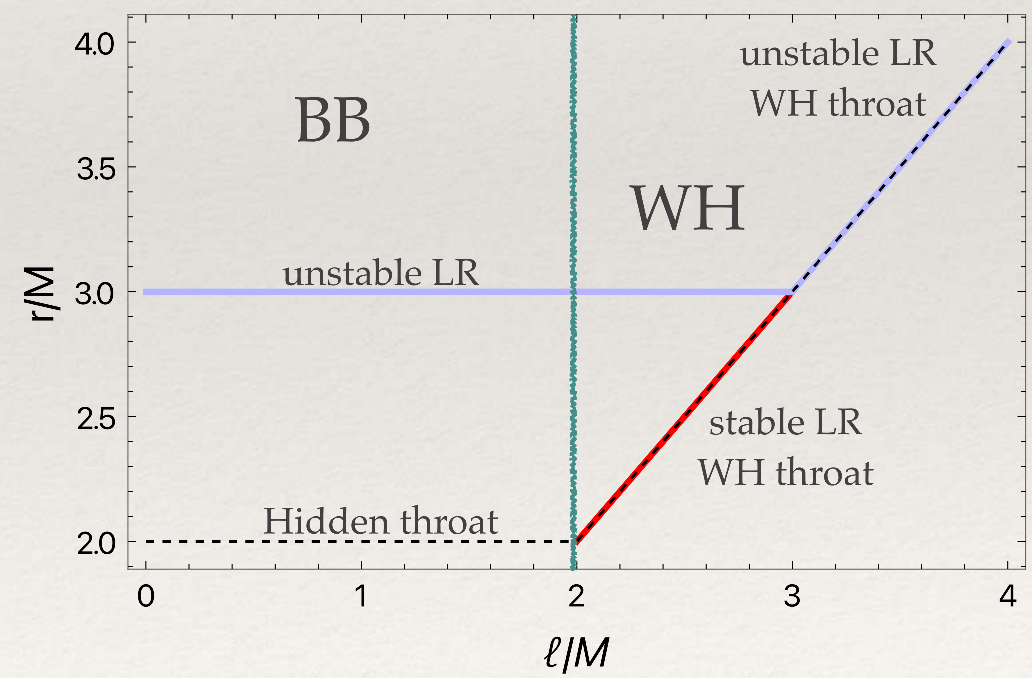
(a) The regular black hole. The maximally extended spacetime continues above and below the portion shown by repetition of this fundamental block.

(b) The null-throat wormhole. The analytically extended spacetime continues above and below by repetition of this fundamental block.

(c) The traversable wormhole.

Figure 1. Penrose diagrams of regular black hole, null-throat wormhole and traversable wormhole. The white area represents "our universe" while the gray area is the "other universe".

- Energy conditions violation at the WH throat
- Also in this case the naked WH will sport for $2M < \ell < 3M$ a stable light-ring at the WH throat.
- For $\ell > 3M$ there is only an unstable light-ring again at the wormhole throat.



Take home message: In spite of being "more exotic" the Black Bounces appear to be less prone to instabilities (e.g. no mass inflation instability if $\ell > r\_$).
Still less studied… more to do.

R. Carballo-Rubio, F. Di Filippo, S. Liberati and M. Visser,
Phys. Rev. D98, no.12, 124009 (2018)

Phenomenology: parametrising the uncertainties

Size, $R = r_S(1 + \Delta)$: the value of the radius below which the modifications to the classical geometry are O(1). $\Delta \geq 0$.
Note the compactness parameter $\mu = \Delta/(1 + \Delta)$. So for $\Delta \ll 1$ one has $\mu \simeq \Delta$

