

Toponium physics at the LHC

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- **PRD 104 (2021) 034023**
- **EPJC 85 (2025) 157**

59th Rencontres de Moriond-EW

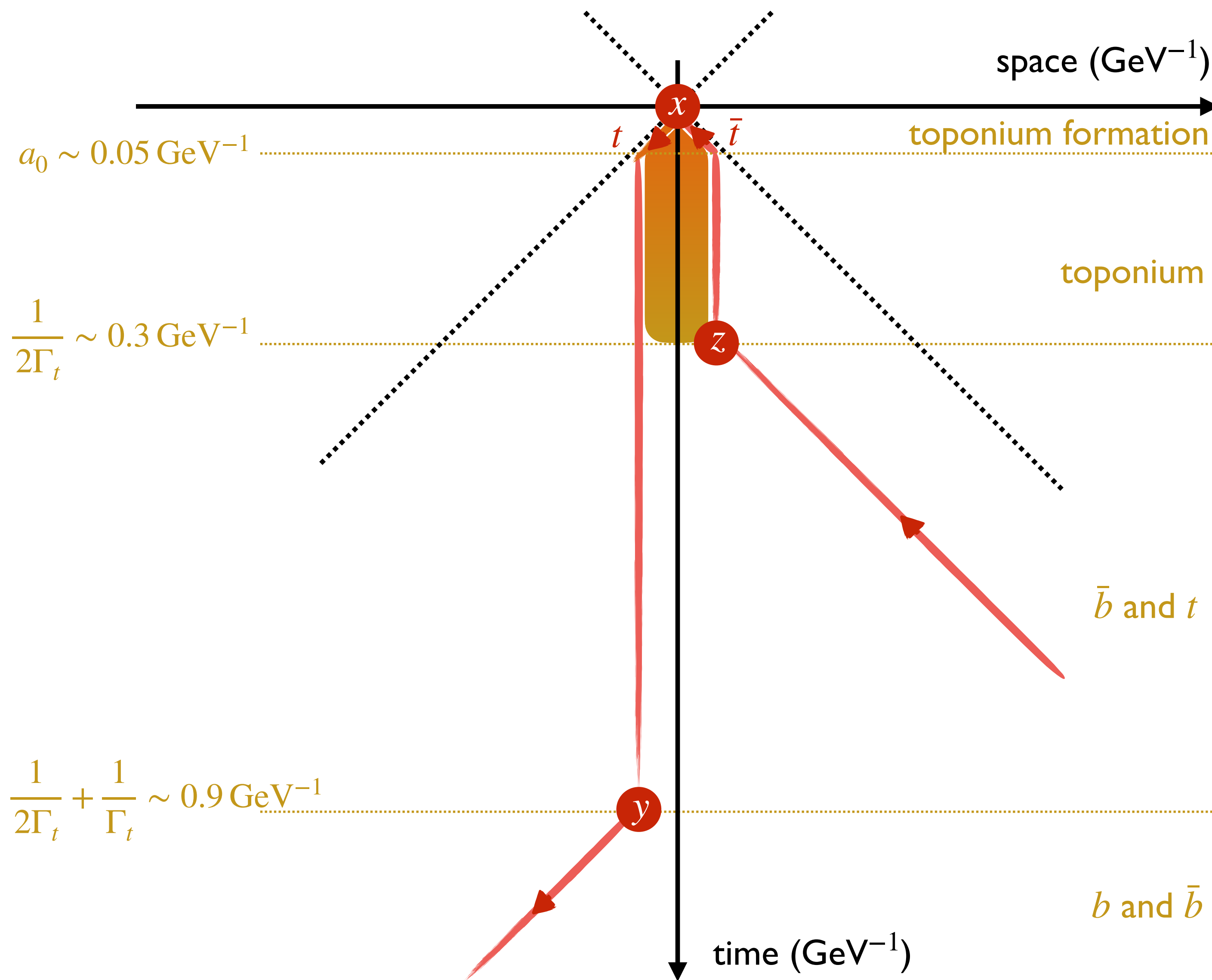
23-30 March 2025, La Thuile (Italia)

Top-antitop production near threshold

[Fadin & Khoze (JETP Lett`87)]

[Fadin, Khoze & Sjöstrand (Z.Phys.C`90)]

[Sumino, Fujii, Hagiwara, Murayama & Ng (PRD`93)]



A position-space picture

- Two-particle state created at $x = (t_0, \vec{x})$
 \approx wave packet propagation to the V_{QCD} barrier
 \rightarrow Typical scale: the Bohr radius

$$a_0 = \frac{1}{C_F \alpha_s m_t / 2} \sim (20 \text{ GeV})^{-1}$$
- Oscillations within the barrier until system decay
 \rightarrow Top [$y = (t_1, \vec{y})$] or antitop [$z = (t_2, \vec{z})$] decay
 \rightarrow Typical scale $\approx (2\Gamma_t)^{-1} \sim (3 \text{ GeV})^{-1}$
- **Probe of the QCD potential**
 \rightarrow Toponium effects
 \rightarrow Currently not included in MC simulations

The toponium Green's function

Three-point correlation function in the non-relativistic limit

$$\begin{aligned}
 K_{abcd}(x, y, z) &= \langle 0 | T \{ t_c(y) \bar{t}_d(z) : \bar{t}_a(x) t_b(x) : \} | 0 \rangle \\
 &= \frac{(1 + \gamma^0)_{ca}}{2} \frac{(1 - \gamma^0)_{bd}}{2} \int d^3 r \left[K_1(y; (z^0, \vec{r})) K_2(z^0, \vec{r}, \vec{z}; x^0, \vec{x}, \vec{x}) + K_1(z; (y^0, \vec{r})) K_2(y^0, \vec{y}, \vec{r}; x^0, \vec{x}, \vec{x}) \right]
 \end{aligned}$$

Antitop-decay first
Top-decay first

↑
↑

←
←
→
→

Non-relativistic spin projection operators
1-particle-state and 2-particle-state propagators

The toponium Green's function (from K_2)

