Controlling hadronic matrix elements in rare b-decays

Moriond EW – 23/03/2025

Méril Reboud

Based on 2206.03797, 2305.06301 [Gubernari, MR, van Dyk, Virto] and ongoing projects

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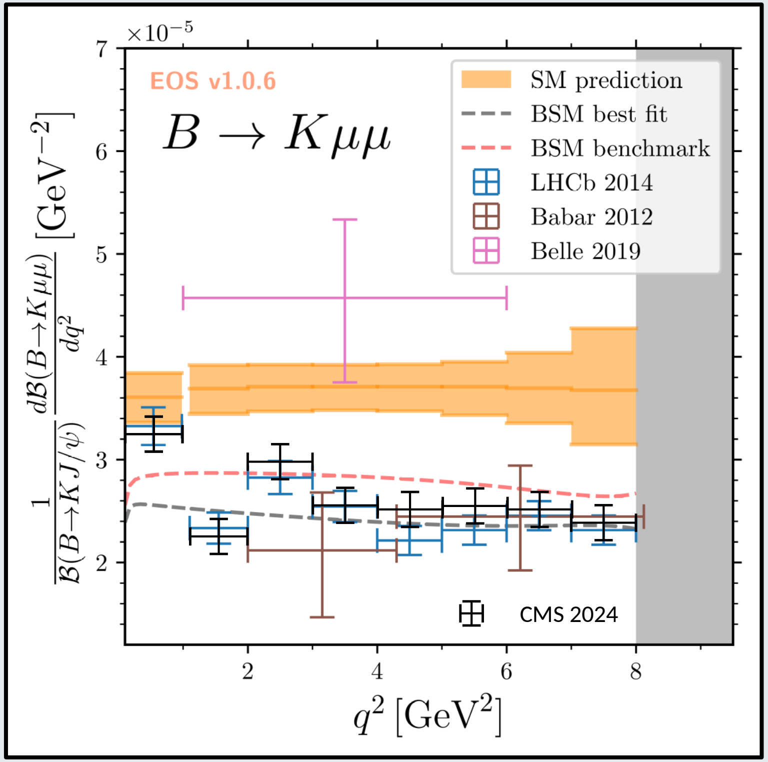
Rare b-decays

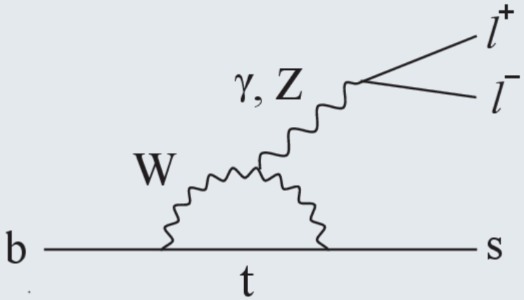
- LHCb and CMS measure b FCNC with an **unprecedented precision**:
  - Mesonic processes $B \to K^{(*)}\ell\ell$, $B_s \to \phi\ell\ell$
  - Baryonic processes $\Lambda_b \to \Lambda^{(*)}\ell\ell$
  - $b \to d$ transitions
- **Large tensions** are still observed
  - $> 4\sigma$ in $B \to K\mu\mu$ and $B_s \to \phi\mu\mu$...
- Hadronic matrix elements dominate the theory uncertainties
  - This talk: (How) can we reduce uncertainties?

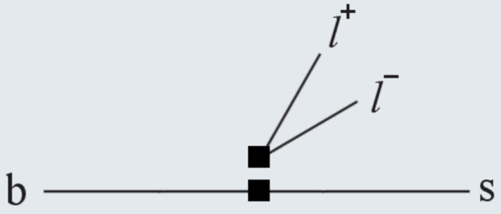


[Gubernari, MR, van Dyk, Virto, ‘22; LHCb ‘14; Babar ‘12; Belle ‘19; CMS ‘24]

Weak Effective Theory

- These processes take place at a scale $m_b < m_W, m_t$





- Allows for a model independent interpretation of the anomalies

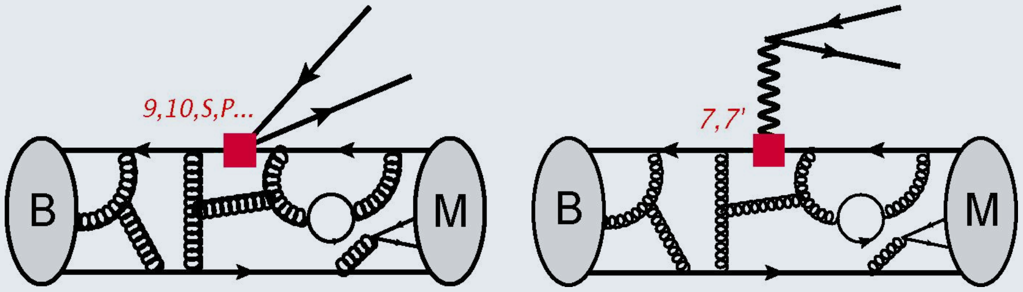
$$\mathcal{H}(b \to s\ell\ell) = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu)\mathcal{O}_i(\mu)$$

$$\mathcal{O}_{9(10)} = \frac{e^2}{16\pi^2}\left(\bar{s}_L \gamma_\mu b_L\right)\left(\bar{\ell}\gamma^\mu(\gamma_5)\ell\right)$$

$$\mathcal{O}_7 = \frac{e}{16\pi^2}\left(\bar{s}_L \sigma_{\mu\nu} b_R\right) F^{\mu\nu}$$

- Avoids the appearance of large logarithm in the calculations of observables

QCD in b → sℓℓ



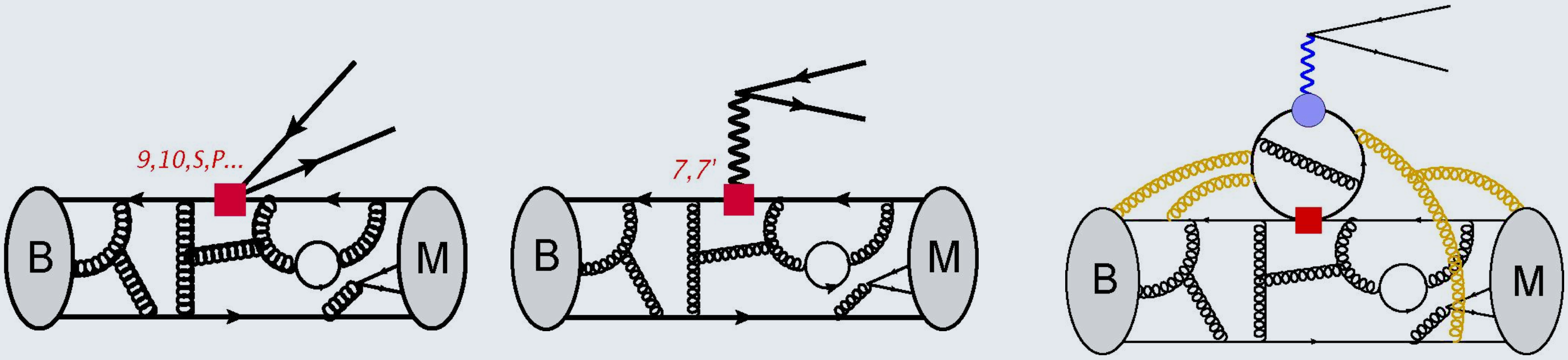
$$\mathcal{A}_\lambda^{L,R}(B \to M_\lambda \ell\ell) = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2m_b M_B}{q^2} \left[C_7 \mathcal{F}_\lambda^T(q^2) \right] \right\}$$

- B → $K^{(*)}$ μμ / ee
- B_s → φ μμ / ee
- Λ_b → $\Lambda^{(*)}$ μμ / ee

Local form-factors, involves e.g.

$$\mathcal{F}_\mu(k,q) = \langle \bar{M}(k) | \bar{s} \gamma_\mu b_L | \bar{B}(q+k) \rangle$$

