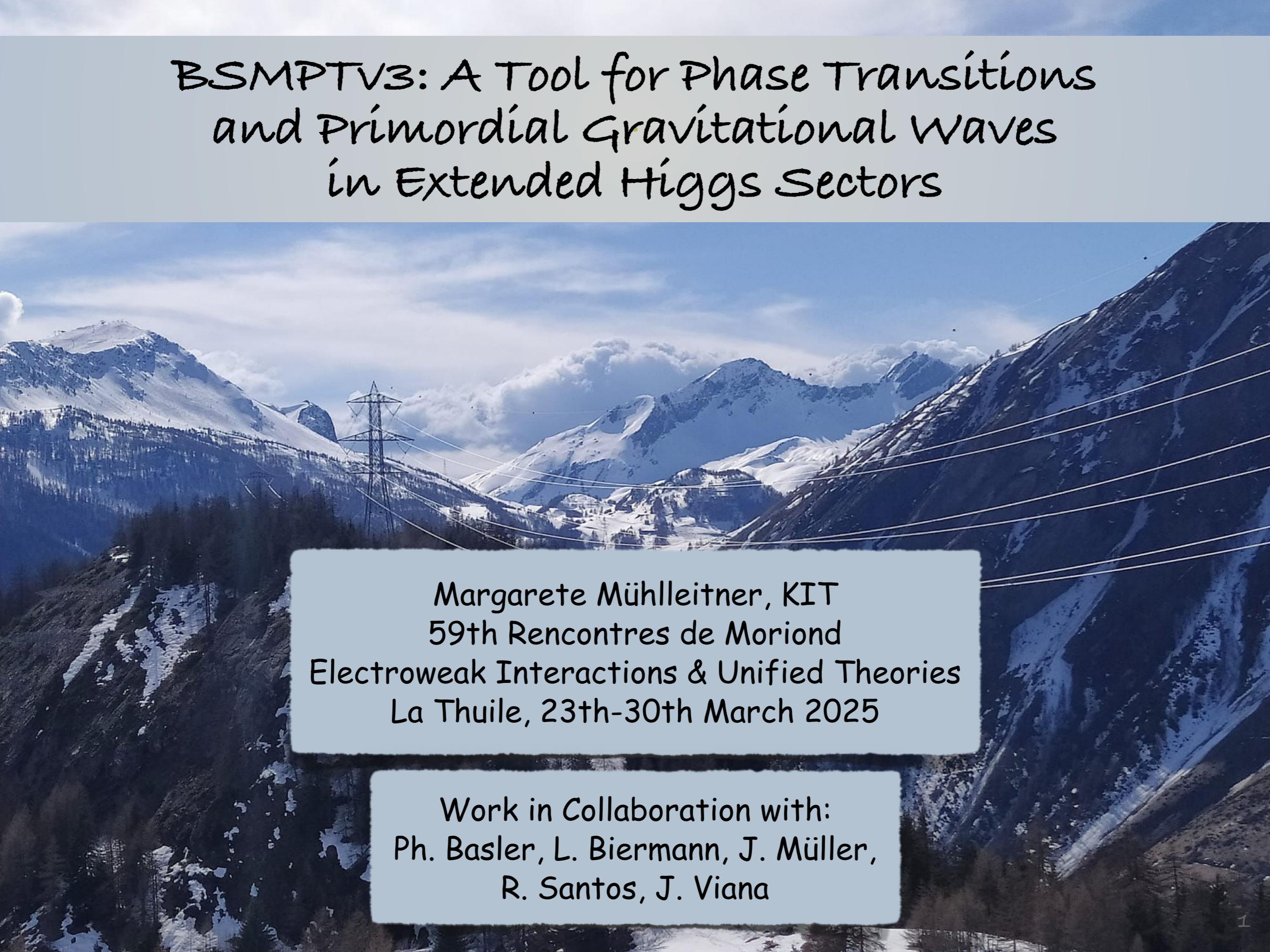


BSMPTV3: A Tool for Phase Transitions and Primordial Gravitational Waves in Extended Higgs Sectors



Margarete Mühlleitner, KIT
59th Rencontres de Moriond
Electroweak Interactions & Unified Theories
La Thuile, 23th-30th March 2025

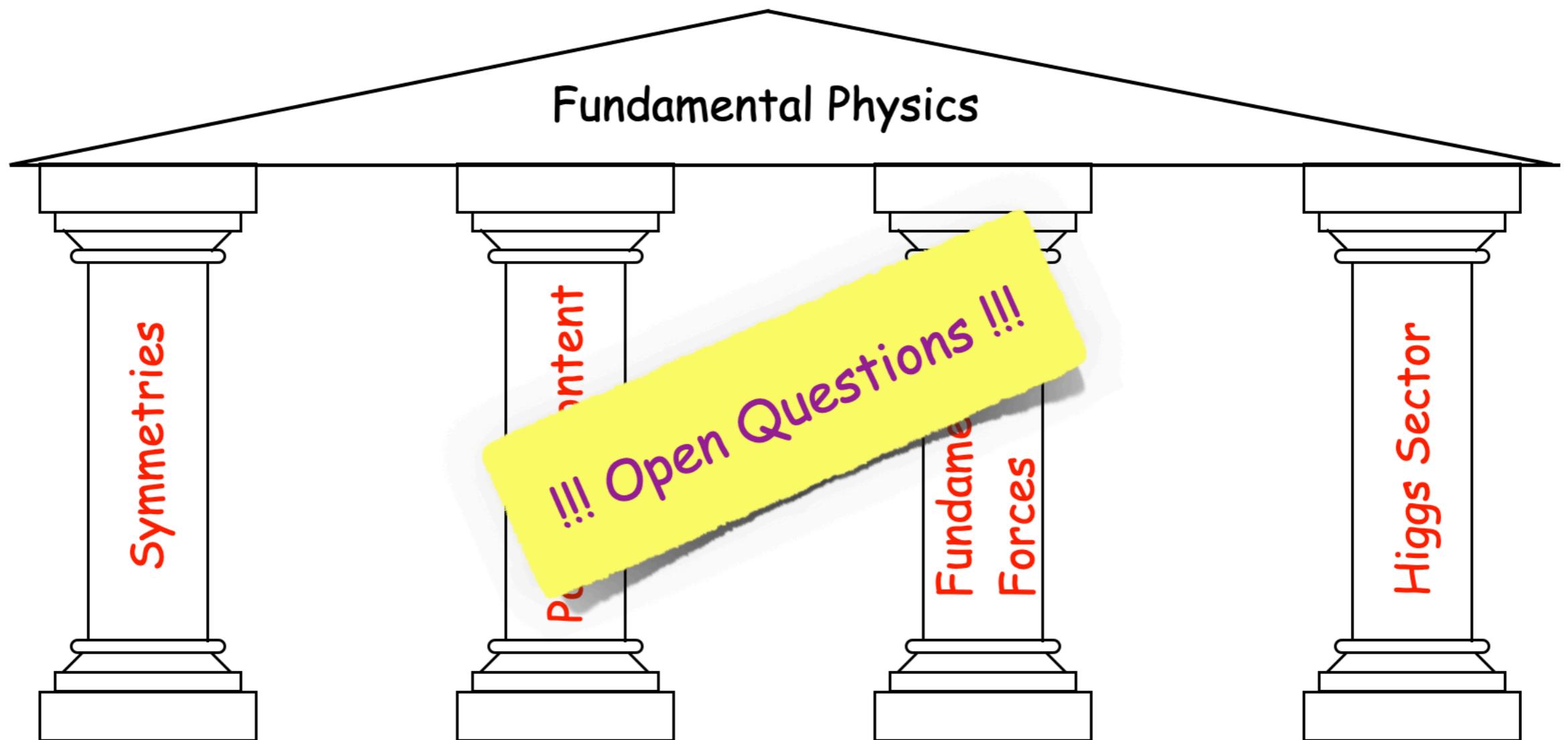
Work in Collaboration with:
Ph. Basler, L. Biermann, J. Müller,
R. Santos, J. Viana

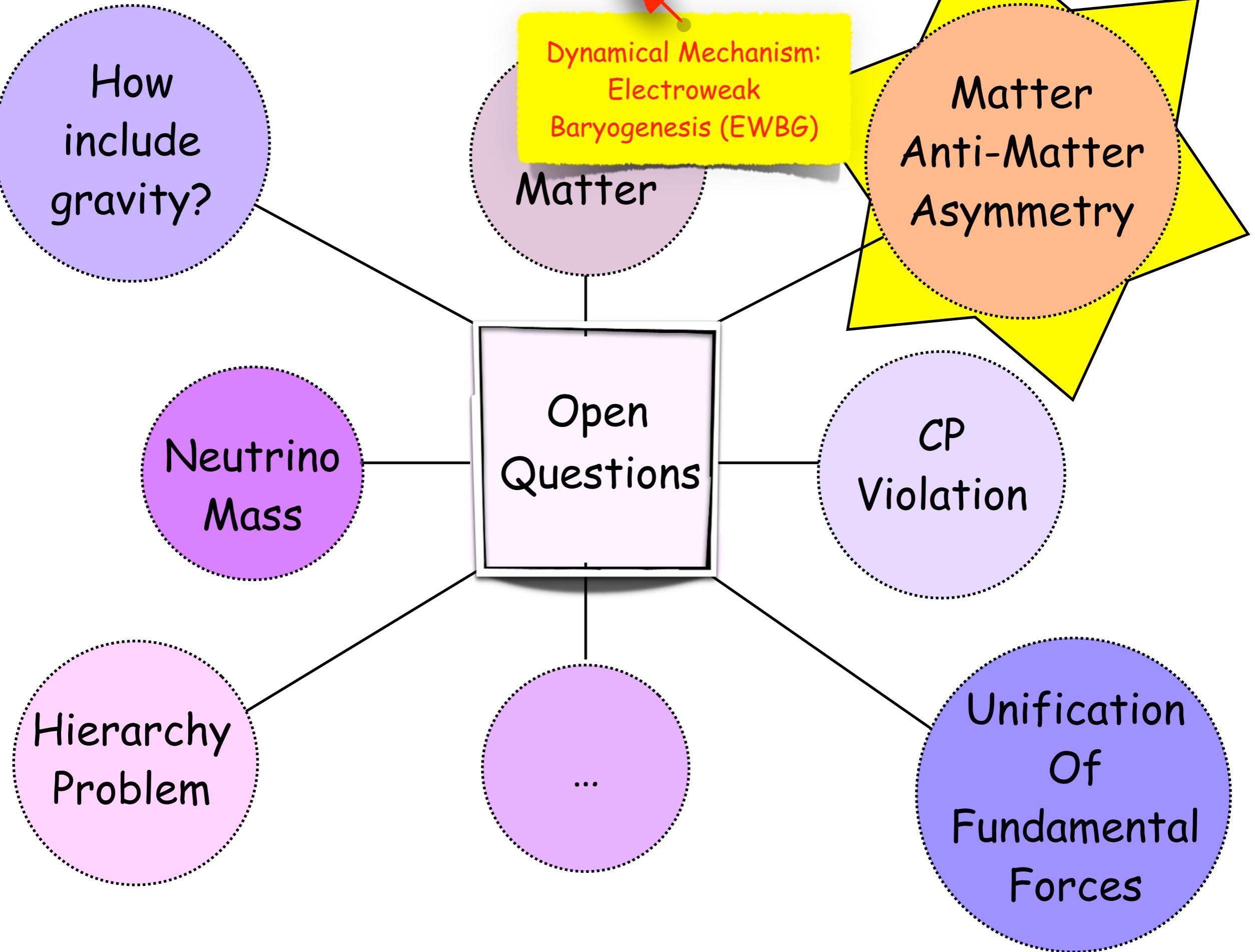
Outline

- Introduction
- Physics Problem
- The code BSMPT
- Gravitational Waves Sourced from FOPT
- Phenomenological Results
- Conclusions



The Standard Model is structurally complete - But





Electroweak Baryogenesis

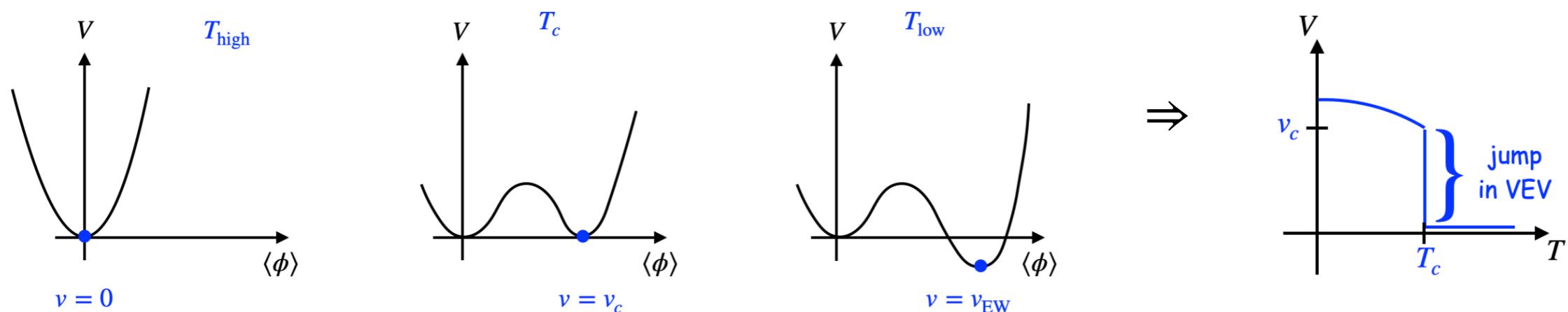
- **Electroweak Baryogenesis (EWBG):** generation of the observed baryon-antibaryon asymmetry in the electroweak phase transition (EWPT) [Riemer-Sorensen, Jenssen '17]

$$5.8 \cdot 10^{-10} < \frac{n_B - n_{\bar{B}}}{n_\gamma} < 6.6 \cdot 10^{-10}$$

- **Sakharov Conditions:** [Sakharov '67]

- * (i) B number violaton (sphaleron processes)
- * (ii) C and CP violation
- * (iii) Departure from thermal equilibrium

- **Additional constraint:** EW phase transition must be strong first order PT [Quiros '94; Moore '99]



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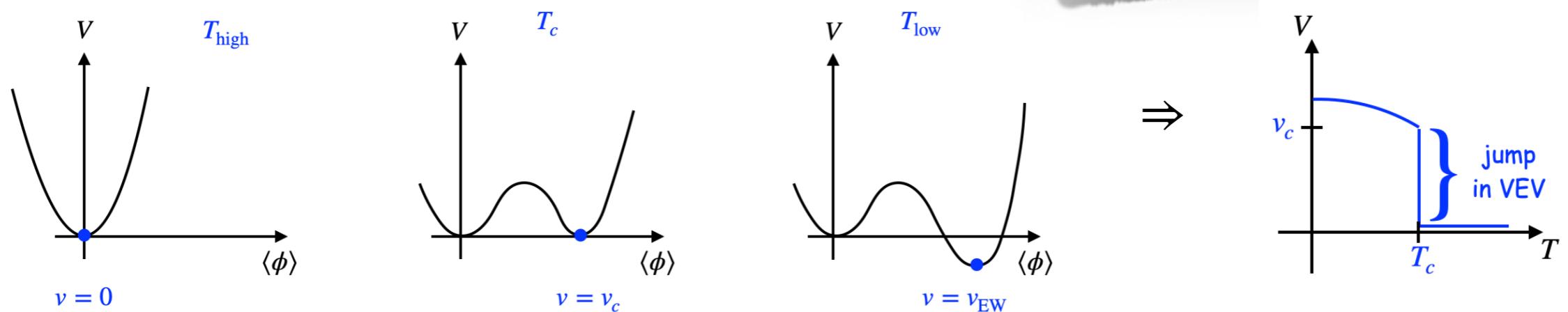
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Strong First-Order Electroweak Phase Transition (SFOEWPT): $\xi_c \equiv \frac{v_c}{T_c} > 1$ [99]



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For $M_H=125$ GeV:
smooth cross-over in SM

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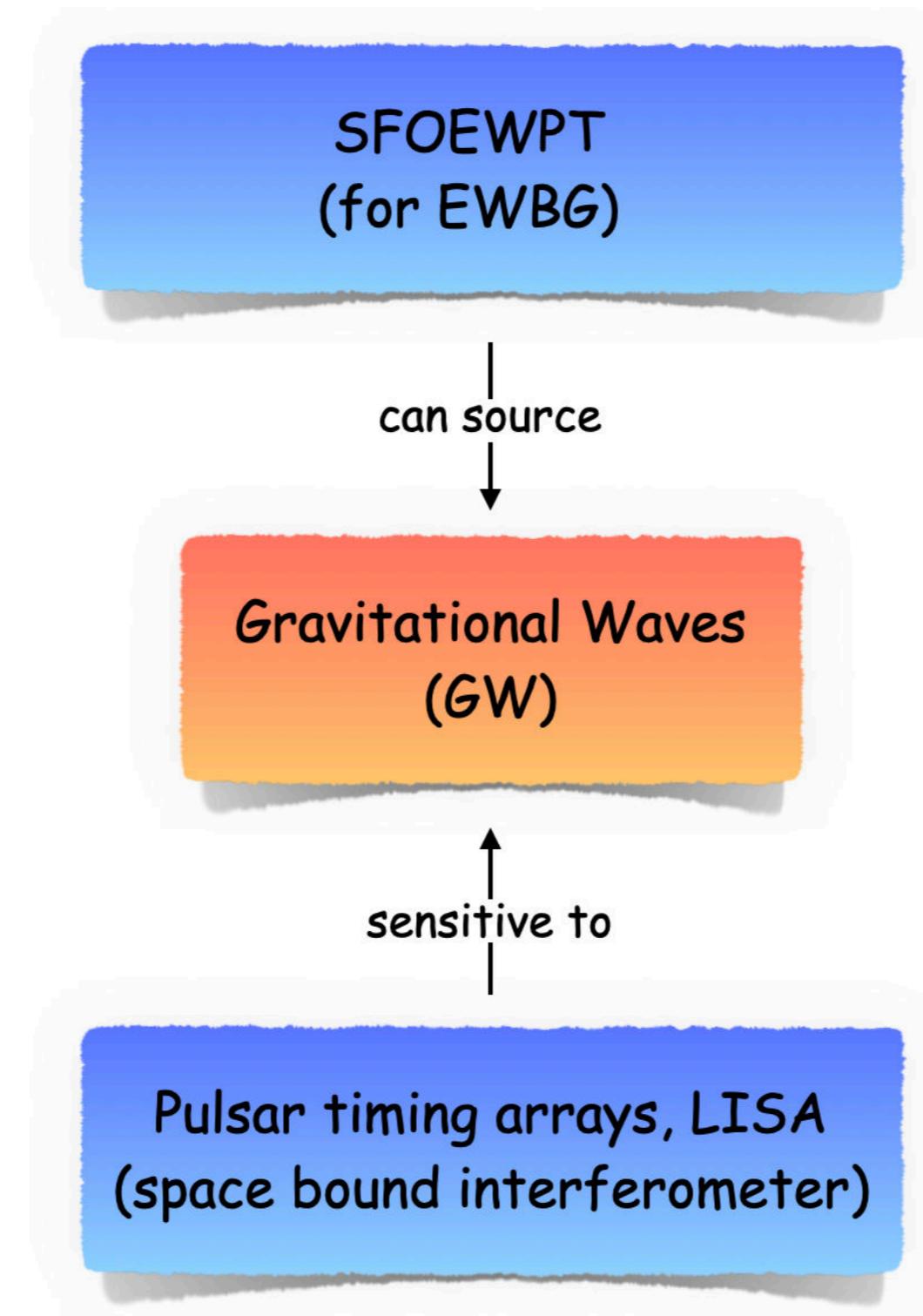
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Requires beyond the
SM (BSM) physics:
extended Higgs sectors

- **Additional constraint:** EW phase transition must be strong first order PT [Quiros '94; Moore '99]

Strong-First-Order Phase Transitions (SFOPT) and Gravitational Waves



Strong-First-Order Phase Transitions (SFOPT) and Gravitational Waves



Directly probe echo of
Cosmological FOPT

Discovery of Physics
Beyond the SM

SFOEWPT
(for EWBG)

can source

Gravitational Waves
(GW)

sensitive to

Pulsar timing arrays, LISA
(space bound interferometer)

Physics Problem

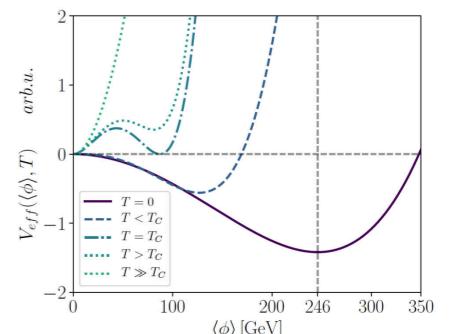


From Particle Physics Model to Gravitational Waves

- ♦ BSM model with extended Higgs sector at T=0:
derive allowed parameter regions compatible w/ theoretical and experimental constraints