- **Solution to the Lippmann-Schwinger equation**
 - Fourier transform of the QCD potential
 - S-wave contributions
- To be solved numerically

$$\widetilde{G}(E; p) = \widetilde{G}_0(E; p) + \int \frac{d^3 q}{(2\pi)^3} \widetilde{V}_{\text{QCD}}(\vec{p} - \vec{q}) \widetilde{G}(E; q)$$

↑
 Free Green's
 function

[Jezabek, Kuhn & Teubner (Z.Phys.C'92)]

Green's function ratio as a seed for toponium modelling

- Valid near threshold → non-relativistic matrix elements
- Conventional MC generators: standard matrix elements
 - Assumption 1: **relativistic effects** negligible
 - Assumption 2: **higher-spin contributions** negligible
- **Matrix-element re-weighting**
 - Colour singlet projection
 - Validity: $E < 4 \text{ GeV}$ and $p^* < 50 \text{ GeV}$

$$i\mathcal{M} \rightarrow i\mathcal{M} \times \frac{\widetilde{G}(E; p^*)}{\widetilde{G}_0(E; p^*)}$$

$t\bar{t}$ production near threshold with MG5aMC

Process: $gg \rightarrow t\bar{t} \rightarrow b\ell^+\nu_\ell \bar{b}\ell'^-\bar{\nu}'_\ell$

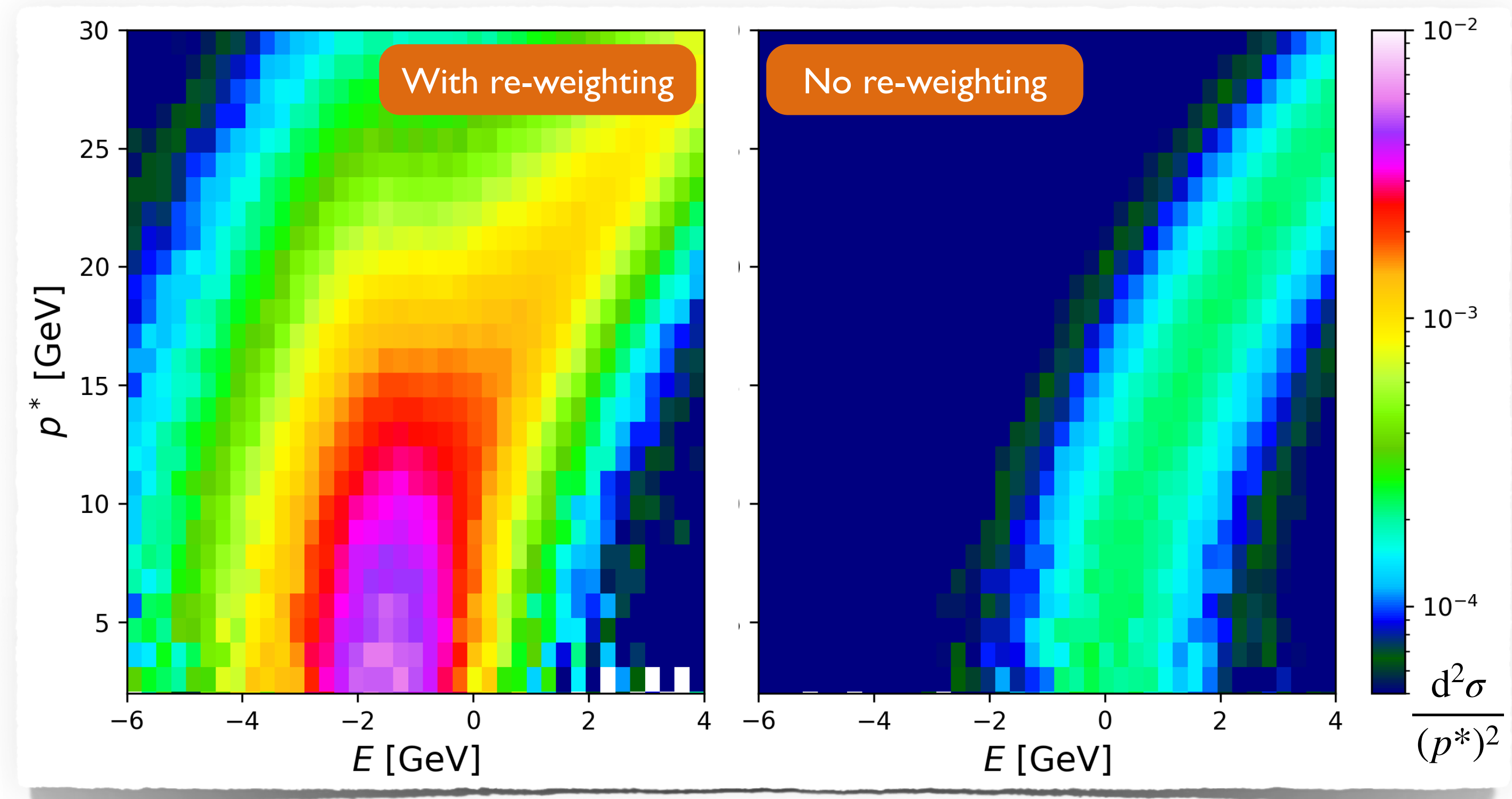
- Six-body final state: spin correlations included
- Projection on the colour singlet
- Matrix-elements with and without re-weighting
- No matching with parton showers

Without re-weighting

- Rates \equiv phase space \times Breit-Wigner
- Similar heat map as for \widetilde{G}_0

With re-weighting

- Similar heat map as for \widetilde{G}
- Normalisation and shape affected
- Access to the QCD Green's function
 \rightarrow ratio of the re-weighted/non-re-weighted predictions



Typical top momentum in the toponium rest frame

$$\langle p(E) \rangle = \frac{\int p d^3p \frac{d\sigma}{p^2 dp dE}}{\int d^3p \frac{d\sigma}{p^2 dp dE}} \quad \leftrightarrow \text{for } E \simeq -2 \text{ GeV: } 20 \text{ GeV (the Bohr radius!)}$$

[Hagiwara, BF, Ma & Zheng (EPJC'25)]