QCD in $b \to s\ell\ell$



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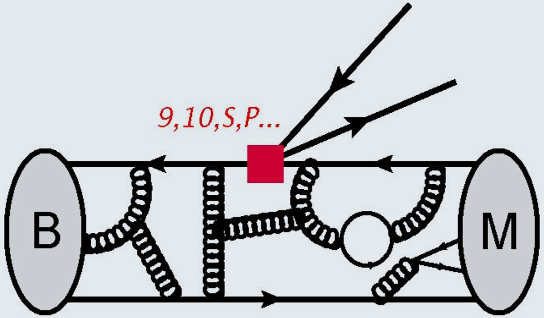
$$\mathcal{H}_\mu(k,q) = i \int d^4x \, e^{iq\cdot x} \langle \bar{M}(k) | T\{\mathcal{J}_\mu^{\mathrm{em}}(x), \mathcal{C}_i \mathcal{O}_i\} | \bar{B}(q+k) \rangle$$

Non-local form-factors

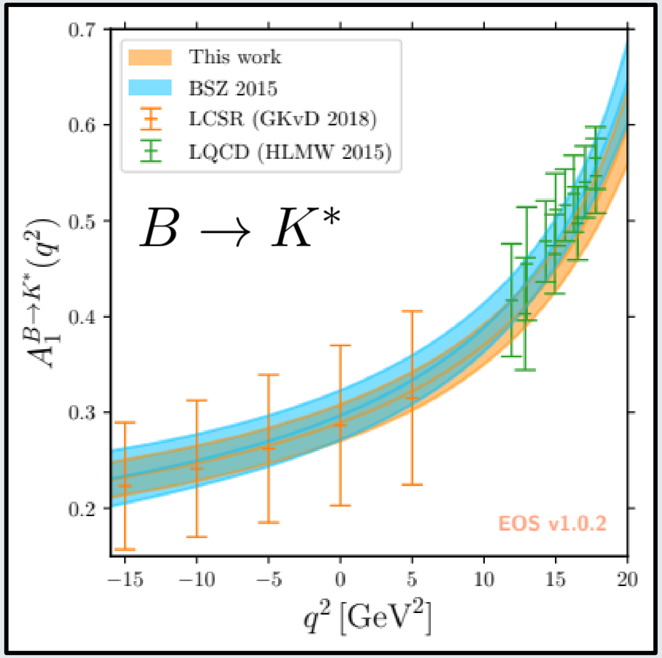
→ Main contributions: the “**charm-loops**” $\mathcal{O}_{2(1)}^c = (\bar{s}_L \gamma_\mu (T^a) c_L)(\bar{c}_L \gamma^\mu (T^a) b_L)$

I. Local Form Factors



Local form factors

- Conceptually easy, but still a dominant source of uncertainties
- 2 **complementary** approaches
  - Light-cone sum rules → most feasible at **small q^2**
    - Lengthy calculations
    - Requires experimental inputs (LCDAs)
    - Large (irreducible?) systematic uncertainties



[Gubernari, MR, van Dyk, Virto, '23; Horgan, Liu, Meinel, Wingate '15; Gubernari, Kokulu, van Dyk '18]

Local form factors

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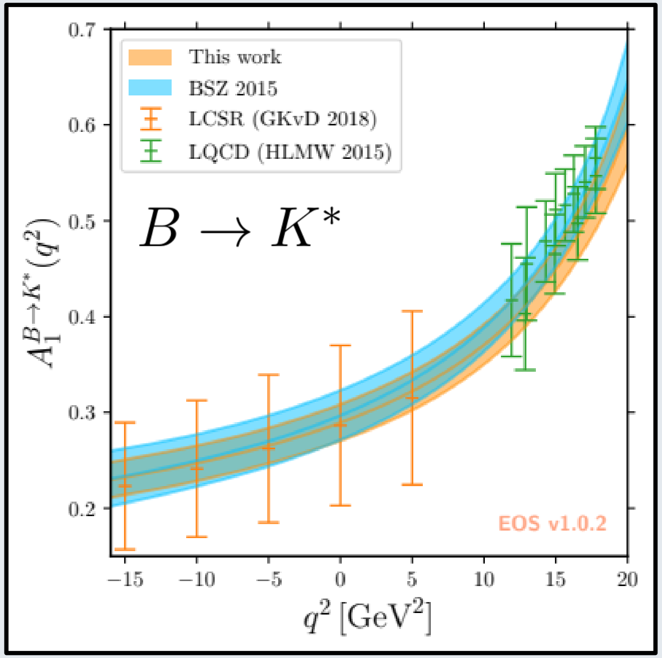
- 2 **complementary** approaches
  - Light-cone sum rules → most feasible at **small q^2**
  - Lattice QCD → most feasible at **large q^2**
    - CPU very expensive
    - Can now also probe small q^2 region
    - Difficulties with unstable mesons ρ, K*, D*, Λ*…



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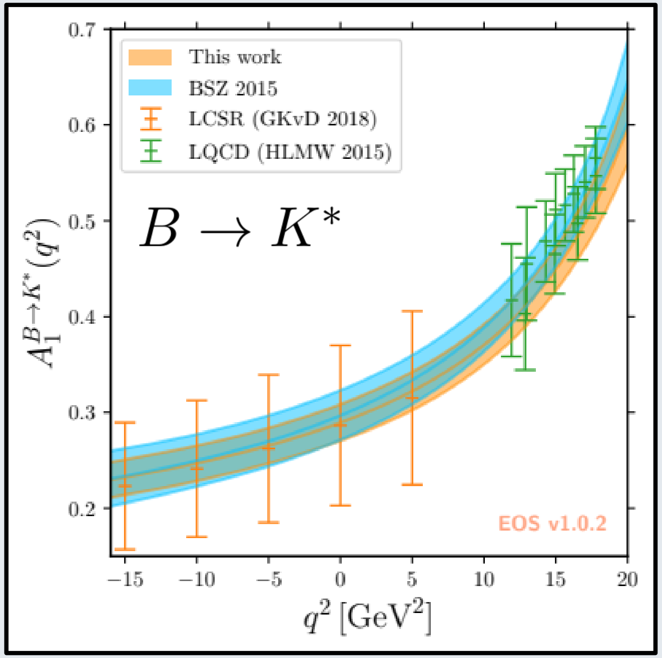
Local form factors

- Conceptually easy, but still a dominant source of uncertainties
- 2 **complementary** approaches
  - Light-cone sum rules → most feasible at **small q^2**
  - Lattice QCD → most feasible at **large q^2**
- Interpolation/Extrapolation requires a **parametrization**

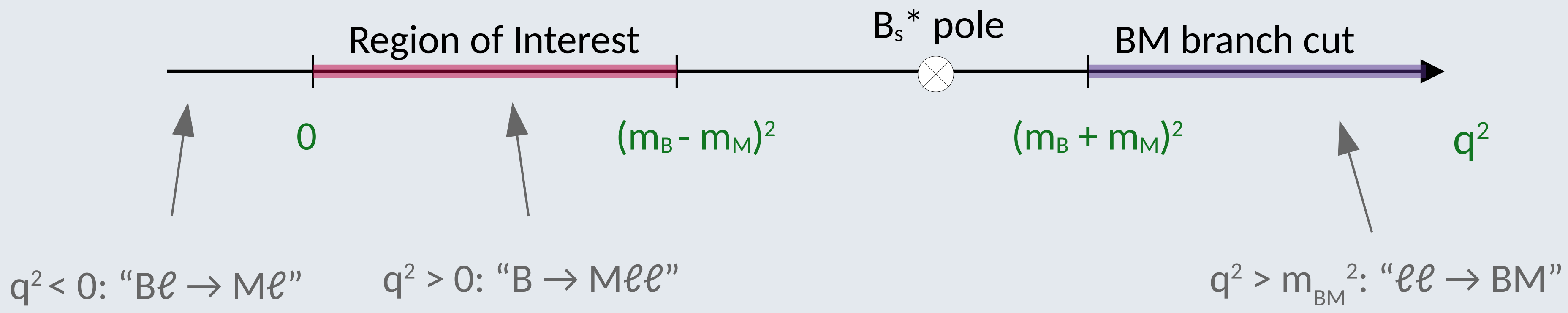
  → Adapt the parametrization to the **analytic properties** of the form factors



[Gubernari, MR, van Dyk, Virto, ‘23; Horgan, Liu, Meinel, Wingate ‘15; Gubernari, Kokulu, van Dyk ‘18]

