SEARCHING FOR NEW PHYSICS USING MUONIC TRANSITIONS

JURE ZUPAN
U. OF CINCINNATI

based on Haxton, McElvain, Menzo, Rule, JZ, 2406.13818;
Fox, Hostert, Menzo, Pospelov, JZ, 2407.03450; 2306.15631;
Bigaran, Fox, Gouttenoire, Harnik, Krnjaic, Menzo, JZ, 2503.07722

Moriond EW, Mar 27 2025

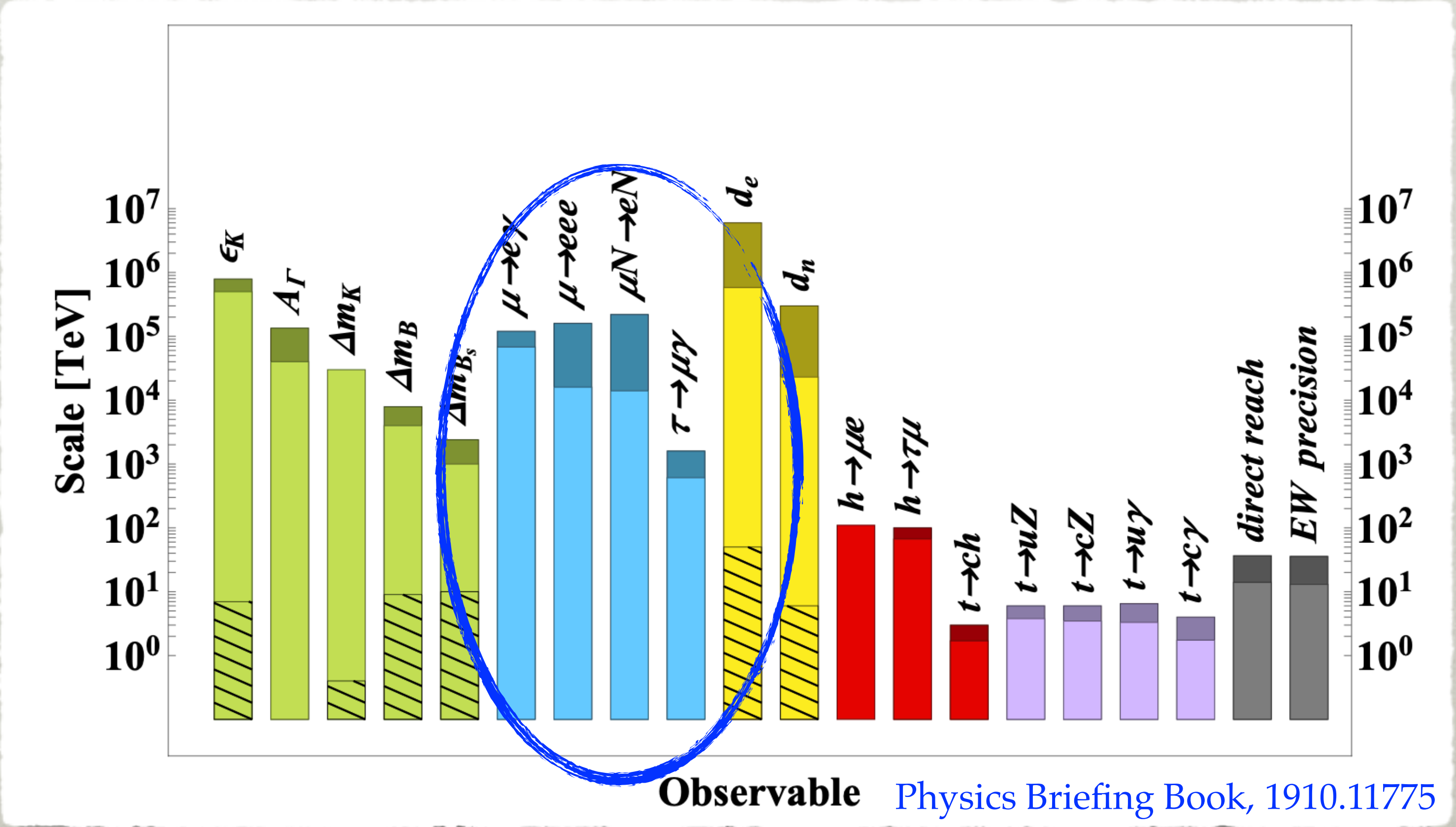
MUONS AND NEW PHYSICS

- muon the lightest unstable particle in the SM
- relatively easy to produce ⇒ large samples available
- can use muons to search for
  - heavy new physics
  - light new physics

SEARCHING FOR HEAVY NEW PHYSICS

see Ana Teixeira's talk for an in depth overview

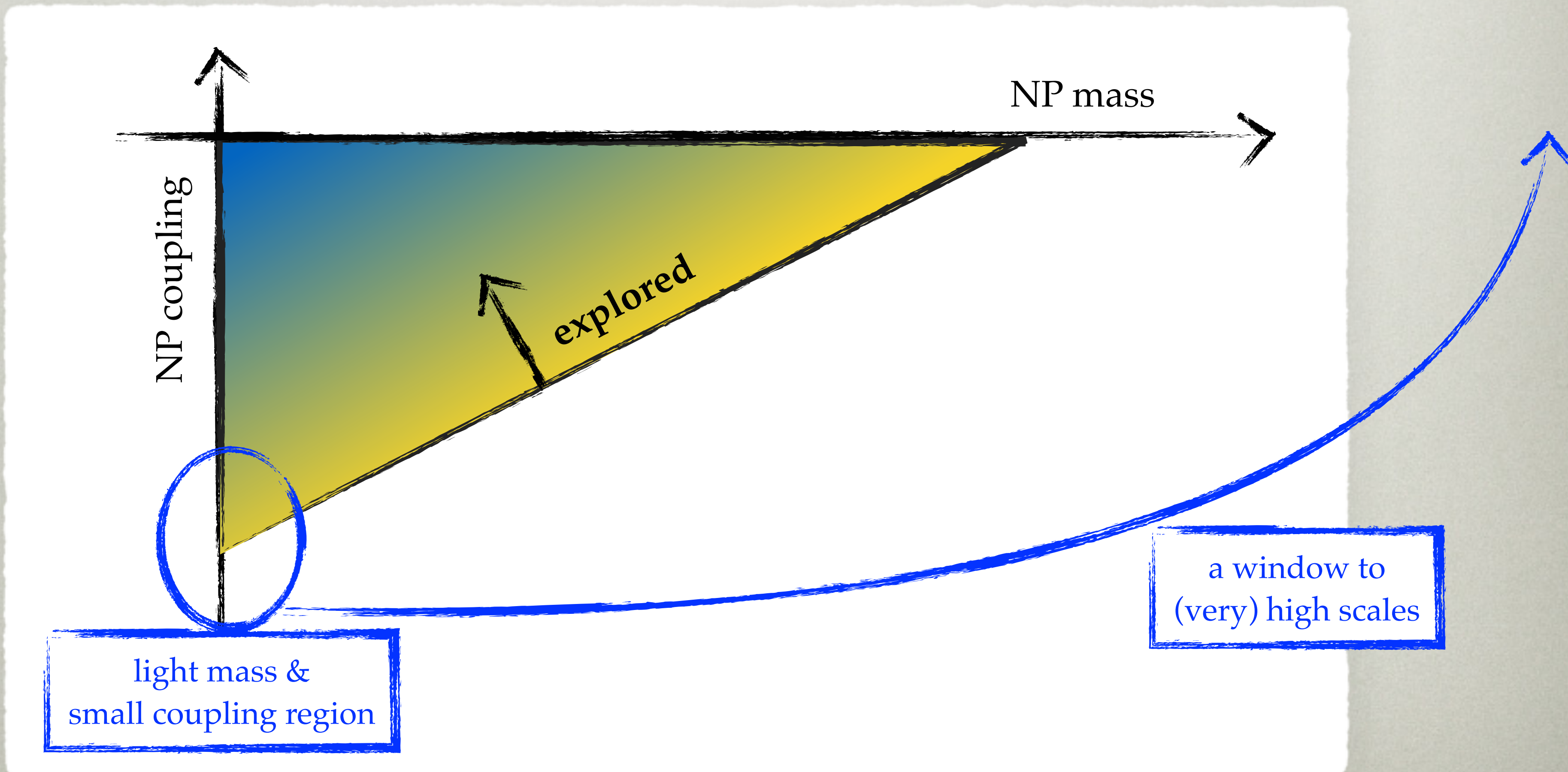
- high effective scales probed



Physics Briefing Book, 1910.11775

SEARCHING FOR LIGHT NEW PHYSICS

- light particles: a window to high UV dynamics
  - the gain comes from very small SM muon decay width