Invariant masses: tops & toponium

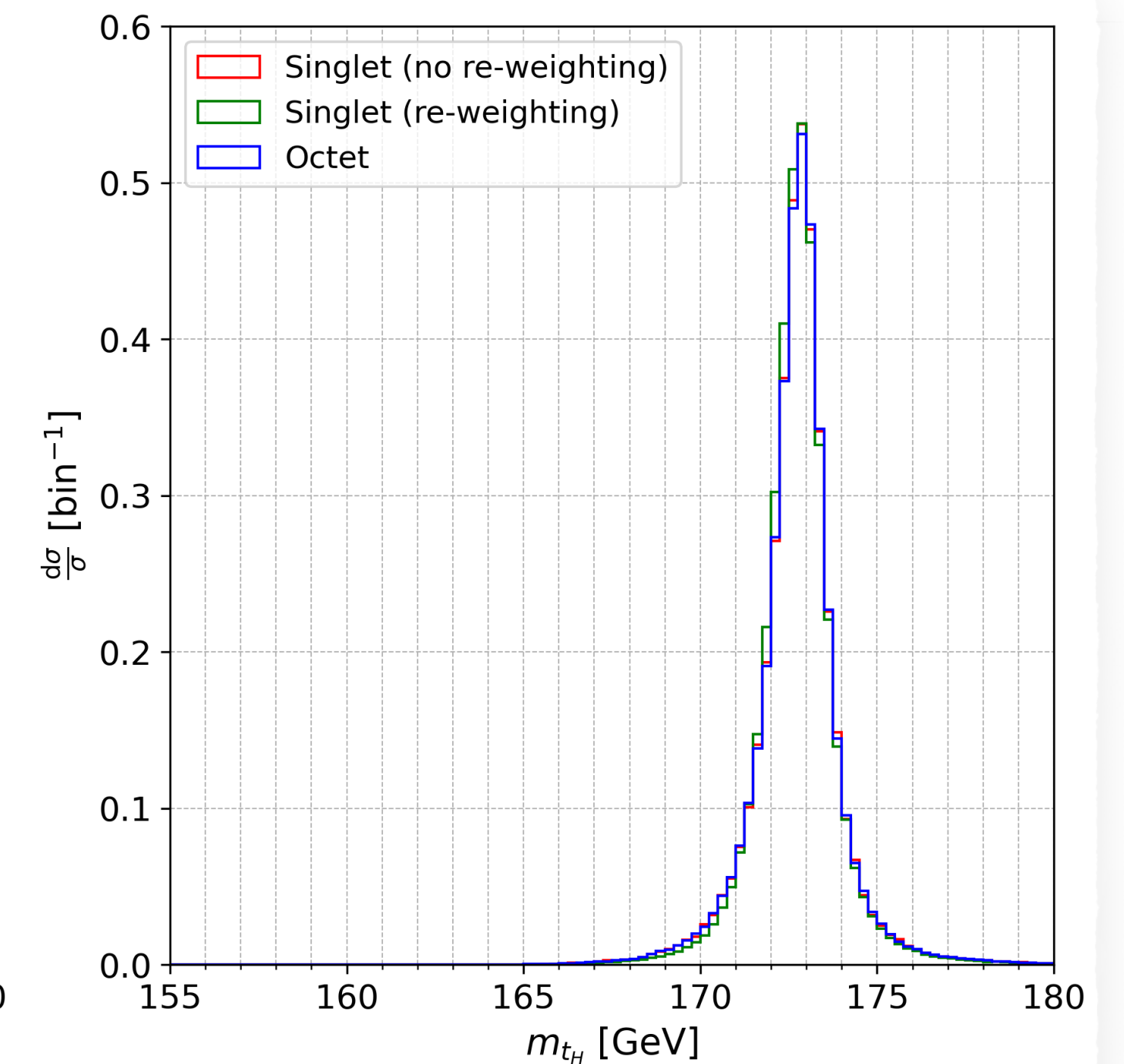
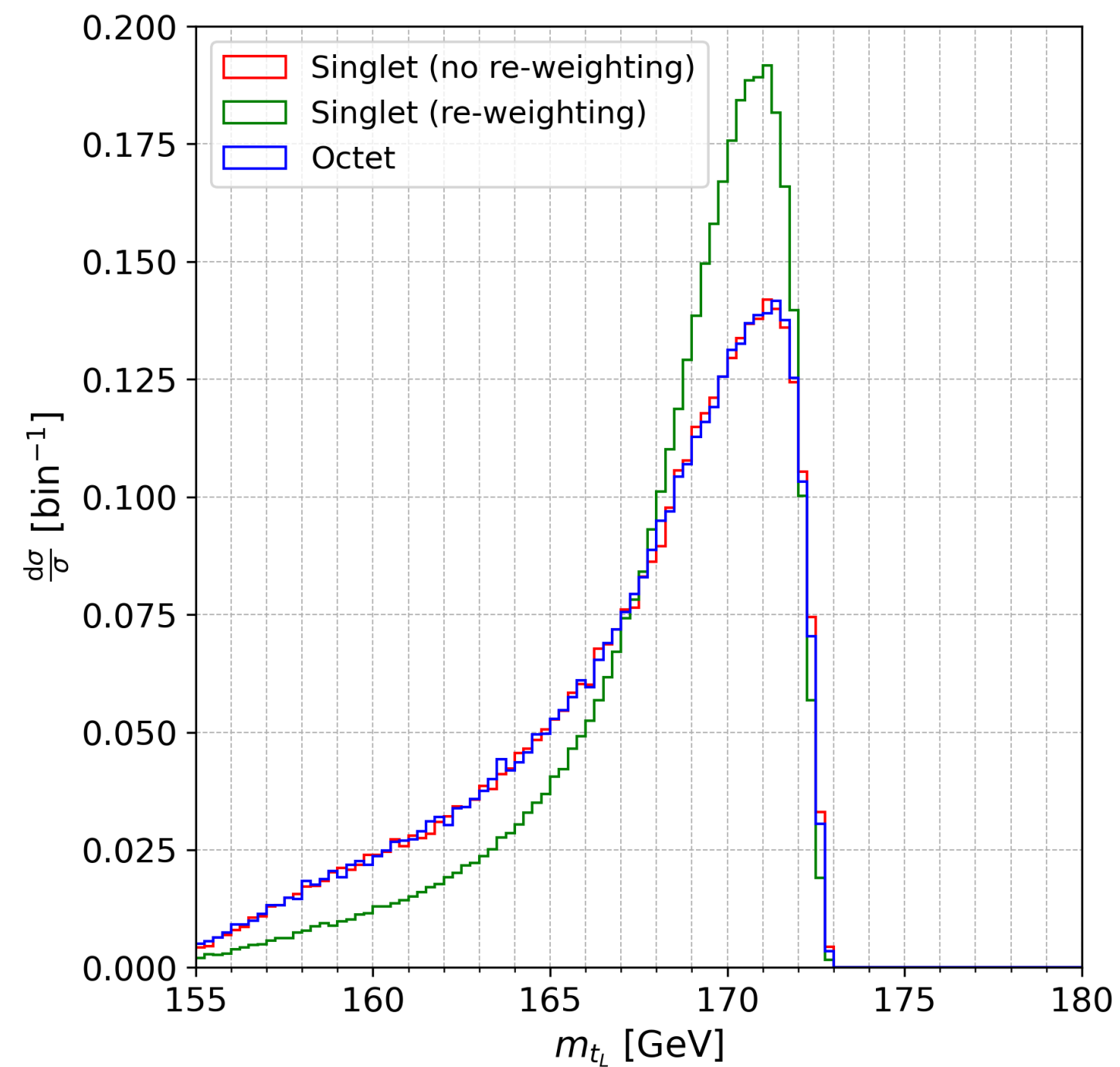
