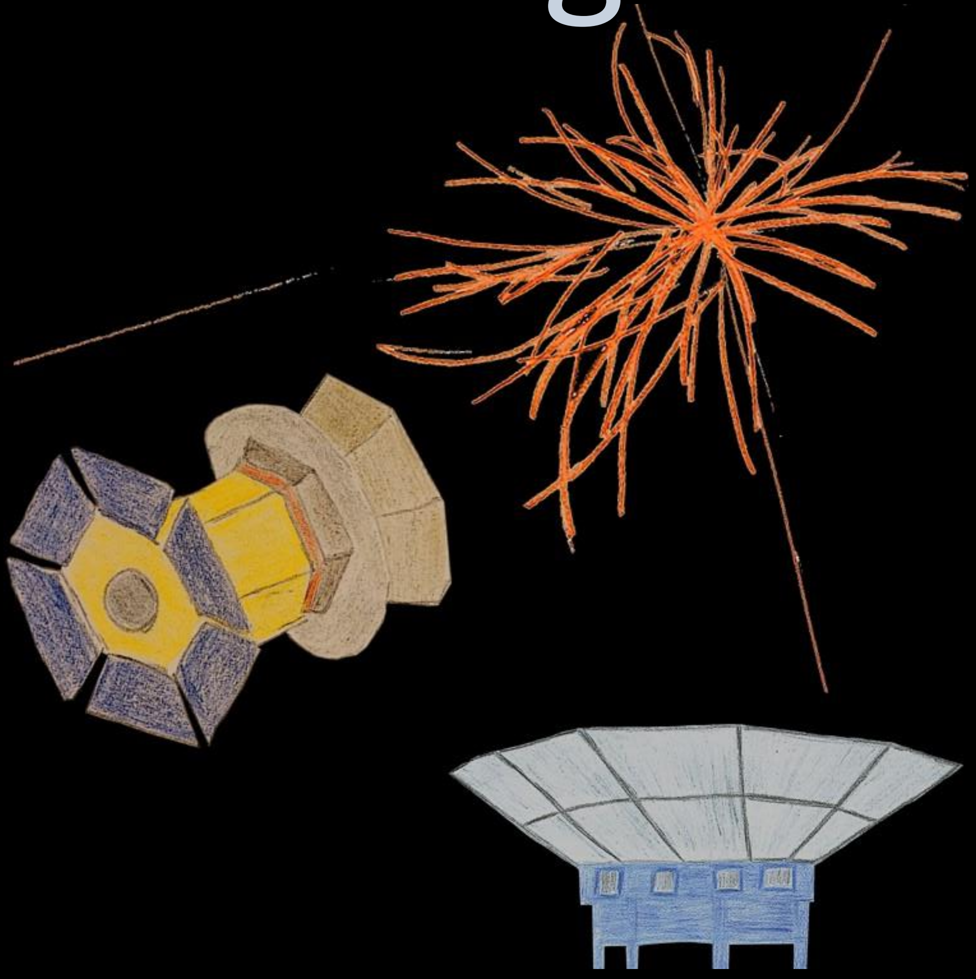
Connecting Cosmic Inflation to Particle Physics with Multi Wavelength Observations



Marco Drewes
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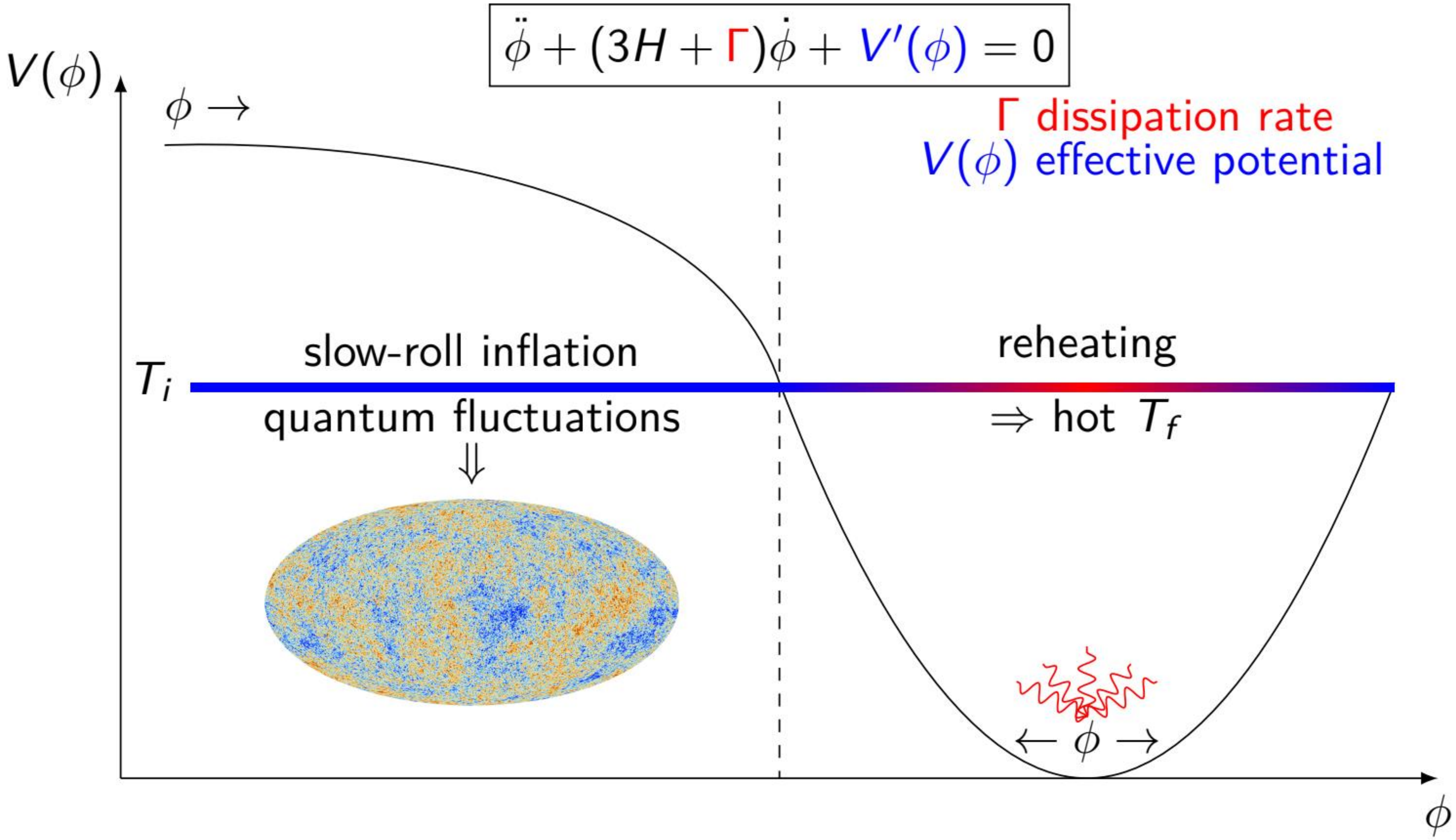
Moriond EW 2025

How hot was the Big Bang?
Measuring the Reheating Temperature

Observable Imprint of Reheating



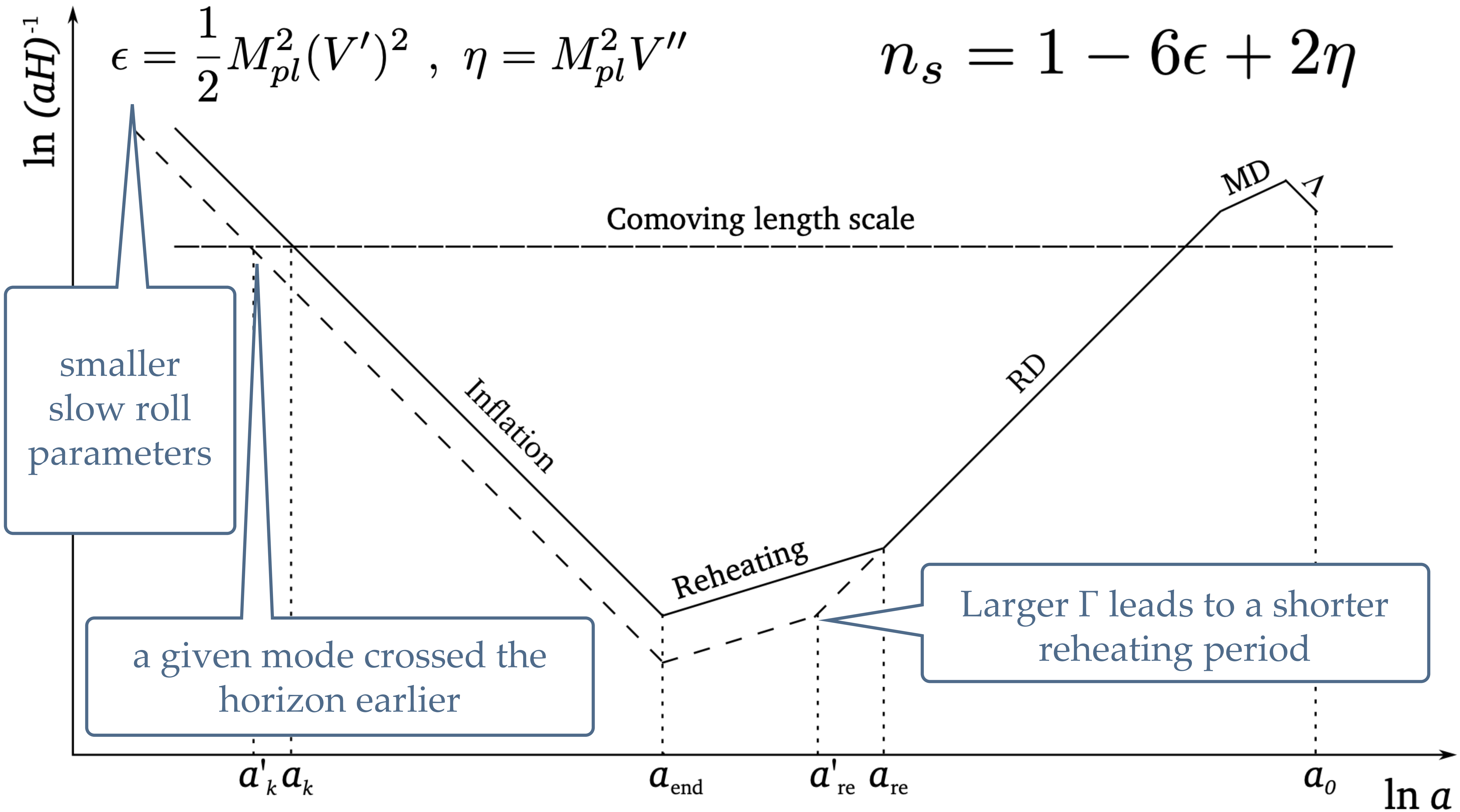
- Inflaton potential determines primordial cosmic perturbations after inflation
- Redshifting of perturbations during reheating affects observed CMB, amount of redshifting depends on the duration of the reheating epoch
- **This makes the CMB sensitive to the reheating temperature**

Martin/Ringeval 1004.5525., Adshead et al 1007.3748, Easther/Peiris 1112.0326, ...

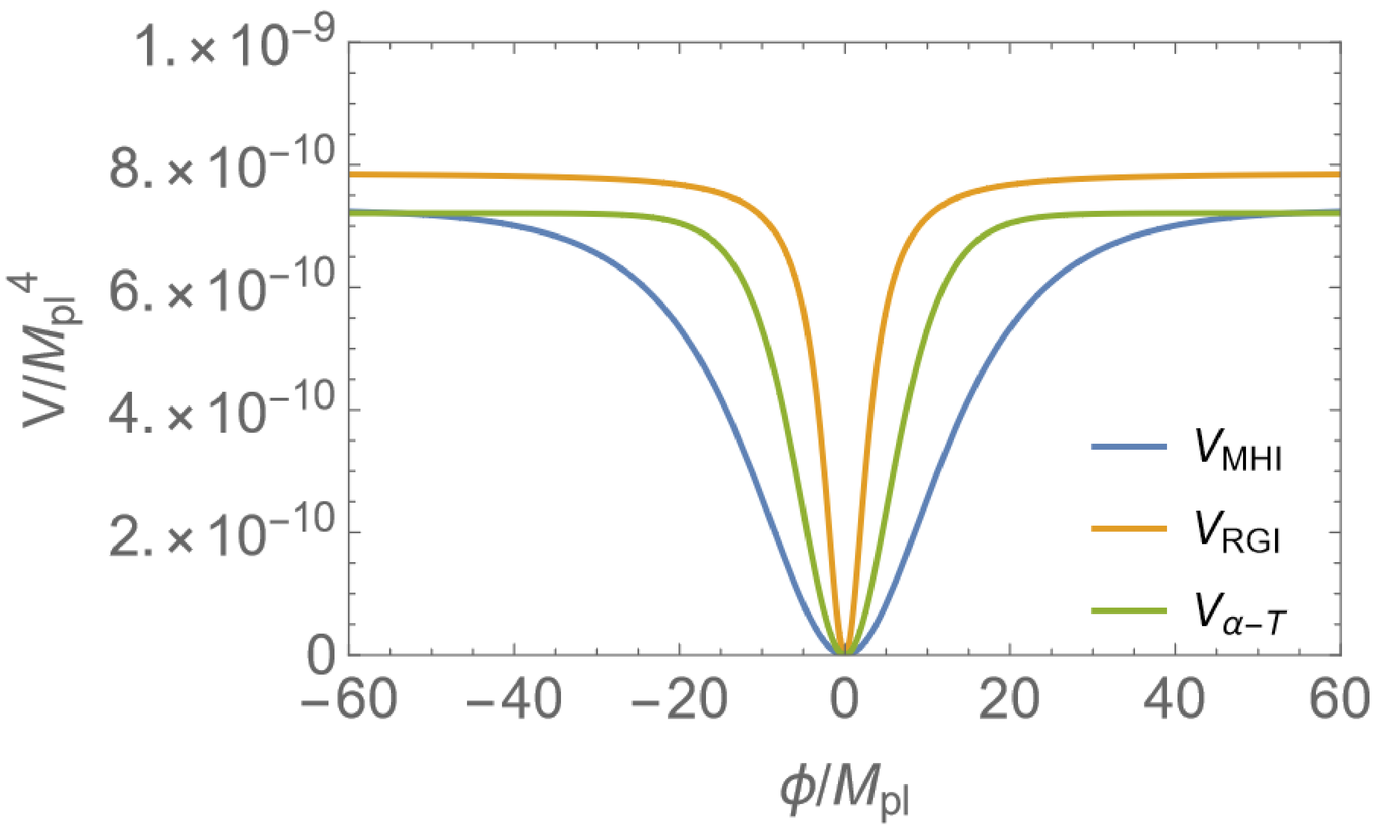
Reheating Effect on CMB Modes

$$\epsilon = \frac{1}{2} M_{pl}^2 (V')^2 \ , \ \eta = M_{pl}^2 V''$$

$$n_s = 1 - 6\epsilon + 2\eta$$



Benchmark Models



Mutated Hilltop Inflation (MHI)

$$\mathcal{V}(\varphi) = M^4 \left[1 - \frac{1}{\cosh(\alpha\varphi/M_{pl})}\right]$$

Radion Gauge Inflation (RGI)

$$\mathcal{V}(\varphi) = M^4 \frac{(\varphi/M_{pl})^2}{\alpha + (\varphi/M_{pl})^2}$$

α -attractor T model (α-T)

