Flavour hierarchies, extended groups and composites

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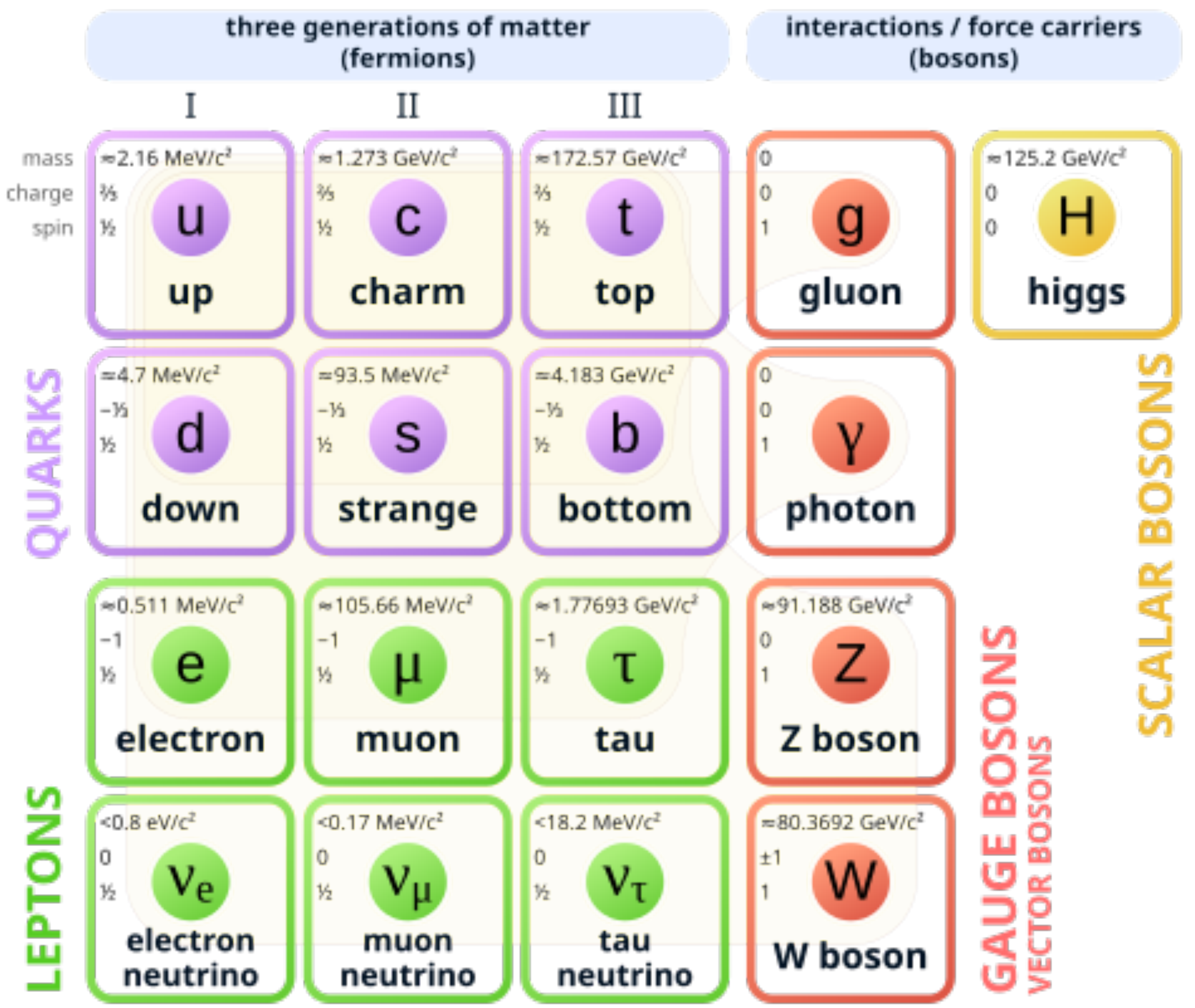
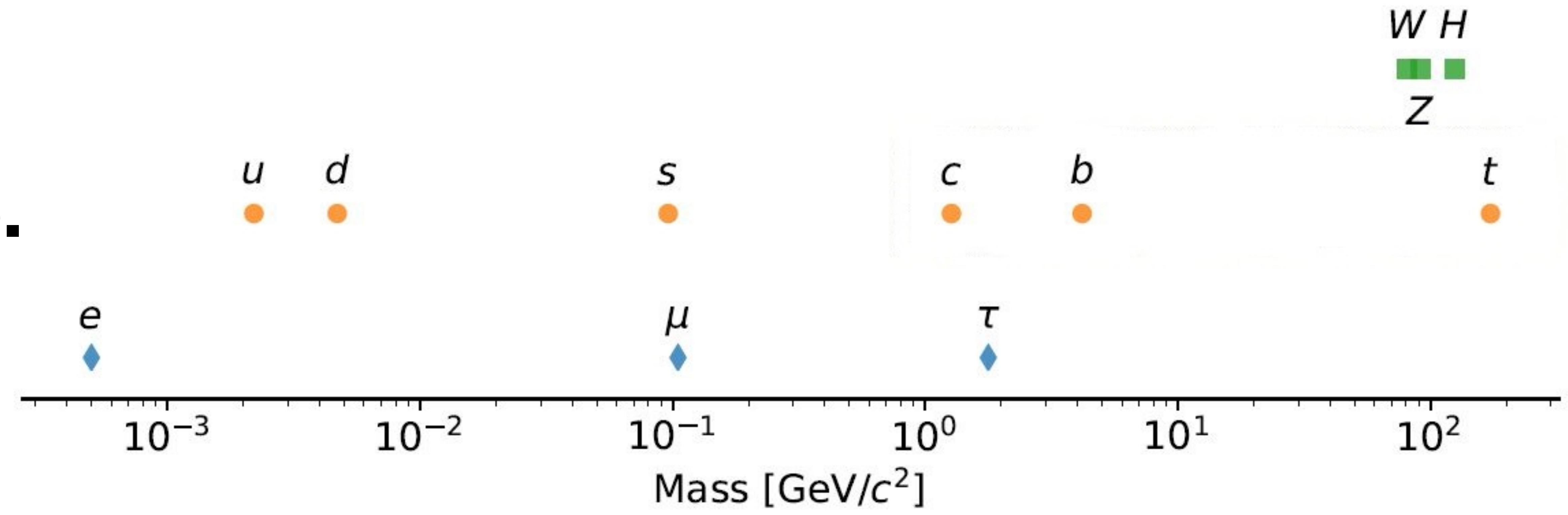
Based on:
[2412.14243]

59th Rencontres de Moriond, 2025

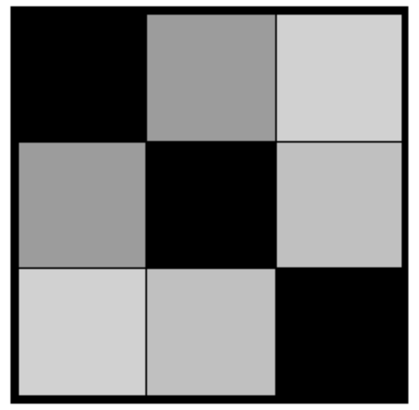
The Standard Model

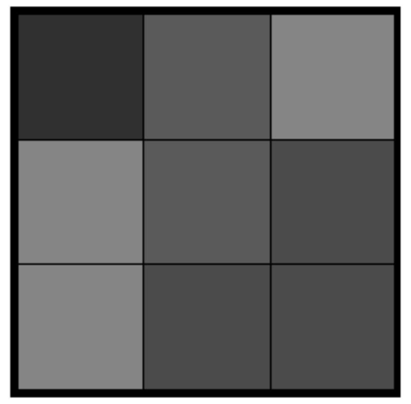


- The SM is a great theory.
- But… Flavor: while gauge sector is universal, Higgs couples to fermions in a funny way:
  - Why hierarchically increasing masses of the families?
  - Why hierarchical CKM but anarchic PMNS?
- Also: Higgs hierarchy problem.
- **A model to address these questions at once.**



[JML, 2412.14243]

$V_{\mathrm{CKM}} \sim$ 

$V_{\mathrm{PMNS}} \sim$ 

(For more exotic approaches, see Sokratis T.'s talk)

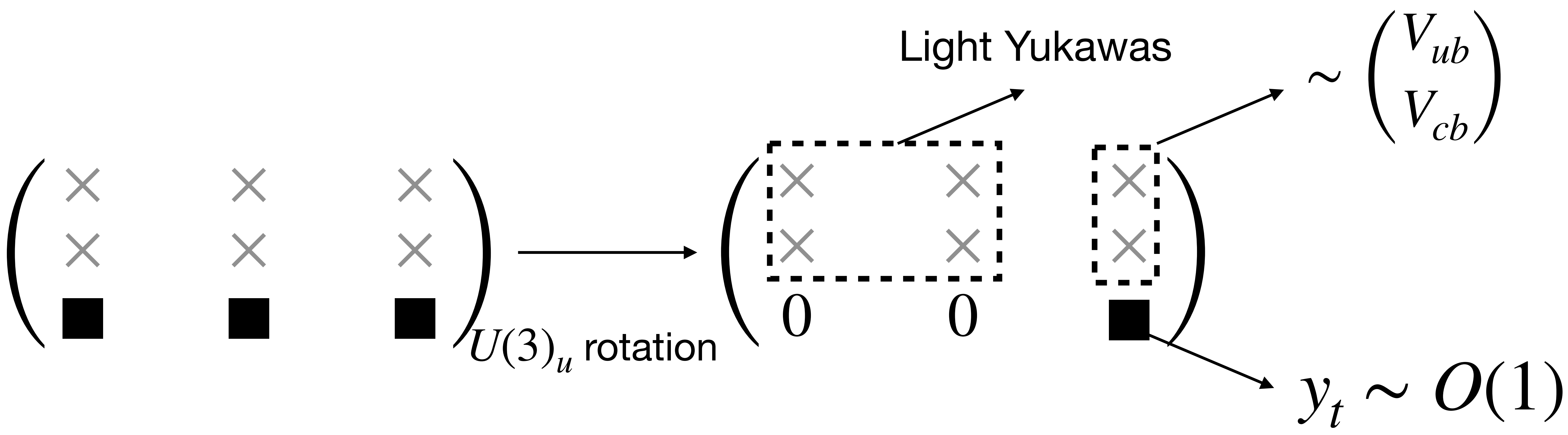
Motivation (quarks)

$$\mathscr{L} \supset Y^{(u)}_{nm}\, \bar{q}^n_L\, H^c\, u^m_R$$

$$\Delta^u_{in}\, \bar{q}^i_L\, H^c\, u^n_R + \bar{q}^3_L\, H^c\, y^t_n u^n_R$$

$i, j = 1,2$

$n, m = 1,2,3$



[Antusch, Greljo, Stefanek, Thomsen, 2311.09288]

