# Cosmological
# Large Scale Structure Observations

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# Outline

# How do we observe LSS

Presently, we use four main ways to investigate LSS

- Galaxy number counts

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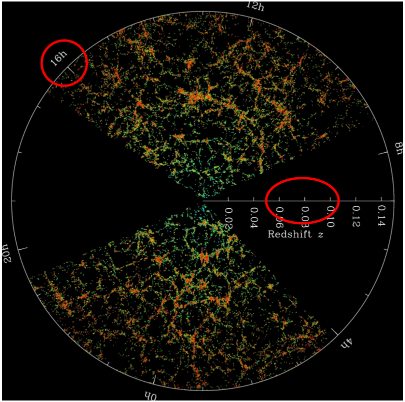
- Galaxy number counts
- Weak lensing shear
- The Lyman-$\alpha$ forest
- Intensity mapping (especially 21cm)

# Galaxy number counts

We count the number of galaxies is an small redshift bin and solid angle

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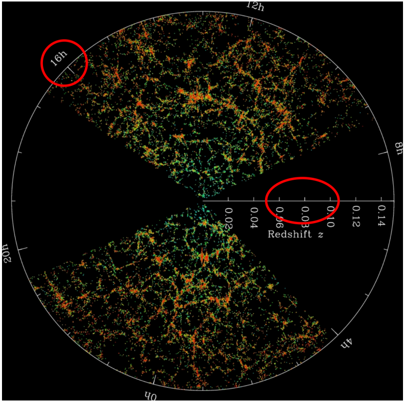


$$\Delta(\mathbf{n}, z) = \frac{N(\mathbf{n}, z) - \bar{N}(z)}{\bar{N}(z)}.$$

$$\xi(\theta, z, z') = \langle \Delta(\mathbf{n}, z)\Delta(\mathbf{n}', z')\rangle$$

$$\mathbf{n} \cdot \mathbf{n}' = \cos\theta\,.$$

This quantity is directly measurable.

When we compute it, already at first order perturbation theory, we must consider:

- We have to take fully into account that all observations are made on our past lightcone which is itself perturbed.
  We see density fluctuations which are further away from us, further in the past.
  We cannot observe 3 spatial dimensions but 2 spatial and 1 lightlike, more precisely we measure 2 angles and a redshift.

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# Computing galaxy number counts

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- The measured redshift is perturbed by peculiar velocities and by the gravitational potential.
- Not only the number of galaxies but also the volume is distorted.
- The angles we are looking into are not the ones into which the photons from a given galaxy arriving at our position have been emitted.

# The galaxy density fluctuation per redshift bin, per sold angle

Putting the density and volume fluctuations together one obtains the galaxy number density fluctuations from scalar perturbations to 1st order as function of the observed redshift $z$ and direction $\mathbf{n}$

$$\begin{aligned}\Delta(\mathbf{n},z) \quad &= \quad bD - 3\mathcal{H}V - (2-5s)\Phi + \Psi + \frac{1}{\mathcal{H}}\left[\dot{\Phi} + \partial_r(\mathbf{V}\cdot\mathbf{n})\right] \\ &+ \left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2-5s}{r(z)\mathcal{H}} + 5s\right)\left(\Psi + \mathbf{V}\cdot\mathbf{n} + \int_0^{r(z)} dr(\dot{\Phi}+\dot{\Psi})\right) \\ &- \frac{2-5s}{2r(z)}\int_0^{r(z)} dr\left[\frac{r(z)-r}{r}\Delta_\Omega(\Phi+\Psi) - 2(\Phi+\Psi)\right].\end{aligned}$$

( Bonvin & RD '11, Challinor & Lewis '11)

# The total galaxy density fluctuation per redshift bin, per sold angle

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( C. Bonvin & RD '11, Challinor & Lewis '11)

Main problem: Galaxy bias (non-linear, scale dependent)

