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ROYAL COMMISSION EXHIBITION OF 1851

Neutrino Cross-Section Theory:
Uncertainties and Implications for Oscillation Measurements

Minoo Kabirnezhad

m.kabirnezhad@imperial.ac.uk

Vietnam Flavour Physics Conference 2025

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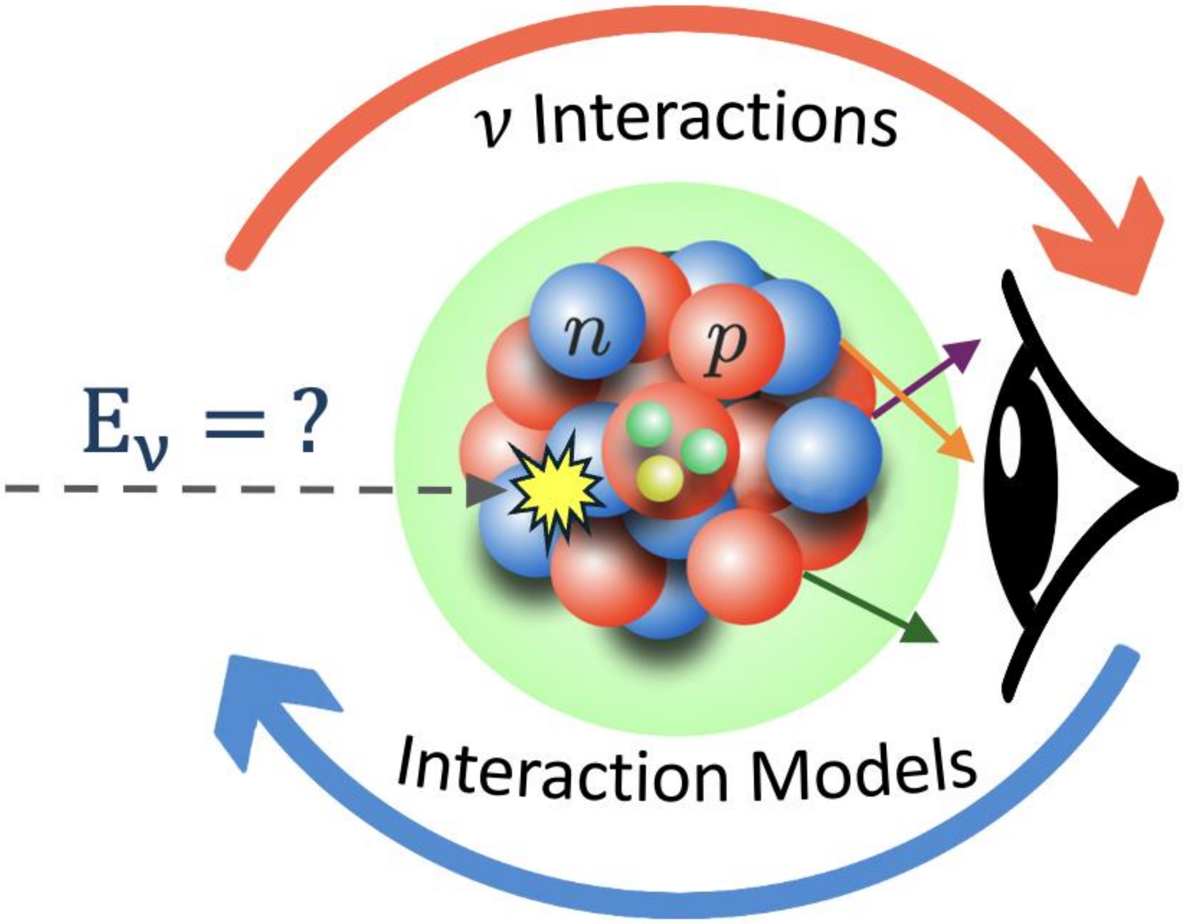
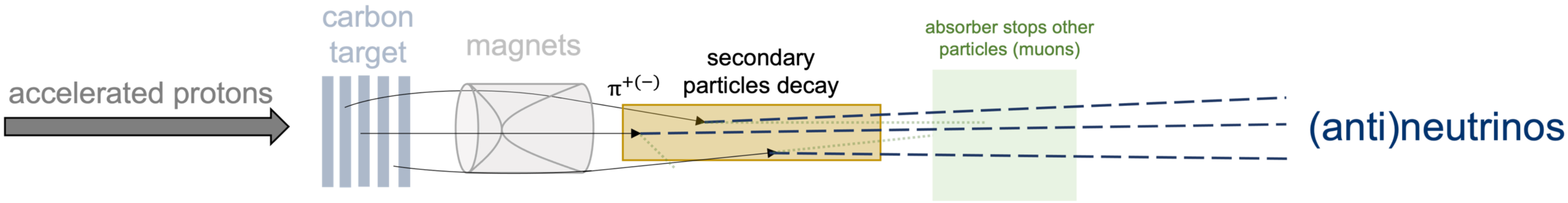
Unveiling the neutrino