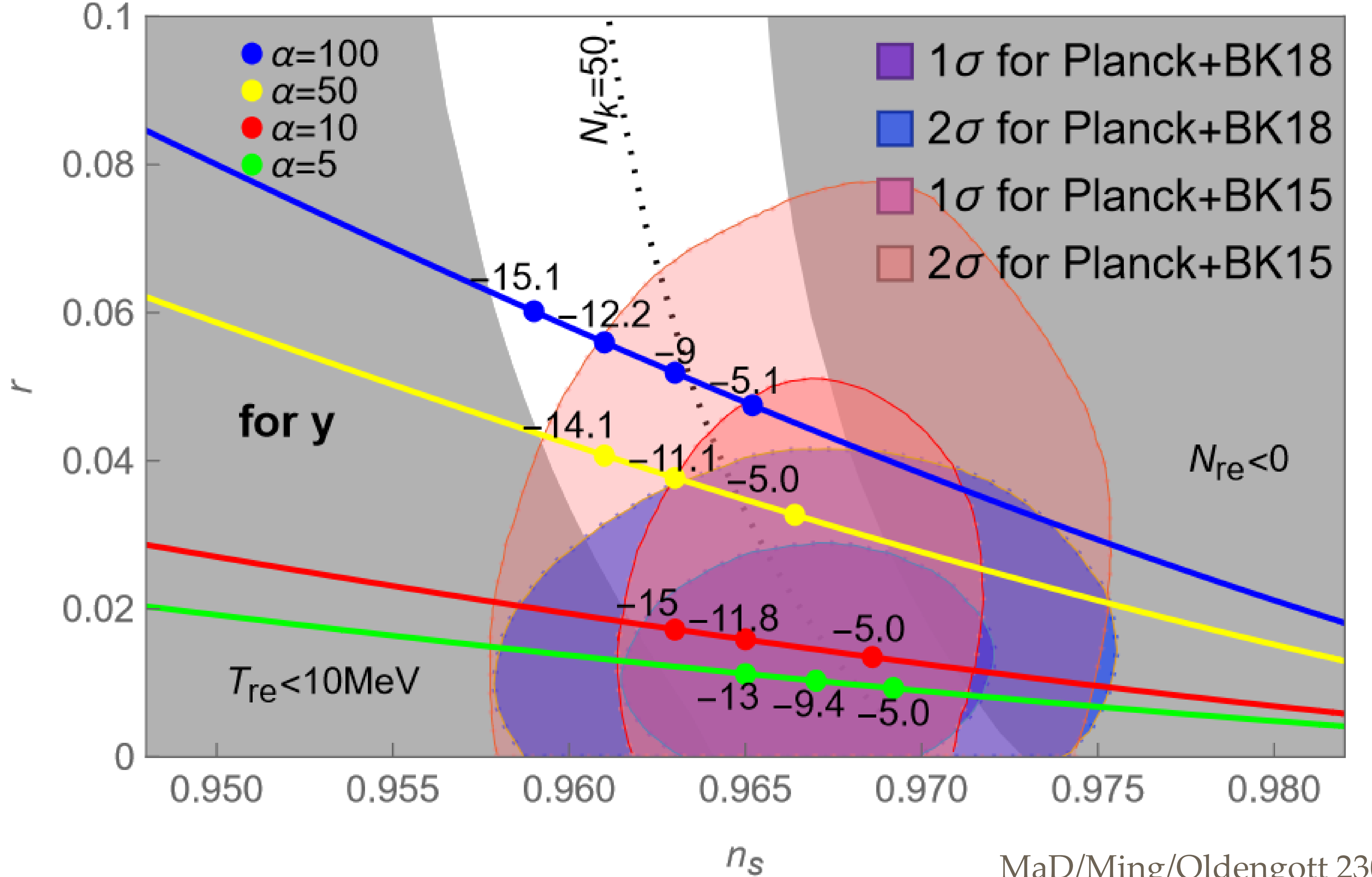
$$\mathcal{V}(\varphi) = M^4 \tanh^{2n}\left(\frac{\varphi}{\sqrt{6\alpha}M_{pl}}\right)$$

- Potentials have two parameters
  - M determines the scale of inflation
  - α determines the inflaton mass
- Together with *Tre*: three parameters…
- …and in principle three observables

$$(A_s, n_s, r)$$

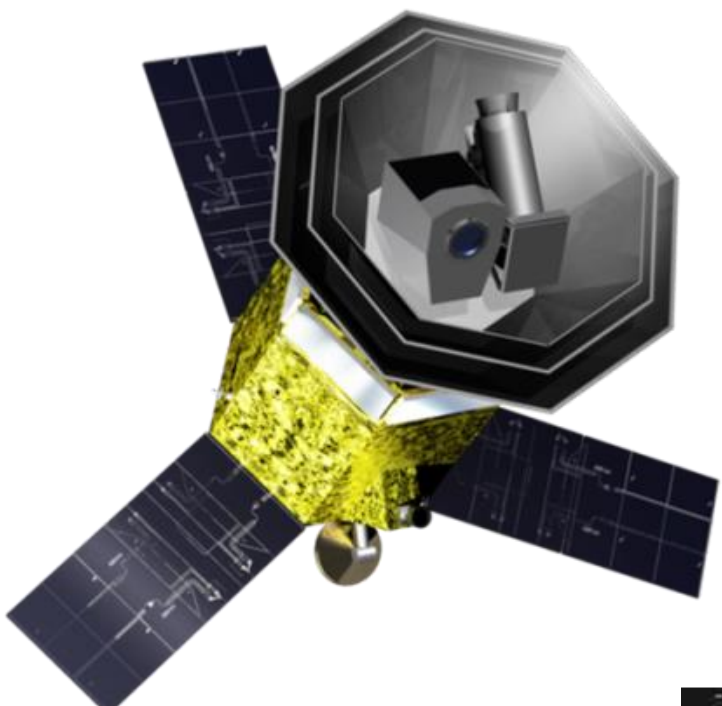
- In practice fix α due to size of σ_{ns}

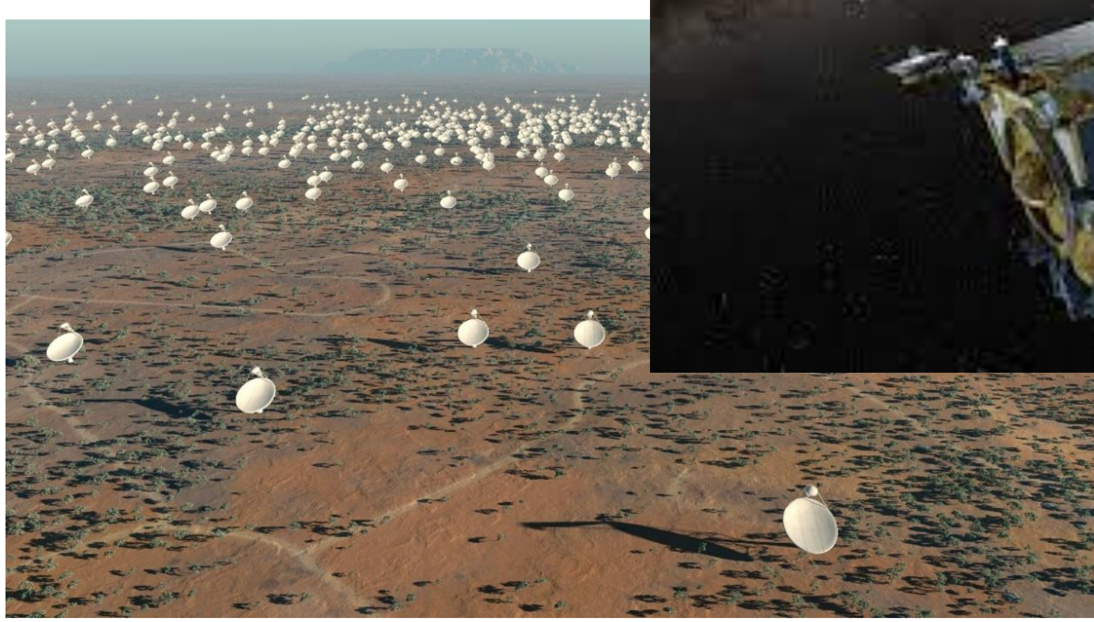
CMB Prediction in RGI Models



MaD/Ming/Oldengott 2303.13503

Future Observations





Measuring *Tre* in the RGI Model

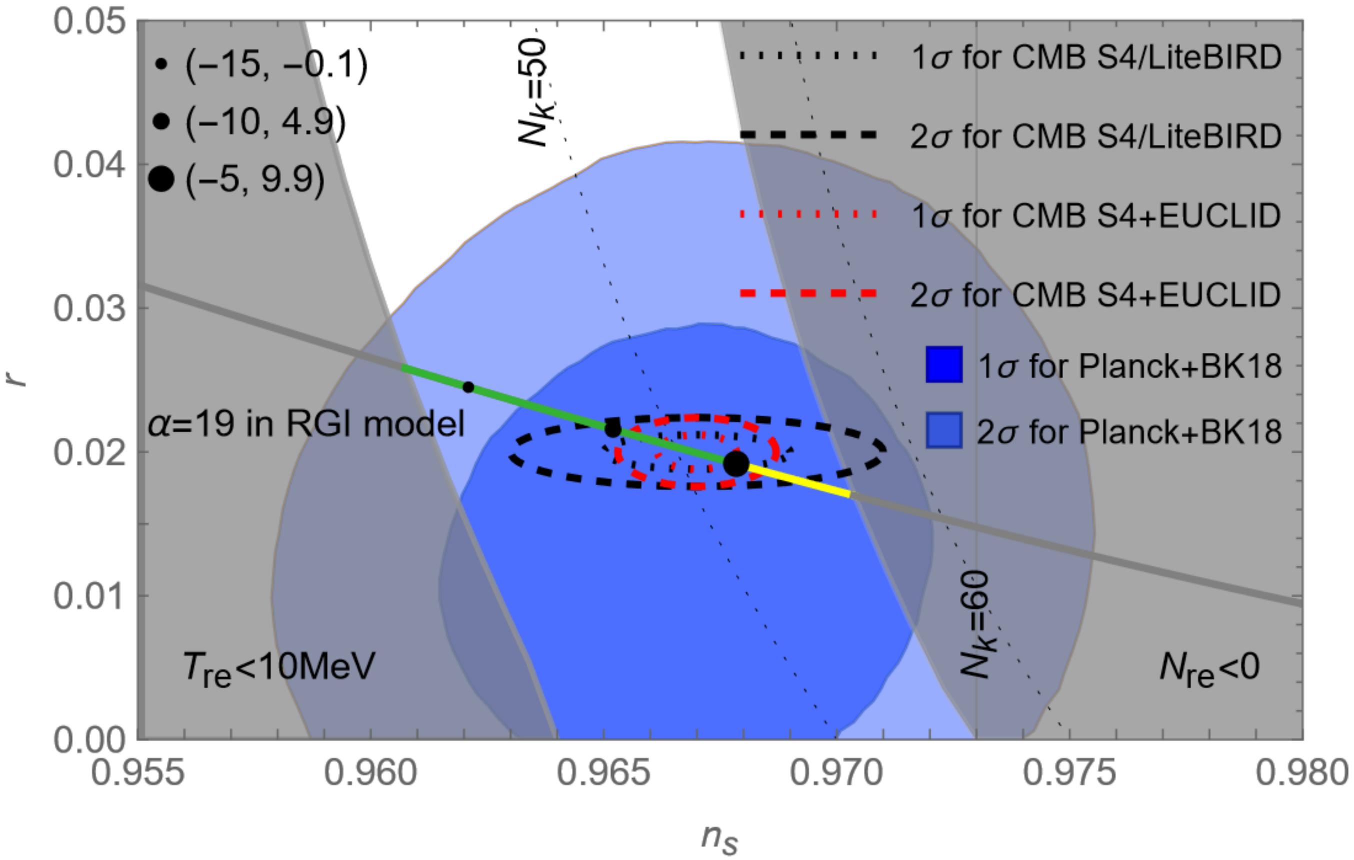
Simple analytic method introduced in MaD/Ming 2208.07609

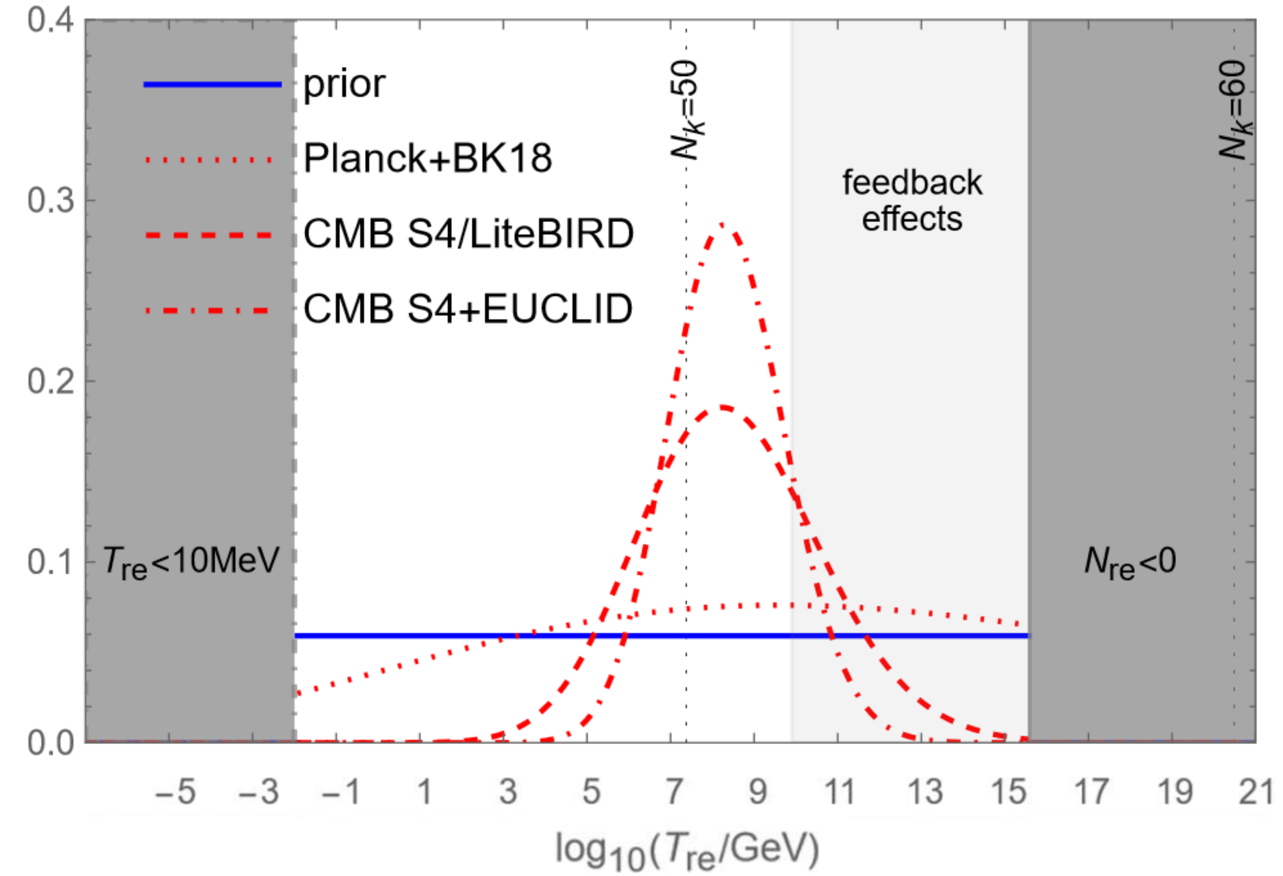
Flat prior in $x = \log_{10}(y)$ $$P(x) = C_1\theta[T_{\rm re}(x) - T_{\rm BBN}]\gamma(x)\theta[N_{\rm re}(x)]$$

Construct likelihood as $$P(\mathcal{D}|x) = C_2\mathcal{N}(n_s, r|\bar{n}_s, \sigma_{n_s}; \bar{r}, \sigma_r)\theta(r)\tilde{\gamma}(x)$$