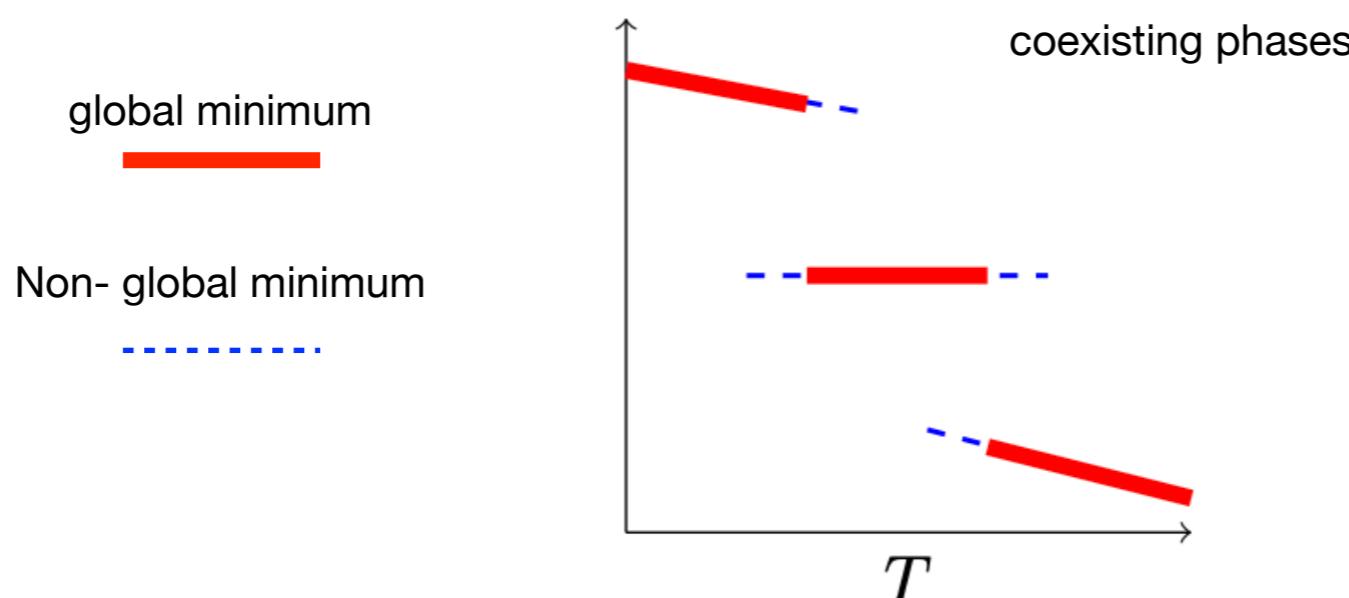
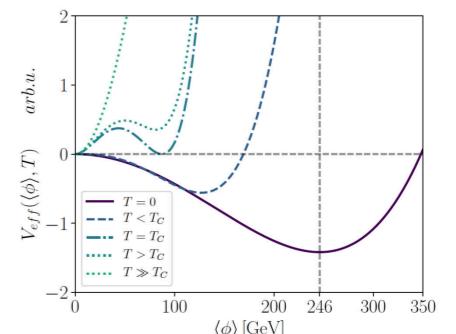
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- ♦ BSM model with extended Higgs sector at $T=0$:
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- ♦ Vacuum phases (= vacuum structure) at non-zero temperatures T :
 - determine effective potential at non-zero temperature
 - trace vacuum phases as function of T
 - collect all coexisting phase pairs w/ their critical temperatures T_c



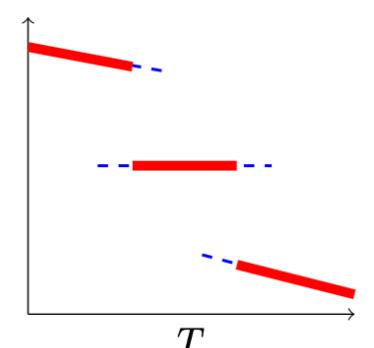
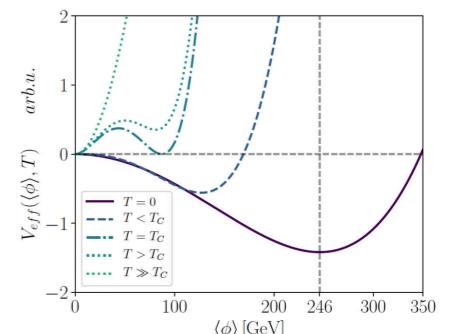
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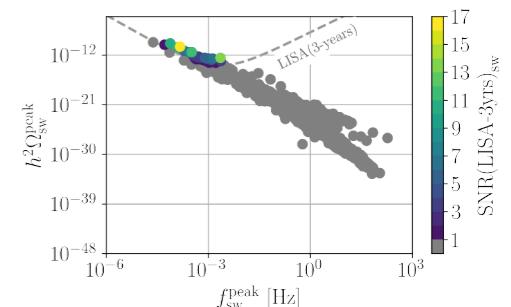
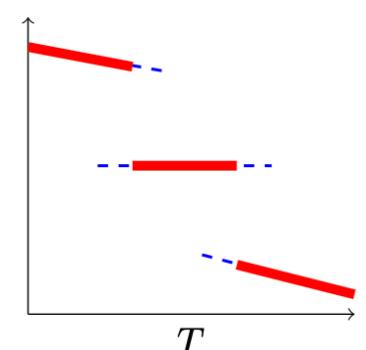
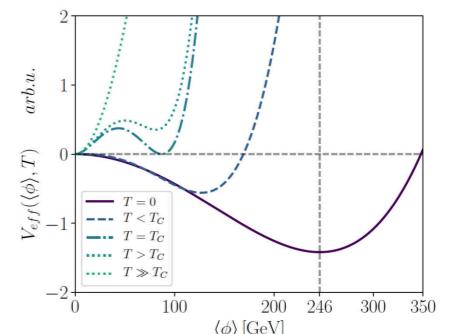
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- ♦ Does the phase transition between false and true minimum really happen?:
 - computation of bounce action and tunneling rate for coexisting pairs w/ T_c



From Particle Physics Model to Gravitational Waves

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 - computation of bounce action and tunneling rate for coexisting pairs w/ T_c
- ♦ Determination of temperatures characteristic for phase transition:
 - nucleation T_n , percolation T_p , completion T_f , reheating T_{reh} temperature
- ♦ Stochastic Gravitational Waves Background SGWB:
 - relate thermodynamics parameters to geometric parameters of SGWB
 - signal-to-noise ratio at LISA



[For a review cf. Athron, et al., 2305.02357]

From Particle Physics Model to Gravitational Waves

- ♦ BSM model with extensive parameter space → derive allowed parameter regions
- ♦ Vacuum phases (= vacuum expectation values)
 - determine effective theory
 - trace vacuum phase evolution
 - collect all coexisting vacua
- ♦ Does the phase transition occur?
 - computation of boundary conditions
- ♦ Determination of temperature scale
 - nucleation T_n , percolation
- ♦ Stochastic Gravitational Waves
 - relate thermodynamics to GWs
 - signal-to-noise ratio

Combine collider phenomenology and cosmological observations

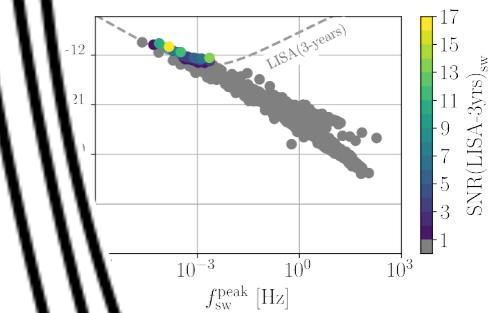
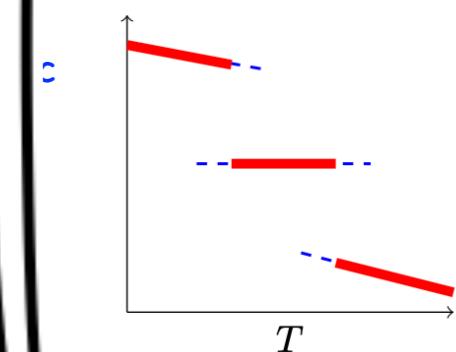
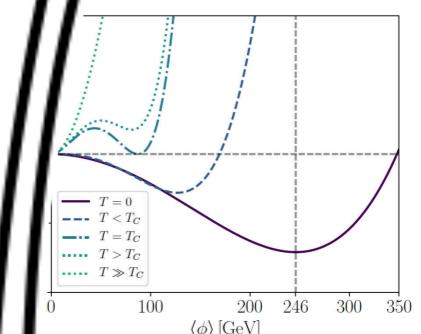
Challenge: large range of energy scales involving different physics

Requires: consistent combination of collider and GWs information

Complexity: numerical solution of problem

Needed: code performing whole chain from particle physics model to GWs

constraints



Available Public Codes

- CosmoTransitions [Wainwright '11]: phase tracing, bounce solution, T_c , T_n^{approx}
- Vevacious, VevaciousPlusPlus [Camargo-Molino eal '13, '14, '15]: finding minima
- AnyBubble [Masoumi eal '17]: bounce solution
- EVADE [Hollik eal '18, '19]: finding minima, bounce solution
- BubbleProfiler [Athron eal '19]: bounce solution
- PhaseTracer/+2 [Athron eal '20, +'24]: phase tracing, T_c /+ bounce solution, GW spectra (of 2019)
- SimpleBounce [Sato '19]: bounce solution
- FindBounce [Guada eal '18, '20]: bounce solution
- OptiBounce [Bardsley '21]: bounce solution
- TransitionListener [Kahlhöfer, Tasillo '21]: bounce solution, characteristic temperatures, GW spectra
- WallGo [Ekstedt eal '24]: wall velocity

BSMPTv3: phase tracing, bounce solution, thermal parameters, GW spectra (of 2024)

for arbitrary extended Higgs sectors

The Code BSMPT

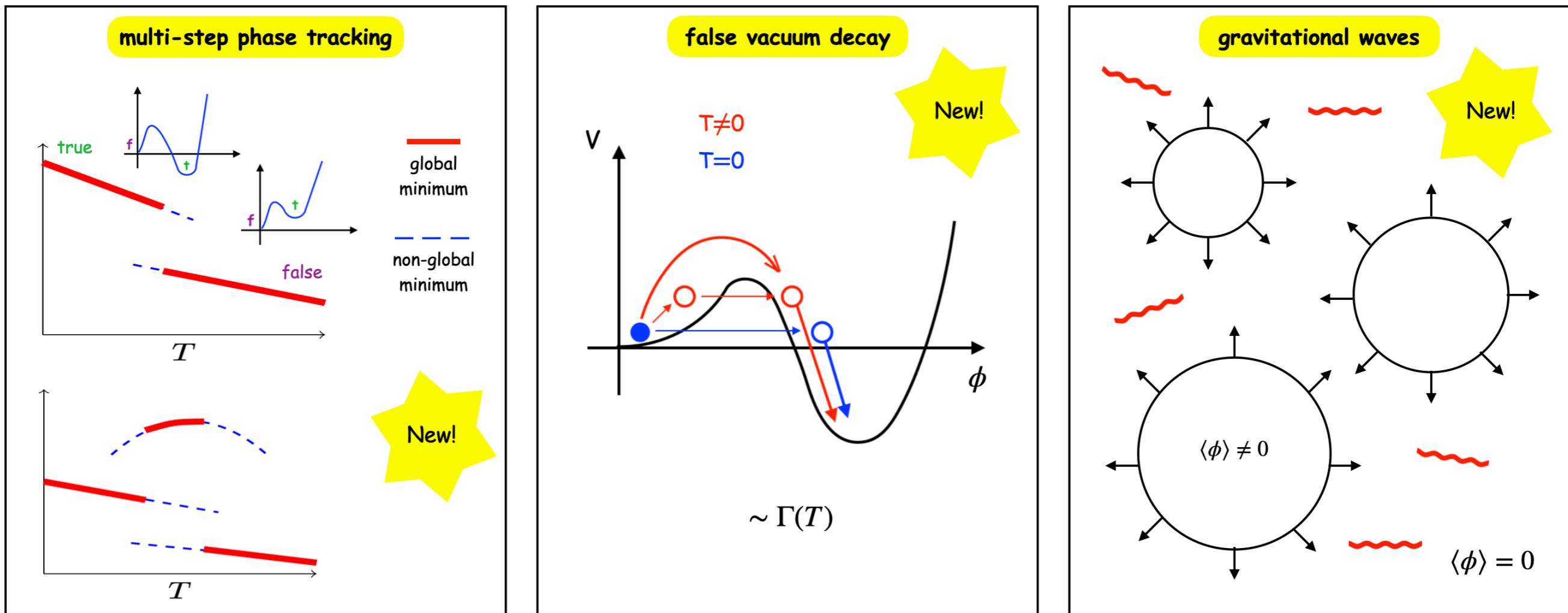


B.SMPT: Beyond the Standard Model Phase Transitions

v1: [Basler,MM,1803.02846] v2: [Basler,MM,Müller, 2007.01725] v3: [Basler,Biermann,MM,Müller,Santos,Viana, 2404.19037]

<https://github.com/phbasler/BSMPT/>

- ◆ Model-independent implementation of the one-loop resummed, T-dependent effective potential
- ◆ Local minimum tracking, identification of phase overlaps and multi-step PT histories
- ◆ Calculates decay rate of false into true vacuum
- ◆ Derivation of the gravitational waves spectrum from strong first-order phase transitions



Further Remarks: Implemented Effective Potential

$$V^{(1)}(\vec{\omega}, T) = \underbrace{V^{(0)}(\vec{\omega}, T)}_{\text{tree-level}} + \underbrace{V^{\text{CW}}(\vec{\omega})}_{\substack{\text{T-indep.} \\ \text{Coleman-Weinberg} \\ \text{potential} \\ \text{MSbar renormalized}}} + \underbrace{V^{\text{T}}(\vec{\omega}, T)}_{\substack{\text{T-dep. UV fin.} \\ \text{IR fin. after resumm.} \\ m^2 \rightarrow m^2 + \Pi^{(1)}(0)}} + \underbrace{V^{\text{CT}}(\vec{\omega})}_{\substack{\text{finite shift of} \\ \text{scalar masses} \\ \& \text{mixing angles}}}$$

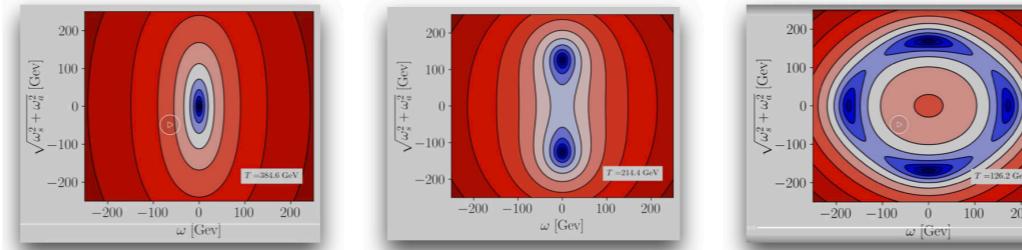
[Coleman, Weinberg, '73]
 [Carrington, '92]
 [Basler et al, '17]
[Parwani, '92]
[Arnold, Espinosa, '93]

BSMPT: Global minimization of loop-corrected effective potential
in all possible field directions $\vec{\omega}$ at $T \neq 0$ GeV

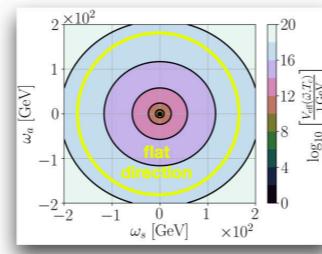
V^{CT} : On-shell-like renormalization scheme (masses & couplings tree-level-like)
 ⇒ Allows for efficient parameter scans to check viability of the models!

Further Remarks: Additional Features

- ♦ Discrete symmetries: models w/ discrete \mathbb{Z}_2 symmetries increase # of possible minima°

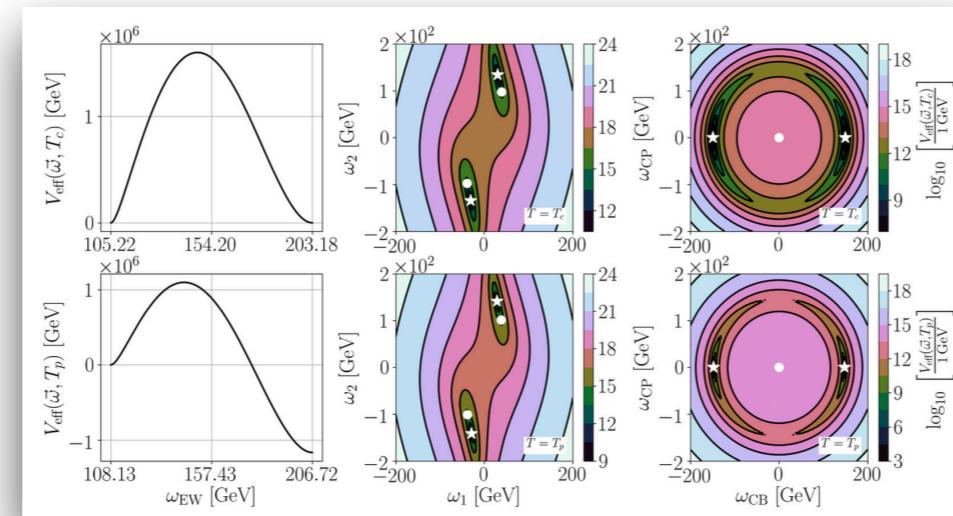


- ♦ Flat directions: multidimensional potentials can have flat directions



- ♦ Check of EW symmetry restoration: symmetry might not be restored at high T
cf. e.g. [Meade,Ramani,'18;Baldes,Servant,'18;Matsedonskyi,Servant,'20;Carena eal,'21;Biekötter eal,'21,'22]

- ♦ Executable PotPlotter: visualization of multi-dimensional potential contours

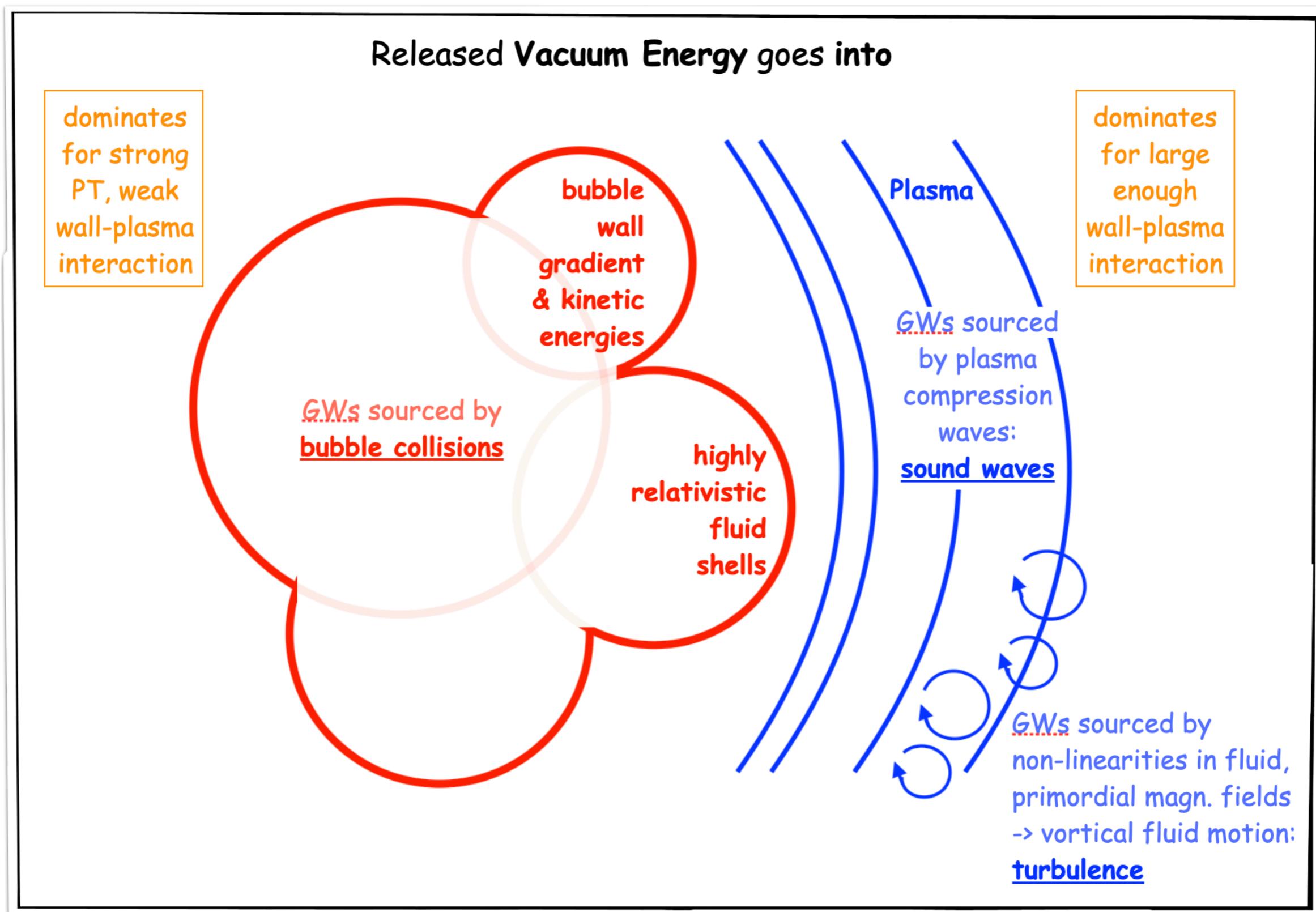


°Model w/ spontan. broken discrete symmetries \sim domain walls; domain wall effects not considered presently.