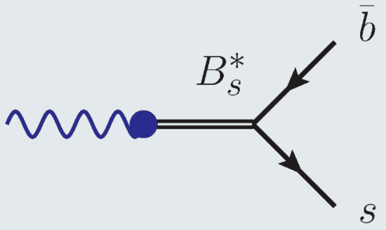
Form Factor Properties

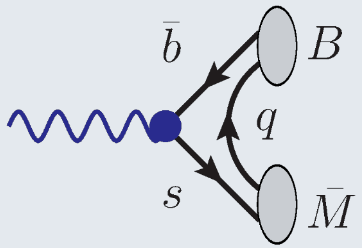
$$\mathcal{F}_\mu(k,q) = \langle \bar{M}(k)|\bar{s}\gamma_\mu b_L|\bar{B}(q+k)\rangle$$



Analytic properties of the form factors:

- Pole due to $\bar{b}s$ **bound state**
- **Branch cut** due to on-shell BM production





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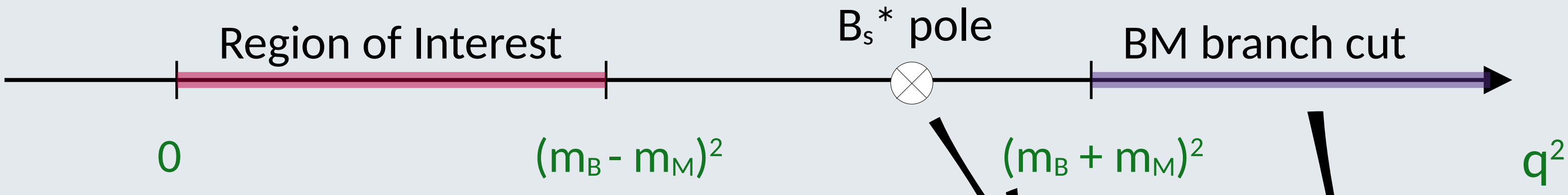

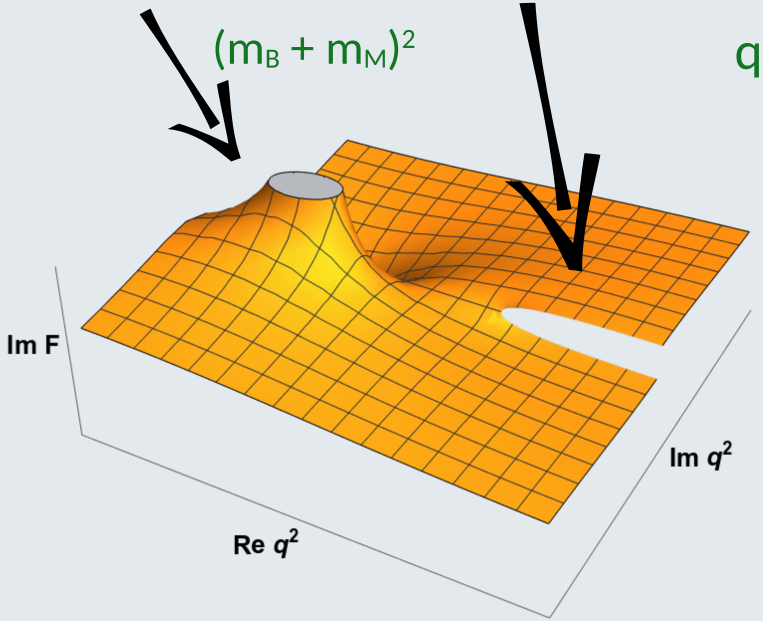

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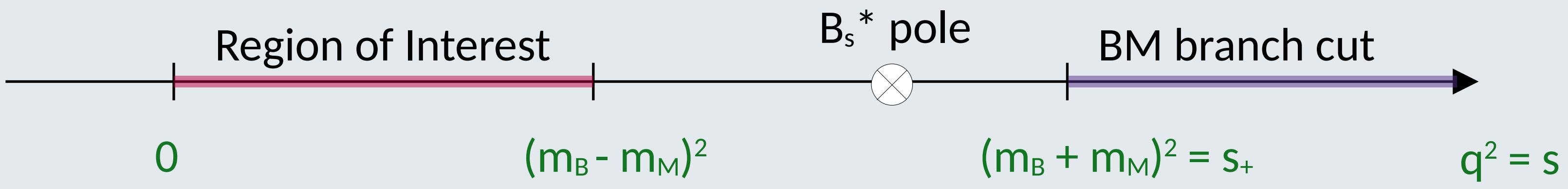
Form Factor Parametrization



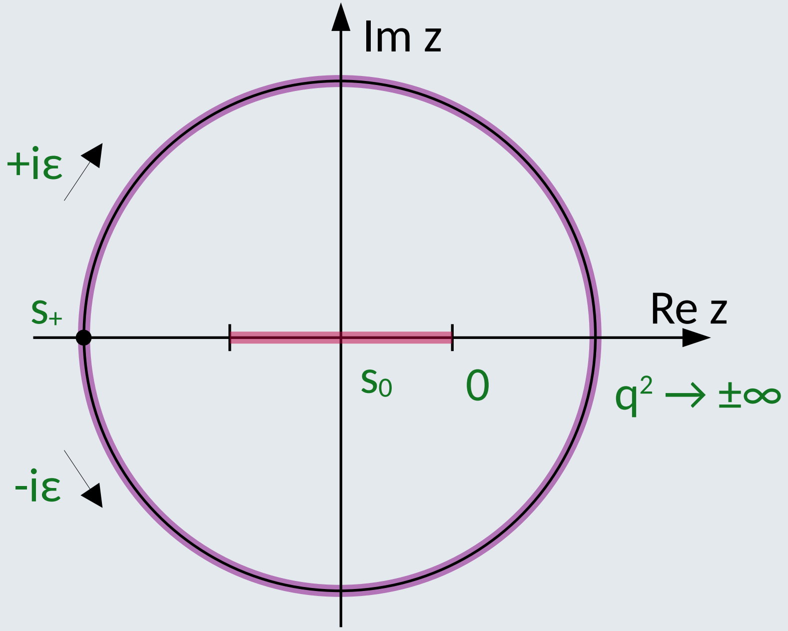
Conformal mapping [Boyd, Grinstein, Lebed '97]

$$z(s) \equiv \frac{\sqrt{s_+ - s} - \sqrt{s_+ - s_0}}{\sqrt{s_+ - s} + \sqrt{s_+ - s_0}}$$

Simplified Series expansion [Bourrely, Caprini, Lellouch, '08; Bharucha, Feldmann, Wick '10]

$$\mathcal{F}_\lambda^{(T)}(q^2) = \frac{1}{q^2 - m_{B_s^*}^2} \sum_{k=0}^{N} \alpha_{\lambda,k} z^k$$

N = 2 is usually enough to provide an **excellent description of the data** (p-values > 70%), but what about the *truncation error*?



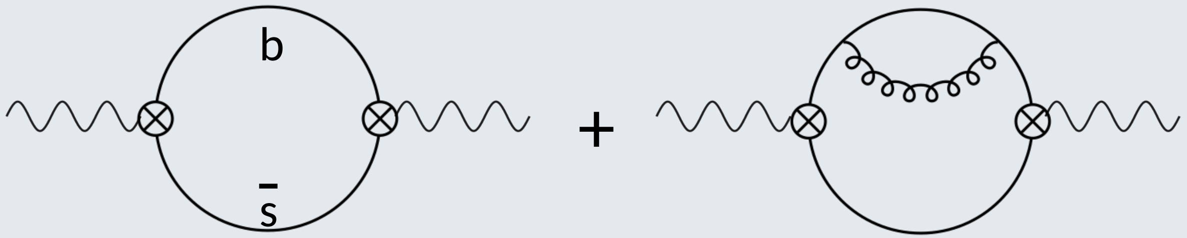
Dispersive bounds

- **Main idea:** Compute the inclusive $e^+e^- \to \bar{b}s$ cross-section and relate it to the form factors [Bharucha, Feldmann, Wick '10]

$$\Pi^{\mu\nu}_{\Gamma}(q) \equiv i \int d^4x \, e^{iq\cdot x} \, \langle 0 | \mathcal{T} \left\{ J^{\mu}_{\Gamma}(x) J^{\dagger,\nu}_{\Gamma}(0) \right\} |0\rangle$$

<u>1) Partonic calculation</u>

Insertion of a scalar, vector or tensor current →



Also done on the lattice for b → c now! [Martinelli et al '21; Harrison '24]

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$$\Pi_\Gamma^{\mu\nu}(q) \equiv i \int d^4x\, e^{iq\cdot x} \langle 0|\mathcal{T}\left\{ J_\Gamma^\mu(x) J_\Gamma^{\dagger,\nu}(0)\right\}|0\rangle$$