THE REST OF THE TALK

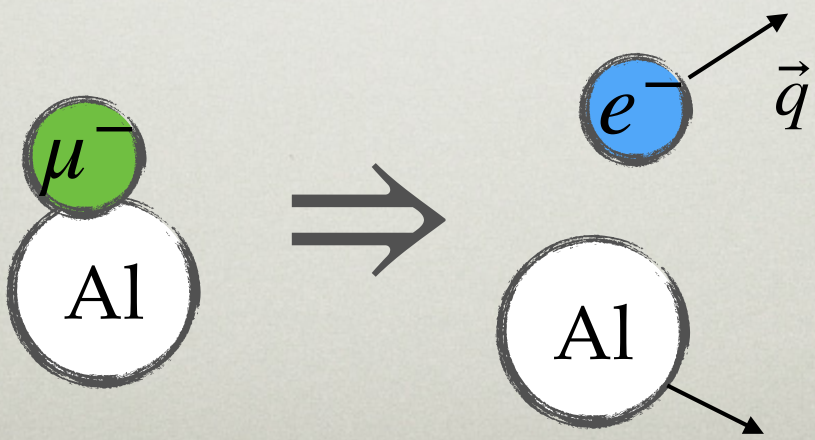
- heavy new physics
  - EFT based $\mu \to e$ predictions
- searching for light NP using muons
  - flavor violating QCD axion in $\mu \to ea$
  - time dependent searches in $\mu \to e\phi$
  - $\mu \to 5e$
  - ....

EFT based $\mu \to e$ predictions

$\mu \to e$ KINEMATICS

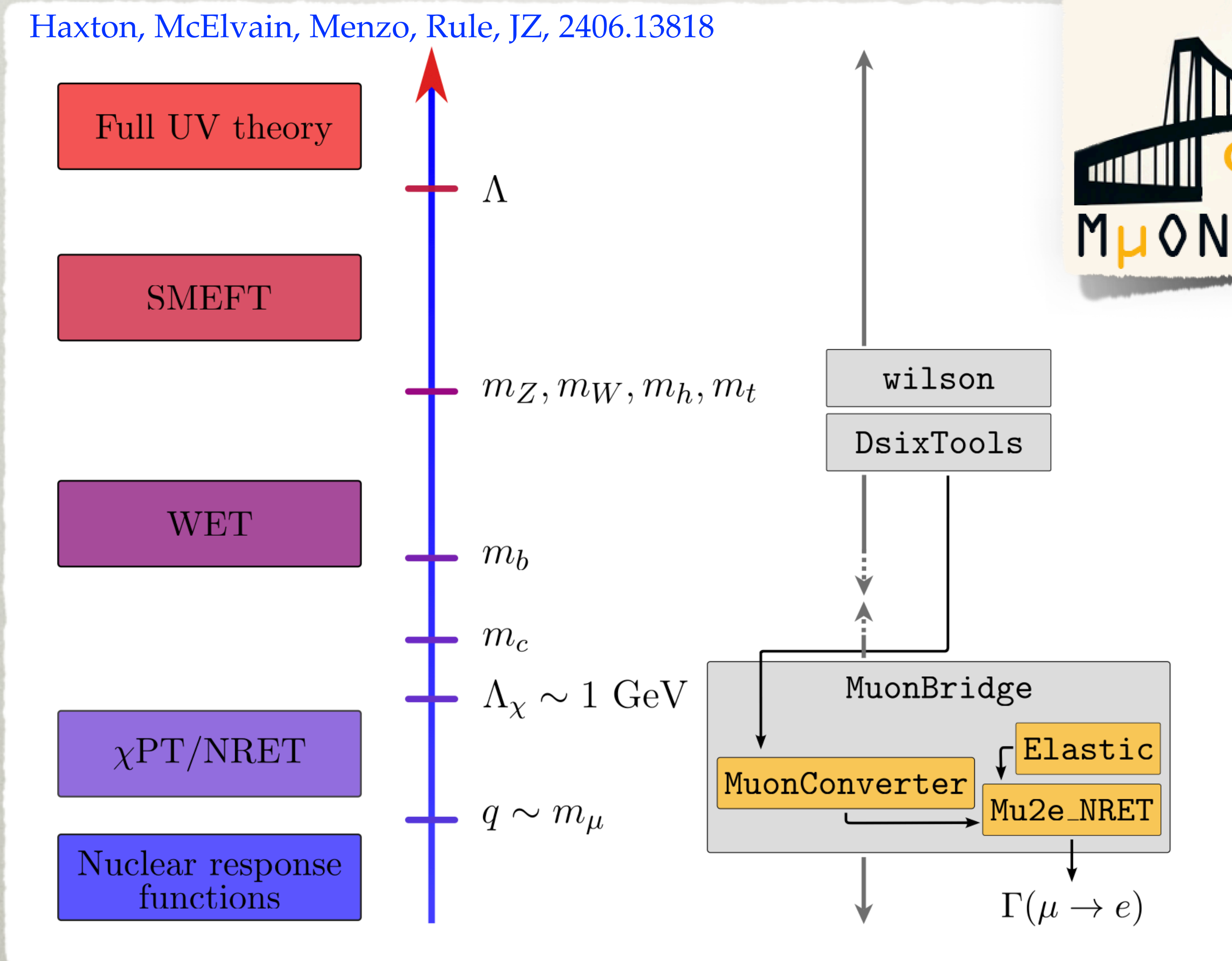
Haxton, McElvain, Menzo, Rule, JZ, 2406.13818

- initial state: μ^- in 1s orbital
- final state: relativistic e^- with three momentum
  - $E_\mu^{\rm bind} \ll m_\mu$ (for ^{27}Al $E_\mu^{\rm bind} \simeq 0.463\,\text{MeV}$) $\Rightarrow |\vec{q}| \sim \mathcal{O}(100\,\text{MeV})$



TOWER OF EFTS ⇒ MUONBRIDGE CODE

Haxton, McElvain, Menzo, Rule, JZ, 2406.13818



MμON BRIDGe

Haxton, McElvain, Ramsey-Mussolf, Rule, 2208.07945
Haxton, McElvain, Rule, 2109.13503
Haxton, McElvain, Menzo, Rule, JZ, 2406.13818

NON-RELATIV. EXPANSION

- a hierarchy of small parameters

$$y \equiv \left(\frac{qb}{2}\right)^2 > |\vec{v}_N| > |\vec{v}_\mu| > |\vec{v}_T|$$

