

# Measurements of semileptonic and leptonic $B$ decays at Belle II

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on behalf of the Belle II collaboration

# Introduction

- New Belle II leptonic and semileptonic results using the **full Run 1 dataset of  $365 \text{ fb}^{-1}$  or  $390\text{M } B\bar{B}$  pairs** collected at an  $e^+e^-$  centre-of-mass energy at the  $Y(4S)$  resonance mass (10.58 GeV)

- **One new leptonic  $B$  decay result**

[arXiv:2502.04885](https://arxiv.org/abs/2502.04885)  
Submitted to PRD

- Measurement of the  $B^+ \rightarrow \tau \nu$  branching fraction with a hadronic tagging method
  - See Giovanni Gaudino's YSF talk this evening for more details

- **Two new semileptonic  $B$  decay results**

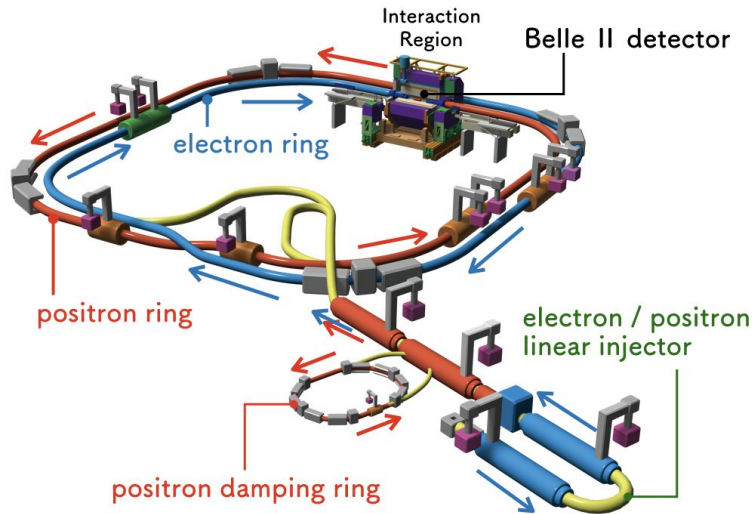
**New for Moriond !**

**Preliminary**

- Determination of  $|V_{cb}|$  using  $B \rightarrow D \ell \nu$  decays with an inclusive tagging method
- Test of lepton flavour universality with measurements of  $R(D^+)$  and  $R(D^{*+})$  using semileptonic  $B$  tagging
  - First result using semileptonic  $B$  tagging !
  - First combined  $R(D)$  and  $R(D^*)$  Belle II measurement !

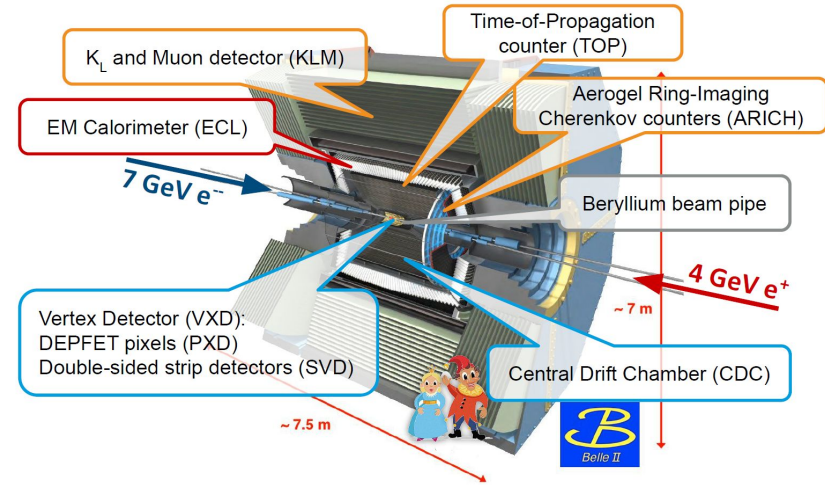
# Experimental setup

## SuperKEKB



Asymmetric-energy  $e^+e^- \rightarrow Y(4S) \rightarrow B\bar{B}$   
Centre-of-mass energy = 10.58 GeV  
World record instantaneous luminosity  
 $5.1 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$  (27/12/2024)

## Belle II

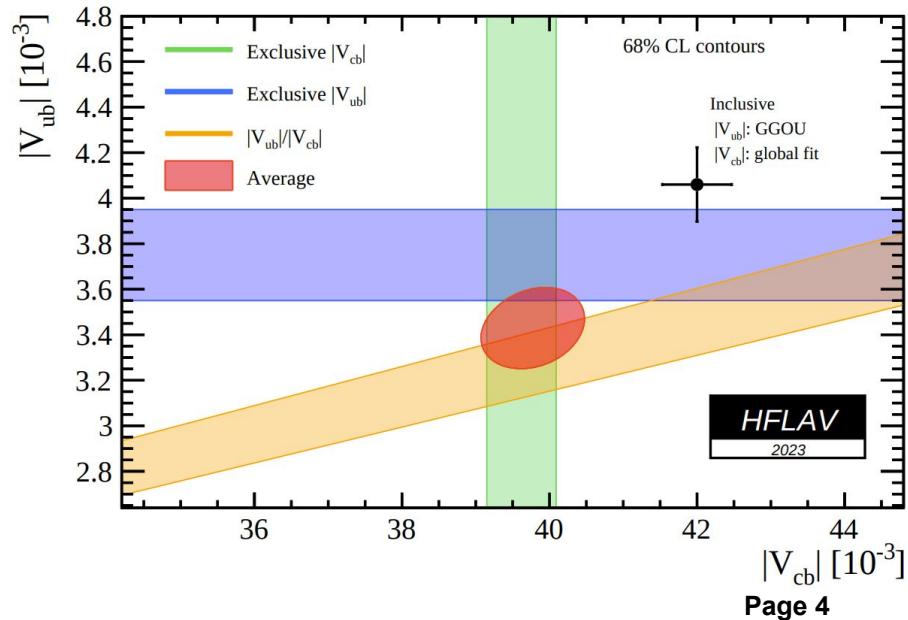


$\sim 4\pi$  spatial coverage  
Well known initial state  
 $\Rightarrow$  Measurements with missing energy  
Run 1 luminosity:  $365.37 \pm 1.70 \text{ fb}^{-1}$

# $|V_{qb}|$ measurements

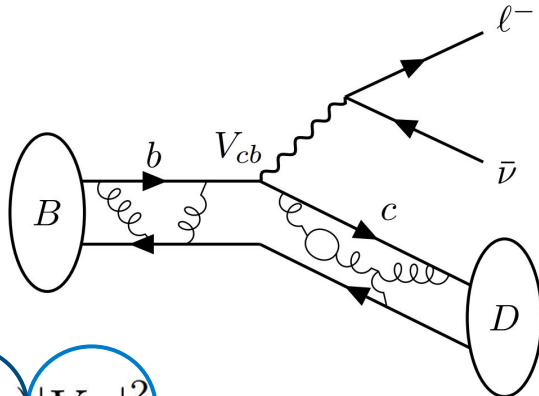
- Measurements of  $|V_{qb}|$  are crucial to **constrain the CKM matrix**
- They are usually measured using semileptonic  $B$  decays
  - Via **exclusive decays**
    - $B \rightarrow \pi / \nu, B \rightarrow D / \nu \dots$
  - Or via **inclusive decays** where no explicit requirements are applied on the hadronic system
- The two methods yield values which **differ by  $\sim 3\sigma$**  for both  $|V_{ub}|$  and  $|V_{cb}|$