2) Relation to form factors

$$\mathrm{Im}\,\Pi_I^X(q^2) = \frac{1}{2}\sum_\Gamma \int d\rho_\Gamma\,(2\pi)^4\,\delta^4(q-p_\Gamma)\,P_I^{\mu\nu}\,\langle 0|\,j_\mu^X\,|\Gamma\rangle\langle\Gamma|\,j_\nu^{\dagger X}\,|0\rangle$$

~ |form factor|^2

Sum over all the $\bar{s}b$ states: $\bar{B}_s$, $\bar{B}K$, $\bar{B}K^*$, $\bar{B}K\pi$, baryons…

Dispersive bounds

- **Main idea:** Compute the inclusive $e^+e^- \to \bar{b}s$ cross-section and relate it to the form factors [Bharucha, Feldmann, Wick '10]

- Assuming global quark-hadron duality we have

$$\chi_\Gamma^{(\lambda)}\big|_{\mathrm{OPE}} = \chi_\Gamma^{(\lambda)}\big|_{\mathrm{1pt}} + \chi_\Gamma^{(\lambda)}\big|_{\bar{B}K} + \chi_\Gamma^{(\lambda)}\big|_{\bar{B}K^*} + \chi_\Gamma^{(\lambda)}\big|_{\bar{B}_s\phi} + \dots\,.$$

Known terms

Sum of positive quantities

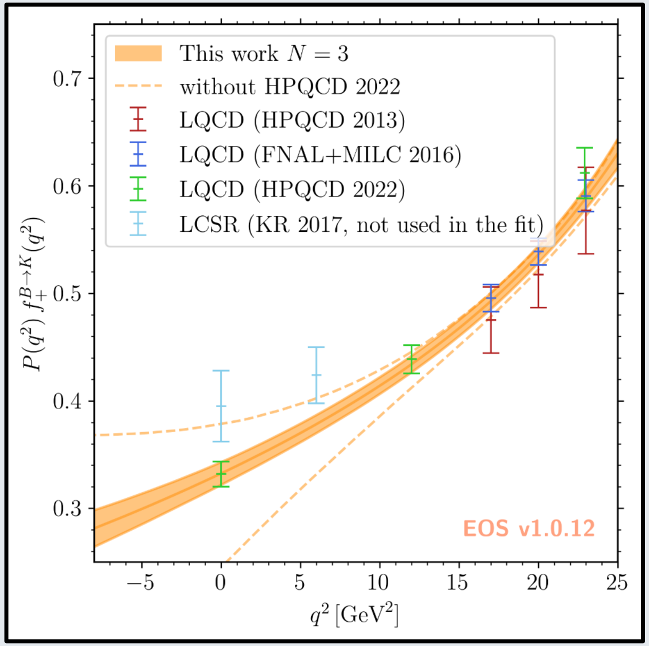
Further contributions such as $B \to K\pi\pi$ or $\Lambda_b \to \Lambda^{(*)}$.

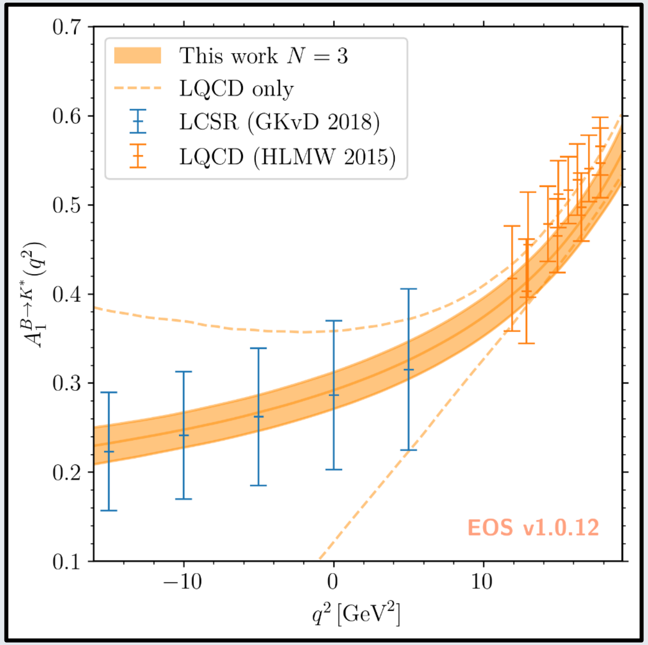
Any new terms *strengthens* the bound.

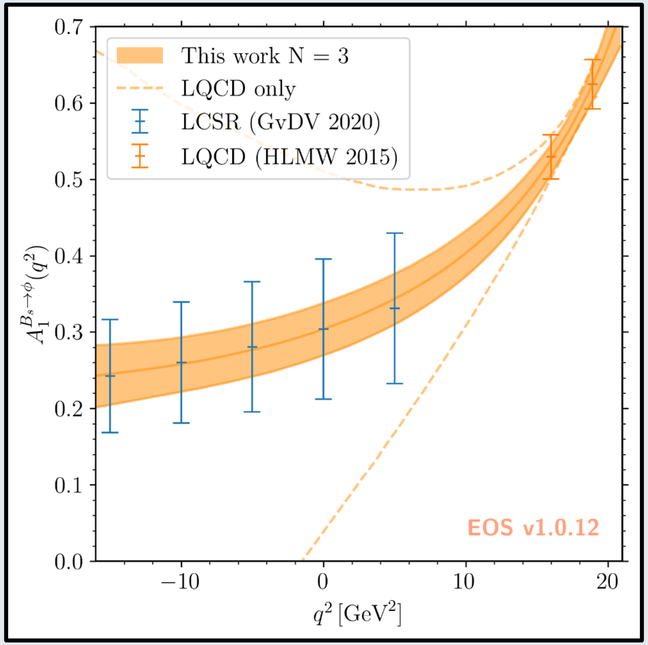
Results for mesonic form-factors

Global fit of $B \to K$, $B \to K^*$ and $B_s \to \phi$

- Fits are very good already at N = 2 (**p-values > 77%**)
- LCSR and LQCD combine nicely but still dominate the uncertainties
- Progresses in LQCD will gradually replace LCSR







II. Beyond narrow-width approximation

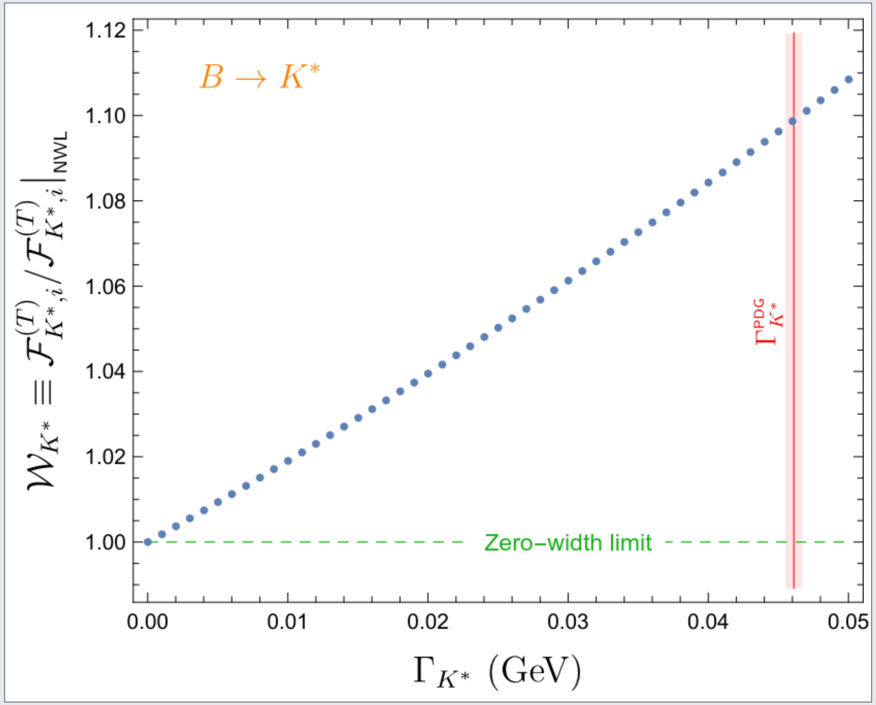
Caveat: finite width effects in B → K*

- $\Gamma_{K^*} / M_{K^*} \sim 5\%$ is not very small

- **Finite width effects** have to be accounted for in the LQCD and LCSR calculations
  - Universal 20% correction to the observables [Descotes-Genon, Khodjamirian, Virto '19]
  - Computable in LQCD [Leskovec '24]