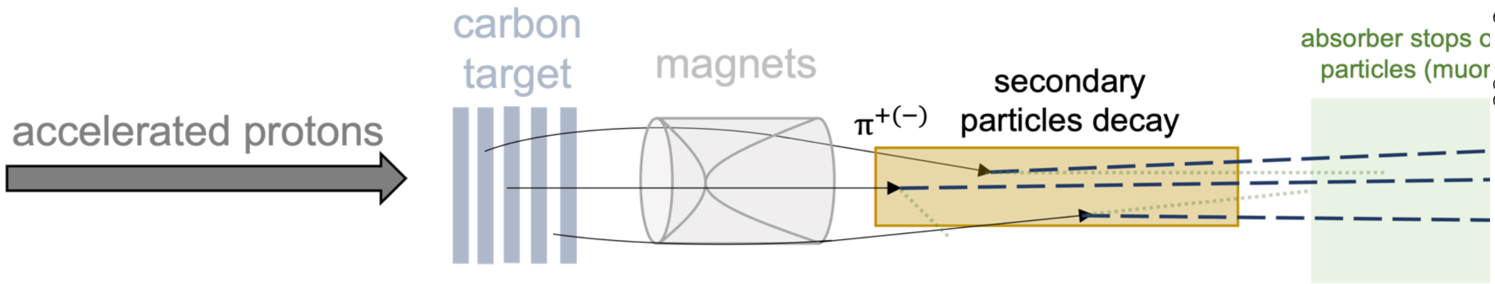
Accelerator-based neutrino experiments

- **ν_μ Beam Source:** Produced from accelerated protons

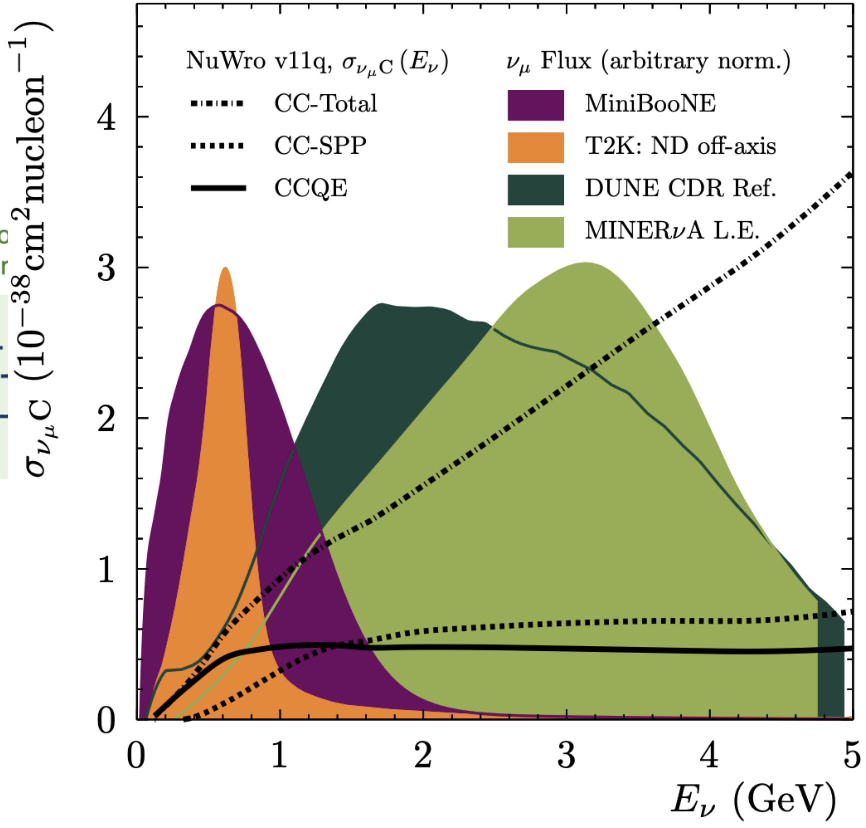


Accelerator-based neutrino experiments

- **ν_μ Beam Source:** Produced from accelerated protons
- **Energy Requirement:** Must be in the few GeV range



