Deforming black holes and ultracompact objects from the outside and from the inside

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Black holes and their symmetries, Tours, June 2-4, 2025

Outline

- Some basics on black hole uniqueness
- Deforming black holes from the outside and from the inside: the axisymmetric case
- Ultracompact objects vs black holes

Some basics on black hole uniqueness

Black holes are highly symmetric

- There can be isolated stars and planets with different shapes. In fact, that is the case.
- Intuition tell us that to generate a Montaigne of a Valley is more difficult for objects with bigger surface gravities: $\kappa = GM/R^2$
- In General Relativity a special limit occurs when compactness $\frac{2GM}{c^2R} \to 1$
- All theoretical evidence points to a non-hair result for black holes

Black holes are highly special

- Isolated black holes are characterised by only 2 numbers (Mass and angular momentum; 3 if they were charged)
- As opposed to their progenitors, Black holes comes in only one shape.
- Chandrasekhar: "The black holes of nature are the most perfect macroscopic objects there are in the universe: the only elements in their construction are our concepts of space and time."

Why is that so?

- The simplest situation in which one can start understanding what is going on is the static case.
- There we have the first uniqueness result due to Werner Israel 1967.
- Loosely speaking: the only static geometry representing a black hole (i.e. with a regular horizon) in an asymptotically flat spacetime is the Schwarzschild geometry.

Israel theorem

W. Israel, Phys. Rev. 164, 1776 (1967)

The set of geometries that are:

- a) Static
- b) Vacuum
- c) Asymptotically flat
- d) With spheroidal equipotential surfaces, labelled by $\epsilon \in (0^+, 1)$
- e) With a regular horizon (non-divergent Kretschmann)
- f) The area of the equipotential surfaces taking a finite limit as $\epsilon \to 0^+$

$$ds^2 = -\mathscr{V}^2(x)dt^2 + g_{ij}(x)dx^i dx^j$$

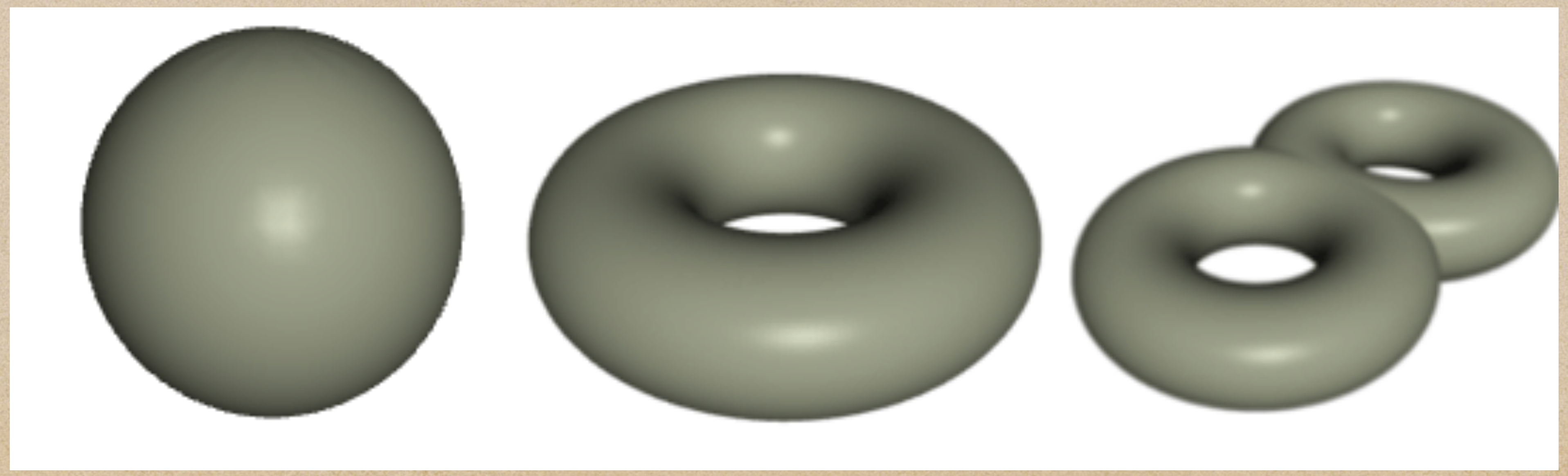
contains a single member: the Schwarzschild geometry

Why is that so?

- By looking just at its enunciation, one would not know what is the reason for what
- There is a mixture of global and local issues at stake. E.g. regularity of the horizon (l); vacuum (g)
- In addition it is not clear the role played by assumptions like (d) or (f)

Israel theorem as a shooting

- One can devise different forms for the local geometry of a regular horizon
- In fact, it would be possible even to have black holes with arbitrary topologies
- The problem comes when integrating Einstein equations outwards assuming vacuum equations. Only the purely spherical horizon case matches with a well defined asymptotically flat region.
- If the integration where made with arbitrary matter contents, then everything would be possible



Deforming from the outside and from the inside: the axisymmetric case

Axisymmetry

Analysing the simpler situation of axisymmetry provides much clarity

$$ds^2 = -\, e^{2U} dt^2 + e^{-2U} \left[e^{2V} \left(dr^2 + dz^2 \right) + r^2 d\varphi^2 \right]$$

with $U(r, z),\ V(r, z)$ satisfying

$$\nabla^2 U = 0$$

Associated Laplace Problem

$$\partial_r V = r \left[(\partial_r U)^2 - (\partial_z U)^2 \right]$$

Solvable by quadrature

$$\partial_z V = 2r \partial_r U \partial_z U$$

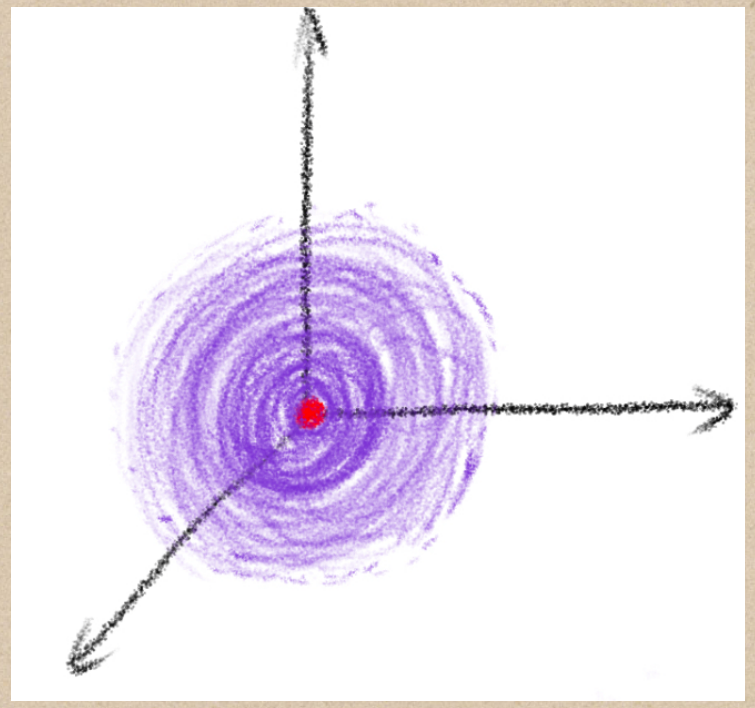
Bach, Weyl,
Mathematische Zeitschrift
13 (1922) 134; Gen Relativ
Gravit 44 (2012) 817