# The angular power spectrum of galaxy number fluctuations

For fixed $z$, we can expand $\Delta(\mathbf{n}, z)$ in spherical harmonics,

$$\Delta(\mathbf{n}, z) = \sum_{\ell m} a_{\ell m}(z) Y_{\ell m}(\mathbf{n}), \qquad C_\ell(z, z') = \langle a_{\ell m}(z) a^*_{\ell m}(z') \rangle.$$

$$\xi(\theta, z, z') = \langle \Delta(\mathbf{n}, z) \Delta(\mathbf{n}', z') \rangle = \frac{1}{4\pi} \sum_\ell (2\ell + 1) C_\ell(z, z') P_\ell(\cos\theta)$$

$$\cos\theta = \mathbf{n} \cdot \mathbf{n}'$$

# The 3d power spectrum of galaxy number fluctuations

For spectroscopic surveys the 3d power spectrum in z-bins can be used in order not to loose redshift information or statistics.

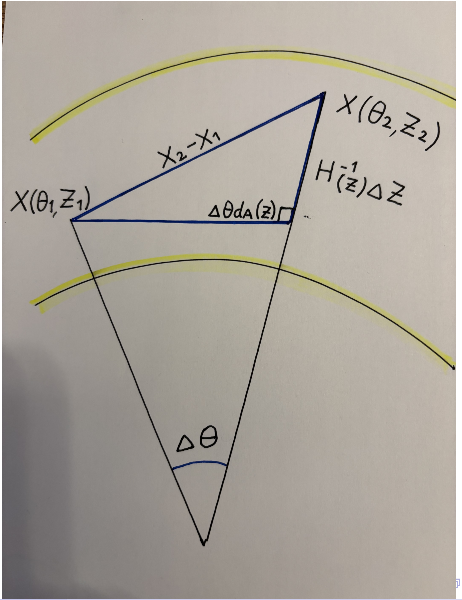
Converting the observed angles and redshifts into distances depends on the cosmology:
Consider a small redshift bin at mean redshift $\bar{z}$

$$|\mathbf{x}(\theta_1, z_1) - \mathbf{x}(\theta_2, z_2)| = \sqrt{d_A(\bar{z})^2\Delta\theta^2 + H^{-2}(\bar{z})\Delta z^2} = \frac{1}{H(\bar{z})}\sqrt{AP(\bar{z})\Delta\theta^2 + \Delta z^2}$$

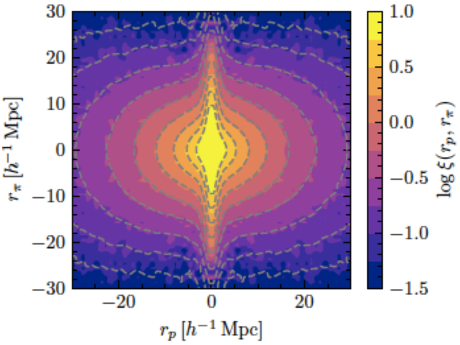
$$AP(z) = d_A(z)H(z) = H(z)\int_0^z \frac{dz'}{H(z')} \,.$$

RSD give raise to a quadrupole ($\ell = 2$) and hexadecapole ($\ell = 4$) in the 3d power spectrum.
It is more difficult to include lensing in the 3d power spectrum (see Castorina & Di Dio, 2021, Jelic-Cizmek et al. 2023).

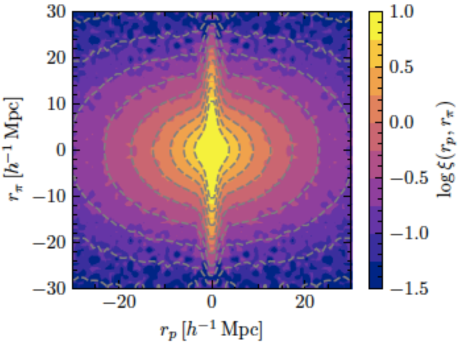
# The 3d power spectrum of galaxy number fluctuations



(from Lange et al. '21)

