Theory progress on $D^+ \to \pi^+ \ell^+ \ell^-$

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24/03/2025

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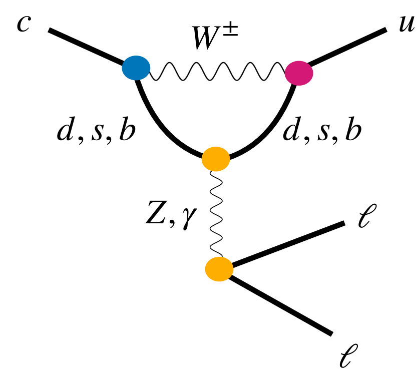
P H

since 1966 Rencontres de Moriond

Why $D \to \pi \ell^+ \ell^-$?

- Simplest Flavour Changing Neutral Current (FCNC) transition in charm sector $(c \to u \ell^+ \ell^-)$: Probe for New Physics?

Penguin diagram (Leading short distance contribution)



$$\implies \mathcal{A}(c \to u) \propto \frac{1}{16\pi^2} \underbrace{V^*_{cs} V_{us}}_{\mathcal{O}(\lambda)} \left(f\left(\frac{m_s^2}{m_W^2}\right) - f\left(\frac{m_d^2}{m_W^2}\right) \right) + \frac{1}{16\pi^2} \underbrace{V^*_{cb} V_{ub}}_{\mathcal{O}(\lambda^5)} \left(f\left(\frac{m_b^2}{m_W^2}\right) - f\left(\frac{m_d^2}{m_W^2}\right) \right)$$

Very strong GIM & CKM suppression.

- Unlike B-FCNCs, charm FCNC is dominated by long distance effects.
- Charm FCNCs are both theoretically and experimentally challenging due to intermediate resonances.

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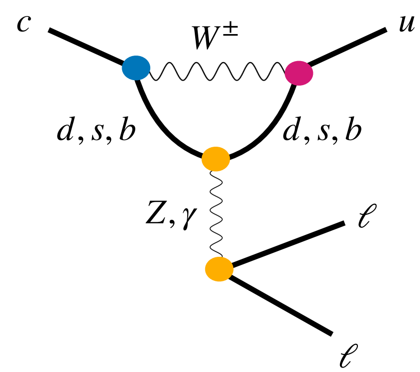

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- The effective Hamiltonian:

$$\mathscr{H}_{eff}^{\Delta S=0} = \frac{4G_F}{\sqrt{2}} \sum_{\mathscr{D}=d,s} \underbrace{\lambda_{\mathscr{D}}}_{\mathcal{O}(\lambda)} \left[C_1(\mu) O_1^{\mathscr{D}} + C_2(\mu) O_2^{\mathscr{D}} \right] - \underbrace{\lambda_b}_{\mathcal{O}(\lambda^5)} \sum_{i=3}^{10} C_i(\mu) O_i$$

$\ll C_{1,2} @ \mathcal{O}(m_c)$

- Hadronic amplitude:

$$\mathscr{A}_\mu^{D^+ \to \pi^+ \gamma^*}(p,q) = i \int d^4x e^{iq.x} \langle \pi^+(p) | T \left\{ j_\mu^{em}(x), \mathscr{H}_{eff}^{(\Delta_s=0,\lambda_b=0)} \right\} | D^+(p+q) \rangle$$

Object of interest

Non-local form factor : **dominated by long distance effects in physical region of q^2.**

- Vector meson can be created from non-leptonic weak decay before γ^* : Resonance contributions

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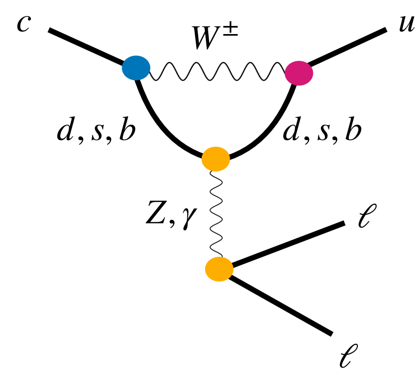

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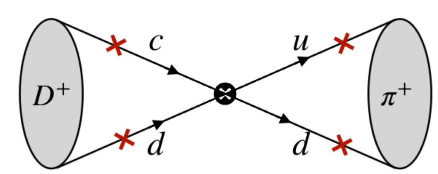
Quark Topologies for $\mathscr{A}_\mu^{D^+ \to \pi^+ \gamma^*}(p,q)$

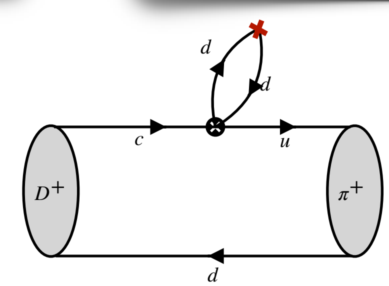
Annihilation Topology

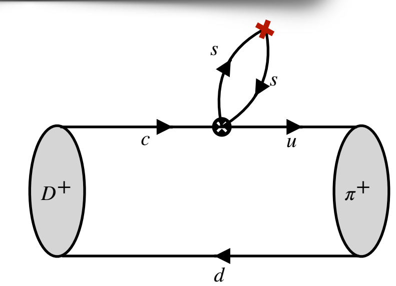


Only d contribution : No GIM cancellation.