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & \boxed{V_{ub}} \\ V_{cd} & V_{cs} & \boxed{V_{cb}} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



# **$B \rightarrow D / \nu$ MEASUREMENT AT BELLE II**

# $B \rightarrow D \ell \nu$ measurement at Belle II



$$w = \frac{M_B^2 + M_D^2 - q^2}{2M_B M_D} \quad q^2 = (p_\ell + p_{\nu_\ell})^2$$

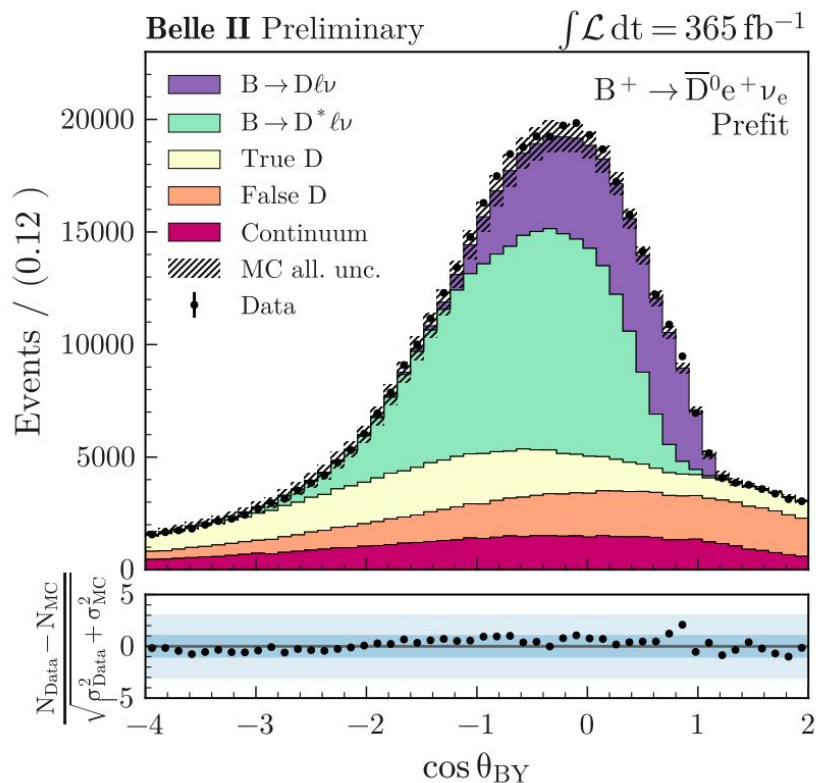
$$\frac{d\Gamma(B \rightarrow D \ell \nu)}{dw} = \frac{G_F^2 m_D^3}{48\pi^3} (M_B + M_D)^2 (w^2 - 1)^{3/2} \eta_{EW}^2 \mathcal{G}^2(w) |V_{cb}|^2$$

- The differential decay rate as a function of the recoil parameter  $w$  is proportional to  $|V_{cb}|^2$  and the  $B \rightarrow D$  form factors
- Studying  $B \rightarrow D \ell \nu$  has 3 main advantages compared to  $B \rightarrow D^* (\rightarrow D^0 \pi^+) \ell \nu$ 
  - Both isospin states  $D^0/D^+$  are accessed
  - The measurement doesn't depend on the reconstruction of a low-momentum  $\pi$ 
    - Leading systematic uncertainty for  $B \rightarrow D^* \ell \nu$  measurements
  - The form factor calculation is more precise
- Belle II has already measured  $|V_{cb}|$  via  $B \rightarrow D^* \ell \nu$  (PRD 108, 092013)
  - $|V_{cb}|_{BGL} = (40.57 \pm 1.16) \times 10^{-3}$

# $B \rightarrow D / \nu$ measurement at Belle II

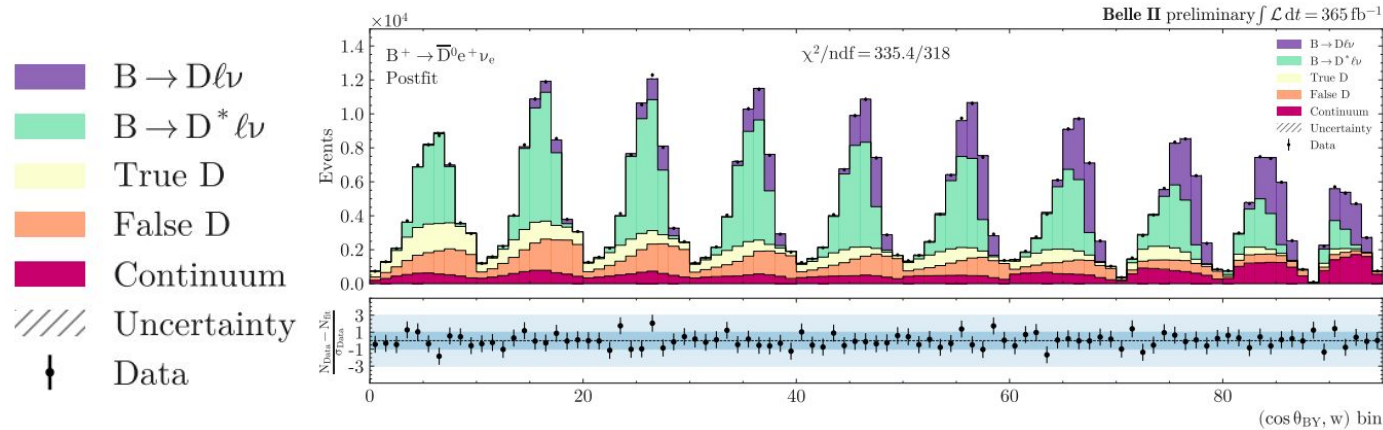
- The Belle II measurement is performed using  $B^0$  and  $B^+$  decays **without explicitly reconstructing the partner  $B$  meson** from the  $Y(4S) \rightarrow B\bar{B}$  decay
  - $D^- \rightarrow K^+ \pi^- \pi^-$  and  $D^0 \rightarrow K^- \pi^+$
- The signal is extracted using the  **$\cos\theta_{BY}$  variable** where  $Y$  represents the  $D/\nu$  system

$$\cos\theta_{BY} = \frac{2 E_B^* E_Y^* - M_B^2 - M_Y^2}{2 |p_B^*| |p_Y^*|}$$



# $B \rightarrow D \ell \nu$ measurement at Belle II

- The signal is **extracted from a 2D binned template fit of  $\cos\theta_{BY}$** :  $w$  split in 10 bins each
- The fit is performed simultaneously on **4 separate channels  $D^0 e^-$ ,  $D^0 \mu^-$ ,  $D^+ e^-$  and  $D^+ \mu^-$**  to extract the individual **branching fractions** and a **lepton flavour universality test**



10  $\cos\theta_{BY}$   
distributions  
in 10  $w$  bins

[arXiv:2411.18639](https://arxiv.org/abs/2411.18639)

	Belle II	HFLAV
$\mathcal{B}(B \rightarrow D^- \ell^+ \nu)$	$2.06 \pm 0.12\%$	$2.12 \pm 0.06\%$
$\mathcal{B}(B \rightarrow \bar{D}^0 \ell^+ \nu)$	$2.31 \pm 0.10\%$	$2.21 \pm 0.06\%$

