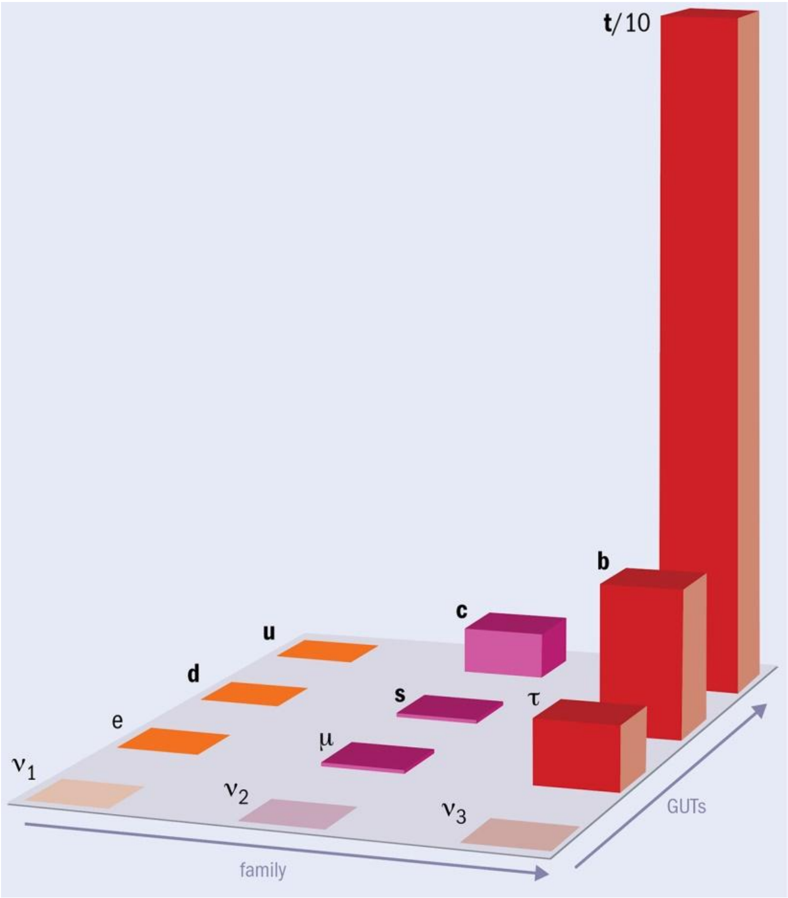
Flavour patterns from Entanglement Minimization?

Sokratis Trifinopoulos

Moriond, Electroweak
24 March 2025

The flavor sector of the Standard Model



- The SM (+ GR) are arguably our most celebrated intellectual achievements in fundamental science.
- It is a *gauge theory* with **19** (+7 for the νSM) input parameters

  leads to **thousands** of accurate predictions!
- 13(+7) of these parameters concern the *flavor* sector:
  - 9(+3) fermion masses
  - 4(+4) mixing parameters
- The mixing parameters are organized in the Cabibbo–Kobayashi–Maskawa (**CKM**) and Pontecorvo–Maki–Nakagawa–Sakata (**PMNS**) matrices, each parametrized by three angles $\theta_{12}, \theta_{13}, \theta_{23}$ and a CP-violating phase δ.

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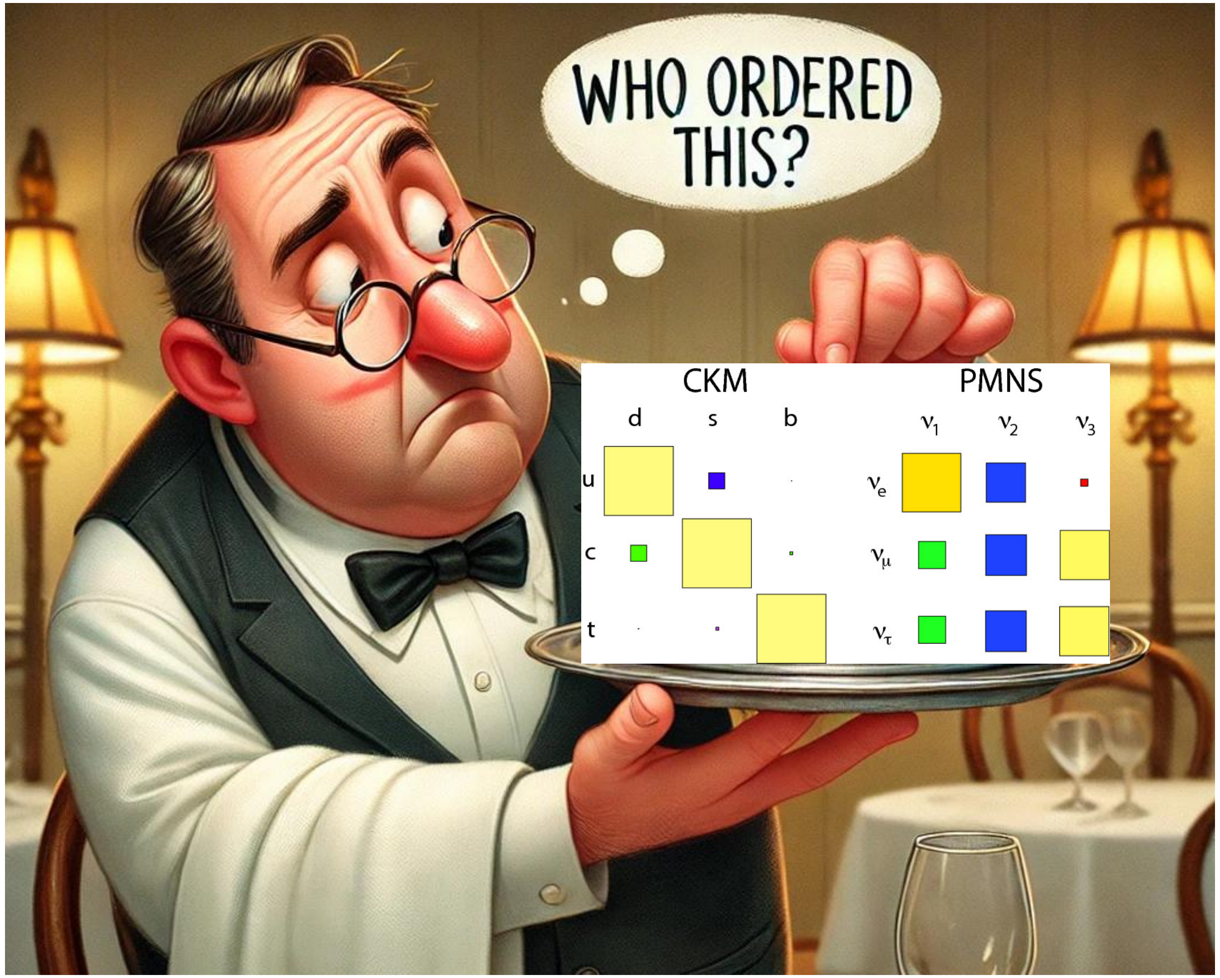
But flavor seems ad-hoc!