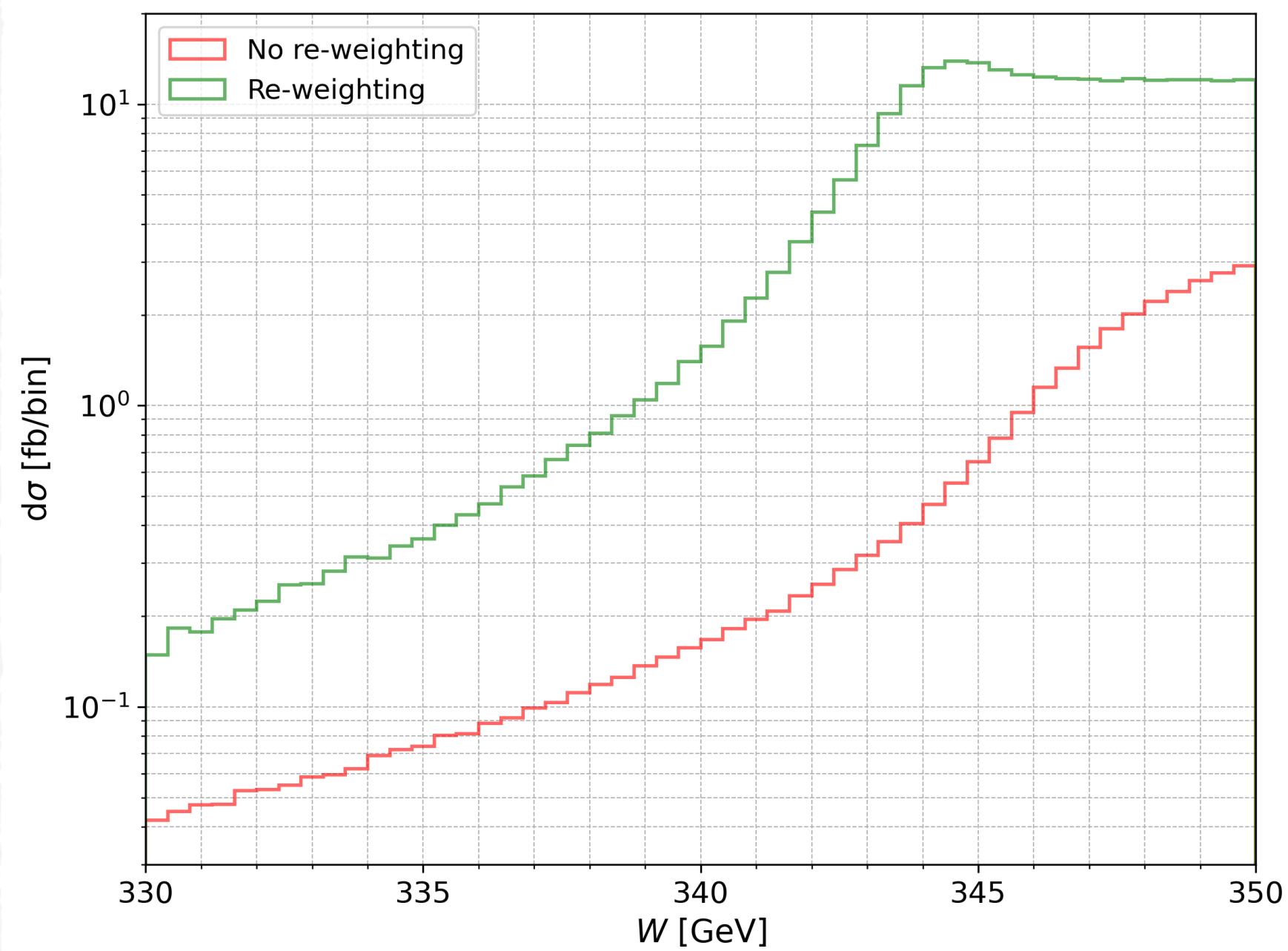
$m_{t\bar{t}}$ invariant mass distribution