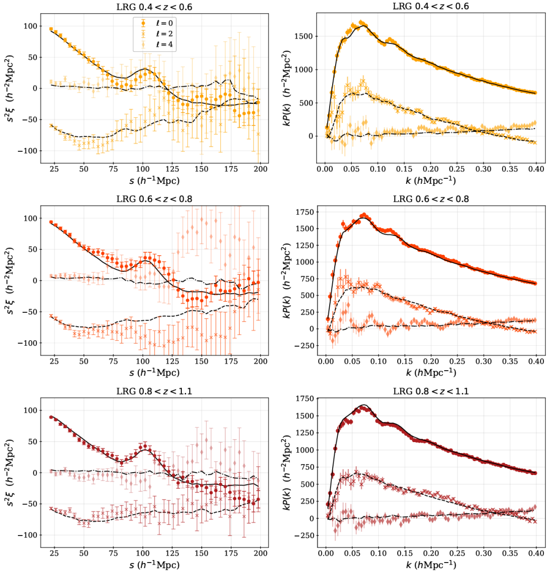



$0.18 < z < 0.3$




$0.3 < z < 0.42$

# The 3d power spectrum from DESI LRG (2024)

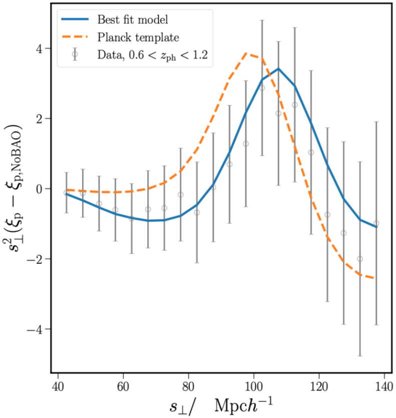



—- monopole ($\ell = 0$)

- - - quadrupole ($\ell = 2$)

-·-·- hexadecapole ($\ell = 4$)

[arXiv:2411.12020]




DES 6year BAO, 2024

[2402.10696]

$$s_P(z) \propto \int_0^z \frac{dz'}{H(z')}$$

$$\frac{d\log H}{d\log(1+z)} = \frac{3}{2}(1+w_{\rm de}\Omega_{\rm de})$$

# Measuring galaxy shapes, the shear power spectrum

The Jacobi map maps two angles, $(\theta^1, \theta^2)$ into an image in the sky $(x^1, x^2)$

$$x^a = D^a_b \theta^b$$

$$D^a_b(z, \mathbf{n}) = d_A(z) \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 + \omega \\ \gamma_2 - \omega & 1 - \kappa + \gamma_1 \end{pmatrix}$$

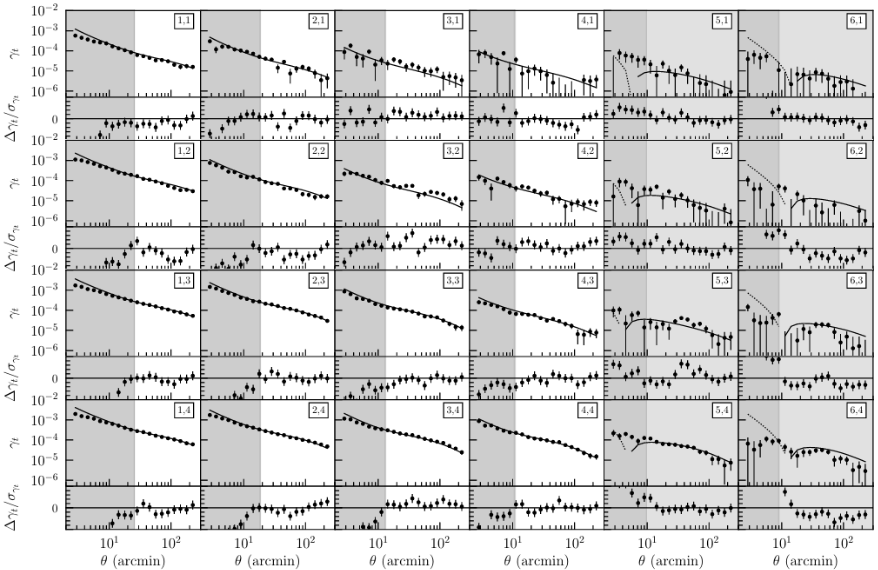
$\kappa$ is the convergence and $\gamma = \gamma_1 + i\gamma_2$ is the complex shear, a spin-2 quantity. The rotation $\omega$ vanishes at first order.
For purely scalar perturbations, neglecting relativistic corrections at very large scales

