Do minimal 331 models still hold up?

Giulia Ricciardi
University of Naples Federico II
& INFN, Sezione di Napoli

21st Rencontres of Vietnam
ICISE, Quy Nhon, Vietnam
23/08/2025

331 Models in a nutshell

Gauge extension

$$SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times SU(3)_L \times U(1)_X$$

$$\mathcal{L}^{331} = \mathcal{L}_{\text{strong}} + i \sum_j \bar{\psi}_{L,j} \gamma_\mu D^{331\,\mu}_{\text{ew}} \psi_{L,j} + \sum_j (D_\mu \phi_j)^\dagger D^\mu \phi_j - V(\phi) + \text{Yukawa}$$

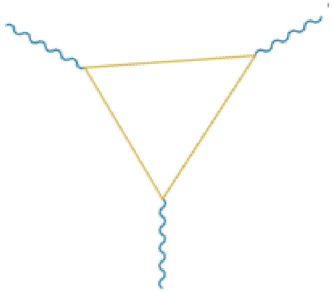
$$D^{331\,\mu}_{\text{ew}} = \partial^\mu - i\, g_L\, W^{a,\mu} T^a - i g_X X B_\mu \mathbb{I}.$$

$$\begin{pmatrix} u_i \\ d_i \\ q_{exotic} \end{pmatrix}_L \quad \begin{pmatrix} \nu_i \\ e_i \\ l_{exotic} \end{pmatrix}_L$$

$$W^\pm, A_\mu, Z_\mu, Z'_\mu, V^{\pm Q_V}, Y^{\pm Q_Y}$$

Overall anomalies cancellations

anomalies
$SU(3)_L^3$
$SU(3)_L^2 \times U(1)_X$



Constrain the number of families in the different representations, e.g.

$$[SU_L(3)]^2 \times U_X(1)]$$

$$\sum_{\text{all triangles}} \mathrm{Tr}(T^a T^b \mathbb{1})X = 0$$

$$N_c\left[N_q^3 X_q^3 + N_q^{\bar{3}} X_q^{\bar{3}}\right] + N_l^3 X_l^3 + N_l^{\bar{3}} X_l^{\bar{3}} = 0$$

Gell-Mann Nishijma relation

$$Q = T_3 + \beta T_8 + X_1 = T_3 + Y$$

331 models are characterized by β value and matter content

Two SSB stages

$SU(3)_L \times U(1)_X \xrightarrow{\mu_{331}} SU(2)_L \times U(1)_Y \xrightarrow{\mu_{EW}} U(1)_{em}$

Reasonable assumptions

- Fermions in triplet or antitriplet

$\Longrightarrow$ Higgs in triplet, sextet, singlet

- $\bar{\psi}_L \psi_R \Phi$, with $\Phi \sim 3$;
- $\bar{\psi}_L (\psi_L)^c \Phi$, with $\Phi \sim 6$;
- $\bar{\psi}_R (\psi_R)^c \Phi$, with $\Phi \sim 1$;
- $\bar{\psi}_L (\psi'_L)^c \Phi$, with $\Phi \sim \bar{3}$;

- Exotic gauge bosons with integer charge value
- Matching with SM Z coupling

$\Longrightarrow \beta = \pm\sqrt{3}, \pm 1/\sqrt{3}$

$\beta = \pm\dfrac{1}{\sqrt{3}}$ vs $\beta = \pm\sqrt{3}$

new fields with non-SM charges.

Why 331 Models

F. Pisano and V. Pleitez, Phys. Rev. D 46, 410 (1992),
P. H. Frampton, Phys. Rev. Lett. 69, 2889 (1992)

- ❑ *Number of families*
  - QCD asymptotic free + fully outside the $SU(3)_C$ conformal window
  - each exotic family replica of SM
  - each $SU(3)_L$ charged fermion has right-handed counterpart

$$\Longrightarrow N_f = 3$$

- ❑ *Small number of free parameters*
  - diversity in matter content
- ❑ *GUT embedding*
  - $E_6 \to SU(2) \times SU(6) \to SU(3) \times SU(3) \times U(1) \to SU(3) \times U(1)$
- ❑ *Peccei-Quinn symmetry*
  - naturally implemented (strong CP problem)
- ❑ *Rich distinctive phenomenology*
  - $\beta = \sqrt{3}$ doubly charged gauge bosons Y^{++}, Y^{--}
  - TeV scale, searches at LHC
  - flavour physics