- Peak at $E \simeq -2 \text{ GeV} \oplus$ extended bound state effects
- Shape in agreement with pNRQCD
- **!** Comparison (relativistic pieces, octet, α_s , PDFs, etc.)

[Sumino & Yokoya (JHEP'10)]

Top mass distributions

- Light top \leftrightarrow invariant mass shifted to lower values
 - \rightarrow Governed by the QCD Green's function
 - \rightarrow Distortion of the Breit-Wigner
 - \rightarrow Bound state effects within the Coulomb potential generated by t_H
- Heavy top \leftrightarrow Breit-Wigner
 - \rightarrow Effectively stable until t_L decays
 - \rightarrow No QCD potential from the decayed top (free propagation)



[Hagiwara, BF, Ma & Zheng (EPJC'25)]

Comparison with pseudo-scalar toy models

A toponium toy Lagrangian with a pseudo-scalar state

$$\mathcal{L}_{\eta_t} = \frac{1}{2} \partial_\mu \eta_t \partial^\mu \eta_t - \frac{1}{2} m_{\eta_t}^2 \eta_t^2 - \frac{1}{4} g_{gg} \eta_t G_a^{\mu\nu} \tilde{G}_{\mu\nu}^a - i g_{tt} \eta_t \bar{t} \gamma_5 t$$

- No free parameters

$$\sigma(\eta_t) \equiv \int_{-8 \text{ GeV}}^{4 \text{ GeV}} dE \left(\frac{d\sigma_{\text{full}}}{dE} - \frac{d\sigma_{\text{NLO}}}{dE} \right) \rightarrow \sigma_\eta(13 \text{ TeV}) \approx 6.5 \text{ pb}$$

→ To be compared with $\sigma_{t\bar{t}} \approx 810 \text{ pb}$

(Non-perfect) Breit-Wigner fit → $m_{\eta_t} \simeq 344 \text{ GeV}$; $\Gamma_{\eta_t} \simeq 7 \text{ GeV}$

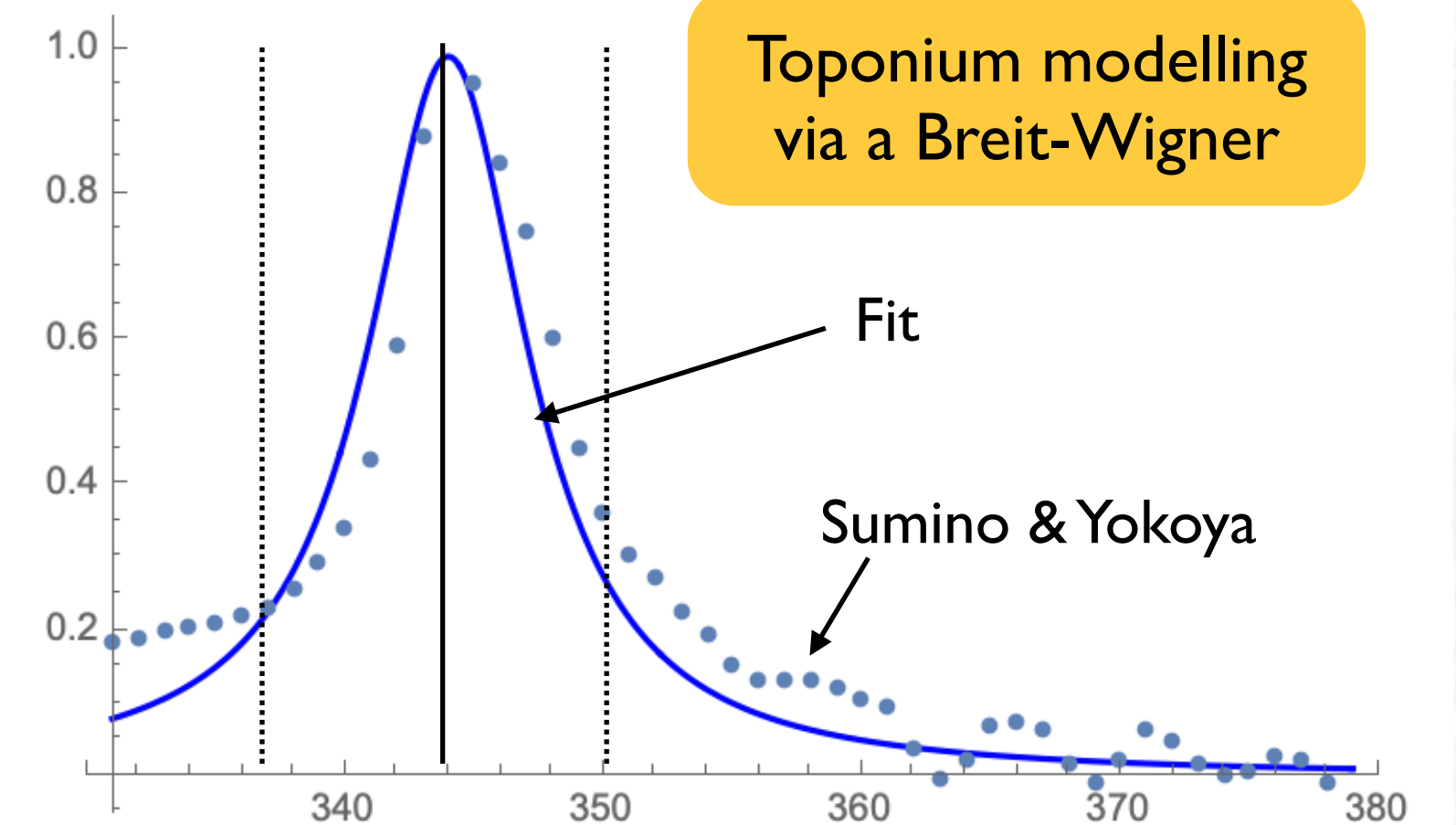
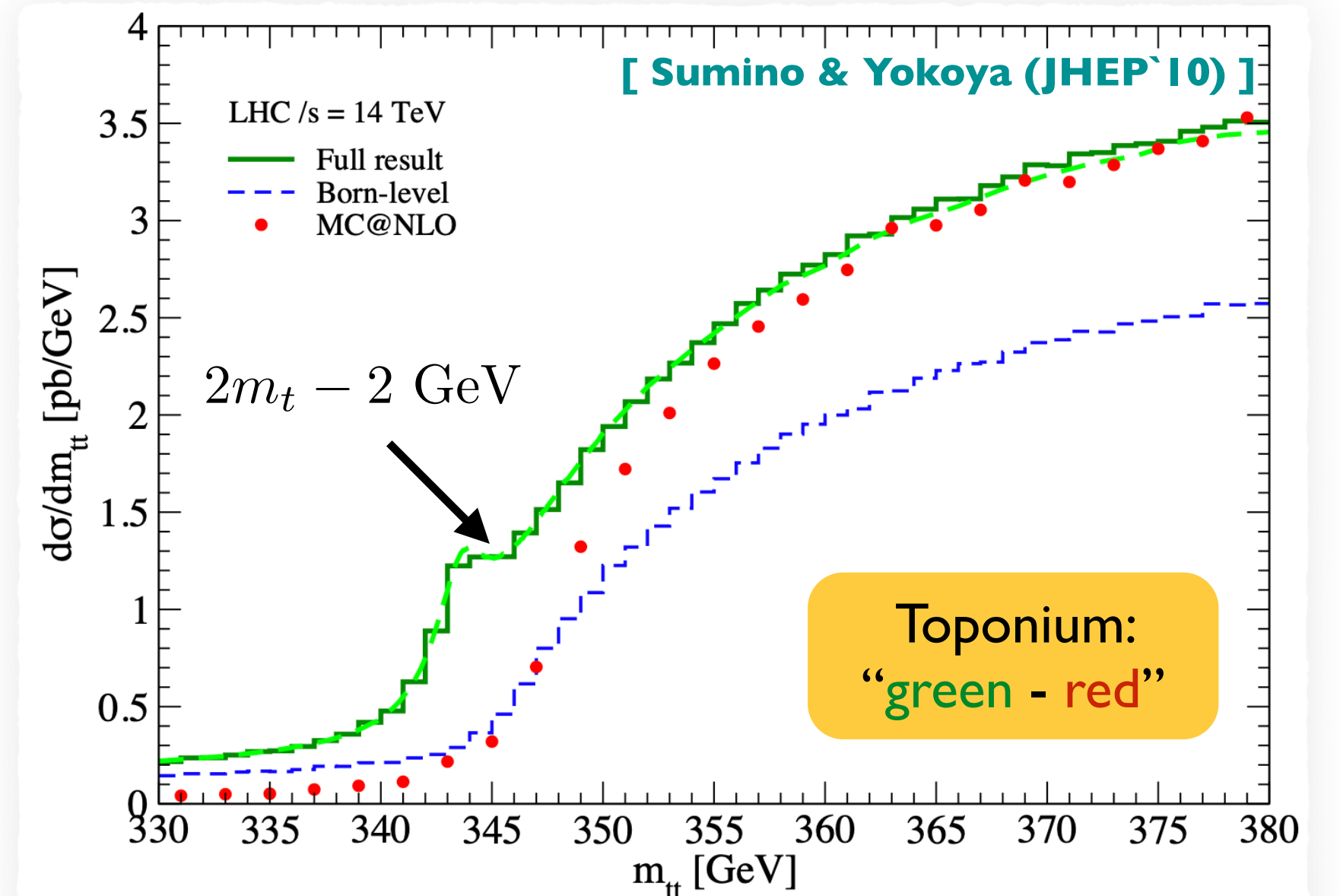
⊕ Green's functions re-weighting