- The mixing angles for quark flavors are hierarchical, i.e. the CKM is almost diagonal:

$$45^\circ \gg \theta_{\mathrm{CKM},12} > \theta_{\mathrm{CKM},13} \approx \theta_{\mathrm{CKM},23} \approx 0$$

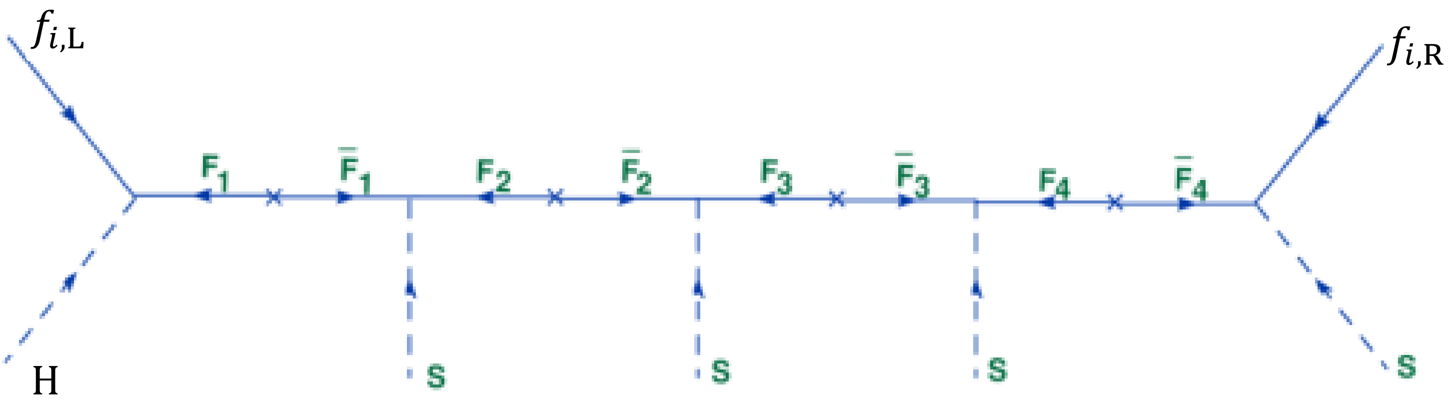
- The parameters of the neutrino mixing appear to be of comparable size and no new relation is known among them, i.e. the PMNS appears to be anarchic:

$$45^\circ > \theta_{\mathrm{PMNS},12} \sim \theta_{\mathrm{PMNS},23} > \theta_{\mathrm{PMNS},13} \gg 0$$

Traditional approach: Flavor symmetries

- Assume that there is an exact symmetry in the UV, which appears broken in the IR.

- Archetypical example: Froggatt-Nielsen $U(1)$



f_i, H: SM
S: flavon
F_i: heavy new fermions

- The mechanism yields a mass term: $O(1)\varepsilon^{Q_i+Q_j} f_{i,L} f_{j,R} H$ with $\varepsilon = \frac{\langle S \rangle}{M_F}$ (spurion).

❖ Advantage: working **within** an established paradigm, *i.e.* **QFT**s with broken symmetries.

❖ Drawbacks: **i)** new UV degrees of freedom (often) lie beyond experimental reach **ii)** *conservation of free parameters* **iii)** spurion analysis of CKM is incompatible with PMNS.

What if there is another way?

…to reduce the SM input parameters without new symmetries in the UV or/and new heavy particles?

[Thaler, **Trifinopoulos**]
2410.23343

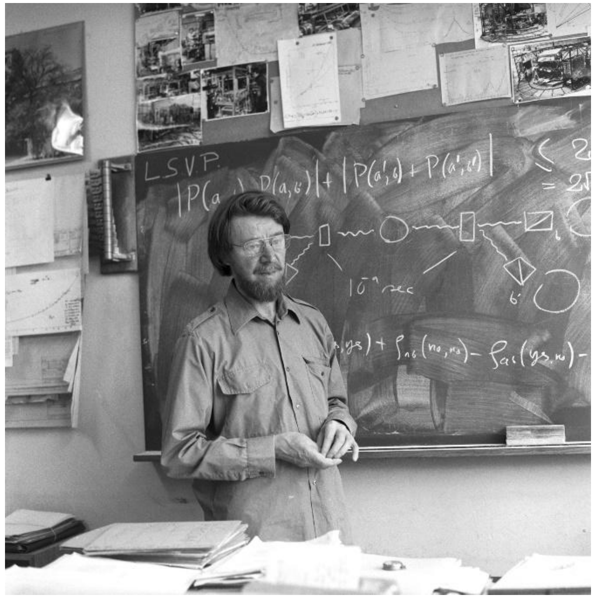
- What we have (so far): Numerical observations (from various fronts) that may hint towards a new principle:

The quantum entanglement generated in $2 \to 2$ elastic fermion scattering induced by electroweak interactions is minimized when the flavor parameters assume (roughly) their νSM values.

- What we don't have (yet):
  i) Any *fundamental* justification for this principle,
  ii) a unique choice of entanglement measure.

Quantum Entanglement

- Another fundamental physical *resource* is: **entanglement.** Similarly to energy, it is a tangible measurable quantity that can be transferred, stored, and consumed.
- *What **is** entanglement?*



1. a property of (at least) two particles: the quantum state of each particle cannot be described independently of the state of the others no matter the distance between them.
   - If two particles A and B get entangled, then: $|\psi_{\mathrm{AB}}\rangle \neq |\psi_{\mathrm{A}}\rangle \otimes |\psi_{\mathrm{B}}\rangle$ (*non-seperable*)
2. inherently quantum & non-local: there is no classical equivalence as proven by **Bell's theorems**; the correlations exist even when the measurements are space-like separated!
3. a carrier of information: central to QIS tasks like quantum teleportation & cryptography.

Measures of entanglement (states)

- Quantum information (or better lack thereof) is quantified by the

  von Neuman entropy: $S[\rho] = -\mathrm{Tr}(\rho \log \rho)$, ($S[\rho] = 0$ for pure states)

- Entanglement is quantified by the information contained in the subsystems via the

  Entanglement entropy: $S_E[\rho] = -\mathrm{Tr}\ (\rho_R \log \rho_R)$, ($\rho_R = \mathrm{Tr}_A\rho$ or $\mathrm{Tr}_B\rho$, for bipartite systems)

- $S_E[\rho]$ is a formal *measure of entanglement*. For pure states it is the unique measure (every other is monotonically related to it). [Plenio,Virmani] quant-ph/0504163

- A more convenient quantity to characterize entanglement of pure states (*entanglement witness*) is the

  Linear entropy: $E[\rho] = \frac{d}{d-1}\left|1 - \mathrm{Tr}\ \rho_R^2\right|$, $(0 \leq E[\rho] \leq 1)$

  d → Hilbert space dim

  0 → separable

  1 → maximally entangled (Bell states)

Measures of entanglement (operators)

- How is entanglement generated at the fundamental level? ➡ scattering & decay processes!
- Scattering is described by means of the unitary $\mathcal{S}$ operator that connects the Fock spaces $\mathcal{F}$ of the incoming and outgoing asymptotic states: $|\mathrm{out}\rangle = \mathcal{S}\,|\mathrm{in}\rangle$.