Minimal 331 models $\beta = \sqrt{3}$

F. Pisano and V. Pleitez, Phys. Rev. D 46, 410 (1992),
P. H. Frampton, Phys. Rev. Lett. 69, 2889 (1992)

Matter content

$$\mathbf{3}\ \begin{pmatrix} u_L^{1,2} \\ d_L^{1,2} \\ D_L^{1,2} \end{pmatrix} \quad \begin{pmatrix} b_L \\ -t_L \\ T_L \end{pmatrix} \begin{pmatrix} \ell_L^a \\ -\nu_L^a \\ (\ell_R^a)^c \end{pmatrix} \ \bar{\mathbf{3}} \qquad \begin{matrix} u_R^a \\ d_R^a \\ D_R^{1,2} \\ T_R \end{matrix}$$

- **Charged BSM fermions** $Q(D^{1,2}) = -4/3$ and $Q(T)=5/3$
  Vector Like Quarks (VLQs),
  left and right-handed components transform under the same representation of the SM gauge group.
- **Only SM leptons**

$$(D^{1,2})^{-4/3},\ T^{5/3},\ Z'^0,\ V^{\pm},\ Y^{\pm\pm}$$

additional gauge bosons, Distinctively,

- **bileptons** (Y^{--}, Y^{++}) of charge ±2 and lepton number ±2
- Neutral Z'

SSB

$$SU_L(3) \times U_X(1) \xrightarrow{\mu_{331}} SU_L(2) \times U_Y(1)$$

$$SU_L(2) \times U_Y(1) \xrightarrow{\mu_{\rm EW}} U_{\rm e.m.}(1)$$

$$\langle\chi\rangle = \begin{pmatrix} 0 \\ 0 \\ \frac{u}{\sqrt{2}} \end{pmatrix} \quad \mathbf{3}$$

$$\langle\rho\rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \\ 0 \end{pmatrix} \quad \langle\eta\rangle = \begin{pmatrix} \frac{v'}{\sqrt{2}} \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{6} \quad \tilde{S} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & w \\ 0 & w & 0 \end{pmatrix}$$

Peccei Quinn symmetry

$$[Q^1, Q^2, Q^3] \equiv [3^1_Q, 3^2_Q, \bar{3}^3_Q] \qquad [L^1, L^2, L^3] \equiv [\bar{3}^1_\ell, \bar{3}^2_\ell, \bar{3}^3_\ell].$$

$$\begin{aligned}\mathcal{L}_{Yukawa} = &+ \lambda^d_{i,a}\overline{Q}_i \rho d_{a,R} + \lambda^u_{i,a}\overline{Q}_i \eta u_{a,R} + \lambda^d_{3,a}\overline{Q}_3 \eta^* d_{a,R} + \lambda^u_{3,a}\overline{Q}_3 \rho^* u_{a,R} + \\ &+ \lambda^J_{i,j}\overline{Q}_i \chi D_{j,R} + \lambda^J_{3,3}\overline{Q}_3 \chi^* T_R \\ &+ \lambda^\ell_{a,b}\epsilon_{ijk}\overline{L}_{ai} L_{bj} \rho^*_k\end{aligned}$$

$$a, b = 1, 2, 3 \qquad i, j = 1, 2.$$

automatic extra global U(1) symmetry
($\neq$ charges of left and right chiral fermions)

	$Q_i, Q_3, \chi, \eta, \rho$
U(1) charge	1 -1 1 1 1

when extending to the whole Lagrangian

- violating terms in the Higgs sector (e.g. $\rho\chi\eta$) $\Longrightarrow$ no accidental symmetry
- SSB at the weak scale $\Longrightarrow$ visble axion ruled out by experiments

Possible solutions by introducing new fields and discrete symmetries

P. Pal, *Phys.Rev.D* 52 (1995) 1659
Dias et al. Phys. Rev. D 68 (2003) 11,
Rev. D 69 (2004) 07

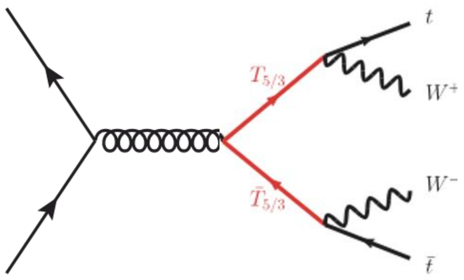
Distinctive signatures at colliders: VLQ

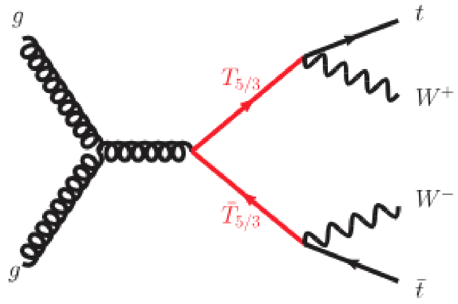
- **VLQ** Left-handed and right-handed components interact equally within the weak interaction
  - VLQ appears in several models (Composite Higgs, Little Higgs, RS, GUTs…)