⊕ cut on the $t\bar{t}$ invariant mass

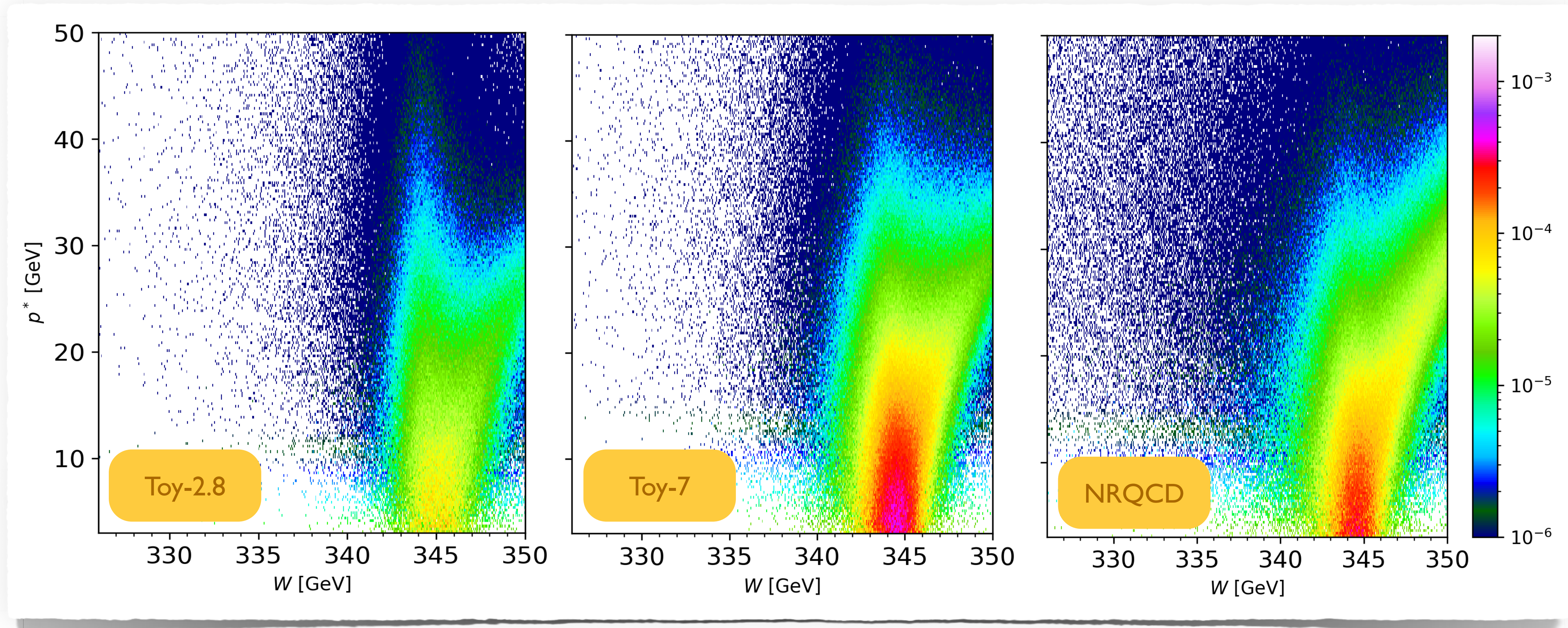
[Hagiwara, BF, Ma & Zheng (PRD'21)]