$$\gamma = \eth^2 \varphi\,, \qquad \kappa = \Delta\varphi = \frac{1}{2}(\eth\eth^* + \eth^*\eth)\varphi$$

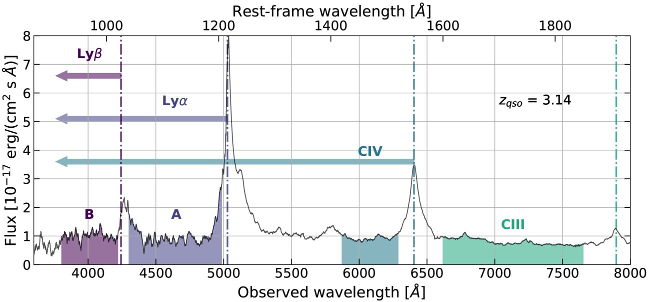
$$\varphi \simeq \frac{1}{2}\int_0^{r(z)} dr' \frac{(r(z) - r')}{rr'}(\Phi + \Psi)\,.$$

Main problem: Intrinsic alignment.

# DES 3Y : $\gamma\Delta$ correlations from the 3x2 analysis (2022) [2105.13549]



The correlation of Ly-$\alpha$ absorption lines give access to the 1d (radial) correlation function of hydrogen clouds.




DESi y1 [2404.03001v4]

Advantage: hi z.
Disadvantage: rather small scales, only 1d.

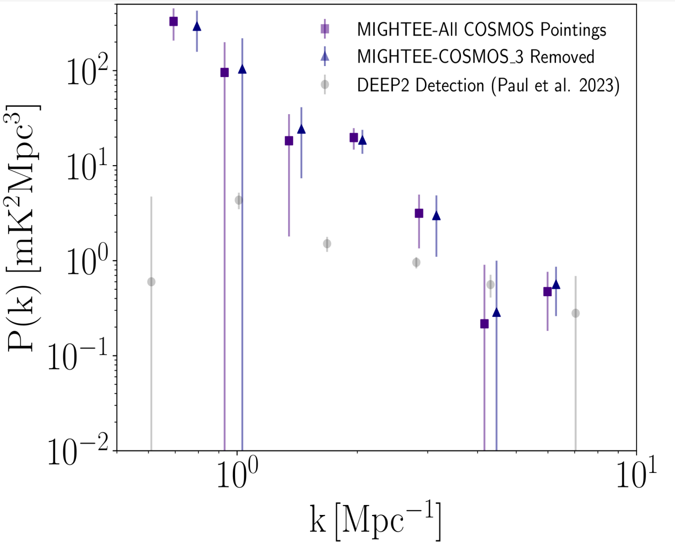
# Intensity mapping

Instead of counting galaxies, we can just measure the intensity coming from a certain emission line, e.g. H-spin flip transition (21cm) of CO lines and more. The intensity of the 21cm line at a given redshift is proportional to the density of hydrogen corrected for redshift distortions,

$$
\begin{aligned}
T_b &= \frac{3}{32\pi}\frac{h^3 A_{10}}{E_{10}} n_b x_{\rm HI}(1+z)\left|\frac{d\lambda}{dz}\right| + T_R e^{-\tau_\nu} \\
\Delta T_b &= \delta T_b(\mathbf{n}) - \delta T_b(\mathbf{n}') = \delta \bar{T}_b(z)[\Delta_T(\mathbf{n}) - \Delta_T(\mathbf{n}')] \\
\Delta_T(\mathbf{n}, z) &= b_T D - (3-b_e)\mathcal{H}V + \frac{1}{\mathcal{H}}\left[\dot{\Phi} + \partial_r(\mathbf{V}\cdot\mathbf{n})\right] + \Psi + \\
&\quad \left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + 2 - b_e\right)\left(\Psi + \mathbf{V}\cdot\mathbf{n} + \int_0^{r_s} dr(\dot{\Phi} + \dot{\Psi})\right)
\end{aligned}
$$