Wide energy neutrino beam



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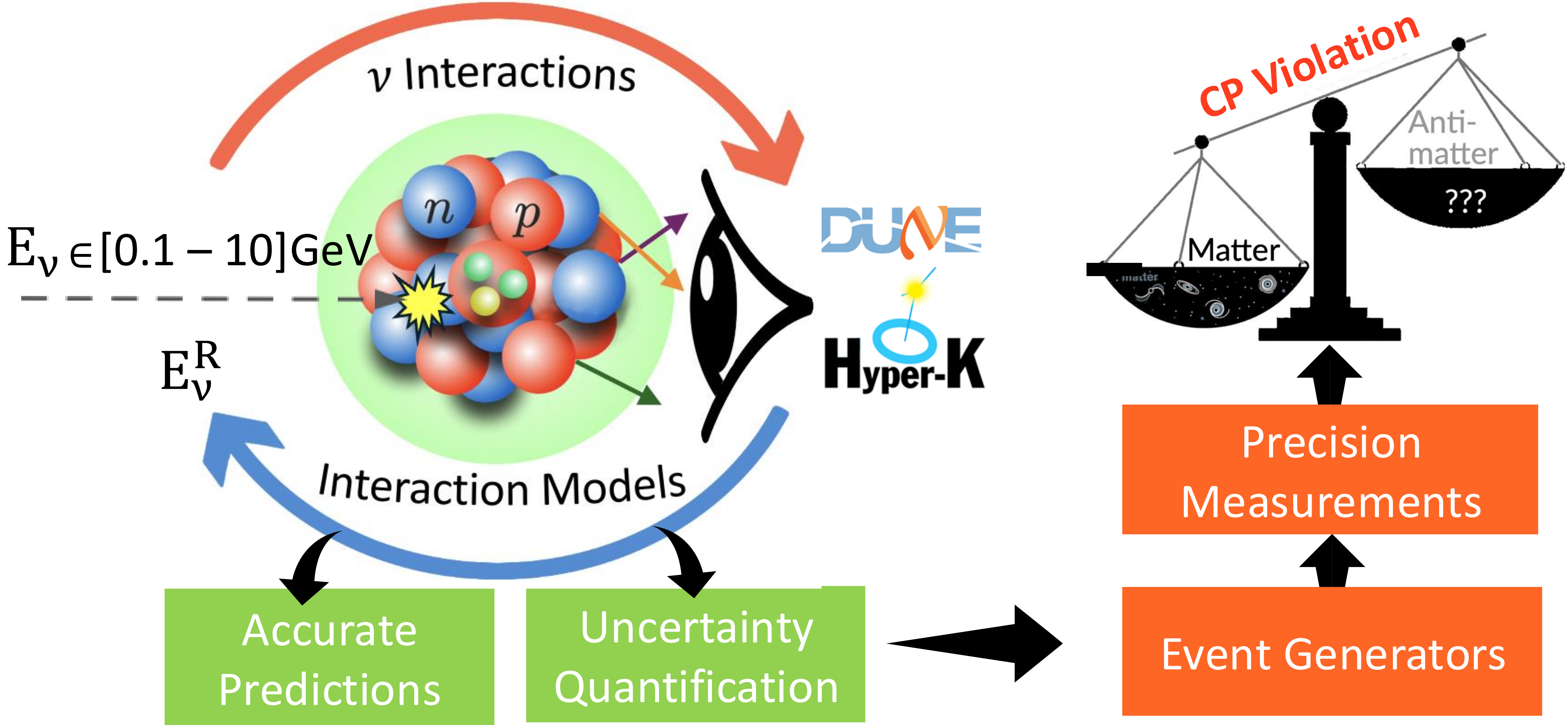
Unveiling the Neutrino: Customised Modelling for Future Discoveries