[Balasubramanian et al] 1108.3568
[Peschanski, Seki] 1602.00720

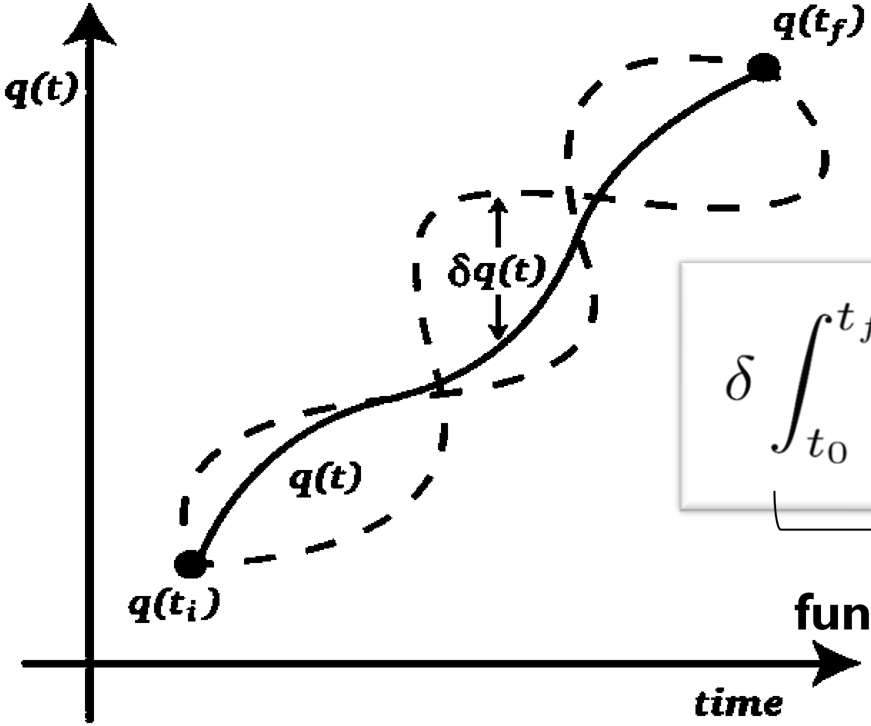
- We can ask how much entanglement is generated by $\mathcal{S}$. The answer depends on the initial states, e.g. $\mathrm{CNOT}\,|00\rangle = |00\rangle$, $\mathrm{CNOT}\,|10\rangle = |10\rangle$, but $\mathrm{CNOT}\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}} \otimes |0\rangle\right) = \frac{|00\rangle+|11\rangle}{\sqrt{2}}$.
- We define the **entangling power**: $\mathcal{E}(\mathcal{S}) \equiv \overline{E\left(\mathcal{S}\,|i\rangle \otimes |j\rangle\right)}$

[Zanardi, Zalka, Faoro] quant-ph/0005031

…and find its extrema with respect to the input parameters of the theory!

Nature already chooses to extremize a functional…

> Nature always uses the simplest means to accomplish its effects.
>
> *Pierre Louis Maupertuis, 1744*



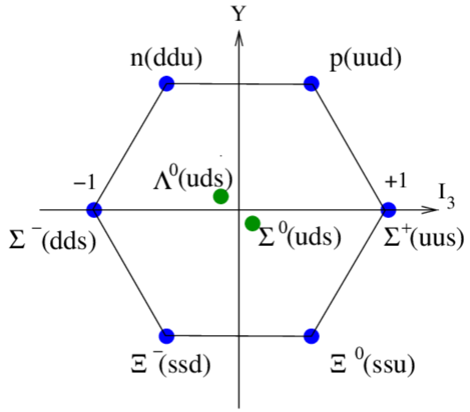
$$\delta \int_{t_0}^{t_f} \mathcal{L} dt = 0$$

Legendre transform of H (total energy)

functional of $q(t)$

Emergent Symmetries from MinEnt

- Minimization of $\mathcal{E}(\mathcal{S})$ had been attempted twice in the literature:



1. The Seattle group [Beane, Kaplan, Klco, Savage] 1812.03138 studied spin-1/2 octet baryon $2 \to 2$ scattering in low-energy QCD and found:

   spin-flavor symmetries ⇔ MinEnt

   ❖ Later, [Low, Mehen] 2104.10835 showed that the $\mathcal{S}$ operator produces no entanglement, when: $\mathcal{S} \sim [\mathbf{1}]$ (⇒ Wigner) or $\mathcal{S} \sim [\mathrm{SWAP}]$ (⇒Shrödinger)

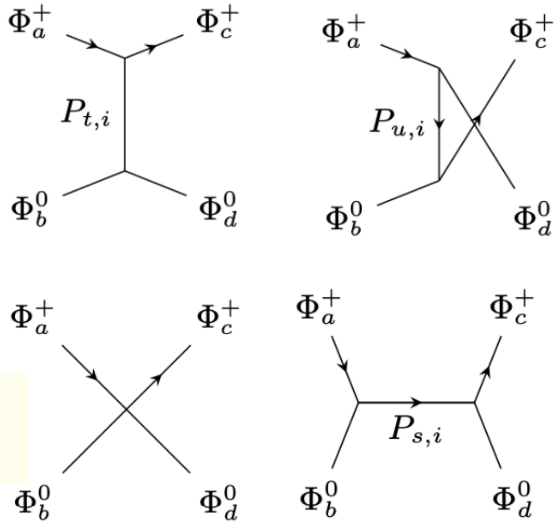
2. [Carena, Low, Wagner, Xiao] 2307.08112 studied tree-level scattering within the 2HDM and found:

   $SO(8)$ symmetry ⇔ MinEnt

   ✓ *natural alignment* limit with a SM-like Higgs

   ❖ But this result depends on the choice of channel. [Chang, Jacobo] 2409.13030, [Kowalska, Sessolo] 2404.13743

   choose the channel that produces the minimum entanglement



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Flavor lives in discrete Hilbert spaces

- Let us consider the G-dimensional quark Hilbert spaces H_u and H_d. For $G = 3$, the quark states are qutrits with the following basis elements (corresponding to the 6 quark flavors):

$$\begin{aligned} H_u : \quad & |1\rangle_u \,, \quad |2\rangle_u \,, \quad |3\rangle_u \,, \\ H_d : \quad & |1\rangle_d \,, \quad |2\rangle_d \,, \quad |3\rangle_d \,. \end{aligned}$$

- Similarly, for leptons and neutrinos we define H_ℓ and H_ν (we really mean mass eigenstates).