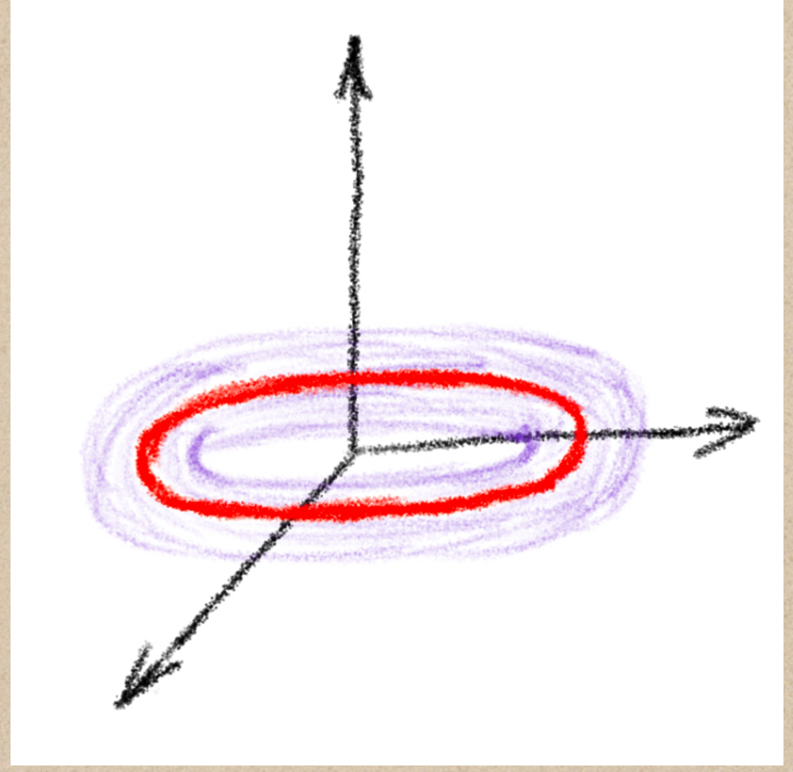
Single objects with a putative horizon

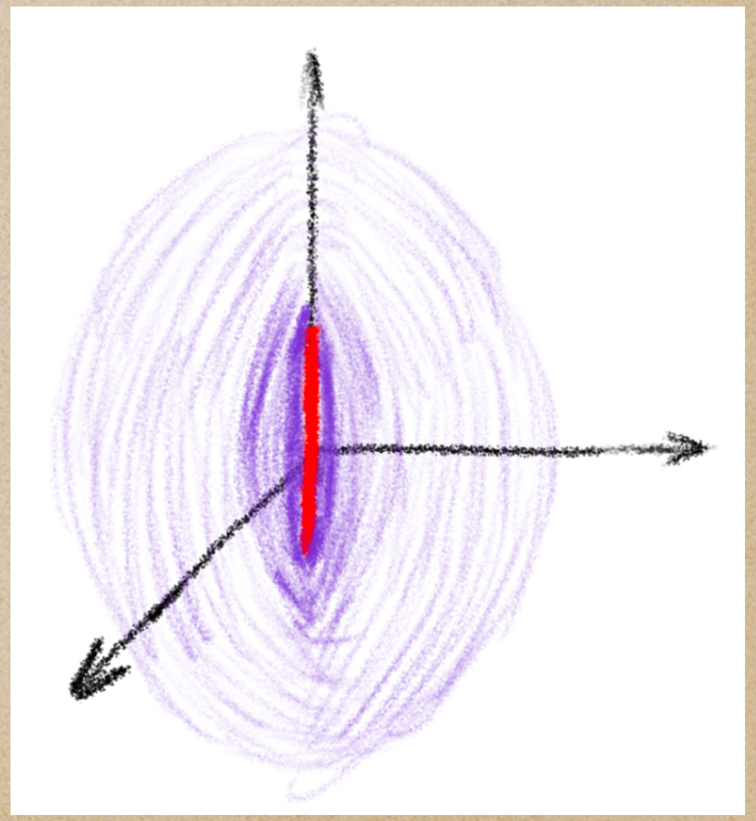
A putative horizon appears when $U(r, z) \to -\infty$ somewhere

Given the Laplacian equation, we know that this can only happen for:

- For a delta point source
- For a delta ring source
- For a delta rod source





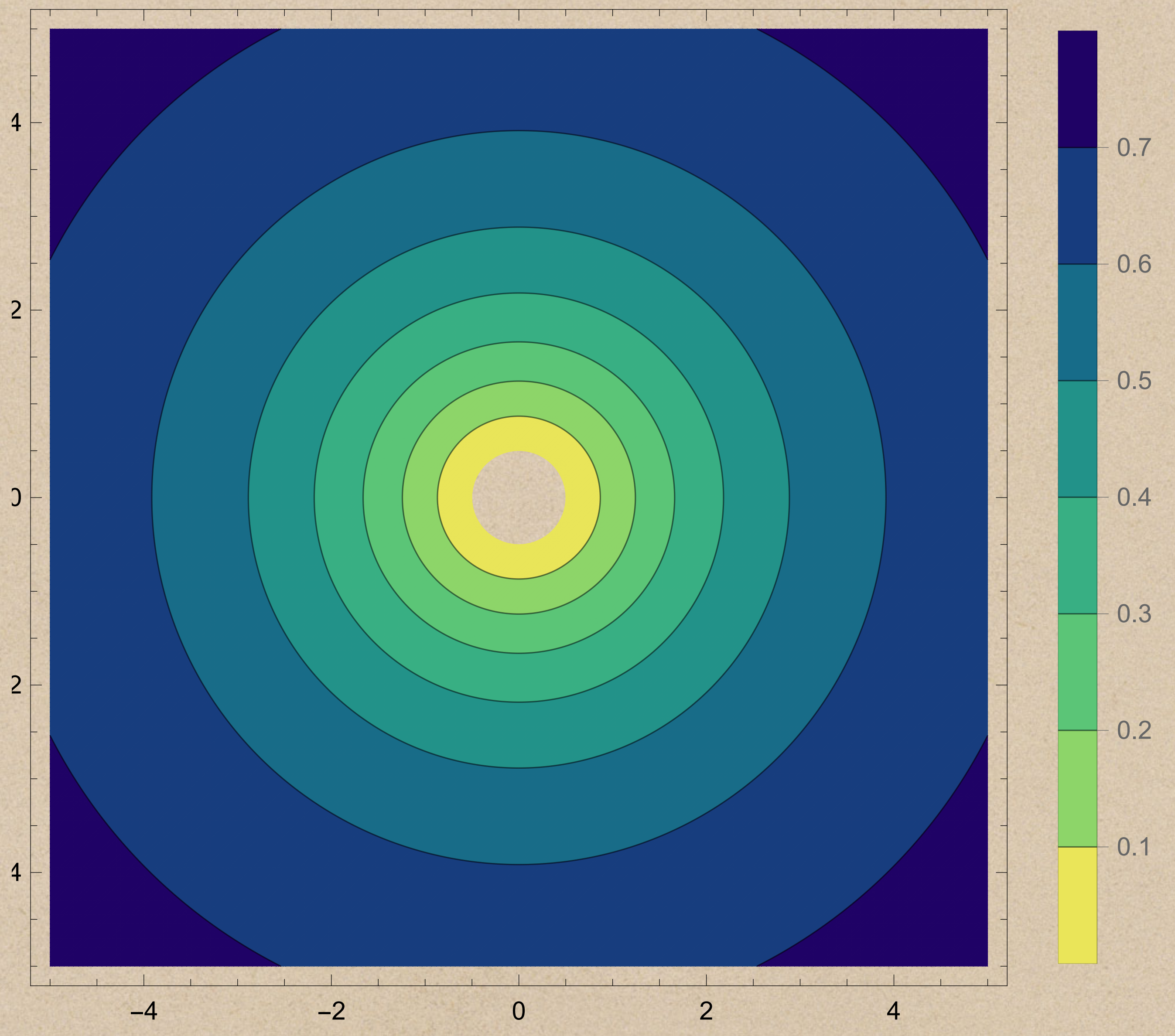


Geroch & Hartle, J. Math. Phys. 23, 680 (1982)