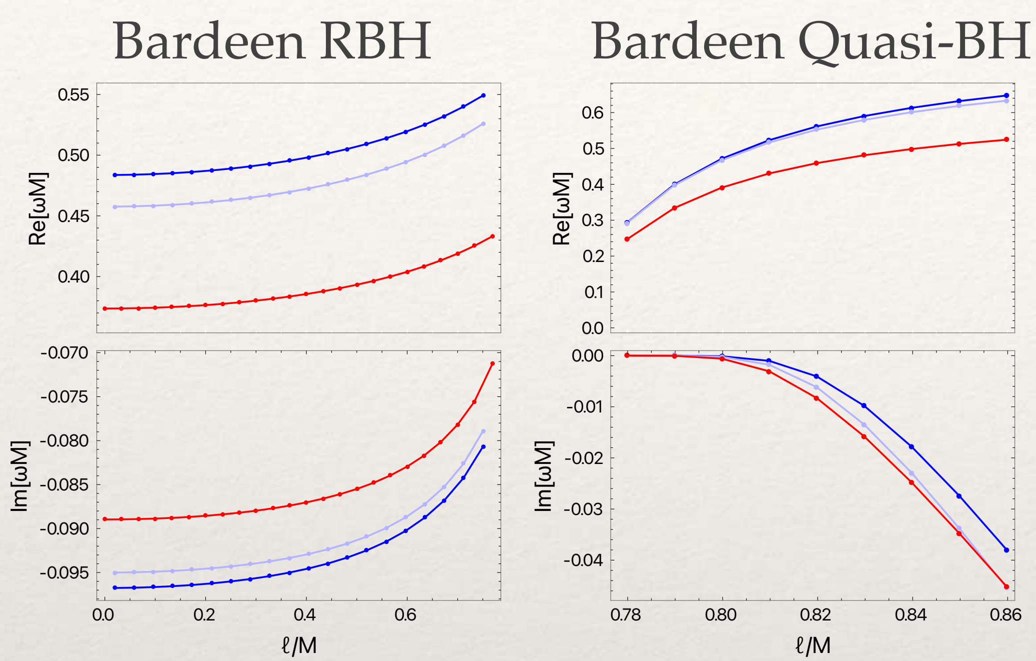
	τ_+ - Lifetime	τ_- -formation time	μ- compactness	κ-Absorption Coeff.	Γ-Elastic reflection Coeff.	$\tilde{\Gamma}$-Inelastic reflection Coeff.	$\varepsilon(r)$- Tails
Classical GR BH	∞	~10 M	0	1	0	0	0
Trapped regions (RBH+Hidden WH)	undertermined	~10 M	0	1	0	0	Non-zero
Quasi-BH	∞	Model dependent	Model dependent	Model dependent	Model dependent	Model dependent	Model dependent
Bouncing Geometries (long lived)	$\mathcal{T}^{(i)}$	Model dependent	0	1	0	0	non-zero and $r_* = O(r_S)$
Traversable Wormholes	∞	unknown	>0	Model dependent	1-κ	0	Model dependent

Note: one of the parameters is not independent: e.g. inelastic interaction parameter must satisfy $\tilde{\Gamma} = 1 - \kappa - \Gamma$

Including additional independent parameters would provide more freedom to play with the observational data but less constraining power. The set introduced is minimal, but still able to asses the observational status of black holes.

Cases 1 & 2: Quasi-normal modes analysis

Let us we study test- field and linear gravitational perturbations in such spacetimes, varying the regularization parameters so to pass smoothly from RBHs to the ultracompact horizonless objects.