Main problem: Foregrounds

# Intensity mapping 21cm cross-power spectrum from MeerKAT (MIGHTEE 2025), $z \sim 0.4$




[2501.17564]

# What can we learn from LSS ?

- Cosmological parameters: $H_0$, $\Omega_m$, $\Omega_b$, curvature, $\cdots$
- What is dark matter ?
- What is dark energy ?
- Neutrino properties
- Physics of inflation (primordial non-Gaussianity)
- Testing GR

# General Relativity

Einstein's theory of gravity has been tested in many ways and passed all the tests with flying colors:

- Light deflection
- Perihel advance of mercury & many other binary systems
- Shapiro time delay
- . . .
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All these observations test GR on solar system size scales. Furthermore, they essentially test vacuum solutions of Einstein's equations,

$$R_{\mu\nu} = 0\,.$$

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$$R_{\mu\nu} = 0\,.$$

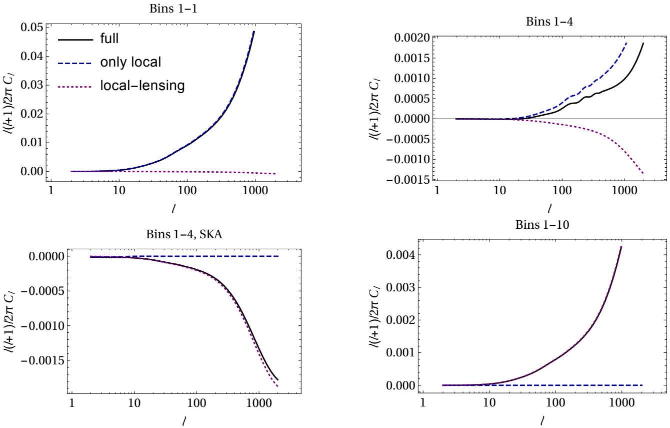
Can we also test these equations with matter,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = G_{\mu\nu} = 8\pi G T_{\mu\nu} \qquad ?$$

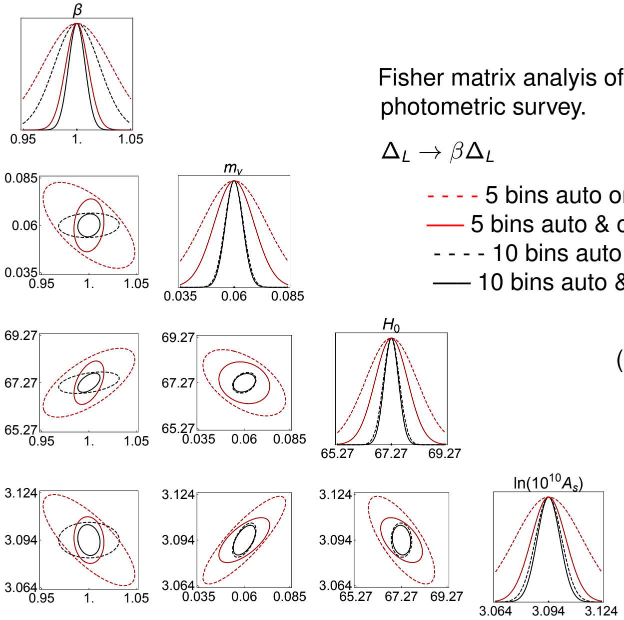
Well separated redshift bins measure mainly the lensing-density correlation:

$$\langle \Delta(\mathbf{n}, z)\Delta(\mathbf{n}', z')\rangle \simeq \langle \Delta^L(\mathbf{n}, z)\delta(\mathbf{n}', z')\rangle \quad z > z'$$

$$\Delta^L(\mathbf{n}, z) = (2 - 5s(z))\kappa(\mathbf{n}, z)$$




(Montanari & RD) [1506.01369]




Fisher matrix analyis of an Euclid-like photometric survey.

