Flavor hierarchies and quark-lepton unification

Based on Greljo, Thomsen, Tiblom; [2406.02687]

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Introduction

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- In the SM, the masses originate from the Yukawa couplings between the fermions and the Higgs
- Naively these couplings should be similar, but instead there is a hierarchical structure between generations
- This is known as the *Flavor Puzzle*

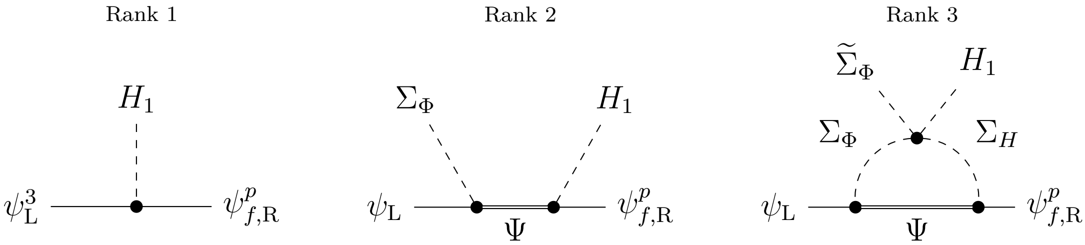

Figure: To provide suitable flavor hierarchies, we make use of three independent rank-one contributions to the Yukawa matrices.

Model Content

Field	$\mathrm{SU}(4)$	$\mathrm{SU}(2)_{\mathrm{L}}$	$\mathrm{U}(1)_{\mathrm{R}}$	$\mathrm{SU}(2)_{q+\ell}$
ψ_{L}	$\mathbf{4}$	$\mathbf{2}$	0	$\mathbf{2}$
ψ_{L}^{3}	$\mathbf{4}$	$\mathbf{2}$	0	$\mathbf{1}$
$\psi_{u,\mathrm{R}}^{p}$	$\mathbf{4}$	$\mathbf{1}$	1/2	$\mathbf{1}$
$\psi_{d,\mathrm{R}}^{p}$	$\mathbf{4}$	$\mathbf{1}$	-1/2	$\mathbf{1}$
$\Psi_{\mathrm{L,R}}$	$\mathbf{4}$	$\mathbf{2}$	0	$\mathbf{1}$
χ	$\mathbf{4}$	$\mathbf{1}$	1/2	$\mathbf{1}$
H_1	$\mathbf{1}$	$\mathbf{2}$	1/2	$\mathbf{1}$
Σ_{H}	$\mathbf{15}$	$\mathbf{2}$	1/2	$\mathbf{1}$
Σ_{Φ}	$\mathbf{15}$	$\mathbf{1}$	0	$\mathbf{2}$

Table: The matter field content of the model and their representations under the gauge group. χ is used to break the gauge group down to $G_{\mathrm{SM}} \times \mathrm{SU}(2)_{q+\ell}$, and Σ_{Φ} is used to break the flavor symmetry.

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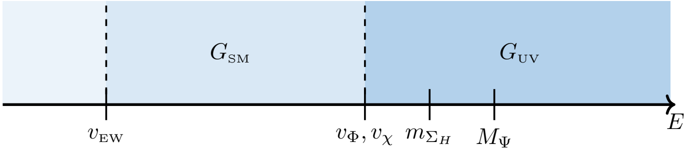

- Rank 1:

$$\mathcal{L}_{\mathrm{UV}} \supset -x_{\mathrm{d}}^{p}\overline{\psi}_{\mathrm{L}}^{3} H_1 \psi_{d,\mathrm{R}}^{p} - X_{\mathrm{d}}^{p}\overline{\psi}_{\mathrm{L}}^{3}\Sigma_H \psi_{d,\mathrm{R}}^{p} + \text{h.c.}$$

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- Rank 2:

$$\mathcal{L}_{\mathrm{UV}} \supset -y_{\mathrm{d}}^{p}\overline{\Psi}_{\mathrm{L}} H_1 \psi_{d,\mathrm{R}}^{p} - Y_{\mathrm{d}}^{p}\overline{\Psi}_{\mathrm{L}} \Sigma_H \psi_{d,\mathrm{R}}^{p} - Y_{\Phi}\overline{\psi}_{\mathrm{L}} \Sigma_{\Phi} \Psi_{\mathrm{R}} + \text{h.c.}$$