A-topology (main contribution)

Loop Topology (Only possible in SCS decays)





In $SU(3)_f$ limit (along with $\lambda_b \approx 0$), a complete GIM cancellation

L-topology (non-zero only due to $m_s \neq m_d$)

- Vector meson can be created from non-leptonic weak decay before γ^* : Resonance contributions

$D \to \pi \ell^+ \ell^-$ using LCSR supported Dispersion relation

[In preparation, AB, Alexander Khodjamirian and Thomas Mannel]

- **Benefits:**
  - An independent alternative to QCDf. [A. Bharucha et. al., (2011.12856)]

    In QCDf, $BR(D^+ \to \pi^+ \mu^+ \mu^-)_{q^2 \in [0.250^2, 0.525^2]} = (8.1^{5.9}_{-6.1}) \times 10^{-9}$

    $BR(D^+ \to \pi^+ \mu^+ \mu^-)_{q^2 > 1.25^2} = (2.7^{+4.0}_{-2.6}) \times 10^{-9}$

    Experimental bounds $\left(< 6.7 \times 10^{-8} \right)$ approaching theory predictions
    $\Longrightarrow$ Imp. to look for alternate QCD methods.

  - Finite m_c : No $\dfrac{1}{m_c}$ corrections unlike QCDf.

  - Possibility to fix ρ, ω, ϕ resonance phases and heavier states (Major source of uncertainties in literature). [G. Hiller et al. 1510.00311, 1909.11108, 2410.00115], [S. Fajfer, N. Kośnik, 1510.00965]

Our methodology: LCSR-supported dispersion relation

Ways to compute $\mathscr{A}_{\mathscr{D}}^{D^+\to\pi^+\gamma^*}(q^2)$

Dispersion relation
(Valid for all q^2 values)

Light Cone Sum Rules
(Only valid in space-like q^2)

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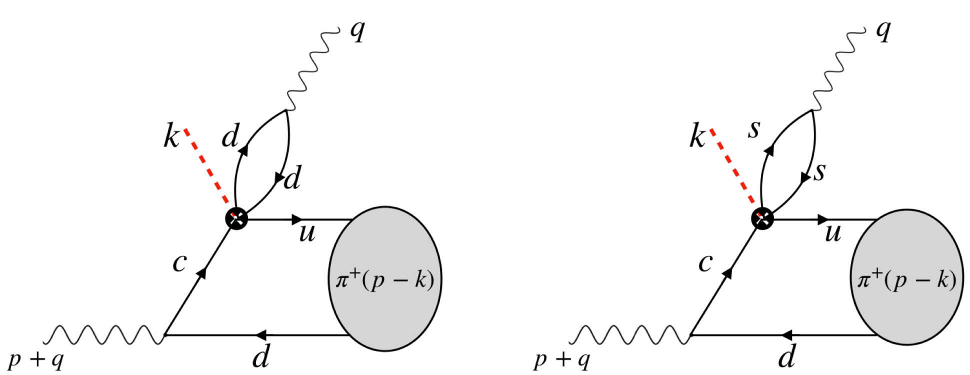