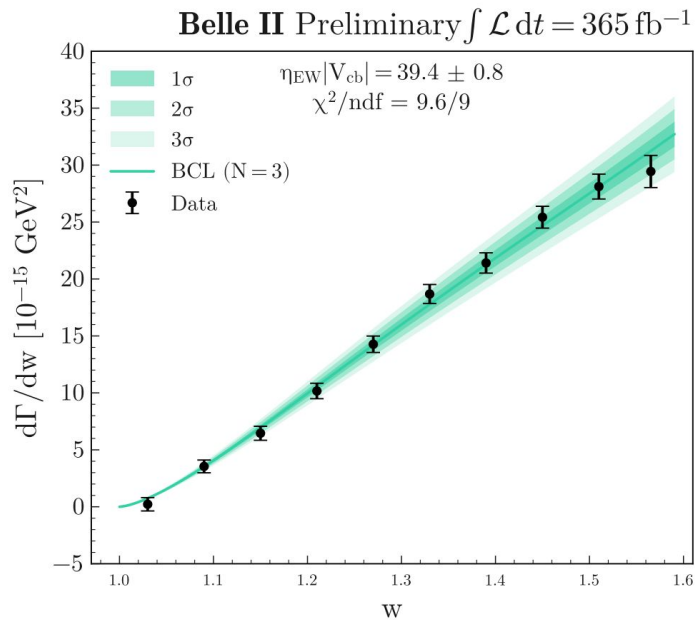
$$\frac{\mathcal{B}(B \rightarrow D e \nu)}{\mathcal{B}(B \rightarrow D \mu \nu)} = 1.02 \pm 0.03$$



# $B \rightarrow D / v$ measurement at Belle II

## Results

- The differential decay rate  $\Delta\Gamma/\Delta w$  in 10  $w$  bins is obtained from the **same fit**
- The obtained values of  $\Delta\Gamma/\Delta w$  are **fitted** to the differential rate expressed using the Bourrely, Caprini, Lellouch (BCL) form factor parametrisation with a  $\chi^2$  fit with lattice QCD constraints  $\rightarrow$  **extraction of  $|V_{cb}|$  and BCL form factor parameters**



$$\chi^2 = \sum_{i,j} \left( \frac{\Delta\Gamma_i}{\Delta w} - \frac{\Delta\Gamma_{i,BCL}}{\Delta w} \right) \mathbf{C}_{ij}^{-1} \left( \frac{\Delta\Gamma_j}{\Delta w} - \frac{\Delta\Gamma_{j,BCL}}{\Delta w} \right) + \sum_{k,l} (c_k - c_{k,FLAG}) \mathbf{D}_{kl}^{-1} (c_l - c_{l,FLAG})$$

Measured values  
 Predicted values

LQCD constraints  
 ( [FNAL/MILC](#) + [HPQCD](#) )

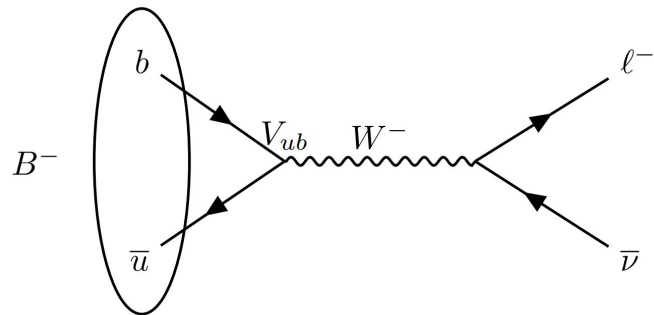
$$|V_{cb}|_{BCL} = (39.2 \pm 0.8) \times 10^{-3}$$

$$|V_{cb}|_{\text{excl.}} = (39.77 \pm 0.46) \times 10^{-3} \quad |V_{cb}|_{\text{incl.}} = (41.97 \pm 0.48) \times 10^{-3}$$

# **$B \rightarrow TV$ MEASUREMENT AT BELLE II**

# Leptonic $B$ decays

## Introduction



$$\mathcal{B}(B^+ \rightarrow \ell^+ \nu_\ell) = \frac{G_F^2 m_B}{8\pi} m_\ell^2 \left(1 - \frac{m_\ell^2}{m_B^2}\right)^2 f_B^2 |V_{ub}|^2 \tau_B$$

$f_B = 190.0 \pm 1.3 \text{ MeV}$   
[FLAG]

- Purely leptonic  $B$  decays are the cleanest channels to measure  $|V_{ub}|$
- Their branching ratio depends on the  $B$  meson decay constant which can be extracted precisely from lattice QCD
- However, they are strongly helicity suppressed and therefore hard to study
- All individual measurements are below the  $5\sigma$  discovery threshold

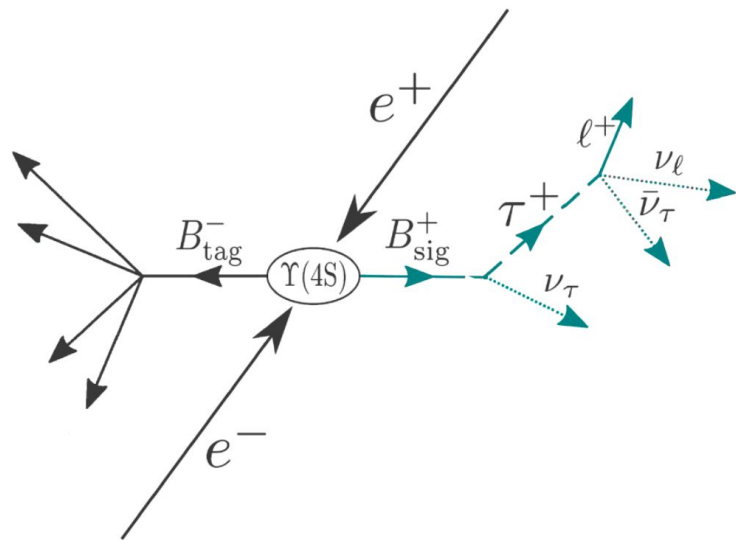
### $B \rightarrow TV$

Experiment	Tag	$\mathcal{B}(10^{-4})$
Belle	Hadronic	$0.72_{-0.25}^{+0.27} \pm 0.11$
BABAR	Hadronic	$1.83_{-0.49}^{+0.53} \pm 0.24$
Belle	Semileptonic	$1.25 \pm 0.28 \pm 0.27$
BABAR	Semileptonic	$1.8 \pm 0.8 \pm 0.2$
PDG		$1.09 \pm 0.24$

# $B \rightarrow \tau\nu$ measurement at Belle II

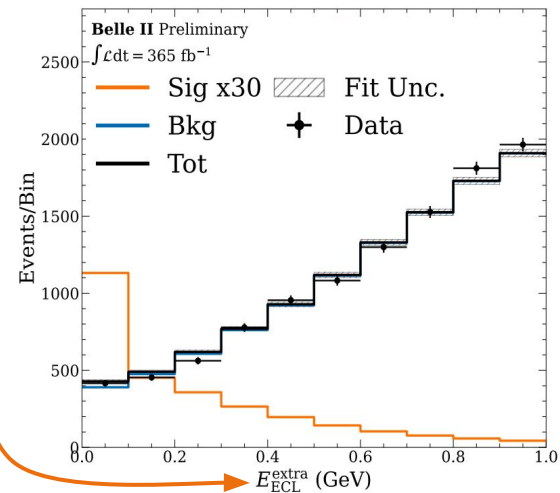
[arXiv:2502.04885](https://arxiv.org/abs/2502.04885)  
Submitted to PRD