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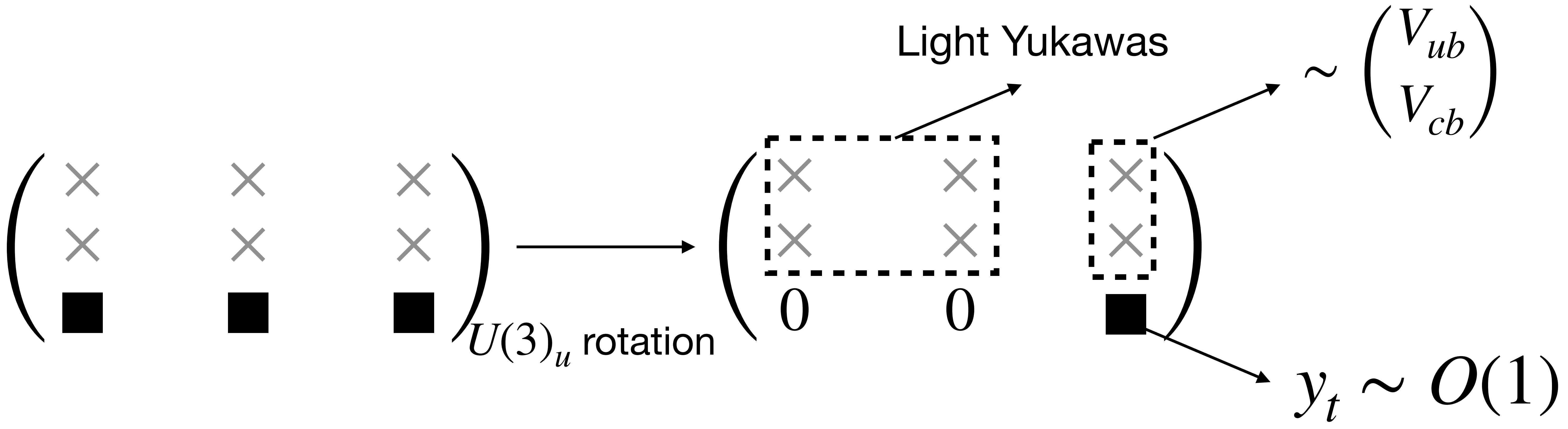

[Antusch, Greljo, Stefanek, Thomsen, 2311.09288]

Motivation (leptons)

$$\mathscr{L} \supset Y^{(e)}_{nm}\, \bar{\ell}^n_L\, H\, e^m_R$$

$$\Delta^e_{ni}\, \bar{\ell}^n_L\, H\, e^i_R + y^\tau_n \bar{\ell}^n_L\, H\, \tau_R$$

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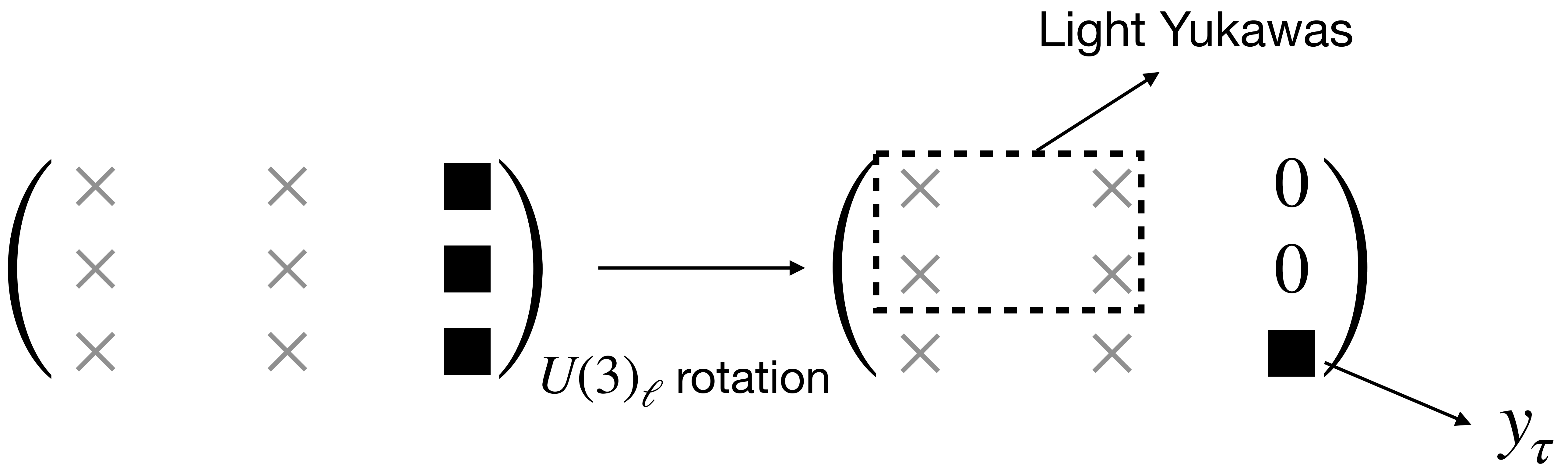

$$\mathscr{L} \supset \frac{\lambda_{nm}}{\Lambda_\nu}(\bar{\ell}^n_L H^c)(H^\dagger \ell^{mc}_L) \longrightarrow \text{Anarchic PMNS}$$

[Antusch, Greljo, Stefanek, Thomsen, 2311.09288]

Motivation (leptons)

$$\mathscr{L} \supset Y^{(e)}_{nm}\,\bar{\ell}^n_L\,H\,e^m_R \;\to\; \Delta^e_{ni}\,\bar{\ell}^n_L\,H\,e^i_R + y_\tau \bar{\ell}^3_L\,H\,\tau_R$$

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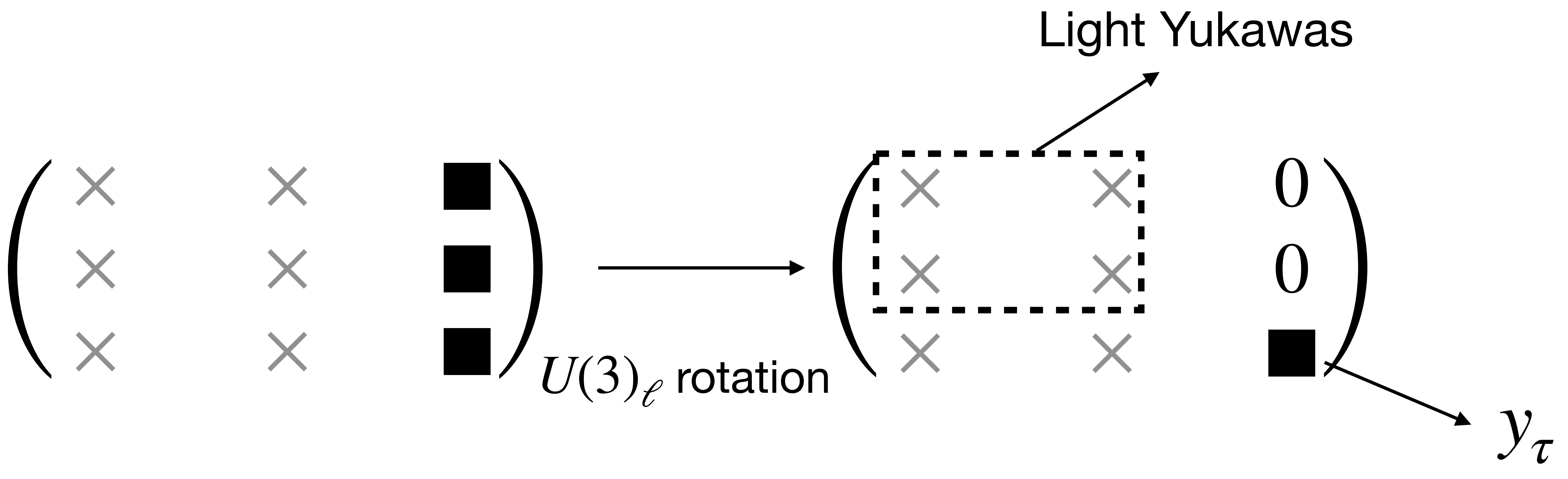

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[Antusch, Greljo, Stefanek, Thomsen, 2311.09288]

Motivation (quark & leptons)

Addressing flavor hierarchies can be effectively done by NP differentiating among LH quarks doublets and among RH charged leptons

[Antusch, Greljo, Stefanek, Thomsen, 2311.09288]