Light Cone Sum Rules
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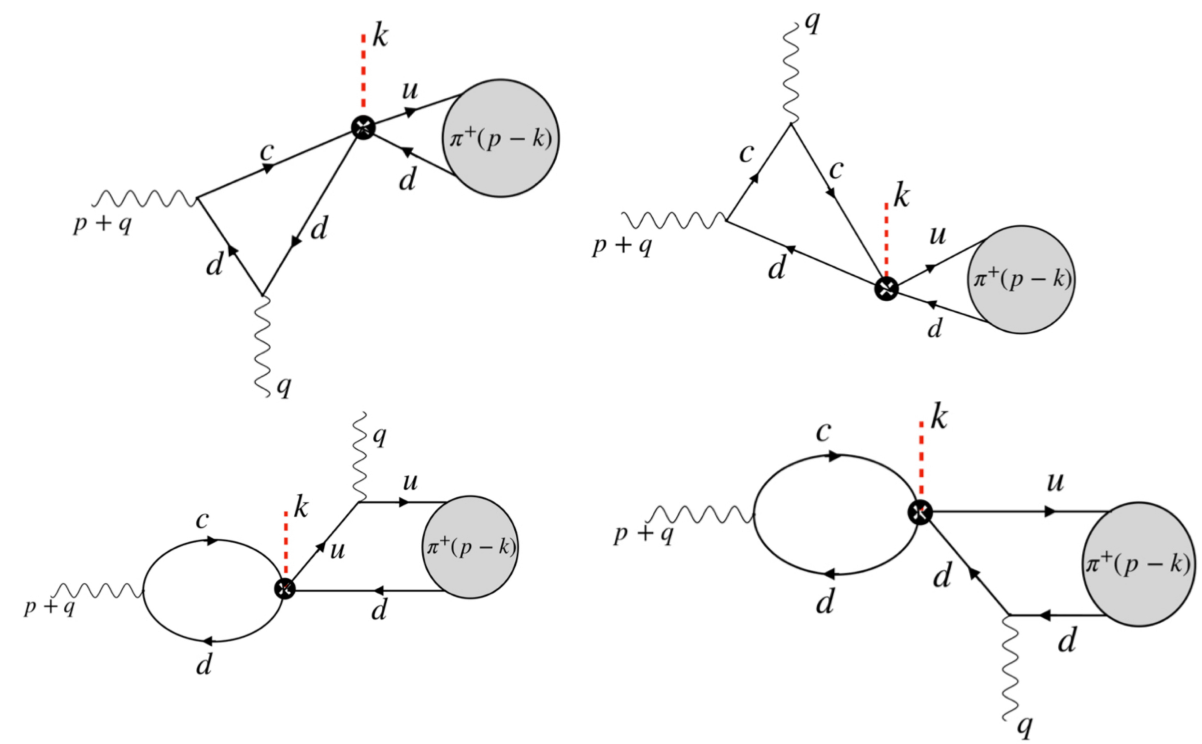
- Can be systematically computed as an expansion in twist using Light Cone Operator Product Expansion.

Weak Annihilation Diagrams (LO) in terms of pion DAs



- Contribution from twist-2 distribution amplitude (DA) of pion.

⁕ Artificial momentum k is introduced at four vertex to avoid parasitic contributions in dispersion relation.

(Method used before in LCSR analysis of $B \to 2\pi$ and $D \to 2\pi, K\bar{K}$)

[A. Khodjamirian et. al, hep-ph/0304179, hep-ph/0509049,1706.07780, hep-ph/0012271]

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$=$ at $q^2 < 0$

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Main idea in a nutshell :

Step-1: Compute $\mathscr{A}^{(D^+\to\pi^+\gamma^*)}(q^2)$ using Light Cone Sum Rules (valid only for $q^2 < 0$)

Step-2: Write the hadronic dispersion relation in terms of unknown phases and parametrize the spectral density (valid for all values of q^2).

Step-3: Match the LCSR results with the dispersion relation at $q^2 < 0$ and estimate the unknown phases and the parametrization parameters.

Step-4: Estimate $\mathscr{A}^{(D^+\to\pi^+\gamma^*)}(q^2)$ in the physical region using dispersion relation.

(Resembling partly the analysis of nonlocal effects in $B \to K^{(*)}\ell^+\ell^-$)

[A. Khodjamirian, T. Mannel, A. Pivovarov, Y. Wang, 1211.0234]

[A. Khodjamirian, A. V. Rusov, 1703.04765] , N. Gubernari, M. Rebound, D. van Dyk, J. Virto, 2011.09813

Hadronic dispersion models and Preliminary Results

[In preparation, AB, A. Khodjamirian and T. Mannel]

$$\mathscr{A}^{(D^+\to\pi^+\gamma^*)}(q^2) = \sum_{V=\rho,\omega,\phi} \frac{\kappa_V f_V |A_{D^+V\pi^+}| e^{i\varphi_V}}{(m_V^2 - q^2 - i\sqrt{q^2}\Gamma_V(q^2))} + \int_{s_0^h}^{\infty} ds \frac{\rho_h(s)}{(s - q^2 - i\epsilon)}$$

- **3-resonance Model:** Most commonly adopted in literature.

$$\int_{s_0^h}^{\infty} ds \frac{\rho_h(s)}{(s - q^2 - i\epsilon)} = 0$$

[G. Hiller et al. 1510.00311, 1909.11108, 2410.00115], [S. Fajfer, N. Kośnik, 1510.00965]