- The Belle II measurement is performed by reconstructing **tag-side  $B$  mesons** in their **hadronic decay channels**
  - This is necessary to constrain the event kinematics despite the presence of multiple undetected neutrinos
- The **signal-side  $\tau$  is reconstructed in 4 channels** to maximise the reconstruction efficiency
  - $\text{BR}(\tau \rightarrow e\nu\nu) = 17.8\%$
  - $\text{BR}(\tau \rightarrow \mu\nu\nu) = 17.4\%$
  - $\text{BR}(\tau \rightarrow \pi\nu) = 10.8\%$
  - $\text{BR}(\tau \rightarrow \rho\nu) = 25.5\%$
  - Total = 71.5%



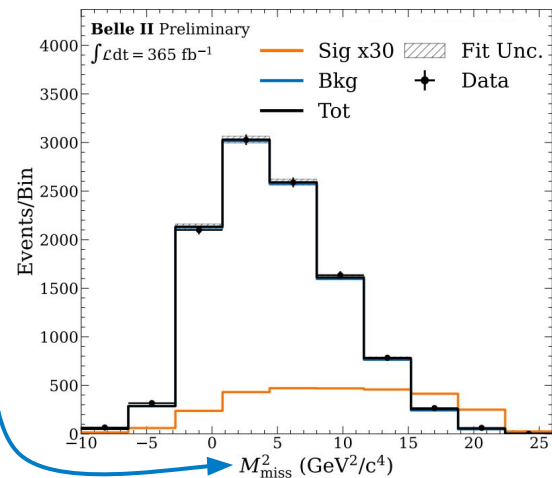
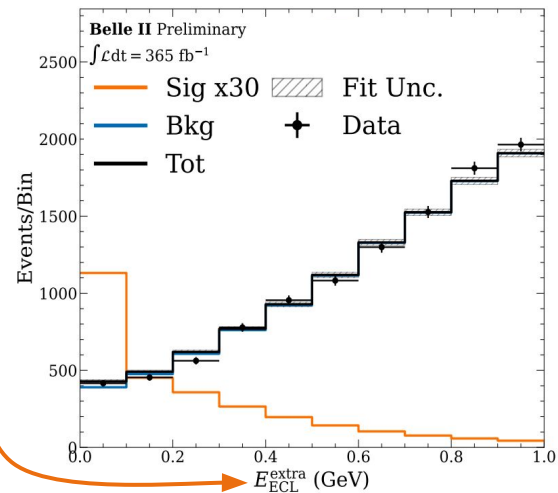
# $B \rightarrow \tau\nu$ measurement at Belle II

- The total energy from neutral clusters not associated with either  $B$  mesons is calibrated using 3 separate control samples to correct  $B\bar{B}$  backgrounds, signal with  $\tau$  leptonic modes and signal with  $\tau$  hadronic modes



# $B \rightarrow \tau \nu$ measurement at Belle II

- The total energy from neutral clusters not associated with either  $B$  mesons is calibrated using 3 separate control samples to correct  $B\bar{B}$  backgrounds, signal with  $\tau$  leptonic modes and signal with  $\tau$  hadronic modes
- The branching fraction is extracted from **2D fit** of the cluster energy and the event squared missing mass
- Simultaneous binned maximum likelihood fit of all 4 signal  $\tau$  decay channels



# $B \rightarrow \tau \nu$ measurement at Belle II

## Results

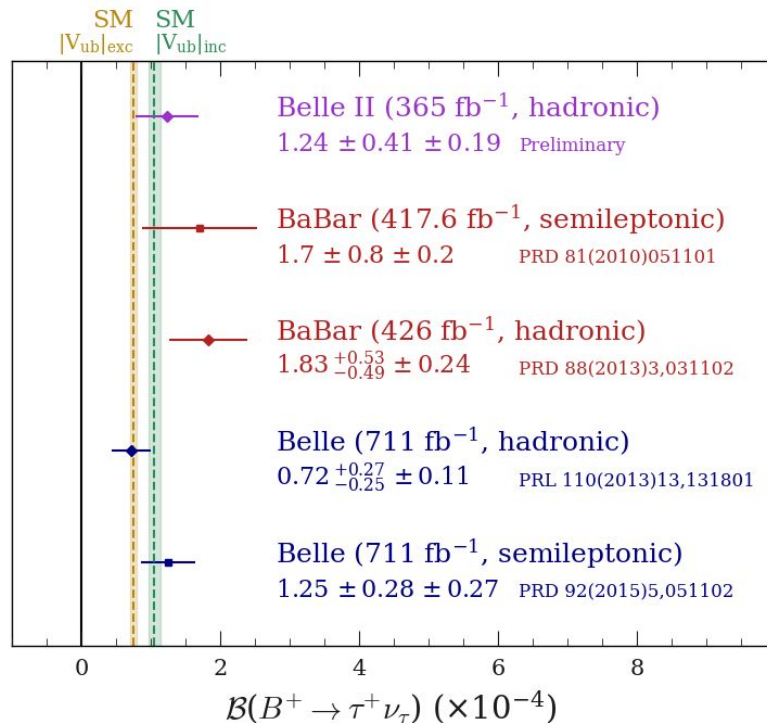
arXiv:2502.04885  
Submitted to PRD

$$\mathcal{B}(B^+ \rightarrow \tau^+ \nu_\tau) = [1.24 \pm 0.41(\text{stat.}) \pm 0.19(\text{syst.})] \times 10^{-4}$$

- **Significance  $\rightarrow 3.0\sigma$** 
  - Expected significance  $\rightarrow 2.7\sigma$
- The measurement is limited by statistics
- The leading systematic uncertainties come from
  - The finite size of the simulated samples
  - The neutral cluster calibration
- The extracted value of  $|V_{ub}|$  is compatible with the exclusive and inclusive averages

$$|V_{ub}|_{B^+ \rightarrow \tau^+ \nu_\tau} = [4.41_{-0.89}^{+0.74}] \times 10^{-3}$$

$$|V_{ub}|_{\text{excl.}} = (3.43 \pm 0.12) \times 10^{-3} \quad |V_{ub}|_{\text{incl.}} = (4.06 \pm 0.16) \times 10^{-3}$$



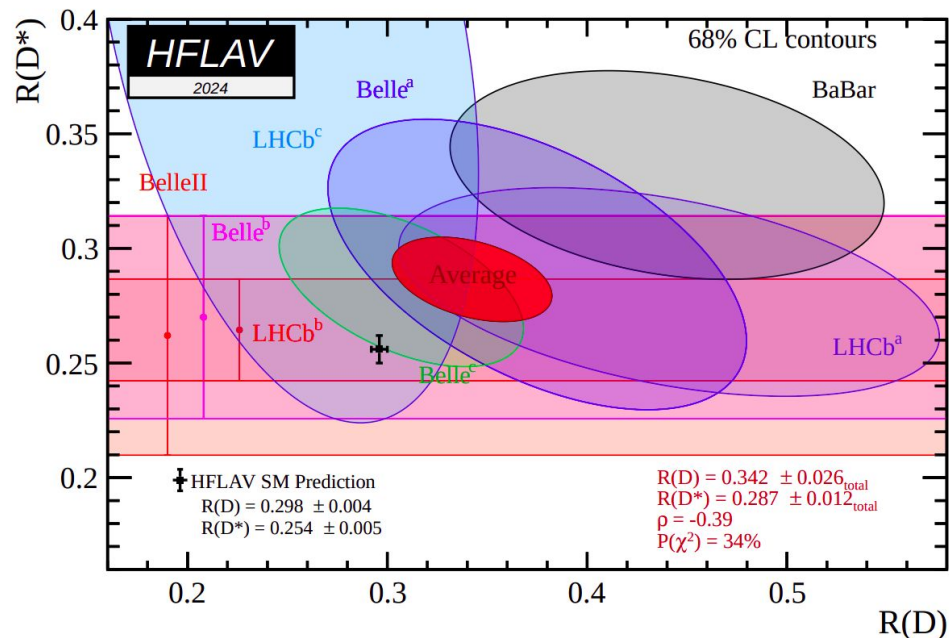
# $R(D^{(*)})$ MEASUREMENT AT BELLE II



# $R(D^{(*)})$ measurement at Belle II

- Test **lepton flavour universality** by studying decays to **heavy  $\tau$  leptons** versus **light  $e, \mu$  leptons**
  - An observation of lepton flavour universality violation would be a clear signature of non-SM couplings with the 3<sup>rd</sup> fermion generation
- Such ratios have been measured by BaBar, Belle, LHCb and now Belle II
- Long standing **discrepancy** currently standing at  **$3.1\sigma$**  for the combined value of  $R(D^{(*)})$