Symmetries

[Pati, Salam, PRD 10 (1974); Li, Ma, PRL 47 (1981)]
[Bordone, Cornella, Fuentes-Martin, Isidori, 1712.01368; Greljo, Stefanek, 1802.04274; Fernández-Navarro, King, 2305.07690; Davighi, Stefanek, 2305.16280; Davighi, Gosnay, Miller, Renner 2312.13346; Capdevila, Crivellin, JML, Pokorski, 2401.00848; Covone, Davighi, Isidori, Pesut, 2407.10950, …]

Get inspired by the idea of flavor deconstruction:

$$G_1 \times G_2 \times G_3 \times G'_U \xrightarrow{\gtrsim \text{PeV}} G_{1,2} \times G_3 \times G_U \xrightarrow{\gtrsim \text{TeV}} G_{\text{SM}}$$

Left-right unification:

$$SU(2)_L \times U(1)_Y \subset SU(2)_L \times U(1)_{B-L} \times SU(2)_R$$

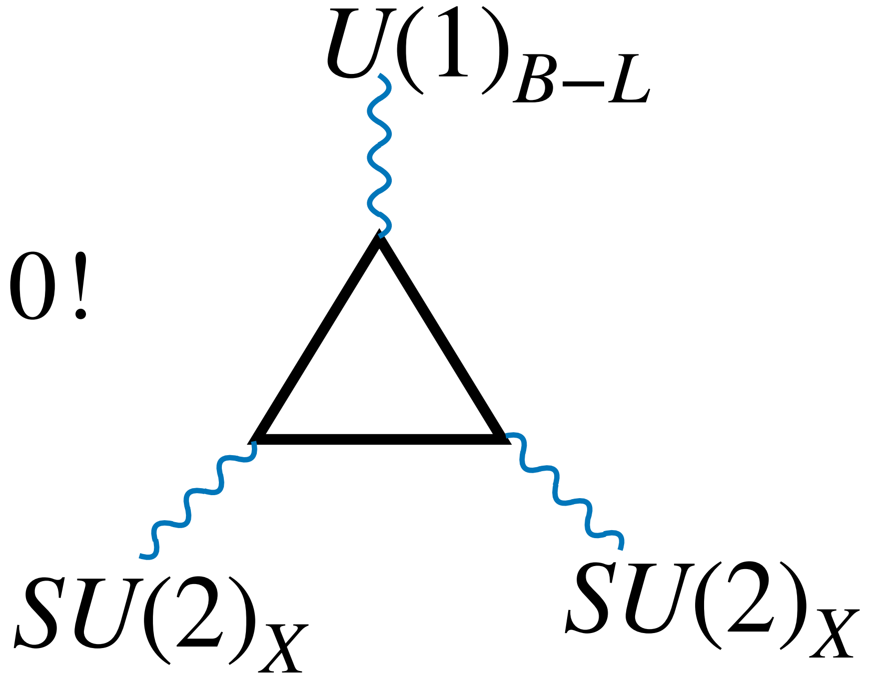
Global approx. symm: $\overbrace{SU(2)_{L1} \times SU(2)_{L2}} \times U(1)_{B-L} \times \overbrace{SU(2)_{R1} \times SU(2)_{R2}}$

Gauge symm: $SU(2)_{L1} \times SU(2)_{L2} \times U(1)_X \times SU(2)_{R2}$

Fermion arrangement

	Site 1	Site 2
$SU(2)_L$	$q_L^{1,2}$ $\ell_L^{1,2,3}$ $\mathscr{A}_{B-L} = 1$	q_L^3 $\mathscr{A}_{B-L} = -1$
$SU(2)_R$	$\ell_R^{1,2}$ $q_R^{1,2,3}$ $\mathscr{A}_{B-L} = 1$	ℓ_R^3 $\mathscr{A}_{B-L} = -1$

- Mixed anomalies: $\mathscr{A}_{B-L} = \sum_{f \sim SU(2)_X} q^R_{B-L} - q^L_{B-L} \neq 0!$
- Witten anomaly: Odd number of doublets!



Breaking sector

- We need a sector breaking the UV gauge symmetry to the SM.
- Let´s do it like in QCD:

$$\langle \bar{q}_L q_R \rangle \sim \Lambda_{\text{QCD}}\, f_\pi^2 \Rightarrow SU(3)_L \times SU(3)_R \to SU(3)_V$$

- Add an hyper-QCD sector:
  - Gauge group: $SU(N_{\text{HC}})$
  - Hyper-fermions: $\zeta_{L,R}$ $\quad \langle \bar{\zeta}_L \zeta_R \rangle \sim \Lambda_{\text{HC}}\, f_\zeta^2$

[Fuentes-Martín, Stangl, 2004.11376;
Fuentes-Martín, JML, 2402.09507]

Breaking and anomaly cancelation

$$\langle \bar{\zeta}_L^{(L)} \zeta_R^{(L)} \rangle \sim \Lambda_{\rm HC} f_\zeta^2 \Rightarrow SU(2)_{L1} \times SU(2)_{L2} \to SU(2)_L$$

$$\langle \bar{\zeta}_L^{(R)} \zeta_R^{(R)} \rangle \sim \Lambda_{\rm HC} f_\zeta^2 \Rightarrow SU(2)_{R1} \times SU(2)_{R2} \to SU(2)_R$$

	Site 1	Site 2
$SU(2)_L$	$q_L^{1,2}$ $\ell_L^{1,2,3}$ $\zeta_L^{(L)}$ $\mathscr{A}_{B-L} = 0$	q_L^3 $\zeta_R^{(L)}$ $\mathscr{A}_{B-L} = 0$
$SU(2)_R$	$\ell_R^{1,2}$ $q_R^{1,2,3}$ $\zeta_L^{(R)}$ $\mathscr{A}_{B-L} = 0$	ℓ_R^3 $\zeta_R^{(R)}$ $\mathscr{A}_{B-L} = 0$

If $(B-L)_\zeta = \dfrac{1}{N_{\rm HC}}$ and $N_{\rm HC}$ is odd $\Rightarrow$ Anomaly-free

[Fuentes-Martín, JML, 2402.09507]

Symmetry-breaking pattern

$$\begin{pmatrix} \zeta_L^{(L)} \\ \zeta_L^{(R)} \end{pmatrix} \sim \mathbf{4} \text{ of } SU(4)_1 \supset SU(2)_{L1} \times SU(2)_{R1}$$

$$\begin{pmatrix} \zeta_R^{(L)} \\ \zeta_R^{(R)} \end{pmatrix} \sim \mathbf{4} \text{ of } SU(4)_2 \supset SU(2)_{L2} \times SU(2)_{R2}$$

$$SU(4)_1 \times SU(4)_2 \xrightarrow{\langle \bar{\zeta}_L \zeta_R \rangle} SU(4)_V$$

(15 broken generators)

$$\mathbf{15} = (\mathbf{1}, \mathbf{1}) + 2 \times (\mathbf{2}, \mathbf{2}) + (\mathbf{1}, \mathbf{3}) + (\mathbf{3}, \mathbf{1})$$

(of $SU(2)_L \times SU(2)_R$)

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$$SU(4)_1 \times SU(4)_2 \xrightarrow{\langle \bar{\zeta}_L \zeta_R \rangle} SU(4)_V$$

(15 broken generators)

Physical pNGBs

Eaten by new massive gauge bosons

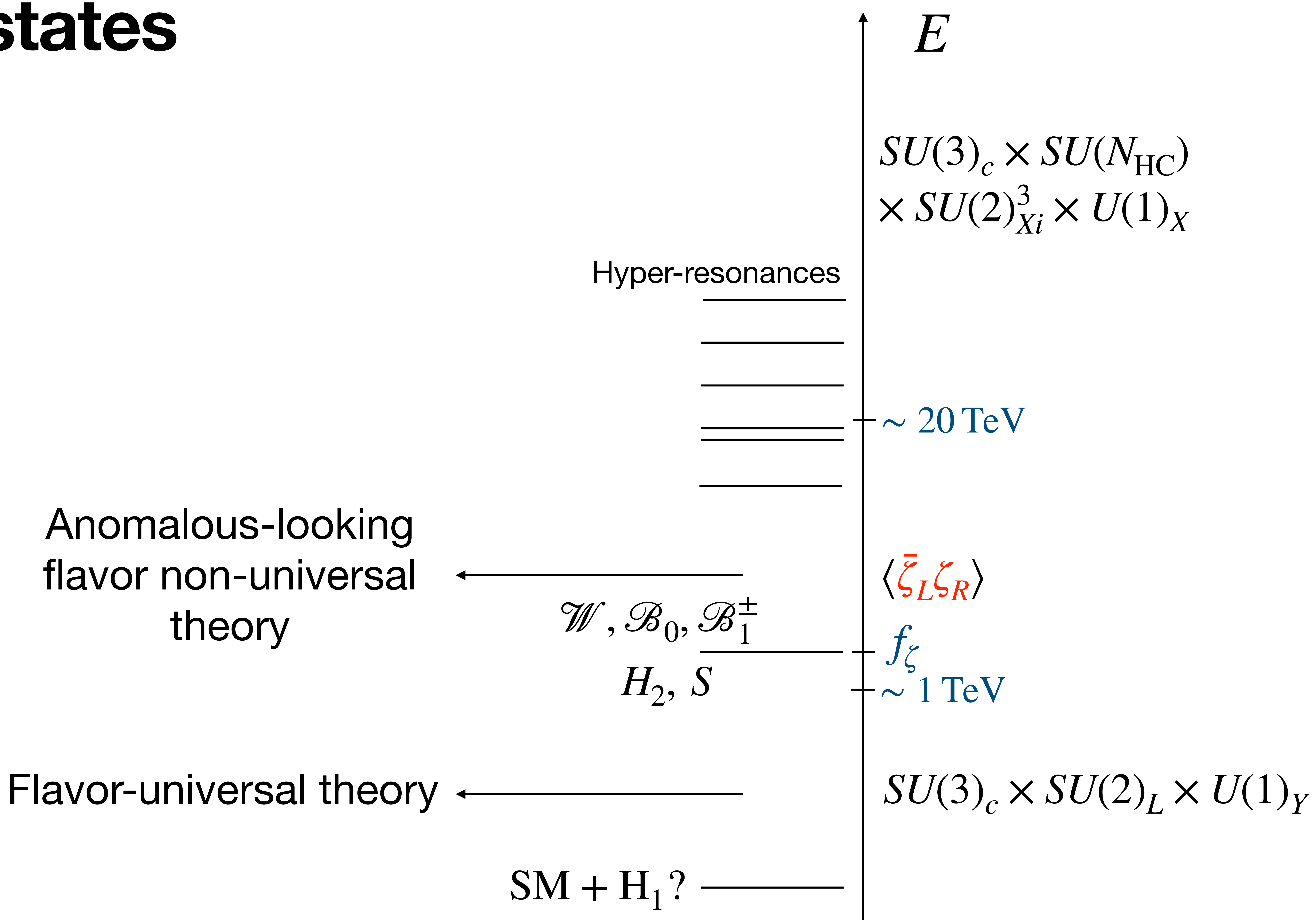
$$\mathbf{15} = (\mathbf{1},\mathbf{1}) + 2 \times (\mathbf{2},\mathbf{2}) + \cancel{(\mathbf{1},\mathbf{3}) + (\mathbf{3},\mathbf{1})}$$