Spin 2
Test Field
perturbation

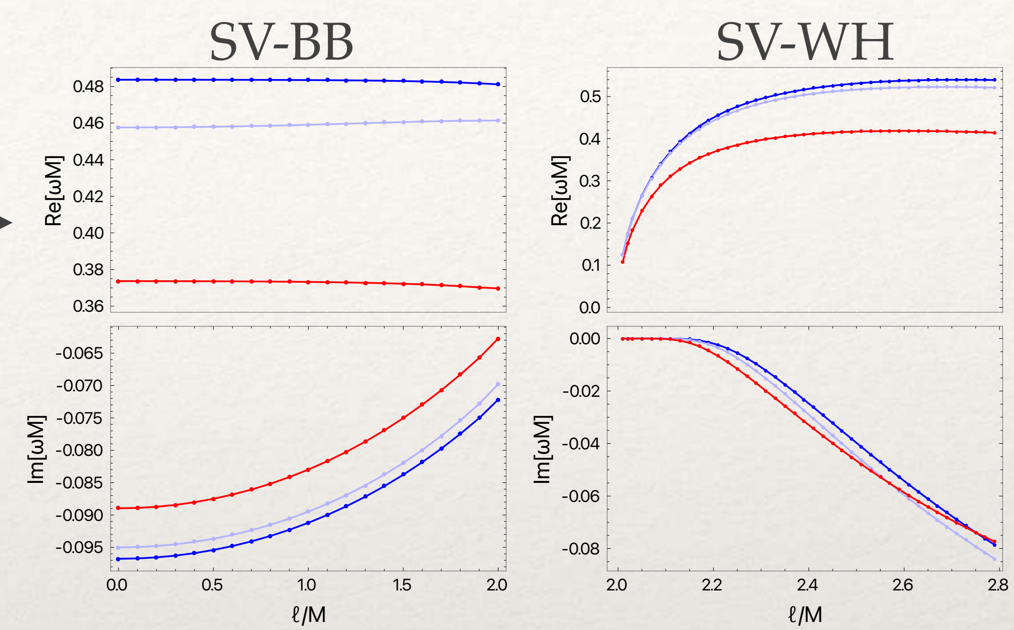


Figure 2. Quadrupolar $l = 2$ fundamental QNMs of the Bardeen metric for test-field perturbations, $s = 0$ (blue), $s = 1$ (light purple) and $s = 2$ (red). On the left results for values of ℓ in the RBH branch that is from $\ell = 0$ (Schwarzschild) to $\ell = \ell_{\rm ext} = \frac{4}{3\sqrt{3}}M$ (extremal RBH). On the right results for values of ℓ in the horizonless branch. Note that for values of the regularization parameter near the extremal case the imaginary part is extremely small and thus we have very long living modes.

Figure 3. Quadrupolar $l = 2$ fundamental QNMs of the SV metric for test-field perturbations, $s = 0$ (blue), $s = 1$ (light purple) and $s = 2$ (red). On the left results for values of ℓ in the RBH branch, that is from $\ell = 0$ (Schwarzschild) to $\ell = 2M$ (one-way wormhole with an extremal null throat). On the right results for values of ℓ in the horizonless branch. It is worth noticing the relative flatness of the real part curves which highlights weak deviations from the singular GR solution behaviour recovered for $\ell = 0$. On the left results for values of the regularization parameter near the extremal case ($\ell_{\rm ext} = 2M$): the imaginary part is extremely small and thus we have very long living modes.

Regular Black holes		Bardeen			Simpson–Visser		
		Test $s=2$	Axial	Polar	Test $s=2$	Axial	Polar
$\ell/M = 0.2$							
	Δ_R	0.0075	−0.0012	0.0037	$-3 \cdot 10^{-5}$	−0.0002	−0.0005
	Δ_I	0.0045	0.0090	0.0090	0.0022	0.0044	0.0044
$\ell/M = 0.6$							
	Δ_R	0.0808	0.0069	0.0297	−0.0003	−0.0163	−0.0067
	Δ_I	0.0674	0.0776	0.0810	0.0236	0.0292	0.0292
$\ell/M = 1.6$							
	Δ_R				−0.0053	−0.0690	−0.0428
	Δ_I				0.1798	0.1854	0.1776

Horizonless compact objects		Bardeen	Simpson–Visser
		Test $s=2$	Test $s=2$
$\delta = 0.05$			
	Δ_R	0.1380	−0.1801
	Δ_I	0.9712	0.9970
$\delta = 0.10$			
	Δ_R	0.3613	−0.0310
	Δ_I	0.6441	0.9015
$\delta = 0.20$			
	Δ_R		0.0482
	Δ_I		0.5913