Model $\mathcal{N}$ with 2dim Gaussian in *ns* and *r*, variances fitted to published sensitivities

Verified by MCMC forecast using CLASS+MontyPython MaD/Ming/Oldengott 2303.13503





Next generation observations can probe *Tre*

MaD/Ming 2208.07609

Connection to Particle Physics: Measuring the Inflaton Coupling

Inflaton Decay

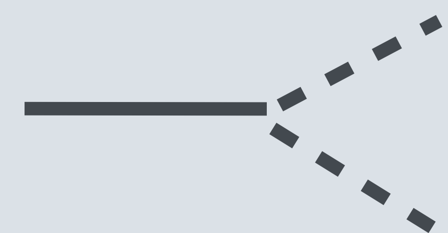
- Reheating ends when $\Gamma = H$. Using this and standard redshifting yields

$$\Gamma|_{\Gamma=H} \simeq \frac{T_{\rm re}^2}{M_{pl}} \frac{\sqrt{g_*}}{3}$$

- Hence we can constrain Γ from observation….
  …and Γ in principle is calculable in terms of microphysical parameter,
  so we can constrain microphysical parameters! MaD 1511.03280, 1903.09599
  CMB sensitive to microphysical parameters that connect inflation to particle physics

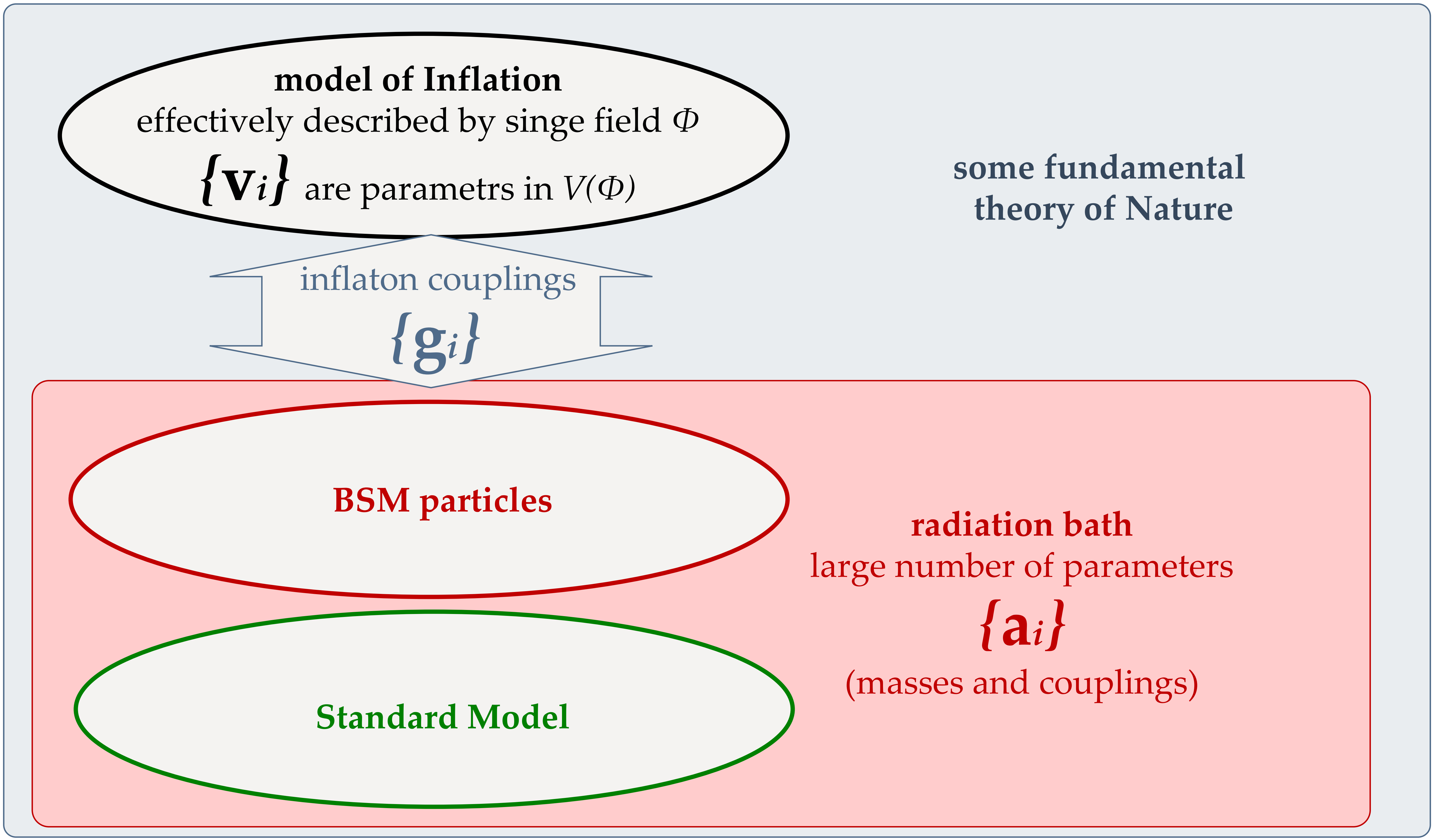
- If inflaton decays via 1 → 2 or 1 → 3 decays then Γ has the form $\Gamma = \mathrm{g}^2 m_\phi / \#$.
- We assume Yukawa coupling, simple rescaling allows to constrain other interactions

	Yukawa	scalar	axion-like	scalar
interaction	$y\Phi\bar{\psi}\psi$	$g\Phi\chi^2$	$\frac{\sigma}{\Lambda}\Phi F_{\mu\nu}\tilde{F}^{\mu\nu}$	$\frac{h}{3!}\Phi\chi^3$
g	y	$\tilde{g} = g/m_\phi$	$\tilde{\sigma} = \sigma m_\phi/\Lambda$	h
#	8π	8π	4π	$3!64(2\pi)^3$
rescaling factor	1	1	$\frac{1}{\sqrt{2}}$	$8\sqrt{6}\pi$



- But: in general **backreaction** of the produced particles introduces dependence of Γ on many microphysical parameters, **making it impossible to constrain any of them without specifying full particle physics model**! e.g. Kofman/Linde/Starobinski …

Inflation in BSM Theories



What about feedback?

- **Reheating is in general a highly nonlinear process**
- **feedback of the produced particles: Γ depends on many parameters**
- **Impossible to constrain any individual microphysical parameter from limited data**

(i) Small inflaton coupling:

- thermal history unaffected by feedback; no impact on relics

- expansion history unaffected by feedback; no impact on relics

(ii) Medium inflaton coupling:

- thermal history affected by feedback; impact on relics

- expansion history unaffected by feedback; no impact on CMB

(iii) Large inflaton coupling:

- thermal history affected by feedback; impact on relics

- expansion history unaffected by feedback; impact on CMB

Conditions for Measuring Inflaton Couplings

MaD 1903.09599

Expand potential $\mathcal{V}(\varphi) = \sum_j \frac{\mathsf{v}_j}{j!} \frac{\varphi^j}{\Lambda^{j-4}}$

General interactions with other fields $\mathsf{g}\Phi^j \Lambda^{4-D} \mathcal{O}[\{\mathcal{X}_i\}]$

I) feedback due to interactions can be avoided if

$$\mathsf{v}_i \ll \left(\frac{m_\phi}{\varphi_{\text{end}}}\right)^{j-\frac{5}{2}} \min\left(\sqrt{\frac{m_\phi}{M_{pl}}}, \sqrt{\frac{m_\phi}{\varphi_{\text{end}}}}\right) \left(\frac{m_\phi}{\Lambda}\right)^{4-}$$

II) Consider general coupling to other fields

$$\mathsf{g} \ll \left(\frac{m_\phi}{\varphi_{\text{end}}}\right)^{j-\frac{1}{2}} \min\left(\sqrt{\frac{m_\phi}{M_{pl}}}, \sqrt{\frac{m_\phi}{\varphi_{\text{end}}}}\right) \left(\frac{m_\phi}{\Lambda}\right)^{4-D}$$

- For plateau models, the above roughly simplifies to

$$|\mathsf{v}_j| \ll \left(3\pi^2 r A_s\right)^{(j-2)/2} \quad , \quad |\mathsf{g}| \ll \left(3\pi^2 r A_s\right)^{j/2}$$

Inflaton coupling
g ≤ electron Yukawa

- It can be compared to the requirement to ensure successful reheating

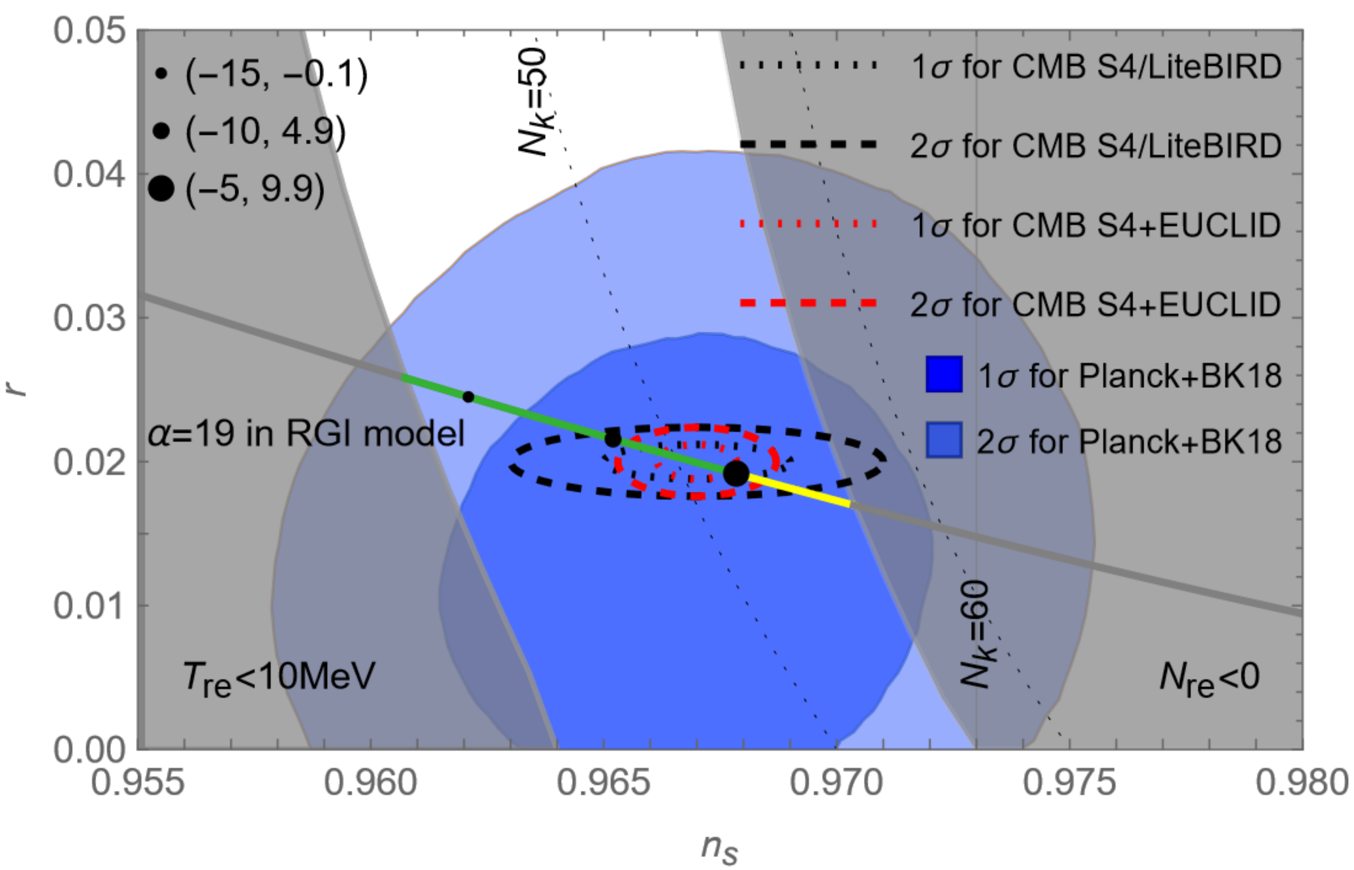
$$\mathsf{g} > g_*^{1/4} \frac{T_{\text{BBN}}}{\sqrt{m_\phi M_{pl}}} \sqrt{\#} \simeq \frac{T_{\text{BBN}}}{M_{pl}} \left(\frac{g_*}{A_s r}\right)^{1/4} \sqrt{\#}.$$

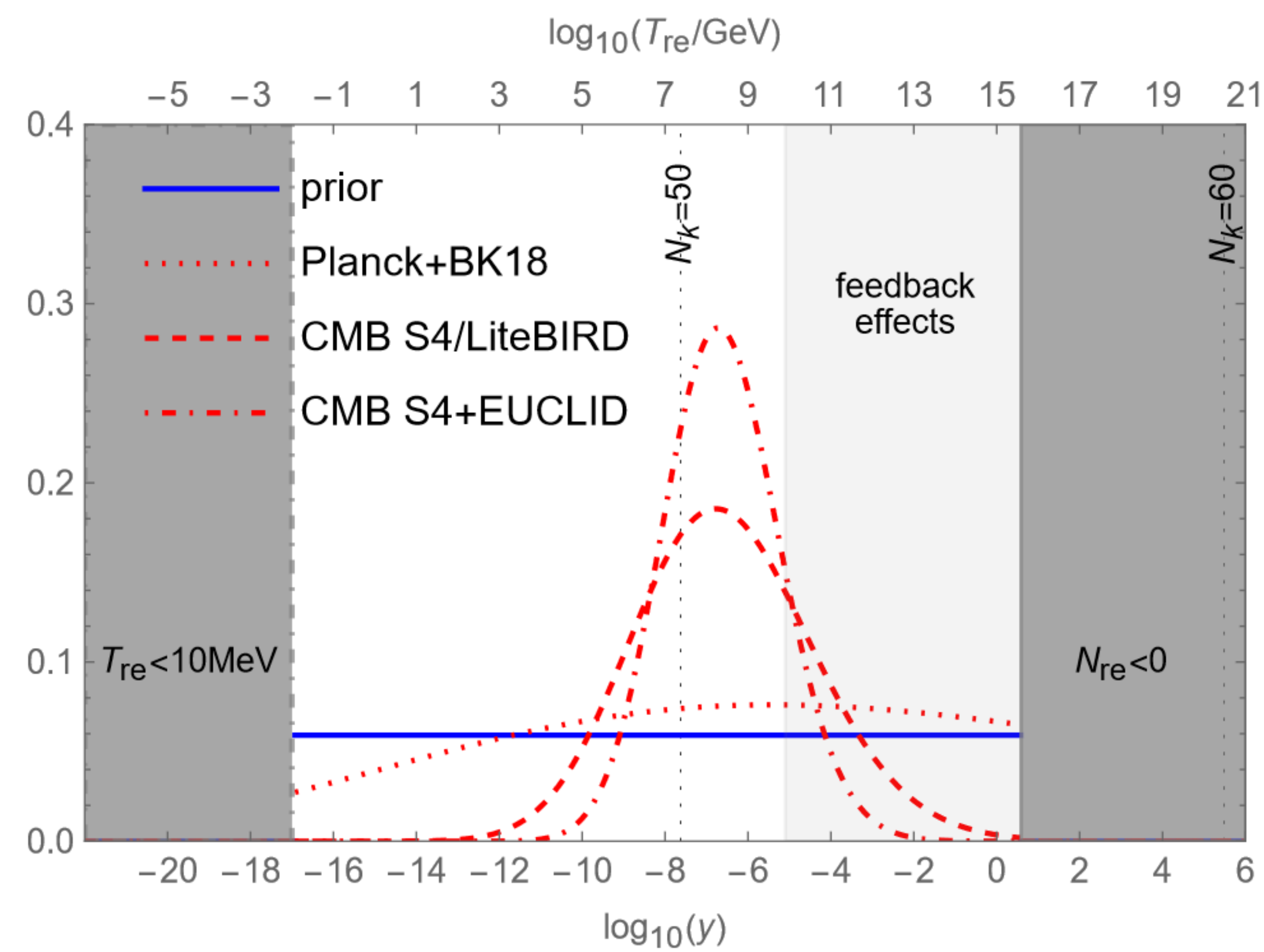
Inflaton mass
$m_\varphi \geq 10^5$ GeV

Inflaton Coupling in the RGI Model

- Consider Yukawa coupling y between inflaton and matter
- In regimes (i) and (ii) the measurement of Tre can be translated into one of y

$$y \simeq 3g_*^{1/4}\frac{T_{\rm re}}{\sqrt{m_\phi M_{pl}}} \quad \text{with} \quad m_\phi = \frac{M^2}{\sqrt{3\alpha}M_{pl}} \quad \text{if} \quad |y| \ll \left(3\pi^2 r A_s\right)^{1/2}$$