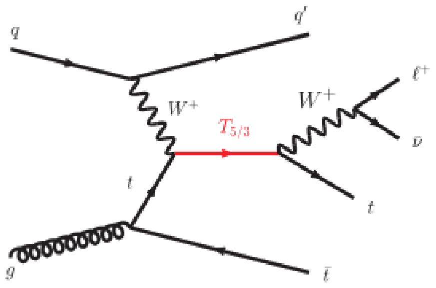
- **Singly** produced via electroweak interaction or **pair-produced** via strong interaction
  - Pair production involves only the SM QCD coupling, then tree-level cross section independent of VLQ properties, other than its mass
  - EW production model dependent on couplings

- More than **60 studies** at ATLAS and CMS, mostly on pair production
  - VLQ decay into SM states

Exotic VLQs →q $W^{\pm}$







- 331 models dominant modes into **exotic bosons**

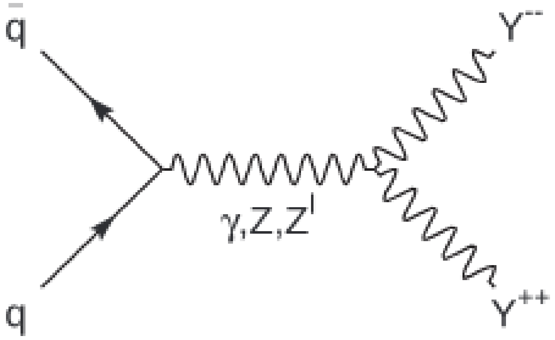
Exotic VLQs →q $V^{\pm}$, q $Y^{\pm\pm}$

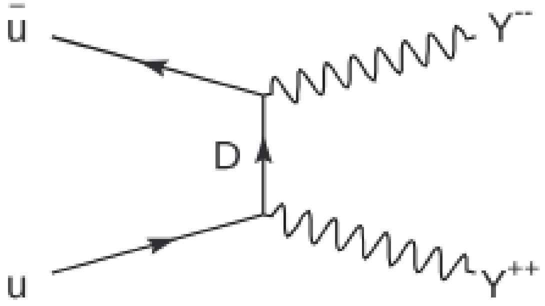
$\Longrightarrow$ $m(T_{5/3}) \gtrsim 1.4$ TeV

G, Corcella et al., *JHEP* 10 (2021) 108

Distinctive signatures at colliders: bileptons

- **Bileptons** doubly charged gauge bosons coupling at tree level to SM lepton pairs but not to SM quark pairs





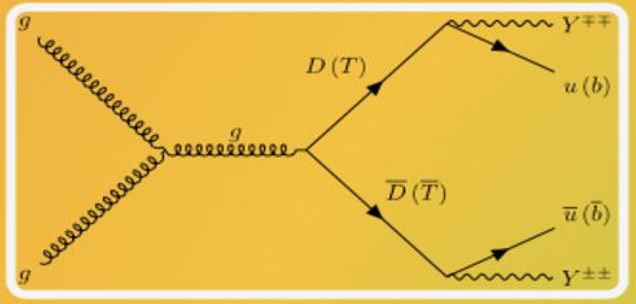
- **4 leptons** processes searches $Y^{++} \to \ell^+\ell^+, Y^{--} \to \ell^-\ell^- \implies m_Y \gtrsim$ 878 GeV

G, Corcella et al., *Phys. Lett. B 2022, 826, 136904*

- ATLAS search for **doubly charged Higgs bosons**, at the 13 TeV LHC with 139 fb^{-1}, focus on **multilepton final states with at least one same-sign lepton pair**

$$pp \to Y^{++}Y^{--}$$

each bilepton decays into a pair of same sign charged leptons



Recast to extract a lower limit on the Y mass $M_Y > 1.3$ TeV

G. Aad et al. (ATLAS), Eur. Phys. J. C 83, 605 (2023),
R. Calabrese, A.O.M. Iorio, S. Morisi, G.R., N. Vignaroli, Phys.Rev.D 109 (2024), 055030

BSM flavour structure

❑ Tree level FCNC

general » one can avoid FCNCs due to massive boson exchange *when* fields ∈ to the same representation under all the *unbroken* & the *broken* generators ⟹ their coupling to the massive gauge boson is *universal*

Not so » in 331 models

$$\mathcal{L}^{Z'} = J_\mu Z'^\mu \,,$$

$$J_\mu = \bar{u}_L \gamma_\mu U_L^\dagger \begin{pmatrix} a & & \\ & a & \\ & & b \end{pmatrix} U_L u_L + \bar{d}_L \gamma_\mu V_L^\dagger \begin{pmatrix} a & & \\ & a & \\ & & b \end{pmatrix} V_L d_L$$