- We build the product Hilbert space: $H_f = H_u \otimes H_d$. A generic state can be written as:

$$|\alpha\rangle = \sum_{i,j=1}^{G} \alpha_{ij} \, |ij\rangle_{ud} \,, \qquad |ij\rangle_{ud} \equiv |i\rangle_u \otimes |j\rangle_d \,, \qquad \mathrm{tr}(\alpha^\dagger \alpha) = 1 \,.$$

α_{ij} → $G \times G$ matrix

$\mathrm{tr}(\alpha^\dagger \alpha) = 1$ → normalization

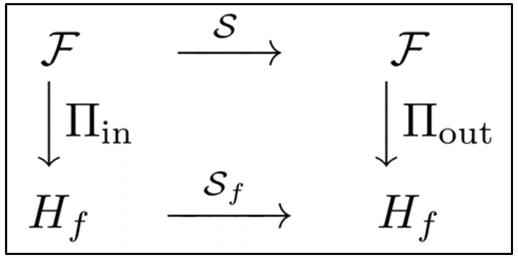
Isolating H_f in elastic scattering

- We want to characterize the flavor entanglement generated by **2 → 2 elastic**, fermion scattering.

flavor indices

$$u_{Li}(p_1)d_{Lj}(p_2) \rightarrow u_{Lk}(p_3)d_{L\ell}(p_4)$$

negative helicity
(≈left-handed chirality)



- To map from the Fock space $\mathcal{F}$ to the flavor Hilbert space H_f via preparation of the initial state and projective measurements of the kinematics of the final state:

perturbative amplitude

scattering angle

$$|\text{out}\rangle_{ij} = \frac{\Pi_{\text{out}}\mathcal{S}\,|\text{in}\rangle_{ij}}{\left|\Pi_{\text{out}}\mathcal{S}\,|\text{in}\rangle_{ij}\right|} = \frac{1}{\mathcal{N}_{ij}} \sum_{k,\ell=1}^{G} \mathcal{M}_{k\ell ij}\,(s,\Theta)\,|p_3,k;p_4,\ell\rangle$$

center-of-mass energy

normalization

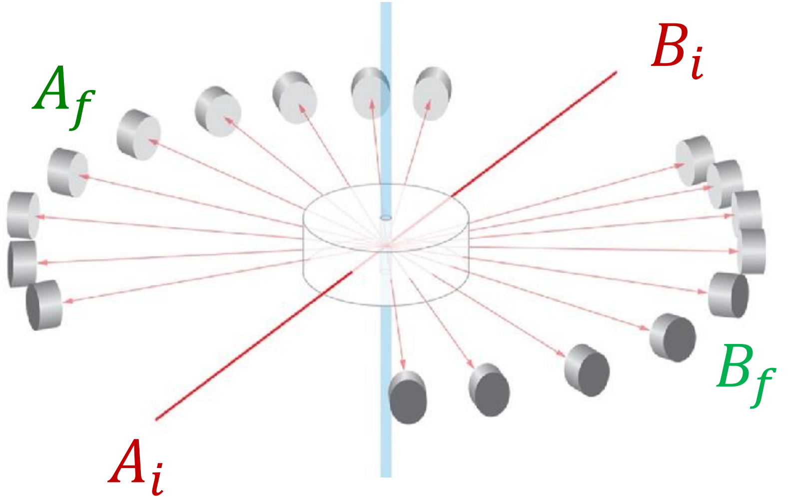
- The operator $\mathcal{S}_f$ ($G^2 \times G^2$matrix) is non-unitary, but still preserves normalization: $\text{diag}(\mathcal{S}_f\mathcal{S}_f^\dagger) = \mathbb{I}\,.$

Perpendicular entangling power

- Averaging over the product states of definite fermion generation, the entangling power reads:

$$\mathcal{E}(\mathcal{S}_f) \equiv \overline{E\left(\mathcal{S}_f\,|i\rangle_u \otimes |j\rangle_d\right)} = \frac{1}{G^2}\sum_{i,j=1}^{G} E\left(\mathcal{S}_f\,|ij\rangle_{ud}\right)$$

$$E(\rho) \equiv \frac{G}{G-1}\left|1 - \mathrm{tr}\,\rho_R^2\right| \,, \qquad \langle k|_u\, \rho_{R,ij}\, |k'\rangle_u = \frac{1}{|\mathcal{N}_{ij}|^2}\sum_{\ell=1}^{G} \mathcal{M}_{k\ell ij}(s,\Theta)\,\mathcal{M}^*_{k'\ell ij}(s,\Theta)\,.$$



- Alice and Bob initiate their beams at A_i and B_i and place their detectors at A_f and B_f, respectively.
- They can each decide to send either up or down quarks, but they can't measure final state flavor. Consequently, there is one unambiguous position for A_f and B_f, which is at $\Theta = 90^\circ$ (invariance under $A_f \leftrightarrow B_f$) .

- We define the **perpendicular entangling power** as: $\mathcal{E}^{\perp}_{\min}(\mathcal{S}^{\perp}_f) \equiv \mathcal{E}_{\min}(\mathcal{S}_f)\Big|_{\Theta=\frac{\pi}{2}}$

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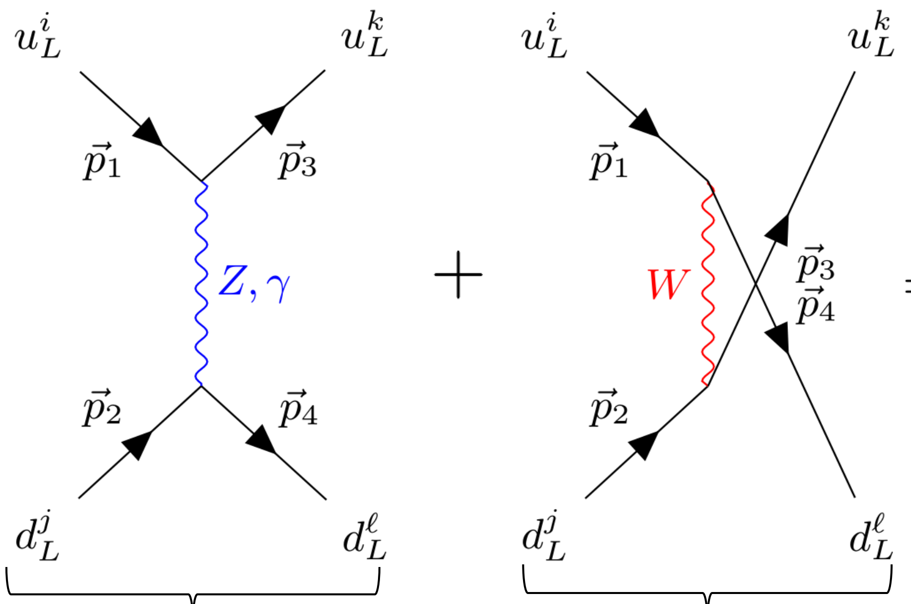
SM flavor-entangling interactions

- Let us start with the two quark generations to gain intuition. In this case there is one flavor parameter, the **Cabibbo** angle $\theta_{\text{CKM},12} = \theta_C \in [0, \pi/4]$. We want to examine:

$$\theta_C^{\min} = \underset{\text{ch},\theta_C}{\arg\min}\, \mathcal{E}^{\perp}_{\text{ch}}[\theta_C]$$