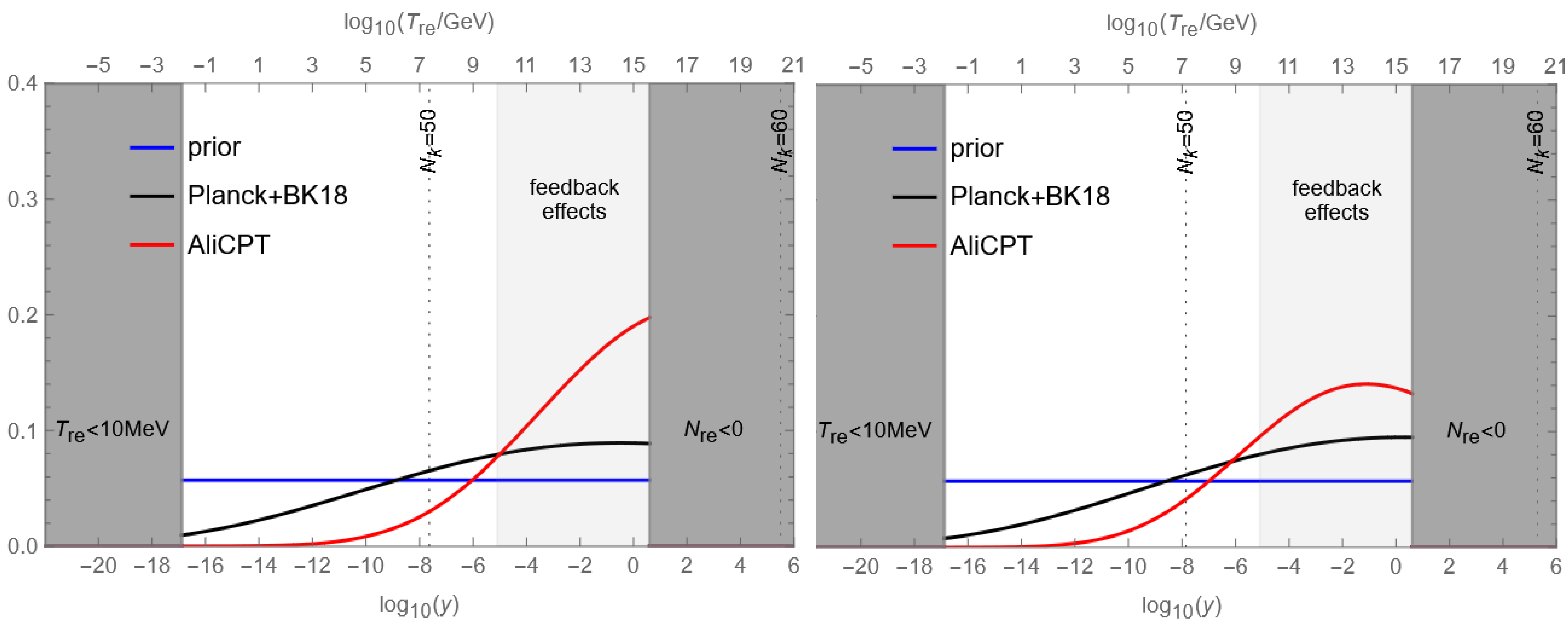
Next generation observations can probe Tre and the inflaton coupling!

MaD/Ming 2208.07609

α-T Model with AliCPT

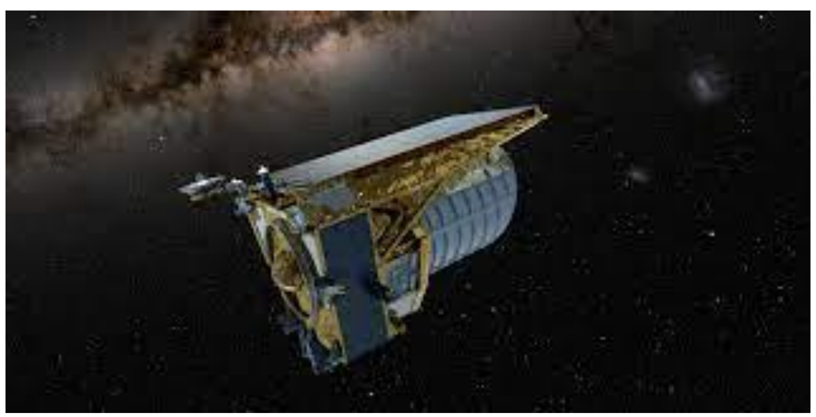
- Under construction in Tibet, China
- Expect 19 modules assembled within 5 years
- Will primary improve *r*
- Less sensitive than CMB-S4, but takes data earlier
- Complementary to CMB-S4, SO, SPT etc due to location in the Northern hemisphere

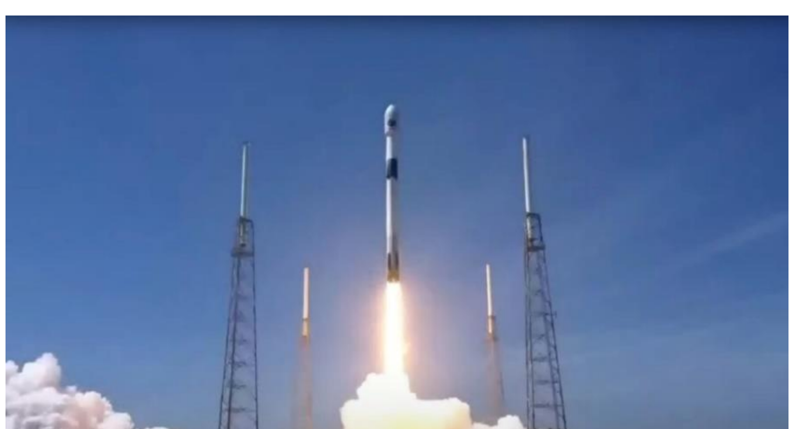
Liu/Ming/MaD/Li 2503.21207

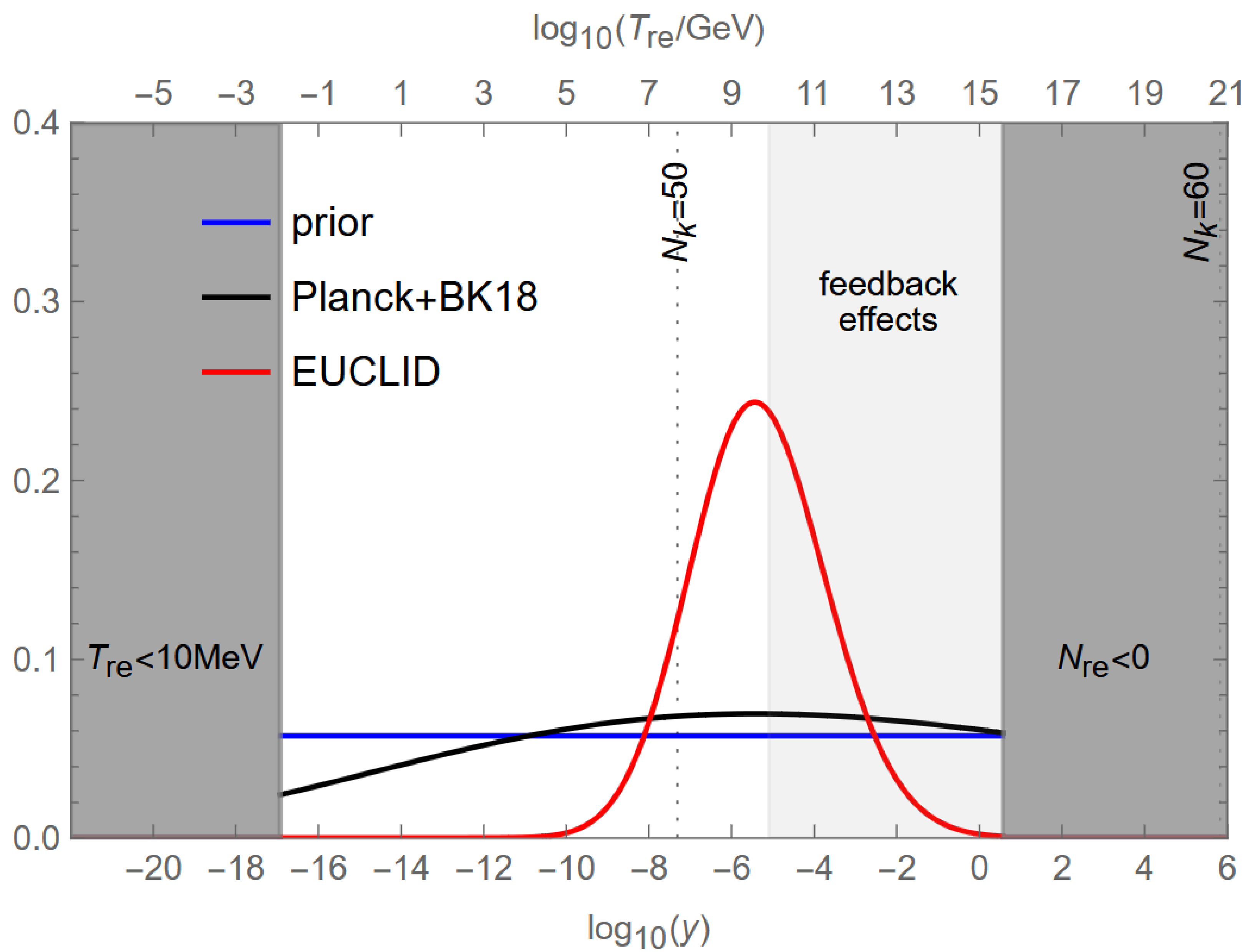


MHI Model with EUCLID

- Improvements on *ns* alone also give access to *Tre* and *y*
- Whether *r* or *ns* is more important depends on the inflation model
- EUCLID's full data set can also measure *Tre* and *y*!
- Complementary probe to CMB







Warm Inflation with the SM

Warm Inflation with the SM

BRAND NEW ★★★ New @ Moriond ★★★ BRAND NEW

Warm inflation: Alternative version of the inflationary paradigm

- Particle production during inflation maintains thermal bath
- Dissipative effects facilitate slow roll
- Thermal fluctuation source scalar CMB perturbations

Berera 9509049

$$\ddot{\phi} + (3H + \Upsilon)\dot{\phi} + V' = 0$$

$$\dot{\rho}_R + 4H\rho_R = \Upsilon\dot{\phi}^2$$

$$\frac{\pi^2}{30} g_* T^4 \approx \frac{\Upsilon}{4H}\left(\frac{V'}{(3H+\Upsilon)}\right)^2$$

Warm Inflation with the SM:

- driven by QCD sphaleron heating
- Dangerous impact of light fermions overcome by Hubble dilution

$$\Upsilon_{\rm eff} = N_c^5 \alpha_s^5 \frac{T^3}{2f^2} \Big/ \left(1 + \frac{2N_f N_c^4 \alpha_s^5 T}{\sqrt{V(\phi)/(3M_{\rm pl}^2)}}\right)$$

Berghaus/MaD/Zell 2503.18829

model of Inflation
effectively described by singe field Φ

$$V(\phi) = \lambda\phi^4$$

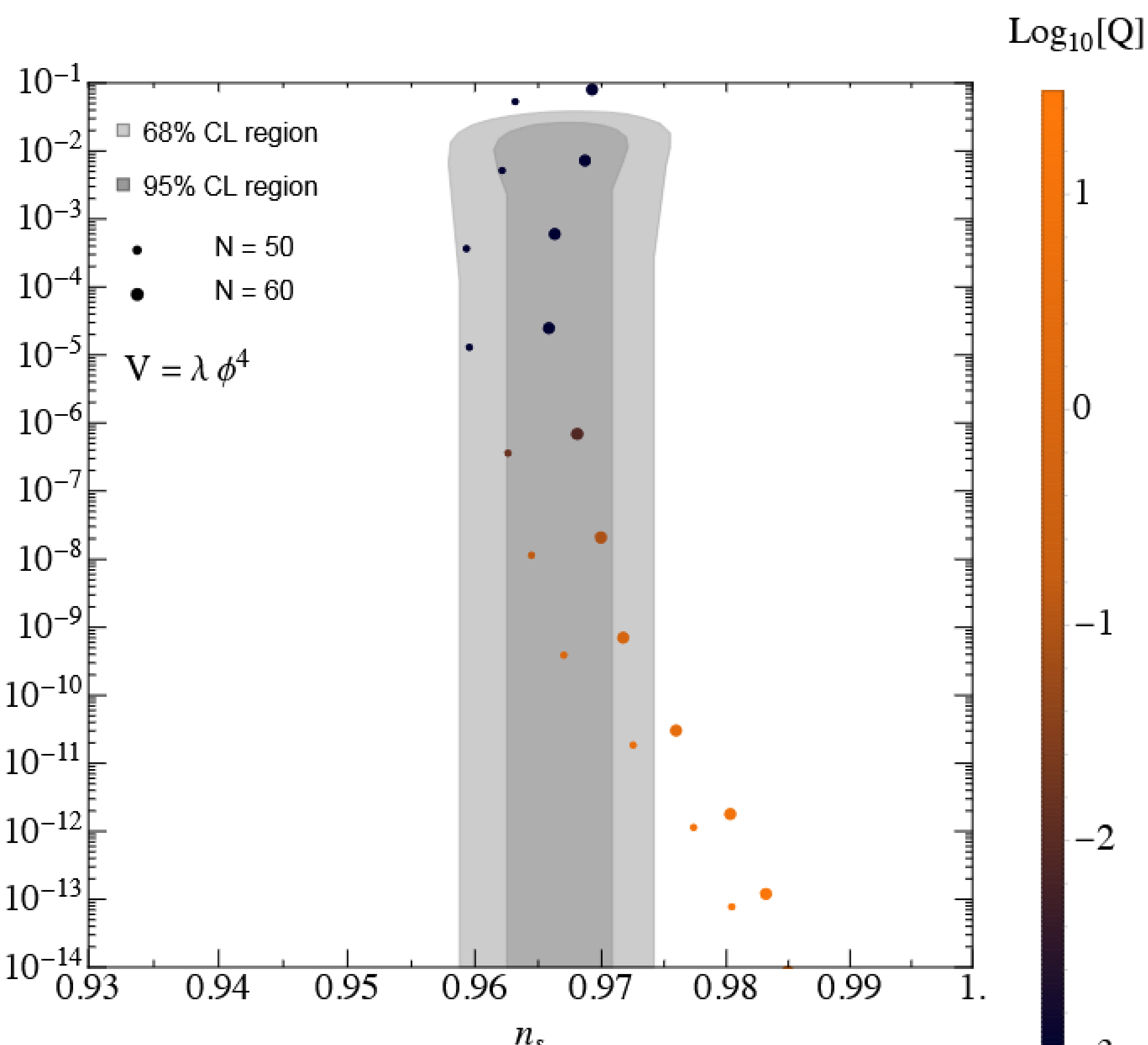
Axion-like
Coupling to gluons

$$\frac{\alpha_s}{8\pi}\frac{\phi}{f} G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$$

Standard Model

Warm Inflation with the SM

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \lambda\phi^4 - \frac{\alpha_s}{8\pi}\frac{\phi}{f}G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$$



$$f \sim 10^{10} - 10^{13}\,\mathrm{GeV}$$

$$\lambda \lesssim 10^{-13}$$

$$T \lesssim 6 \times 10^{14}\,\mathrm{GeV}$$

Warm Inflation with the SM pheno:

- Only one free combination of parameters f and λ after fitting CMB amplitude A_s
- inflaton coupling to gluons can be predicted with future CMB observations
- inflaton can potentially be found in ALP searches
- works with a simple φ^4 potential
- no reheating needed, inflation can directly produce quark-gluon plasma

Berghaus/MaD/Zell 2503.18829

Conclusions

- Cosmological perturbations are affected by *reheating epoch* after cosmic inflation
- **Next generation observations can measure reheating temperature *Tre***
- Sensitivity to *Tre* can approximately be related to that to *r* or *ns*
- Measuring *Tre* via detection of *r*:
  - Best sensitivity can be achieved by CMB-S4 or LiteBIRD
  - Other observatories can already measure *Tre* with larger error earlier:
    Southern Hemisphere: Simmons Observatory and South Pole Telescope
    Northern Hemisphere: AliCPT
- Measuring *Tre* via improvement on *ns*:
  - Optical surveys: DESI, EUCLID…
  - In the future radio data from SKA provide complementary probe
- Constraint on *Tre* can be translated into measuring the inflaton coupling
  ⇒ measure microphysical parameter that connects inflation to particle physics!
- Specific example (first model I built in my life):
  QCD-driven warm inflation: inflaton may be discovered in ALP searches
  ⇒ complementary probe in the laboratory and in the sky!