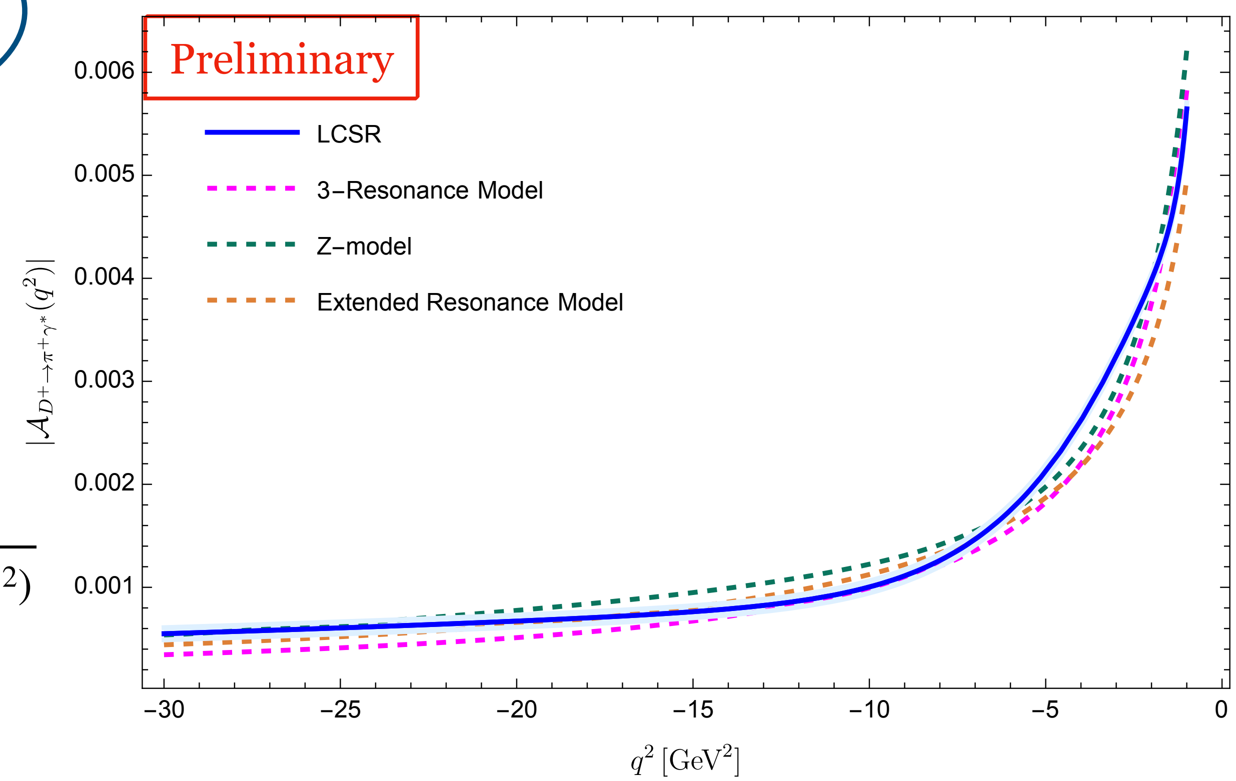
- **Extended Resonance Model**

$$\int_{s_0^h}^{\infty} ds \frac{\rho_h(s)}{(s - q^2 - i\epsilon)} = \sum_{V=\rho',\phi'} \frac{r_V |A_{D^+V\pi^+}| e^{i\varphi_V}}{(m_V^2 - q^2 - i\sqrt{q^2}\Gamma_V(q^2))} + \frac{r_{eff}\, e^{i\varphi_{eff}}}{m_{eff}^2 - q^2 - i\sqrt{q^2}\Gamma_{eff}(q^2)}$$

- **Z-parametrisation model** (valid only for $q^2 < s_0^h$):
  - Using once subtracted dispersion relation with q_0^2 as subtraction point.

$$\int_{s_0^h}^{\infty} ds \frac{(q^2 - q_0^2)\,\rho_h(s)}{(s - q_0^2)(s - q^2 - i\epsilon)} = \sum_{k=1}^{K} a_k \left([z(q^2)]^k - [z(q_0^2)]^k\right)$$

with, $z(q^2) = \dfrac{\sqrt{s_0^h - q^2} - \sqrt{s_0^h}}{\sqrt{s_0^h - q^2} + \sqrt{s_0^h}}$ and $a_k =$ Complex coefficients