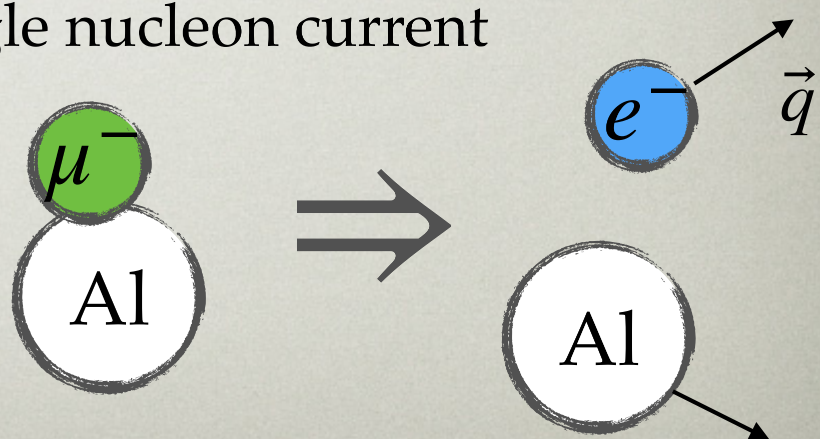
$b \sim$ nuclear size

$\vec{v}_N = (\vec{k}_1 + \vec{k}_2)/2$ average nucleon velocity

bound muon velocity

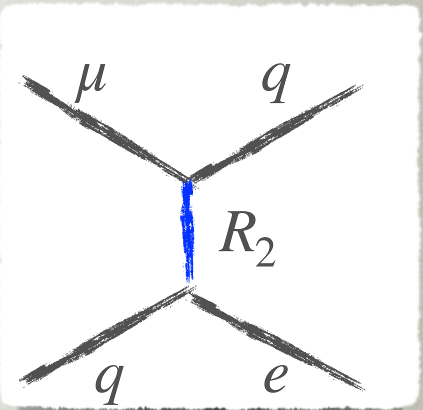
velocity of outgoing target nucleus

- $y \sim 0.2 - 0.5 \Rightarrow$ nuclear scales are being probed
- Chiral EFT: interactions with single nucleon current dominate
- can expand in v_N and v_μ
  - we keep $\mathcal{O}(v_N)$, $\mathcal{O}(v_\mu)$ terms



Haxton, McElvain, Menzo, Rule, JZ, 2406.13818

LEPTOQUARK EXAMPLE



- scalar leptoquark R_2 in the (3, 2, 7/6) of the SM gauge group

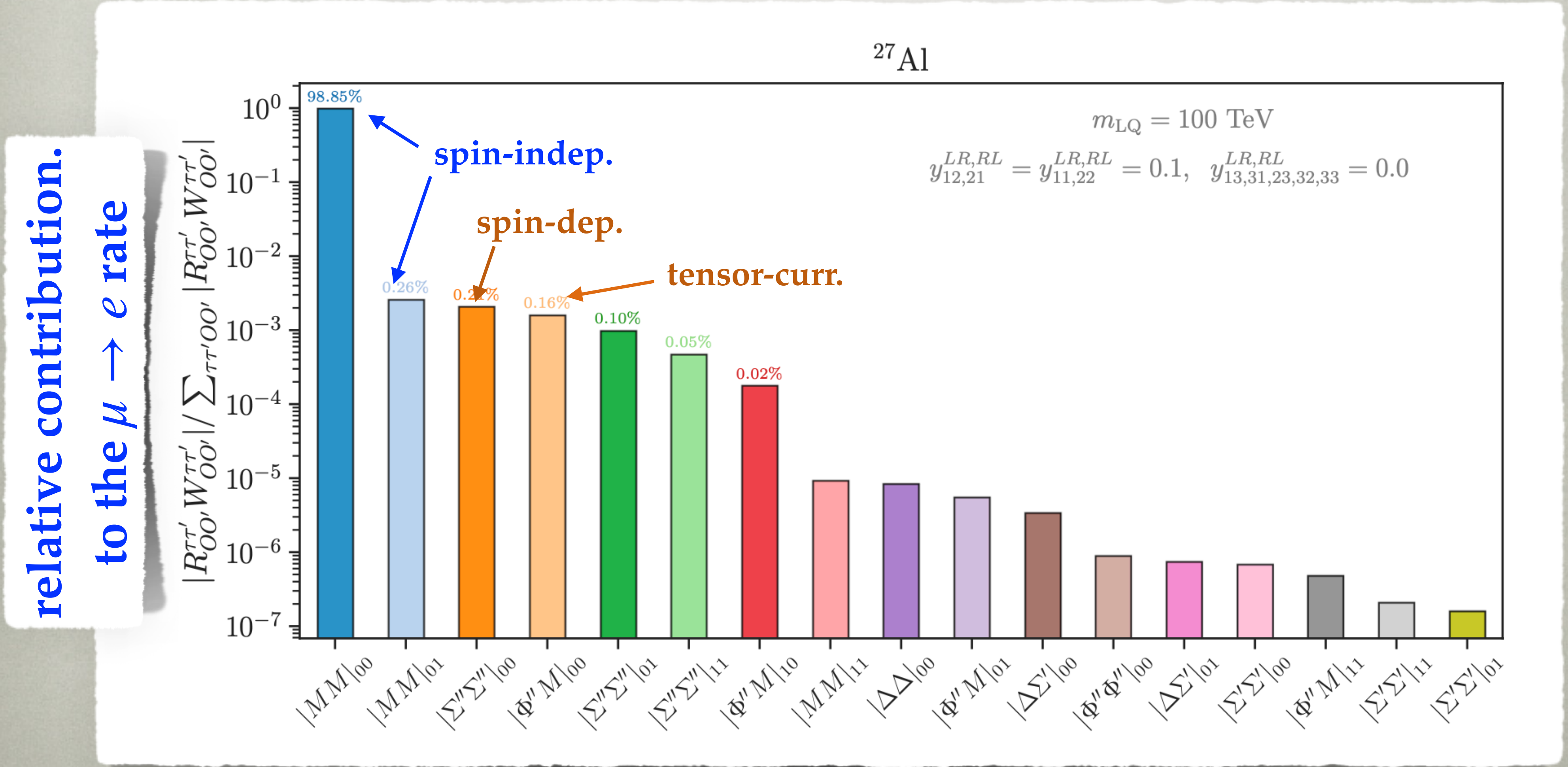
$$\mathcal{L} \supset y_{2\,ij}^{RL}\, \bar{u}_R^i R_2 L_L^j + y_{2\,ij}^{LR}\, \bar{e}_R^i R_2^* Q_L^j + \text{h.c.},$$

- integrating out $R_2 \Rightarrow$ generates all 10 dim 6 ops in WET
    - including tensor currents
    - these have coherently enhanced contribs. at subleading powers in v_N, $v_\mu \Rightarrow$ kept in **MuonBridge**

Haxton, McElvain, Menzo, Rule, JZ, 2406.13818

DIFFERENT CONTRIBS.

- typical point in the parameter space is dominated by spin independent contrib.



SEARCHING FOR LIGHT NEW PHYSICS

SEARCHING FOR LIGHT NEW PHYSICS

- can we use large datasets of stopped muons for light NP searches?
  - answer experiment dependent
- three examples
  - QCD axion in $\mu \to ea$
  - time dependent $\mu \to e\phi$
  - $\mu \to 5e$ at Mu3e

QCD axion

see, e.g., Calibbi, Redigolo, Ziegler, JZ, 2006.04795

AXION LIKE MODELS

- any spontaneously broken global symmetry ⇒ (p)NGB
  - if "light enough" can be DM
- in general couplings to gluons, photons, SM fermions