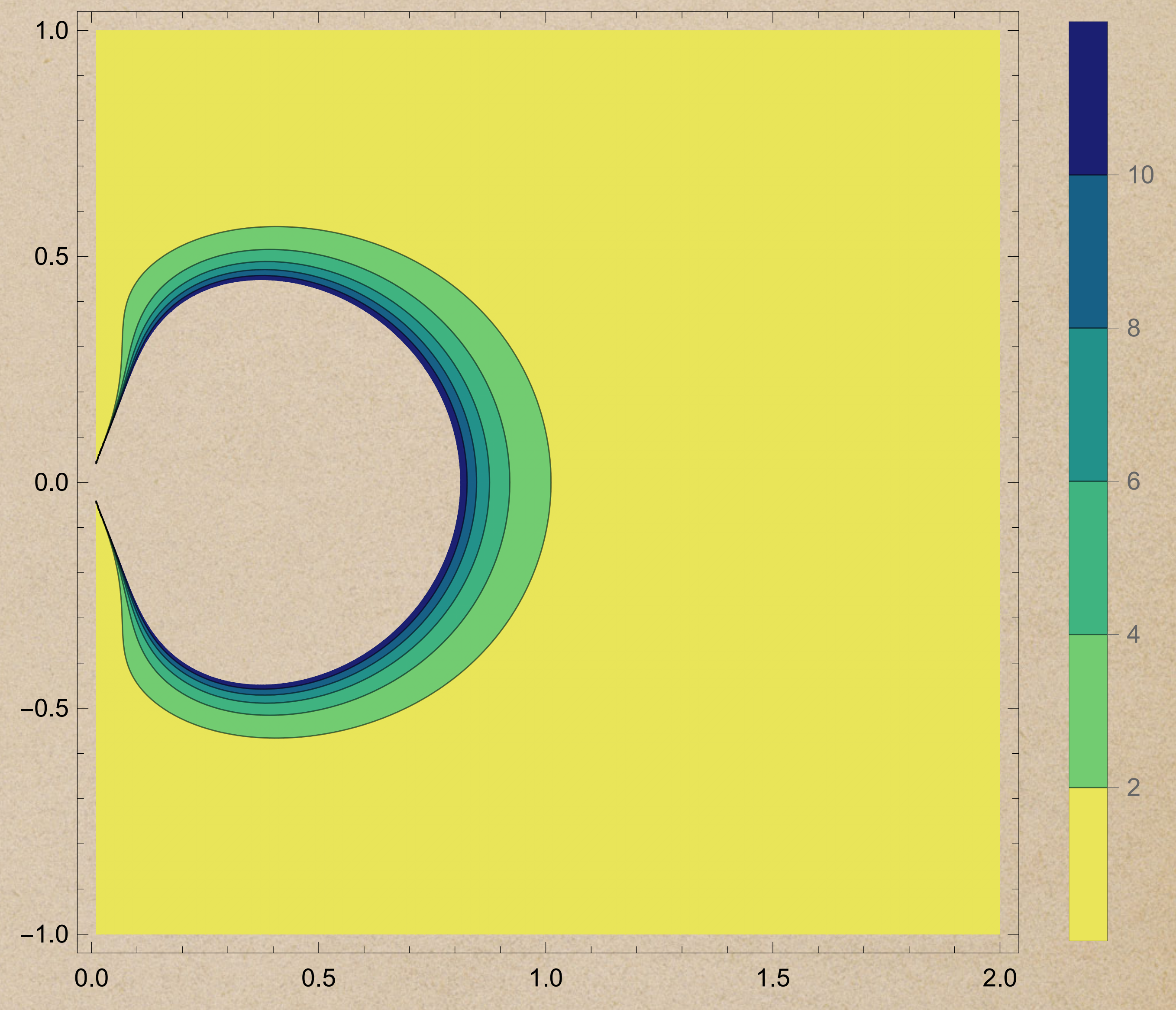
Delta point source

$$U(R) = -\frac{M}{R}$$

$$V(R,\theta) = -\frac{M^2 \sin^2\theta}{R^2}$$



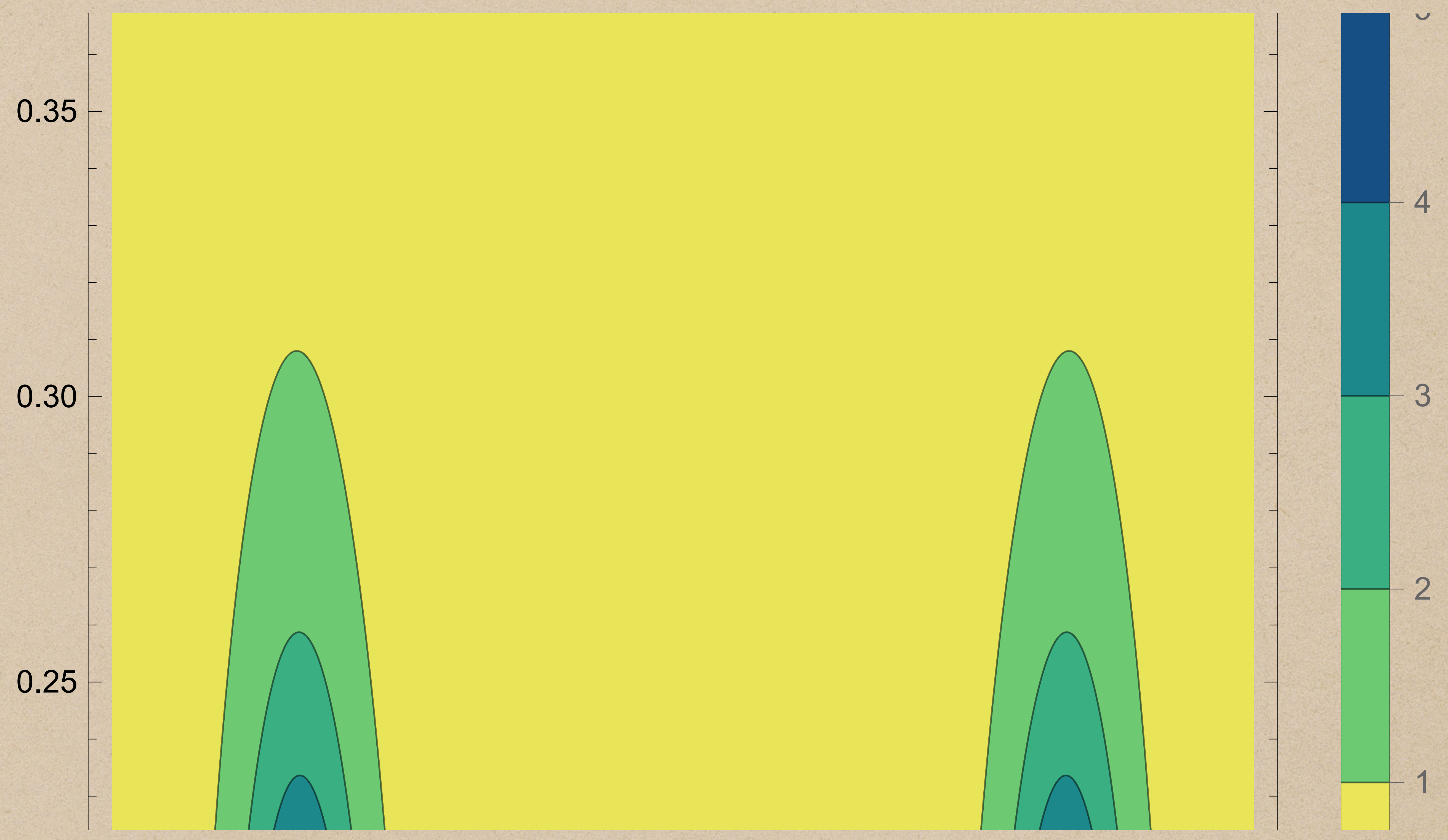
Red-shift function



Kretschmann scalar

Curzon geometry

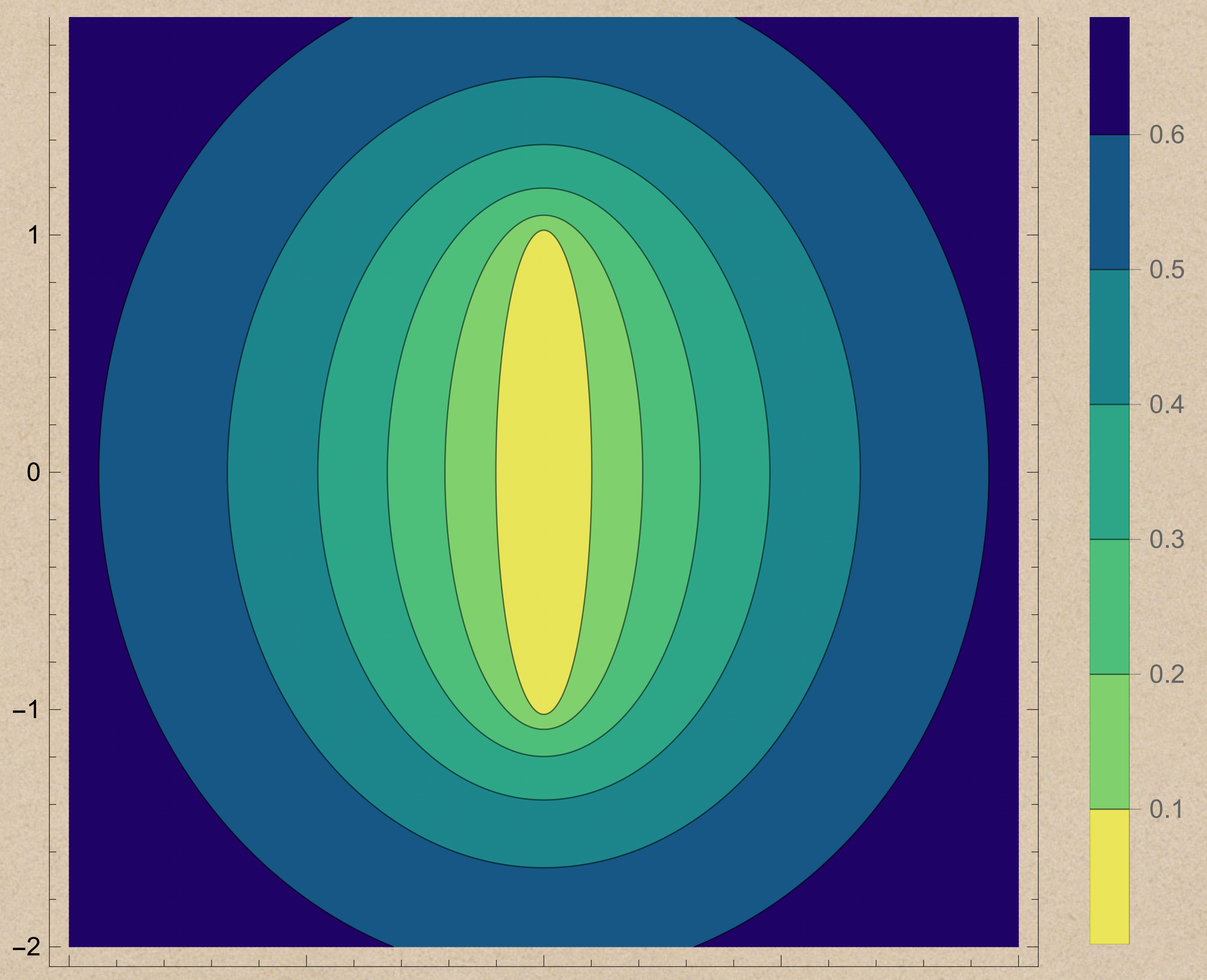
Delta ring source



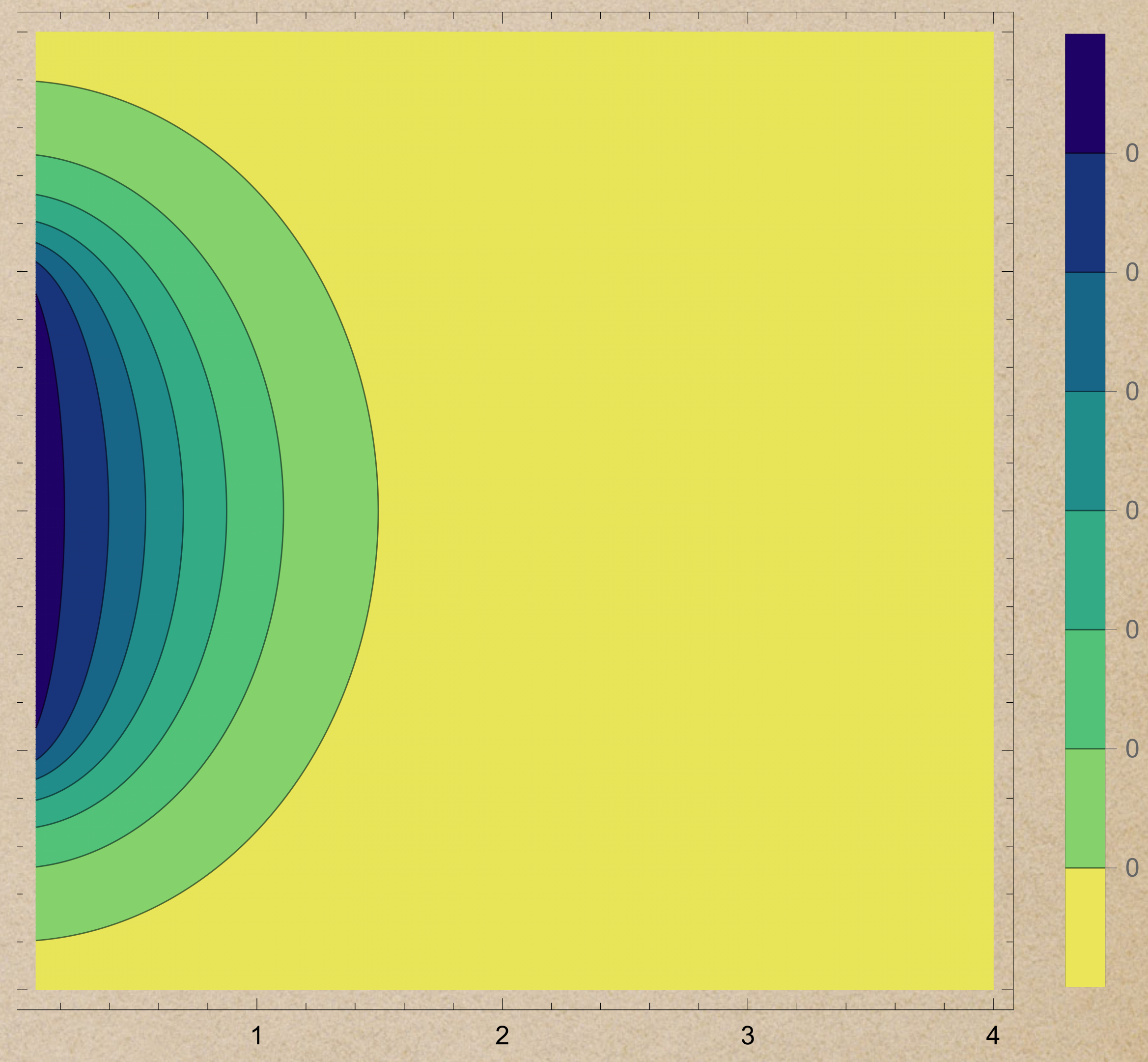
Delta rod source

$$U_S(r,z) = \frac{1}{2}\log\left(\frac{R_+ + R_- - 2M}{R_+ + R_- + 2M}\right)$$

$$V_S(r,z) = \frac{1}{2}\log\left(\frac{(R_+ + R_-)^2 - 4M^2}{4R_+R_-}\right)$$



Red-shift Schwarzschild



Kretshmann scalar

Deforming a BH from inside

The only real black hole is the rod one, i.e. the Schwarzschild black hole.

Now, the only possible deformation of a Schwarzschild black hole is:

$$U = f(z)\log r + o\left(\log r\right) \quad \text{with} \quad f(z) = 2\lambda(z)$$