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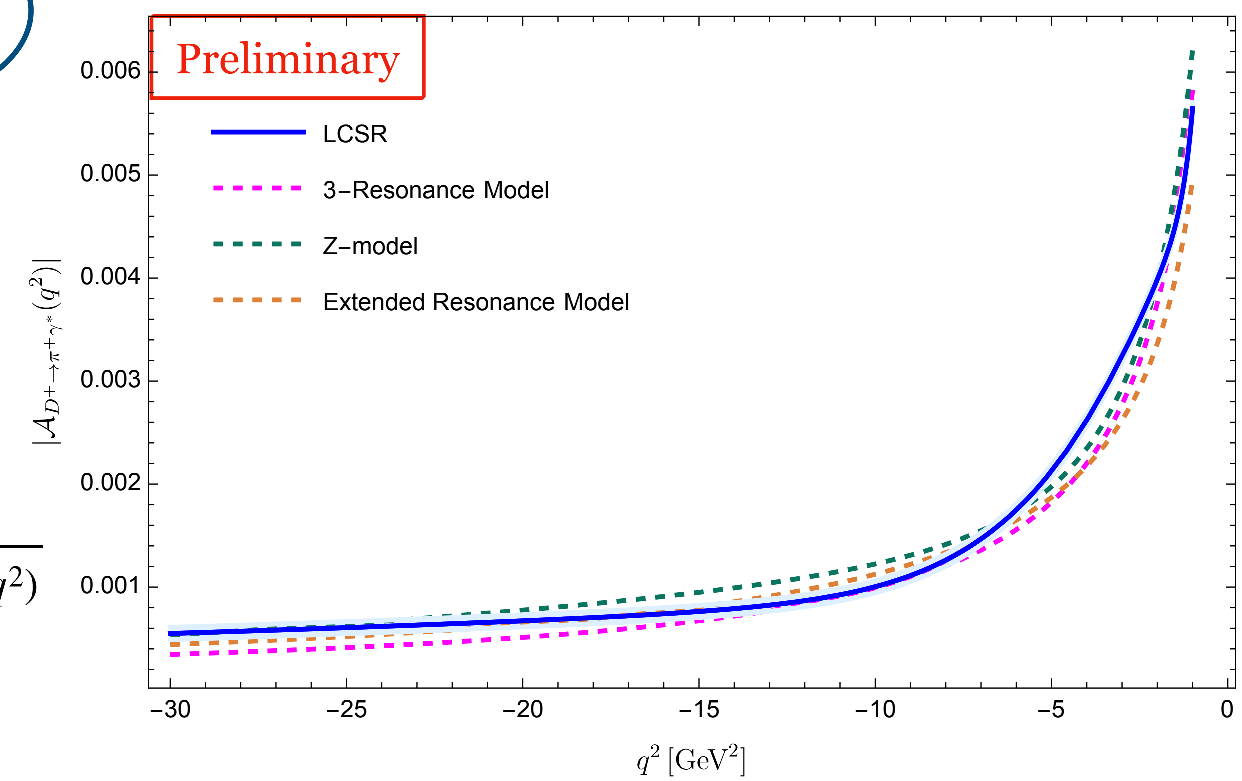