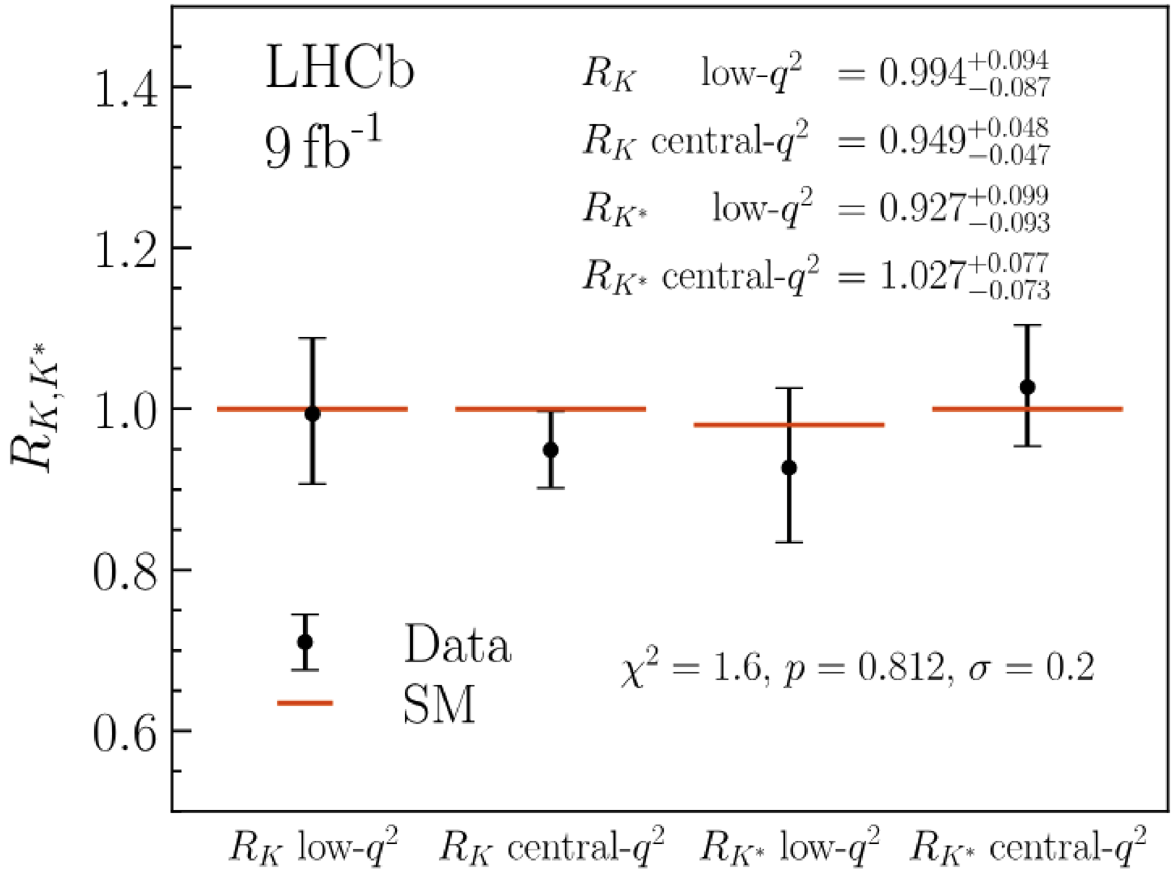
❑ New sources of CP violation

currents mediated by new gauge bosons depend on the elements of the *two mass rotation matrices (not only CKM)* ⟹ new CP violating phases

Anomalies in quark flavour sector

$$R_{K,K^*}(q_a^2, q_b^2) = \frac{\int_{q_a^2}^{q_b^2} \frac{d\Gamma(B^{(+,0)} \to K^{(+,*0)}\mu^+\mu^-)}{dq^2} dq^2}{\int_{q_a^2}^{q_b^2} \frac{d\Gamma(B^{(+,0)} \to K^{(+,*0)}e^+e^-)}{dq^2} dq^2}$$

LHCb Collab. Phys.Rev.D 108 (2023) 3, 032002



Practically disappeared.