It's a 2HDM!

$$\left\{ \begin{array}{l} \mathscr{W}_\mu^{\pm,0} \sim (\mathbf{1},\mathbf{3})_0 \\ \mathscr{B}_\mu^0 \sim (\mathbf{1},\mathbf{1})_0 \\ \mathscr{B}_\mu^\pm \sim (\mathbf{1},\mathbf{1})_{\pm 1} \end{array} \right.$$

W'_L, W'_R and $2\,Z'$

New states

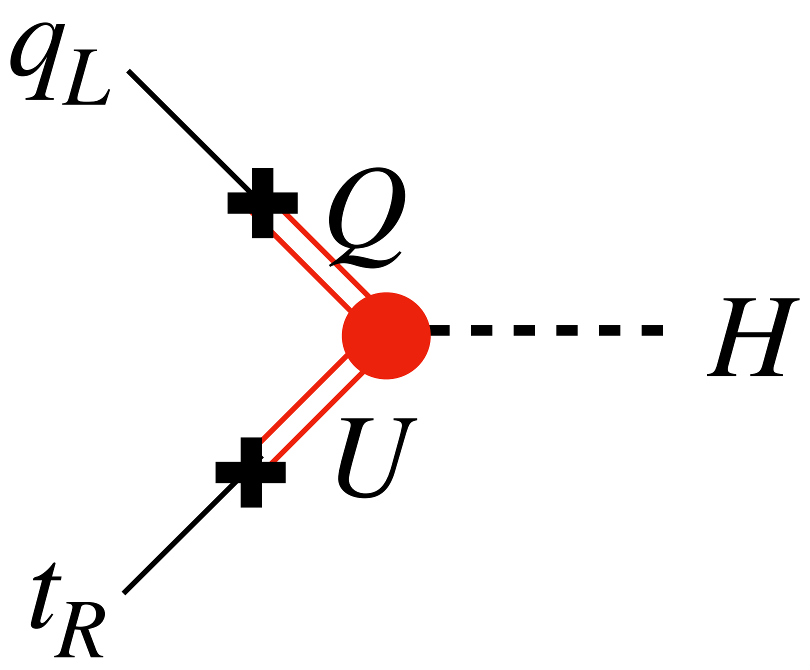


E
$SU(3)_c \times SU(N_{\rm HC}) \times SU(2)^3_{Xi} \times U(1)_X$
Hyper-resonances
~ 20 TeV
Anomalous-looking flavor non-universal theory
$\langle \bar{\zeta}_L \zeta_R \rangle$
$\mathscr{W}, \mathscr{B}_0, \mathscr{B}_1^\pm$
f_ζ
$H_2,\ S$
~ 1 TeV
Flavor-universal theory
$SU(3)_c \times SU(2)_L \times U(1)_Y$
$\rm SM + H_1$?

Yukawa couplings

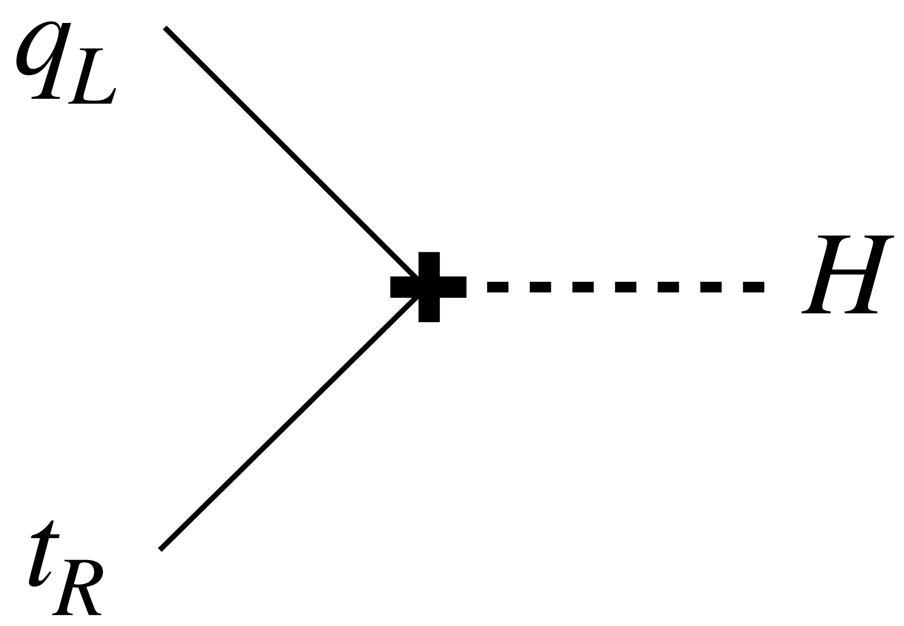
- Partial compositeness:

$$\mathscr{L} \supset \bar{t}_R O_T + q_L O_Q$$

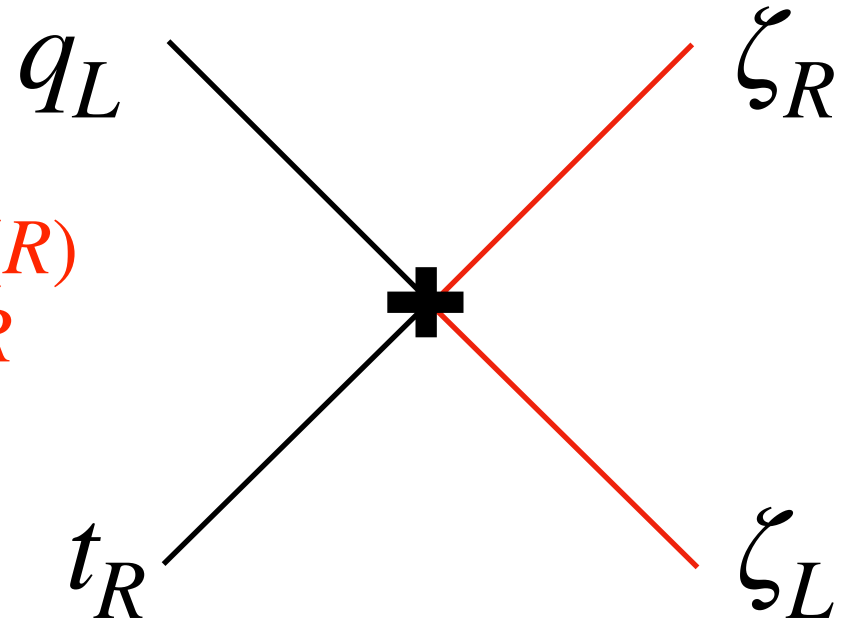


- Bilinears (à la technicolor):

$$\mathscr{L} \supset \bar{q}_L t_R O_H$$



$$O_H = \bar{\zeta}_L^{(R)} \zeta_R^{(L)},\ \bar{\zeta}_L^{(L)} \zeta_R^{(R)}$$



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- Extended gauge addresses flavor hierarchies:

	Site 1	Site 2
$SU(2)_L$	$q_L^{1,2}$ $\ell_L^{1,2,3}$ $\zeta_L^{(L)}$	q_L^3 $\zeta_R^{(L)}$
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Third family (e.g. top):

Light families (e.g. μ):

$$\mathscr{L} \propto \quad (\bar{q}_L^3 t_R)(\bar{\zeta}_L^{(R)} \zeta_R^{(L)}) \qquad (\bar{\ell}_L^2 \mu_R)(\bar{\zeta}_L^{(R)} \zeta_R^{(R)})(\bar{\zeta}_R^{(R)} \zeta_L^{(L)})$$

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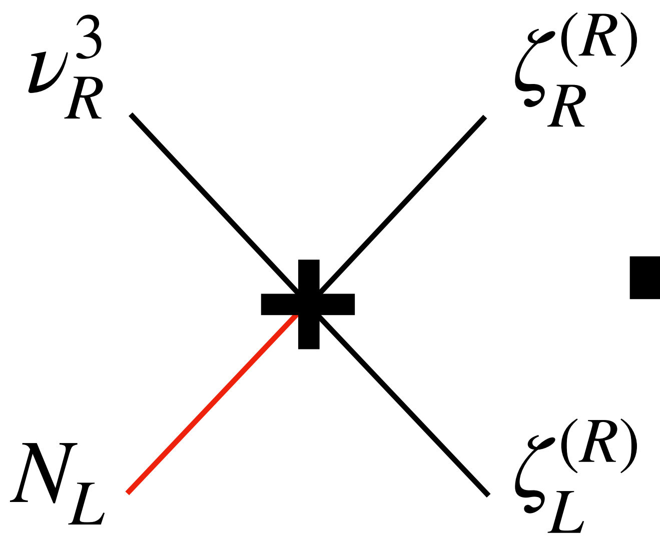
CKM & PMNS ✔