Table I. Relative deviations from the quadrupolar fundamental Schwarzschild frequency $\Delta_{R/I} = \frac{\omega_{R/I}-\omega^S_{R/I}}{|\omega^S_{R/I}|}$ with $\omega^S M = 0.37367 - 0.08896i$, for $s = 2$ test-field and linear gravitational perturbations, both in the axial and polar sectors, for selected valued of the regularization parameter. Results are shown for the Bardeen and SV spacetimes, on the left for the RBH branch and on the right for horizonless configurations. For the Bardeen metric there are no results for $\ell/M = 1.6$ and $\delta = 0.2$, with $\delta \equiv \ell/\ell_{\rm ext} - 1$, since for those values of compactness the spacetime not only lose the presence of the horizon but even of a photon sphere. For both spacetimes results for axial and polar gravitational perturbations are not reported for horizonless configurations because of the numerical issues present in this branch. Looking at the test field case, it is easy to see the large increment Δ_I passing from the RBH configurations to the horizonless ones for small δs.

Summary

- For $\ell \lesssim M$ both the RBH and SV BB show deviations for the Schwarzschild QNM
- SV BB tends to show smaller deviations.
- **Third generation GW detector with enough statistics might see this if $\ell/M \sim O(10^{-1})$!**
- Quasi-BH configurations show marked longer perturbations lifetimes (tiny imaginary part) for $\ell \gtrsim \ell_{\rm ext}$
- This is a sufficient condition to expect non-linear instability and appears to be related to the presence of the inner-stable-light-ring
- However, note that the imaginary part becomes comparable with the Schwarzschild one very rapidly as the compactness decreases even before the inner-light ring disappear.
- A non-linear analysis is definitely needed…
- (and what about matter interactions?)

GW channel: Echoes from BH Mimickers

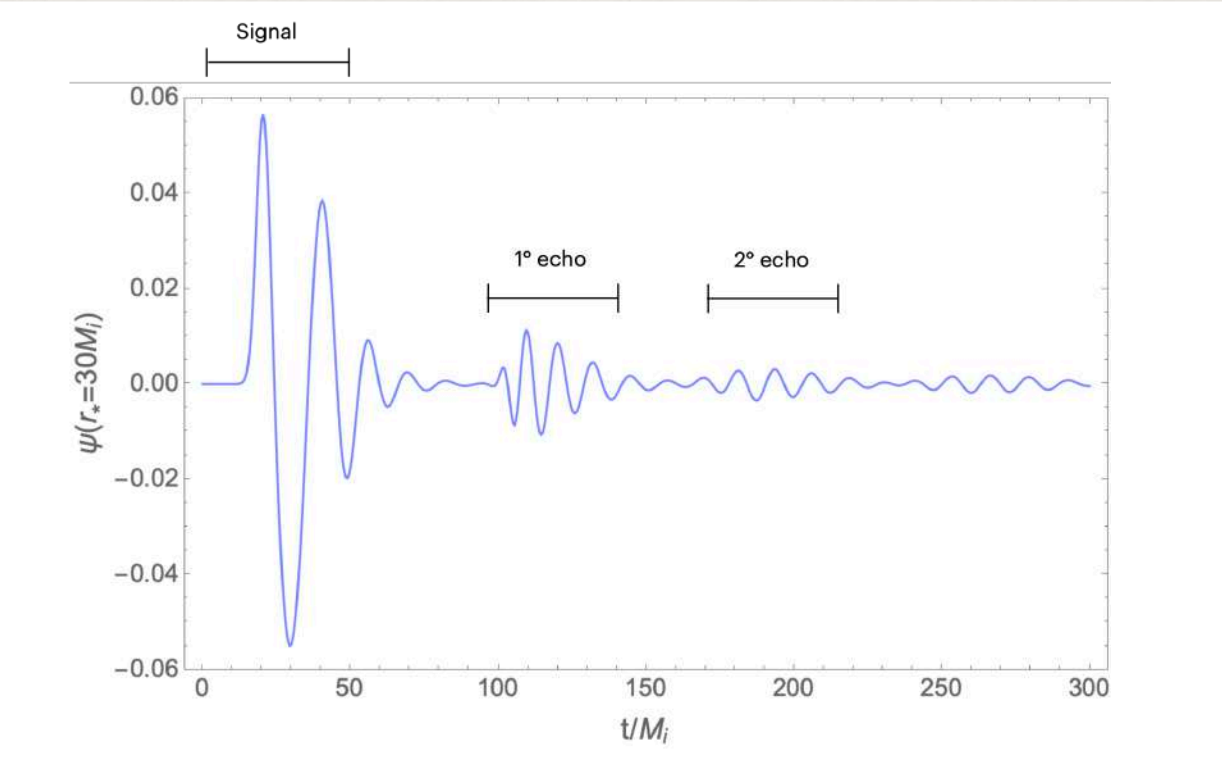
- In the case of a black hole GW scattered back at the potential barrier (usually close to the light ring) are lost inside the horizon.
- For an horizonless object (quasi-BH or traversable wormhole) instead the wave can go through the center and bounce again at the potential barrier with a part transmitted at infinity and one par reflected.