$$\mathcal{K} = \frac{12(f(z)-1)^2 f(z)^2}{r^4} + \frac{8f'(z)^2\left[3f(z)^2\log^2(r) - 3f(z)\log^2(r) + \log^2(r) + 2\log(r) + 2\right]}{r^2} + \text{finite terms}$$

It is divergent unless $f(z) = 1$. Thus, one cannot deform from inside!

Deforming a BH from outside

- Realistic black holes would not be isolated, they will be surrounded by matter
- Gürlebeck showed a non-hair result for this case [PRD90,224041 (2014); PRL 114, 151102 (2015)]
- Result: to the asymptotic multipolar structure of a spacetime containing black holes, the black holes contribute as if the were isolated Schwarzschild black holes.
- In this way, even though they are distorted by surrounding matter they do not acquire proper hair
- We have given a different version of this result

Deforming a BH from outside

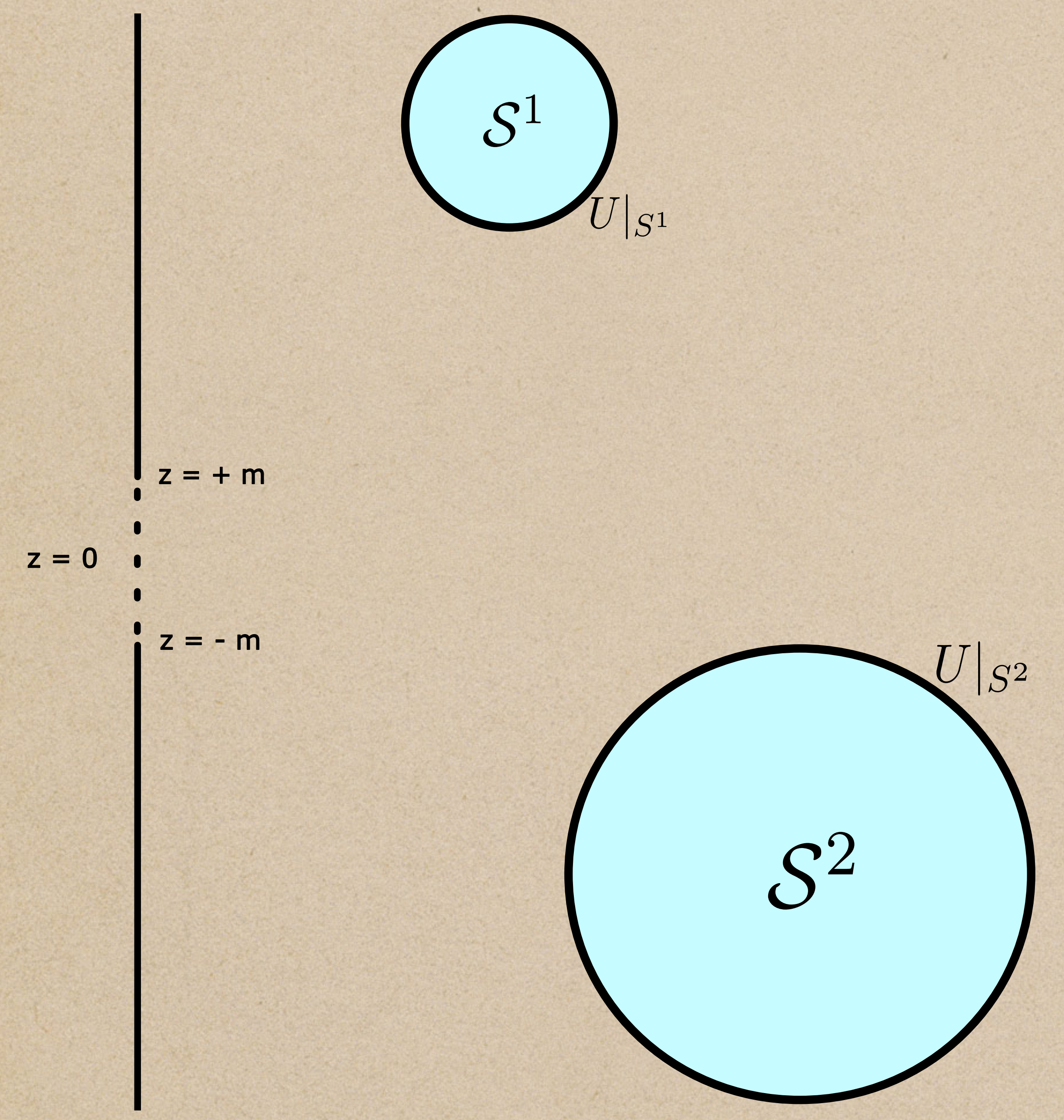
The presence of external sources add a new analytic piece to the potential. This piece is always finite at the horizon.