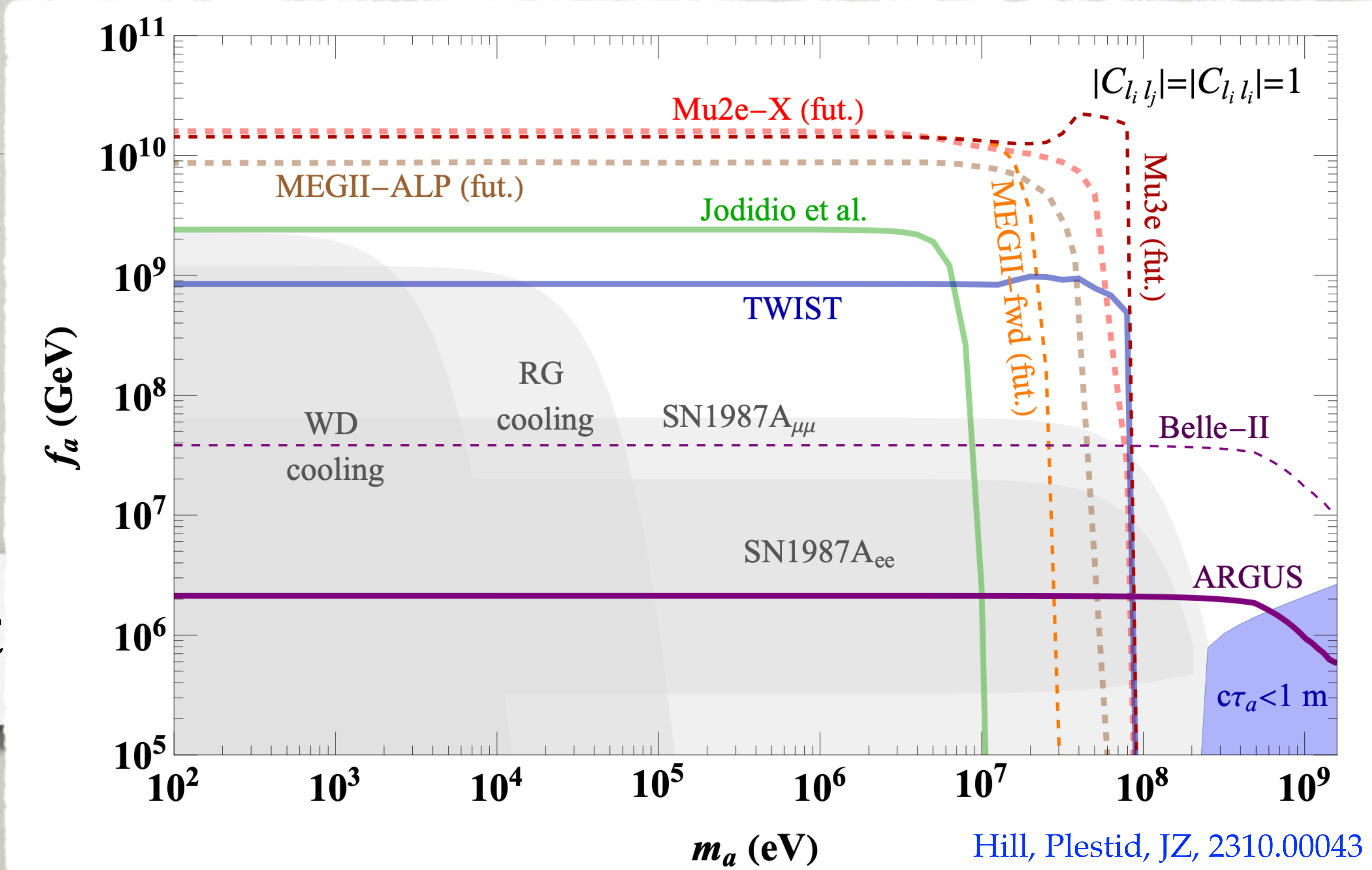
$$\mathcal{L}_{\rm eff} = \frac{\alpha_s}{8\pi}\frac{a}{f_a}G\tilde{G} + \frac{E}{N}\frac{\alpha_{\rm em}}{8\pi}\frac{a}{f_a}F\tilde{F} + \frac{\partial_\mu a}{2f_a}\,\bar{f}_i\gamma^\mu(C^V_{f_if_j} + C^A_{f_if_j}\gamma_5)f_j$$

$$F^{V,A}_{f_if_j} \equiv \frac{2f_a}{c^{V,A}_{f_if_j}}$$

$$F_{\ell_i\ell_j} = \frac{2f_a}{\sqrt{|C^V_{\ell_i\ell_j}|^2 + |C^A_{\ell_i\ell_j}|^2}}$$

- in general ALPs will have flavor violating couplings
  - here focus on enhanced couplings to leptons

see, e.g., Calibbi, Redigolo, Ziegler, JZ, 2006.04795



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TIME DEPENDENT

$$\mu \to e\phi$$

NON-ABELIAN pNGB

Bigaran, Fox, Gouttenoire, Harnik, Krnjaic, Menzo, JZ, 2503.07722

- if DM a non-Abelian pNGB the interactions with the SM of the form

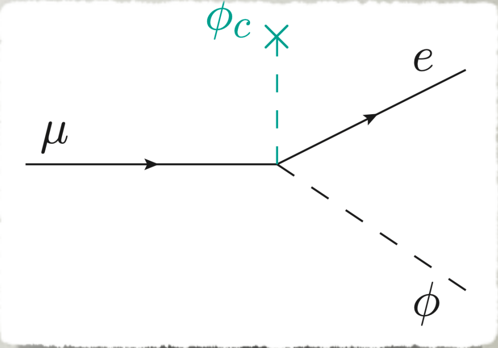
$$\mathcal{L}_{\rm int} \supset \frac{i\phi_c(\partial_\mu \phi)}{2f^2}\,\bar{\ell}_i\gamma^\mu(C^V_{ij}+C^A_{ij}\gamma_5)\ell_j,$$

  - example in the SM: $\pi^\pm$ interacting with leptons via photon exchange

- classical ϕ background induces time dependent $\mu \to e\phi$ decays

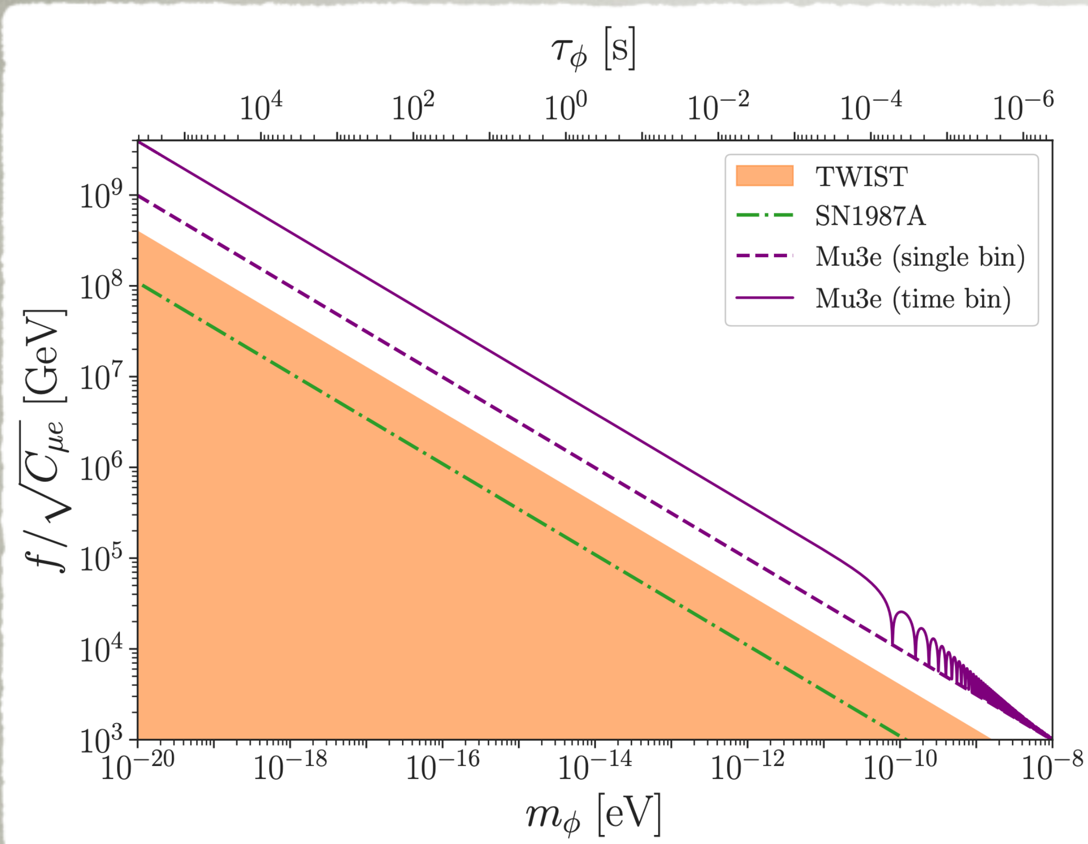
$$\phi_c(t) = \phi_0 \cos(m_\phi t + \delta)$$

$$\phi_0 = \frac{\sqrt{2\rho_\phi}}{m_\phi}$$



  - time dep. searches can be more sensitive ⇔ for systematics dominated searches
  - time dep. a smoking gun signal of DM

- can do the same type of search at MEG-II, Mu2e-X, COMET-X
  - can do time dep. $\tau \to \ell\phi$ search at Belle II