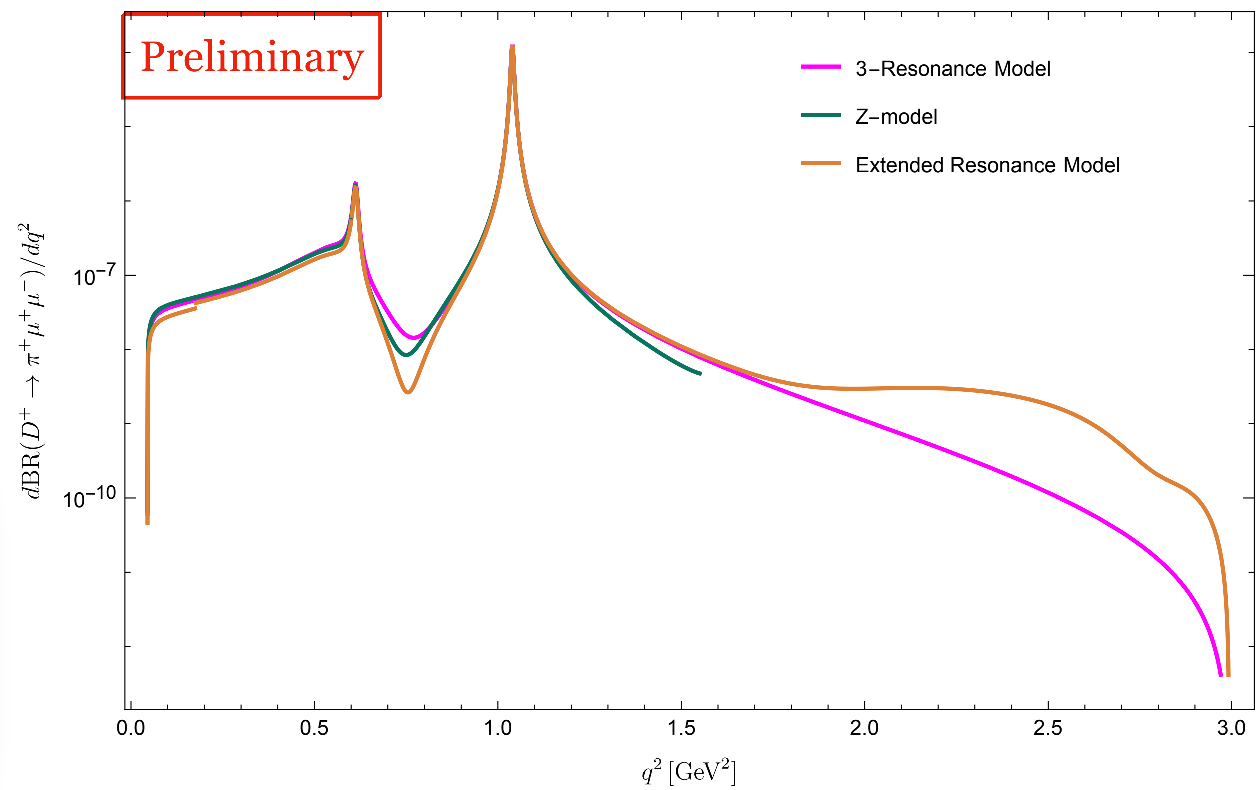

Comments:

- Excited states influence intermediate and high q^2 regions.
- Fit suggest constructive interference between ρ/ω and ϕ resonances.
- Uncertainty estimates are under progress.

Summary and Outlook

- Amplitude for $D^+ \to \pi^+\ell^+\ell^-$ is mainly dominated by weak annihilation topology generated by $O_{1,2}$, "Loop" and "Short distance" contributions e.g. due to O_9 are tiny.
- Contribution from higher resonances is important especially for high q^2 region.
- Preliminary fits suggest a constructive interference between ρ/ω and ϕ resonances.
- We also perform U-spin analysis to relate these Singly Cabibo Suppressed modes to Cabibbo Favoured (CF) modes.
- CF modes includes only annihilation topologies $\Longrightarrow$ Can be helpful to understand QCD dynamics involved.
- As a byproduct we also look at CF $D_s^+ \to \pi^+\ell^+\ell^-$ and $D^0 \to K^0\ell^+\ell^-$ modes.

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Thank you for your attention !!

Back up!

Highlights from literature!

- Treat resonances as a correction to C_9 : using Breit Wigner parametrization [G. Hiller et al. 1510.00311, 1909.11108, 2410.00115], [S. Fajfer, N. Kośnik, 1510.00965]
- Major source of uncertainties : unknown strong phases

What about Weak annihilation and higher resonances?

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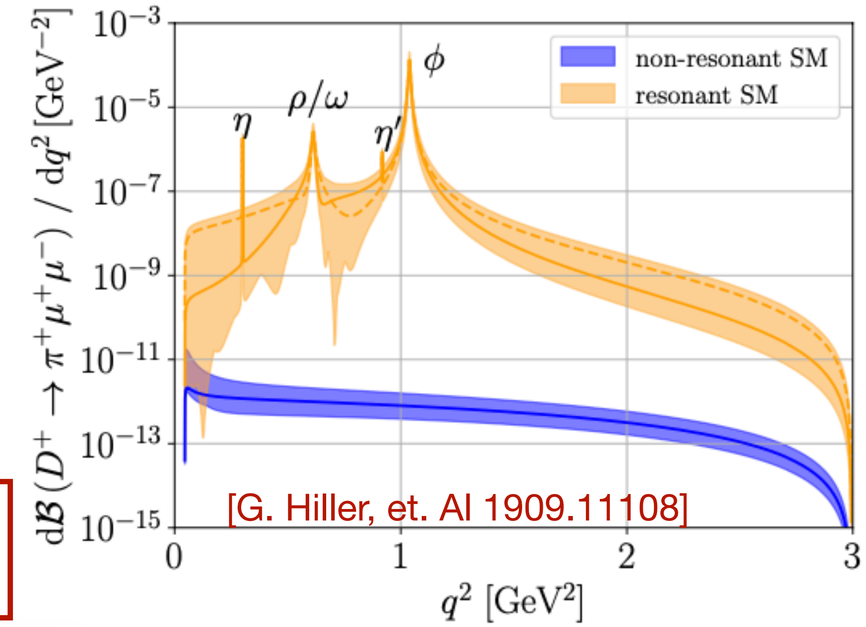

- vetoing a certain q^2- region do not remove resonances from amplitude.
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$\Longrightarrow$ Proper QCD based study is important!!