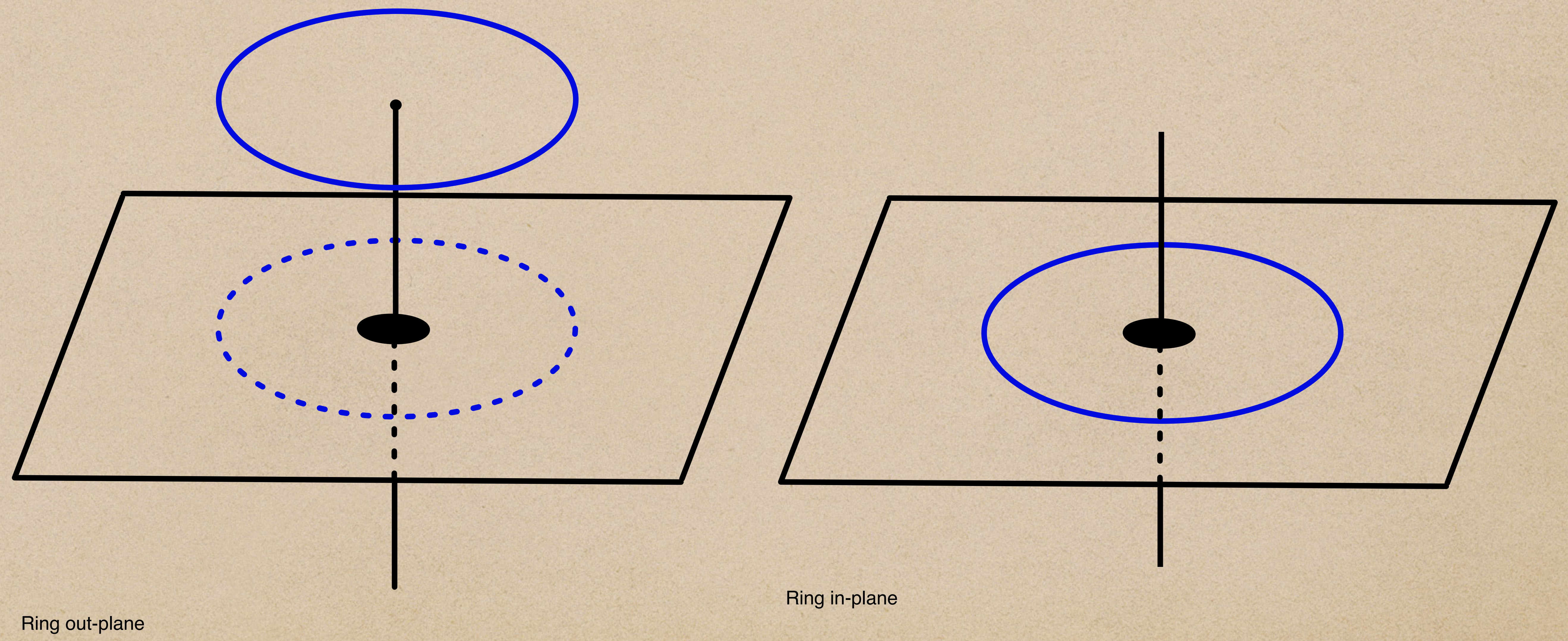
$$U(r, z) = U_{\text{Sch}}(r, z) + U_{\text{Dist}}(r, z)$$

Theorem: Given a regular gravitational environment, there exists only one static, axisymmetric and asymptotically flat geometry containing a black hole which is non-singular. Moreover, the horizon of the black hole will depart from spherical symmetry (the shape when the environment is trivial) in a unique and specific manner that is completely dictated by the environment.

Regular gravitational environment

$$\oint_\gamma \boldsymbol{\omega} = \int_S dzdr\ r\ \partial_z U \nabla^2 U = 0$$