AN PNGB

n, Fox, Gouttenoire, Harnik, Krnjaic, Menzo, JZ, 2503.07722

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$$ {}^V_{ij} + C^A_{ij}\gamma_5)\ell_j, $$

with leptons via photon exchange

$$\phi_c(t) = \phi_0 \cos(m_\phi t + \delta)$$

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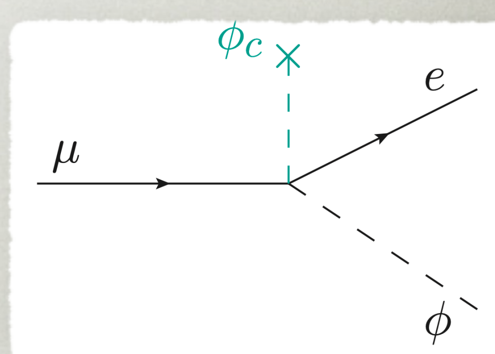

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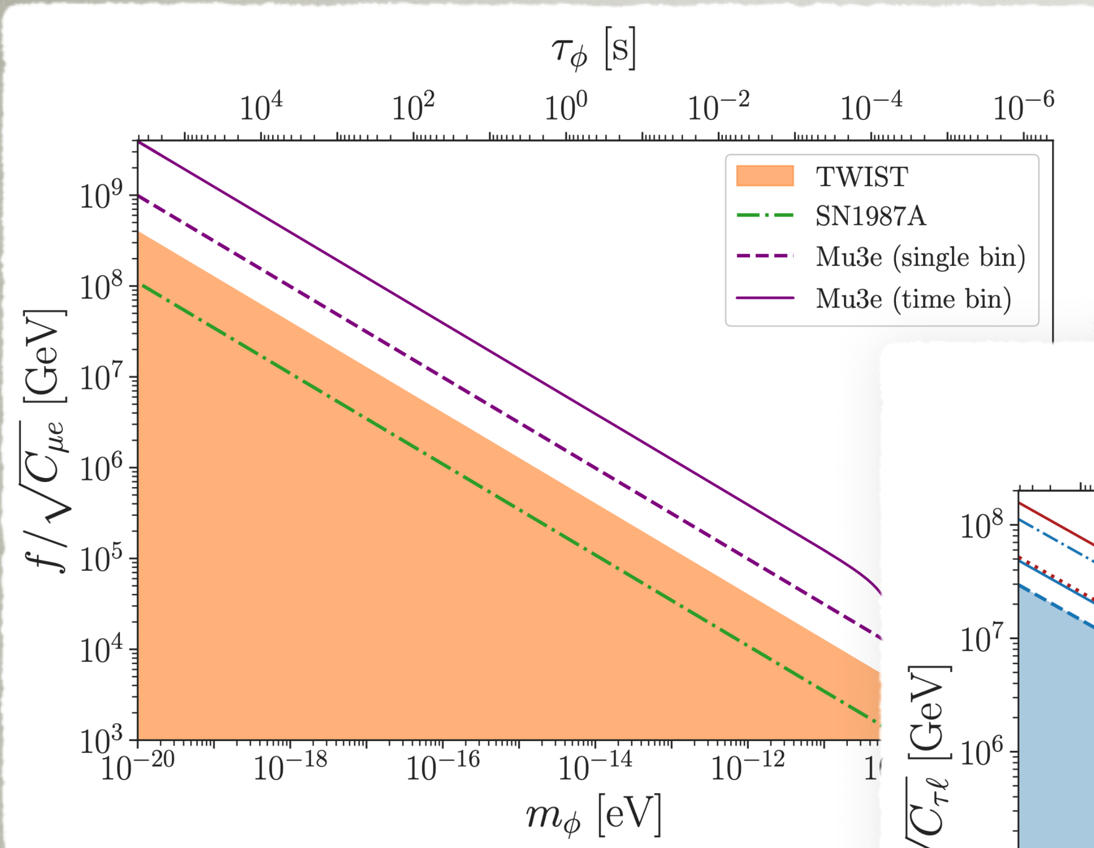
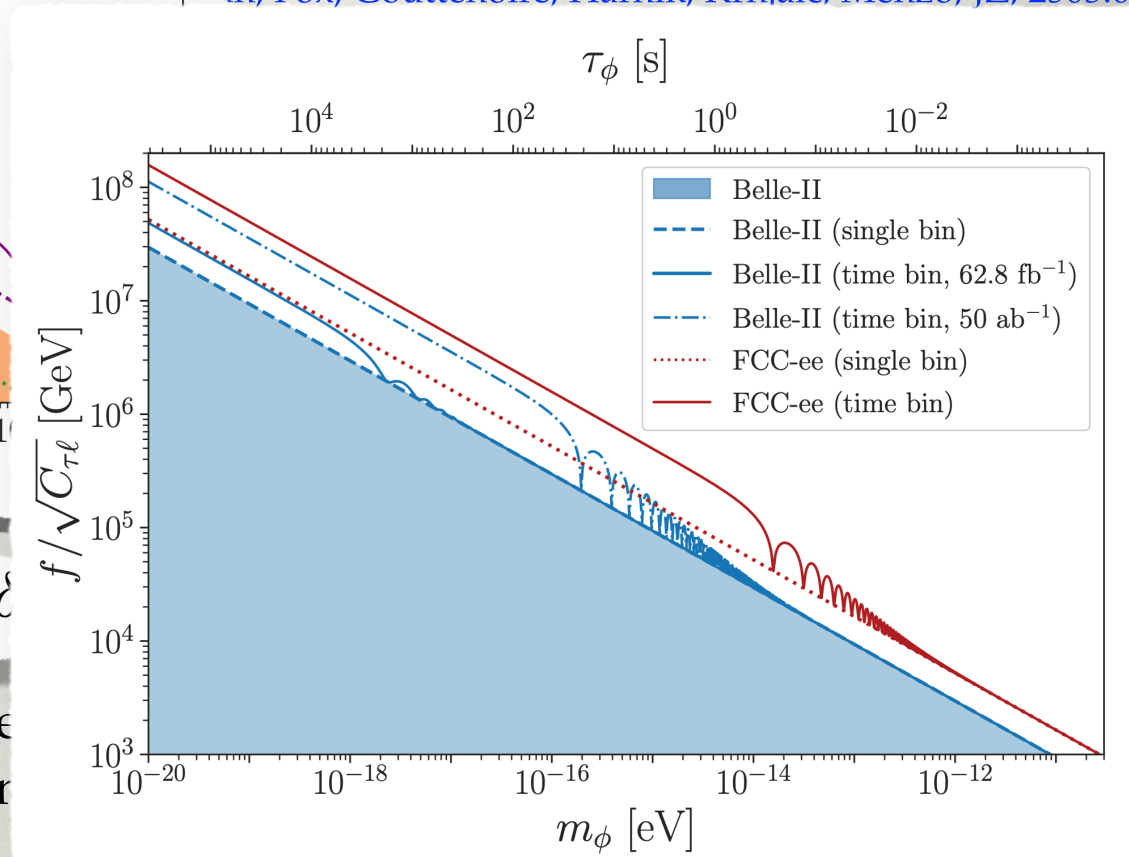