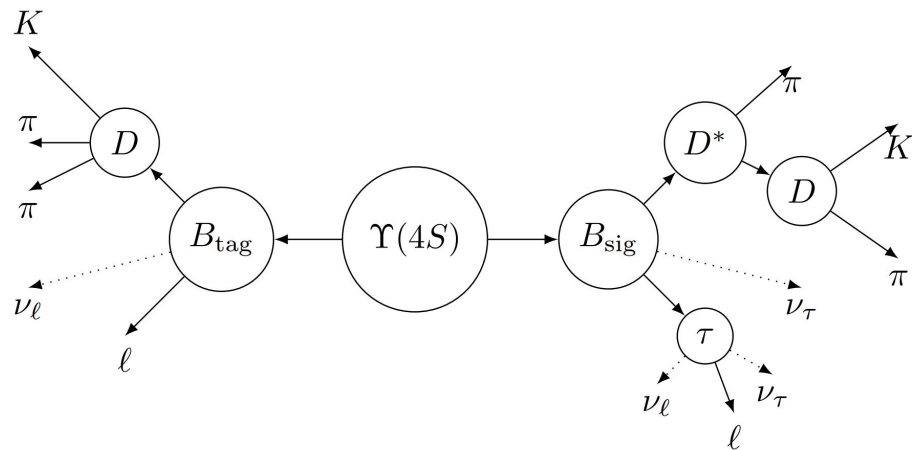
$$R(D^{(*)}) = \frac{\mathcal{B}(B \rightarrow D^{(*)} \tau \nu)}{\mathcal{B}(B \rightarrow D^{(*)} \ell \nu)}$$



# $R(D^{(*)})$ measurement at Belle II

## Reconstruction

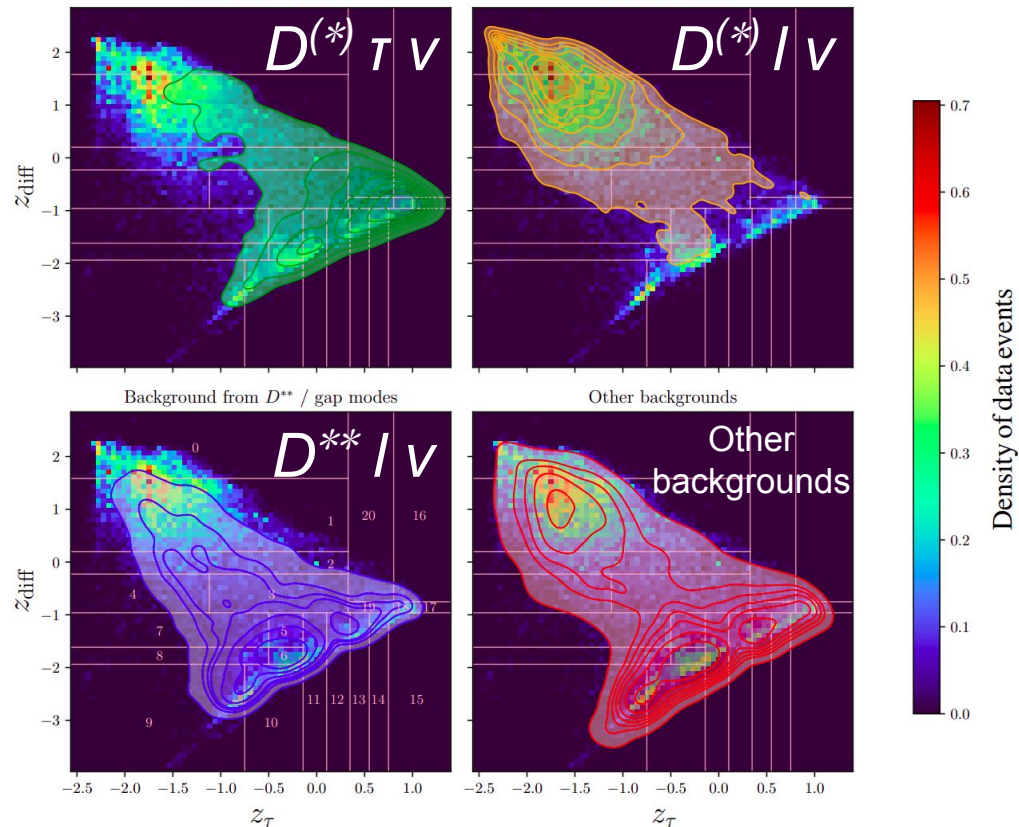
- The Belle II measurement is performed with  $B^0$  decays
- The Belle II measurement is performed by reconstructing **tag-side  $B$  mesons** in their **semileptonic decay channels**
  - $B_{tag} \rightarrow D/D^* l \nu$
- $\tau$  are reconstructed in their **leptonic decay channels**
  - $\tau \rightarrow l \nu \nu$
- $D$  mesons on both sides are reconstructed through **various decays to  $K^+$ ,  $K_S$ ,  $\pi^+$ ,  $\pi^0$** 
  - Tag side: 26 decay modes
  - Signal side: 13 decays modes



# $R(D^{(*)})$ measurement at Belle II

- A **BDT** algorithm is used to separate the **events in 3 classes**
  - Semitauonic signal events
  - Semileptonic signal events
  - Background events
- Most **discriminating** input feature is  $\cos\theta_{BY}$
- Each event is assigned a BDT score  $z_T, z_l, z_{bkg}$
- The signal is extracted in a **2D binned template fit** of  $z_T$  and  $z_{diff} = z_l - z_{bkg}$

Belle II Preliminary

 $\bar{B}^0 \rightarrow D^{(*)} + \tau^- \bar{\nu}$  $\bar{B}^0 \rightarrow D^{(*)} + \ell^- \bar{\nu}$  $\int \mathcal{L} dt = 365 \text{ fb}^{-1}$ 