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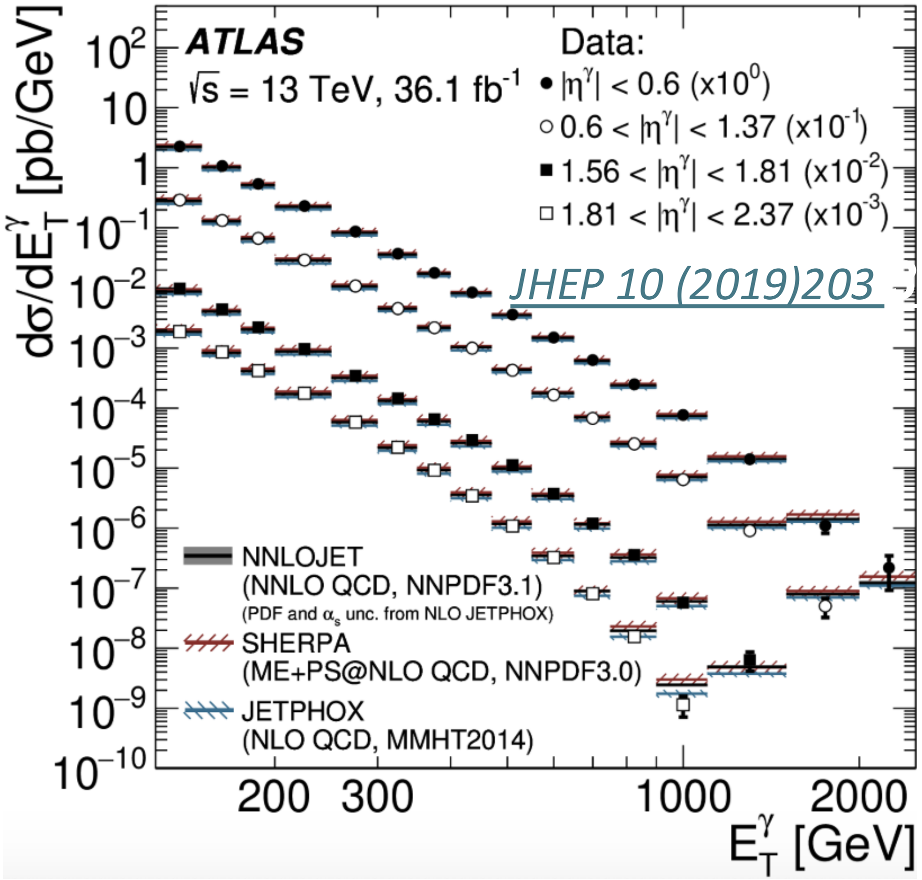
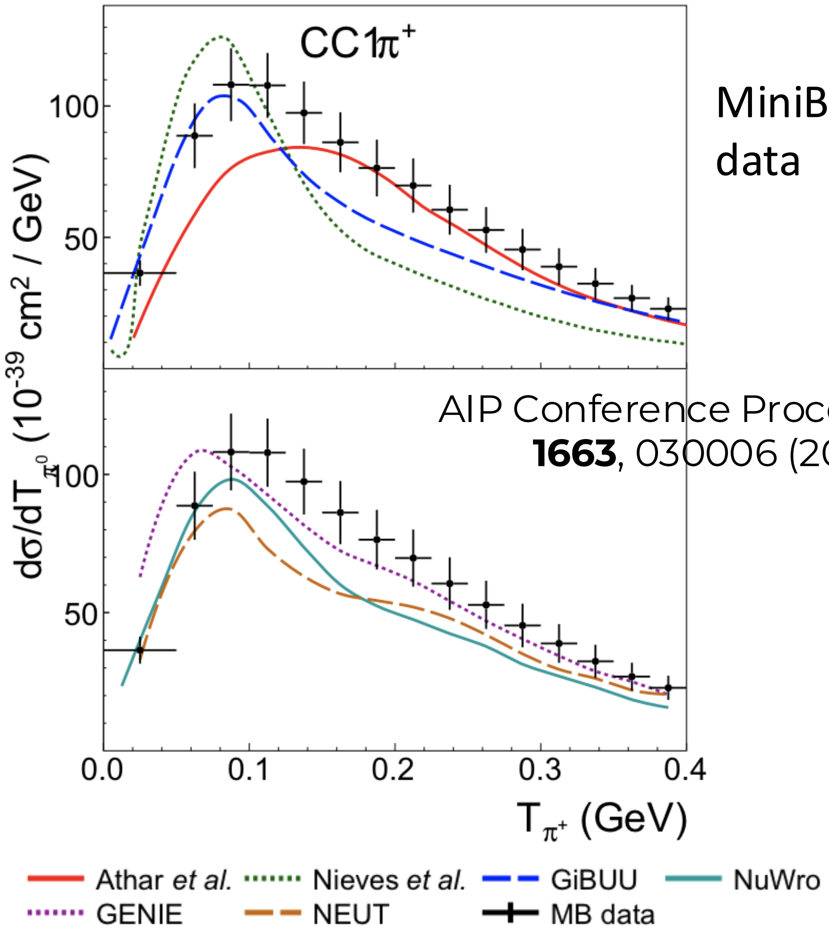
Cross-section uncertainties remain a major challenge



