# DYNAMICAL FORMATION OF REGULAR BLACK HOLES

#### PABLO BUENO

"BLACK HOLES AND THEIR SYMMETRIES"

TOURS — JULY 2025







#### Based on:

- [PB, Cano, Hennigar] *arXiv*:2403.04827. PLB 861 (2025) 139260
- [PB, Cano, Hennigar, Murcia] arXiv:2412.02740. PRD 111 (2025) 10, 104009
- [PB, Cano, Hennigar, Murcia] arXiv:2412.02742. PRL 134 (2025) 18, 181401
- [PB, Cano, Hennigar, Murcia, Vicente-Cano] arXiv:2505.09680. PRD xxx (2025) xx, xxxxxx
- + work in progress

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  - ⇒ Fundamental question: how do they get resolved?

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- Many models proposed [Sakharov; Bardeen; Poisson, Israel; Dymnikova; Hayward; ...]
  - $\Rightarrow$  Not actual GR solutions: postulated metrics as theoretical test beds...
  - ...In the absence of a dynamical framework, all these ideas are very poorly justified

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All curvature invariants remain finite everywhere (in particular at r = 0).

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- → Highly ad hoc matter required
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   ⇒ regular black holes are not the general solutions of these theories
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- Understanding such effects is in general completely out of reach...

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- First dynamical model of matter collapse leading to the formation of regular black holes!

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- O Quasi-topological theories constructed at curvature orders: n=3 [Oliva, Ray; Myers, Robinson], n=4 [Dehghani, Bazrafshan, Mann, Mehdizadeh, Ghanaatian, Vahidinia], n=5 [Cisterna, Guajardo, Hassaine, Oliva] and  $\forall n$  (and  $\forall D \geq 5$ ) [PB, Cano, Hennigar; Moreno, Murcia].

Let  $W_{abcd}$  denote the Weyl tensor and  $Z_{ab}$  the traceless part of the Ricci tensor, then:

$$\begin{split} & \mathcal{Z}_{(1)} = R \,, \\ & \mathcal{Z}_{(2)} = \frac{1}{(D-2)} \left[ \frac{W_{abcd} W^{abcd}}{D-3} - \frac{4 Z_{ab} Z^{ab}}{D-2} \right] + \frac{Z_{(1)}^2}{D(D-1)} \,, \\ & \mathcal{Z}_{(3)} = \frac{24}{(D-2)(D-3)} \left[ \frac{W_{ac}^{bd} Z_b^2 Z_c^d}{(D-2)^2} - \frac{W_{acde} W^{bcde} Z_b^a}{(D-2)(D-4)} + \frac{2(D-3) Z_b^a Z_c^b Z_a^c}{3(D-2)^3} + \frac{(2D-3) W^{ab}_{\ \ cd} W^{cd}_{\ \ eff} W^{ef}_{\ \ ab}}{12(D((D-9)D+26)-22)} \right] + \frac{3 Z_{(1)} Z_{(2)}}{D(D-1)} - \frac{2 Z_{(1)}^3}{D^2(D-1)^2} \,, \\ & \mathcal{Z}_{(4)} = \frac{96}{(D-2)^2(D-3)} \left[ \frac{(D-1) \left(W_{abcd} W^{abcd}\right)^2}{8D(D-2)^2(D-3)} - \frac{(2D-3) Z_e^f Z_f^e W_{abcd} W^{abcd}}{4(D-1)(D-2)^2} - \frac{2 W_{acbd} W^{cefg} W^d_{\ \ \ efg} Z^{ab}}{D(D-3)(D-4)} \right] \\ & - \frac{4 Z_{ac} Z_{de} W^{bdce} Z_b^a}{(D-2)^2(D-4)} + \frac{(D^2-3D+3) \left(Z_a^b Z_b^a\right)^2}{D(D-1)(D-2)^3} - \frac{Z_a^b Z_b^c Z_c^d Z_d^a}{(D-2)^3} + \frac{(2D-1) W_{abcd} W^{aecf} Z^{bd} Z_{ef}}{D(D-2)(D-3)} \right] + \frac{4 Z_{(1)} Z_{(3)} - 3 Z_{(2)}^2}{D(D-1)} \,, \end{split}$$