This generates "echoes".

Key point: even for ultra compact objects the delay between such echoes is macroscopic (logarithmic scaling).

Time delay for an object of compactness $\Delta = r/2M_0 - 1$

$$\Delta t_{\text{echo}} = 2\int_{r_0=2M_0(1+\Delta)}^{r_{\text{peak}}\approx 3M_0} \frac{\mathrm{d}r}{1-2M_0/r} \approx 2M_0\left[1-2\Delta-2\ln(2\Delta)\right]$$



- **Note that we neglect above the time to cross the object, but there are example of very "deep potential well" cores which would generate a huge time delay and make the effect observationally irrelevant: Arrechea, SL, vellucci. *JCAP* 12 (2024) 004, Semiclassical stars with AdS cores.**

- **The amplitude of gravitational wave echoes would be proportional to Γ. A non-observation of echoes can only constrain this parameter. A positive detection of echoes could be used in order to determine also Δ.**

So far searches for quasi-periodic signals…

Echos and Non-linear back reaction

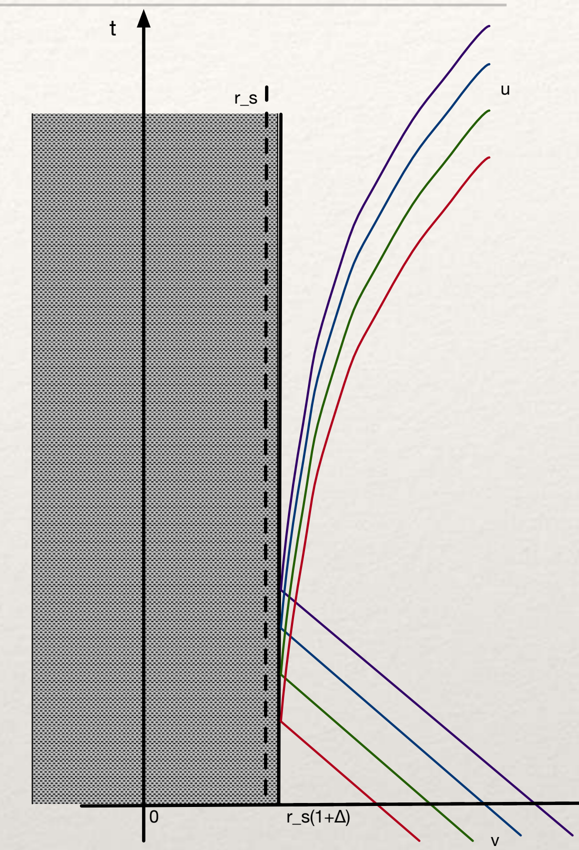
Non-linear interactions between the GW and the central object

- These are neglected in extant analyses. However, this appears to be inconsistent
- **For quasi-BH even modest amounts of accretion will generate a trapped region**
- The formation of a trapping horizon might be avoided by nonlinear interactions