...AN PNGB

...an, Fox, Gouttenoire, Harnik, Krnjaic, Menzo, JZ, 2503.07722



$$\phi_c(t) = \phi_0 \cos(m_\phi t + \delta ...$$

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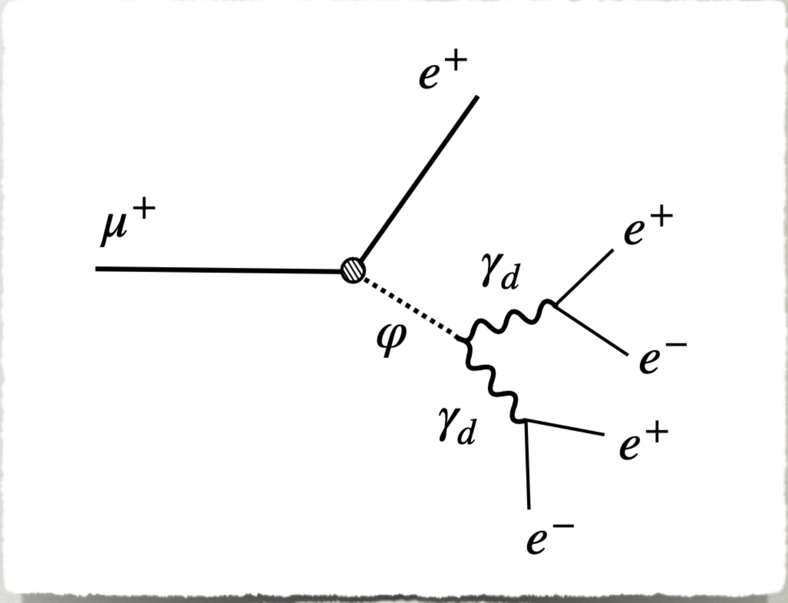
$$\mu \to 5e$$

A SIMPLE DARK SECTOR MODEL

Hostert, Menzo, Pospelov, JZ, 2306.15631

- higgsed dark abelian gauge group $U(1)_d$
  - dark photon γ_d
  - light dark Higgs h_d
- coupling to SM
  - kinetic mixing
  - flavor violating high dim op.,e.g., $\frac{C_{ij}}{\Lambda}\bar{\ell}_i e_j H\phi$
- very conservative reach at Mu3e: $\mathrm{Br}(\mu \to 5e) \sim 10^{-12}$
  - accepting all accidental bckgs.
  - $\Rightarrow$ reach on effective UV scale, $\Lambda \sim 10^{15}$ GeV



OTHER LIGHT NEW PYSICS SIGNATURES

- muon can only decay to e, γ, inv $\Rightarrow$ a "finite" numb. of differ. observable signatures

MEG-II MEG, 2005.00339

$e \backslash \gamma$	0	1	2	3	4
1		$e\gamma$	$e2\gamma$	$e3\gamma$	$e4\gamma$
3	$3e$	$3e\gamma$	$3e2\gamma$	$\ldots$	
5	$5e$	$5e\gamma$	$\ldots$		
7	$7e$	$\ldots$			
$\ldots$					

Mu3e (→ $3e$)

2306.15631 (→ $5e$)

Greljo, Palavrić, Tunja, JZ, work in progress

- other options
  - displaced vertices in $\mu \to 3e$ Knapen, Opferkuch, Redigolo, Tammaro, 2410.13941
  - $\mu \to 3ea$ Knapen, Langhoff, Opferkuch, Redigolo, 2311.17915
  - BNV annihilation $\mu^- p \to$ dark sect. Fox, Hostert, Menzo, Pospelov, JZ, 2407.03450
  - ....

CONCLUSIONS

- EFT approach well suited for predicting the $\mu \to e$ conversion rates
  - results available in the form of a public code **MuonBridge**
- rare muon decays can be use to search for light NP
  - QCD axion, time dep. npNGB DM, $\mu \to 5e$,...

BACKUP SLIDES

LIGHT NEW PHYSICS $\Rightarrow$ PROBE OF HIGH SCALES

- rare decays into a light state, X, e.g., $\mu \to e\phi$,
  - exquisite probes of UV physics
- parametric gains compared to probing NP through dim-6 ops
  - the reason is that the SM decay widths are power suppressed $\Gamma_\ell \propto m_\ell^5/m_W^4$
- if $\mu \to e\phi$ through dim 5 op. suppressed by $1/f_\phi \Rightarrow Br(\mu \to e\varphi) \propto (m_W^2/f_\phi m_\mu)^2$

WET

Haxton, McElvain, Menzo, Rule, JZ, 2406.13818

- 10 dimension 6 ops

$$\mathcal{Q}_{1,q}^{(6)} = (\bar{e}\gamma_\alpha\mu)(\bar{q}\gamma^\alpha q)\,, \qquad \mathcal{Q}_{2,q}^{(6)} = (\bar{e}\gamma_\alpha\gamma_5\mu)(\bar{q}\gamma^\alpha q)\,,$$

$$\mathcal{Q}_{3,q}^{(6)} = (\bar{e}\gamma_\alpha\mu)(\bar{q}\gamma^\alpha\gamma_5 q)\,, \qquad \mathcal{Q}_{4,q}^{(6)} = (\bar{e}\gamma_\alpha\gamma_5\mu)(\bar{q}\gamma^\alpha\gamma_5 q)\,.$$

$$\mathcal{Q}_{5,q}^{(6)} = (\bar{e}\mu)(\bar{q}q)\,, \qquad \mathcal{Q}_{6,q}^{(6)} = (\bar{e}i\gamma_5\mu)(\bar{q}q)\,,$$

$$\mathcal{Q}_{7,q}^{(6)} = (\bar{e}\mu)(\bar{q}i\gamma_5 q)\,, \qquad \mathcal{Q}_{8,q}^{(6)} = (\bar{e}i\gamma_5\mu)(\bar{q}i\gamma_5 q)\,,$$

$$\mathcal{Q}_{9,q}^{(6)} = (\bar{e}\sigma^{\alpha\beta}\mu)(\bar{q}\sigma_{\alpha\beta}q)\,, \qquad \mathcal{Q}_{10,q}^{(6)} = (\bar{e}i\sigma^{\alpha\beta}\gamma_5\mu)(\bar{q}\sigma_{\alpha\beta}q)\,.$$

- additional 16 operators at dimension 7
- related to other WET bases used in the literature by linear transf.
- note: tensor currents appear already at dimension 6

Haxton, McElvain, Ramsey-Mussolf, Rule, 2208.07945
Haxton, McElvain, Rule, 2109.13503

NON-RELATIVISTIC LIMIT

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bound muon velocity

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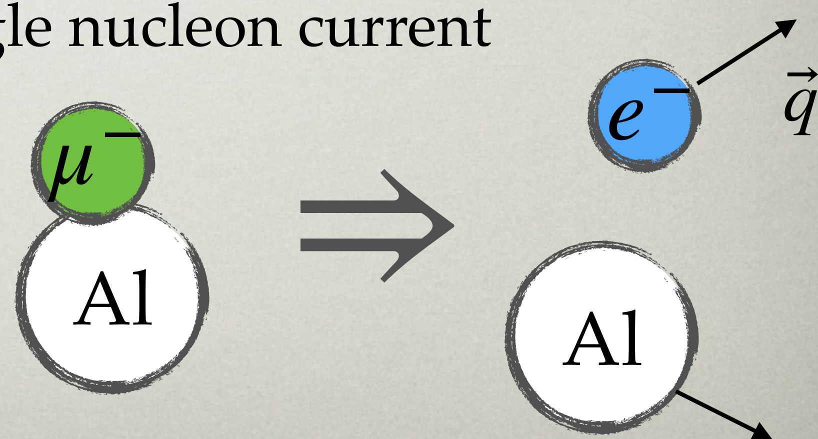

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NON-RELATIVISTIC LIMIT

$$\mathcal{O}_1 = 1_L\, 1_N, \qquad \mathcal{O}_2' = 1_L\, i\hat{q}\cdot\vec{v}_N,$$
$$\mathcal{O}_3 = 1_L\, i\hat{q}\cdot[\vec{v}_N\times\vec{\sigma}_N], \qquad \mathcal{O}_4 = \vec{\sigma}_L\cdot\vec{\sigma}_N,$$
$$\mathcal{O}_5 = \vec{\sigma}_L\cdot(i\hat{q}\times\vec{v}_N), \qquad \mathcal{O}_6 = i\hat{q}\cdot\vec{\sigma}_L\, i\hat{q}\cdot\vec{\sigma}_N,$$
$$\mathcal{O}_7 = 1_L\, \vec{v}_N\cdot\vec{\sigma}_N, \qquad \mathcal{O}_8 = \vec{\sigma}_L\cdot\vec{v}_N,$$
$$\mathcal{O}_9 = \vec{\sigma}_L\cdot(i\hat{q}\times\vec{\sigma}_N), \qquad \mathcal{O}_{10} = 1_L\, i\hat{q}\cdot\vec{\sigma}_N,$$
$$\mathcal{O}_{11} = i\hat{q}\cdot\vec{\sigma}_L\, 1_N, \qquad \mathcal{O}_{12} = \vec{\sigma}_L\cdot[\vec{v}_N\times\vec{\sigma}_N],$$
$$\mathcal{O}_{13}' = \vec{\sigma}_L\cdot(i\hat{q}\times[\vec{v}_N\times\vec{\sigma}_N]), \qquad \mathcal{O}_{14} = i\hat{q}\cdot\vec{\sigma}_L\, \vec{v}_N\cdot\vec{\sigma}_N,$$
$$\mathcal{O}_{15} = i\hat{q}\cdot\vec{\sigma}_L\, i\hat{q}\cdot[\vec{v}_N\times\vec{\sigma}_N], \qquad \mathcal{O}_{16}' = i\hat{q}\cdot\vec{\sigma}_L\, i\hat{q}\cdot\vec{v}_N.$$