$$\begin{split} \mathcal{Z}_{(5)} &= \frac{960(D-1)}{(D-2)^4(D-3)^2} \left[ \frac{(D-2)W_{ghij}W^{ghij}W_{ab}c^dW_{cd}^{ef}W_{ef}^{ef}^{ab}}{40D(D^3-9D^2+26D-22)} + \frac{4(D-3)Z_b^bZ_c^cZ_c^dZ_e^aZ_e^a}{5(D-1)(D-2)^2(D-4)} \right. \\ &\quad - \frac{(3D-1)W^{ghij}W_{ghij}W_{acde}W^{bcde}Z_b^a}{10D(D-1)^2(D-4)} - \frac{4(D-3)(D^2-2D+2)Z_a^bZ_b^aZ_c^dZ_e^dZ_c^e}{5D(D-1)^2(D-2)^2(D-4)} \\ &\quad - \frac{(D-3)(3D-1)(D^2+2D-4)W^{ghij}W_{ghij}Z_c^dZ_e^dZ_c^e}{10D(D-1)^2(D+1)(D-2)^2(D-4)} + \frac{(5D^2-7D+6)Z_g^hZ_g^hW_{abcd}Z^{ac}Z^{bd}}{10D(D-1)^2(D-2)} \\ &\quad + \frac{(D-2)(D-3)(15D^5-148D^4+527D^3-800D^2+472D-88)W_{ab}^{cd}W_{cd}^{ef}W_{ef}^{ab}Z_g^hZ_g^b}{40D(D-1)^2(D-4)(D^5-15D^4+91D^3-277D^2+418D-242)} \\ &\quad - \frac{2(3D-1)Z^{ab}W_{acbd}Z^{ef}W_e^{ef}^gZ_g^d}{D(D^2-1)(D-4)} - \frac{Z_a^bZ_b^cZ_{cd}Z_{ef}W^{eefd}}{(D-1)(D-2)} + \frac{(D-3)W_{acde}W^{bcde}Z_b^aZ_f^2Z_g^f}{5D(D-1)^2(D-4)} \\ &\quad - \frac{(D-2)(D-3)(3D-2)Z_b^aZ_b^cW_{daef}W^{efgh}W_{gh}^{dc}}{(D-1)(D-2)} + \frac{W_{ghij}W^{ghij}Z^{ac}Z^{bd}W_{abcd}}{20D(D-1)^2} \\ &\quad + \frac{5Z_{(1)}Z_{(4)}-2Z_{(2)}Z_{(3)}}{D(D-1)} + \frac{6Z_{(1)}Z_{(2)}^2-8Z_{(1)}^2Z_{(3)}}{D^2(D-1)^2} \,. \end{split}$$

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$$\mathcal{Z}_{(n+5)} = \frac{3(n+3)\mathcal{Z}_{(1)}\mathcal{Z}_{(n+4)}}{D(D-1)(n+1)} - \frac{3(n+4)\mathcal{Z}_{(2)}\mathcal{Z}_{(n+3)}}{D(D-1)n} + \frac{(n+3)(n+4)\mathcal{Z}_{(3)}\mathcal{Z}_{(n+2)}}{D(D-1)n(n+1)} \, .$$

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Here we will go **beyond** the perturbative regime...

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• Hence, N(r,t) = N(t), f(r,t) = f(r) and without loss of generality we can set  $N(t) = 1 \Rightarrow$  our theories satisfy a **Birkhoff theorem**: every spherically symmetric vacuum solution of our theories is also static.

[Oliva, Ray; PB, Cano, Hennigar, Murcia]

• The full non-linear EOM for a general spherically symmetric ansatz reduce to an algebraic equation for f(r):

$$\boxed{\frac{1-f(r)}{r^2} + \sum_{n=2}^{n_{\text{max}}} \alpha_n \frac{(D-2n)}{(D-2)} \left[\frac{1-f(r)}{r^2}\right]^n = \frac{2M}{r^{D-1}}}$$

where M is an integration constant related to the mass.

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$$f(r) \stackrel{r \to 0}{=} 1 - \left(\frac{2\mathsf{M}}{\alpha_{\max}}\right)^{1/n_{\max}} r^{2 - \frac{(D-1)}{n_{\max}}} + \dots$$
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Note that exponent tends to 2 as  $n_{
m max} o \infty$ ...