Gravitational Waves Sourced From FOPT



Sources of Gravitational Waves from PTs



Implementation of GWs in BSMPTv3

- ♦ Spectra of gravitational waves sourced by first-order phase transitions: [Caprini et al., 2403.03723]

$$\Omega_{\text{GW}}(f) = \Omega_{\text{GW}}^{\text{coll}}(f) + \Omega_{\text{GW}}^{\text{sw}}(f) + \Omega_{\text{GW}}^{\text{turb}}(f)$$

Modelling by broken power law (BPL) or double broken power law (DBPL)

Updated implementation in BSMPTv3 in 2025

- ♦ BPL for GWs sourced by bubble collisions & highly relativistic fluid shells $\Omega_{\text{GW}}^{\text{coll}}$:

$$\Omega_{\text{GW}}^{\text{BPL}}(f) = \Omega_{\text{peak}} \frac{(n_1 - n_2)^{\frac{n_1 - n_2}{a_1}}}{\left[-n_2 \left(\frac{f}{f_{\text{peak}}} \right)^{-\frac{n_1 a_1}{n_1 - n_2}} + n_1 \left(\frac{f}{f_{\text{peak}}} \right)^{-\frac{n_2 a_1}{n_1 - n_2}} \right]^{\frac{(n_1 - n_2)}{a_1}}}$$

- ♦ DBPL for GWs sourced by sound waves $\Omega_{\text{GW}}^{\text{sw}}$ and for GW sourced by turbulence $\Omega_{\text{GW}}^{\text{turb}}$:

$$\Omega_{\text{GW}}^{\text{DBPL}}(f) = \Omega_{\text{int}} \times S(f) = \Omega_2 \times S_2(f)$$

$$S(f) = N \left(\frac{f}{f_1} \right)^{n_1} \left[1 + \left(\frac{f}{f_1} \right)^{a_1} \right]^{\frac{-n_1 + n_2}{a_1}} \left[1 + \left(\frac{f}{f_2} \right)^{a_2} \right]^{\frac{-n_2 + n_3}{a_2}}$$

The Signal-to-Noise Ratio

- Signal-to-noise ratio at LISA:

$$\text{SNR} = \sqrt{\mathcal{T} \int_{f_{\min}}^{f_{\max}} df \left[\frac{h^2 \Omega_{\text{GW}}(f)}{h^2 \Omega_{\text{Sens}}(f)} \right]^2}$$

$h^2 \Omega_{\text{Sens}}$ nominal sensitivity of a given LISA configuration to stochastic sources
 \mathcal{T} experimental acquisition time in seconds (4 years, min. duty cycle of 75%)
 f_{\min}, f_{\max} minimum and maximum frequency to which LISA is sensitive

- In BSMPTv3:

SNR in BSMPTv3: $\text{SNR}(3 \text{ years})$

for \mathcal{Y} years:

$$\text{SNR}(\mathcal{Y}) = \sqrt{\frac{\mathcal{Y}}{3}} \text{SNR}(3 \text{ years})$$

Phenomenological Results



Phase Transitions in the 2HDM

♦ 2-Higgs-Doublet-Model: four minimum directions

$$V_{\text{tree}} = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - [m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}] + \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 \\ + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \left[\frac{1}{2} \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \text{h.c.} \right].$$

$$\Phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} \rho_1 + i \eta_1 \\ \zeta_1 + \omega_1 + i \psi_1 \end{pmatrix}, \quad \Phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \rho_2 + \omega_{\text{CB}} + i \eta_2 \\ \zeta_2 + \omega_2 + i (\psi_2 + \omega_{\text{CP}}) \end{pmatrix}$$

$$\{\omega_{\text{CB}}, \omega_1, \omega_2, \omega_{\text{CP}}\}|_{T=0} = \{0, v_1, v_2, 0\}, \text{ with}$$

$$\omega_{\text{EW}}|_{T=0} \equiv \sqrt{\omega_1^2 + \omega_2^2 + \omega_{\text{CB}}^2 + \omega_{\text{CP}}^2} \Big|_{T=0} = \sqrt{v_1^2 + v_2^2} \equiv v = 246 \text{ GeV}$$

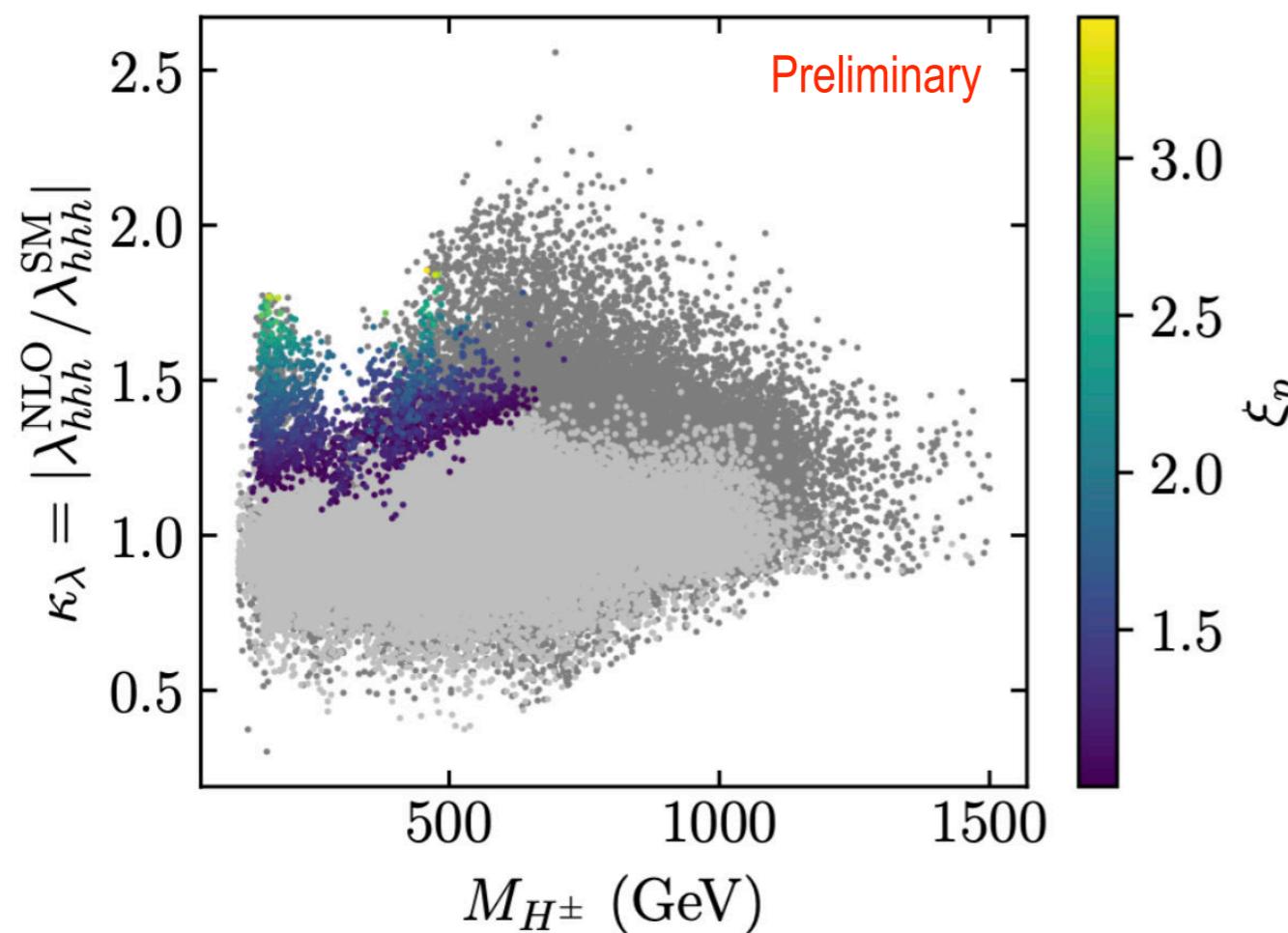
For further work on SFOEWPT collider imprints in 2HDM+EFT cf. eg. [Anisha,Biermann,Englert,MM,'22], [Anisha,Azevedo,Biermann,Englert,MM,'23], [Biermann,Borschensky,Englert,MM,Naskar,'24] & this appendix

Collider implications of an SFOEWPT

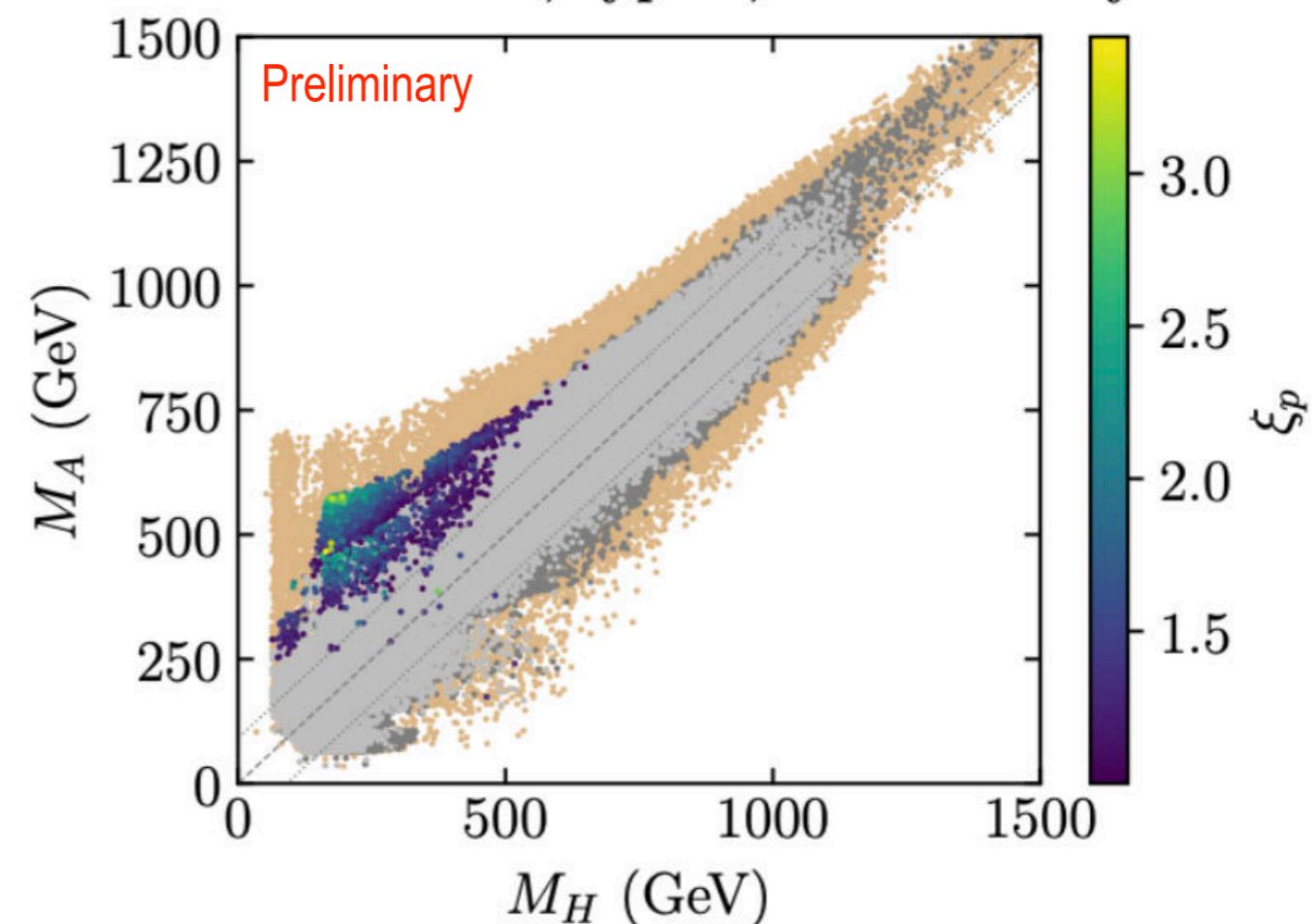
[Biermann, Borschensky, MM, Santos, Viana, to appear]

dark gray: no EW symmetry restoration (EWSR) at high T
 light gray: EWSR at high T, $\xi_p < 1$
 colored: EWSR at high T + $\xi_p \geq 1$, i.e. SFOEWPT

R2HDM, type I, S \rightarrow EW only



R2HDM, type I, S \rightarrow EW only



2HDM SFOEWPT requires enhanced trilinear
Higgs self-coupling of the SM-like Higgs

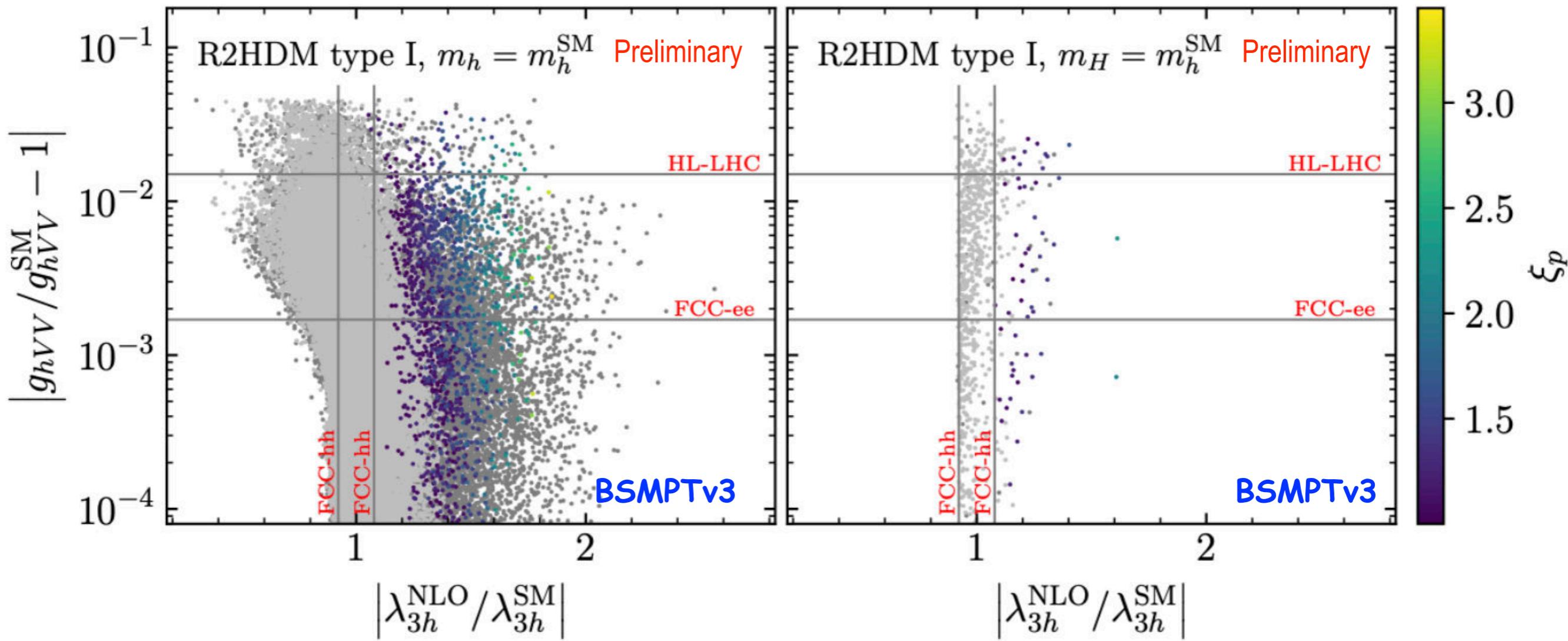
2HDM SFOEWPT typically: **A-H mass gap**
 $\rightsquigarrow A \rightarrow ZH$ signatures (need not, however)
 Similar findings e.g. [Dorsch et al.'13, '14; Biekötter et al.'23]

Single-Di-Higgs Relations

colored: EWSR at high $T + \xi_p \geq 1$, i.e. SFOEWPT

2HDM Type I

[Biermann,Borschensky,MM,Santos,Viana, to appear]

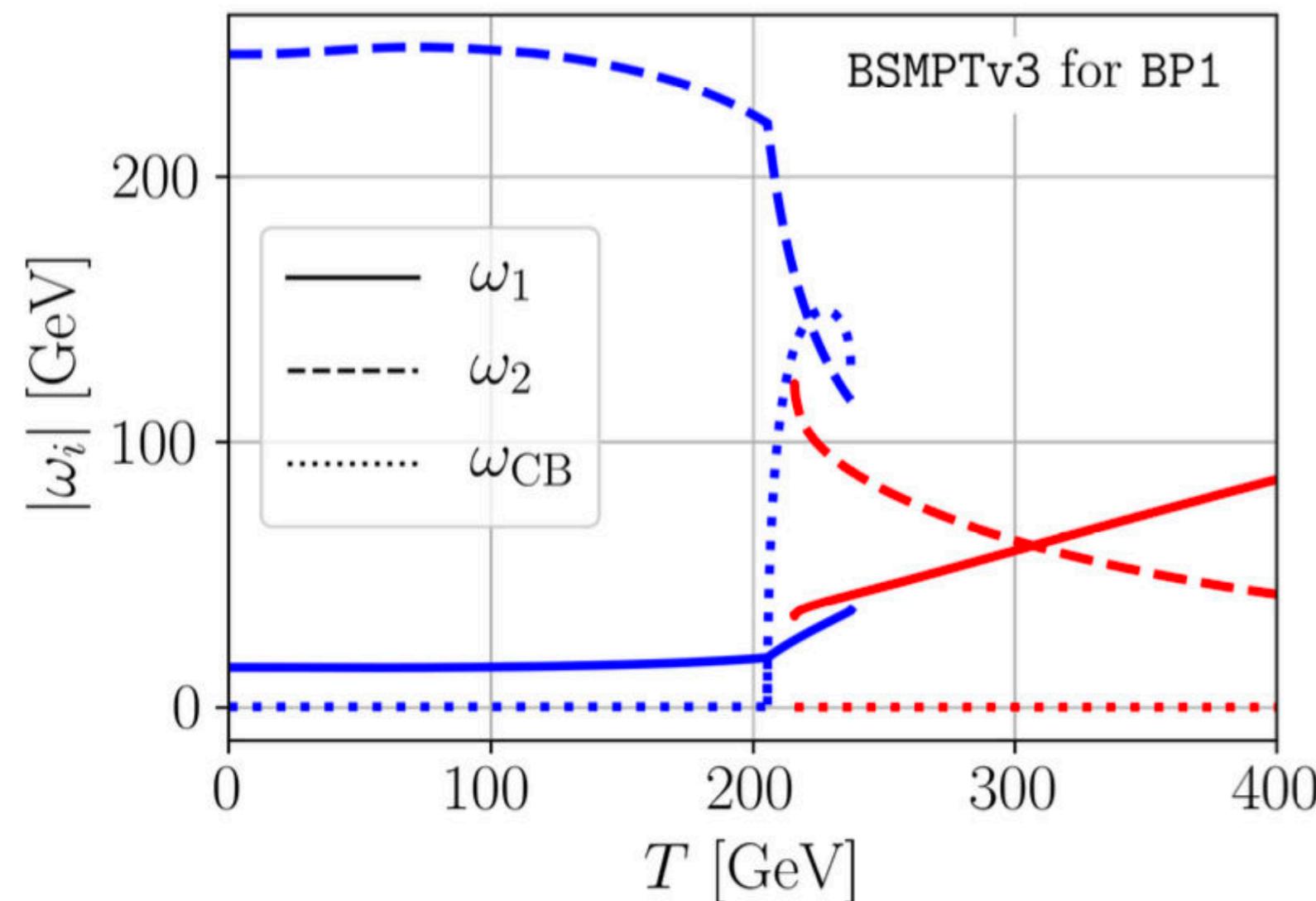


Precise single Higgs coupling measurements at FCC-ee:
no further constraint on trilinear Higgs self-coupling