# $R(D^{(*)})$ measurement at Belle II

## Results

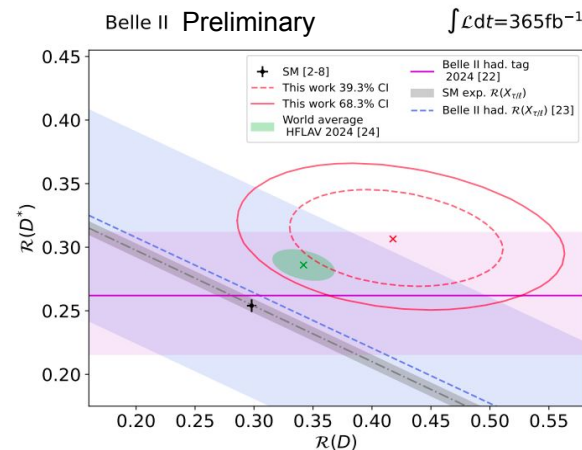
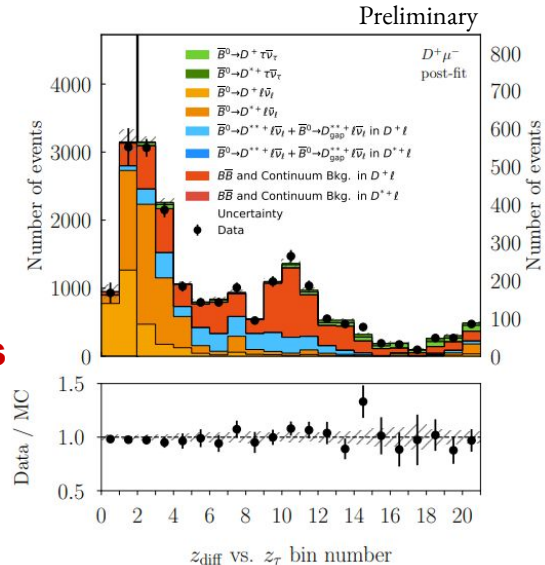
- The fit is performed over **4 separate channels**:  $D^+e^-$ ,  $D^+\mu^-$ ,  $D^{*+}e^-$ ,  $D^{*+}\mu^-$  and The measurement is statistically limited
- The leading systematic uncertainties are coming from
  - The finite size of the simulated samples
  - The lepton identification efficiency and fake rate corrections
- The addition of  $B^+$  modes will improve the precision

$D^{(*)} \tau \nu$

$D^{(*)} l \nu$

$D^{*+} l \nu$

Backgrounds



$$\mathcal{R}(D^+) = 0.418 \pm 0.074 \text{ (stat)} \pm 0.051 \text{ (syst)}$$

$$\mathcal{R}(D^{*+}) = 0.306 \pm 0.034 \text{ (stat)} \pm 0.018 \text{ (syst)}$$

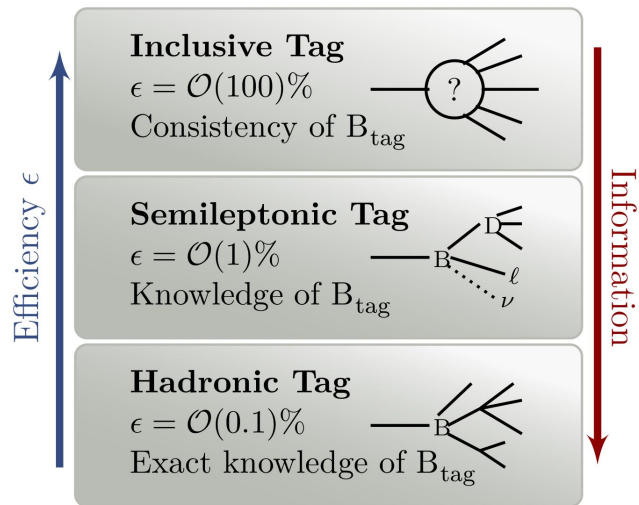
# Summary

- **3 very recent high-profile Belle II analyses** were presented here
  - Measurement of  $|V_{cb}|$  via  $B \rightarrow D / \nu$  decays
    - Competitive with previous measurements of  $|V_{cb}|$  via  $B \rightarrow D^* / \nu$  decays which are usually preferred because of a branching fraction about twice higher
  - $B^+ \rightarrow \tau \nu$  branching fraction measurement
    - Competitive with previous measurements
    - Measurements of  $|V_{ub}|$  with negligible theoretical uncertainty
  - Combined  $R(D^+)$  and  $R(D^{*+})$  measurement
    - First Belle II result with semileptonic tagging method
    - First Belle II combined  $R(D)$ - $R(D^*)$  measurement

# BACKUP

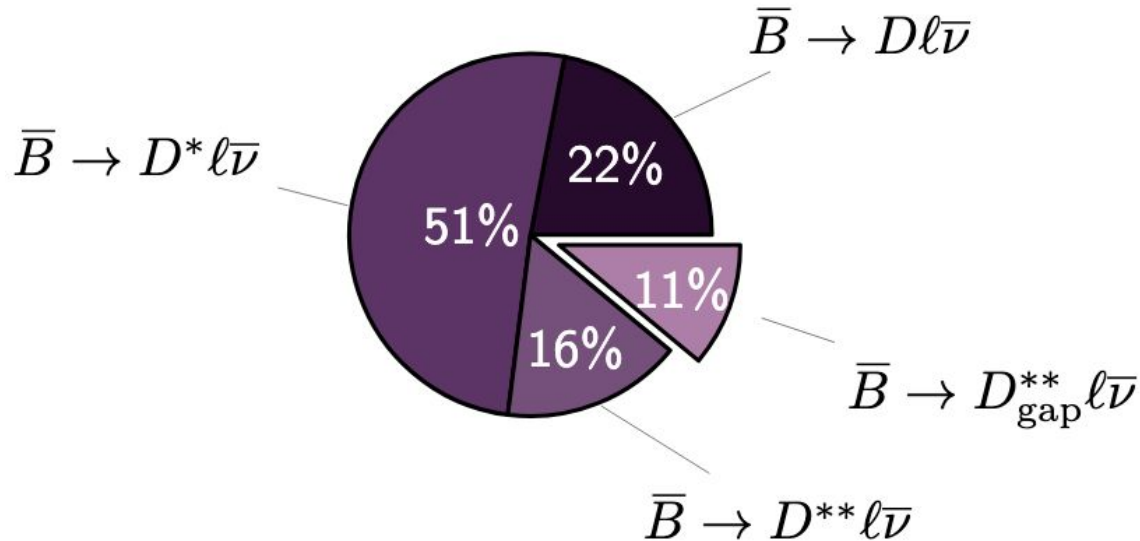
# Tagging methods

- At B-factories it is possible to reconstruct both  $B$  mesons coming from the  $Y(4S) \rightarrow B_{sig} \bar{B}_{tag}$  decay
- Three possible tagging strategies
  - Inclusive tagging (untagged)
    - Only simple consistency selections are applied on the  $B_{tag}$
    - Offers high efficiency for statistically limited measurements
  - Semileptonic tagging
    - Reconstruct the  $B_{tag}$  in its  $B \rightarrow D/D^* / \nu$  decays
    - Relatively low efficiency but more kinematically constrained events
  - Hadronic tagging
    - Reconstruct the  $B_{tag}$  in its hadronic decays in a total of  $O(10,000)$  channels
    - Very low efficiency but precise reconstruction of the full event
    - Particularly useful for measurements with undetected particles and/or inclusive systems



# $B \rightarrow X_c / \nu$ decays

- The sum of branching fractions of semileptonic  $B$  decays to  $D$ ,  $D^*$  and  $D^{**}$  doesn't match the measured inclusive  $B \rightarrow X_c / \nu$  branching fraction
- To fill the gap, unmeasured decays are added referred to as gap modes
  - $B \rightarrow D^{(*)} \eta / \nu$





# $B \rightarrow D / v$ measurement at Belle II

BCL expansion: PRD 79, 013008 (2009)

BCL expansion

$$r = M_D/M_B \quad \mathcal{G}^2(w) = \frac{4r}{(1+r)^2} f_+^2(w) \quad f_0(w_{\max}) = f_+(w_{\max})$$

$$f_+(q^2) = \frac{1}{1 - q^2/M_+^2} \sum_{k=0}^{N-1} a_k \left[ z^k - (-1)^{k-N} \frac{k}{N} z^N \right] \quad f_0(q^2) = \frac{1}{1 - q^2/M_0^2} \sum_{k=0}^{N-1} b_k z^k$$