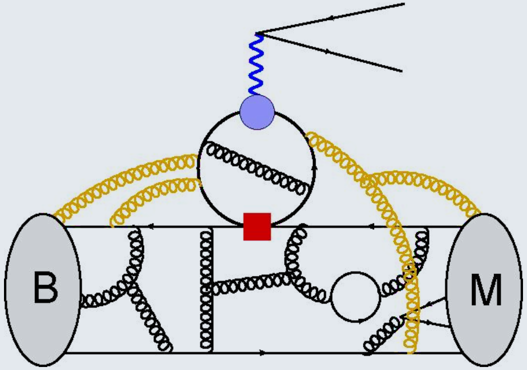
- B → Kπμμ decays also have a large **S-wave component** [LHCb '16]
  - LCSR inputs for the S-wave are now available [Descotes-Genon, Khodjamirian, Virto, Vos '23]

- Need for a generic parametrization for B → Kπ form factors [Gustafson, Herren *et al* '23, Herren, Kubis *et al* '25]



[Descotes-Genon, Khodjamirian, Virto '19]

III. Non Local Form Factors



Long story short

1) The contribution is **dominated by the charm loops** due to O_{1c} and O_{2c}

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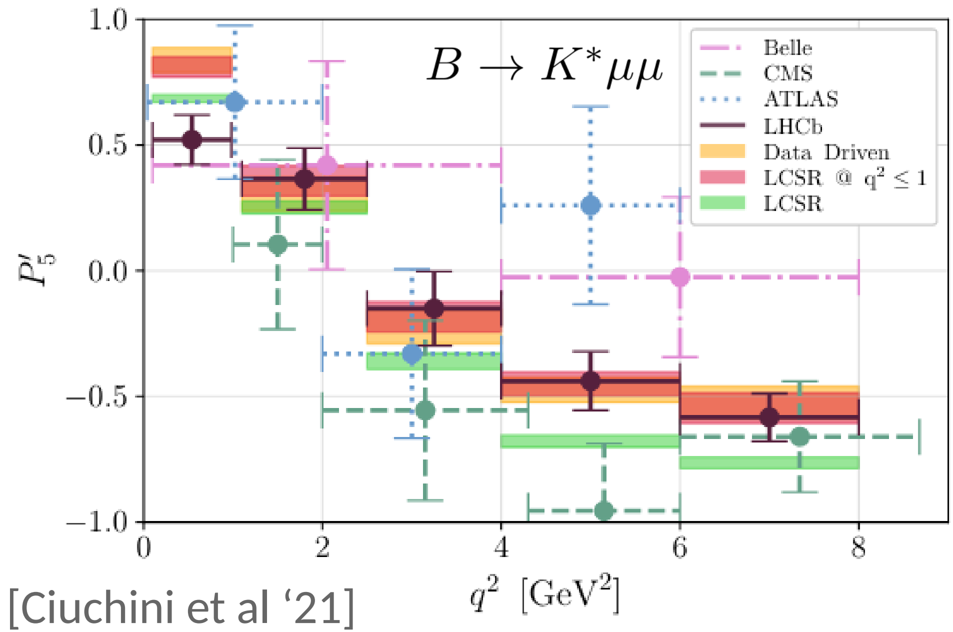
2) The contribution **mimics new physics** by shifting C_9

→ *Pure* data-driven approaches *can't resolve SM and NP* [Ciuchini *et al* '21, '22]



[Ciuchini et al '21]

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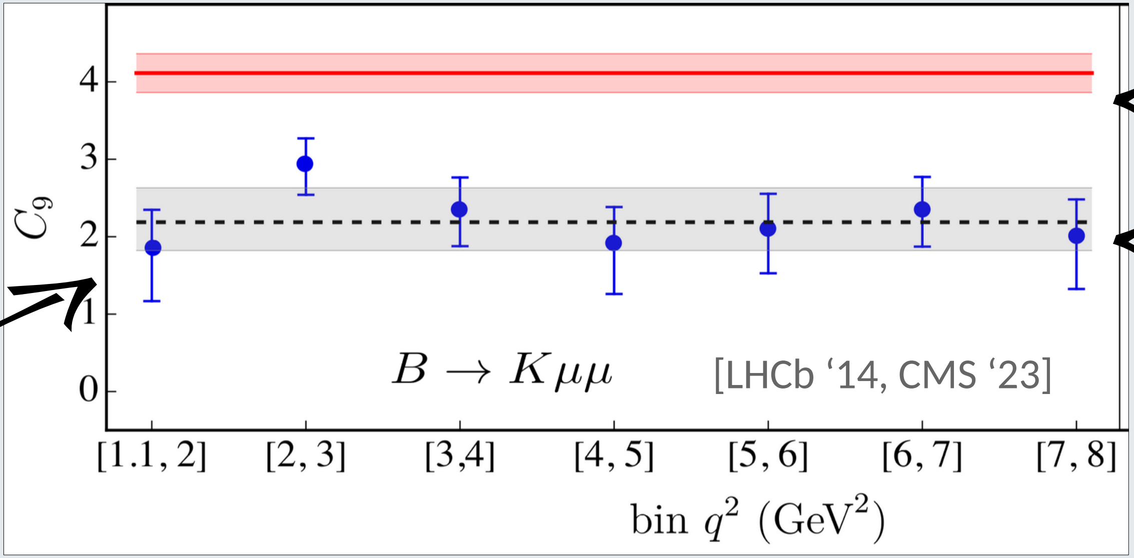
3) Moriond 2025 status:
   - No visible q^2 dependence for NP [Bordone, Isidori, Maechler, Tinari '24]

At low q^2:



SM prediction

Constant-C_9 fit

Data from LHCb and CMS

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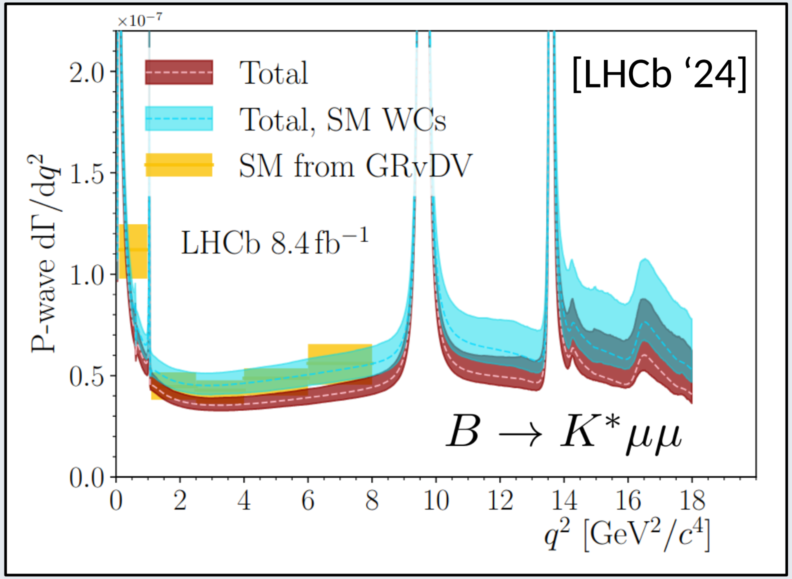
   - Assuming a simple analytic structure, charm loops are small [Gubernari, MR, van Dyk, Virto '22, LHCb '24]



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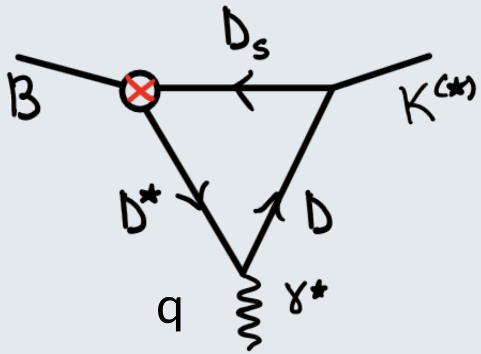
   - ▻ Mesonic calculations hint at possible rescattering effects [Ciuchini *et al* '22, Mutke, Hoferichter, Kubis '24; Isidori, Polonsky, Tinari '24]