Higgs potential

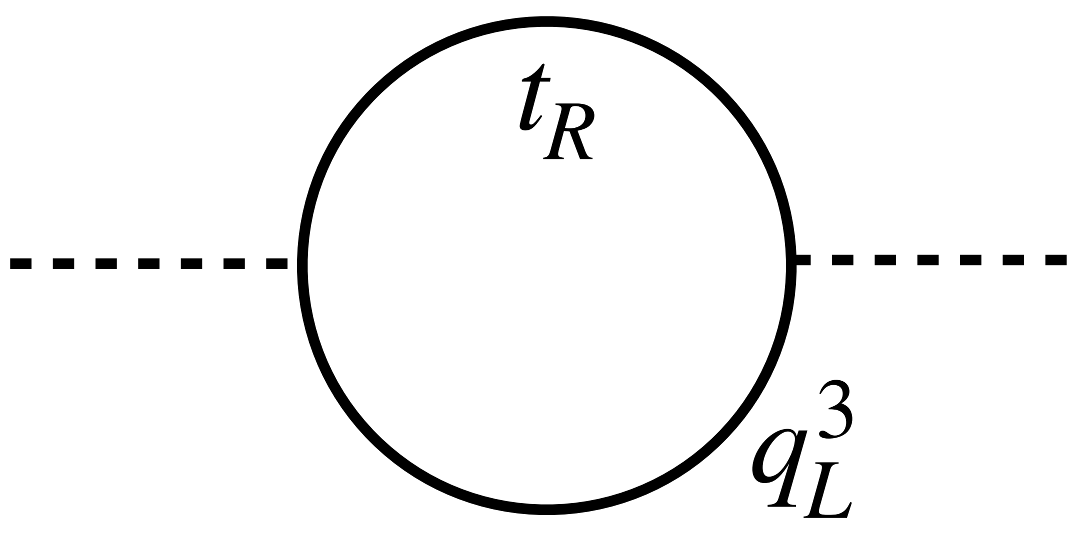
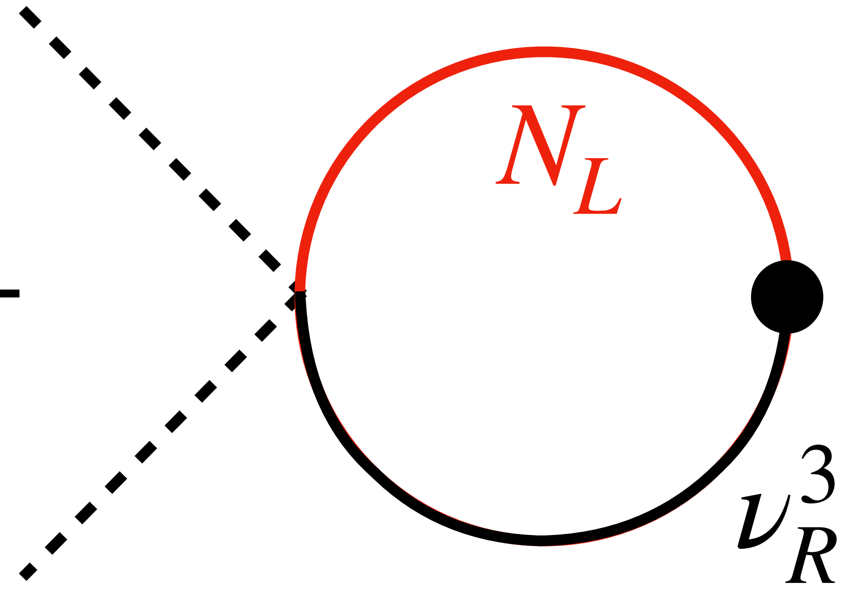
$\ell_R^3 = (\nu_R^3, \tau_R) \sim SU(2)_{R2}$

$N_L \sim \mathbf{1}$



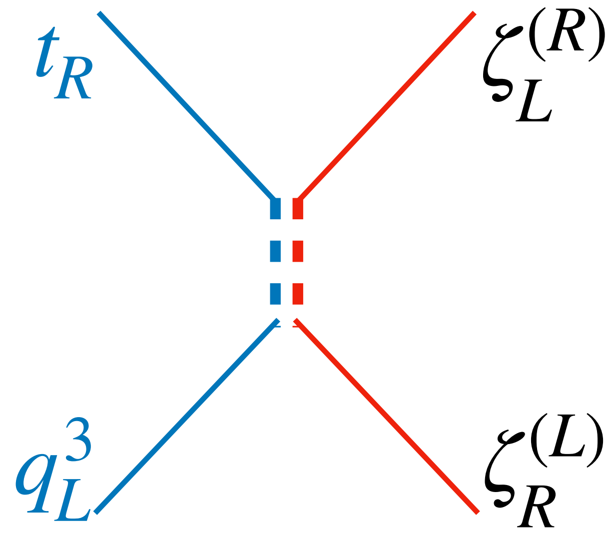
$\Rightarrow m_N \bar{N}_L \nu_R^3$

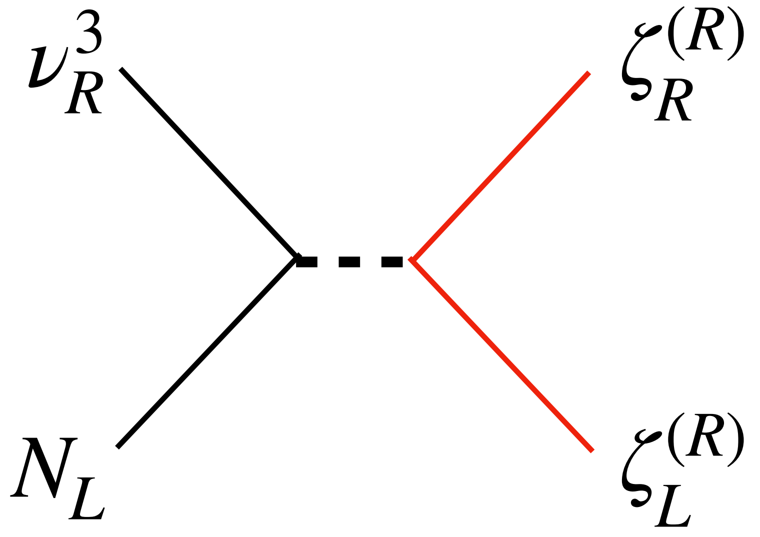
- Every breaking of $SU(4)_1 \times SU(4)_2$ contributes to the potential:

$V(H_1, H_2, S) =$  $+$  $+ \ldots$

$\Delta m_{H_1}^2 < 0$ $\qquad$ $\Delta m_{H_1}^2 > 0$

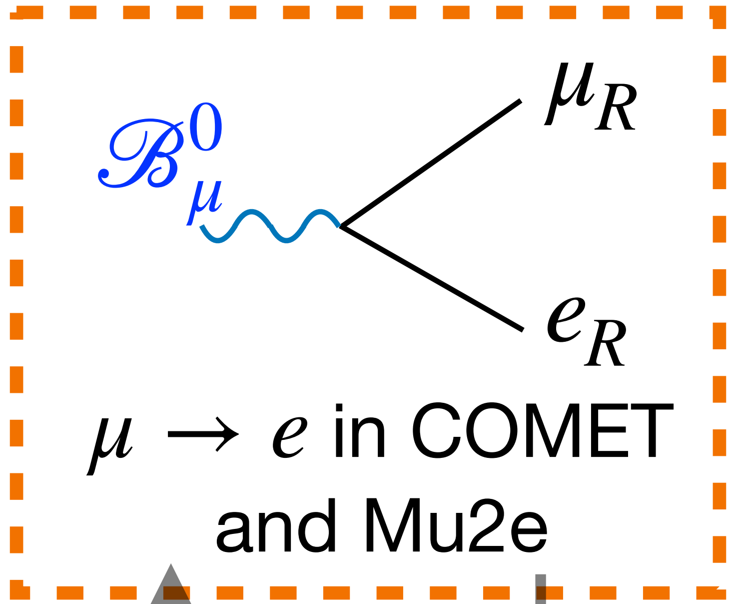
- Benchmark for 4-fermion operators:



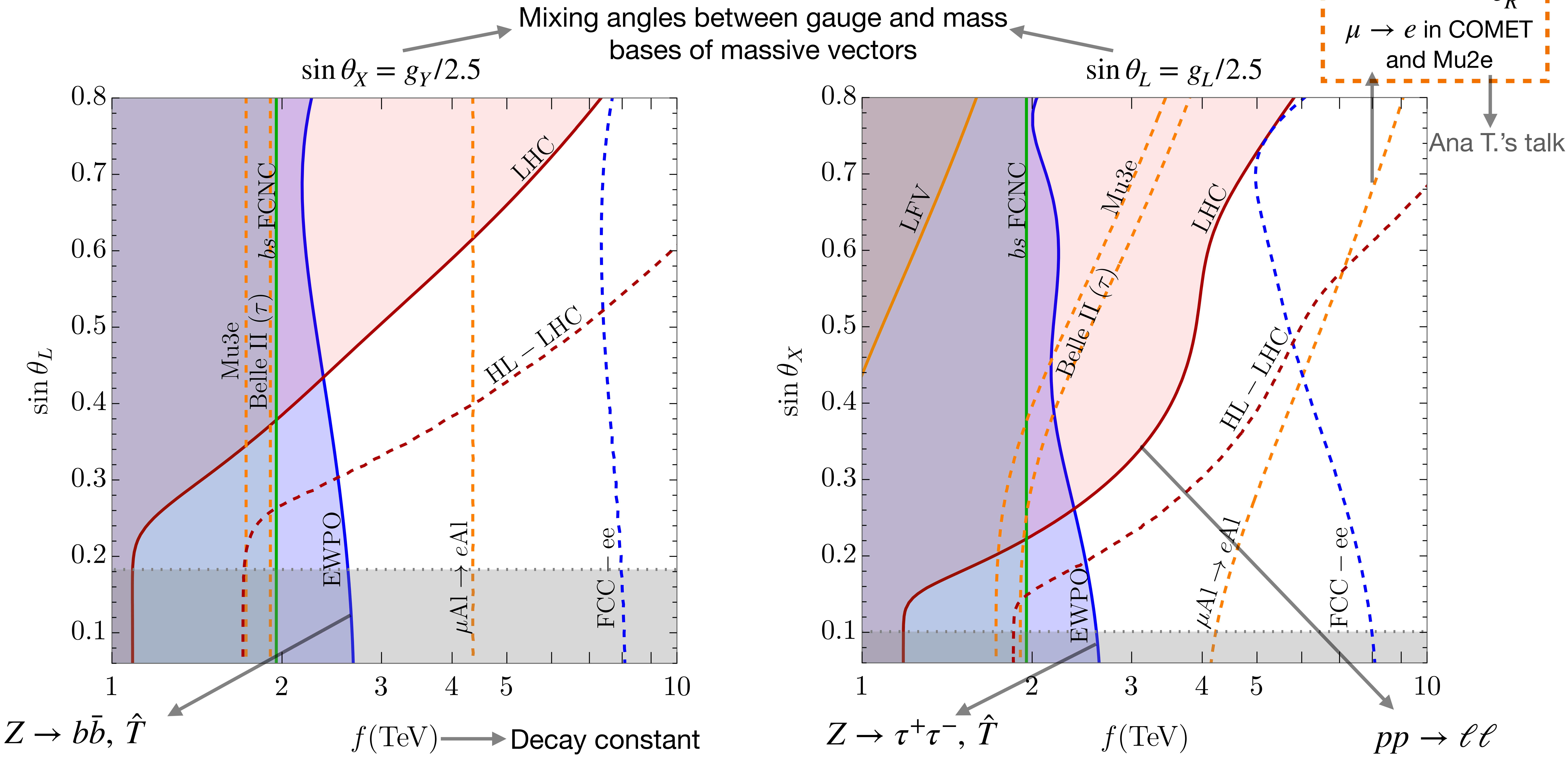


$$\Rightarrow \begin{cases} m_{H_1}^2 \sim \dfrac{M_N^2 - 12 f_\zeta^2 y_t^2}{4N_{\rm HC}} = O(v^2) \\ m_{H_2}^2 = 2m_S^2 \sim \dfrac{M_N^2}{4N_{\rm HC}} = O(\text{TeV}^2) \\ \lambda_H \sim \dfrac{3y_t^2}{8N_{\rm HC}} \sim 0.1 - 0.2 \end{cases}$$

Some pheno:



$\mathscr{B}^0_\mu$
μ_R
e_R
$\mu \to e$ in COMET
and Mu2e
Ana T.'s talk



Mixing angles between gauge and mass
bases of massive vectors
$\sin\theta_X = g_Y/2.5$
$\sin\theta_L = g_L/2.5$
$\sin\theta_L$
$\sin\theta_X$
Mu3e
Belle II (τ)
bs FCNC
EWPO
LHC
HL – LHC
μAl $\to$ eAl
FCC – ee
LFV
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
1
2
3
4
5
10
f(TeV)
Decay constant
$Z \to b\bar{b},\ \hat{T}$
$Z \to \tau^+\tau^-,\ \hat{T}$
$pp \to \ell\ell$

Conclusions

- We have presented a model that in a reasonably minimal way addresses:
  - Hierarchies of third family vs light families.
  - Hierarchy of CKM vs anarchy of PMNS.
  - The emergence of the Higgs as a composite and the breaking of the EW symmetry.
- Rich pheno with interplay between LHC, EW and flavor.
- Future tests of the model from improvements in $\mu \to e$!

Thanks!

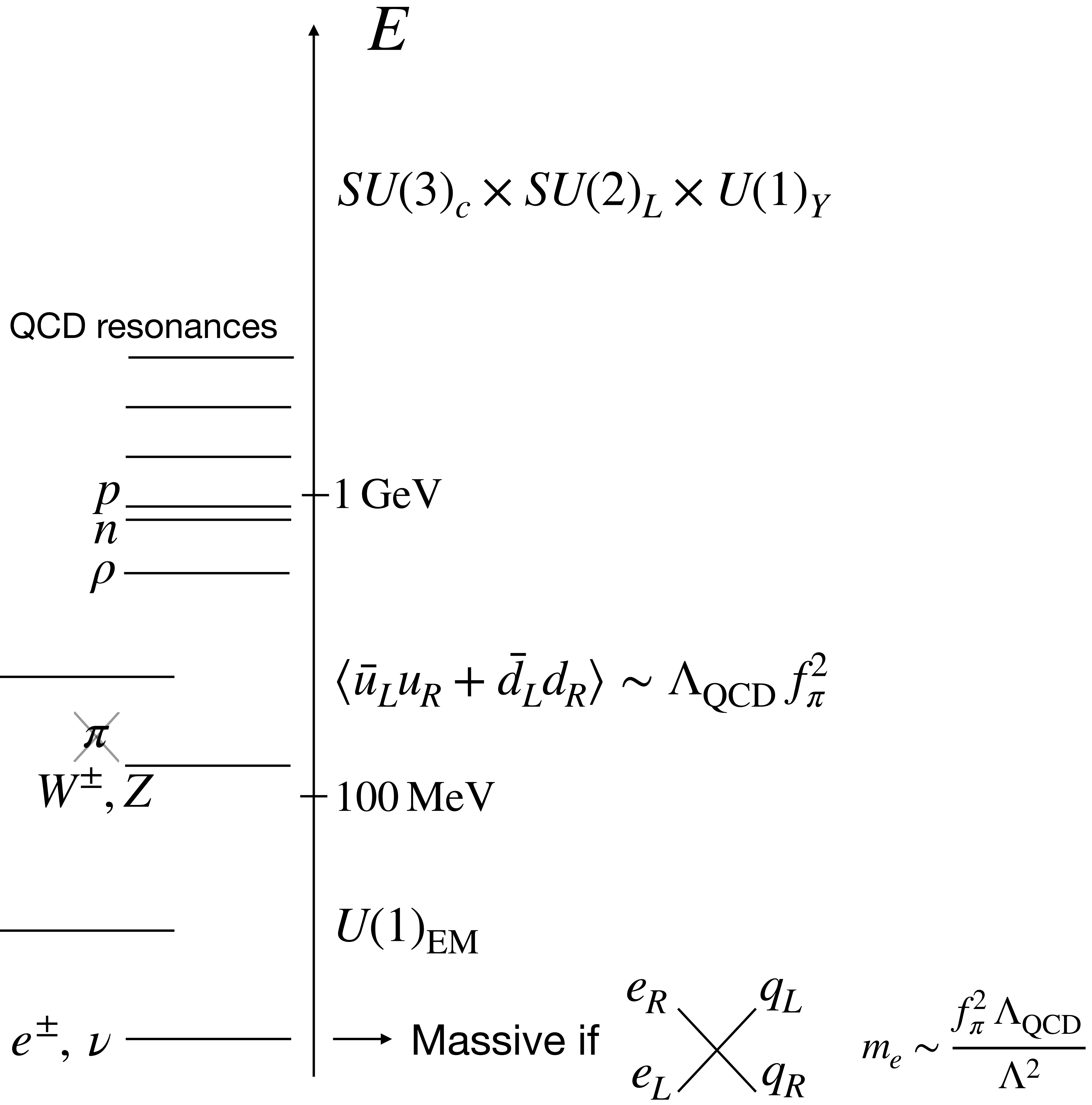
The SM analogy

- Imagine a universe with:

  - One family
  - No Higgs

Anomalous-looking non chiral-universal theory

Chiral-universal theory



LH suppression in leptons

[Greljo, Thomsen, 2309.11547; Davighi, Gosnay, Miller, Renner, 2312.13346; Capdevila, Crivellin, JML, Pokorski, 2401.00848; Isidori, Greljo, 2406.01696]

$$\mathscr{L} \supset Y^{(e)}_{nm}\, \bar{\ell}^n_L\, H\, e^m_R$$

$$\Delta^e_{in}\, \bar{\ell}^i_L\, H\, e^n_R + \bar{\ell}^3_L\, H\, y^\tau_n e^n_R$$

$$\begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \blacksquare & \blacksquare & \blacksquare \end{pmatrix} \xrightarrow{} U(3)_e \text{ rotation} \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & \blacksquare \end{pmatrix}$$

Light Yukawas

y_τ

$$\mathscr{L} \supset \frac{1}{\Lambda_\nu}\left[\lambda_{33}(\bar{\ell}^3_L H^c)(H^\dagger \ell^{3c}_L) + \lambda_{3n}(\bar{\ell}^3_L H^c)(H^\dagger \ell^{nc}_L) + \lambda_{nm}(\bar{\ell}^n_L H^c)(H^\dagger \ell^{mc}_L)\right]$$

$$1 \sim \lambda_{33} \gg \lambda_{3n} \gg \lambda_{nm} \Rightarrow m_{\nu_2}/m_{\nu_3} \ll \theta_{23} \ll 1 \;\Rightarrow\; \text{Hierarchical PMNS}$$

$$\begin{pmatrix} - & - & \times \\ - & - & \times \\ \times & \times & \blacksquare \end{pmatrix}$$