- At LO the minimal elastic entangling channel in the SM happens to be $ud \to ud$ induced by electroweak interactions. In the high-energy limit we have:



flavor-preserving

flavor-changing or preserving

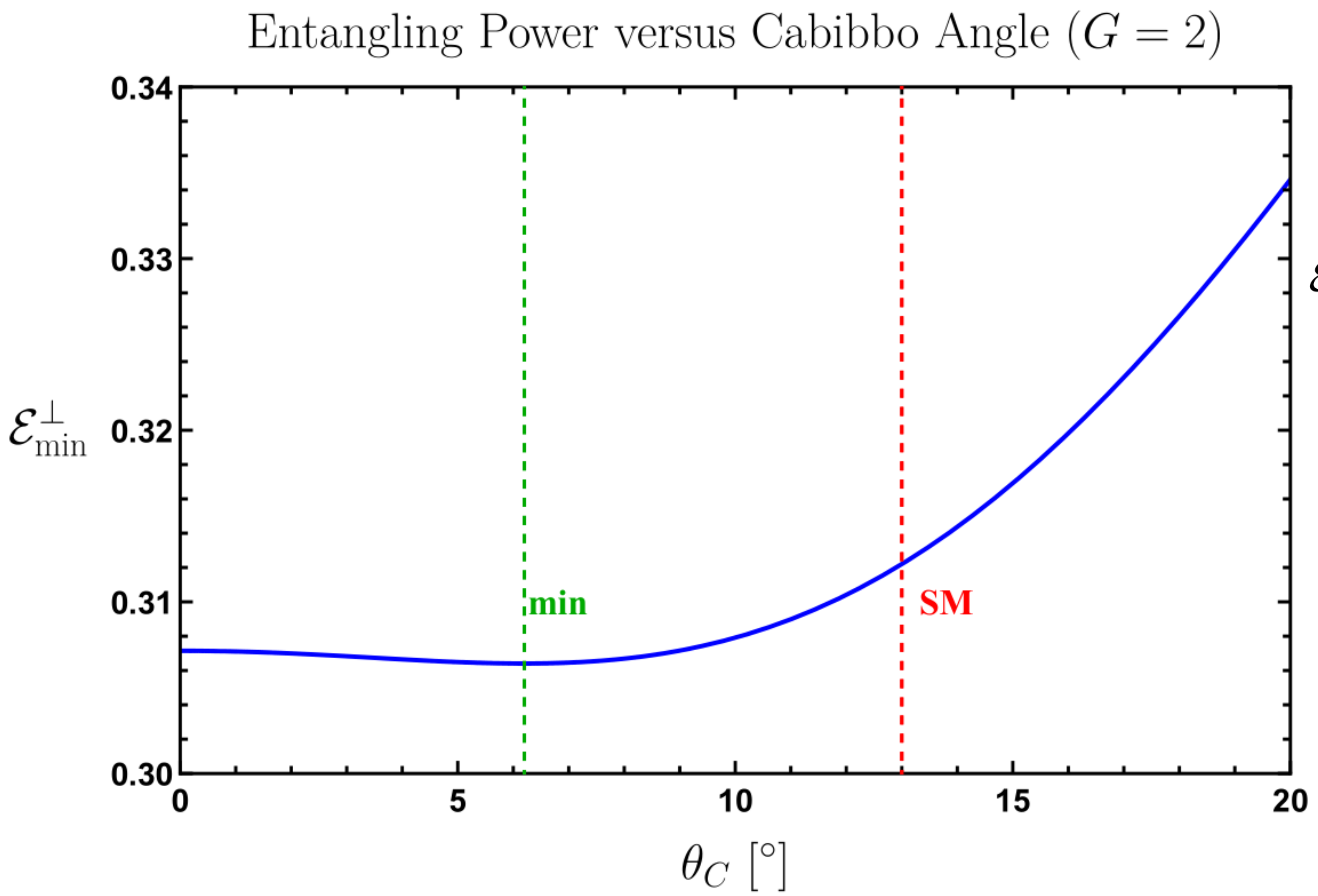
$$= -i\left(P^{t(Z,\gamma)}_{k\ell ij} + P^{u(W)}_{k\ell ij}\right) 2s\,,$$

$$P^{t(Z,\gamma)}_{k\ell ij} = g^2 \delta_{ik}\delta_{j\ell}\left(\frac{Y^u Y^d}{\cos^2\theta_W}\frac{1}{t - m_Z^2} + \frac{\sin^2\theta_W\, Q^u Q^d}{t}\right),$$

$$P^{u(W)}_{k\ell ij} = \frac{g^2 V^*_{i\ell} V_{kj}}{2}\frac{1}{u - m_W^2}\,.$$

- The only input parameter is the **Weinberg** angle: $\cos\theta_W = m_W/m_Z$

Entangling power of EW interactions ($G = 2$)



$$E\left(\mathcal{S}_f^{\perp}|12\rangle_{ud}\right) = E\left(\mathcal{S}_f^{\perp}|21\rangle_{ud}\right)$$

$$\mathcal{E}^{\perp}_{ud} = 8y^2\left[\frac{\cos^4\theta_C}{(1+2y+4y^2-2y\cos 2\theta_C)^2} + \frac{\sin^4\theta_C}{(1+2y+4y^2+2y\cos 2\theta_C)^2}\right]$$

$$E\left(\mathcal{S}_f^{\perp}|11\rangle_{ud}\right) = E\left(\mathcal{S}_f^{\perp}|22\rangle_{ud}\right)$$