- QCD based analysis: QCDf for low q^2 and OPE for high q^2 : [A. Bharucha et. al., (2011.12856)]

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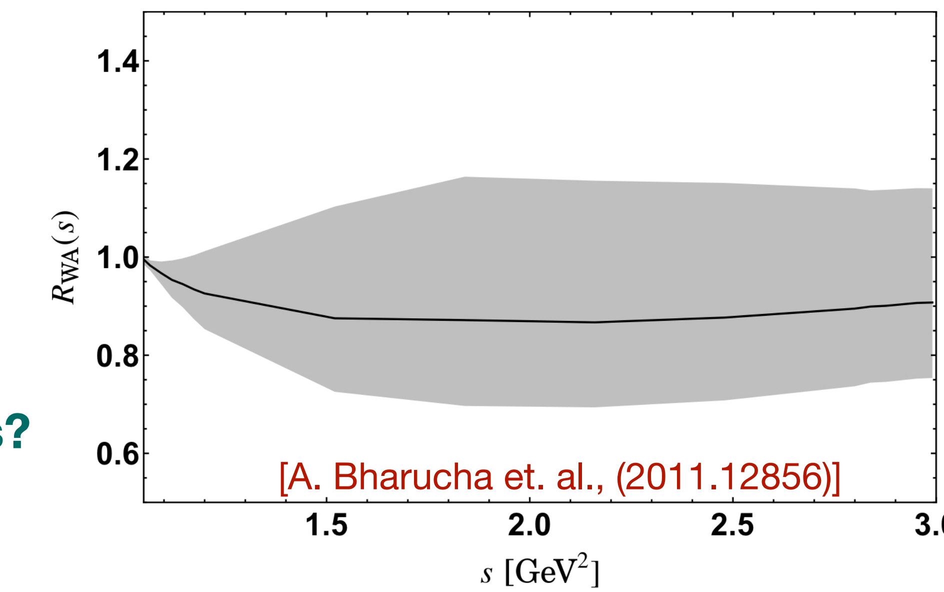

- Still open questions:
  - Annihilation diagrams included in QCDf estimates: emission from initial d-quark
    - **How big are other three contributions?**
  - $\frac{1}{m_c}$ corrections (use of D-meson Distribution Amplitudes) ?

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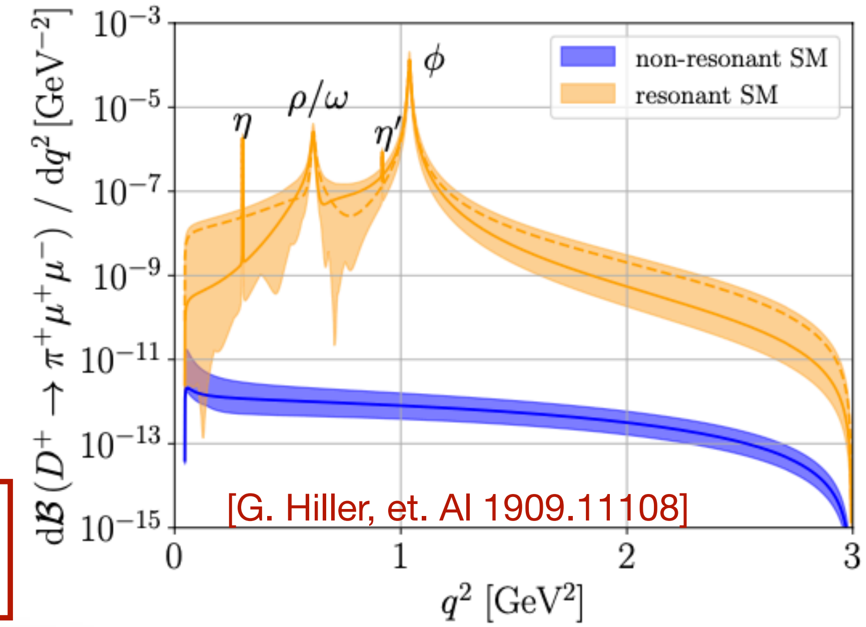

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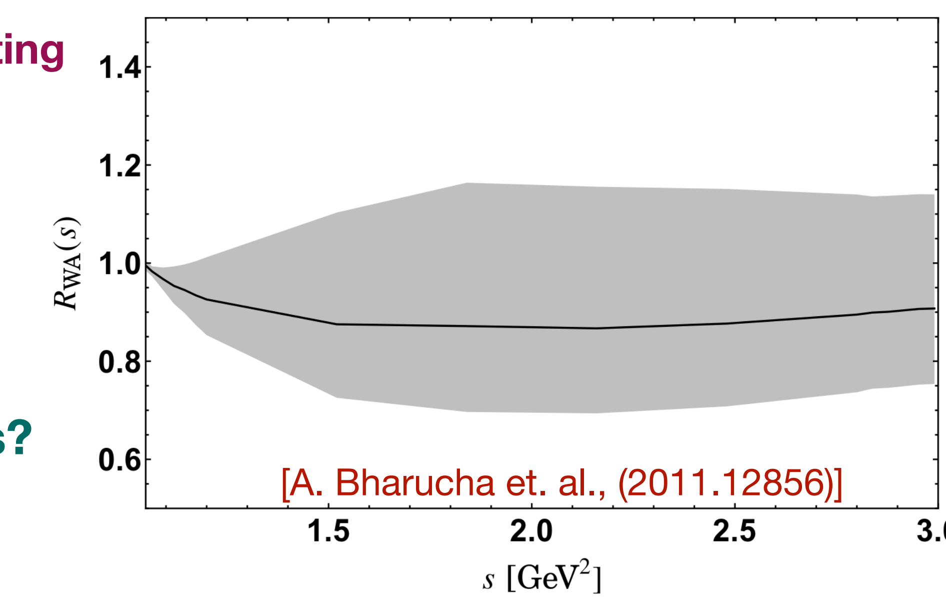