Exotic Phase Transitions: Intermediate Charge-Breaking Minimum

[Basler,Biermann,MM,Müller,Santos,Viana,'24]

high-T phase, low-T phase



From [Aoki,Biermann,Borschensky,Ivanov,MM,Sakurai,'23]

BP1 input parameters, 2HDM type1:

$$\begin{aligned}\lambda_1 &= 6.931, \lambda_2 = 2.631, \lambda_3 = 1.287, \\ \lambda_4 &= 4.772, \lambda_5 = 4.782, \\ m_{12}^2 &= 1.893 \times 10^4 \text{ GeV}^2, \tan \beta = 16.578\end{aligned}$$

History:

BSMPTv3

- first-order PT from neutral (red) to charge-breaking CB phase (blue)
- smooth transition into a neutral minimum

GWs from the Model „CP in the Dark“

- Next-to-Minimal 2-Higgs Doublet Model:

[Azevedo,Ferreira,MM,Patel,Santos,Wittbrodt,'18]

$$\begin{aligned} V^{(0)} = & m_{11}^2 |\Phi_1|^2 + m_{22}^2 |\Phi_2|^2 + \frac{m_S^2}{2} \Phi_S^2 + \left(A \Phi_1^\dagger \Phi_2 \Phi_S + \text{h.c.} \right) \\ & + \frac{\lambda_1}{2} |\Phi_1|^4 + \frac{\lambda_2}{2} |\Phi_2|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2 + \lambda_4 |\Phi_1^\dagger \Phi_2|^2 + \frac{\lambda_5}{2} [(\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_1)^2] \\ & + \frac{\lambda_6}{4} \Phi_S^4 + \frac{\lambda_7}{2} |\Phi_1|^2 \Phi_S^2 + \frac{\lambda_8}{2} |\Phi_2|^2 \Phi_S^2. \end{aligned}$$

- with one discrete \mathbb{Z}_2 symmetry: $\Phi_1 \rightarrow \Phi_1, \quad \Phi_2 \rightarrow -\Phi_2, \quad \Phi_S \rightarrow -\Phi_S$

one SM-like Higgs plus dark sector: h_1, h_2, h_3, H^\pm

- trilinear coupling A is complex: dark sector with explicit CP violation \leftarrow not constrained by electric dipole moment

Vacuum Structure of „CP in the Dark“

→ General vacuum structure at $T \neq 0$:

$$\Phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} \rho_1 + i\eta_1 \\ \zeta_1 + \omega_1 + i\Psi_1 \end{pmatrix}, \Phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \rho_2 + \omega_{\text{CB}} + i\eta_2 \\ \zeta_2 + \omega_2 + i(\Psi_2 + \omega_{\text{CP}}) \end{pmatrix}, \Phi_S = \zeta_S + \omega_S$$

electroweak VEVs: ω_1, ω_2

CP-violating VEV: ω_{CP}

charge-breaking VEV: ω_{CB}

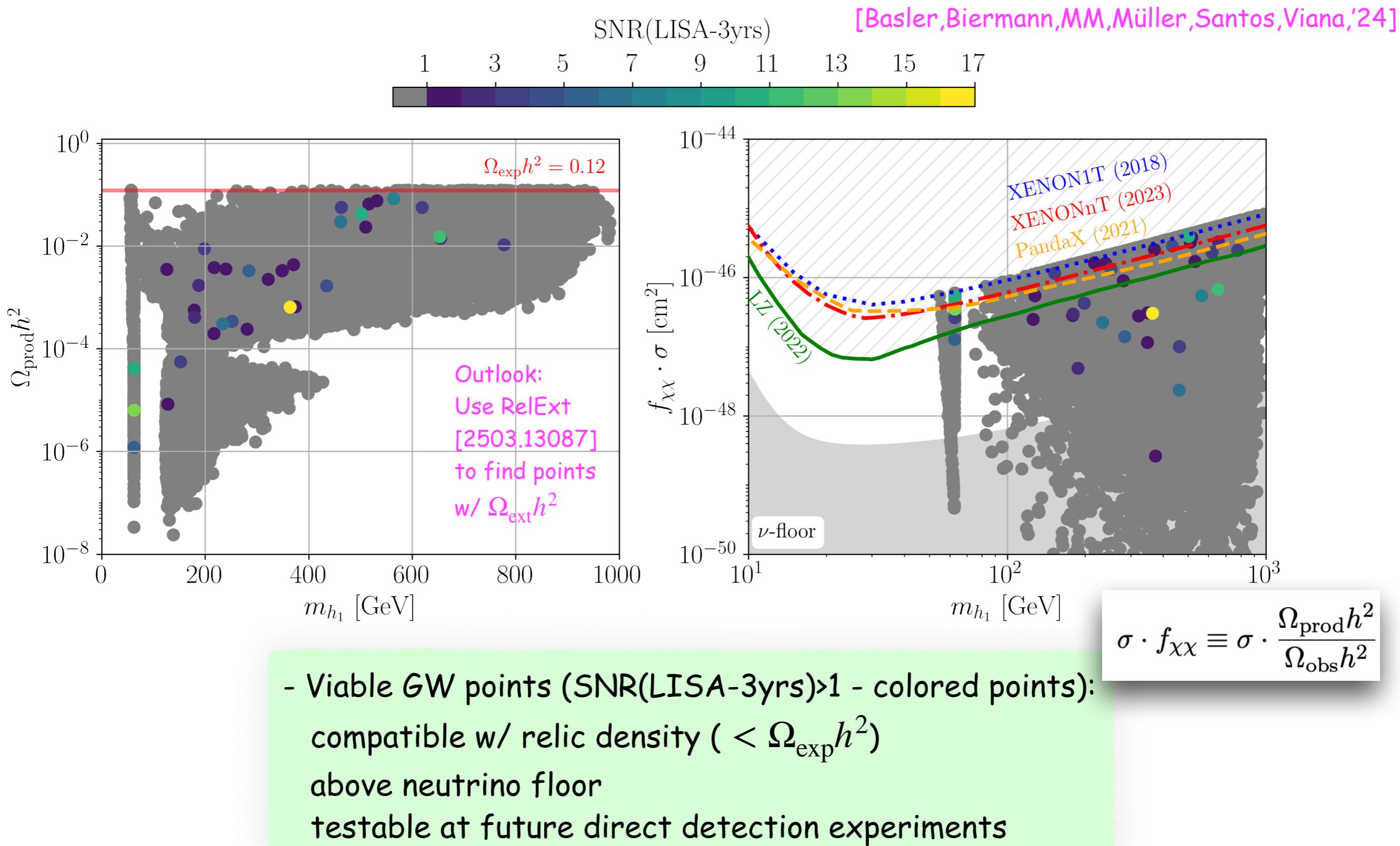
Z_2 -breaking VEV: ω_S

→ General vacuum structure at $T = 0$:

$$\Phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} \rho_1 + i\eta_1 \\ \zeta_1 + v_1 + i\Psi_1 \end{pmatrix}, \Phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \rho_2 + i\eta_2 \\ \zeta_2 + i\Psi_2 \end{pmatrix}, \Phi_S = \zeta_S$$

$$\omega_1|_{T=0} = v_1 \equiv v = 246.22 \text{ GeV}$$

DM Observables and GWs

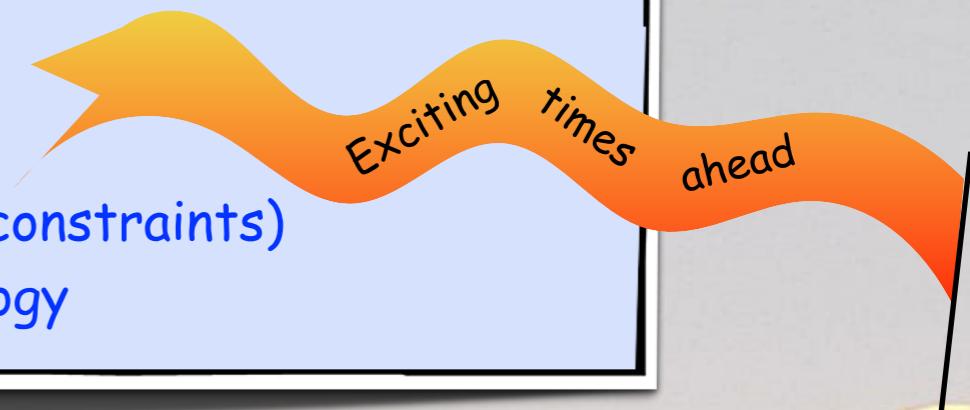


Conclusions

- Gravitational Waves from Strong First Order EW Phase Transitions:
 - great tool to test vacuum history of the Universe
 - to test beyond-SM-physics
- Non-trivial chain:

particle physics - vacuum history of the Universe - gravitational waves
requires sophisticated approaches, tools, precise experimental input
- Vacuum history of extended Higgs sectors:

complicated
exciting & surprising results (in accordance w/ constraints)
leaves testable imprints on collider phenomenology





*Thank you for
your attention!*

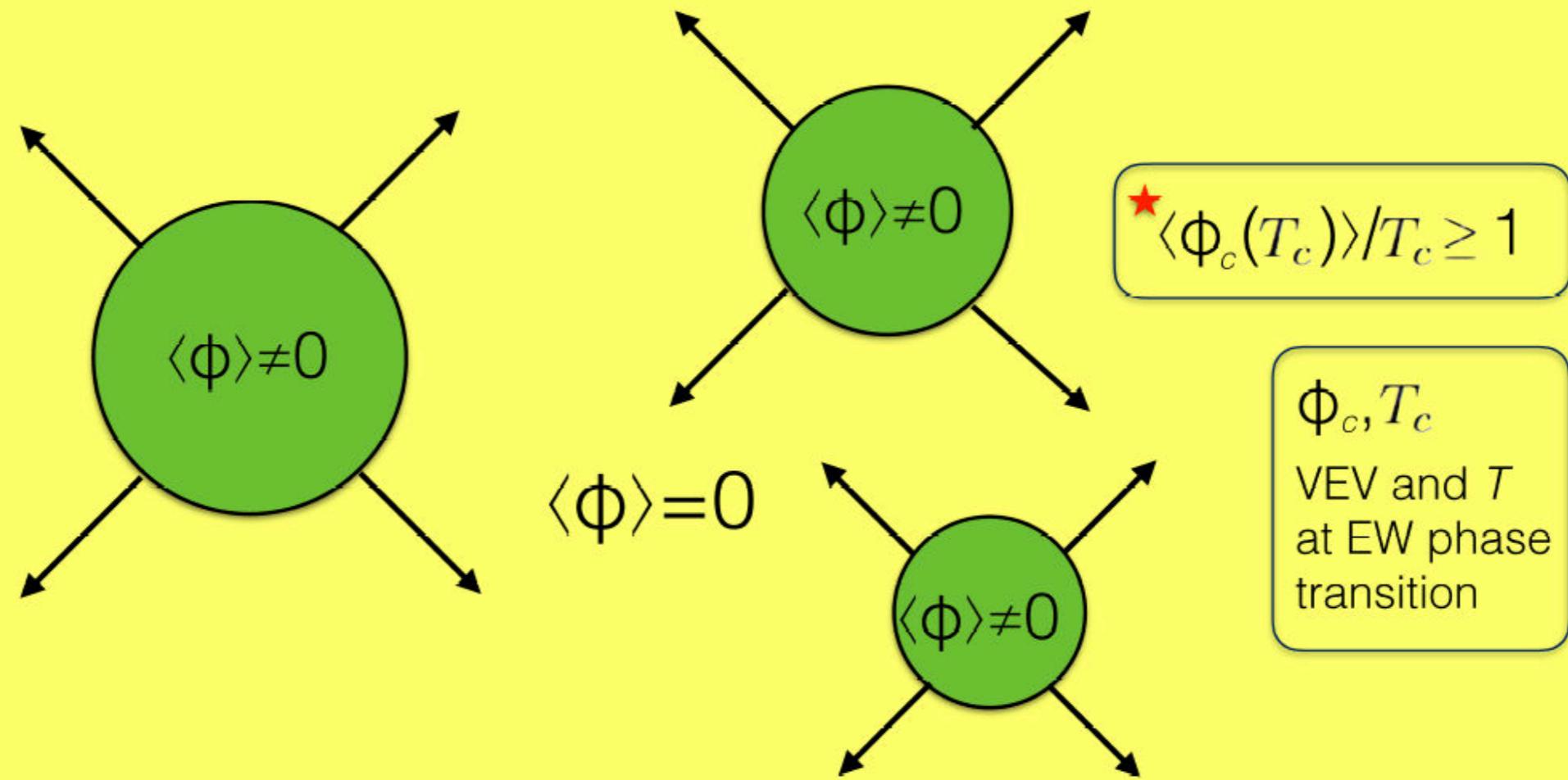
Appendix



Electroweak Baryogenesis

Bubbles of the non-zero Higgs field VEV nucleate from the symmetric vacuum

They expand & particles in plasma interact with the phase interface in a CP-violating way



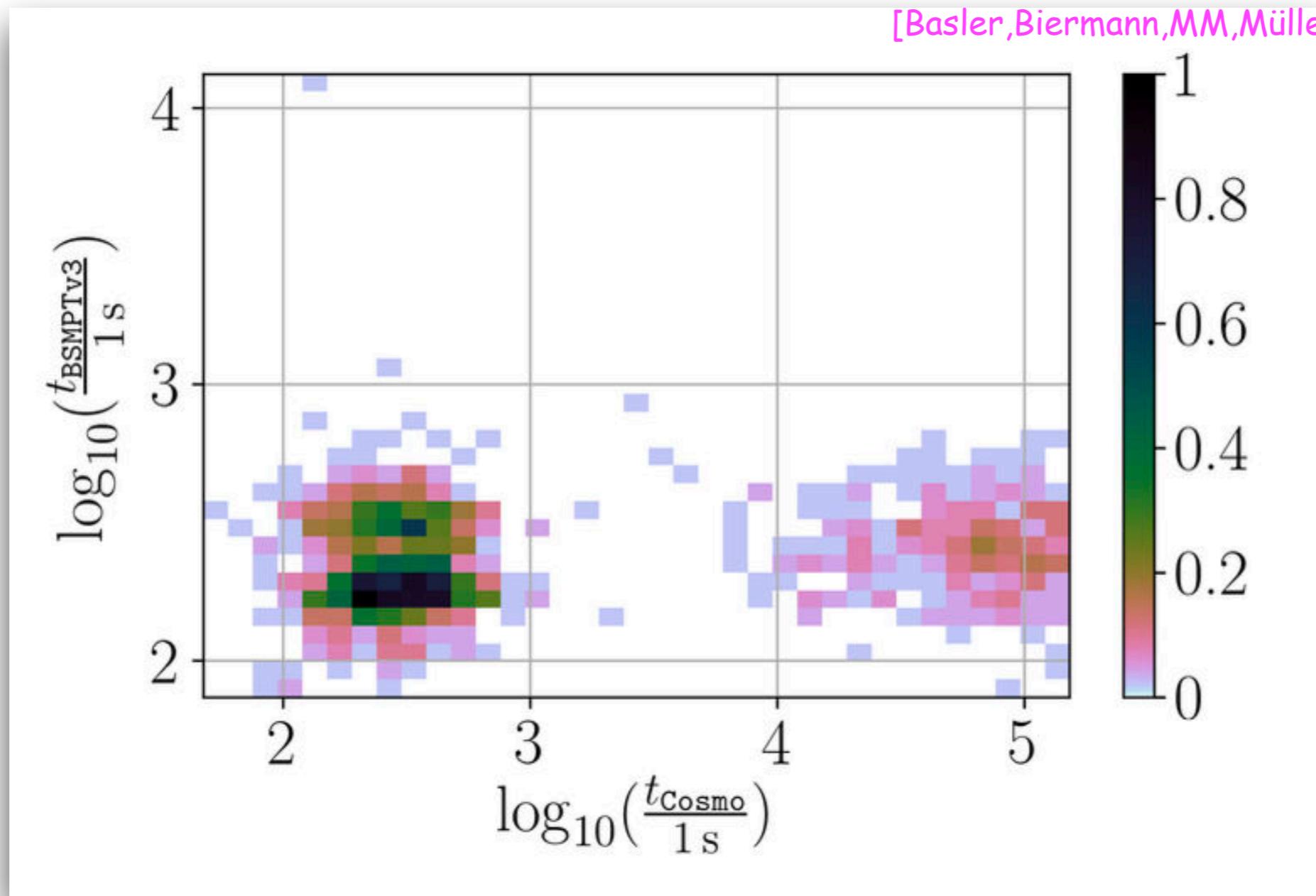
CP-asymmetry is converted into a baryon asymmetry by sphalerons in the symmetric phase in front of bubble wall

Produced baryons must not be washed out by sphaleron processes in symmetric phase in front of bubble wall \star

Broader Comparison based on Parameter Scan

- Scan of 2HDM parameter space: four field directions for minimization
with ScannerS [Coimbra eal,'13;MM eal,'22]
check for relevant theoretical and experimental constraints [cf. Azevedo eal,'23]
- Comparison BSMPTv3 and CosmoTransitions: for subset of points for which both codes find the same transitions

Runtime Comparison



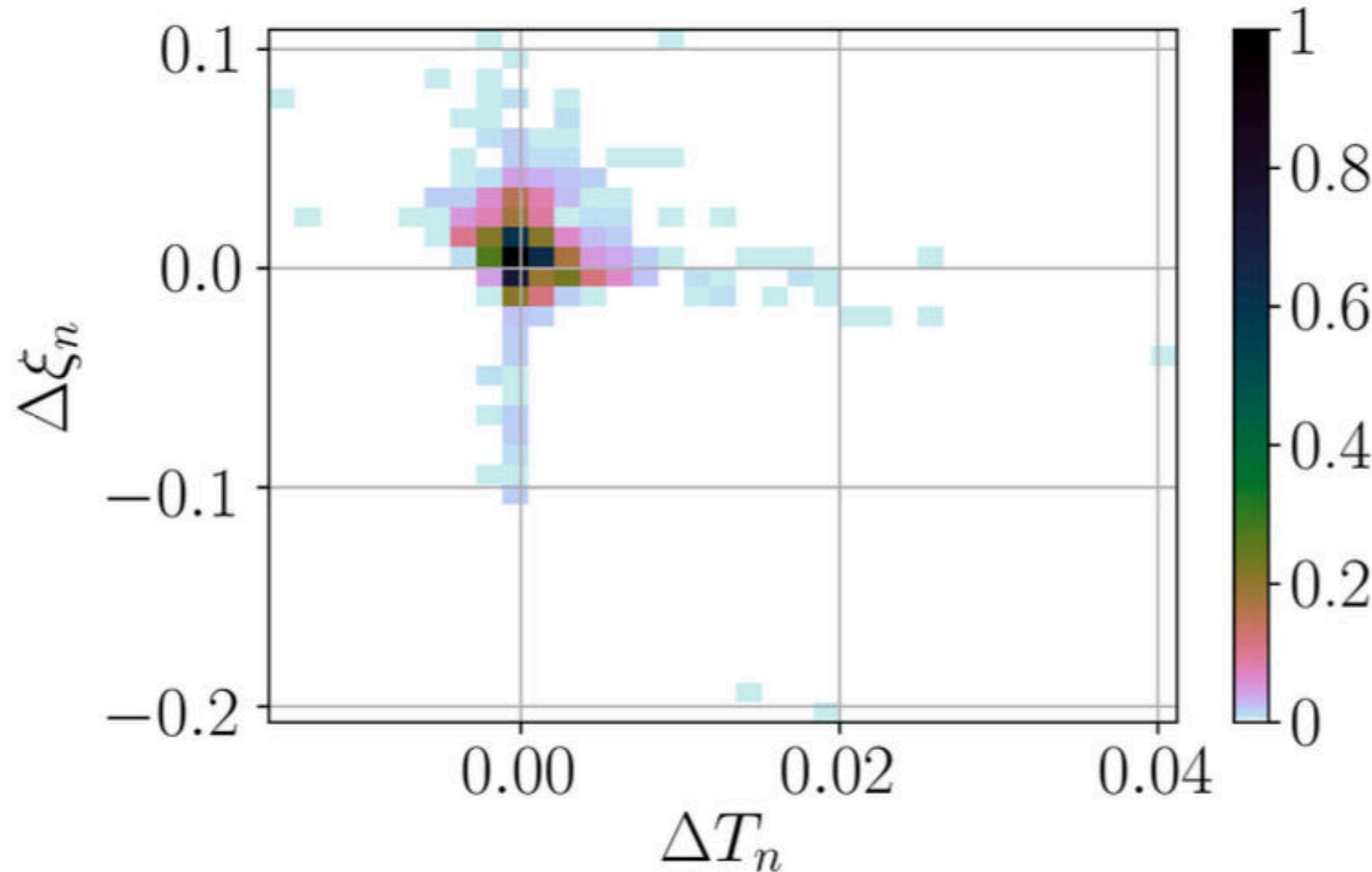
BSMPTv3 up to 10^3 faster than CosmoTransitions