	Values	Correlation coefficients				
$a_0^+$	0.8959(92)	1	0.26	-0.38	0.95	0.51
$a_1^+$	-8.03(15)		1	0.17	0.33	0.86
$a_2^+$	49.3(31)			1	-0.31	0.16
$a_0^0$	0.7813(73)				1	0.47
$a_1^0$	-3.38(15)					1

Measured parameters of the  $N = 3$  BCL expansion

# $B \rightarrow D / \nu$ measurement at Belle II

## Systematics budget

Fractional contributions to the total relative uncertainty of  $|V_{cb}|$

$$|V_{cb}| = (39.2 \pm 0.4 \text{ (stat.)} \pm 0.6 \text{ (syst.)} \pm 0.5 \text{ (theo.)}) \times 10^{-3}$$

	Uncertainty [%]
Statistical uncertainty	0.9
MC Stat. Error	0.5
$N_{bb}$	0.5
$f_{00}/f_{+-}$	0.1
$f_B$	0.3
$\mathcal{B}(D \rightarrow K\pi(\pi))$	0.3
Selection	0.5
$\mathcal{B}(B \rightarrow X_c \ell \nu)$	0.3
Lepton identification	0.2
Kaon identification	0.5
Tracking efficiency	0.3
Signal PDF	0.4
$B \rightarrow D^* \ell \nu$ form factor	0.1
$w$ background modelling	0.5
Background reweighing	0.3
$\tau_{B^{0/\pm}}$	0.1
Total Systematic	1.5
Lattice QCD inputs	1.2
Long-distance QED	0.4
Total theory	1.3
Total	2.1

# $B \rightarrow D / \nu$ measurement at Belle II

## Electroweak and QED corrections

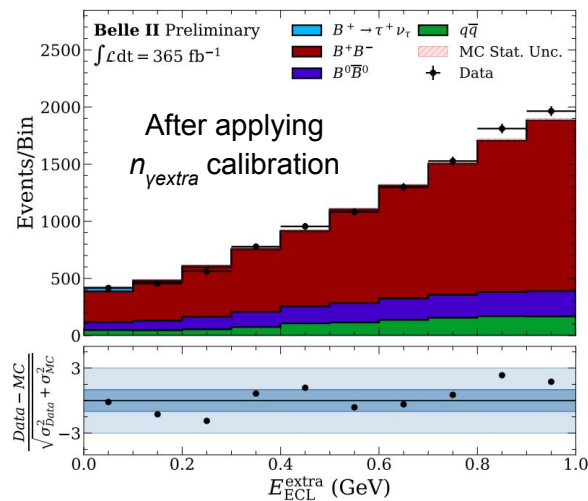
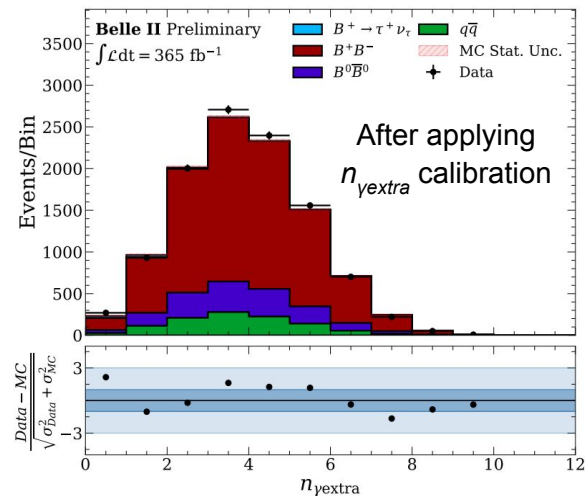
- Short-distance electroweak corrections are well understood
  - $\eta_{EW} = (1.0066 \pm 0.0002)$  [Nucl. Phys. B 196, 83 (1982)]
- Long-distance QED corrections arise from photon exchange between the  $D$  meson and the charged lepton (Coulomb correction)
  - $\delta_{Coulomb} = (1 + \alpha\pi) = 1.023$  [Phys. Rev. D 41, 1736 (1990)]
  - A nuisance parameter  $\theta$  is introduced to take into account the isospin-breaking effect of the Coulomb correction which modifies the  $B$  lifetime ratio
    - $\tau_{0^+} \rightarrow \tau_{0^+}(1 + \alpha\pi\theta)$
  - This is an important information that cannot be accessed in  $B \rightarrow D^* / \nu$  measurements where the  $D^*$  is usually reconstructed via  $D^* (\rightarrow D^0 \pi^+)$

# $B \rightarrow \tau\nu$ measurement at Belle II

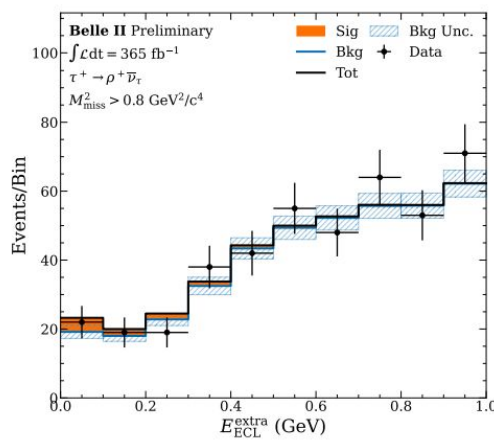
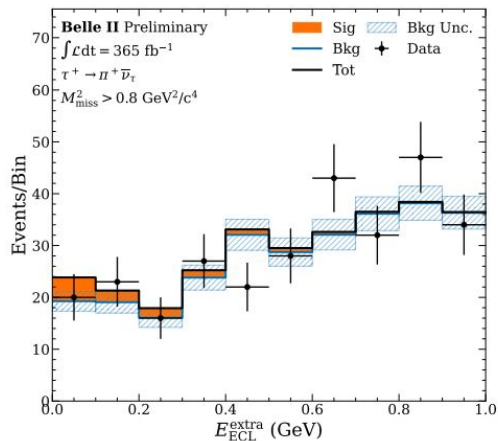
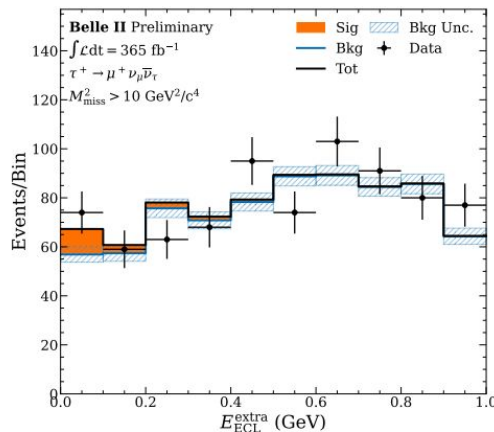
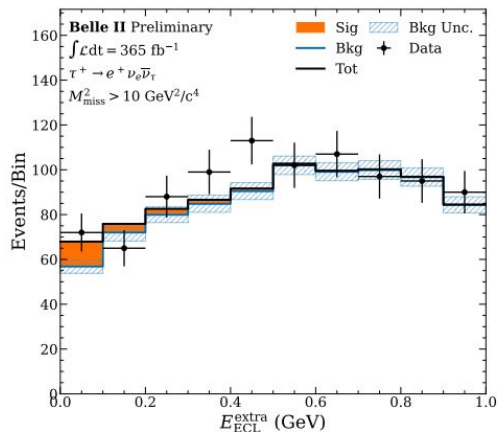
## Calibration and validation

$$w_{\text{cont.}} = \frac{\mathcal{S}_{\text{cont.}}}{1 - \mathcal{S}_{\text{cont.}}}$$