[PB, Cano, Hennigar]

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- Case 3: Quasi-topological gravity with an infinite number of corrections  $n_{\max} \to \infty$ , dS core emerges and resolves the singularity!
- o This occurs quite generically. Sufficient condition:

$$\alpha_n \geq 0 \,\,\forall \,\, n \,, \quad \lim_{n \to \infty} (\alpha_n)^{\frac{1}{n}} = \mathsf{C} > \mathsf{O}$$

[PB, Cano, Hennigar]

- $\circ$  Case 3: Quasi-topological gravity with an infinite number of corrections  $n_{\max} \to \infty$ , dS core emerges and resolves the singularity!
- This occurs quite generically. Sufficient condition:

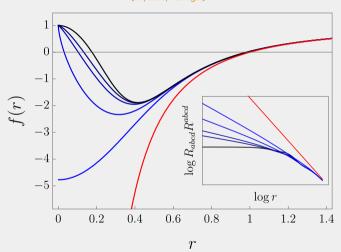
$$\alpha_n \geq 0 \ \forall \ n \ , \quad \lim_{n \to \infty} (\alpha_n)^{\frac{1}{n}} = C > 0$$

Example:

$$\alpha_n = \frac{(D-2)}{(D-2n)} \alpha^{n-1} \quad \Rightarrow \quad f(r) = 1 - \frac{2Mr^2}{r^{D-1} + 2M\alpha}$$
 (Hayward black hole)

## Schwarzschild black hole singularity resolution

[PB, Cano, Hennigar]



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- Our regular black holes are solutions to actual theories, so we can ask now: how do they form?

[PB, Cano, Hennigar, Murcia]

We consider the collapse of a very thin spherical shell of dust

$$T_{ab} = \sigma \delta(\mathbf{r} - \mathbf{R}(\tau)) \mathbf{u}_a \mathbf{u}_b.$$

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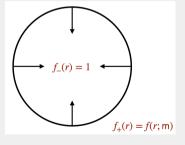
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- $\circ$  Goal: find the trajectory of the shell  $R(\tau)$



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$$m = \int_0^M \frac{\mathrm{d}M'}{\sqrt{\dot{R}^2 + f(R, M')}}$$

#### where:

- 1.  $m \equiv shell's proper mass$
- 2.  $M \equiv \text{spacetime total mass}$
- 3.  $\dot{R} \equiv$  shell's proper velocity
- 4.  $f(R, M) \equiv \text{metric function for a spacetime of mass } M \text{ evaluated at the shell radius}$

o Case 1: Einstein gravity,

$$f(R,M) = 1 - \frac{2M}{R^{D-3}} \Rightarrow \dot{R}^2 + V(R) = \frac{M^2}{m^2} - 1, \quad \left| V(R) = -\frac{M}{R^{D-3}} - \frac{m^2}{2R^{2(D-3)}} \right|$$

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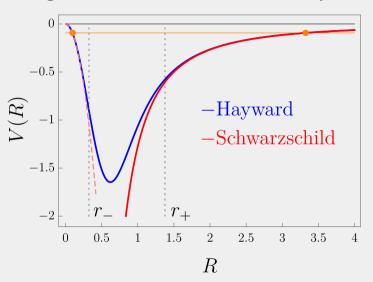
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The potential does not diverge at R = o!



■ A shell that starts collapsing at some finite radius  $R_0 > r_+$  keeps on decreasing its size, eventually giving rise to a regular black hole.

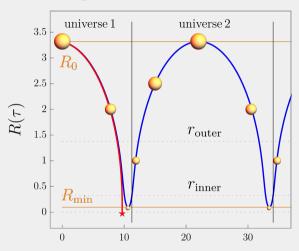
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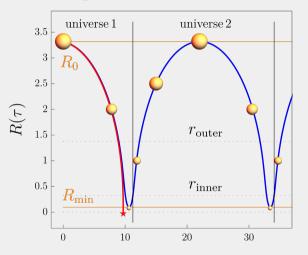
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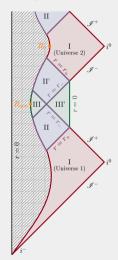
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- The shell grows up to  $r = R_0$ , at which point the process of collapse starts over.







[PB, Cano, Hennigar, Murcia, Vicente-Cano]

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We consider the collapse of a spherical star of pressureless dust

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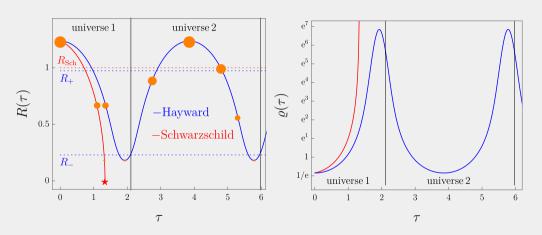
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Cold dark matter bouncing universe model



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- First fully dynamical models in which the collapse of matter leads to the formation of regular black holes!

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