BSMPTv3: mean (median) runtime 4.15 min (3.47 min)

CosmoTransitions: mean (median) runtime 41.46 min (5.61 min)

Temperature Comparison

[Basler,Biermann,MM,Müller,Santos,Viana,'24]



$$\Delta T_i = \frac{(T_i^{\text{BSMPTv3}} - T_i^{\text{Cosmo}})}{T_i^{\text{BSMPTv3}}}$$

$$\xi_i = \frac{\sqrt{\sum_k \omega_k^2(T_i)}}{T_i}$$

$$\omega_k \in \{\omega_{\text{CB}}, \omega_1, \omega_2, \omega_{\text{CP}}\}$$

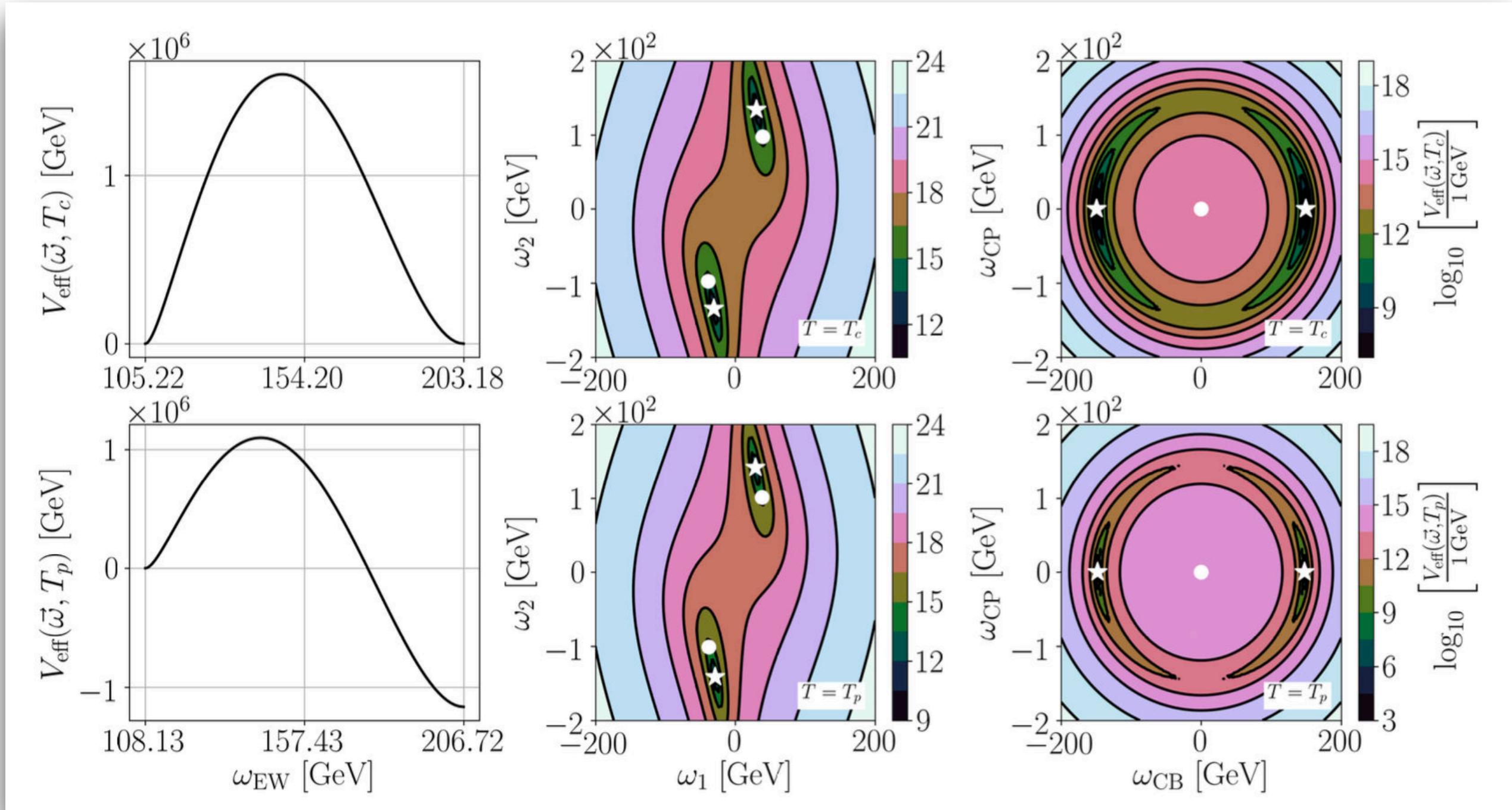
$$\Delta \xi_i = \frac{(\xi_i^{\text{BSMPTv3}} - \xi_i^{\text{Cosmo}})}{\xi_i^{\text{BSMPTv3}}}$$

Outliers: up to 4.1% rel. difference in ΔT_n
up to -20.7% rel. difference in $\Delta \xi_n$
outliers correlated w/ rapidly changing potential in small T interval

Potential Contours generated w/ PotPlotter

- BP1 (2HDM):

[Basler,Biermann,MM,Müller,Santos,Viana,'24]



White dots (asterisks) = wrong (true) minimum

Collider Imprints of an SFOEWPT

[Anisha, Biermann, Englert, MM, '22]

- ♦ 2-Higgs-Doublet-Model type II: struggles with strong first-order PT
- ♦ Add extra dynamics: scalar dimension-6 operators

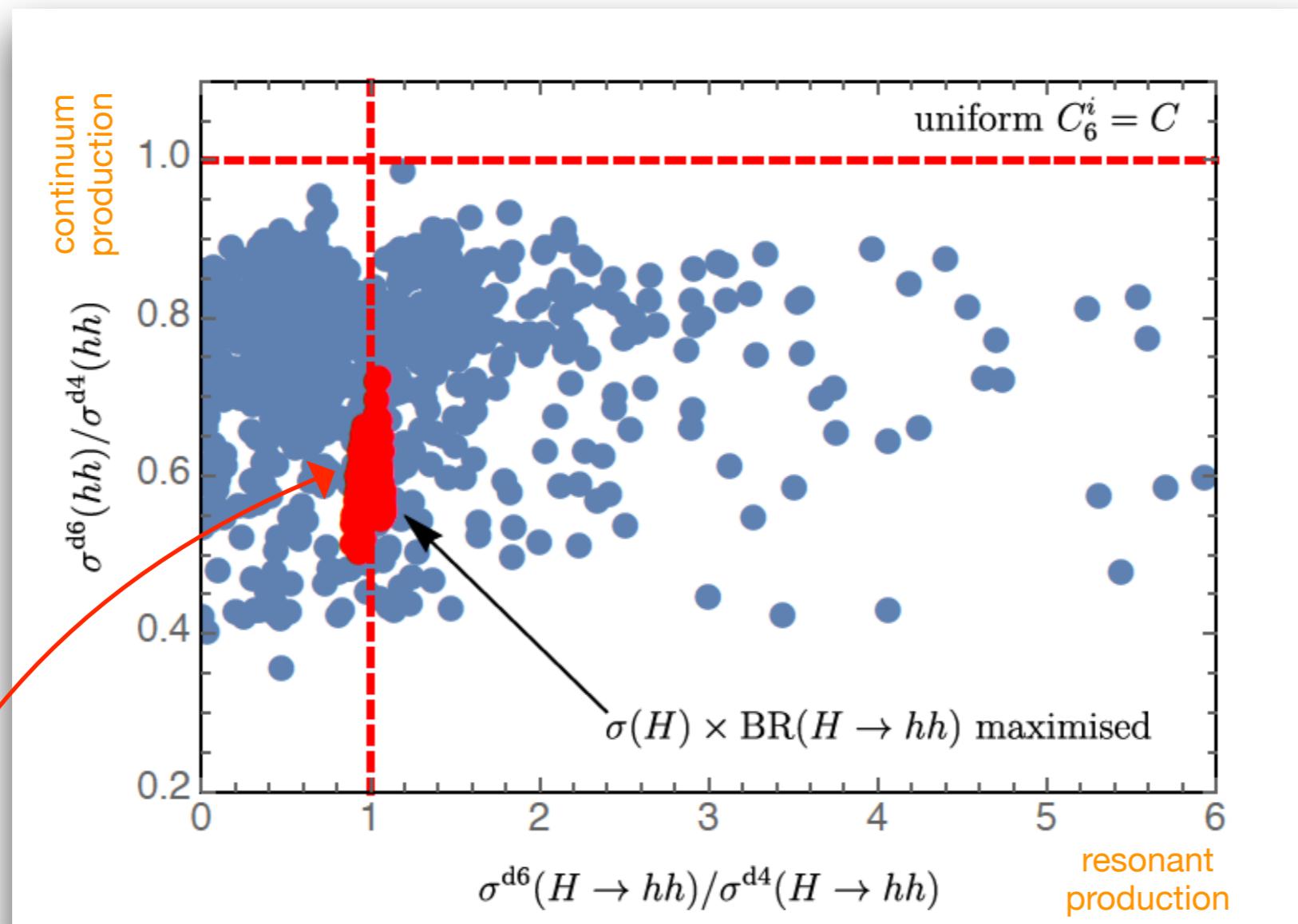
$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{\text{2HDM}} + \sum_i \frac{C_6^i}{\Lambda^2} O_6^i \quad \Rightarrow \quad V_{\text{dim-6}} = - \sum_i \frac{C_6^i}{\Lambda^2} O_6^i$$

O_6^{111111}	$(\Phi_1^\dagger \Phi_1)^3$	O_6^{222222}	$(\Phi_2^\dagger \Phi_2)^3$
O_6^{111122}	$(\Phi_1^\dagger \Phi_1)^2 (\Phi_2^\dagger \Phi_2)$	O_6^{112222}	$(\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2)^2$
O_6^{122111}	$(\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) (\Phi_1^\dagger \Phi_1)$	O_6^{122122}	$(\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2)$
O_6^{121211}	$(\Phi_1^\dagger \Phi_2)^2 (\Phi_1^\dagger \Phi_1) + \text{h.c.}$	O_6^{121222}	$(\Phi_1^\dagger \Phi_2)^2 (\Phi_2^\dagger \Phi_2) + \text{h.c.}$

- ♦ EFT effects: shifts in Higgs self-couplings \rightsquigarrow imprint in multi-Higgs production

Continuum Versus Resonant hh Production

[Anisha,Biermann,Englert,MM,'22]



- Higgs-philic points w/ $\xi_c^{d6} \approx 1$: suffer from low $\xi_c^{d4} \rightsquigarrow$ large Higgs potential modifications required for $\xi_c^{d6} \approx 1 \rightsquigarrow$ decreased di-Higgs continuum ratio (~ 50 %)
- Resonant $H \rightarrow hh$ production enhancement factor of 2.5 possible for cxn in fb range

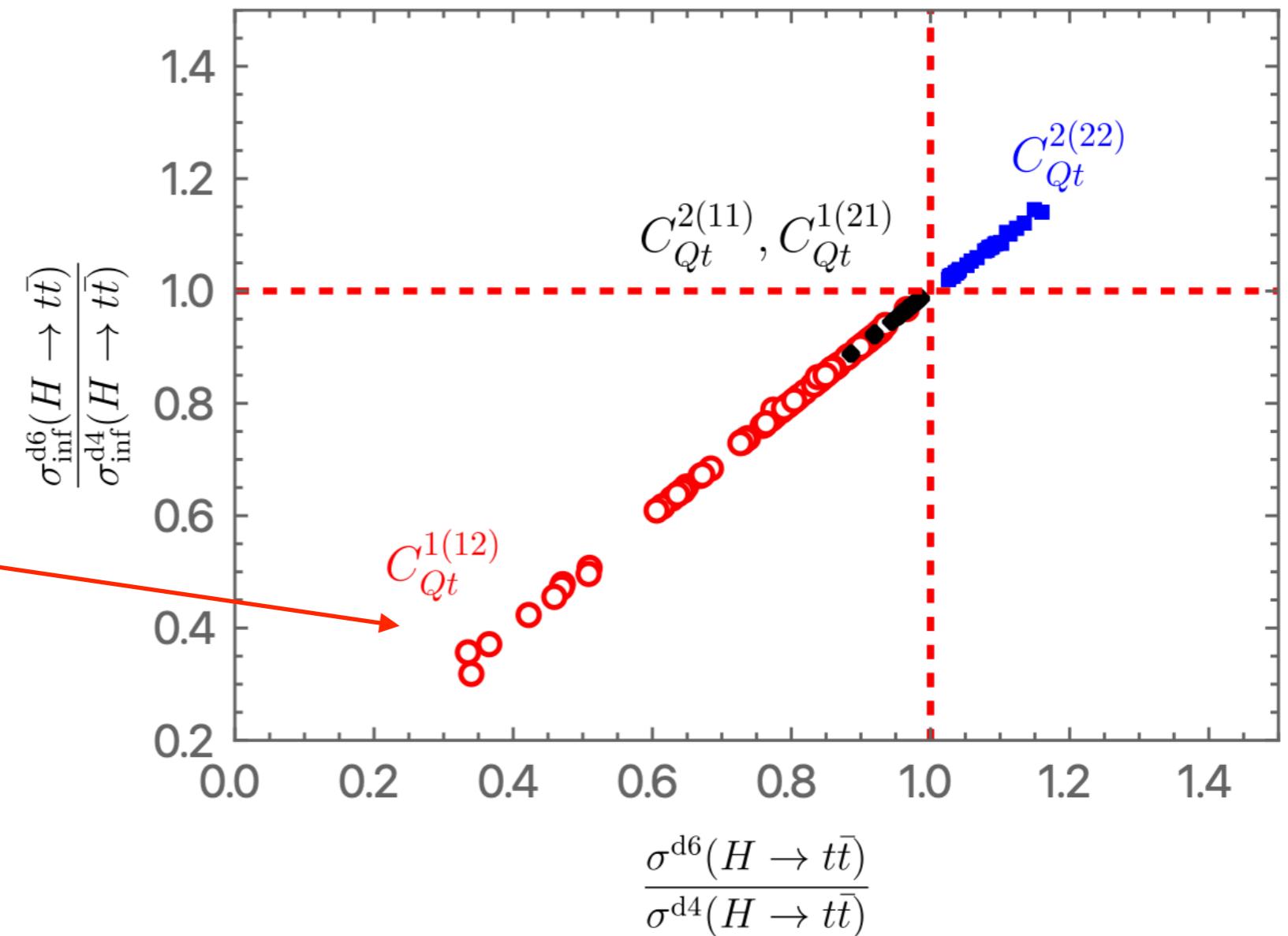
SFOEWPT and Exotic Higgs Searches

- ◆ Additional dim-6 top-Yukawa coupling modification:

[Anisha,Azevedo,Biermann,Englert,MM,'23]

$\mathcal{O}_{Qt}^{1(12)}$	$(\bar{Q}_L t_R \tilde{\Phi}_1)(\Phi_1^\dagger \Phi_2)$
$\mathcal{O}_{Qt}^{1(21)}$	$(\bar{Q}_L t_R \tilde{\Phi}_1)(\Phi_2^\dagger \Phi_1)$
$\mathcal{O}_{Qt}^{2(11)}$	$(\bar{Q}_L t_R \tilde{\Phi}_2)(\Phi_1^\dagger \Phi_1)$
$\mathcal{O}_{Qt}^{2(22)}$	$(\bar{Q}_L t_R \tilde{\Phi}_2)(\Phi_2^\dagger \Phi_2)$

Overestimate of
heavy Higgs searches



Heavy physics parametrized by $C_{Qt}^{1(12)}$ w/ an SFOEWPT: correlated w/
underproduction of heavy 2HDM Higgs bosons in $gg \rightarrow H/A \rightarrow t\bar{t}$

Version 3: <https://github.com/phbasler/BSMPT>

Program: BSMPT version 3.0.7

Released by: Philipp Basler, Lisa Biermann, Margarete Mühlleitner, Jonas Müller, Rui Santos and João Viana



Manual: version 3.0

BSMPT - Beyond the Standard Model Phase Transitions:

The C++ program package BSMPT allows for the detailed study of (multi-step) phase transitions between temperature-dependent minima in the one-loop daisy-resummed finite-temperature effective potential.

The program tracks temperature-dependent minima, calculates the bounce solution, the characteristic temperatures and gravitational wave signals of first-order phase transitions. The code also allows to derive the loop-corrected trilinear Higgs self-couplings and provides the computation of the baryon asymmetry for the CP-violating 2-Higgs Doublet Model (C2HDM).

We apply an 'on-shell' renormalization scheme in the sense that the loop-corrected masses and mixing angles are required to be equal to their tree-level input values. This allows for efficient scans in the parameter space of the models.

The models implemented so far are

- Standard Model (SM)
- CP-conserving 2-Higgs-Doublet Model (R2HDM)
- CP-violating 2-Higgs-Doublet Model (C2HDM)
- Next-to-Minimal 2HDM (N2HDM)
- CP in the Dark ([arXiv 1807.10322](https://arxiv.org/abs/1807.10322), [arXiv 2204.13425](https://arxiv.org/abs/2204.13425))
- Complex Singlet Extension (CxSM)

The code is structured such that users can add their own models.

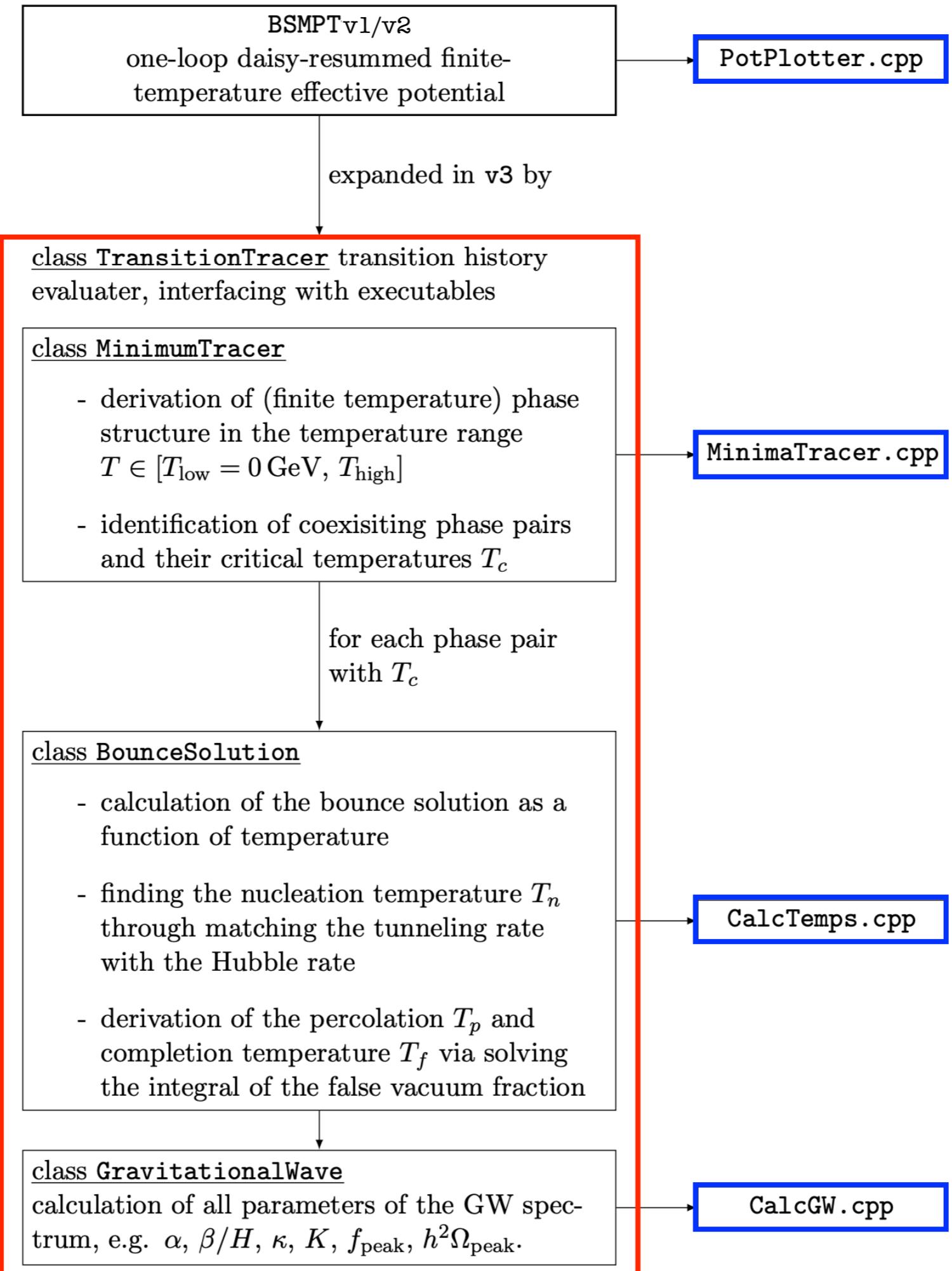
The program package can be downloaded at: <https://github.com/phbasler/BSMPT>

The documentation of the code is provided at <https://phbasler.github.io/BSMPT/documentation>.