- Continuum calibration
  - Use a sample of data collected 60 MeV below the  $Y(4S)$  resonance
  - Train a BDT to distinguish simulated and data events
  - The classifier response  $\mathcal{S}_{\text{cont.}}$  is used to reweight continuum events
- The data-simulation disagreement in  $E^{\text{extra}}$  variable is known to originate from an incorrect modelling of the extra neutral cluster multiplicity  $n_{\text{yextra}}$
- 3 control samples are used to extract calibration factors in bins of  $n_{\text{yextra}}$ 
  - Extra track sample  $\rightarrow B\bar{B}$  background
  - $B \rightarrow D^* / \nu$  sample  $\rightarrow$  signal  $\tau \rightarrow l \nu \nu$
  - Double tag sample  $\rightarrow$  signal  $\tau \rightarrow h \nu$



# $B \rightarrow \tau\nu$ measurement at Belle II



Fit results in signal enriched regions

- $M_{\text{miss}}^2 > 10 \text{ GeV}^2$  for leptonic  $\tau$  channels
- $M_{\text{miss}}^2 > 0.8 \text{ GeV}^2$  for hadronic  $\tau$  channels

# $B \rightarrow \tau\nu$ measurement at Belle II

## Systematics budget

Source	Syst.
Simulation statistics	13.3%
Fit variables PDF corrections	5.5%
Decays branching fractions in MC	4.1%
Tag $B^-$ reconstruction efficiency	2.2%
Continuum reweighting	1.9%
$\pi^0$ reconstruction efficiency	0.9%
Continuum normalization	0.7%
Particle identification	0.6%
Number of produced $\Upsilon(4S)$	1.5%
Fraction of $B^+B^-$ pairs	2.1%
Tracking efficiency	0.2%
Total	15.5%

Fractional contributions to the total relative systematic uncertainty of the  $B \rightarrow \tau\nu$  branching fraction

# $R(D^{(*)})$ measurement at Belle II

## $D$ decay modes

Decay mode	tag side	signal side
$D^0 \rightarrow K^- \pi^+ \pi^0$	✓	✓
$D^0 \rightarrow K^- \pi^+ \pi^+ \pi^-$	✓	✓
$D^0 \rightarrow K^- K^+ K_S^0$	✓	✓
$D^0 \rightarrow K^- K^+$	✓	✓
$D^0 \rightarrow K^- \pi^+$	✓	✓
$D^0 \rightarrow K_S^0 \pi^+ \pi^-$	✓	✓
$D^0 \rightarrow \pi^- \pi^+$	✓	✓
$D^0 \rightarrow K^- \pi^+ \pi^0 \pi^0$	✓	-
$D^0 \rightarrow K^- \pi^+ \pi^+ \pi^- \pi^0$	✓	-
$D^0 \rightarrow \pi^- \pi^+ \pi^0$	✓	-
$D^0 \rightarrow \pi^- \pi^- \pi^+ \pi^0$	✓	-
$D^0 \rightarrow \pi^- \pi^+ \pi^+ \pi^-$	✓	-
$D^0 \rightarrow K_S^0 \pi^0$	✓	-
$D^0 \rightarrow K_S^0 \pi^+ \pi^- \pi^0$	✓	-
$D^0 \rightarrow K^- K^+ \pi^0$	✓	-
<hr/>		
$D^+ \rightarrow K^- \pi^+ \pi^+$	✓	✓
$D^+ \rightarrow K_S^0 \pi^+ \pi^0$	✓	✓
$D^+ \rightarrow K_S^0 \pi^+ \pi^+ \pi^-$	✓	✓
$D^+ \rightarrow K_S^0 \pi^+$	✓	✓
$D^+ \rightarrow K^- K^+ \pi^+$	✓	✓
$D^+ \rightarrow K_S^0 K^+$	-	✓
$D^+ \rightarrow \pi^+ \pi^0$	✓	-
$D^+ \rightarrow K^- \pi^+ \pi^+ \pi^0$	✓	-
$D^+ \rightarrow \pi^+ \pi^+ \pi^-$	✓	-
$D^+ \rightarrow \pi^+ \pi^+ \pi^- \pi^0$	✓	-
$D^+ \rightarrow K^+ K_S^0 K_S^0$	✓	-
$D^0 \rightarrow K^- K^+ \pi^+ \pi^0$	✓	-

# $R(D^{(*)})$ measurement at Belle II

## Systematics budget

Fractional contributions to the total (relative)  
uncertainty of  $R(D)$  and  $R(D^*)$

Systematic Uncertainty	$\Delta\mathcal{R}(D^+)$	$\Delta\mathcal{R}(D^{*+})$
<b>Additive</b>		
MC sample size	0.033 (8.0%)	0.014 (4.7%)
Gap $\mathcal{B}$	0.027 (6.4%)	0.001 (0.1%)
LID efficiency ( $\mu$ )	0.022 (5.1%)	0.001 (0.1%)
Fake rates ( $e$ )	0.012 (2.9%)	0.003 (0.9%)
Continuum fraction	0.002 (0.6%)	0.001 (0.2%)
Gap FFs	0.002 (0.5%)	0.001 (0.2%)
$\bar{B} \rightarrow D^{(*)} \ell \bar{\nu}_\ell / \tau \bar{\nu}_\tau$ FFs	0.002 (0.5%)	0.002 (0.7%)
$\mathcal{B}(\bar{B} \rightarrow D^{**} \ell \bar{\nu}_\ell)$	0.002 (0.5%)	0.001 (0.1%)
$\bar{B} \rightarrow D^{**} \ell \bar{\nu}_\ell$ FFs	0.001 (0.3%)	0.001 (0.2%)
BDT modeling	0.001 (0.3%)	0.001 (0.2%)
LID efficiency ( $e$ )	0.001 (0.1%)	0.001 (0.2%)
Fake rates ( $\mu$ )	0.001 (0.1%)	0.001 (0.1%)
$\pi^\pm$ from $D^* \rightarrow D\pi$	0.003 (0.7%)	0.001 (0.1%)
<b>Total Additive Uncertainty</b>	<b>0.050 (12%)</b>	<b>0.015 (4.8%)</b>
<b>Multiplicative</b>		
$\bar{B} \rightarrow D^{(*)} \ell \bar{\nu}_\ell / \tau \bar{\nu}_\tau$ FFs	0.009 (2.1%)	0.011 (3.5%)
MC sample size	0.007 (1.7%)	0.004 (1.2%)
LID efficiency ( $e$ )	0.001 (0.2%)	0.001 (0.2%)
$\mathcal{B}(\tau^- \rightarrow \ell^- \bar{\nu}_\ell \nu_\tau)$	0.001 (0.2%)	0.001 (0.2%)
LID efficiency ( $\mu$ )	0.001 (0.1%)	0.001 (0.1%)
Tracking efficiency	0.001 (0.1%)	0.001 (0.1%)
$\pi^\pm$ from $D^* \rightarrow D\pi$	– (–)	0.001 (0.2%)
<b>Total Multiplicative Uncertainty</b>	<b>0.012 (2.8%)</b>	<b>0.011 (3.7%)</b>
<b>Total Syst. Uncertainty</b>	<b>0.051 (12%)</b>	<b>0.018 (6.2%)</b>
<b>Total Stat. Uncertainty</b>	<b>0.074 (18%)</b>	<b>0.034 (11%)</b>
<b>Total Uncertainty</b>	<b>0.090 (22%)</b>	<b>0.039 (13%)</b>