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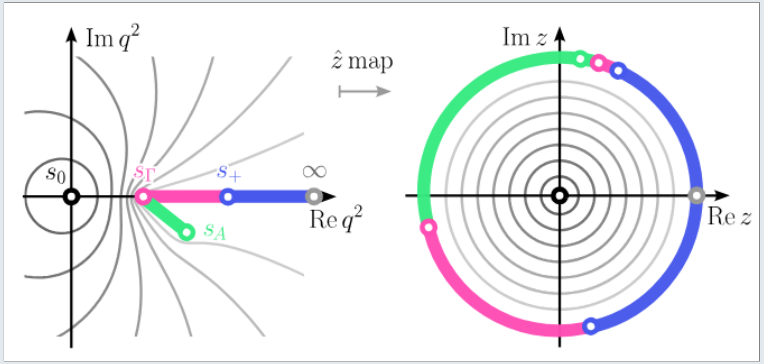
   - Mesonic calculations hint at possible rescattering effects [Ciuchini *et al* '22, Mukte, Hoferichter, Kubis '24; Isidori, Polonsky, Tinari '24]
   - Analytic parametrizations can, in principle, account for *anomalous thresholds* [Gopal, Gubernari '24]

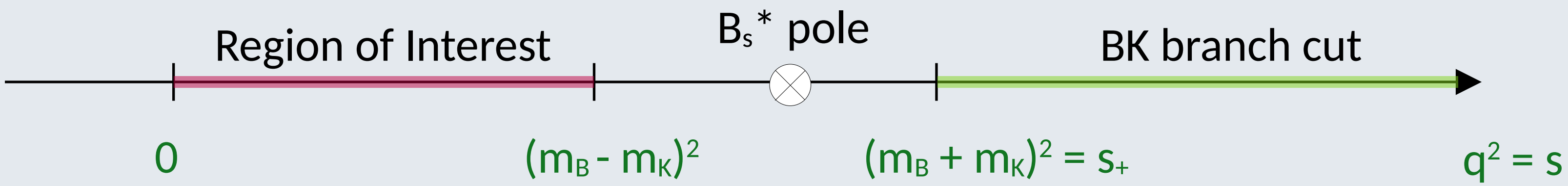


Conclusion

- FCNC are notoriously **hard to predict**
  - Local form-factors are only well-known for a couple of transitions
  - The size of the charm loops is still under investigation
- Recent progresses on lattice QCD, specific calculations and analytic constraints allow for **large numerical analyses** that benefit from unitarity constraints
- **Upcoming data** will be crucial to (1) validate these approaches and (2) further constrain the matrix elements

Back-up

Simple case: B → K

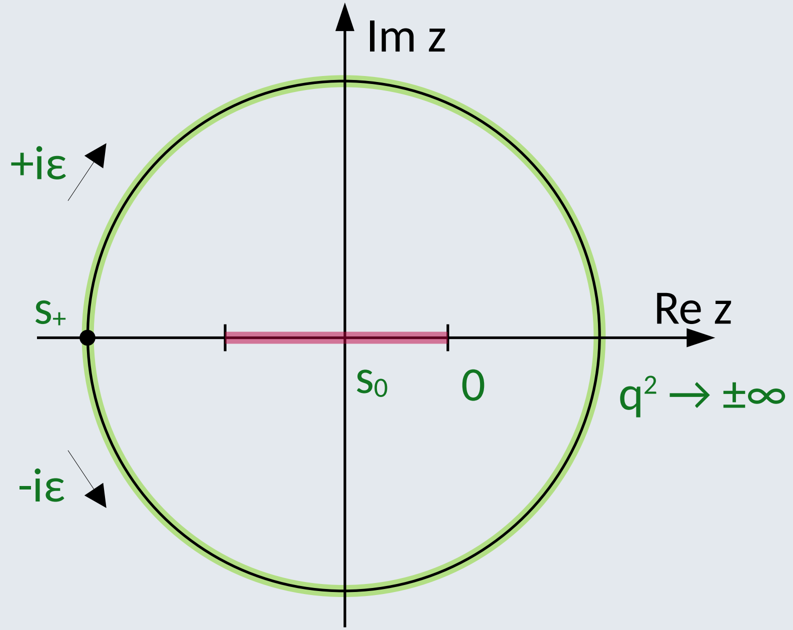


- The branch cut starts **at** the pair production threshold (neglecting $B_s\pi$)
- The monomial z^k are **orthogonal** on the unit circle

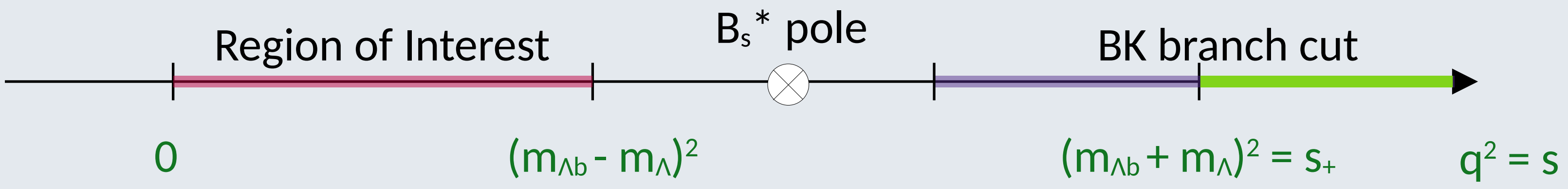
$$\mathcal{F}^{B\to K} = \frac{1}{\mathcal{P}(z)\phi(z)} \sum_{k=0}^{N} \alpha_k z^k$$

Known functions

$$\chi_\Gamma^{(\lambda)}|_{\bar{B}K} = \sum_{k=0}^{N} |\alpha_k|^2$$



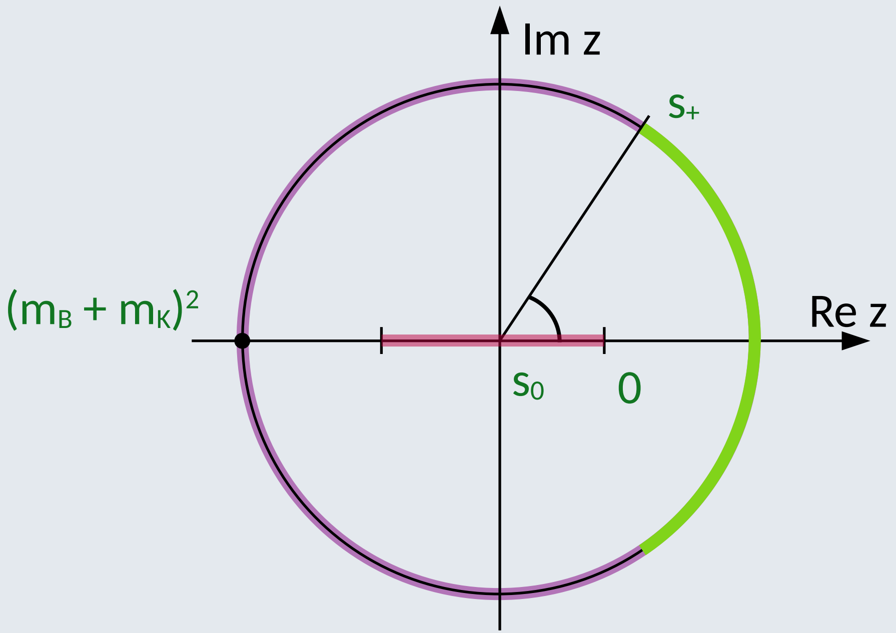
Less simple case, e.g. $\Lambda_b \to \Lambda$



- The first branch cut (BK) starts **before** the pair production threshold
- Introduce orthonormal polynomials of the **arc of the unit circle**

$$\mathcal{F}^{\Lambda_b \to \Lambda} = \frac{1}{\mathcal{P}(z)\phi(z)} \sum_{k=0}^{N} \alpha_k p_k(z)$$