Sample input and output files are provided in the directory 'example'.

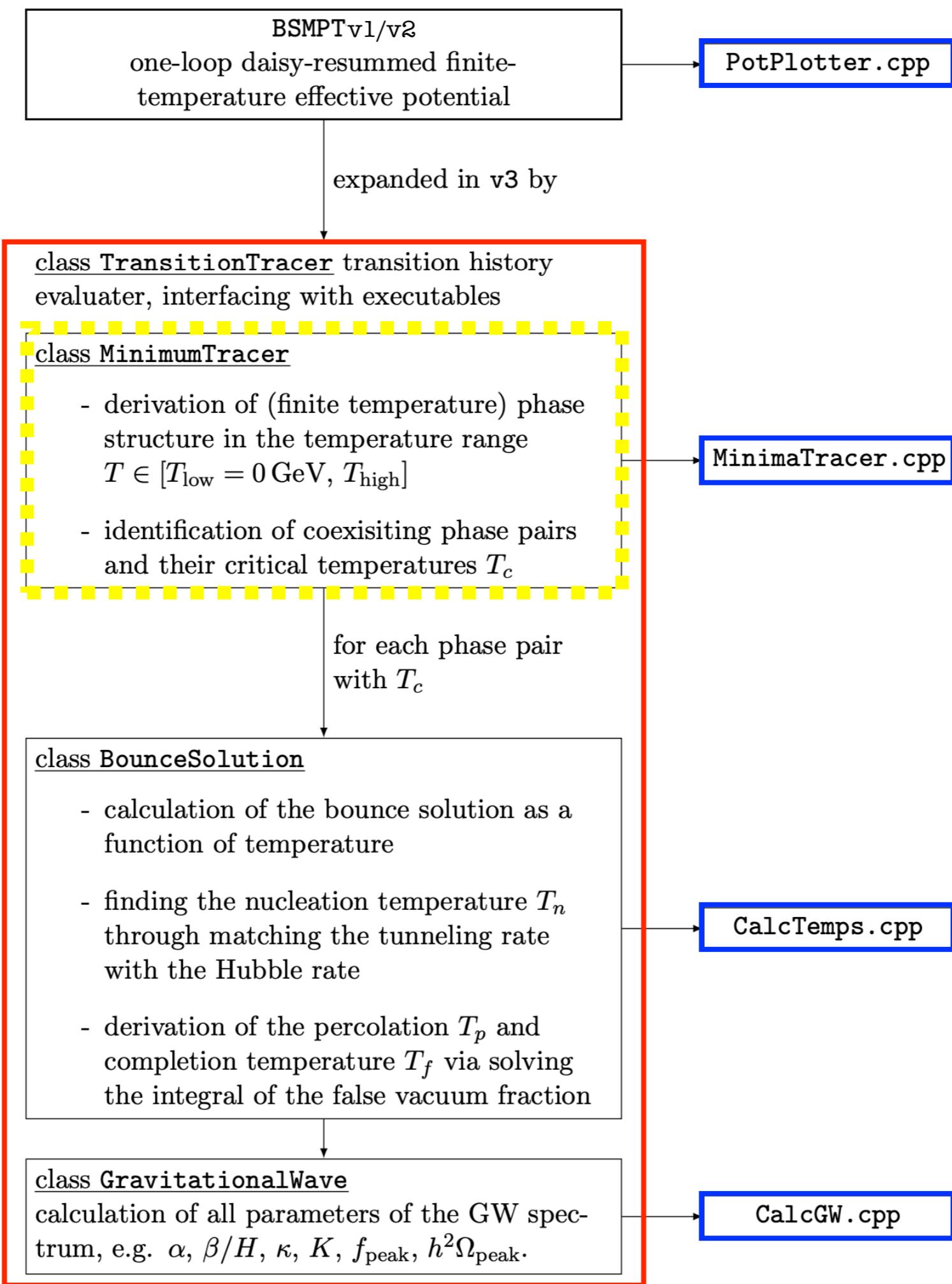
Upgrade to
BSMPTv3

BSMPTv3
4 classes
4 executables

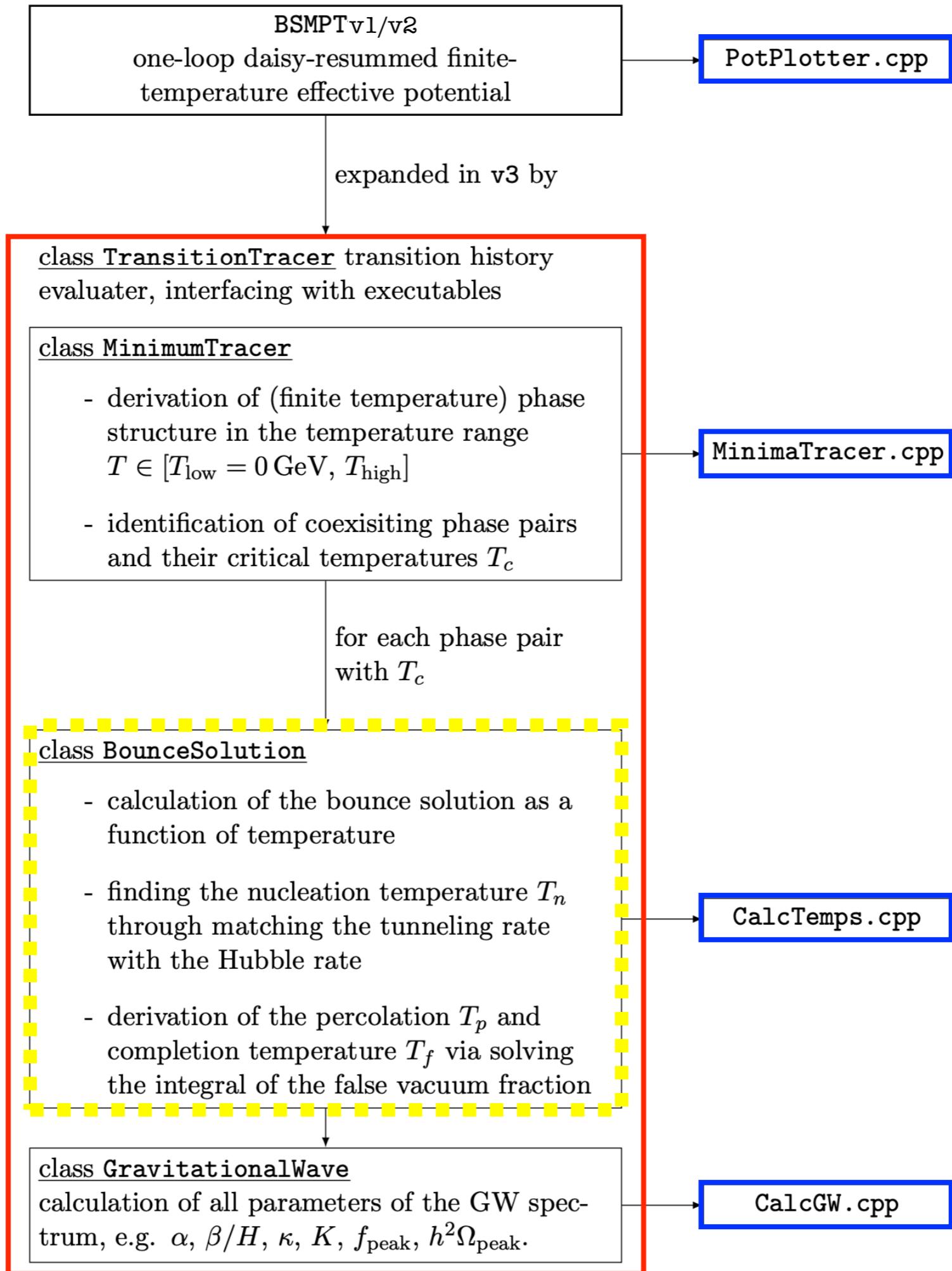


Code structure
Classes
Functionalities

BSMPTv3
4 classes
4 executables

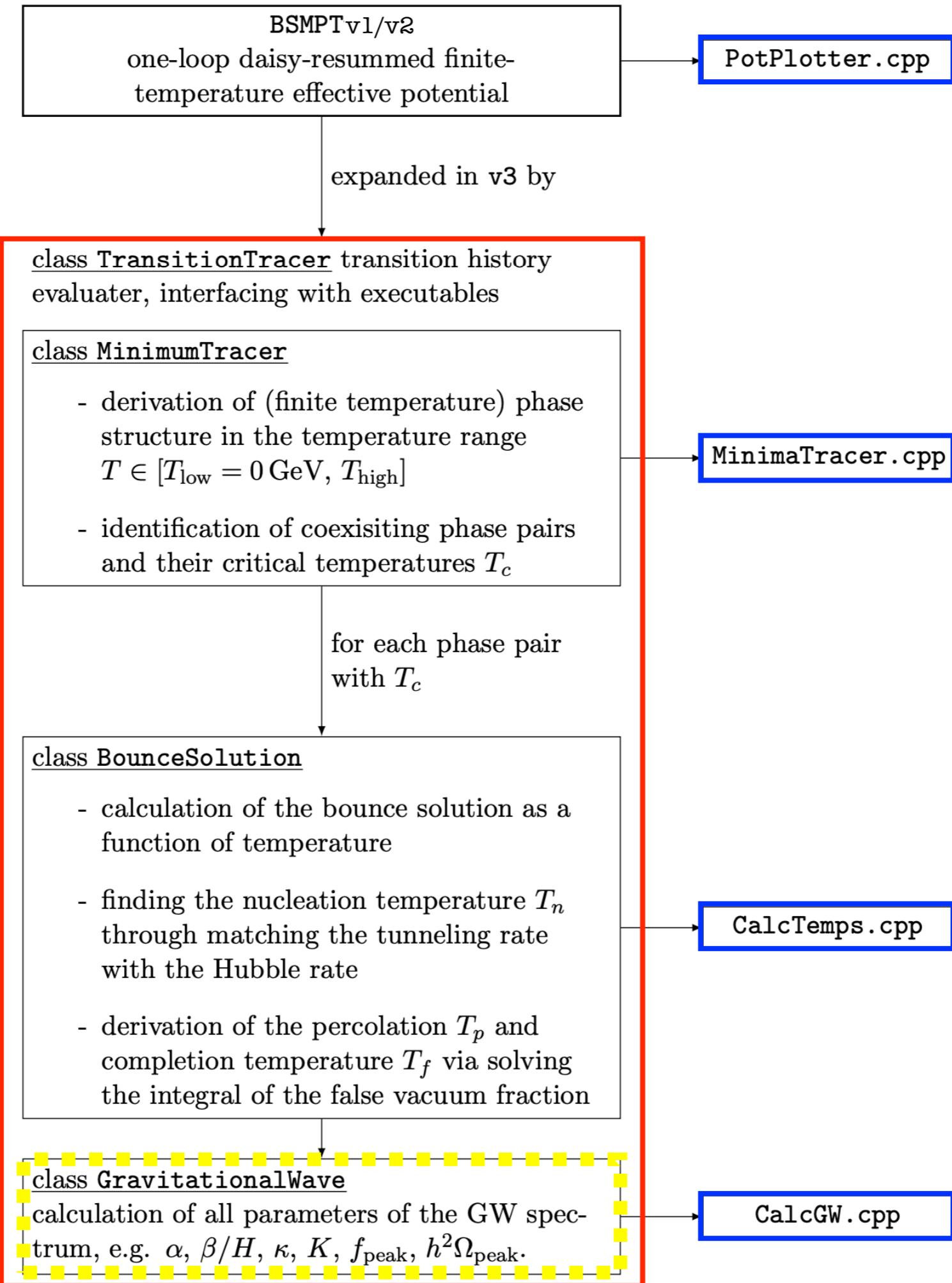


BSMPTv3
4 classes
4 executables



Calculation of the
bounce solution and
characteristic
temperatures for
first-order phase
transitions between
pairs of coexisting
phases

BSMPTv3
4 classes
4 executables



Calculation of the gravitational waves spectrum sourced by first-order phase transitions

wall velocity in BSMPTV3

Slide taken from Lisa Biermann

By default $v_w = 0.95$, or set to user input or one of the following estimates:

- Estimate by [Lewicki et al., '22] (assuming steady-state ($\dot{v}_b = 0$) and local thermal equilibrium):

$$v_b \simeq \begin{cases} \sqrt{\frac{\Delta V}{\alpha \rho_\gamma}} & \text{if } \sqrt{\frac{\Delta V}{\alpha \rho_r(T_*)}} < v_{\text{CJ}} \\ 1 & \text{if } \sqrt{\frac{\Delta V}{\alpha \rho_r(T_*)}} > v_{\text{CJ}} \end{cases}$$

$\rho_r(T_*) = \frac{\pi^2}{30} g^*(T_*) T_*^4$
rel. matter density
$\Psi = \frac{\omega_t}{\omega_f}$ enthalpy ratio
$a = 0.2233$ num. fit result
$b = 1.704$ num. fit result
$p = -3.433$ num. fit result

- Estimate by [Laurent et al., '23] (assuming local thermal equilibrium):

$$v_b = \left(\left| \frac{3\alpha + \Psi - 1}{2(2 - 3\Psi + \Psi^3)} \right|^{\frac{p}{2}} + \left| v_{\text{CJ}} \left(1 - a \frac{(1 - \Psi)^b}{\alpha} \right) \right|^{\frac{p}{2}} \right)^{\frac{1}{p}}$$

with Chapman-Jouguet velocity

$$v_{\text{CJ}} = \frac{1 + \sqrt{3\alpha(1 - c_{s,f}^2 + 3c_{s,f}^2\alpha)}}{1/c_{s,f} + 3c_{s,f}\alpha}$$

[Giese et al., 2004.06995]

- Estimates of v_b in *local thermal equilibrium* serve as **upper bound** as v_b gets reduced by non-equilibrium effects!
- Alternatively use e.g. WallGo [Ekstedt et al., 2411.04970]

Modelling of Stochastic Gravitational Waves Background

- ♦ Spectra of gravitational waves sourced by first-order phase transitions: [Caprini et al., 2403.03723]

$$\Omega_{\text{GW}}(f) = \Omega_{\text{GW}}^{\text{coll}}(f) + \Omega_{\text{GW}}^{\text{sw}}(f) + \Omega_{\text{GW}}^{\text{turb}}(f)$$

Modelling by broken power law (BPL) or double broken power law (DBPL)



- ♦ BPL for GWs sourced by bubble collisions & highly relativistic fluid shells $\Omega_{\text{GW}}^{\text{coll}}$:

$$\Omega_{\text{GW}}^{\text{BPL}}(f) = \Omega_{\text{peak}} \frac{(n_1 - n_2)^{\frac{n_1 - n_2}{a_1}}}{\left[-n_2 \left(\frac{f}{f_{\text{peak}}} \right)^{-\frac{n_1 a_1}{n_1 - n_2}} + n_1 \left(\frac{f}{f_{\text{peak}}} \right)^{-\frac{n_2 a_1}{n_1 - n_2}} \right]^{\frac{(n_1 - n_2)}{a_1}}}$$

[Caprini et al., 2403.03723]

- ♦ Geometric parameters for $\Omega_{\text{GW}}^{\text{coll}}$:

[Caprini et al., 2403.03723] [Lewicki, Vaskonen, 2208.11697]

$$n_1 = 2.4, n_2 = -2.4, a_1 = 1.2$$

$$\Omega_{\text{peak}}^{\text{coll}} \simeq 0.05 F_{\text{GW},0} K_{\text{coll}}^2 \left(\frac{\beta}{H(T_*)} \right)^{-2} \quad f_{\text{peak}}^{\text{coll}} \simeq 0.11 H_{*,0} \frac{\beta}{H(T_*)}$$

Modelling of Stochastic Gravitational Waves Background

- DBPL for GWs sourced by sound waves $\Omega_{\text{GW}}^{\text{sw}}$ and for GW sourced by turbulence $\Omega_{\text{GW}}^{\text{turb.}}$:

$$\Omega_{\text{GW}}^{\text{DBPL}}(f) = \Omega_{\text{int}} \times S(f) = \Omega_2 \times S_2(f)$$

[Caprini et al., 2403.03723]

with the shape function

$$S(f) = N \left(\frac{f}{f_1} \right)^{n_1} \left[1 + \left(\frac{f}{f_1} \right)^{a_1} \right]^{\frac{-n_1 + n_2}{a_1}} \left[1 + \left(\frac{f}{f_2} \right)^{a_2} \right]^{\frac{-n_2 + n_3}{a_2}}$$

- Geometric parameters for $\Omega_{\text{GW}}^{\text{sw}}$:

$$f_1^{\text{sw}} \simeq 0.2 H_{*,0} (H_* R_*)^{-1}, \quad f_2^{\text{sw}} \simeq 0.5 H_{*,0} \Delta_w^{-1} (H_* R_*)^{-1}, \quad S_2(f) = S(f)/S(f_2)$$

$$\Omega_2 = \frac{1}{\pi} \left(\sqrt{2} + \frac{2f_2/f_1}{1+f_2^2/f_1^2} \right) \Omega_{\text{int}}, \quad \text{Norm N from: } S_2(f_2) = 1$$

$$\{n_1, n_2, n_3, a_1, a_2\} = \{3, 1, -3, 2, 4\}$$

Modelling of Stochastic Gravitational Waves Background

- DBPL for GWs sourced by sound waves $\Omega_{\text{GW}}^{\text{SW}}$ and for GW sourced by turbulence $\Omega_{\text{GW}}^{\text{turb.}}$:

$$\Omega_{\text{GW}}^{\text{DBPL}}(f) = \Omega_{\text{int}} \times S(f) = \Omega_2 \times S_2(f)$$

[Caprini et al., 2403.03723]

with the shape function

$$S(f) = N \left(\frac{f}{f_1} \right)^{n_1} \left[1 + \left(\frac{f}{f_1} \right)^{a_1} \right]^{\frac{-n_1 + n_2}{a_1}} \left[1 + \left(\frac{f}{f_2} \right)^{a_2} \right]^{\frac{-n_2 + n_3}{a_2}}$$

- Geometric parameters for $\Omega_{\text{GW}}^{\text{turb.}}$:

$$f_1 = \frac{\sqrt{3\Omega_s}}{2\mathcal{N}} H_{*,0} (H_* R_*)^{-1}, f_2 \simeq 2.2 H_{*,0} (H_* R_*)^{-1}, \Omega_2 = F_{\text{GW},0} A_{\text{MHD}} \Omega_s^2 (H_* R_*)^2$$

$$\{n_1, n_2, n_3, a_1, a_2\} = \{3, 1, -8/3, 4, 2.15\}, \mathcal{N} \approx 2$$

Relations Geometric - Thermodynamic Parameters for $\Omega_{\text{GW}}^{\text{coll}}$

with $h^2 F_{\text{GW},0} = 1.64e^{-5} \left(\frac{100}{g_*} \right)^{1/3}$, $H_{*,0} = 1.65e^{-5} \text{Hz} \left(\frac{g_*}{100} \right)^{1/6} \left(\frac{T_{\text{reh}}}{100 \text{GeV}} \right)$ [Caprini et al., 2403.03723]

reheating temperature $T_{\text{reh}} \leq T_*$ $T_{\text{reh}} = \begin{cases} T_* & , \alpha(T_*) < 1 \\ T_* [1 + \alpha(T_*)]^{1/4} & , \text{else} \end{cases}$ valid only for [Athron et al., 2305.02357]
large enough bubble wall velocities

fractional energy density
of the collision source $K_{\text{coll}} = \frac{\kappa_{\text{coll}} \alpha}{1 + \alpha}$ [Ellis et al., 1903.09642]

efficiency factor
(vacuum energy
fraction in walls) $\kappa_{\text{coll}} = \left(1 - \frac{\alpha_\infty}{\alpha} \right) \left(1 - \frac{1}{\gamma_{\text{eq}}^c} \right) \frac{R_{\text{eq}}}{R_*} \frac{\gamma_*}{\gamma_{\text{eq}}}$, (*) [Kierkla et al.'22; Ellis et al.'19, '20]