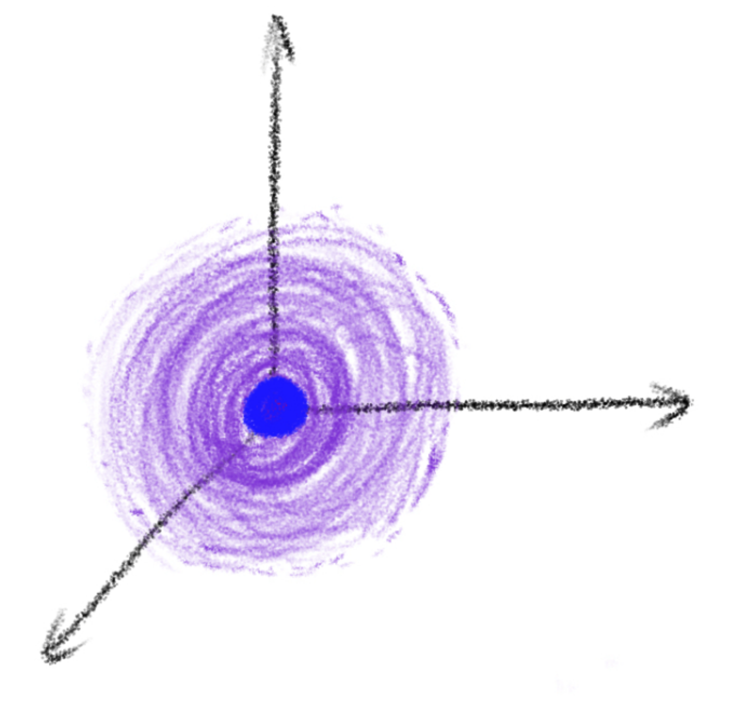


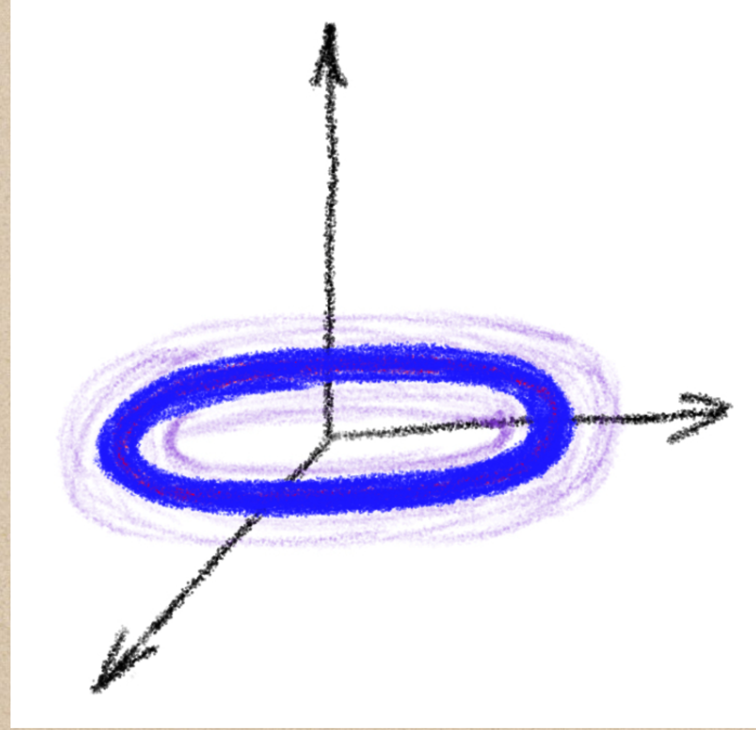
Black holes vs ultracompact objects

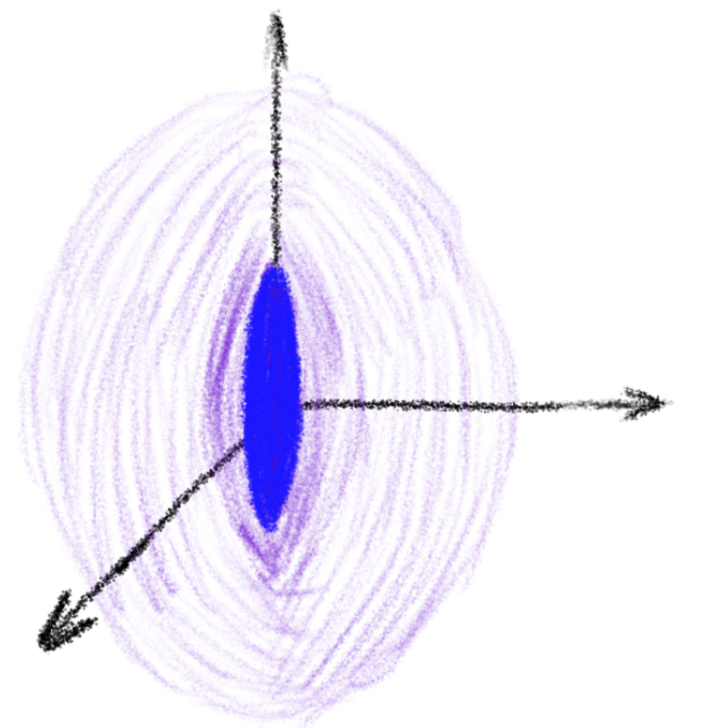
Ultracompact objects

Definition: Objects whose innermost equipotential surface (outside the object itself) has a very small value $\epsilon \ll 1$

- Point-like source
- Ring-like source
- Rod-like source







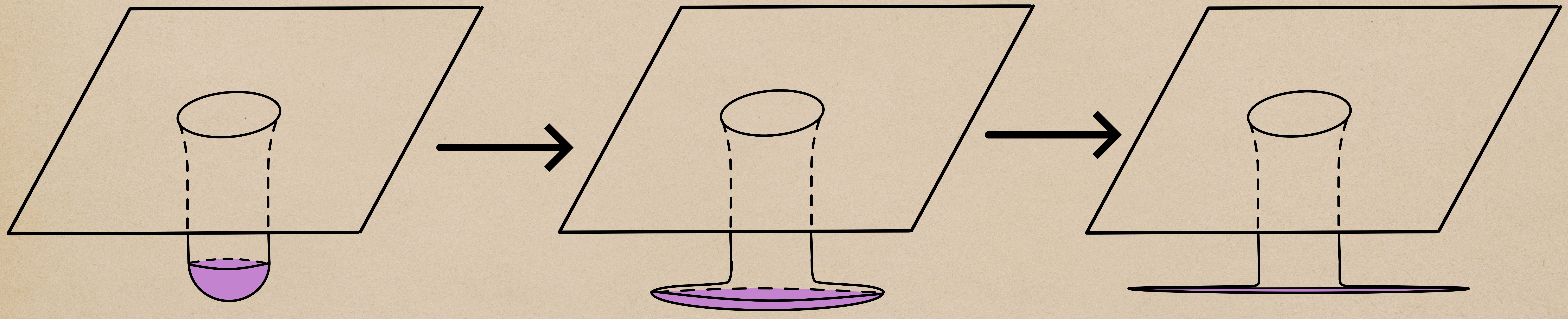
Ultracompact objects with sub-Planckian curvatures

Imposing that an ultracompact object does not exhibit Planckian curvatures results in constraints on the amount of hair they can have