Good news for minimal 331 models where leptons are in the same representation (anomaly cancellation)

S. Descotes-Genon, M. Moscati, G.R. *Phys.Rev.D* 98 (2018) 11, 115030
A. Addazi, G.R., S. Scarlatella, R. Srivastava, J.W.F. Valle, *Phys.Rev.D* 106 (2022) 3, 035030

Landau pole problem

$$\alpha_i(\mu) = \frac{\alpha_i(\mu_0)}{1 - \frac{b_i}{2\pi}\alpha_i(\mu_0)\log\left(\frac{\mu}{\mu_0}\right)} + \mathcal{O}(\alpha_i^2)\,,$$

where μ_0 is some fixed reference value and b_i the first coefficient of the perturbative expansion of the β function. When $b_i > 0$, the coupling diverges at some energy scale (Landau pole).

$$b_i = \frac{2}{3}\sum_{fermions} \mathrm{Tr}(T_a T_a) + \frac{1}{3}\sum_{scalars} \mathrm{Tr}(T_a T_a) - \frac{11}{3}C_2^A.$$

$$U(1) : C_2^A = 0 \rightarrow b_X > 0$$

$$SU(N) : C_2^A = N$$

$\Rightarrow$ U(1) divergence

Landau pole philosophy

the Landau pole arises within **perturbation theory**, which cannot be expected to reliably represent behaviors related to large or even infinite couplings.

Real insight can only be gained using non-perturbative approaches.

It is not unreasonable to assume that physical QFT observables are well-behaved even when the coupling diverges at the Landau pole and beyond.

Landau pole evidence of an energy scale at which the theory **breaks down** (UV-incomplete or effective theories).

More concerning when it approaches energies accessible to present or near future experimental efforts.

331 model Landau pole

μ_{331} μ_{EW}

$$SU(3)_L \times U(1)_X \rightarrow SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$$

the scales μ_{331} and μ_{EW} delimitate energy regimes with different symmetries and matter content, both of which affect the running.

$$\begin{aligned} b_Y &= \frac{20}{9}N_F + \frac{1}{6}N_H \\ b_{2L} &= -\frac{22}{3} + \frac{4}{3}N_F + \frac{1}{6}N_H \\ b_c &= -11 + \frac{4}{3}N_F, \end{aligned}$$

The SM gauge group dictates the running of the gauge couplings g_Y, g_{2L} from the electroweak scale $\mu_{EW} \sim M_Z$ up to the μ_{331} scale.

The different matter content and the unbroken 331-gauge symmetry lead the running of g_X, g_{3L} above the μ_{331} scale.

e.g.

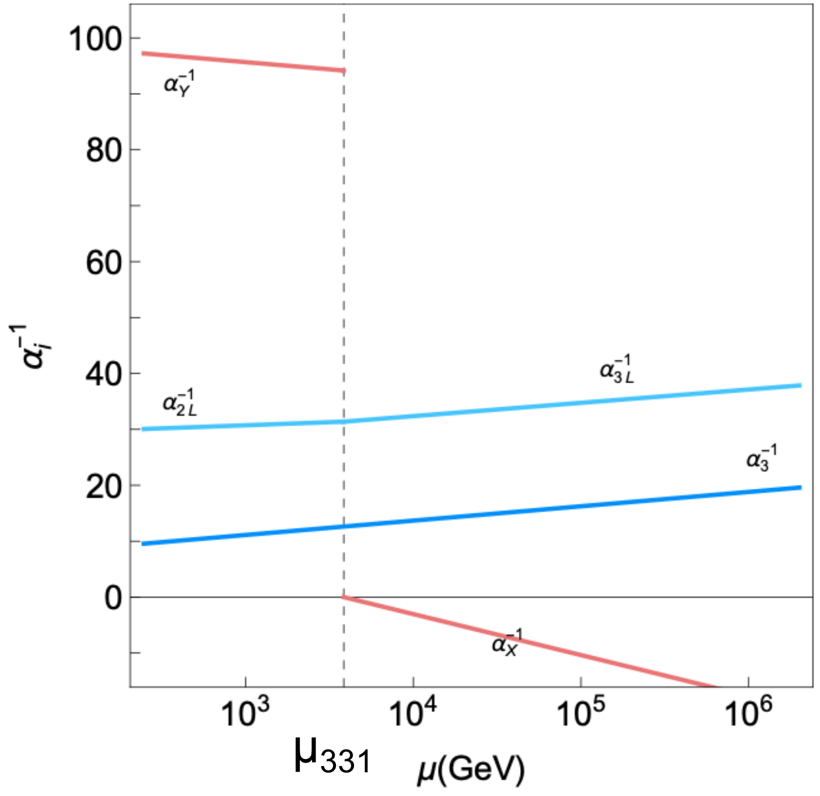
$b_Y = 41/6$	$b_{2L} = -19/6$	$b_c = -7$	SM
$b_Y = 7$	$b_{2L} = -3$	$b_c = -7$	2HDM

331 models Landau pole

Matching conditions

$$g_{2L}(\mu_{331}) = g_{3L}(\mu_{331}) \,, \qquad \frac{1}{g_X^2(\mu_{331})} = \frac{1}{6}\left(\frac{1}{g_Y^2} - \beta^2 \frac{1}{g_L^2}\right)\Bigg|_{\mu=\mu_{331}}$$