A ground-state toponium toy Lagrangian with a pseudo-scalar state

- Only capturing the vicinity of the ground state
 - OK for $E \lesssim -0.5 \text{ GeV}$
- Radial excitations and P-wave states in $-0.5 \text{ GeV} \leq E \leq 0 \text{ GeV}$
 - Not in Green's function (yet)
 - Not captured in the ground-state approximation
- Mixed contributions around threshold ($\Gamma_{\eta_t} \simeq 3 \text{ GeV}$)
- Fit for $E < 0$ only: $\sigma_\eta(13 \text{ TeV}) \approx 4.4 \text{ pb}$



Toy models versus NRQCD: $d^2\sigma/(p^*)^2$



Imperfections of the toy model visible in the key distribution

- The W spectrum seems OK in all cases
 - only the peak is well reproduced...
- Strong distortion with the p^* distribution

Toy models should not be used anymore!

Towards discovery: spin density matrices

Di-leptonic signal properties from $t\bar{t}$ production and t/\bar{t} decay density matrices

- Distribution of the angles of the leptons in their parent top rest frames

$$\sum_{\sigma, \bar{\sigma}, \sigma', \bar{\sigma}'} \rho_{gg \rightarrow (t\bar{t})_1}^{\sigma\bar{\sigma}, \sigma'\bar{\sigma}'} d\rho_{\sigma, \sigma'}^{t \rightarrow b\ell^+\nu_\ell} d\rho_{\bar{\sigma}', \bar{\sigma}}^{\bar{t} \rightarrow \bar{b}\ell'^-\bar{\nu}_{\ell'}} \Rightarrow \frac{d\Gamma}{d\cos\theta d\varphi d\cos\bar{\theta} d\bar{\varphi}} \propto \frac{1 + \sin\bar{\theta} \sin\theta \cos(\bar{\varphi} - \varphi) + \cos\bar{\theta} \cos\theta}{2}$$

[Hagiwara, Yokoya & Zheng (JHEP'18)]

- Toponium characteristics: **small $\Delta\varphi_{\ell\ell'}$, small $|\Delta\eta_{\ell\ell'}|$ and small $m_{\ell\ell'}$**