where $$y = \frac{Y^u Y^d}{\cos^2\theta_W} + \sin^2\theta_W Q^u Q^d\,.$$

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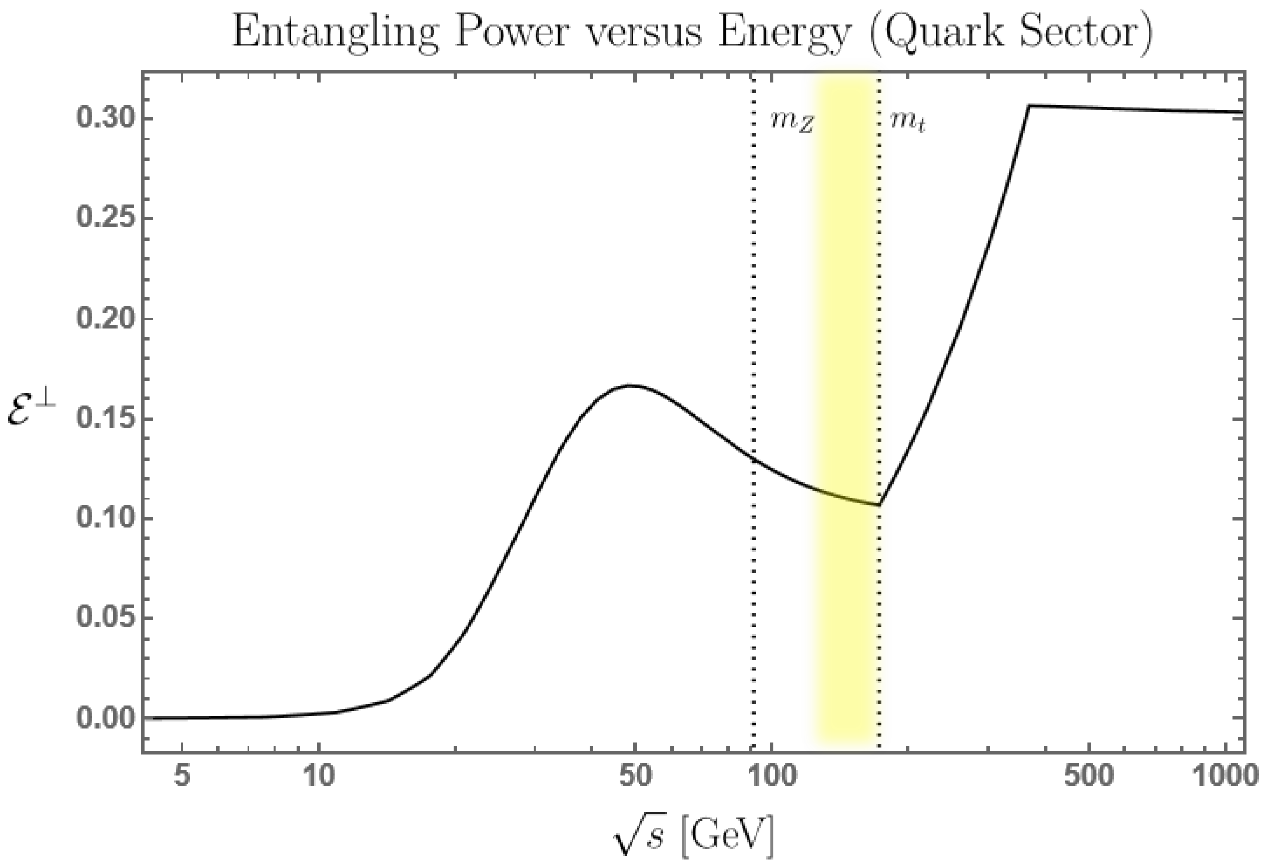
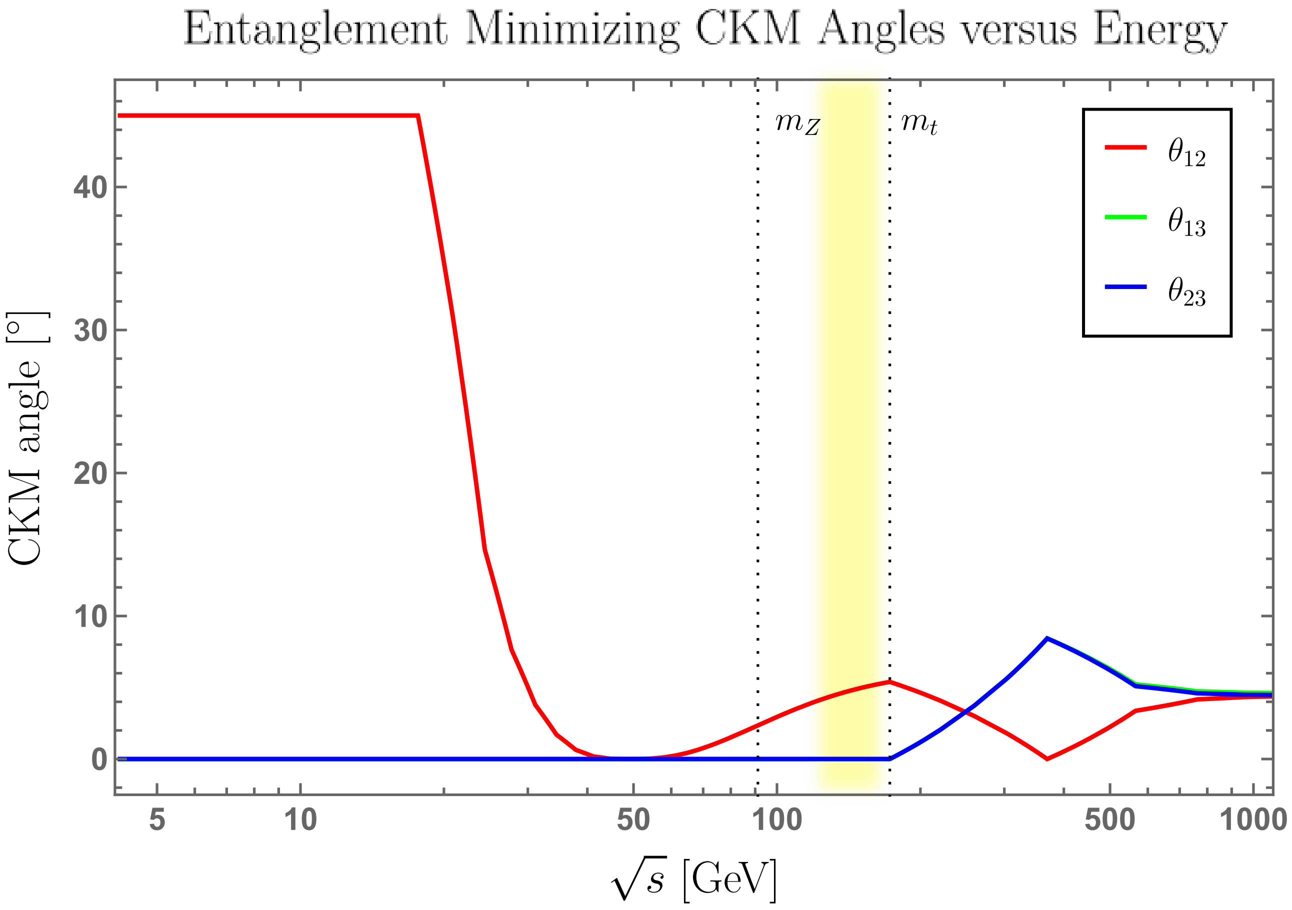
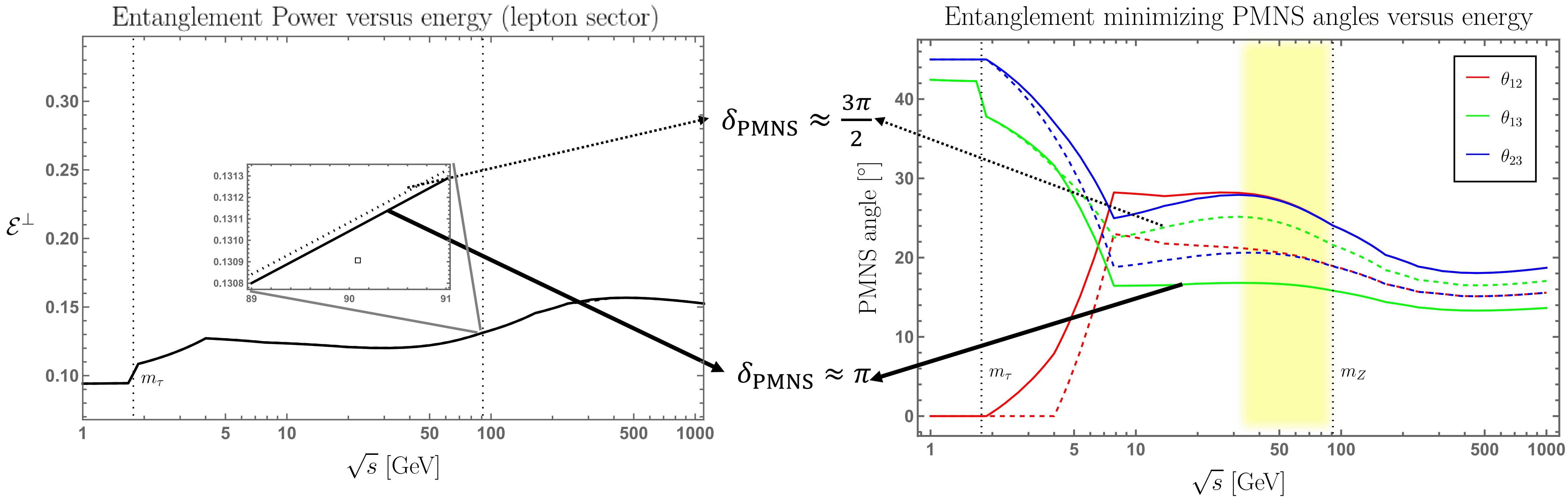
Towards the Full CKM $(ud \to ud)$