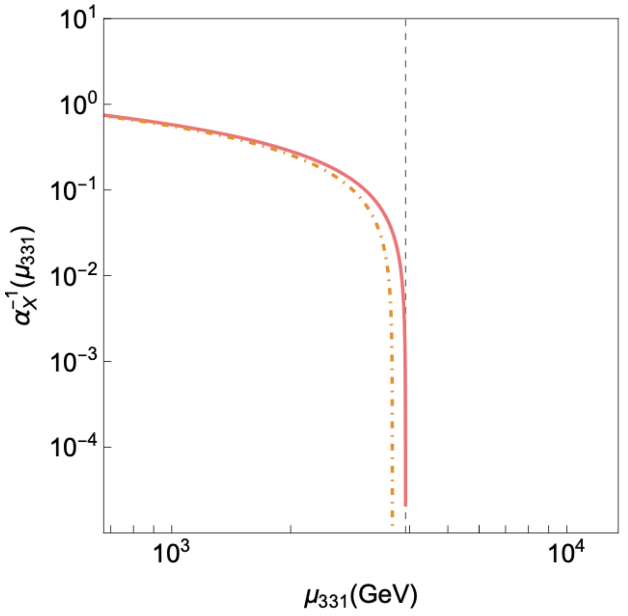
A. Buras, F.De Fazio, J. Girrbach, M.V. Carlucci.JHEP, 02:023, 2013.



S. Morisi, G.P. Perdona', G.R., 2505.15785 [hep-ph]
to be published on Phys Rev D

Minimal 331 models with $\beta = \sqrt{3}$

$$N_{\bar{3}}^{\text{lept fam}} = 3,\ N_{3}^{\text{quark fam}} = 2,\ N_{\bar{3}}^{\text{quark fam}} = 1$$



μ_{331} bounds

- lower from experimental constraint

R. Calabrese, A.O.M. Iorio, S. Morisi, G.R., N. Vignaroli, Phys.Rev.D 109 (2024), 055030

- upper from the condition $\alpha_X^{-1}(\mu_{331})$ ≥0, (2 Higgs mass lower than 331 scale)

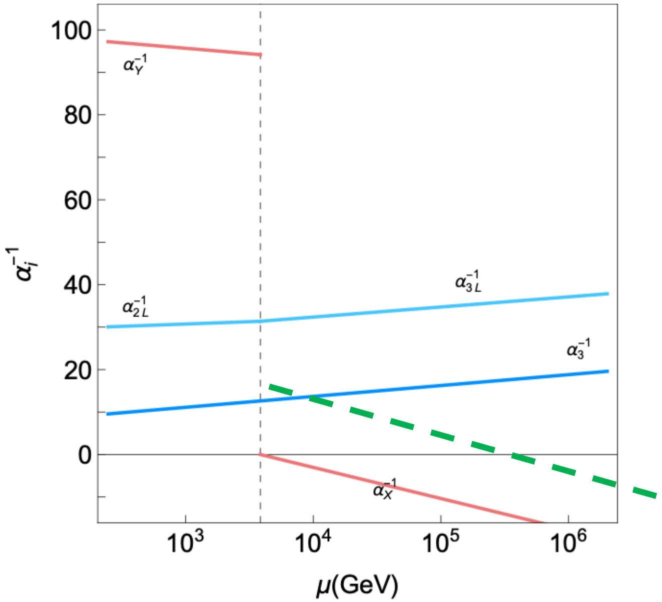
$$3850\,\text{GeV} \lesssim \mu_{331} \lesssim 3908\,\text{GeV}$$

$\beta = \sqrt{3}$

$\alpha_X^{-1}(3850\,\text{GeV}) = 0.006$

$\beta = 1/\sqrt{3}$

$\alpha_X^{-1}(3850\,\text{GeV}) = 13.951$



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$\mu_{LP} \sim 3858$ GeV

Rescuing bileptons from Landau pole

A game of changing matter

1. Assume light masses for the D_1,D_2,T extra quarks;
2. include exotic leptons;
3. extend the scalar sector;
4. enlarge the number of families.