$\Delta_L \to \beta\Delta_L$

- - - - 5 bins auto only
—— 5 bins auto & cross
- - - - 10 bins auto only
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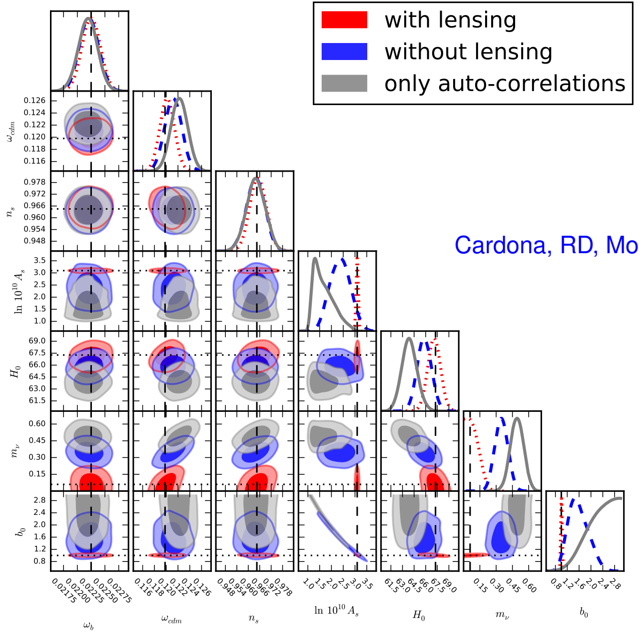
(Montanari & RD 2015)

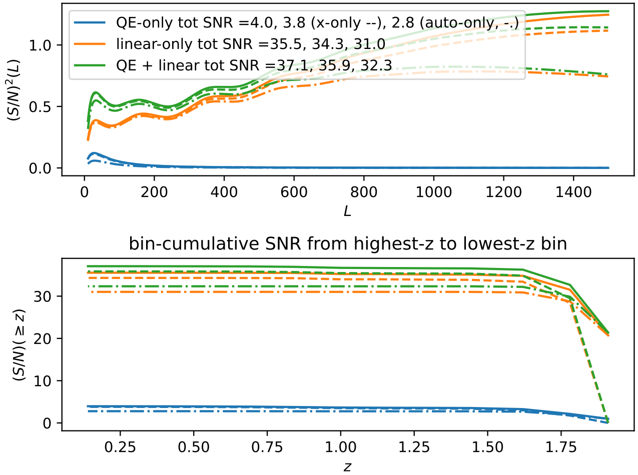
# Neglecting the lensing potential biases cosmological parameters




Cardona, RD, Montanari & Kunz, (2016)

# Estimating the lensing potential

With galaxy surveys we can estimate the lensing potential and in principle generate tomographic maps of it.




Nistane, Jalilvand, Carron, RD, Kunz (2022)

# $E_g$ statistics

In GR photon propagation, which governs weak lensing is sensitive to the sum of the Bardeen potentials, $\Phi + \Psi$; massive non-relativistic particles are accelerated by $\Psi$.