$\alpha_\infty = \frac{P_{1 \rightarrow 1}}{\rho_\gamma}$, $\kappa_{\text{coll}} = 0$ if $\alpha < \alpha_\infty$ plasma friction term [Gouttenoire et al.'21]

initial bubble radius $R_0 = \left[\frac{3S_3}{8\pi\Delta V} \right]^{\frac{1}{3}}$ [Kierkla et al, 2210.07075]

average bubble radius at transition temperature T_* $R_* = \left[T_* \int_{T_*}^{T_c} \frac{dT'}{T^2} \frac{\Gamma(T')}{H(T')} e^{-I(T')} \right]^{-1/3}$ [Enqvist et al.'92;
Turner et al.'92;
Ellis et al.'20]

(*): valid in the regime where
bubble wall is accelerated by $\Delta P = \Delta V - P_{1 \rightarrow 1} - P_{1 \rightarrow N}^{(n)}$ with vacuum pressure ΔV and plasma friction term P
pressure difference:

Geometric - Thermodynamic Parameters for $\Omega_{\text{GW}}^{\text{SW}}$, $\Omega_{\text{GW}}^{\text{turb}}$

with $\Delta_w = \xi_{\text{shell}} / \max(v_w, c_s)$ and sound shell thickness $\xi_{\text{shell}} = |v_w - c_s|$

[Caprini et al., 2403.03723]

Ω_{int} for sound waves: $\Omega_{\text{int}}^{\text{SW}} = F_{\text{GW},0} A_{\text{sw}} K_{\text{sw}}^2 (H_* R_*) \Upsilon$

with the kinetic energy fraction: $K_{\text{sw}} = \frac{0.6 \kappa_{\text{sw}} \alpha_{\text{eff}}}{1 + \alpha_{\text{eff}}} \frac{\alpha_{\text{eff}}}{\alpha}$ where $A_{\text{sw}} = 0.11$ and $\alpha_{\text{eff}} \equiv \alpha(1 - \kappa_{\text{coll}})$

efficiency factor κ_{sw} obtained by using model-independent approach of [Giese et al., 2004.06995; 2010.09744] depending on α_{eff} , v_w and sound speed in the true and false vacuum:

$$c_{s,t}^2 = \frac{1}{T_*} \frac{V'(\vec{\omega}_t, T_*)}{V''(\vec{\omega}_t, T_*)}, \quad c_{s,f}^2 = \frac{1}{T_*} \frac{V'(\vec{\omega}_f, T_*)}{V''(\vec{\omega}_f, T_*)} \quad [\text{Giese et al., 2010.09744}]$$

suppression factor: $\Upsilon = 1 - \frac{1}{\sqrt{1 + 2H_* \tau_{\text{sw}}}}$, where $H_* \tau_{\text{sw}} = \min \left[\frac{H_* R_*}{\sqrt{\bar{v}_f^2}}, 1 \right]$

with the average (relativistic) characteristic fluid velocity $\bar{v}_f^2 = \Gamma^{-1} K = \frac{3}{4} K$

furthermore $A_{\text{MHD}} = 4.37 \times 10^{-3}$ and $\Omega_s = \kappa_{\text{turb}} K_{\text{sw}}$

κ_{turb} : fraction of overall kinetic energy in bulk motion that is converted to MHD turbulence

Implementation of GWs in first BMSPTV3 Release 2024

- Relevant quantities for GW spectrum:

- PT strength, resp. released latent heat during PT

$$\alpha = \frac{1}{\rho_\gamma} \left[V(\vec{\phi}_f) - V(\vec{\phi}_t) - \frac{T}{4} \left(\frac{\partial V(\vec{\phi}_f)}{\partial T} - \frac{\partial V(\vec{\phi}_t)}{\partial T} \right) \right]_{T=T_*}$$

- inverse time scale of the PT

$$\frac{\beta}{H} = T_* \left. \frac{d}{dT} \left(\frac{\hat{S}_3(T)}{T} \right) \right|_{T_*}$$

- bubble wall velocity \mathbf{v}_b

H	Hubble constant
τ_{sh}	fluid turnover time shock formation time
g_*	eff. number of rel. energy d.o.f.
c_s	sound speed
κ	efficiency factor

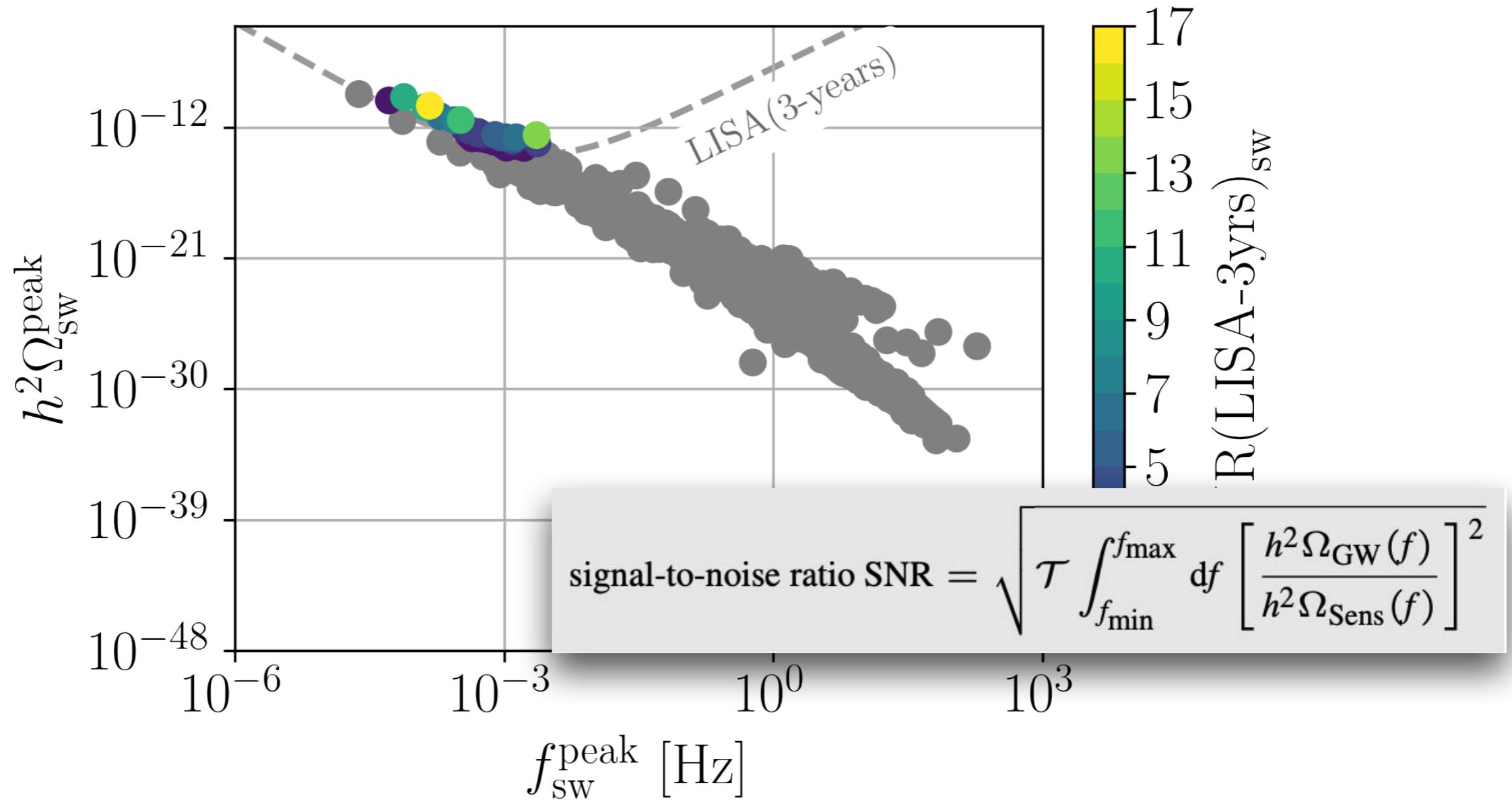
- Peak frequency and amplitude of acoustic GWs [Hindmarsh et al.'17; Caprini et al.'20]

$$f^{\text{peak}} = 26 \times 10^{-6} \frac{\beta}{H} \left(\frac{1}{(8\pi)^{1/3} \max(\mathbf{v}_b, c_s)} \right) \left(\frac{T_*}{100 \text{ GeV}} \right) \left(\frac{g_*}{100} \right)^{1/6} \text{ Hz}$$

$$h^2 \Omega_{\text{GW}}^{\text{peak}} = 4 \times 10^{-7} \left(\frac{100}{g_*} \right)^{1/3} \begin{cases} \frac{(8\pi)^{1/3} \max(\mathbf{v}_b, c_s)}{\beta/H} \left(\frac{\kappa \alpha}{1+\alpha} \right)^2 & \text{if } H\tau_{\text{sh}} \simeq 1 \\ \frac{2}{\sqrt{3}} \left(\frac{(8\pi)^{1/3} \max(\mathbf{v}_b, c_s)}{\beta/H} \right)^2 \left(\frac{\kappa \alpha}{1+\alpha} \right)^{3/2} & \text{if } H\tau_{\text{sh}} < 1 \end{cases}$$

GW from (S)FOEWPT in „CP in the Dark“

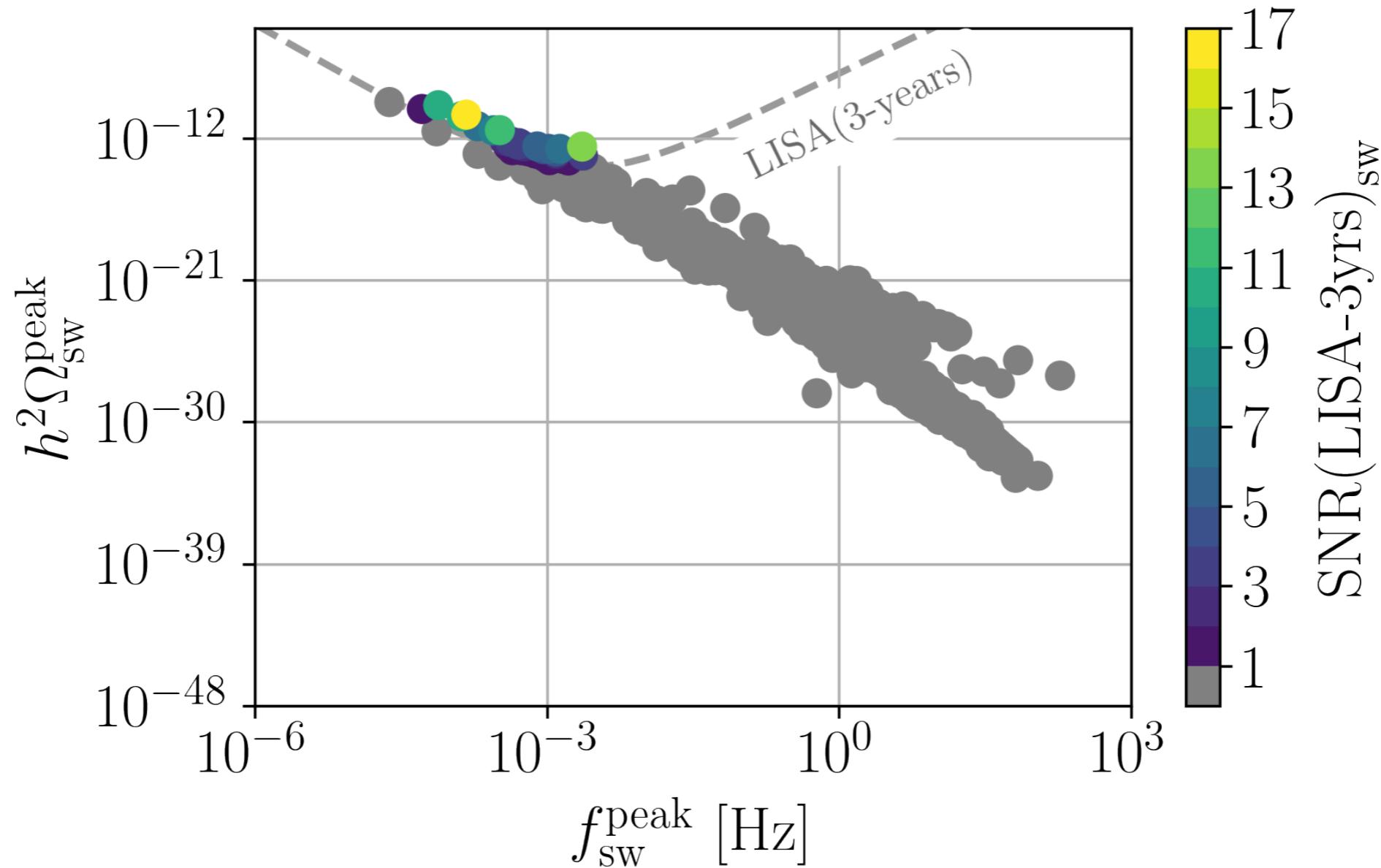
[Basler,Biermann,MM,Müller,Santos,Viana,'24]



- \exists points w/ $\text{SNR}(\text{LISA-3yrs}) > 10$, compatible w/ all relevant theor. and exp. constraints
- all points lead to EW minimum at $T=0$ (no vacuum trapping)
- all of the LISA-sensitive points (colored points) have SFOEWPT: $\xi_c > 1$

GW from (S)FOEWPT in „CP in the Dark“

[Basler,Biermann,MM,Müller,Santos,Viana,'24]

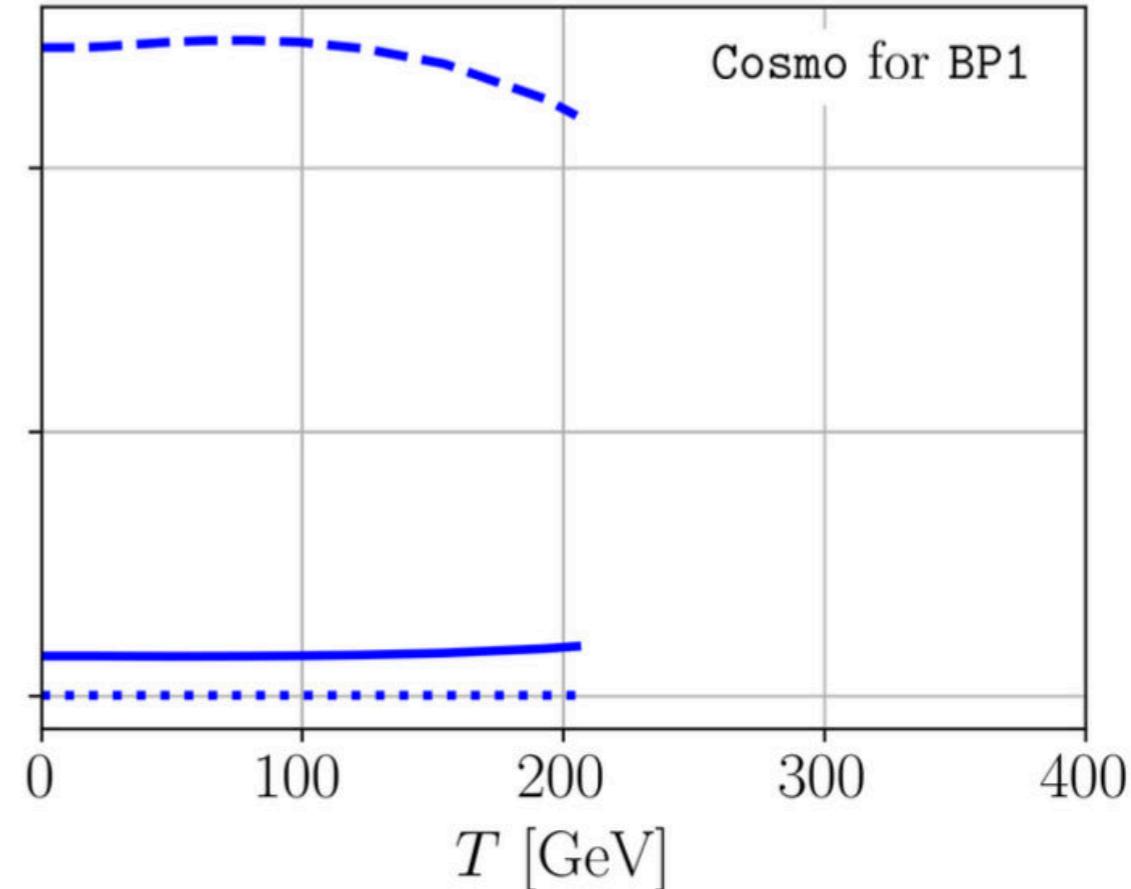
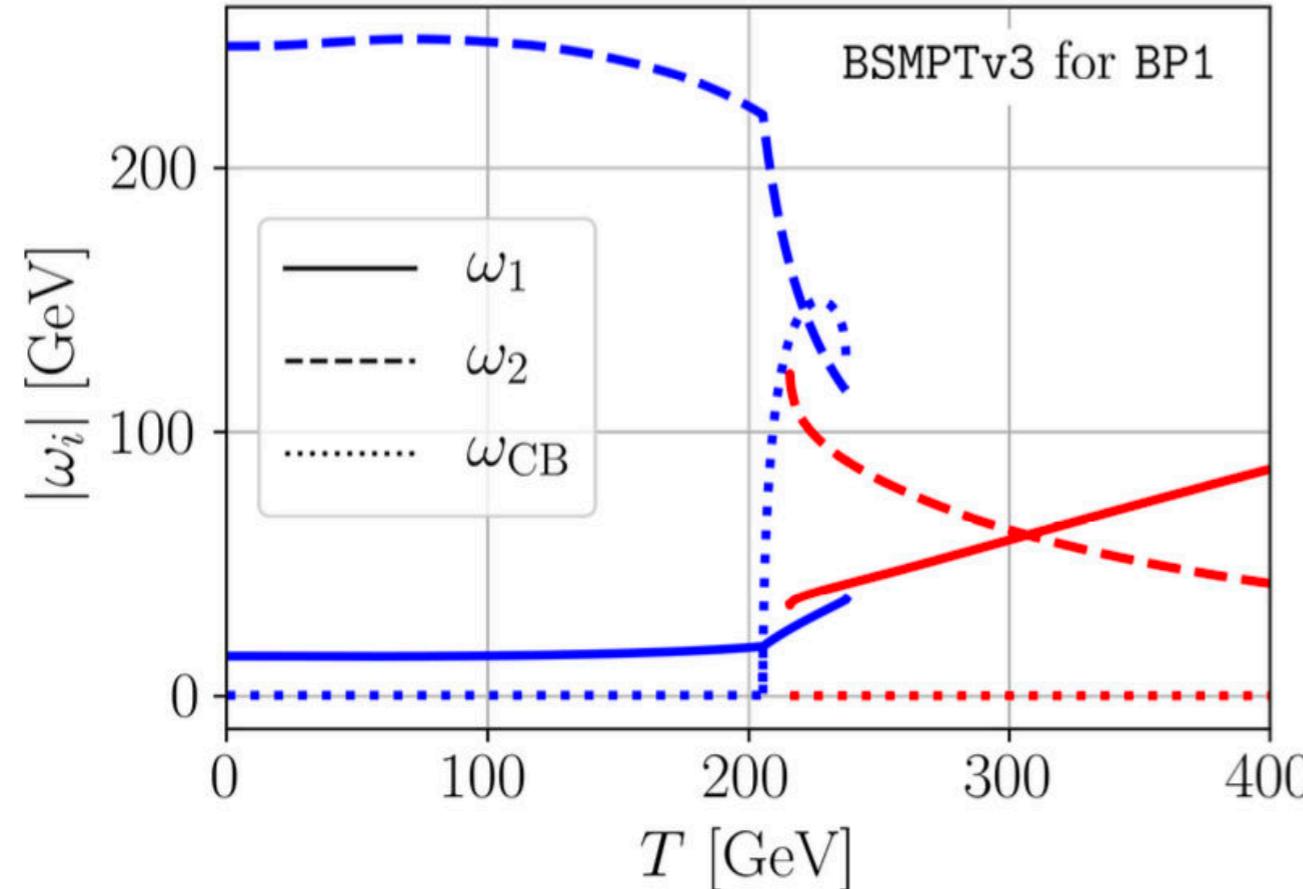


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Transition History - Comparison w/ CosmoTransitions

high-T phase, low-T phase

[Basler,Biermann,MM,Müller,Santos,Viana,'24]



History:

BSMPTv3

- first-order PT from neutral (red) to charge-breaking CB phase (blue)
- second-order PT into a neutral minimum