Only 3) 4) shift the maximal of 331-scale

$$\boxed{m_Y \sim \mu_{331}}$$

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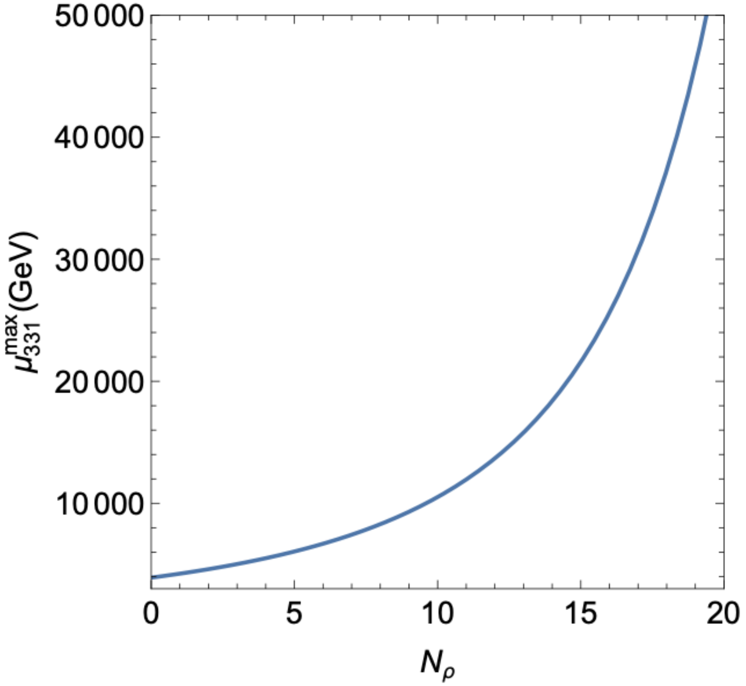
Extending the scalar sector

add an arbitrary number of ρ-like scalar triplets N_ρ which correspond to extra Higgs $SU_L(2)$ doublets and contribute to the running between μ_{EW} and μ_{331}.

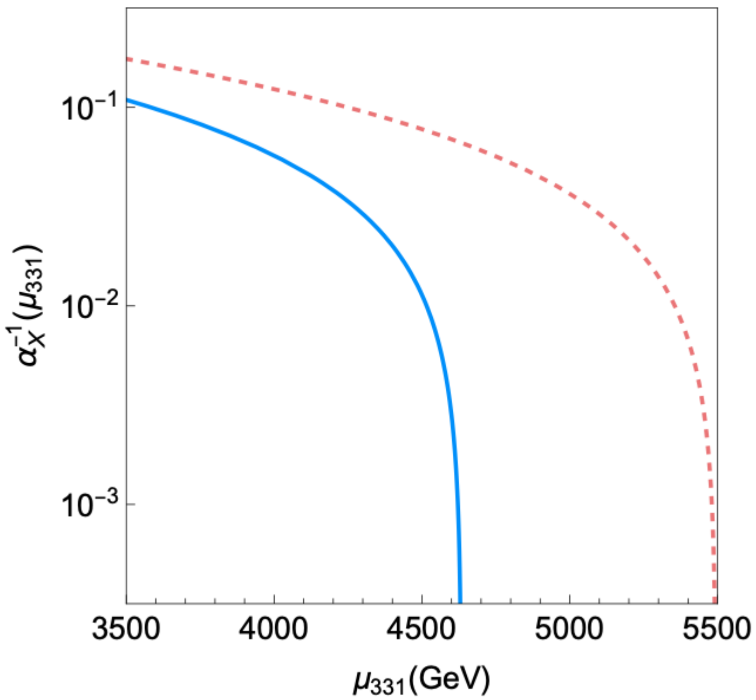


Maximal value of the 331-breaking scale as a function of the number of Higgs doublets

$N_H = 2 + N_\rho = 6$ can arise from an E_6 inspired framework.

add Higgs sextets

$$\mathcal{L}_Y = Y^{ij}\bar{L}^c{}_i S L_j$$



Running of α_X^{-1} in the sextet extension for different values of the mass of the doublet and triplet: electroweak scale (dashed line), 600 GeV (continuous line).