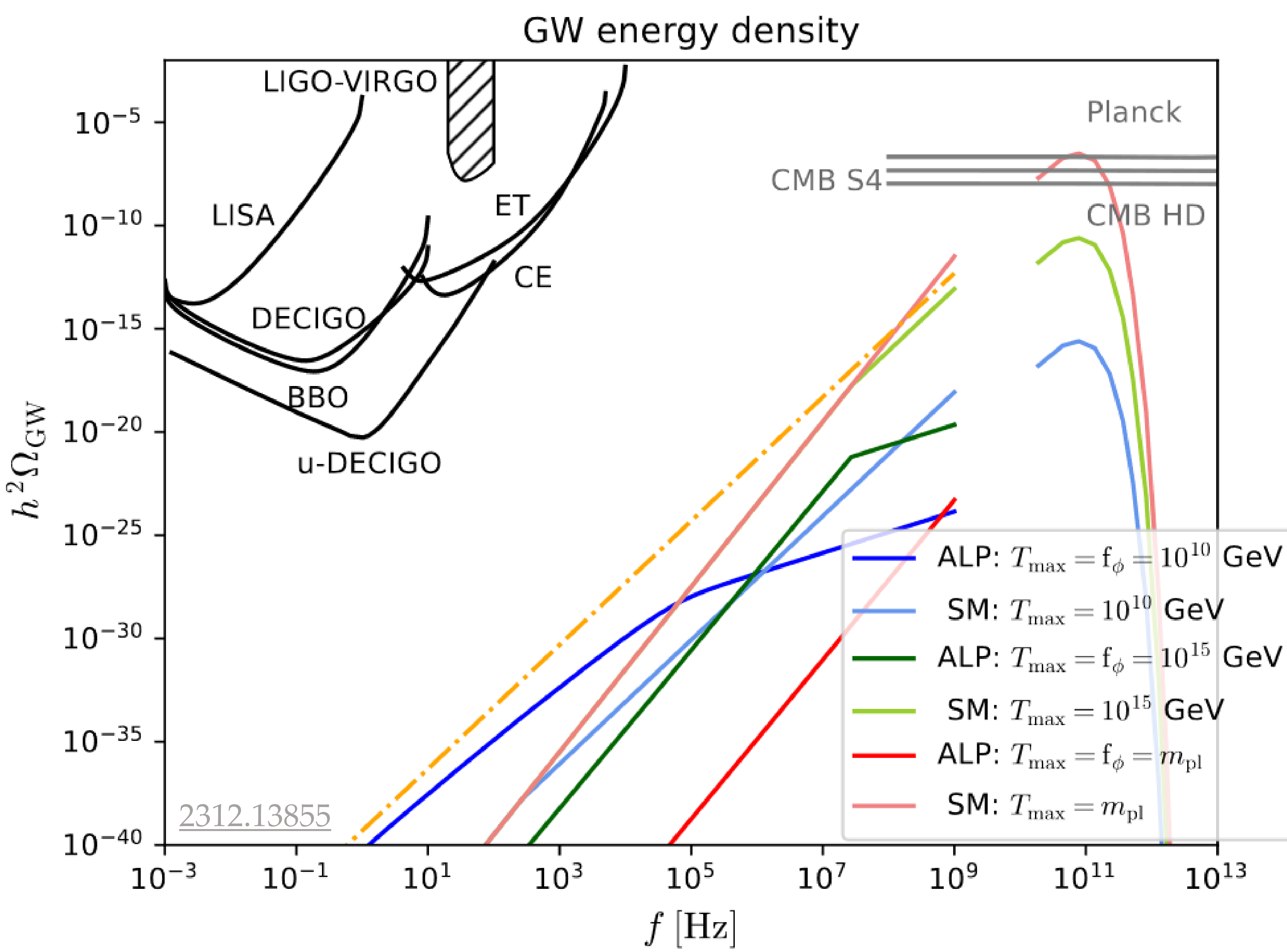
Backup Slides

GW as Big Bang Thermometer

A Big Bang Thermometer

- Primordial plasma emits thermal GWs (GW equivalent to CMB)
- Spectrum is sensitive to reheating temperature Ghiglieri/Laine 1504.02569

  Ghiglieri/Jackson.Laine 2004.11392 Ringwald/Schütte-Engel/Tamarit 2011.04731



- Signature is extremely robust; thermal GW emission is unavoidable
- Generic upper bound: direct detection impossible in SM & minimal extensions

  MaD/Georis/Klaric/Klose 2312.13855
- Indirect detection through *Neff* feasible with CMB-S4 for extremely high *Tre*

What else can we do???

Warm Inflation in the SM

Sphaleron Heating

- From the Lagrangian one obtains $$\ddot{\phi} + 3H\dot{\phi} + V' = -\frac{\alpha_s}{8\pi f}\langle G^a_{\mu\nu}\tilde{G}^{a\mu\nu}\rangle$$
- For massless fermion chiral anomaly gives $$\partial_t j^0_A = -N_f \left\langle \frac{\alpha_s G^a_{\mu\nu}\tilde{G}^{a\mu\nu}}{4\pi} \right\rangle$$
- In the limit of zero frequency this correlator can be related to the sphaleron rate

$$\frac{\alpha_s}{8\pi f}\langle G^a_{\mu\nu}\tilde{G}^{a\mu\nu}\rangle = \Upsilon_{\rm sph}\left(\dot{\phi} + \frac{6 f j^0_A}{N_c T^2}\right) \qquad \Upsilon_{\rm sph} = (\alpha_s N_c)^5 \frac{T^3}{f^2}$$

McLarren/Mottola/Shaposhnikov 91 Moore/Tassler 1011.1167 Laine/Procacci 2102.09913

- This yields coupled set of equations

$$\ddot{\phi} + 3H\dot{\phi} + V' = -\Upsilon_{\rm sph}\left(\dot{\phi} + 2N_f f \mu_A\right)$$

$$\dot{\mu}_A + 3H\mu_A = -\frac{6 f \Upsilon_{\rm sph}}{N_c T^2}\left(\dot{\phi} + 2N_f f \mu_A\right)$$

- Without Hubble expansion there is no stationary solution with dissipation
- Hubble expansion yields effective dissipation coefficient $$\Upsilon_{\rm eff} = \Upsilon_{\rm sph} \Big/ \left(1 + \frac{\Upsilon_{\rm sph}}{3H}\frac{N_f}{N_c}\frac{12 f^2}{T^2}\right)$$

CMB Observables

What the CMB is sensitive to

- Spectrum of perturbations at end of inflation is given by choice of potential
- Energy density is also determined by the potential

$$\rho_{\rm end} \simeq \frac{4}{3}\mathcal{V}_{\rm end}$$

- Impact of reheating is determined by

  - the averaged equation of state
  (also calculable for a given potential, as inflaton dominates during reheating)

$$\bar{w}_{\rm re} = \frac{1}{N_{\rm re}}\int_0^{N_{\rm re}} w(N)dN$$

  - the duration of the reheating epoch *Nre*.

- Equivalently can use $\rho_{\rm re} = \rho_{\rm end}\exp(-3N_{\rm re}(1+\bar{w}_{\rm re}))$ to obtain reheating temperature:

$$\frac{\pi^2 g_*}{30}T_{\rm re}^4 \equiv \rho_{\rm re}. \qquad T_{\rm re} = \exp\left[-\frac{3(1+\bar{w}_{\rm re})}{4}N_{\rm re}\right]\left(\frac{40\mathcal{V}_{\rm end}}{g_*\pi^2}\right)^{1/4}$$

- *Tre* is the only quantity that is not calculable for a given potential

Connection to Observables

- We use only a small number of observables (A_s, n_s, r)
  (in principle the CMB and LSS contain more bytes, but let's start with this…)
- Need relation between observables and potential parameters and

$$T_{\rm re} = \exp\left[-\frac{3(1+\bar{w}_{\rm re})}{4} N_{\rm re}\right]\left(\frac{40\mathcal{V}_{\rm end}}{g_* \pi^2}\right)^{1/4}$$

N_{re} can be written as

$$N_{\rm re} = \frac{4}{3\bar{w}_{\rm re}-1}\left[N_k + \ln\left(\frac{k}{a_0 T_0}\right) + \frac{1}{4}\ln\left(\frac{40}{\pi^2 g_*}\right) + \frac{1}{3}\ln\left(\frac{11 g_{s*}}{43}\right) - \frac{1}{2}\ln\left(\frac{\pi^2 M_{pl}^2 r A_s}{2\sqrt{\mathcal{V}_{\rm end}}}\right)\right]$$

where N_k can be obtained from

$$N_k = \ln\left(\frac{a_{\rm end}}{a_k}\right) = \int_{\varphi_k}^{\varphi_{\rm end}} \frac{H d\varphi}{\dot{\varphi}} \approx \frac{1}{M_{pl}^2}\int_{\varphi_{\rm end}}^{\varphi_k} d\varphi \frac{\mathcal{V}}{\partial_\varphi \mathcal{V}}$$

and φ_k is obtained by solving

$$n_s = 1 - 6\epsilon_k + 2\eta_k\ , \quad r = 16\epsilon_k$$

A subscript k means: evaluated at the moment when the mode k crosses the horizon.

Parameters

Parameters

- We use only a small number of observables (A_s, n_s, r)

- We can therefore only derive a meaningful constraint on microphysical parameters when Γ depends only on a small number of them, ideally only on one.

- We shall distinguish three classes of parameters:

{ vi } the parameters in the inflaton potential *V(φ)* define the "model of inflation".

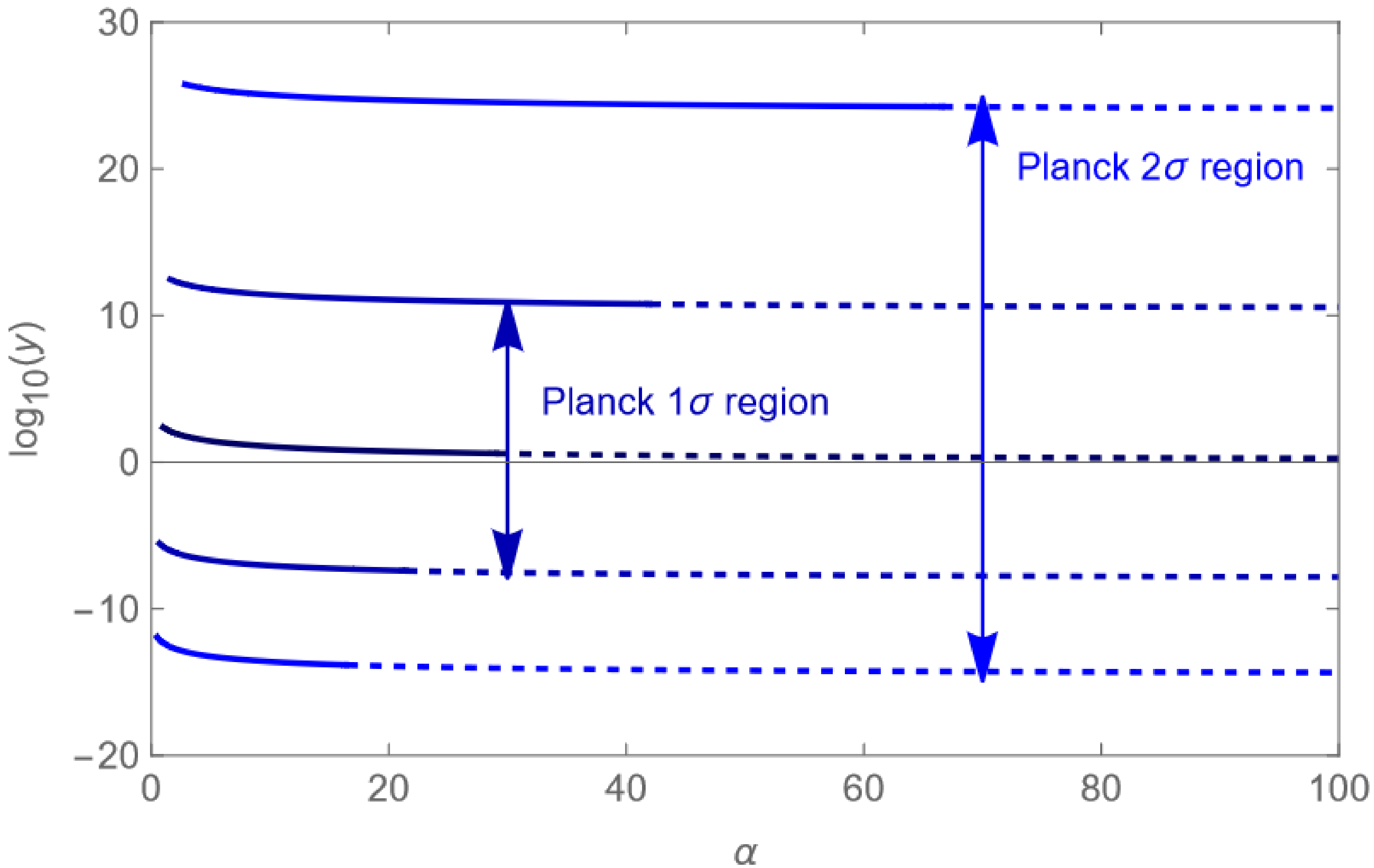
$$\mathcal{V}(\varphi) = \sum_j \frac{\mathsf{v}_j}{j!} \frac{\varphi^j}{\Lambda^{j-4}}$$

{ gi } the inflaton couplings between the inflaton and other fields connect the "model of inflation" to an underlying "model of particle physics".

{ ai } all other parameters of the "particle physics model", e.g. masses and gauge interactions amongst the produced particles..