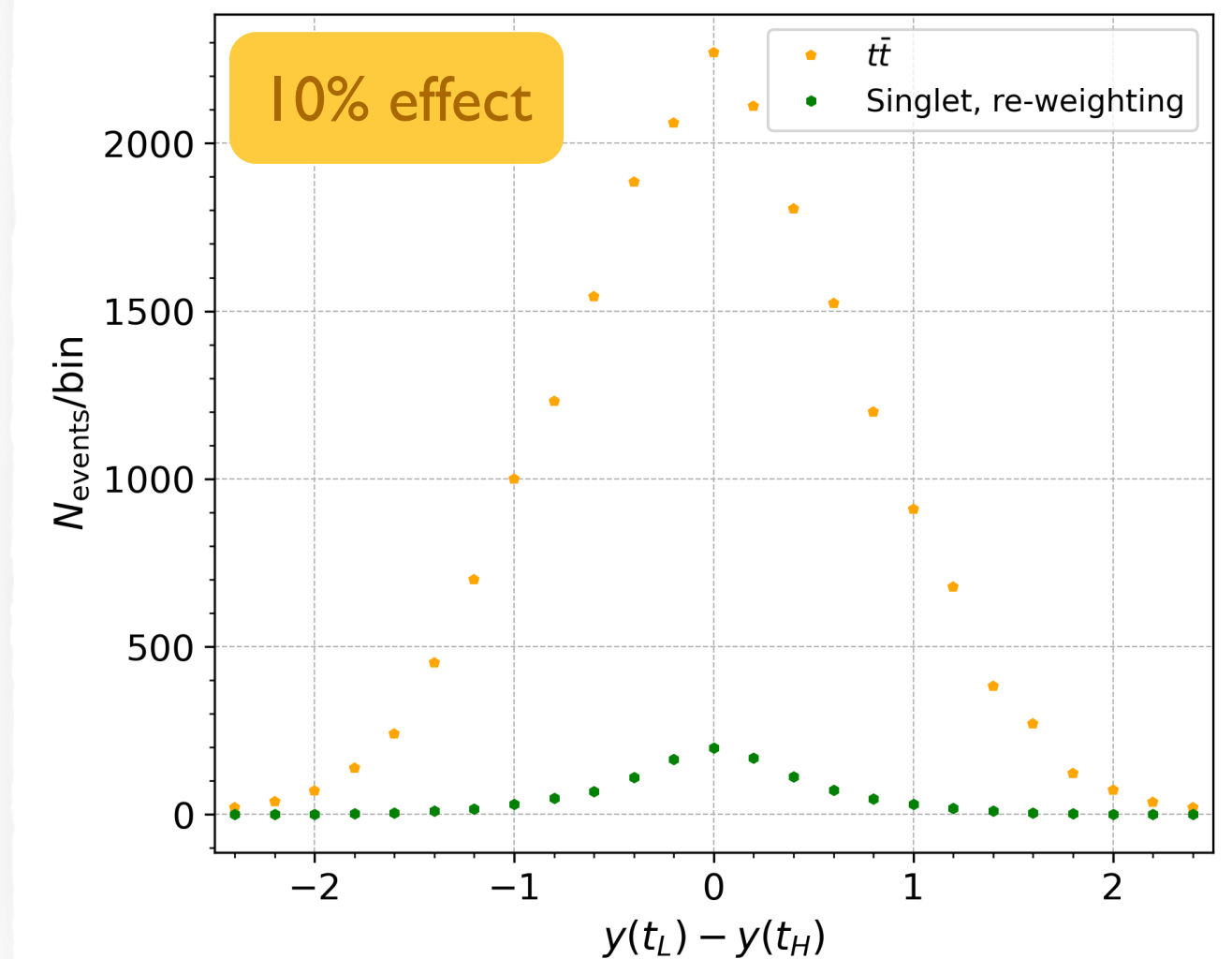
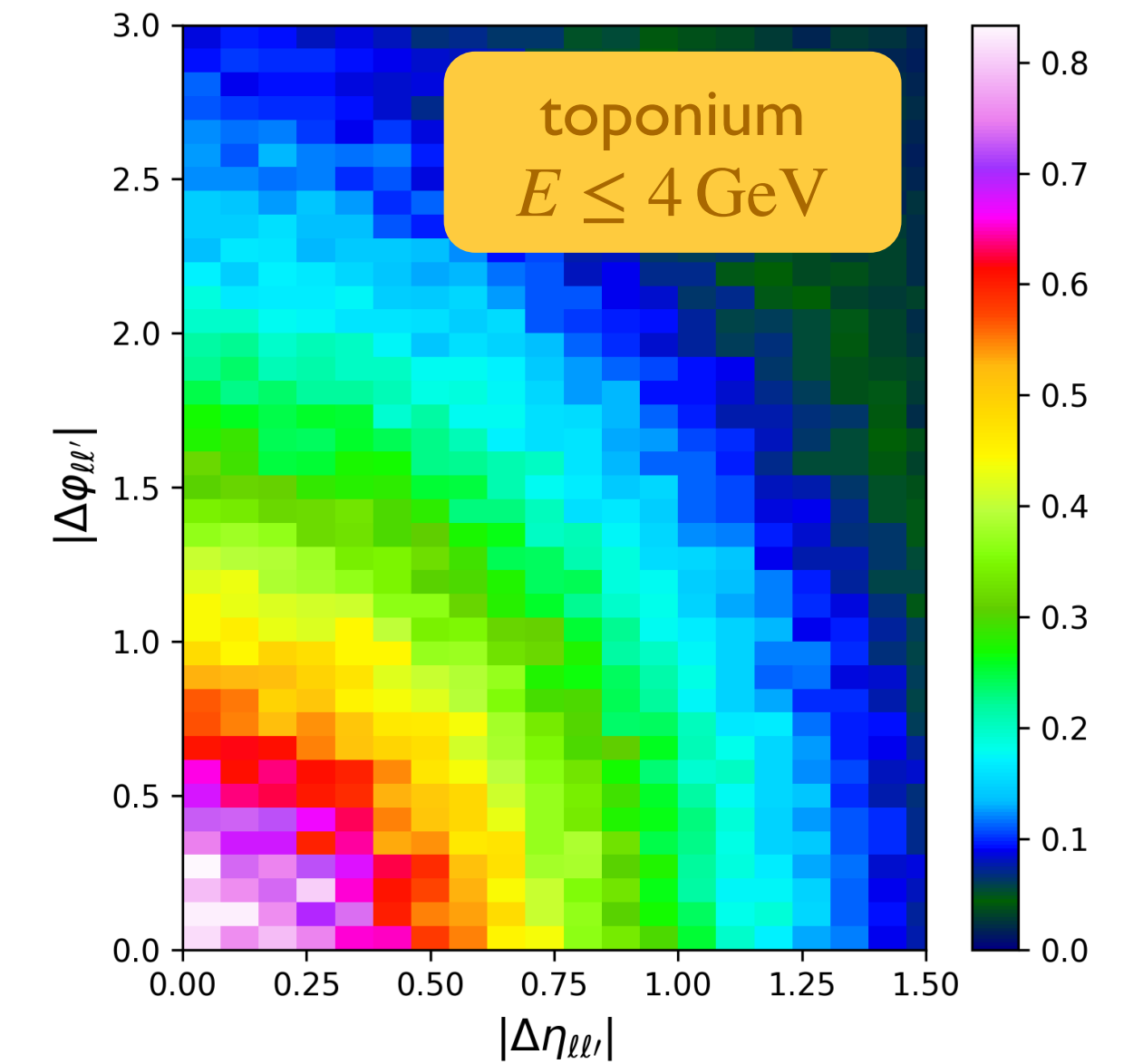
→ Small azimuthal angle separation (survives the lab frame boost)

→ Small di-lepton invariant mass
(cf. tension with data?)

$$m_{\ell\ell'}^2 = 2E_{\bar{\ell}}E_{\ell'} \left(1 - \sin\bar{\theta} \sin\theta \cos(\bar{\varphi} - \varphi) - \cos\bar{\theta} \cos\theta \right)$$

Bulk of toponium events

- **Option A:** $\Delta\varphi_{\ell\ell'} < 0.9$; $\Delta\eta_{\ell\ell'} < 0.9$
→ $N_{t\bar{t}} = 127,000$; $N_{\text{toponium}} = 3,520$ [sensitivity of 9.7σ (S/B \sim 2.8%)]
- **Option B:** $\Delta\varphi_{\ell\ell'} < \pi/5$; $m_{\ell\ell'} < 40$ GeV
→ $N_{t\bar{t}} = 77,100$; $N_{\text{toponium}} = 2,200$ [sensitivity of 7.8σ (S/B \sim 2.9%)]
- **Option C:** $\Delta R_{\ell\ell'}^2 < 0.8$
→ $N_{t\bar{t}} = 99,400$; $N_{\text{toponium}} = 2,980$ [sensitivity of 9.3σ (S/B \sim 3%)]



A few last words...

Top-antitop production near the threshold

- Emergence of a toponium system at a scale of 0.05 GeV^{-1}
- Decay at a time scale of $\sim 0.3 \text{ GeV}^{-1}$
- Occurs well before hadronisation at 5 GeV^{-1}

Possibility to probe the toponium wave function

- 'The smallest hadron in the SM'
- The story just begins...

To-do list

- Higher spins
- Colour octet (no resonant enhancement)
- Matching NRQCD to NLO-QCD + parton showers
- Impact on top mass measurements