Example: If vacuum polarisations supports a Quasi-BH in Boulware vacuum

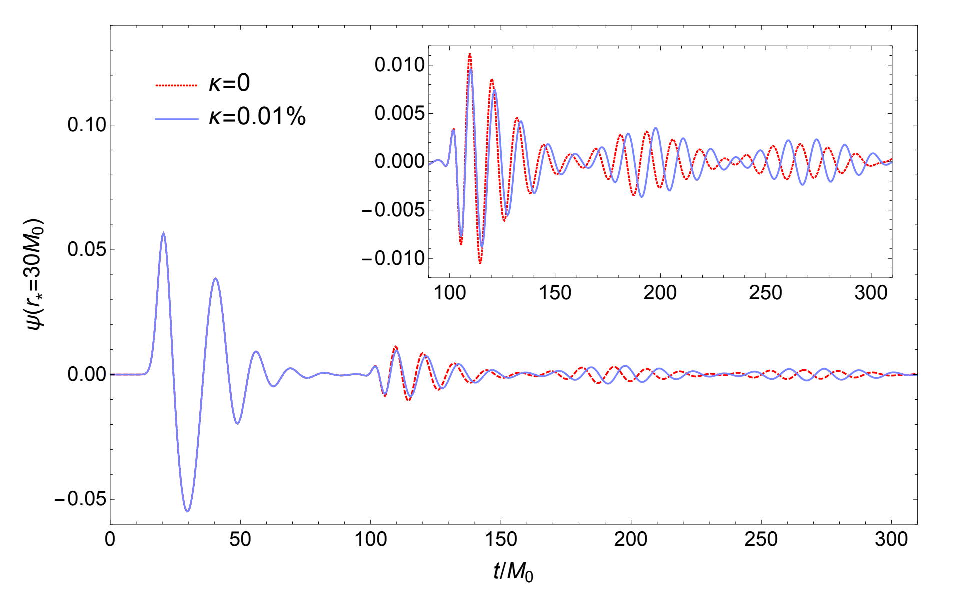
RSET $\propto -\left(1-\frac{2M}{r}\right)^{-2}$ so even tiny change 2M→r can generate huge back-reaction.

- The more compact the central object is, the larger is the fraction of the energy stored in the gravitational waves to be transferred through nonlinear interactions. I.e. large absorption $\kappa = 1 - E_{\text{out}}/E_{\text{in}}$



V. Vellucci, E. Franzin and S. Liberati, ``Echoes from backreacting exotic compact objects,'' arXiv:2205.14170 [gr-qc].

- A model-independent outcome of these interactions has to be the expansion of the central object in order to avoid the formation of trapping horizons.

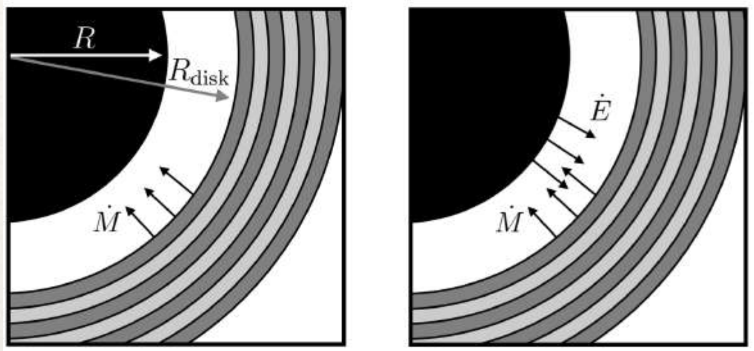


- For very compact objects, very small ΔM corresponds to large variations in the compactness. Changes in compactness are sufficient to destroy the periodicity of the echos…

- So, even for $\kappa \sim 0.01\ \%$ one get noticeable delays between echoes given that the compactness of the object has to increase