Our Goal: Assume that reheating is primarily driven by one interaction with coupling g, identify parameter region where g can be measured without having to specify details of the underlying particle physics model and the *{ai}*

Practical Problems

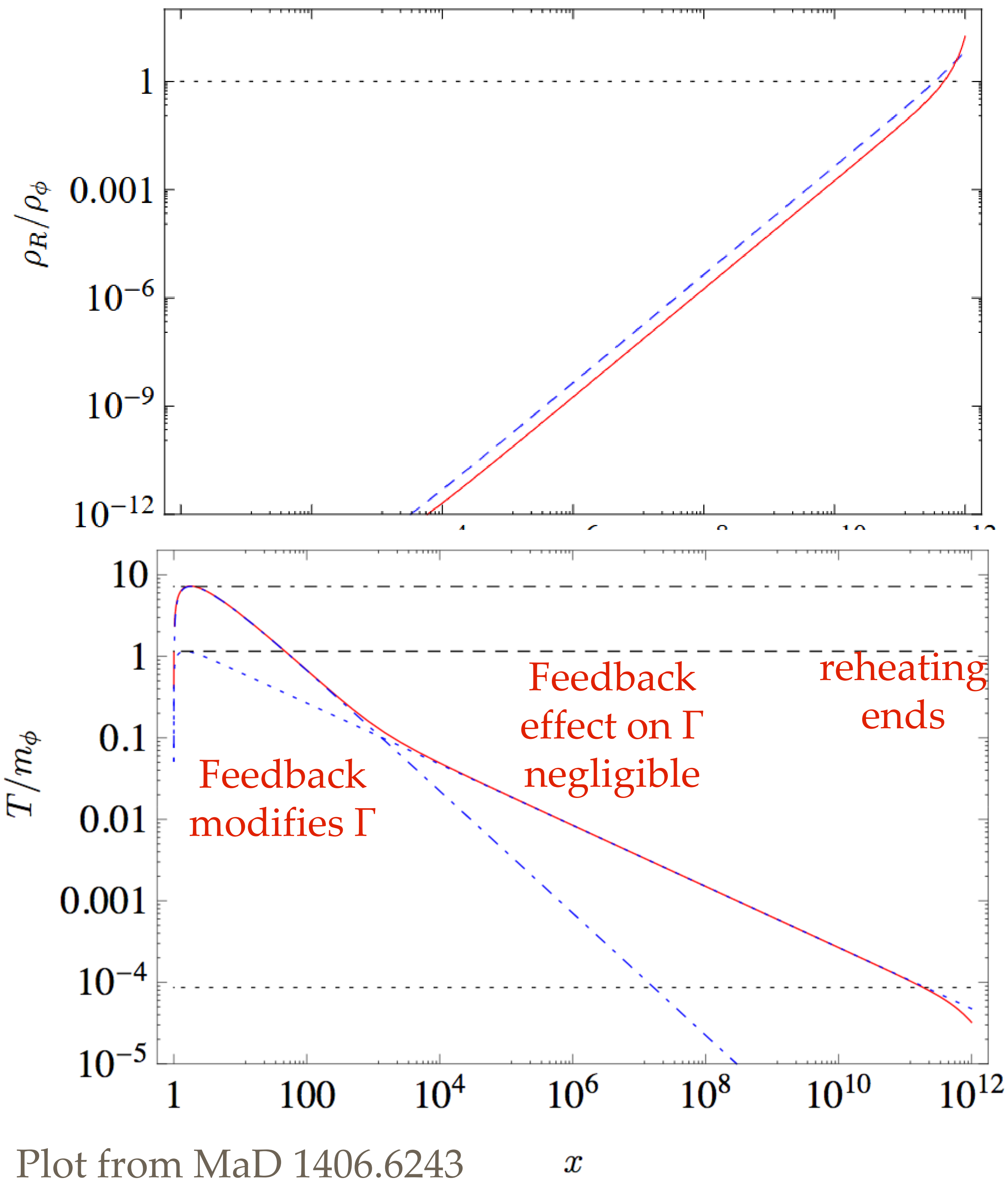


- Error bar on spectral index is too large to fix all three parameters from observation; here we fix α by hand (e.g. by model building), which defines a family of models
- Range of allowed values for α is restricted by requirement to avoid feedback during reheating (conditions from Part I)

MaD/Ming/Oldengott 2303.13503

Feedback Effects

What about feedback?



Plot from MaD 1406.6243

(i) Small inflaton coupling:

- thermal history unaffected by feedback; no impact on relics
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(ii) Medium inflaton coupling:

- thermal history affected by feedback; impact on relics
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Parametric Resonance

Mode equation for produced particles during harmonic oscillations ***$\varphi(t) = \Phi \cos(\omega t)$*** can are rewritten as **Mathieu equation** with z ~ ωt

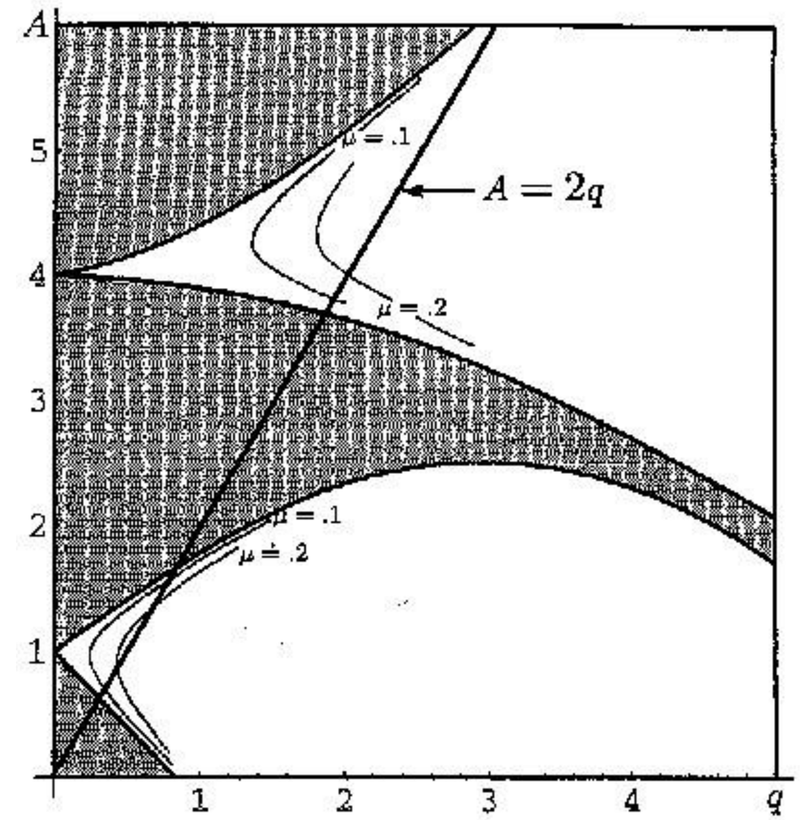
Kofman/Linde/Starobinski

$$\mathcal{X}_k''(z) + \left[A_k - 2q\cos(2z)\right]\mathcal{X}_k(z) = 0$$

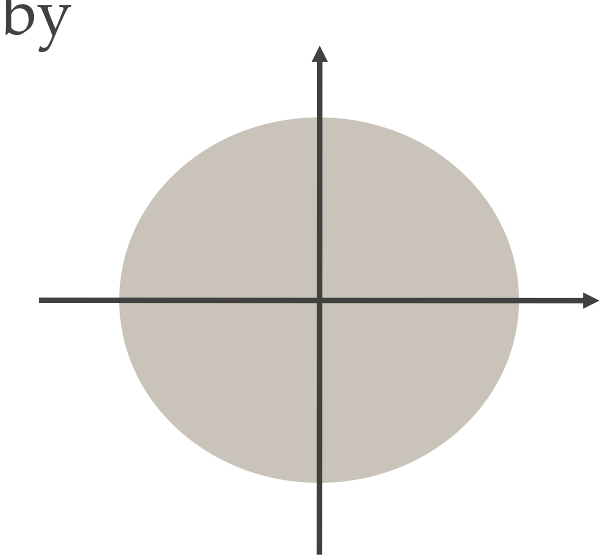
We demand

I) $q < 1$ to avoid "broad resonance"

II) $q^2 m < H$ to make sure that redshifting avoids "narrow resonance"



Note that redshifting depends only on the model of inflation { vi } because φ dominates during reheating. Avoiding the resonance by rescattering would introduce a dependence on { ai }.



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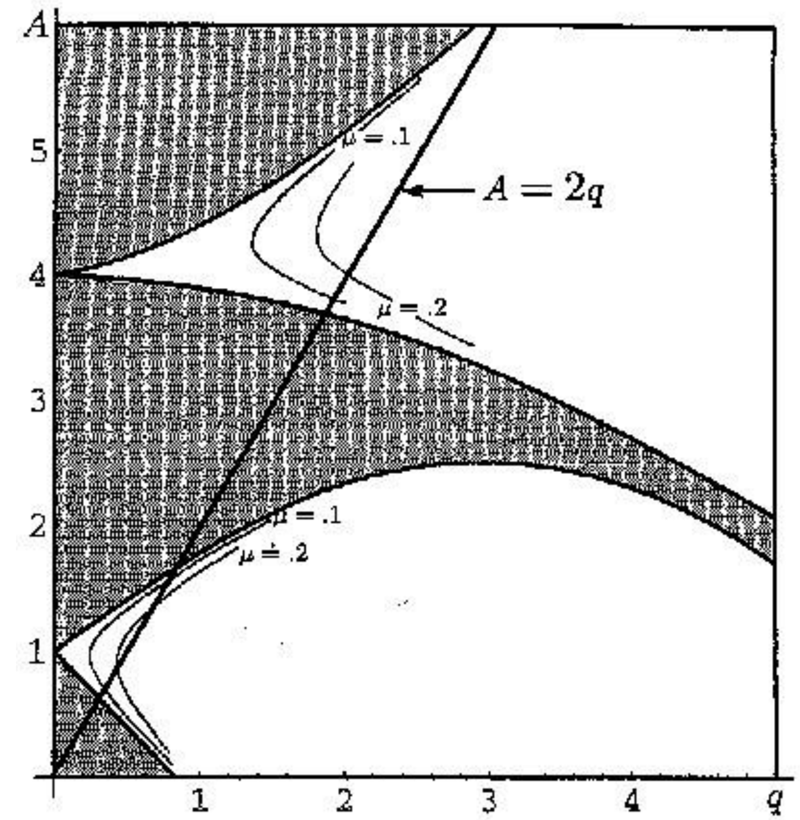

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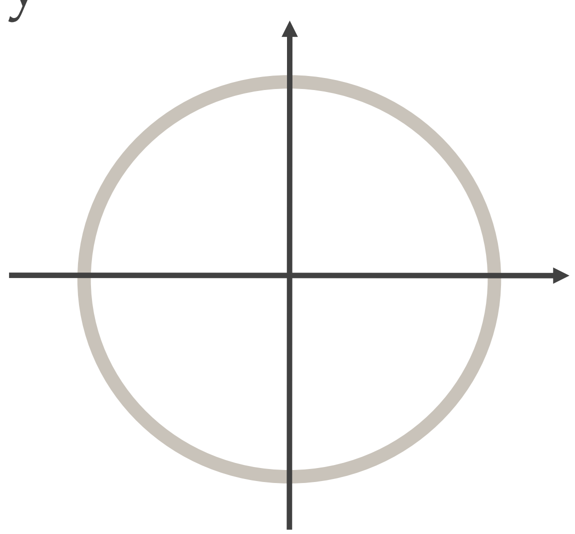

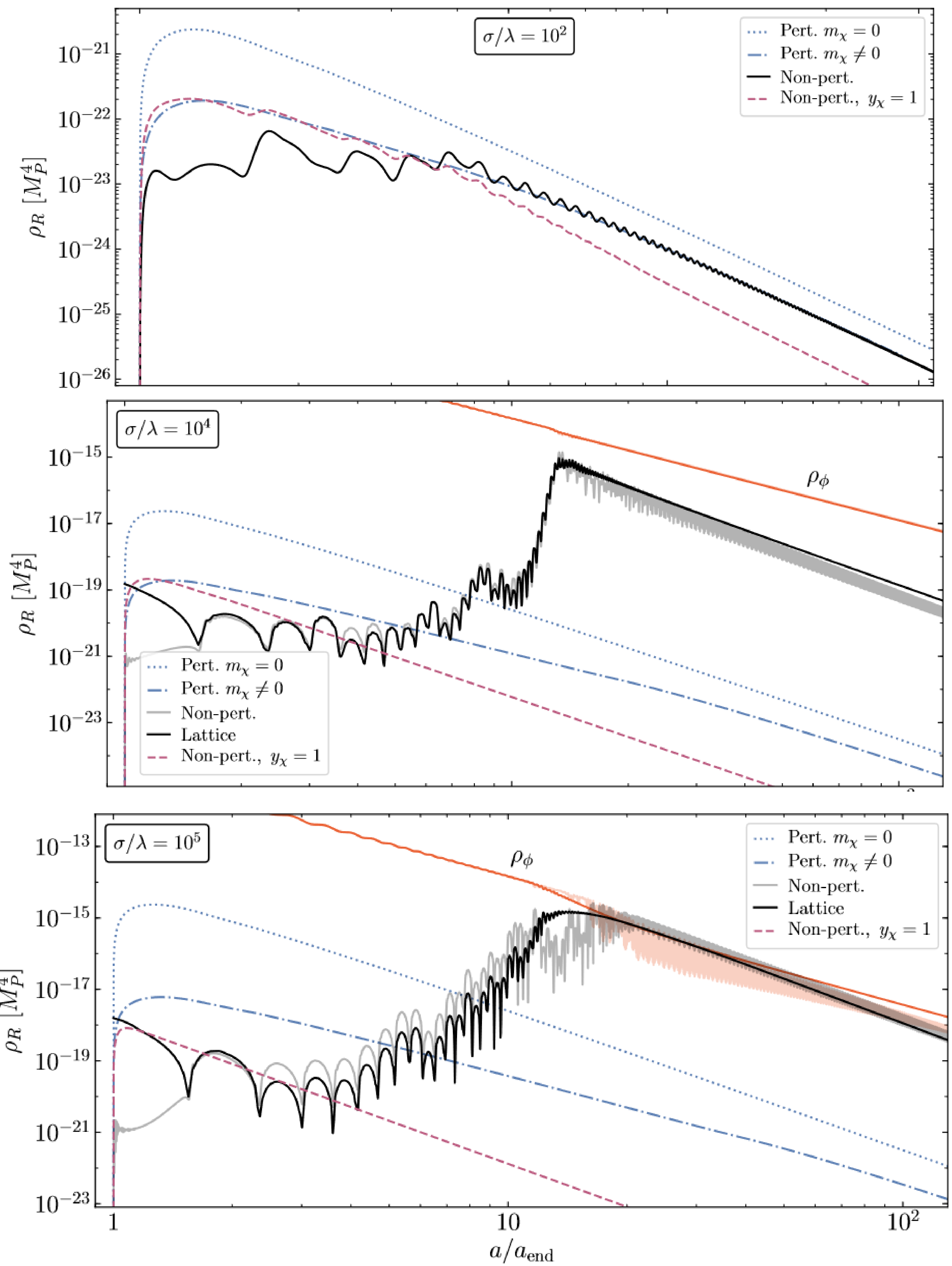

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Garcia/Kaneta/Mambrini/Olive/Verner 2109.13280