Towards the Full PMNS $(\nu\ell \rightarrow \nu\ell)$

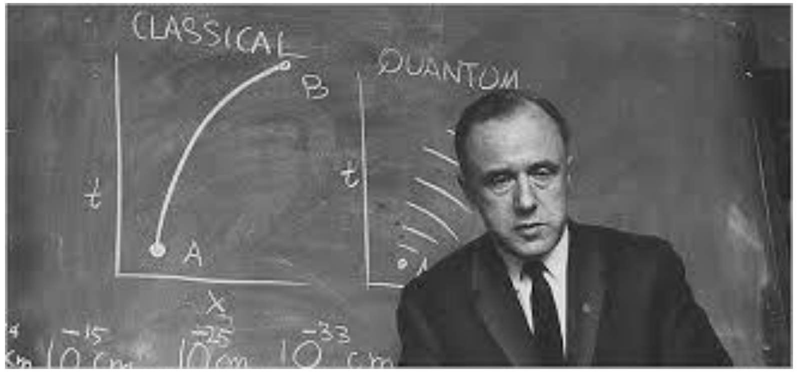
- The only differences between quarks and leptons are: i) the EW charges & ii) the participation/absence of the heaviest fermion (tau/top) in scattering processes at $\sqrt{s} \sim m_Z$.



- δ_{PMNS} is the only flavor parameter which is not yet experimentally determined. In our framework, the preferred value (at LO) is close to π!

Conclusions

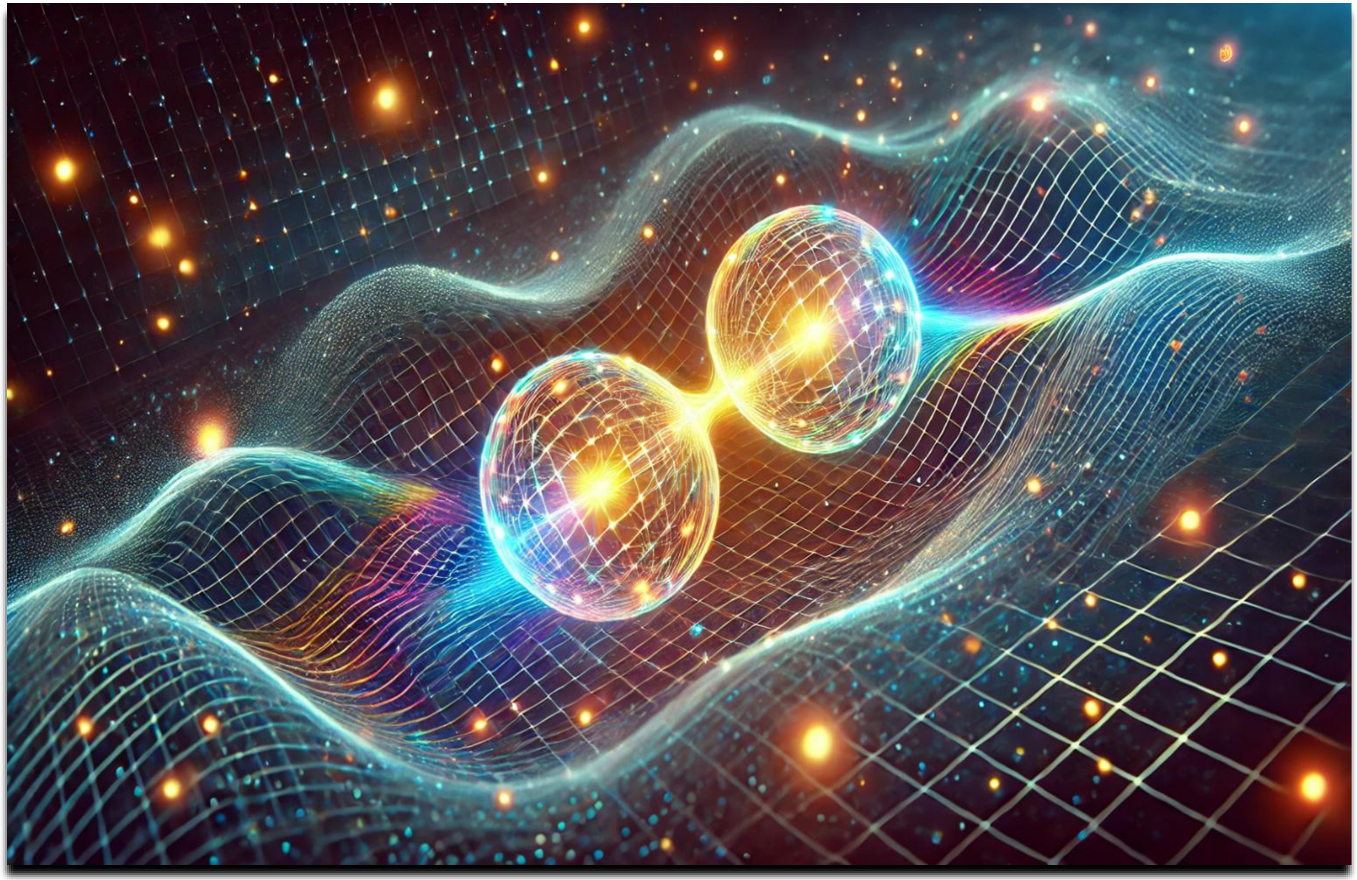
- To our knowledge, this is the first time the differing CKM and PMNS structures have arisen from a common mechanism (without new symmetries).
- Even though one can argue that the experimentally known parameters are postdictions, we (may) have a prediction for the $\delta_{\text{PMNS}} \approx \pi$.
- Further explorations are required to ultimately answer the question:
  Is this all just a numerical coincidence, or could minimization of quantum entanglement really be a fundamental principle of nature?
- Injecting QIS concepts into HEP is speculative but very exciting!



All things physical are information-theoretic in origin and this is a *participatory universe*.