Non-relativistic density fluctuations generate $\Phi$ via the Poisson equation. In standard GR $\Phi = \Psi$ such that the following combination is independent of both, bias and scale:

$$E_g(k, z) \equiv \frac{H(z)(\Phi + \Psi)}{3H_0^2(1+z)V} = f(z) \simeq [\Omega_m(z)]^{0.55} \, .$$

(Zhang et al., 2007) This can be converted to (Pullen et al., 2015)

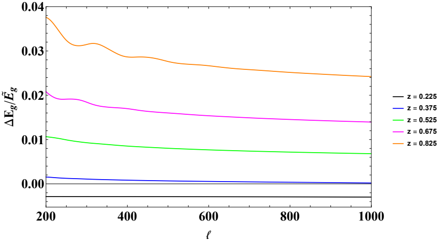
$$E_g(\ell, z) = \Gamma(z) \frac{C_\ell^{\kappa\delta}(z_*, z)}{\beta C_\ell^{\delta\delta}(z, z)}$$

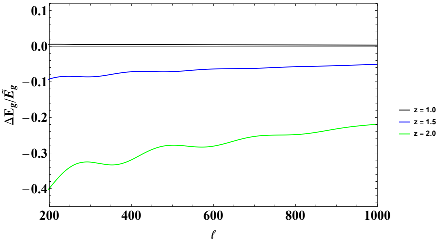
It has, however been pointed out (Moradinezhad Dizgah & RD 2016), that when observing galaxies, we do not directly observe $C_\ell^{\kappa\delta}$ or $C_\ell^{\delta\delta}$ but rather $C_\ell^{\kappa g}$ and $C_\ell^{gg}$.

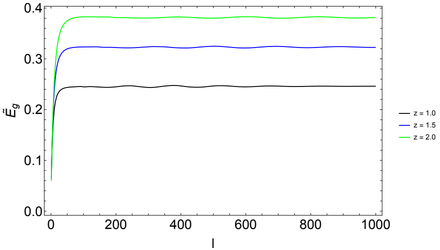
# $E_g$ statistics

Euclid like survey

DES-like survey




(Figures from Ghosh & RD 2019)

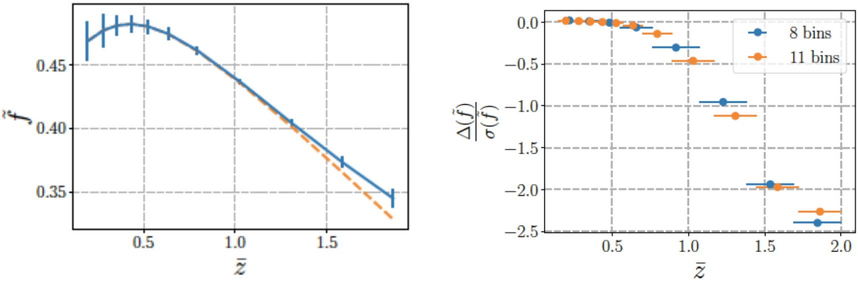
For intensity mapping $s \equiv 0.4$ and the correction terms vanish (Pourtsidou 2016).

# Measuring the growth rate of perturbations

- The growth rate of perturbations is very sensitive to the theory of gravity.
- A cosmological constant is the only form of dark energy which exhibits absolutely no clustering.
- Redshift space distortions are most sensitive to the growth rate. hence to measure it we need good redshift resolution $\rightarrow$ a spectroscopic survey.
- Even though 'lensing convergence' is not relevant for std cosmological parameter estimation with spectroscopic surveys, it does significantly affect the growth rate.

The growth rate is best estimated with RSD. However, in the k-power spectrum lensing is not easily included.
Including lensing, SKA2 will be able to determine it at the few % level (2 - 3% in a Fisher analysis).

$$\tilde{f}(z) = f(z)\sigma_8(z)$$ (neglecting lensing / including lensing in the analysis)




(Lepori, Jelic-Cizmek, Bonvin, RD 2020)

# Conclusions

- So far cosmological LSS data mainly determined $\xi(r)$, or equivalently $P(k)$ or $B(k_1, k_2, k_3)$ $\cdots$.
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- The new generation of 3d galaxy surveys like **Euclid**, **DESI**, **SKA**, **LSST** etc. can determine directly the measured 3d correlation functions and spectra, $\xi(\theta, z, z')$ and $C_\ell(z, z')$ and $b_{\ell_1,\ell_2,\ell_2}(z_1, z_2, z_3)\ \cdots$ from the data.