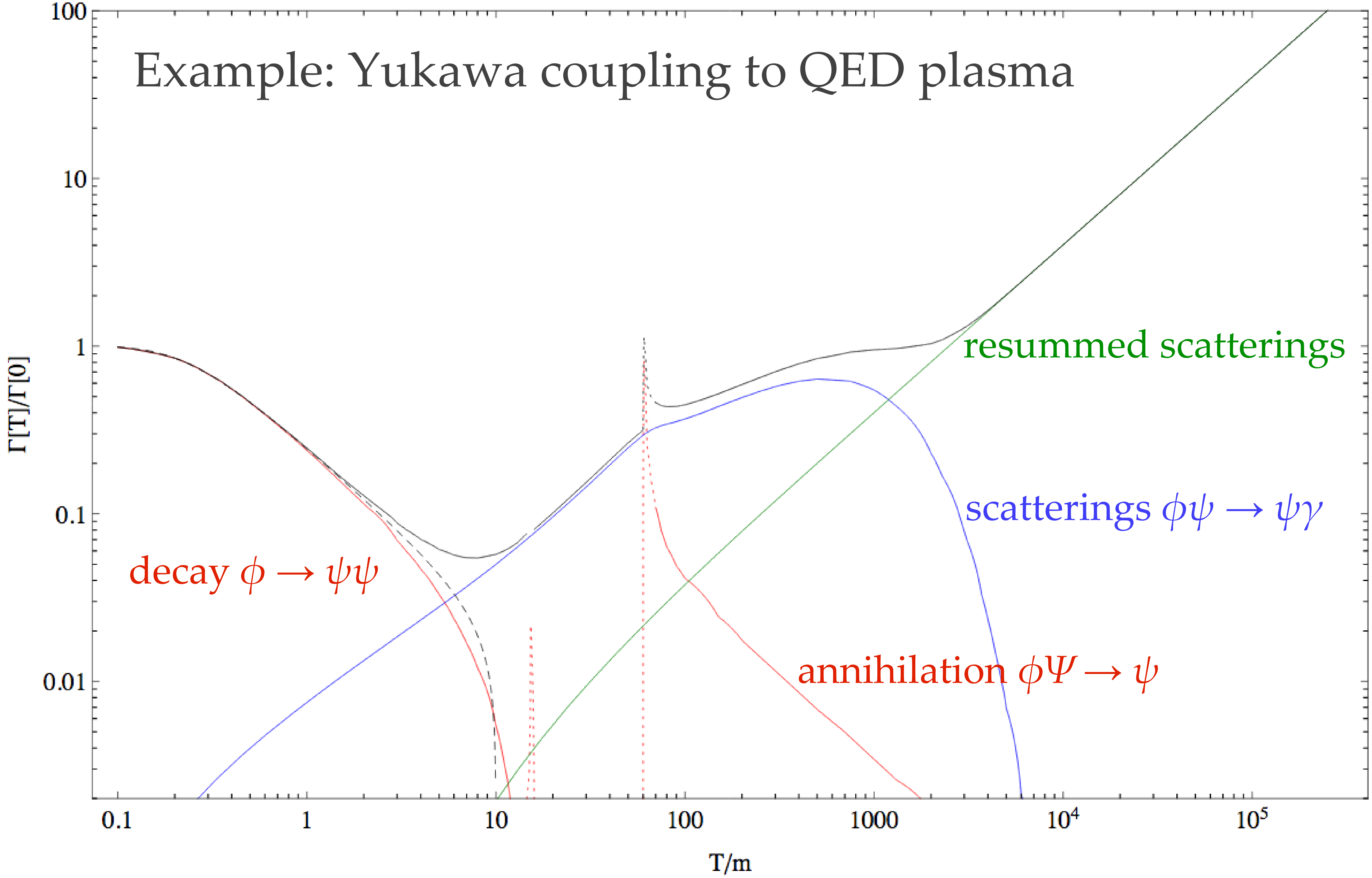
What about thermal feedback?

- Even if there is no resonance, thermal effects can potentially modify Γ

interaction	process	contribution to Γ_φ
$g\Phi\chi^2$	$\varphi \to \chi\chi$	$\frac{g^2}{8\pi M_\phi}\left(1-(2M_\chi/M_\phi)^2\right)^{1/2}\left(1+2f_B(M_\phi/2)\right)\theta(M_\phi-2M_\chi)$
$\frac{h}{4}\Phi^2\chi^2$	$\varphi\varphi \to \chi\chi$	$\frac{h^2\varphi^2}{256\pi M_\phi}\left(1-(M_\chi/M_\phi)^2\right)^{1/2}\left(1+2f_B(M_\phi)\right)\theta(M_\phi-M_\chi)$
$\frac{\alpha}{\Lambda}\Phi F_{\mu\nu}\tilde{F}^{\mu\nu}$	$\varphi \to \gamma\gamma$	$\frac{\alpha^2}{4\pi}\frac{M_\phi^3}{\Lambda^2}\left(1-(2M_\gamma/M_\phi)^2\right)^{1/2}\left(1+2f_B(M_\phi/2)\right)\theta(M_\phi-2M_\gamma)$
$y\Phi\bar{\psi}\psi$	$\varphi \to \psi\bar{\psi}$, $M_\psi \simeq m_\psi$	$\frac{y^2}{8\pi}M_\phi\left(1-(2m_\psi/M_\phi)^2\right)^{3/2}\left(1-2f_F(M_\phi/2)\right)\theta(M_\phi-2m_\psi)$
	$\varphi \to \psi\bar{\psi}$, $M_\psi \gg m_\psi$	$\frac{y^2}{8\pi}M_\phi\left(1-(2M_\psi/M_\phi)^2\right)^{1/2}\left(1-2f_F(M_\phi/2)\right)\theta(M_\phi-2M_\psi)$

- Prefactor typically depends on a single coupling constant $\in \{ g_i \}$
- Phase space given by "thermal masses" depends on the $\{ \alpha_i \}$, becomes relevant when $T > m_\varphi/\alpha_i$
- Quantum statistical effects are relevant for occupation numbers O[1] ($T > m_\varphi$ for equilibrium distributions), depends on $\{ \alpha_i \}$ because rescatterings determine distribution functions

What about thermal feedback?



MaD/Kang 1305.0267

Forecast Method: Details

Future Sensitivities

We employed two methods to estimate the sensitivities of future observations:

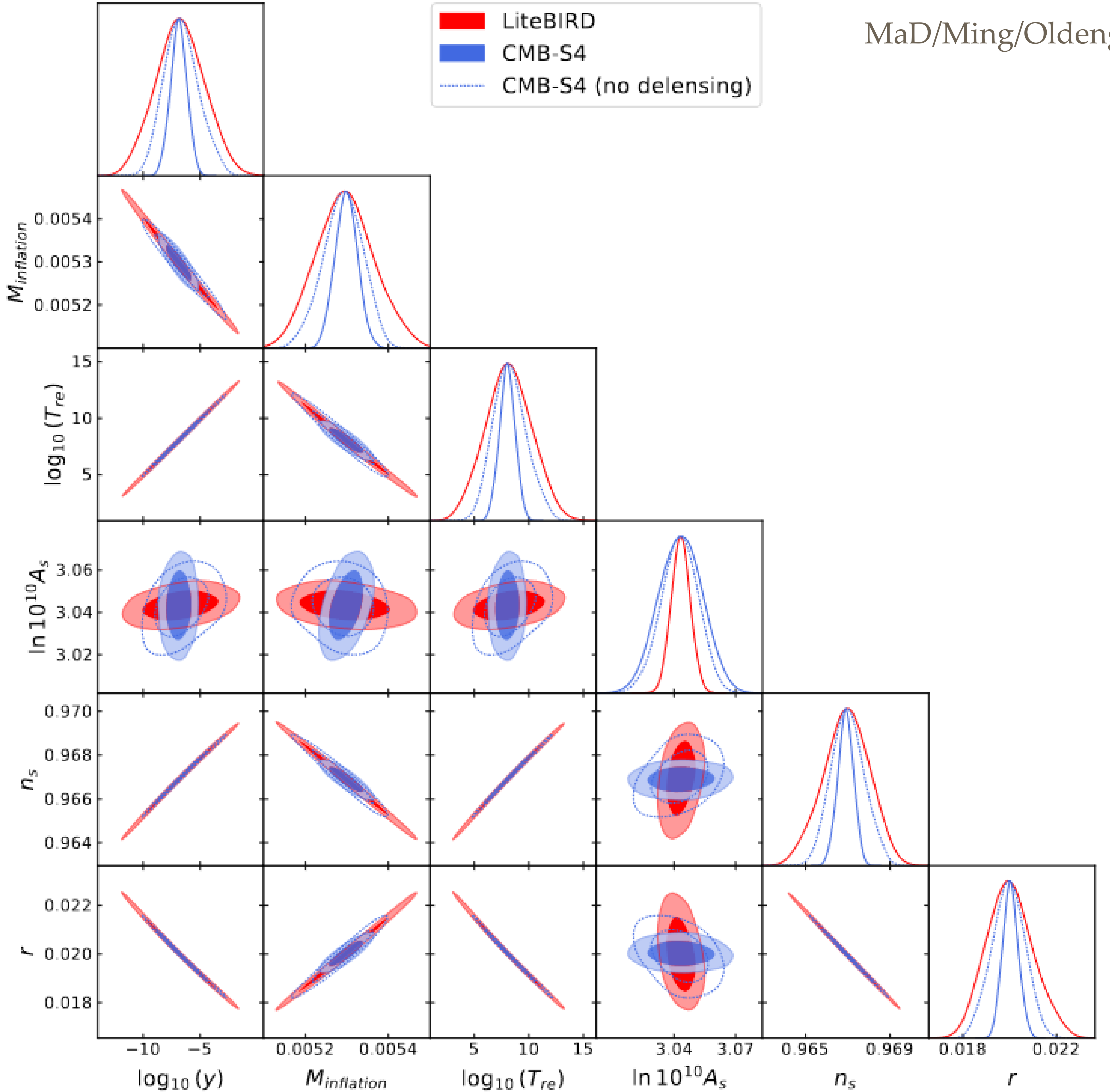
- An analytic method that simply assumes a Gaussian likelihood for the sensitivities in n_s and r MaD/Ming 2208.07609

- An Forecasts with a modified version of CLASS and MontePython with the free parameters MaD/Ming/Oldengott 2303.13503

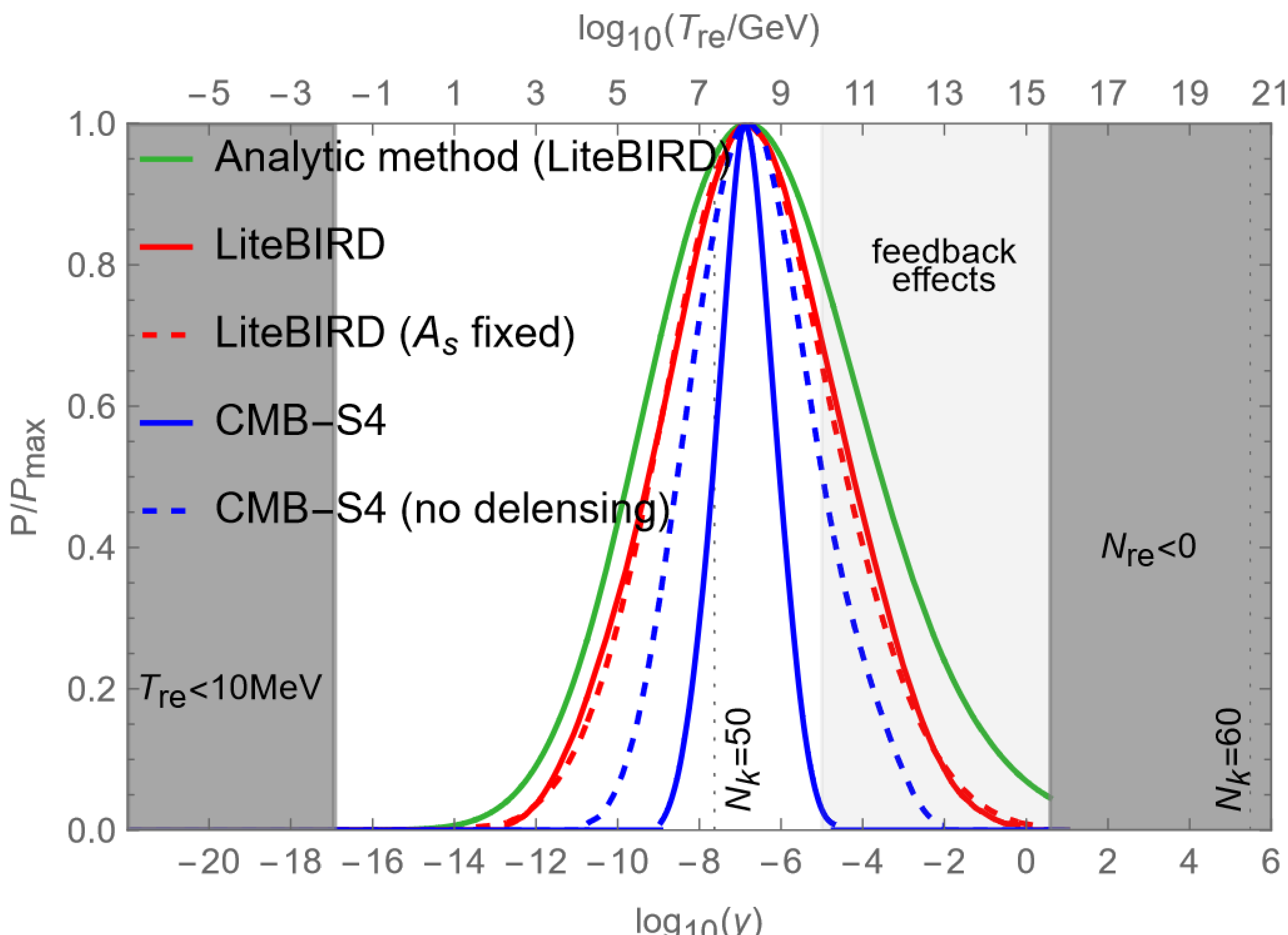
$$X = \{\omega_{\rm b}, \omega_{\rm cdm}, 100\theta_{\rm s}, \tau_{\rm reio}\} + \log_{10}(y) + M \, ;$$

(using build-in functions for LiteBIRD and CMB-S4)