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- y
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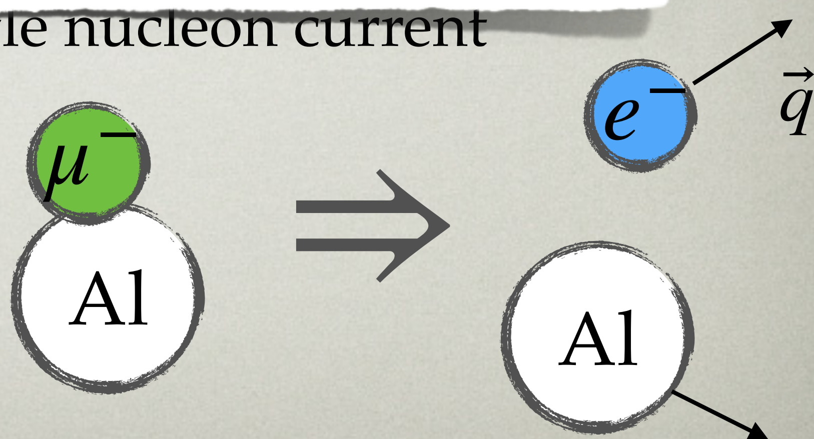


MATCHING ONTO NRET

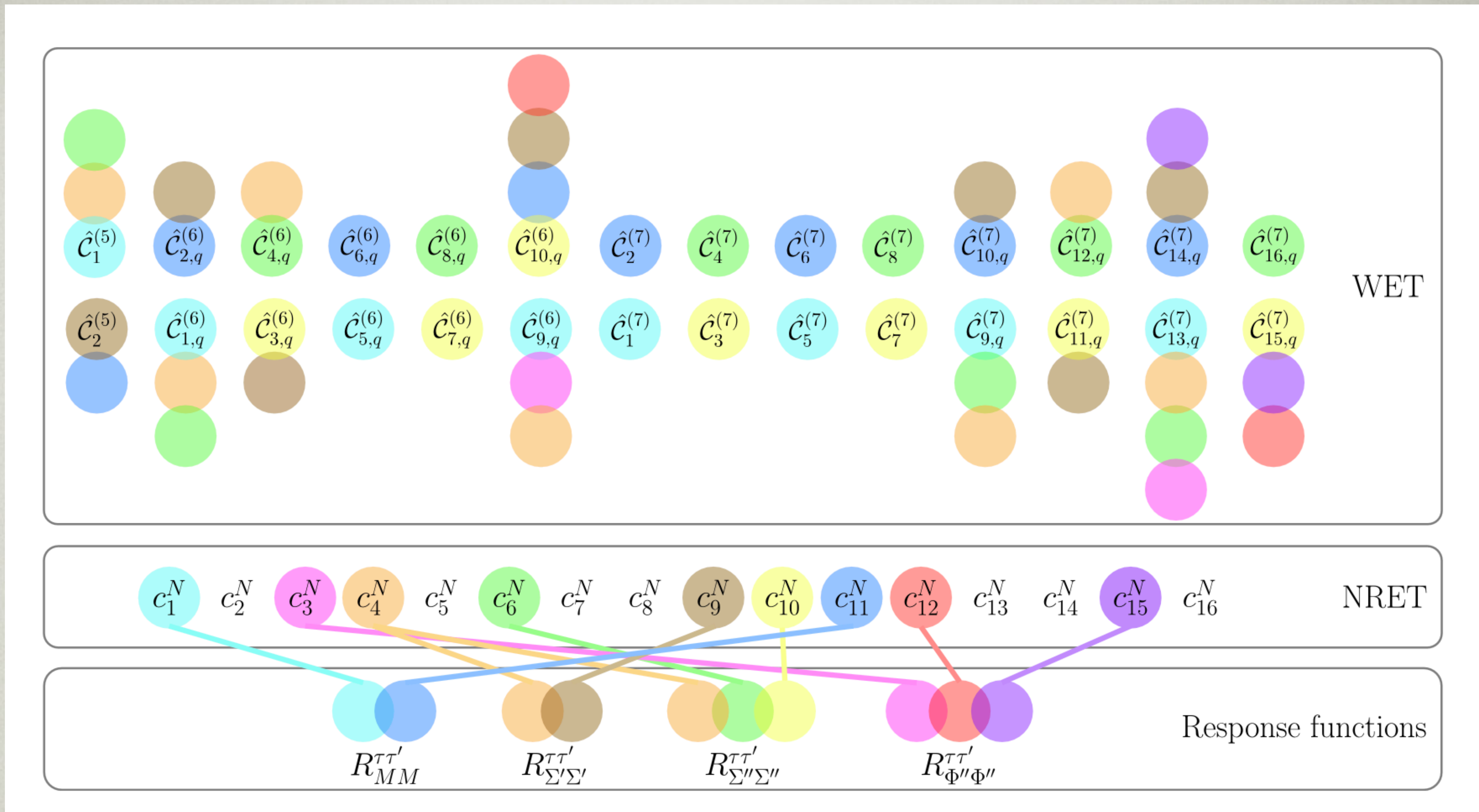
- NRET effective Lagrangian

$$\mathcal{L}_{\rm eff}^{\rm NRET} = \sum_{N=n,p} \sum_{i=1}^{16} c_i^N \mathcal{O}_i^N + \cdots ,$$

- low energy coefficients c_i^N functions of $\vec{q}_{\rm eff}^{\,2}$
  - for $\mu \to e$ this is a constant
  - their values from nonperturbative matching of WET to NRET
    - follow from nucleon matrix elements $\langle N | \mathcal{O}_i | N \rangle$
- for $\mu \to e$ transition rate prediction need nuclear physics $|\langle A | \mathcal{O}_i | A \rangle|^2$
  - nuclear response functions W_i

$$W_M \sim \mathcal{O}(A^2) \gg \left\{ W_{\Sigma'}, W_{\Sigma''}, \frac{q_{\rm eff}^2}{m_N^2} W_{\tilde{\Phi}''} \right\} \gg \left\{ \frac{q_{\rm eff}^2}{m_N^2} W_{\Delta}, \frac{q_{\rm eff}^2}{m_N^2} W_{\tilde{\Phi}'} \right\}$$

  - $\mu \to e$ rate: $\Gamma(\mu \to e) \propto \sum_i R_i W_i$



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  - $\mu \to e$ rate: $\Gamma(\mu \to e) \propto \sum_i R_i W_i$

MUONBRIDGE

- the code/repository **MuonBridge** consists of three modules
    - **MuonConverter:** matches WET to NRET
        - can interface with RG running codes
    - **Mu2e_NRET:** calculates the $\mu \rightarrow e$ rate

$$B(\mu^- \to e^-) = \frac{\Gamma\left[\mu^- + (A,Z) \to e^- + (A,Z)\right]}{\Gamma\left[\mu^- + (A,Z) \to \nu_\mu + (A,Z-1)\right]},$$

        - particle physics input from **MuonConverter**, i.e., WET Wilson coeffs. C_i
    - **Elastic:** a database of shell model density matrices for calculating nuclear form factors
- comes in both **Python** and **Mathematica** versions