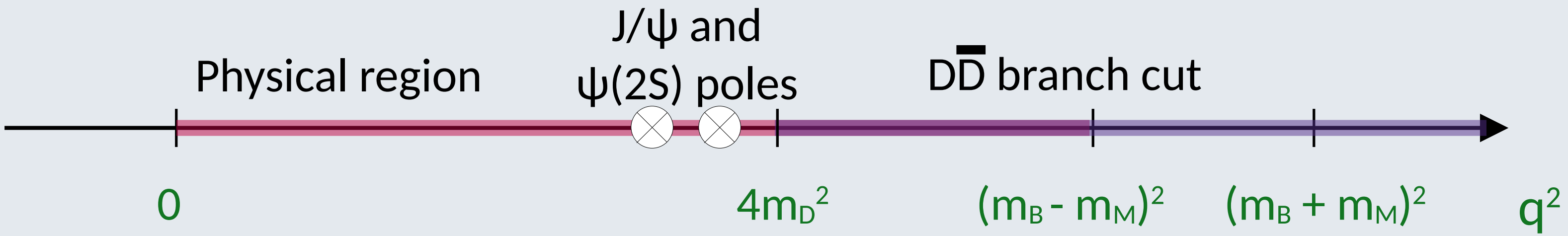
- (Or still expand in z and deal with a more complicated bounds [Flynn, Jüttner, Tsang '23])



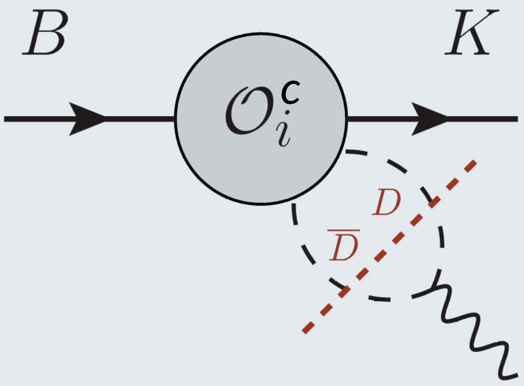
Local form factors fit

- With this framework we perform a **combined fit** of $B \to K$, $B \to K^*$ and $B_s \to \varphi$ LCSR and lattice QCD inputs:
  - $B \to K$:
    - [HPQCD ’13 and ’22; FNAL/MILC ’17]
    - ([Khodjamiriam, Rusov ’17]) → large uncertainties, not used in the fit
  - $B \to K^*$:
    - [Horgan, Liu, Meinel, Wingate ’15]
    - [Gubernari, Kokulu, van Dyk ’18] (B-meson LCSRs)
  - $B_s \to \varphi$:
    - [Horgan, Liu, Meinel, Wingate ’15]
    - [Gubernari, van Dyk, Virto ’20] (B-meson LCSRs)
- Adding $\Lambda_b \to \Lambda^{(*)}$ form factors is possible and desirable

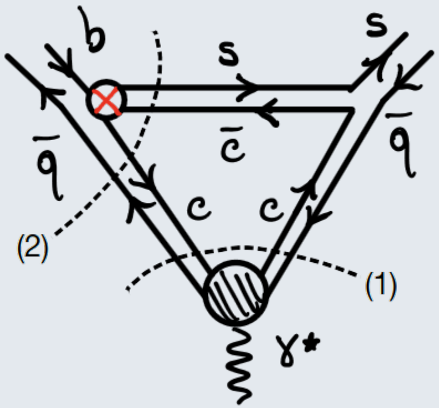
Analyticity properties of H_μ

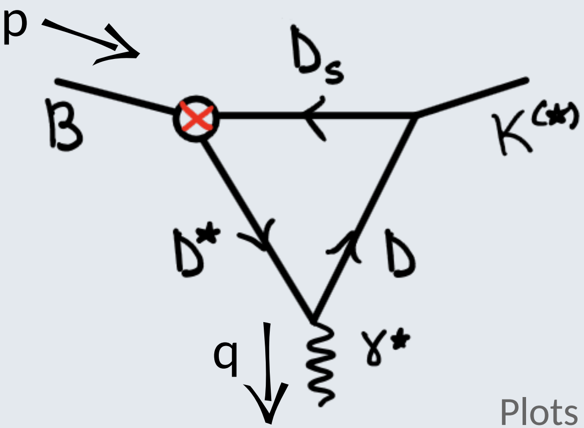


- Poles due to the narrow charmonium resonances
- Branch-cut starting at $4m_D^2$



More involved analytic structure?





Plots from [Ciuchini et al. '22]

- $M_B > M_{D^*} + M_{D_s}$ → The function $H_\lambda(p^2,q^2)$ has a branch cut in p^2 and the physical decay takes place on this branch cut: **H_λ is complex-valued!**
- Triangle diagrams are known to create *anomalous* branch cuts in q^2 [e.g. Lucha, Melikhov, Simula '06] → Not clear if it happen here (no Lagrangian nor power counting)
- Models *implementing* these diagrams find their contribution to be O(10%) [Isidori, Polonsky, Tinari '24; Mutke, Hoferichte, Kubis '24]

GRvDV parametrization

- Nonlocal form factors are expanded using **orthonormal polynomials** of the arc of the unit circle [Gubernari, MR, van Dyk, Virto ‘22]:

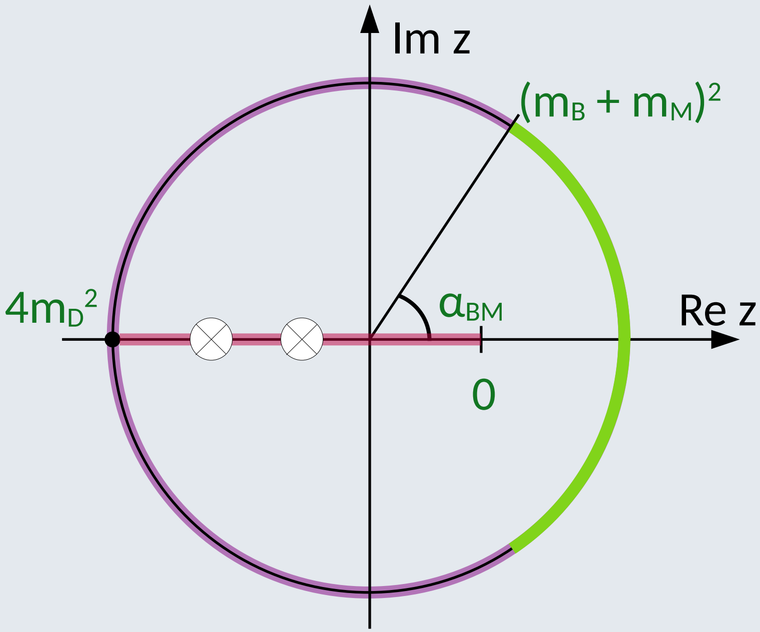
$$\mathcal{H}_\lambda(z) = \frac{1}{\phi(z)\mathcal{P}(z)} \sum_{k=0}^{N} a_{\lambda,k}\, p_k(z)$$

- The coefficients respect a **simple bound** [Gubernari, van Dyk, Virto ‘20]:

$$\sum_{n=0}^{\infty} \left\{ 2\left|a_{0,n}^{B\to K}\right|^2 + \sum_{\lambda=\perp,\parallel,0} \left[2\left|a_{\lambda,n}^{B\to K^*}\right|^2 + \left|a_{\lambda,n}^{B_s\to\phi}\right|^2 \right] \right\} < 1$$

- The series converges on an arc of the unit circle but the convergence is slow and useless in practice

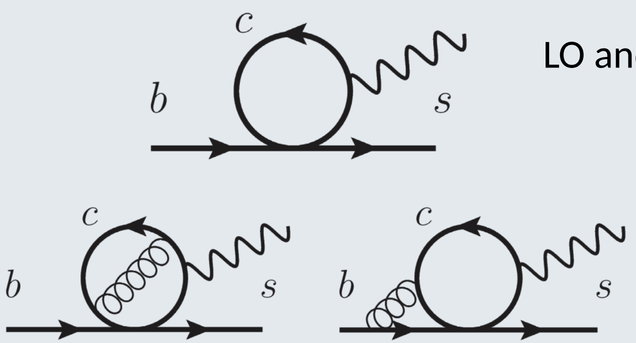
$$z(s) = \frac{\sqrt{4m_D^2 - s} - \sqrt{4m_D^2 - s_0}}{\sqrt{4m_D^2 - s} + \sqrt{4m_D^2 - s_0}}$$



Theory inputs

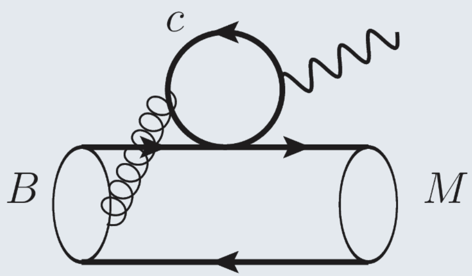
$\mathcal{H}_\lambda$ can be calculated in **two kinematics regions**:

- **Local** OPE $|q|^2 \gtrsim m_b^2$ [Grinstein, Piryol '04; Beylich, Buchalla, Feldmann '11]
- **Light Cone** OPE $q^2 \ll 4m_c^2$ [Khodjamirian, Mannel, Pivovarov, Wang '10]



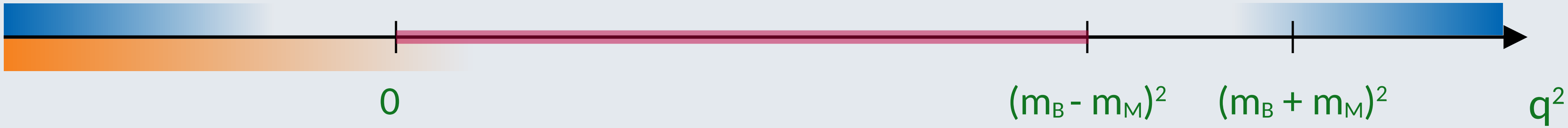
LO and α_s corrections

[Asatrian, Bieri, Greub, Walker '04; de Boer '17; **Asatrian, Greub, Virto '19**]



Non-perturbative soft gluon corrections

[Khodjamirian, Mannel, Pivovarov, Wang '10; **Gubernari, van Dyk, Virto '20**]



- Still focusing on $B \to K$, $B \to K^*$ and $B_s \to \varphi$

  Inputs:

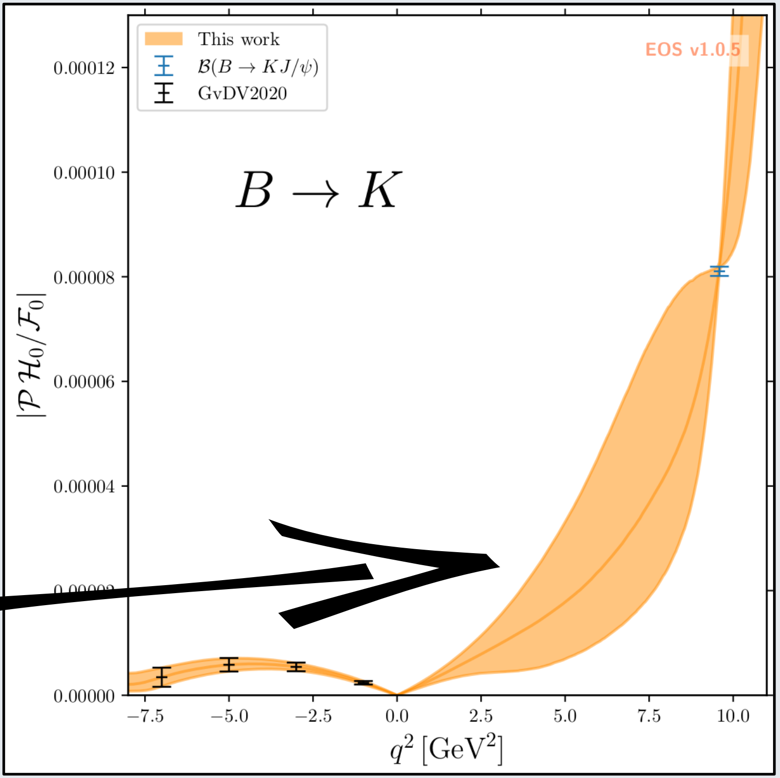
  - 4 theory point at negative q^2 from the light cone OPE
  - Experimental results at the J/ψ (we keep $\psi(2S)$ for future work)

- Use an under-constrained fit (N = 5) and allows for saturation of the dispersive bound

  → The uncertainties are **truncation order independent**, increasing the expansion order does not change their size

  → All p-values are larger than 11%

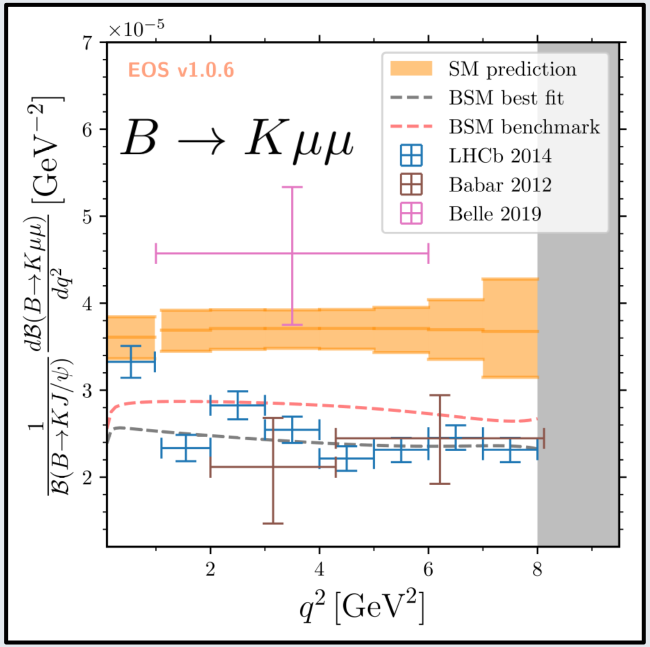
[Gubernari, MR, van Dyk, Virto '22]

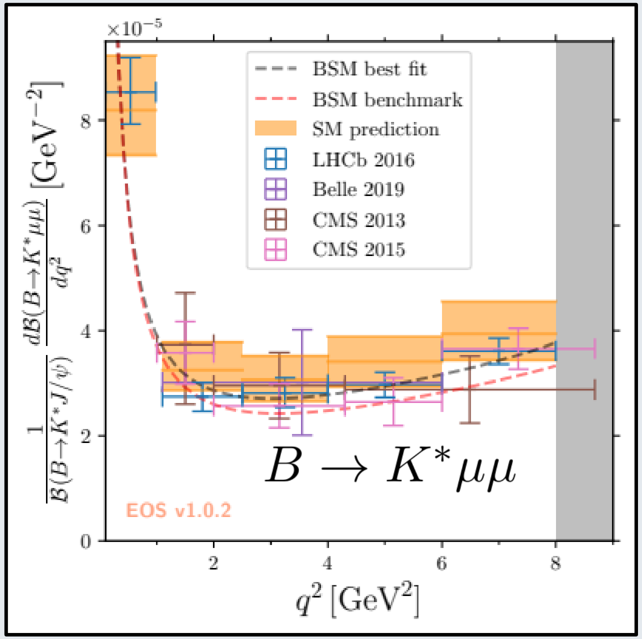


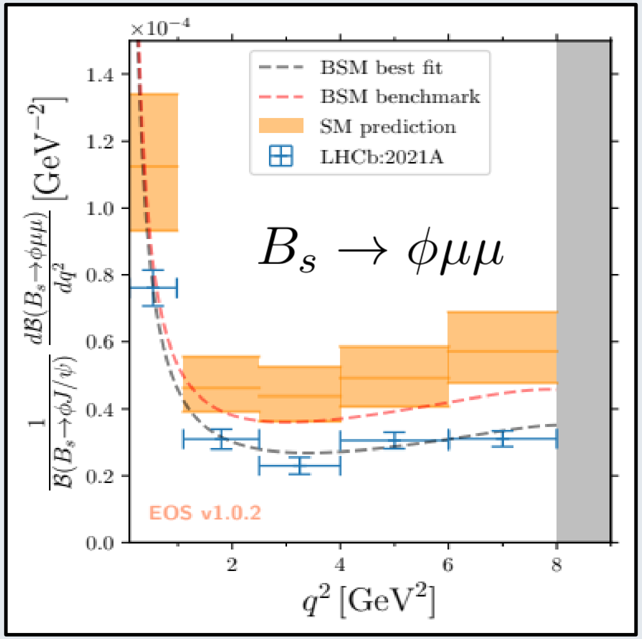
Confrontation with data

- This approach of the non-local form factors **does not solve the "B anomalies"**.
- In this approach, the greatest source of theoretical uncertainty now comes from **local form factors**.

Experimental results:
[Babar: 1204.3933; Belle: 1908.01848, 1904.02440; ATLAS: 1805.04000, CMS: 1308.3409, 1507.08126, 2010.13968, LHCb: 1403.8044, 2012.13241, 2003.04831, 1606.04731, 2107.13428]







Additional plots can be found in the paper: 2206.03797