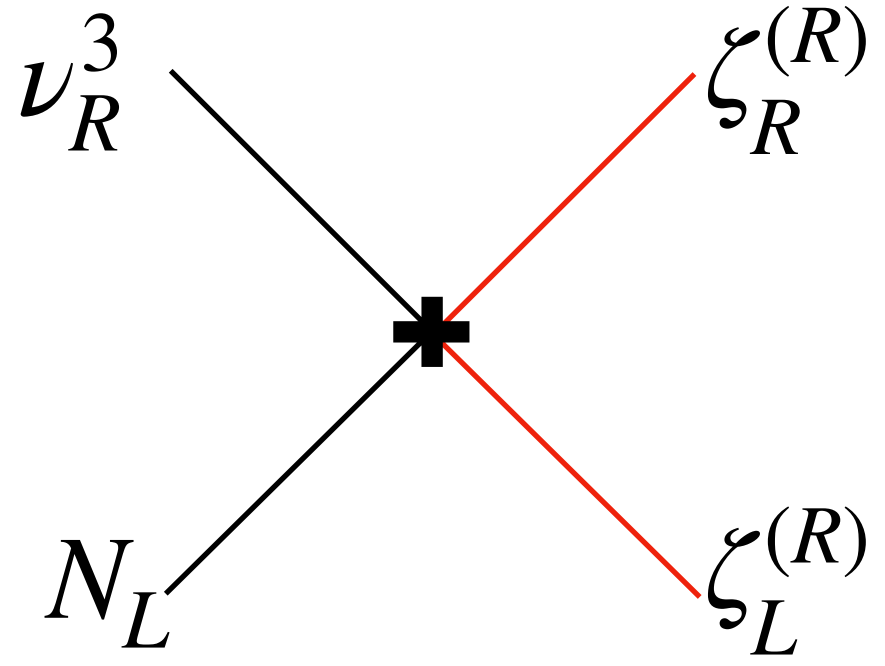
$i, j = 1,2$

$n, m = 1,2,3$

Heavy Neutral Lepton

- The partner of τ_R, ν_R^3 has a τ-size Yukawa. It should become a HNL:

	Site 1	Site 2
$SU(2)_L$	$q_L^{1,2}$ $\ell_L^{1,2,3}$ $\zeta_L^{(L)}$	q_L^3 $\zeta_R^{(L)}$
$SU(2)_R$	$\ell_R^{1,2}$ $q_R^{1,2,3}$ $\zeta_L^{(R)}$	ℓ_R^3 $\zeta_R^{(R)}$



$$\left.\begin{array}{l}(\nu_R^3, \tau_R)\\ N_L \sim \mathbf{1}\end{array}\right\} m_N \bar{N}_L \nu_R^3$$

[JML, 2412.14243]

All representations

Field	$SU(2)_{L1}$	$SU(2)_{R1}$	$SU(2)_{L2}$	$SU(2)_{R2}$	$U(1)_{B-L}$	$SU(3)_c$	$SU(N_{\rm HC})$
q_L^3	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$1/6$	$\mathbf{3}$	$\mathbf{1}$
$q_L^{1,2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$1/6$	$\mathbf{3}$	$\mathbf{1}$
$q_R^{1,2,3}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$1/6$	$\mathbf{3}$	$\mathbf{1}$
$\ell_L^{1,2,3}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$-1/2$	$\mathbf{1}$	$\mathbf{1}$
ℓ_R^3	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	$\mathbf{1}$	$\mathbf{1}$
$\ell_R^{1,2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$-1/2$	$\mathbf{1}$	$\mathbf{1}$
$\zeta_L^{(L)}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$1/2N_{\rm HC}$	$\mathbf{1}$	$\square$
$\zeta_L^{(R)}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$1/2N_{\rm HC}$	$\mathbf{1}$	$\square$
$\zeta_R^{(L)}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$1/2N_{\rm HC}$	$\mathbf{1}$	$\square$
$\zeta_R^{(R)}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$1/2N_{\rm HC}$	$\mathbf{1}$	$\square$
N_L	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	$\mathbf{1}$	$\mathbf{1}$

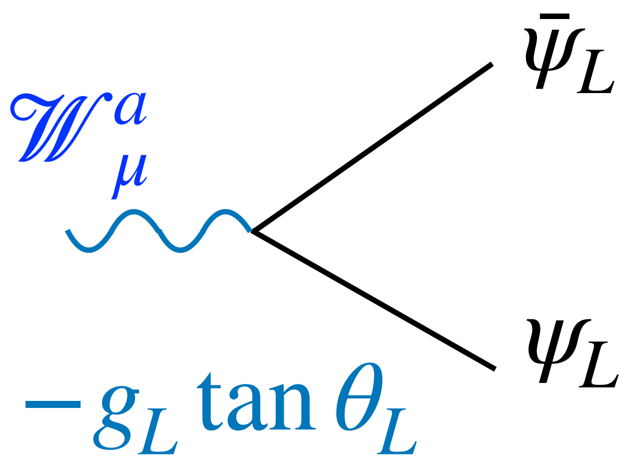
[JML, 2412.14243]