MaD/Ming/Oldengott 2303.13503

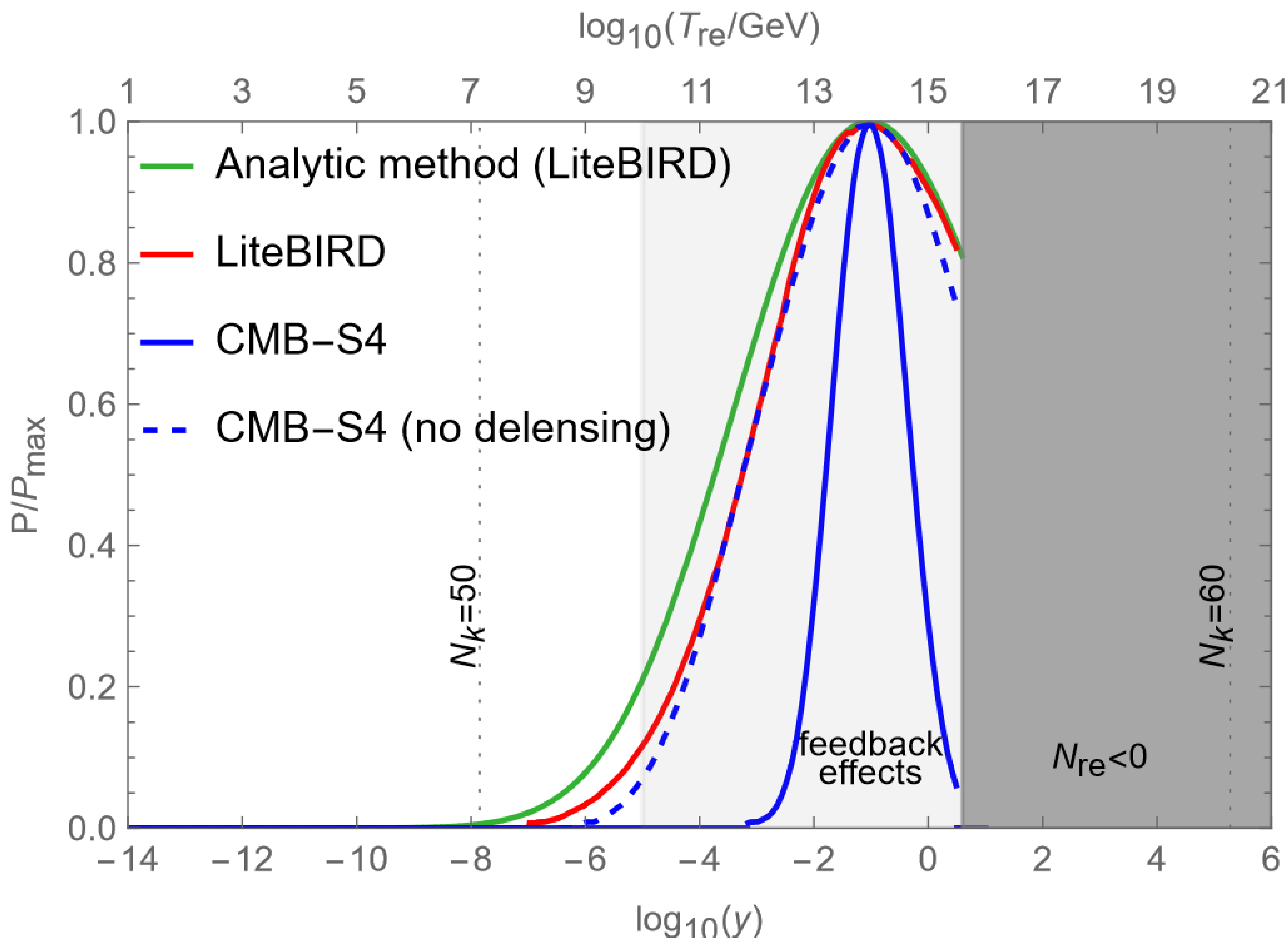


Forecast Method: RGI



MaD/Ming/Oldengott 2303.13503

Forecast Method: α-T



MaD/Ming/Oldengott 2303.13503

Forecast Method: Summary

LiteBIRD

model	x	$M[M_{\rm pl}]$	$\log_{10}(T_{\rm re}[{\rm GeV}])$
MHI	-6.17 ± 2.08	0.00519 ± 0.00007	8.71 ± 2.07
RGI	-6.75 ± 2.19	0.00530 ± 0.00007	8.15 ± 2.18
α-T	-1.67 ± 1.42	0.00521 ± 0.00005	13.16 ± 1.42

CMB-S4

model	x	$M[M_{\rm pl}]$	$\log_{10}(T_{\rm re}[{\rm GeV}])$
MHI	-6.31 ± 0.69 (-6.31 ± 1.59)	0.00519 ± 0.00003 (0.00519 ± 0.00005)	8.57 ± 0.67 (8.57 ± 1.58)
RGI	-6.86 ± 0.74 (-6.64 ± 1.55)	0.00530 ± 0.00003 (0.00530 ± 0.00005)	8.04 ± 0.74 (8.26 ± 1.54)
α-T	-1.04 ± 0.64 (-1.61 ± 1.40)	0.00518 ± 0.00003 (0.00520 ± 0.00005)	13.79 ± 0.64 (13.22 ± 1.40)

MaD/Ming/Oldengott 2303.13503

Quantum Equations of Motion

Correlators and 2PI Effective Action

- Inflaton condensate

$$\varphi(x) \equiv \langle \Phi(x) \rangle$$

- Spectral function

$$\Delta_\phi^-(x,y) \equiv i\langle [\Phi(x), \Phi(y)] \rangle$$

- Statistical propagator

$$\Delta_\phi^+(x,y) \equiv \frac{1}{2}\langle \{\Phi(x), \Phi(y)\} \rangle - \varphi(x)\varphi(y),$$

Feynman propagator

$$\begin{aligned}
\Delta_F(x_1,x_2) &= \Delta^+(x_1,x_2) - \frac{i}{2}\mathrm{sign}(x_1^0 - x_2^0)\Delta^-(x_1,x_2) \\
\Delta_{\bar{F}}(x_1,x_2) &= \Delta^+(x_1,x_2) + \frac{i}{2}\mathrm{sign}(x_1^0 - x_2^0)\Delta^-(x_1,x_2),
\end{aligned}$$

Wightman functions

$$\begin{aligned}
\Delta^>(x_1,x_2) &= \Delta^+(x_1,x_2) - \frac{i}{2}\Delta^-(x_1,x_2) \\
\Delta^<(x_1,x_2) &= \Delta^+(x_1,x_2) + \frac{i}{2}\Delta^-(x_1,x_2).
\end{aligned}$$

Correlators and 2PI Effective Action

- Inflaton condensate

$$\varphi(x) \equiv \langle \Phi(x) \rangle$$

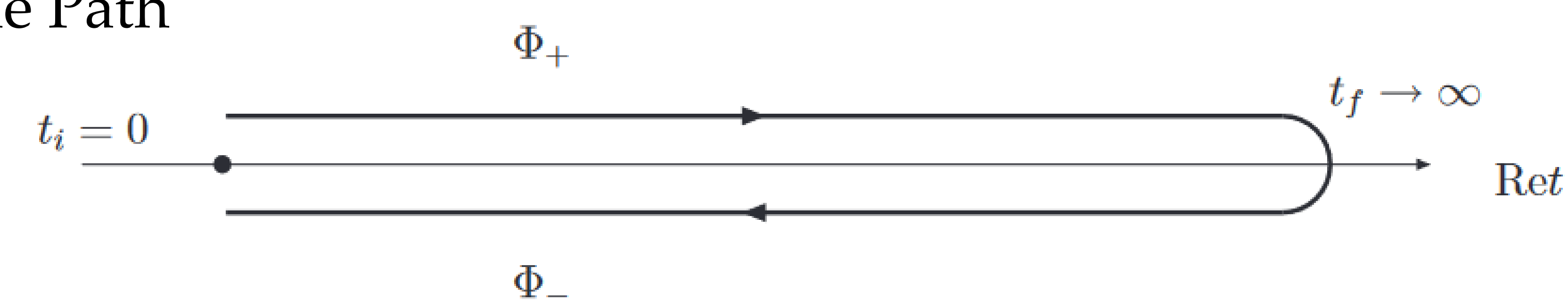
- Spectral function

$$\Delta_\phi^-(x,y) \equiv i\langle [\Phi(x), \Phi(y)] \rangle$$

- Statistical propagator

$$\Delta_\phi^+(x,y) \equiv \frac{1}{2}\langle \{\Phi(x), \Phi(y)\} \rangle - \varphi(x)\varphi(y),$$

- Closed Time Path



- 2PI Effective Action

$$\mathbf{\Gamma}_\varphi[\varphi, \Delta] = S[\varphi] + \mathbf{\Gamma}_{\varphi\mathrm{loop}}[\varphi, \Delta[\varphi]],$$

$$\mathbf{\Gamma}_{\varphi\mathrm{loop}}[\varphi, \Delta[\varphi]] = \frac{i}{2}\mathrm{Tr}\ln\left(\Delta^{-1}\right) + \frac{i}{2}\mathrm{Tr}\left(G^{-1}[\varphi]\,\Delta\right) + \mathbf{\Gamma}_{\varphi 2}[\varphi, \Delta],$$

EoM for Condensate

- Obtain equation of motion for condensate from

$$\frac{\delta \mathbf{\Gamma}_\varphi[\varphi, \Delta]}{\delta \varphi(x)} = 0$$

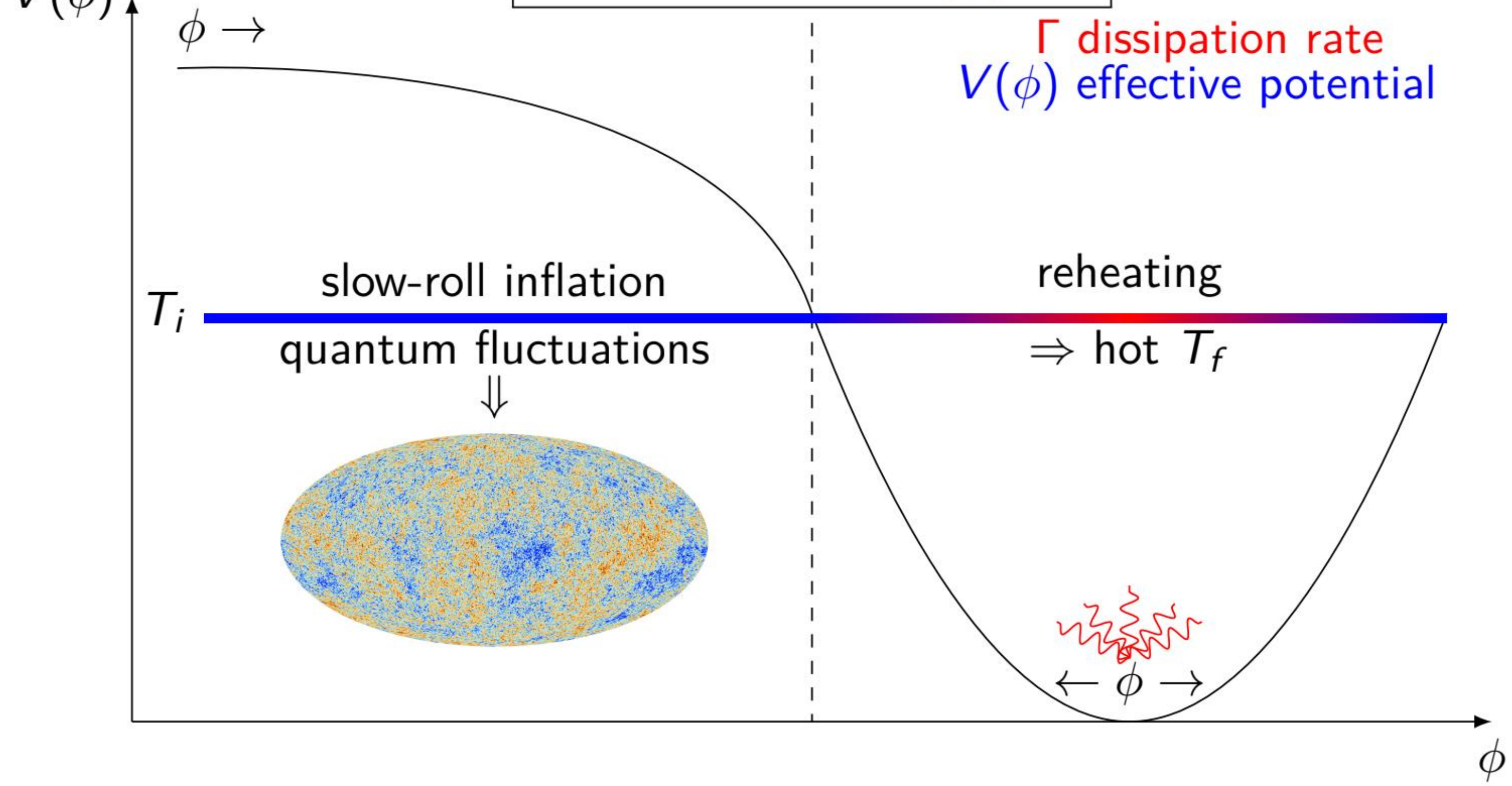
- In general gives non-local Langevin-type equation, can be Markovianised if

"slow roll" $\tau_{\text{int}}^2 \ddot{\varphi}/\varphi \ll (\tau_{\text{int}}\dot{\varphi}/\varphi)^n \ll 1$

"mild non-linear" $\varphi(t') \approx \overline{\varphi}_0 \cos(Mt') + \frac{\overline{\varphi}_0}{M} \sin(Mt')$

Boyanovsky et al 9408214
Yokoyaka/Linde 9809409
Mukaida/Nakayama/Takimoto 1308.4394
Buldgen/MaD/Kang/Kim 1912.02772
Calzetta/Hu000 '89
Greiner/Muller 9605048
Ai//MaD/Glavan/Hajer 2108.00254

$$\ddot{\phi} + (3H + \Gamma)\dot{\phi} + V'(\phi) = 0$$

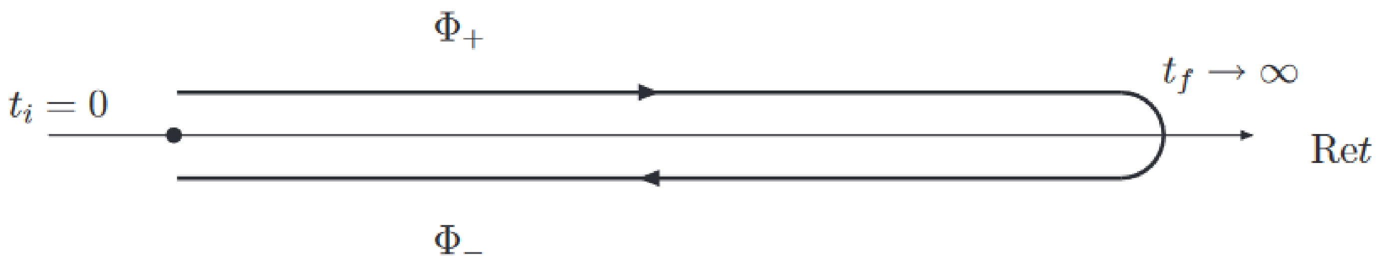


EoM for Propagators

- From the 2PI effective action

$$\frac{\delta \boldsymbol{\Gamma}_{\varphi}[\varphi, \Delta]}{\delta \Delta(x,y)} = 0$$

- …on the closed time path…



- …obtain Kadanoff-Baym equations for the Wightman functions

$$(\Box_1 + m^2)\Delta^<(x_1, x_2) = \int d^4x' \left(-\Pi_{++}(x_1, x')\Delta^<(x', x_2) + \Pi^<(x_1, x')\Delta_{--}(x', x_2)\right)$$

$$(\Box_1 + m^2)\Delta^>(x_1, x_2) = \int d^4x' \left(-\Pi^>(x_1, x')\Delta_{++}(x', x_2) + \Pi_{--}(x_1, x')\Delta^>(x', x_2)\right)$$

EoM for Propagators

- Re-write in terms of spectral function and statistical propagator

$$\left(\partial_{t_1}^2 + \Omega_\phi^2(t_1;\mathbf{k})\right)\Delta_\phi^-(t_1,t_2;\mathbf{k}) = -\int_{t_2}^{t_1} dt'\,\Pi_\phi^-(t_1,t';\mathbf{k})\Delta_\phi^-(t',t_2;\mathbf{k})$$

$$\left(\partial_{t_1}^2 + \Omega_\phi^2(t_1;\mathbf{k})\right)\Delta_\phi^+(t_1,t_2;\mathbf{k}) = -\int_{t_i}^{t_1} dt'\,\Pi_\phi^-(t_1,t';\mathbf{k})\Delta_\phi^+(t',t_2;\mathbf{k}) + \int_{t_i}^{t_2} dt'\,\Pi_\phi^+(t_1,t';\mathbf{k})\Delta_\phi^-(t',t_2;\mathbf{k})$$

- Time dependent frequency

$$\Omega_\phi^2(t;\mathbf{k}) = \mathbf{k}^2 + \mathcal{M}_\phi^{\mathrm{tree}}(t)^2 + \int \frac{d^3\mathbf{q}}{(2\pi)^3}\,\Pi_\phi^{\mathrm{loc}}(t,t;\mathbf{k}) \equiv \mathbf{k}^2 + \mathcal{M}_\phi(t)^2$$

- Zeroth order in $\hbar$: as classical oscillator with time dependent frequency

$$(\partial_{t_1}^2 + \mathbf{k}^2 + \mathcal{M}_\phi^2)\Delta_\phi^+ = 0$$

Instabilities (parametric resonance, tachyonic instability…) unless

$$\dot{\mathcal{M}}_\phi/\mathcal{M}_\phi^2 \ll 1$$

Shtanov/Traschen/Brandenberger
Kofman/Linde/Starobinski
…