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- After integrating out the VLF:

$$\mathcal{L}_{\mathrm{EFT}} \supset \frac{Y_{\Phi}}{M_{\Psi}}\left(y_{\mathrm{d}}^{p}\overline{\psi}_{\mathrm{L}} \Sigma_{\Phi} H_1 \psi_{d,\mathrm{R}}^{p} + Y_{\mathrm{d}}^{p}\overline{\psi}_{\mathrm{L}} \Sigma_{\Phi} \Sigma_H \psi_{d,\mathrm{R}}^{p} + \mathrm{h.c.}\right)$$

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- Rank 3 (calculated with `Matchete`):

$$\mathcal{L}_{\mathrm{EFT}} \supset \frac{15}{32}\frac{1}{16\pi^2}\frac{1}{M_{\Psi}}\left[\log\left(\frac{M_{\Psi}^2}{\mu^2}\right) - 1\right] Y_u^p Y_{\Phi} \lambda_{45}^{*} \,\mathrm{Tr}\left[\widetilde{\Sigma}_H \widetilde{\Sigma}_{\Phi}\right] \overline{\psi}_{\mathrm{L}} \psi_{u,\mathrm{R}}^{p}$$

- Rank 1:

$$\mathcal{L}_{\rm UV} \supset -x_{\rm d}^{p}\overline{\psi}^{3}_{\rm L} H_1 \psi^{p}_{d,\rm R} - X_{\rm d}^{p}\overline{\psi}^{3}_{\rm L}\Sigma_H \psi^{p}_{d,\rm R} + \text{h.c.}$$

- Rank 2:

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- We give VEVs to both Higgses, defining the angle between them as

$$\tan\beta \equiv \frac{v_2}{v_1}\,, \quad v_1^2 + v_2^2 = v_{\rm EW}^2 = (174\,\text{GeV})^2\,.$$

Results

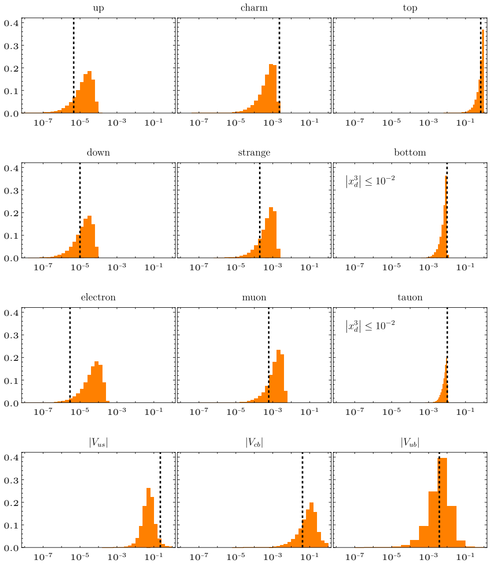

Figure: Histogram showing the probability of obtaining the observed flavor hierarchies when the UV parameters are drawn randomly from a flat distribution between -1 and 1. Only the parameter x_d^3 needs to be fine-tuned.

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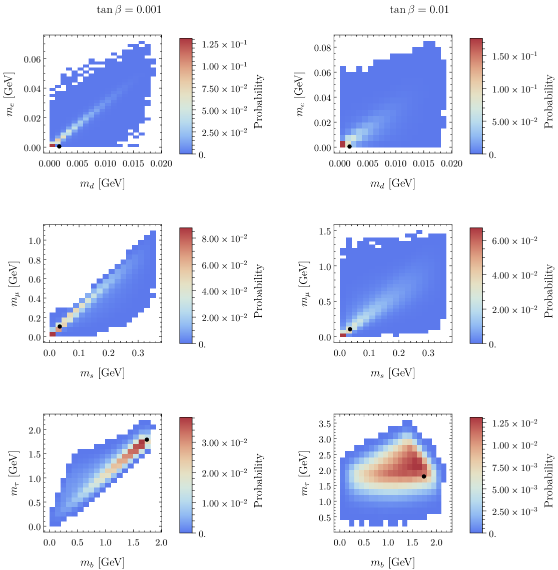

Figure: Correlation between the masses of the down quarks and charged leptons for two different values of β.

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- This model also predicts a strong correlation between the masses of down-type quarks and charged leptons
- The model allows for small neutrino masses through the use of the inverse seesaw mechanism, but to predict large PMNS mixing angles additional structure is needed

Phenomenology

- In Pati-Salam type models vector leptoquarks contribute to $K_L \to \mu e$
- This constrains the breaking scale to a few PeV
- Muon conversion on heavy nuclei also places bounds on the model
- Best present bound is from SINDRUM-II for the process $\mu \, \mathrm{Au} \to e \, \mathrm{Au}$
- Is expected to improve with future experiments such as Mu2e and COMET