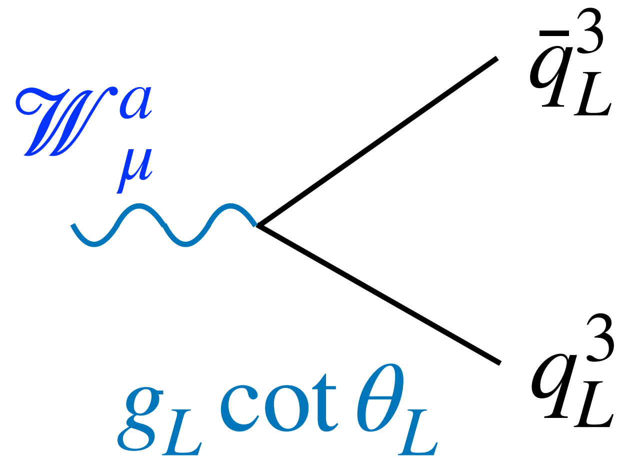
Vector states

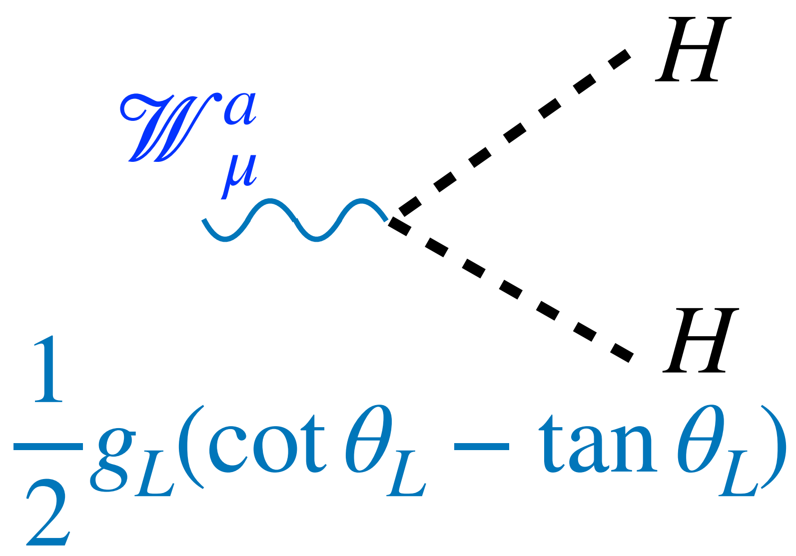
Mixing angles $\theta_L,\ \theta_X$

$\mathscr{W}^a_\mu \sim (\mathbf{1},\mathbf{3})_0$

$m_{\mathscr{W}} = 2 f g_L \csc 2\theta_L$

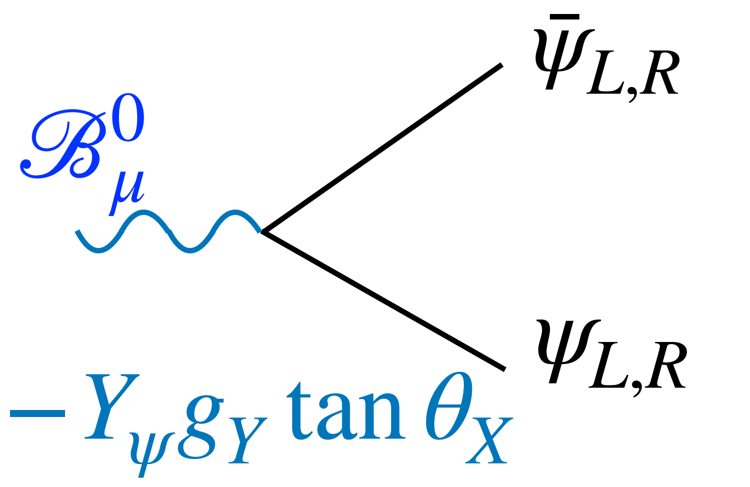


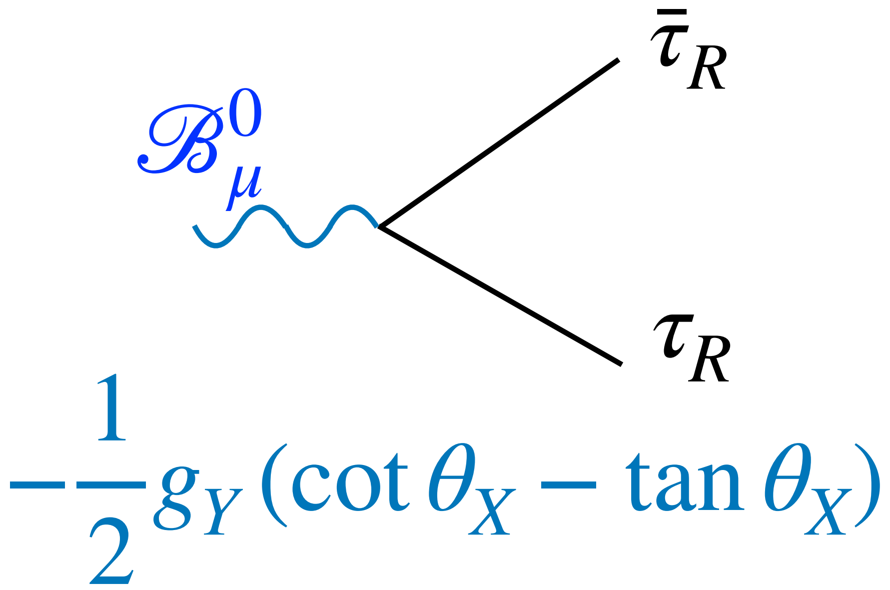


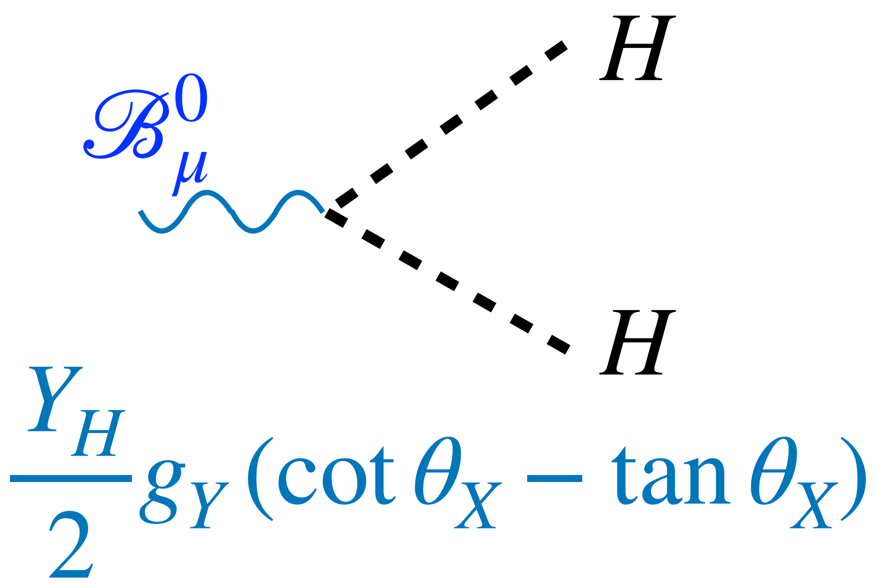


$\mathscr{B}^0_\mu \sim (\mathbf{1},\mathbf{1})_0$

$m_{\mathscr{B}^0} = 2 f g_Y \csc 2\theta_X$

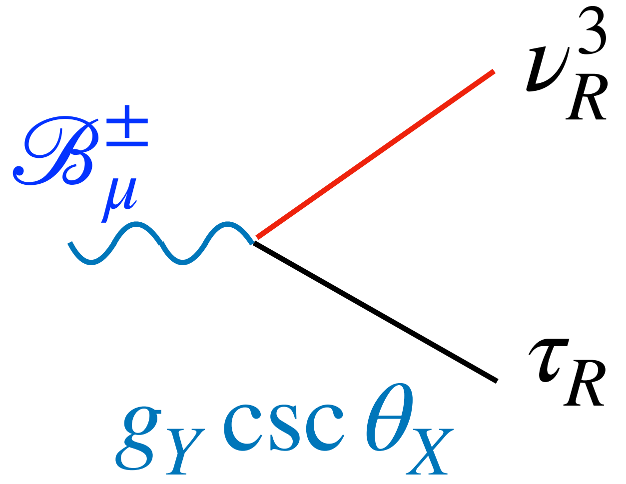


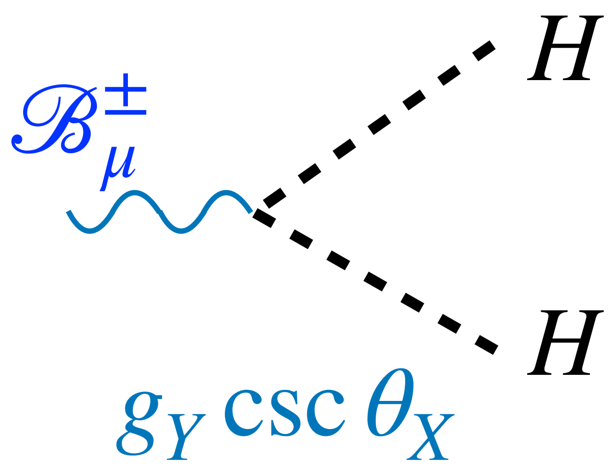




$\mathscr{B}^\pm_\mu \sim (\mathbf{1},\mathbf{1})_{\pm 1}$

$m_{\mathscr{B}^\pm} = f g_Y \csc\theta_X$





[JML, 2412.14243]

2HDM

$$SU(4)_1 \times SU(4)_2 \to SU(4)_V \supset SU(2)_L \times SU(2)_R$$

Goldstone matrix: $U = e^{i\,\Pi/f},\ \Pi = \frac{1}{2}\begin{pmatrix} T^a \sigma_a + \frac{S}{\sqrt{2}} 1_2 & -i\,(H_1 + iH_2) \\ i\,(H_1 + iH_2)^\dagger & \Delta^a \sigma_a - \frac{S}{\sqrt{2}} 1_2 \end{pmatrix}$

H_1 → CP even; H_2 → CP odd

- Assuming CP conservation:

$$V \supset \frac{1}{2} m_S^2 S^2 + m_{H_2}^2 |H_2|^2 + m_{H_1}^2 |H_1|^2 + i\, m_{12}^2 (H_1^\dagger H_2 - H_2^\dagger H_1) + \frac{\lambda}{4} |H|^4$$

$$\langle H_2 \rangle \propto i \langle H_1 \rangle \Rightarrow$$ ~~custodial~~

[JML, 2412.14243]

Semisimple UV-completion

$SU(3)_c \times SU(N_{\text{HC}}) \times U(1)_{B-L}$

Field	$SU(2)_{L1}$	$SU(2)_{R1}$	$SU(2)_{L2}$	$SU(2)_{R2}$	$SU(4)_{\text{PS}}$	$SU(N_{\text{HC}}+1)$
$(q_L^3, L_L^{(L)})$	**1**	**1**	**2**	**1**	**4**	**1**
$(q_L^{1,2}, \ell_L^{1,2})$	**2**	**1**	**1**	**1**	**4**	**1**
$(q_R^3, L_R^{(R)})$	**1**	**2**	**1**	**1**	**4**	**1**
$(q_R^{1,2}, \ell_R^{1,2})$	**1**	**2**	**1**	**1**	**4**	**1**
$(\zeta_L^{(L)}, \ell_L^3)$	**2**	**1**	**1**	**1**	**1**	$\square$
$(\zeta_L^{(R)}, L_L^{(R)})$	**1**	**2**	**1**	**1**	**1**	$\square$
$(\zeta_R^{(L)}, L_R^{(L)})$	**1**	**1**	**2**	**1**	**1**	$\square$
$(\zeta_R^{(R)}, \ell_R^3)$	**1**	**1**	**1**	**2**	**1**	$\square$
N_L	**1**	**1**	**1**	**1**	**1**	**1**

[JML, 2412.14243]

Alternative model

	Site 1	Site 2
$SU(2)_L$	q_L^3 $\ell_L^{1,2,3}$ $\zeta_L^{(L)}$	$q_L^{1,2}$ $\zeta_R^{(L)}$
$SU(2)_R$	ℓ_R^3 $q_R^{1,2,3}$ $\zeta_L^{(R)}$	$\ell_R^{1,2}$ $\zeta_R^{(R)}$

If $(B-L)_\zeta = \dfrac{2}{N_{\rm HC}}$ and $N_{\rm HC}$ is even $\Rightarrow$ Anomaly-free

[JML, 2412.14243]

Alternative model

Field	$SU(2)_{L1}$	$SU(2)_{R1}$	$SU(2)_{L2}$	$SU(2)_{R2}$	$U(1)_{B-L}$	$SU(3)_c$	$SU(N_{\rm HC})$
q_L^3	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$1/6$	$\mathbf{3}$	$\mathbf{1}$
$q_L^{1,2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$1/6$	$\mathbf{3}$	$\mathbf{1}$
$q_R^{1,2,3}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$1/6$	$\mathbf{3}$	$\mathbf{1}$
$\ell_L^{1,2,3}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$-1/2$	$\mathbf{1}$	$\mathbf{1}$
ℓ_R^3	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$-1/2$	$\mathbf{1}$	$\mathbf{1}$
$\ell_R^{1,2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	$\mathbf{1}$	$\mathbf{1}$
$\zeta_L^{(L)}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$1/N_{\rm HC}$	$\mathbf{1}$	$\square$
$\zeta_L^{(R)}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$1/N_{\rm HC}$	$\mathbf{1}$	$\square$
$\zeta_R^{(L)}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$1/N_{\rm HC}$	$\mathbf{1}$	$\square$
$\zeta_R^{(R)}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$1/N_{\rm HC}$	$\mathbf{1}$	$\square$

[JML, 2412.14243]