R. Carballo-Rubio, F. Di Filippo, S.L. and M.Visser,
JCAP08 (2022) no.08, 055.[arXiv:2205.13555 [astro-ph.HE]].

EHT Constraints from Reemission



The minimum surface luminosity expected at infinity L∞ can be estimated as

$$L_\infty > \eta \dot{M} \ \text{ where } \ \eta = \dot{E}/\dot{M}$$

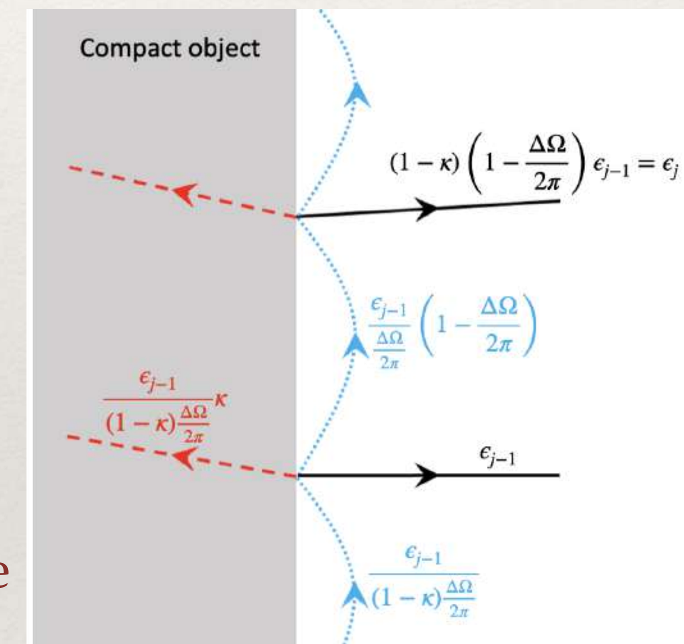
An upper bound on the observed luminosity can then be translated into a constraint on the η parameter. From ETH we know $\eta < 10^{-2}$

How this translates on a bound on the relative compactness $\mu = 1 - 2M/r_*$?

Assuming that all the kinetic energy of infalling matter is converted to outgoing radiation, leads to the naive result $\eta = 1 - \sqrt{\mu}$

However, this does not take into account lensing ($\Delta\Omega/2\pi = 27\mu/8 + O(\mu^2)$) and the possibility that part of the radiation is absorbed by the Quasi-BH. I.e. the case $\kappa \neq 0$.

Indeed, one can model the quasi-BH—matter interaction as a series of bounces of the radiation over the surface which have to be summed up.



The net effect is $$\eta(t) = \frac{\dot{E}}{\dot{M}} = \frac{\Delta\Omega}{2\pi}\frac{(1-\kappa)}{\kappa + \frac{\Delta\Omega}{2\pi}(1-\kappa)}\left\{1 - (1-\kappa)^{t/\tau}\left(1 - \frac{\Delta\Omega}{2\pi}\right)^{t/\tau}\right\}.$$

t=time over which SGrA* has been accreting $t \approx T_{\rm Edd} \approx 3.8 \times 10^8$ yr
τ = time for each bounce $\approx O(10M) \sim 10^2$ s

For the physical limit $\tau/T \ll \kappa < 1$ $$\eta = \frac{\Delta\Omega}{2\pi}\frac{1-\kappa}{\kappa + \frac{\Delta\Omega}{2\pi}(1-\kappa)}.$$

So e.g. for $\tilde{\Gamma} = 1 - \kappa = 10^{-5} \Longrightarrow \mu \lesssim 10^{-7}$
or for $\tilde{\Gamma} = 1 - \kappa = 10^{-2} \Longrightarrow \mu \lesssim 1$

So no meaningful upper-bound constraints can be placed for objects with large absorption coefficients

Closure

- BH are the new frontier for testing classical and quantum deviations from GR
- Basic arguments from Penrose singularity theorem show that regular spacetime resolutions of singularities are divide in two families depending on the absence/presence of a minimal radius
- For both these families there are related horizonfull and horizonless solutions.
- Ensuing instabilities of inner trapping and extremal horizons are crucial to understand the actual long living end points of such models…
- In any case: avoiding the central singularity appears to generically lead to long range effects (in time or space).
- The resulting black hole mimickers are very hard to exclude with current observations but they are not hopeless and better modelling plus multimessanger astrophysics will be the key to test them.

Are we at the dawn of a new form of QG phenomenology based on BH observations?

THANK YOU!


we have seen many ultra
compact object in the sky
sure we have seen so many
GR black holes!
Haven't we?