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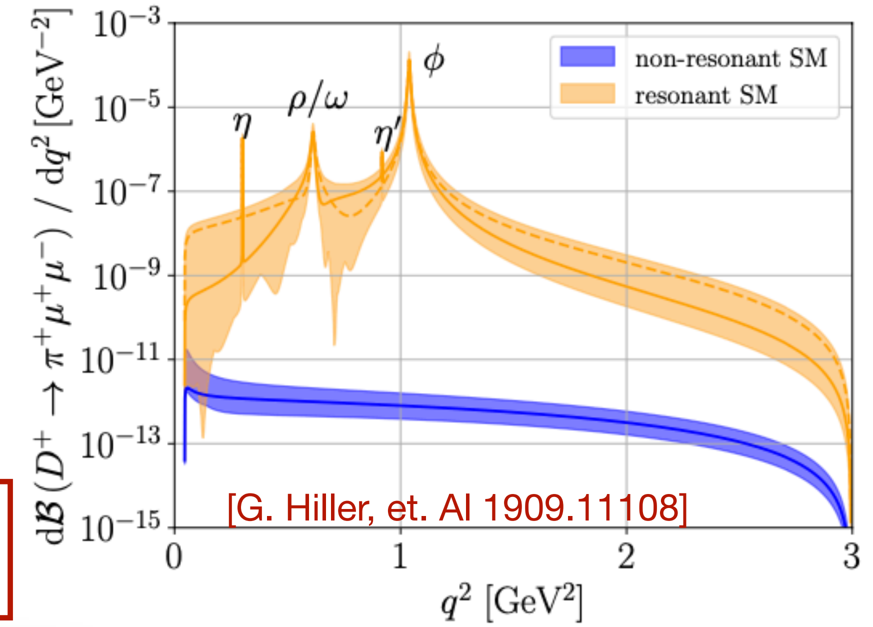

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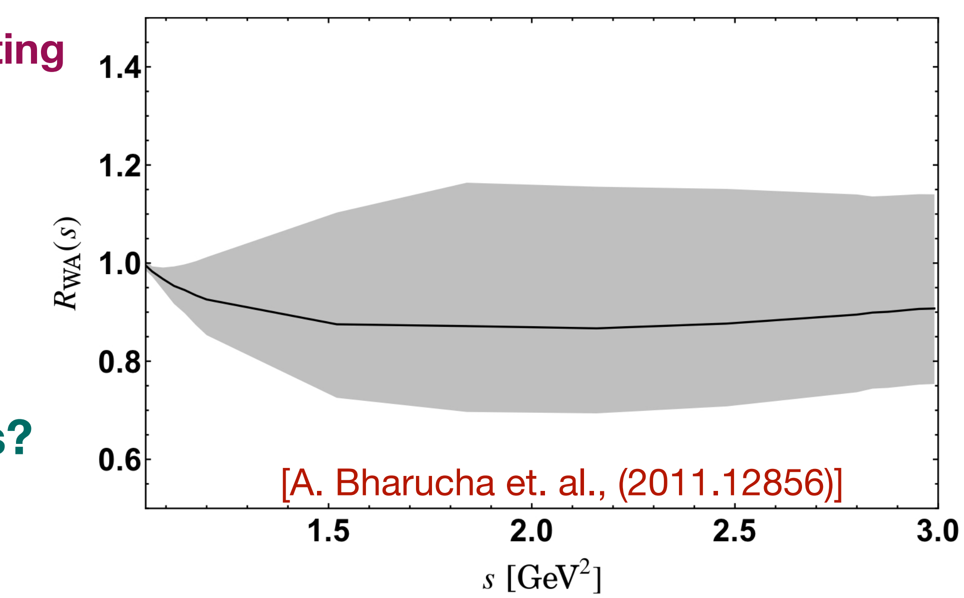

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