$$\mathscr{K} \leq \mathscr{K}_{\mathrm{P}}$$

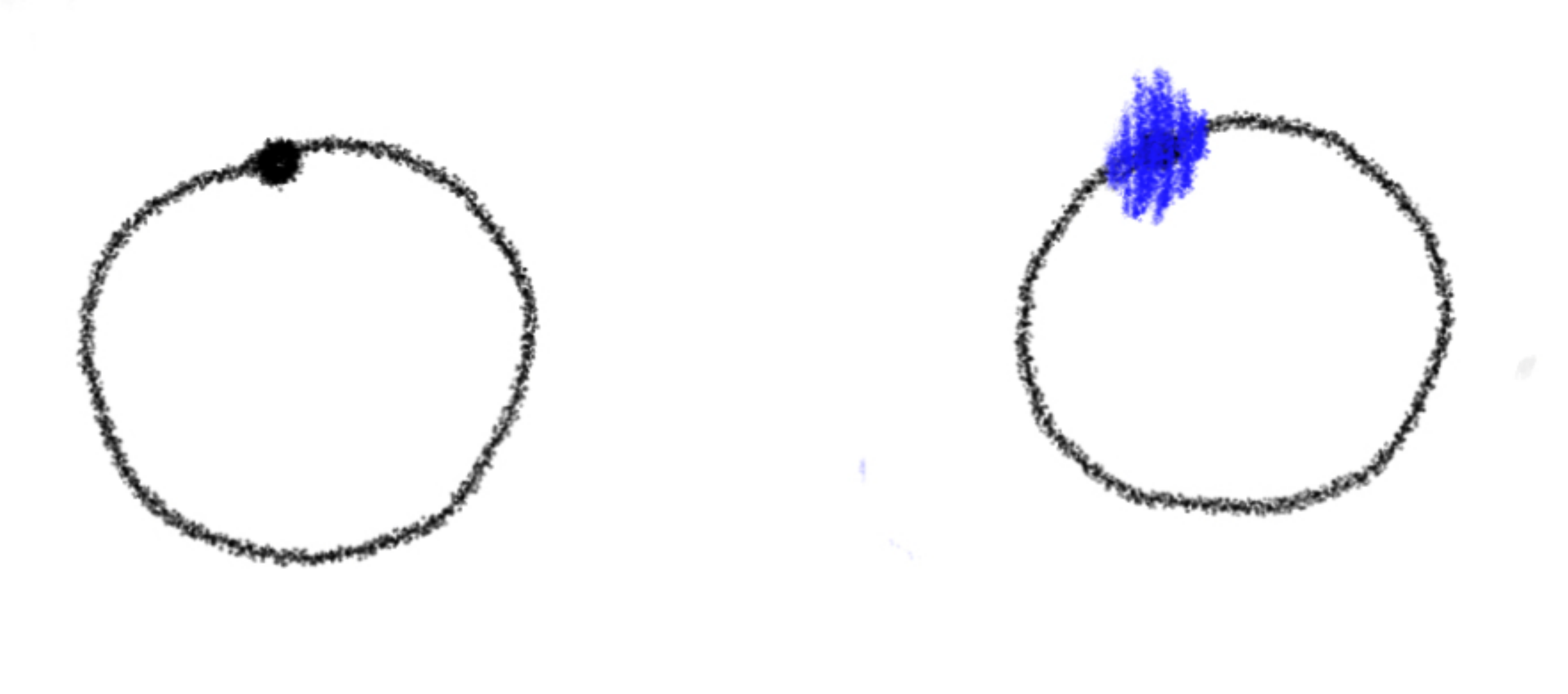
Sub-Planckian-curvature ultracompact objects need to be massive (with respect Planck mass), and in the case of Schwarzschild deformations, very close to spherical.

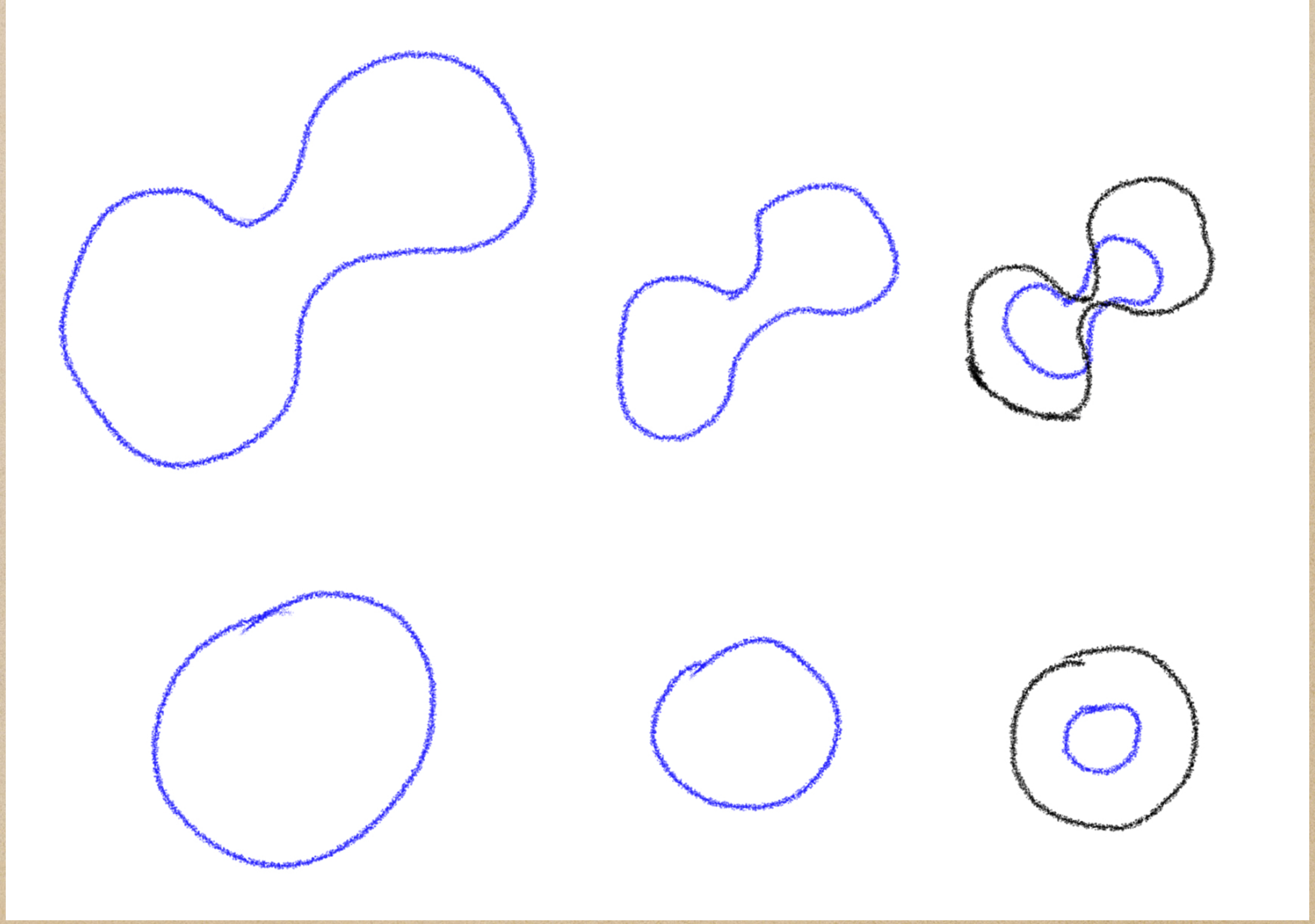
Curzon-like collapse and UC objects



$$\epsilon = \exp\left(-\frac{2M^{(0)}}{R_0}\right) \qquad M^{(0)} = \frac{1}{2}R_0\,|\log\epsilon\,| \qquad \mathscr{K}_{\max} \sim \frac{1}{\mathscr{R}_0^4}\exp\left(\frac{2[M^{(0)}]^2}{R_0^2}\right)$$

$$\frac{\mathscr{R}_0}{\ell_P} \gtrsim \exp\left(\frac{[M^{(0)}]^2}{2R_0^2}\right) = \exp\left(\frac{1}{8}\,|\log\epsilon\,|^2\right) \qquad \frac{R_0}{\ell_P} \gtrsim \frac{1}{\sqrt{2}}\,|\log\epsilon\,|^{3/2}\exp\left(\frac{1}{8}\,|\log\epsilon\,|^2\right)$$





Conclusions and final remarks

- We have cleared up the difference between deforming a black hole from inside vs deforming from outside
- Black holes react to external deformations but cannot be deformed from inside without creating non-regular horizons
- Ultracompact objects have to be also quite hairless. Here there is no sharp distinction between deformations from the outside or from the inside.

Thanks for your attention