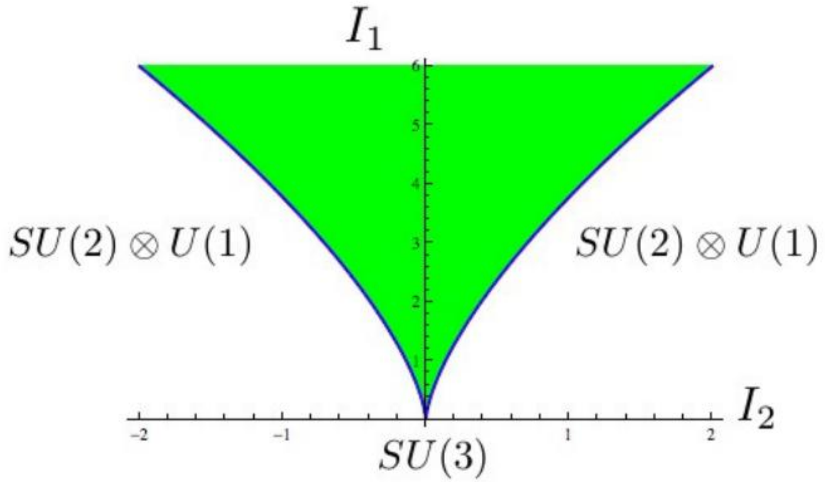
[J.A.Wheeler] *"Information, Physics, Quantum: The Search for Links" in Complexity, Entropy and the Physics of Information (1990)*

Thank you!



Flavor from a Minimization (Energy) Principle

- There is already an attempt in the literature of invoking a Minimization principle for explaining the flavor structures. [Alonso, Gavela, Isidori, Maniani] 1306.5927

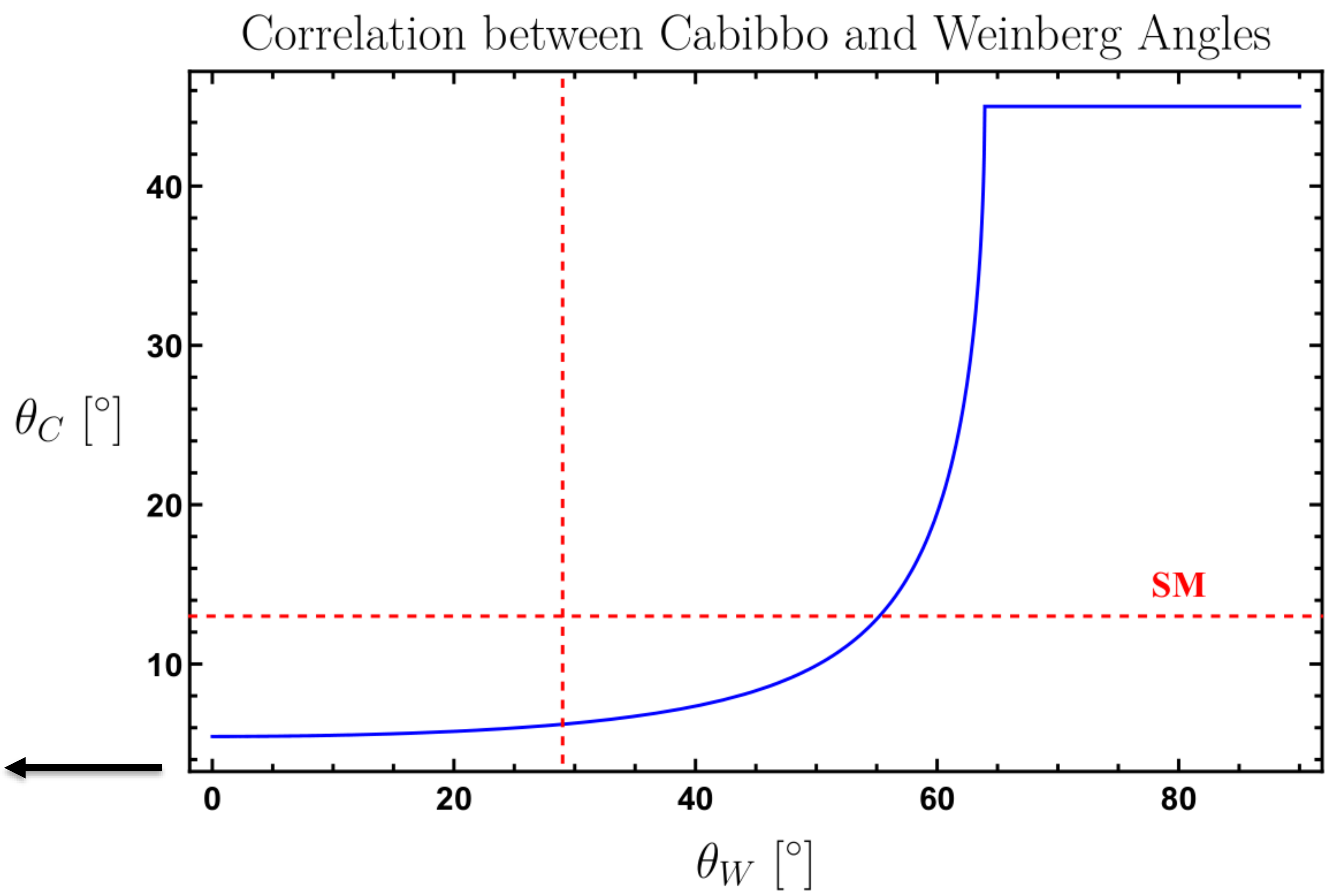


- Group theoretical methods are employed to identify the *natural* extrema of a generic potential *V* invariant under the SM flavor symmetry (in the massless limit).

- The extrema correspond to specific maximal subgroups and thus to symmetry-breaking patterns that generate the texture of the resulting Yukawa matrices (at $O(1)$ accuracy).

- Discrete flavor symmetries, e.g. A_4 provide better numerical *postdictions*. However, the required symmetry breaking has different sources between quarks and leptons and the vacuum alignment is problematic. [He, Keum, Volkas] hep-ph/0601001

Entangling power of EW interactions $(G = 2)$

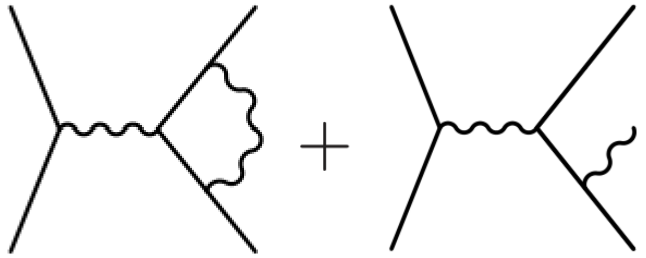


W-exchange dominates

Z-exchange dominates

What is next?

- A 10% increase in the charged-current contribution leads to $\theta_c \approx 13°$!
  Historically, the **one-loop** level has been highly illuminating!



$\sim$ IR finite cross-sections (*Bloch–Nordsieck theorem*)

but Π_2 restricts to 2-particle final state?

- We need to develop an IRC safe entanglement measure for bypartiite systems.
  - other QIS concepts might prove to be useful!

- Revisit the nucleon-nucleon scattering results in the presence of θ_{QCD}. Are the CP-violating terms producing entanglement in spin-flavor space? → Spoiler: yes!

- Intriguing fact: **EntMax** in helicity space wrt the gauge couplings in tree-level EW scattering yields $\theta_W = \frac{\pi}{6}$. [Cervera-Lierta et al] 1703.02989

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