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- These 3d quantities will of course be more noisy, but they also contain more information.
- These spectra are not only sensitive to the matter distribution (density) but also to the velocity via (redshift space distortions) and to the perturbations of spacetime geometry (lensing) .

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- To test GR e.g. with the growth rate of perturbations it is important to include lensing even in the analysis of spectroscopic surveys.

- The spectra $C_\ell^{gg}(z,z')$, $C_\ell^{\kappa\kappa}(z,z')$ and $C_\ell^{g\kappa}(z,z')$, but also $C_\ell^{\gamma\gamma}(z,z')$, $C_\ell^{\kappa\gamma}(z,z')$ and $C_\ell^{g\gamma}(z,z')$ etc depend sensitively and in several different ways on the theory of gravity (growth factor, relation between $\Psi$ and $\Phi$), on the matter and baryon densities, and on the velocity.
  Their measurements provide a new route to not only to estimate cosmological parameters but also to test general relativity on cosmological scales and in the presence of matter.

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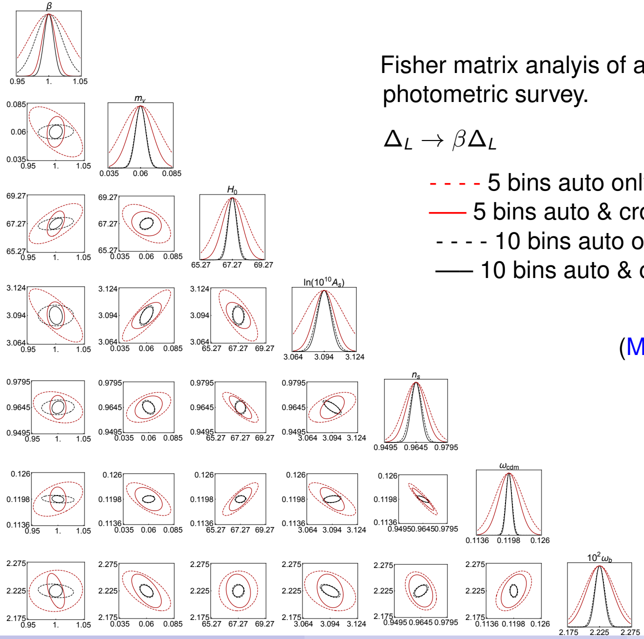


Fisher matrix analyis of an Euclid-like photometric survey.

$\Delta_L \to \beta\Delta_L$

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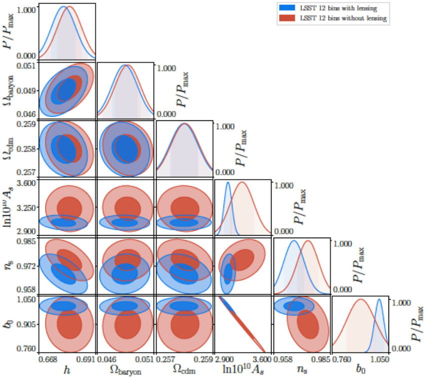
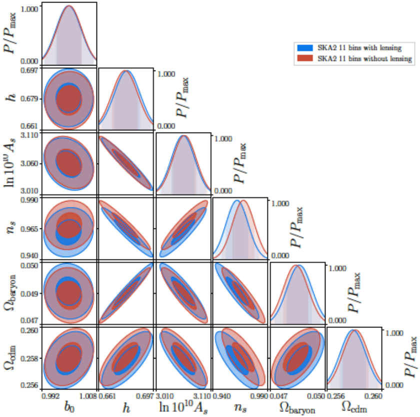
(Montanari & RD 2015)

# Standard parameter estimation from Vera Rubin Observatory (LSST) and SKA2 galaxy number counts




(Lepori, Jelic-Cizmek, Bonvin, RD 2020)

Errobars on std parameters from LSST will be similar to those from SKA2

$h_0$, $n_s$ and $\Omega_{\rm cdm}$ will even be better determined with LSST than with SKA2 !