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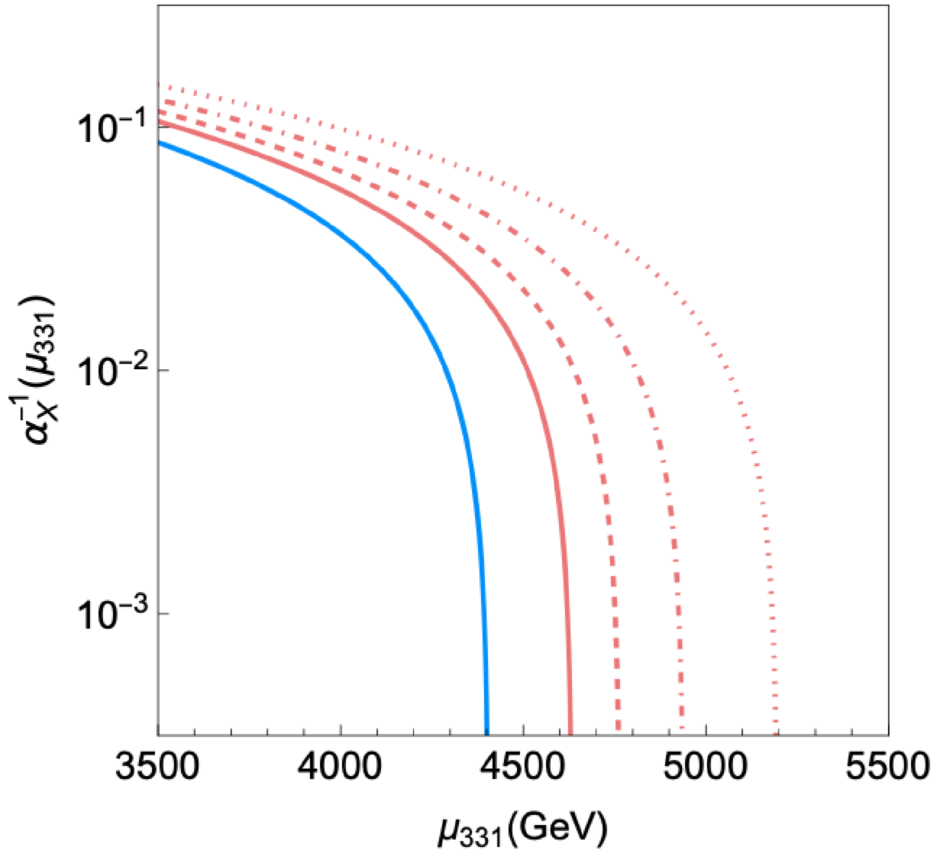
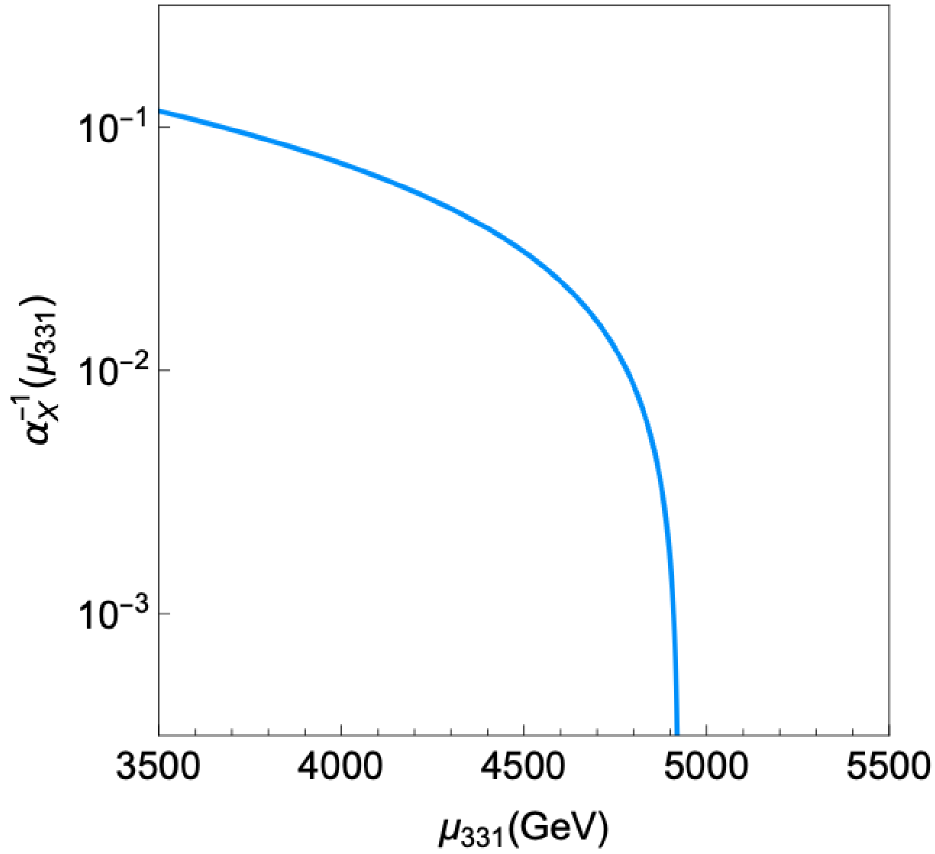
4th Family Extension Model





Running of α_X^{-1} with the energy scale μ_{331}. (Left plot) 4th family model for different values of the mass scale m_{NP} 1500 GeV (blue continuous), 1000 GeV (red continuous), 800 GeV (dashed), 600 GeV (dot-dashed) and 400 GeV (dotted). (Right plot) 4th family model + sextet with m_{NP} ~1500 GeV.

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Take home message:

- ✓ Analysis in TeV range based on PT calculation are <u>debatable</u> in minimal models with $\beta = \sqrt{3}$
- ✓ Landau pole value can be increased (but not too much) by losing minimality