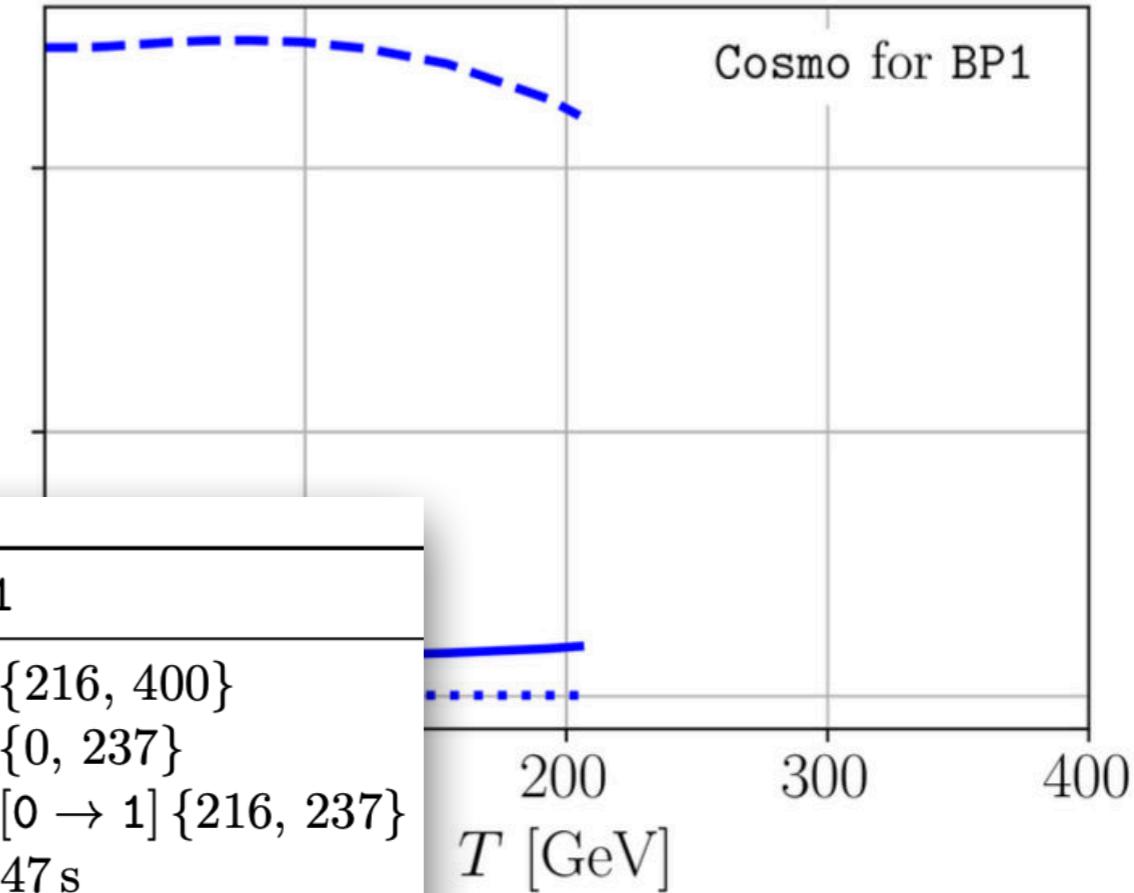
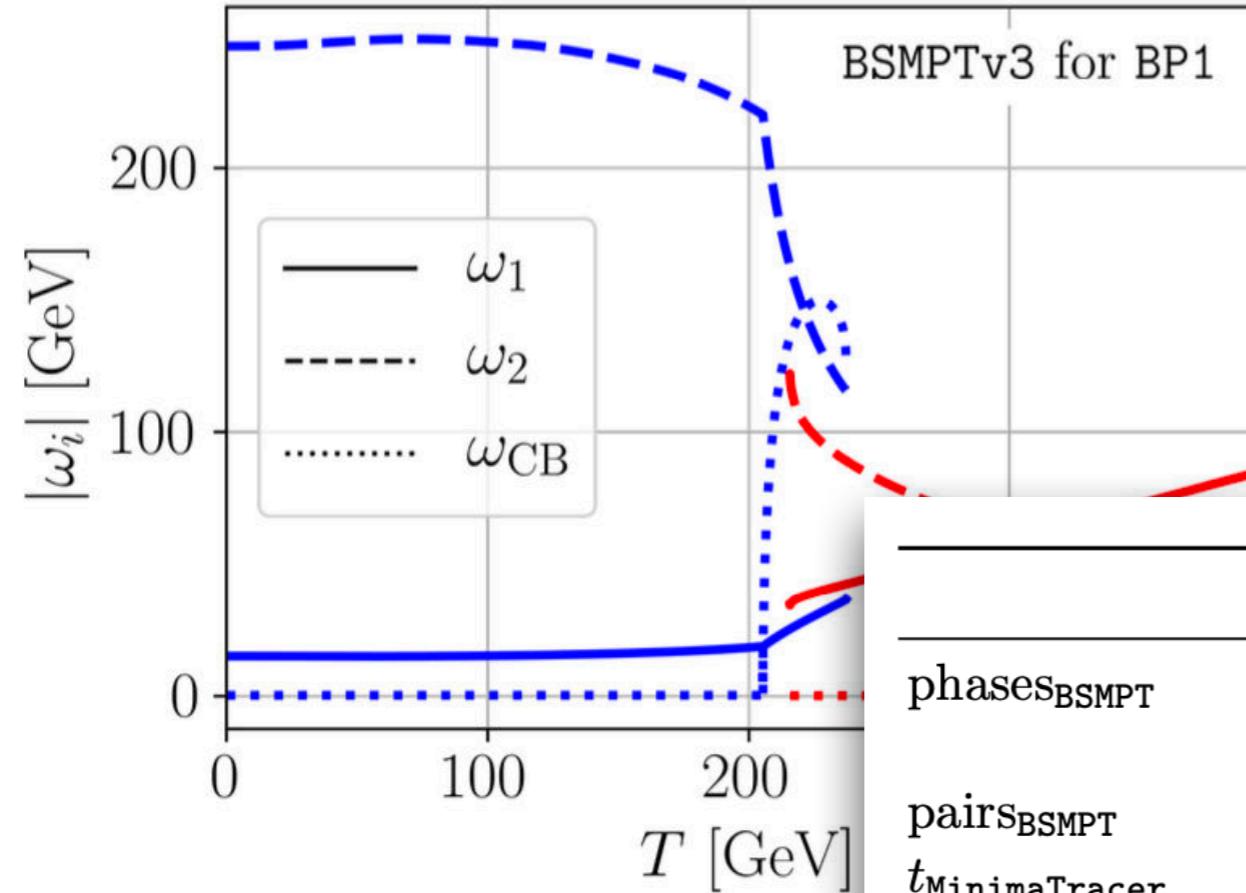
CosmoTransitions

- agrees w/ low-T phase until $T \sim 200$ GeV
- fails to trace any minima for higher temperatures

Transition History - Comparison w/ CosmoTransitions

high-T phase, low-T phase

[Basler,Biermann,MM,Müller,Santos,Viana,'24]



BSMPTv3

- first-order PT from neutral to charge-breaking CB phase
- second-order PT into a minimum

BP1	
$\text{phases}_{\text{BSMPT}}$	0: {216, 400} 1: {0, 237}
$\text{pairs}_{\text{BSMPT}}$	0: [0 → 1] {216, 237}
$t_{\text{MinimaTracer}}$	41.47 s
T_c	{226.3}
T_n	{222.9, 222.9}
T_p	{222.6}
T_f	{222.6}
$t_{\text{CalcTemps}}$	6.87 min
history	0 – (0) → 1
$\text{phases}_{\text{Cosmo}}$	{0, 206}
$T_{\text{crit, Cosmo}}$	{–}
$T_{\text{approx, Cosmo}}$	{–}
t_{Cosmo}	3.95 s

CosmoTransitions
uses w/ low-T phase
 $T \sim 200$ GeV
to trace any minima
at higher temperatures

only finds
one phase
though

The CP-violating 2HDM (C2HDM)

- ❖ C2HDM Higgs potential: w/ softly broken \mathbb{Z}_2 symmetry ($\Phi_1 \rightarrow \Phi_1, \Phi_2 \rightarrow -\Phi_2$)

[Ginzburg,Krawczyk,Osland,'02]

$$V = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left(m_{12}^2 \Phi_1^\dagger \Phi_2 + h.c. \right) + \frac{\lambda_1}{2} \left(\Phi_1^\dagger \Phi_1 \right)^2 + \frac{\lambda_2}{2} \left(\Phi_2^\dagger \Phi_2 \right)^2 \\ + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \left[\frac{\lambda_5}{2} \left(\Phi_1^\dagger \Phi_2 \right)^2 + h.c. \right]$$

All parameters are real except for m_{12}^2 and λ_5 : $m_{12}^2 = |m_{12}^2| e^{i\phi(m_{12}^2)}$, $\lambda_5 = |\lambda_5| e^{i\phi(\lambda_5)}$

The two complex phases are not independent of each other

$$2\text{Re}(m_{12}^2) \tan \phi(m_{12}^2) = v_1 v_2 \text{Re}(\lambda_5) \tan \phi(\lambda_5)$$

Ensure explicit CP violation (both phases cannot be removed simultaneously) by choosing:

$$\boxed{\phi(\lambda_5) \neq 2\phi(m_{12}^2)}$$

The CP-Violating 2HDM (C2HDM)

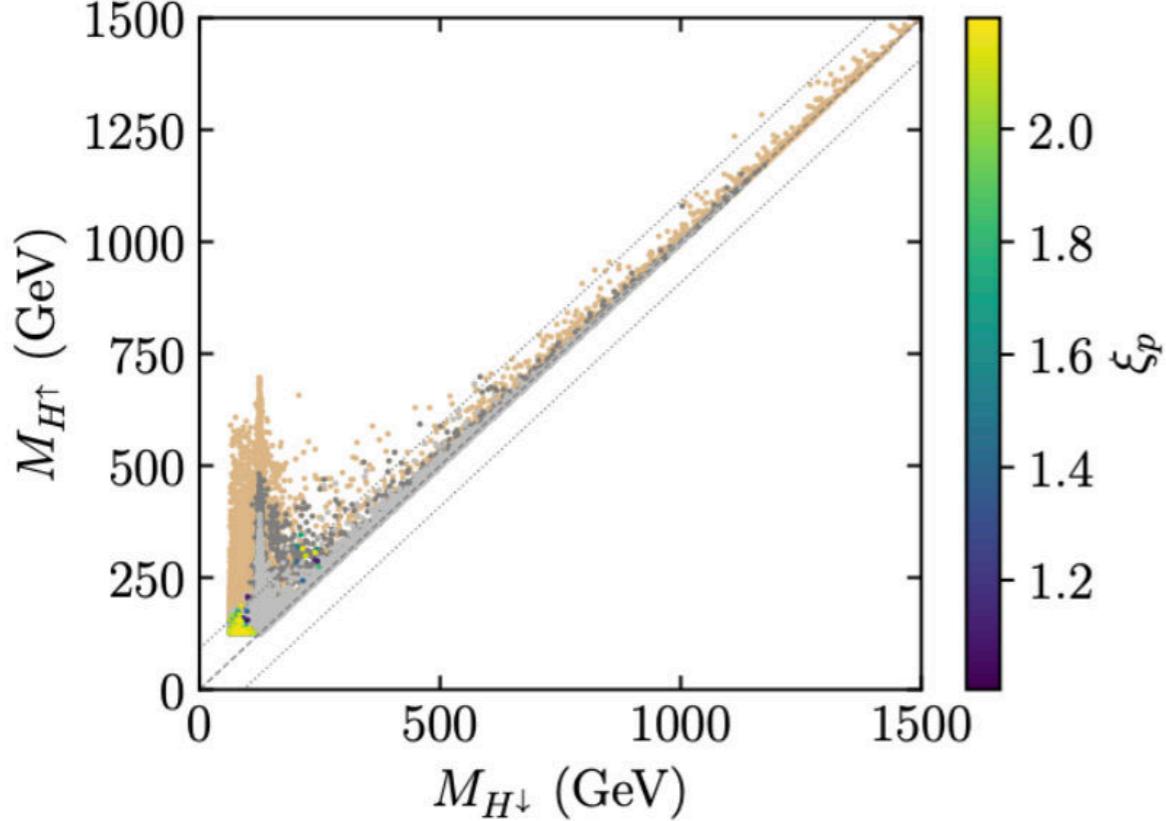
- ❖ Mass spectrum: CP violation \sim neutral formerly CP-even (h, H) and CP-odd (A) states mix to mass eigenstates H_i ($i = 1, 2, 3$) with indefinite CP quantum number.
Charged Higgs sector is unchanged.

3 neutral CP-mixed Higgs bosons: H_1, H_2, H_3 ,
with $m_{H_1} \leq m_{H_2} \leq m_{H_3}$
2 charged Higgs bosons: H^+, H^-

- ❖ Allowed amount of CP violation: stringently constrained by EDM measurements

SFOEWPT in the C2HDM?

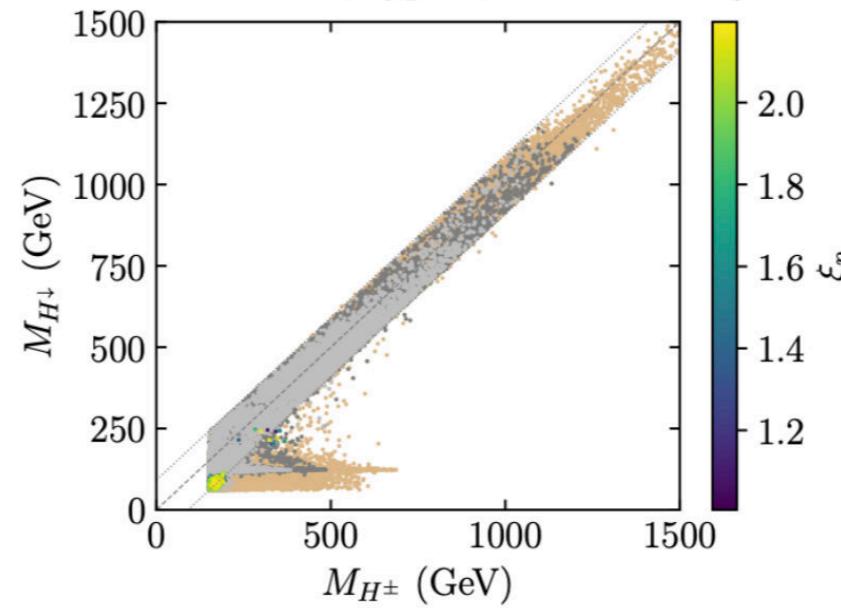
C2HDM, type I, all \rightarrow EW only



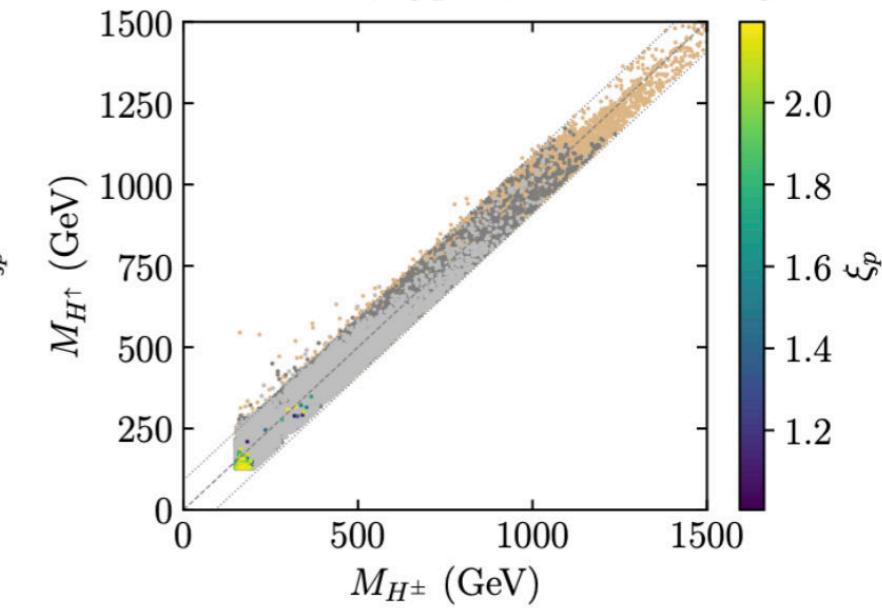
[Biermann,Borschensky,MM,Santos,Viana,to appear]

C2HDM SFOEWPT:
compressed light Higgs spectrum

C2HDM, type I, all \rightarrow EW only



C2HDM, type I, all \rightarrow EW only

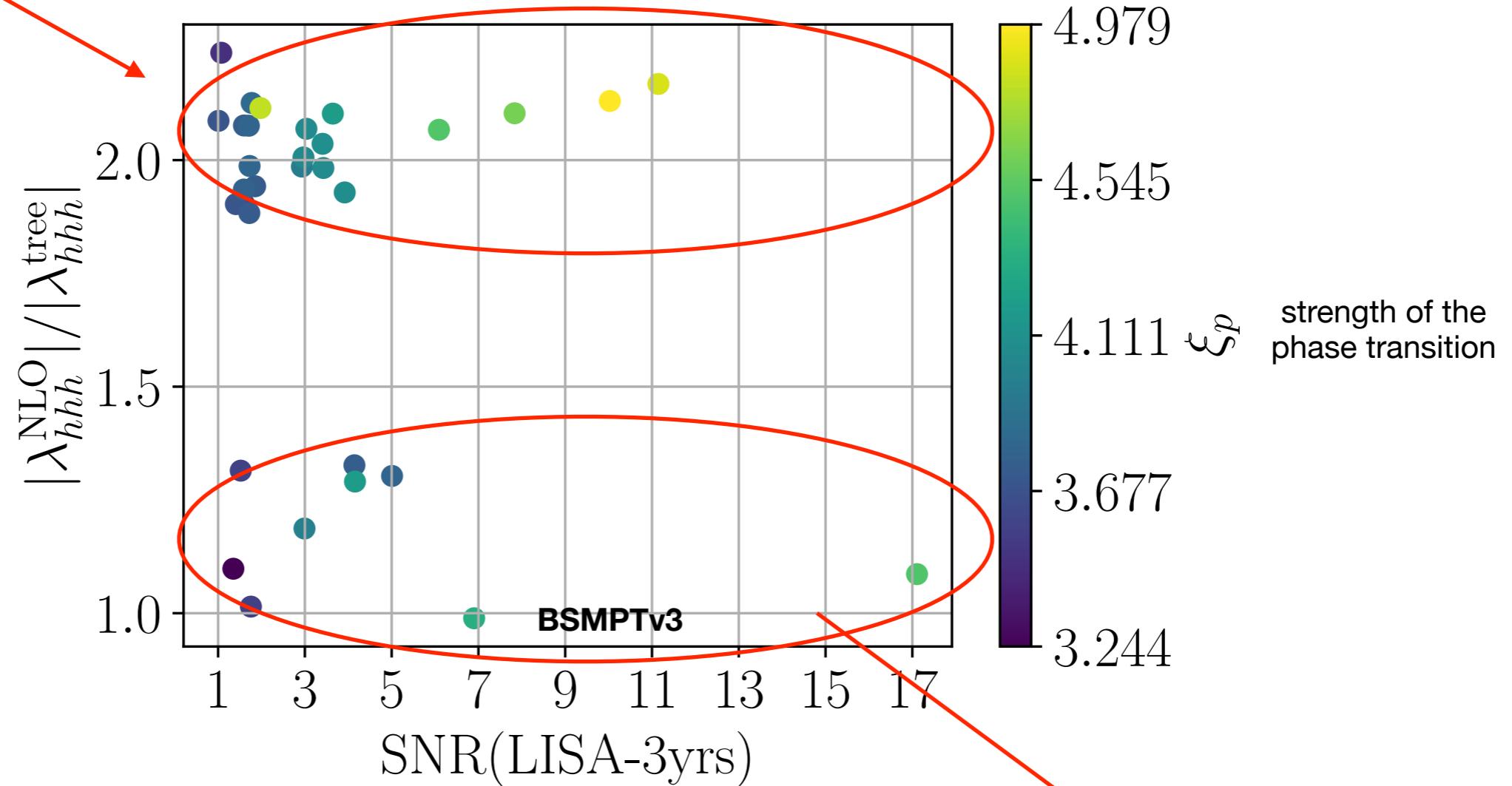


Higgs Self-coupling & Evolution of the Universe

- Model CP in the Dark: SM-Higgs ($\lambda_{hhh}^{tree} = \lambda_{hhh}^{SM}$) + Dark Sector; GW signal from SFOPT

[Basler,Biermann,MM,Müller,Santos,Viana,'24]

SFOPT typically requires enhanced λ_{HHH}



[Biermann,Borschensky,Erhardt,MM,Santos,Viana, to appear]

- Vector DM Model: two visible Higgs bosons + Dark photon

Strong first-order phase transition: $\delta\lambda_{hhh}^{NLO}/\lambda_{hhh}^{LO} = 8\%$

SFOPT possible also for
 $\delta\lambda_{HHH} < 50\%$

Strong First-Order Electroweak Phase Transition (SFOEWPT)