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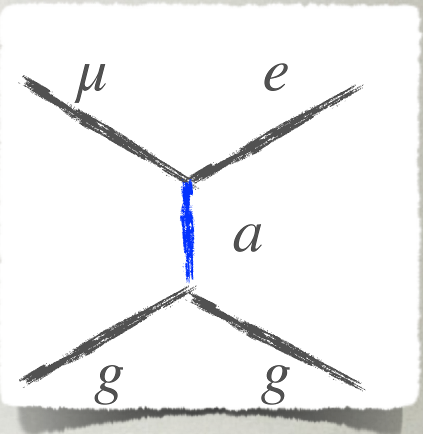
  - $\mu \to e$ rate: $\Gamma(\mu \to e) \propto \sum_i R_i W_i$

LIGHT ALP



- the same formalism trivially extends to light mediators
- example light ALP coupling to μe and gluons
- strictly speaking WET no longer an appropriate EFT
  - but trivial fix, allow Wilson coeffs to be q^2 dependent, $C_i \propto 1/(m_a^2 - q^2)$
  - since a only weakly couples to gluons: corrections to QCD can be neglected, i.e., just an external probe
  - in $\mu \to e$ the q is fixed, so C_i are even constants

MATCHING ONTO NRET

- NRET effective Lagrangian

$$\mathcal{L}_{\rm eff}^{\rm NRET} = \sum_{N=n,p} \sum_{i=1}^{16} c_i^N \mathcal{O}_i^N + \cdots ,$$

- low energy coefficients are functions of $\vec{q}_{\rm eff}^{\,2}$ in general
  - for $\mu \to e$ this is a constant
- their values from nonperturbative matching of WET to NRET
  - follow from nucleon matrix elements
  - for instance

$$c_1^N = \sum_q \frac{1}{m_q} \hat{\mathcal{C}}_{5,q}^{(6)} F_S^{q/N} + \hat{\mathcal{C}}_1^{(7)} F_G^N + \hat{\mathcal{C}}_5^{(7)} F_\gamma^N ,$$
$$+ \sum_q \hat{\mathcal{C}}_{1,q}^{(6)} F_1^{q/N} + m_+ \sum_q \hat{\mathcal{C}}_{9,q}^{(7)} F_1^{q/N} - \sum_q \frac{q^2}{2 m_N m_q} \hat{\mathcal{C}}_{13,q}^{(7)} \Big(F_{T,1}^{q/N} - 4 F_{T,2}^{q/N} \Big)$$

N

- NR

$$\langle N'|\bar q\gamma^\mu q|N\rangle = \bar u'_N\Big[F_1^{q/N}(q^2)\gamma^\mu - \frac{i}{2m_N}F_2^{q/N}(q^2)\sigma^{\mu\nu}q_\nu\Big]u_N\,,$$

$$\langle N'|\bar q\gamma^\mu\gamma_5 q|N\rangle = \bar u'_N\Big[F_A^{q/N}(q^2)\gamma^\mu\gamma_5 - \frac{1}{2m_N}F_{P'}^{q/N}(q^2)\gamma_5 q^\mu\Big]u_N\,,$$

$$\langle N'|m_q\bar q q|N\rangle = F_S^{q/N}(q^2)\,\bar u'_N u_N\,,$$

$$\langle N'|m_q\bar q i\gamma_5 q|N\rangle = F_P^{q/N}(q^2)\,\bar u'_N i\gamma_5 u_N\,,$$

$$\langle N'|\frac{\alpha_s}{12\pi}G^{a\mu\nu}G^a_{\mu\nu}|N\rangle = F_G^N(q^2)\,\bar u'_N u_N\,,$$

$$\langle N'|\frac{\alpha_s}{8\pi}G^{a\mu\nu}\tilde G^a_{\mu\nu}|N\rangle = F_{\tilde G}^N(q^2)\,\bar u'_N i\gamma_5 u_N\,,$$

$$\langle N'|m_q\bar q\sigma^{\mu\nu}q|N\rangle = \bar u'_N\Big[F_{T,0}^{q/N}(q^2)\,\sigma^{\mu\nu} - \frac{i}{2m_N}\gamma^{[\mu}q^{\nu]}F_{T,1}^{q/N}(q^2) - \frac{i}{m_N^2}q^{[\mu}k_{12}^{\nu]}F_{T,2}^{q/N}(q^2)\Big]u_N\,,$$

$$\langle N'|\frac{\alpha}{12\pi}F^{\mu\nu}F_{\mu\nu}|N\rangle = F_\gamma^N(q^2)\,\bar u'_N u_N\,,$$

$$\langle N'|\frac{\alpha}{8\pi}F^{\mu\nu}\tilde F_{\mu\nu}|N\rangle = F_{\tilde\gamma}^N(q^2)\,\bar u'_N i\gamma_5 u_N\,.$$

- low
  - f
- the NR
  - f
  - for instance

$$c_1^N = \sum_q \frac{1}{m_q}\hat{\mathcal C}^{(6)}_{5,q}F_S^{q/N} + \hat{\mathcal C}^{(7)}_1 F_G^N + \hat{\mathcal C}^{(7)}_5 F_\gamma^N,$$

$$+\sum_q \hat{\mathcal C}^{(6)}_{1,q}F_1^{q/N} + m_+\sum_q \hat{\mathcal C}^{(7)}_{9,q}F_1^{q/N} - \sum_q \frac{q^2}{2m_N m_q}\hat{\mathcal C}^{(7)}_{13,q}\Big(F_{T,1}^{q/N} - 4F_{T,2}^{q/N}\Big)$$

CONVERSION RATE

- matrix elements squared of NRET operators leads to nuclear response functions W_i

$$\Gamma(\mu \to e) = \frac{G_F^2}{\pi} \frac{q_{\rm eff}^2}{1+\frac{q}{M_T}} |\phi_{1s}^{Z_{\rm eff}}(\vec{0})|^2 \sum_{\tau=0,1} \sum_{\tau'=0,1} \Big\{ \Big[\tilde{R}_{MM}^{\tau\tau'} W_{MM}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\Sigma''\Sigma''}^{\tau\tau'} W_{\Sigma''\Sigma''}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\Sigma'\Sigma'}^{\tau\tau'} W_{\Sigma'\Sigma'}^{\tau\tau'}(q_{\rm eff})\Big] + \frac{q_{\rm eff}^2}{m_N^2}\Big[\tilde{R}_{\Phi''\Phi''}^{\tau\tau'} W_{\Phi''\Phi''}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\tilde{\Phi}'\tilde{\Phi}'}^{\tau\tau'} W_{\tilde{\Phi}'\tilde{\Phi}'}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\Delta\Delta}^{\tau\tau'} W_{\Delta\Delta}^{\tau\tau'}(q_{\rm eff})\Big] - \frac{2q_{\rm eff}}{m_N}\Big[\tilde{R}_{\Phi''M}^{\tau\tau'} W_{\Phi''M}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\Delta\Sigma'}^{\tau\tau'} W_{\Delta\Sigma'}^{\tau\tau'}(q_{\rm eff})\Big]\Big\}$$

$$\tilde{R}_{MM}^{\tau\tau'} = \tilde{c}_1^\tau \tilde{c}_1^{\tau'*} + \tilde{c}_{11}^\tau \tilde{c}_{11}^{\tau'*},$$

$$\tilde{R}_{\Sigma''\Sigma''}^{\tau\tau'} = (\tilde{c}_4^\tau - \tilde{c}_6^\tau)(\tilde{c}_4^{\tau'*} - \tilde{c}_6^{\tau'*}) + \tilde{c}_{10}^\tau \tilde{c}_{10}^{\tau'*}.$$

$$\tilde{R}_{\Sigma'\Sigma'}^{\tau\tau'} = \tilde{c}_4^\tau \tilde{c}_4^{\tau'*} + \tilde{c}_9^\tau \tilde{c}_9^{\tau'*}$$

$$c_i \equiv \tilde{c}_i/v^2 = \sqrt{2} G_F \tilde{c}_i,$$

- particle physics is in products of Wilson coefficients R_i
  - for instance the terms not vanishing in $v_N \to 0$ limit