JHEP 10 (2019)203

ν_μ CH$_2 \rightarrow \mu^- \pi^+ X$



MiniBooNE data

AIP Conference Proceedings **1663**, 030006 (2015)

Phenomenological Form factors

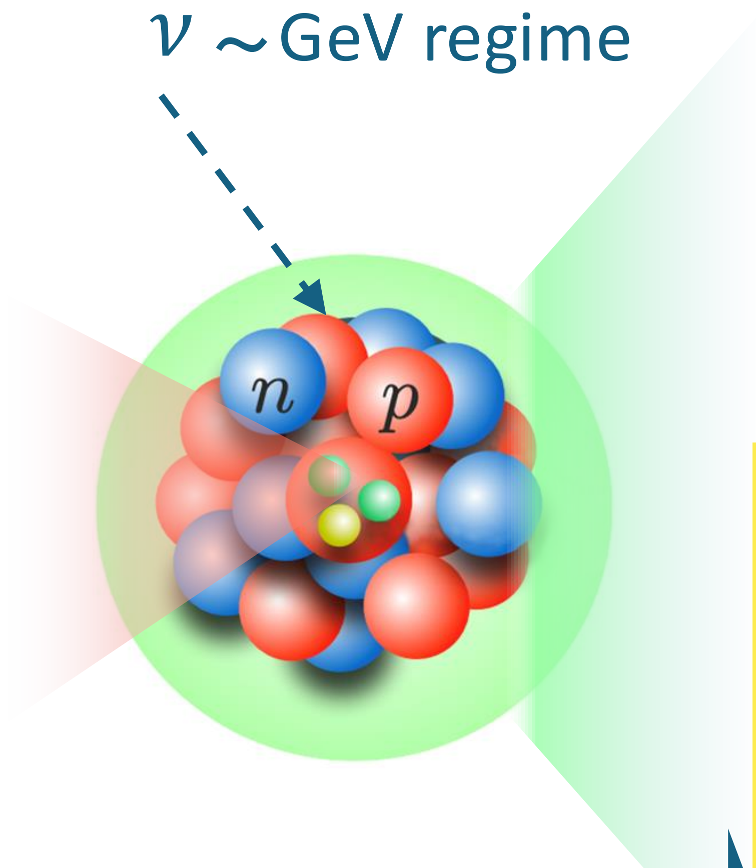
- Several parameters to fit using symmetries and experimental data

Nucleon Structure

Lattice QCD

- Form-factors exist only for Quasi-elastic channels and depend on the Form-factor models

Effective approach



First-principle approach

Independent-Particle Models

- Assumptions required for a realistic simulation
- Including leading-order correlations

Nuclear Medium

Quantum Simulation

- Solving many-particle quantum systems

Challenges in Modelling Neutrino Interactions at GeV Energies

- **Low Event Rates:** Historically discouraged precision calculations.
- **Complex Scattering Problem:** Slowed theoretical and experimental progress.
  - **Kinematic Coverage:** Neutrino beams (0.1–10 GeV) span a broad energy range, including a critical transition region, while available cross-section **data remains sparse**.
  - **Nuclear Effects:** Scattering occurs within complex multi-particle quantum systems, posing significant **computational challenges**.

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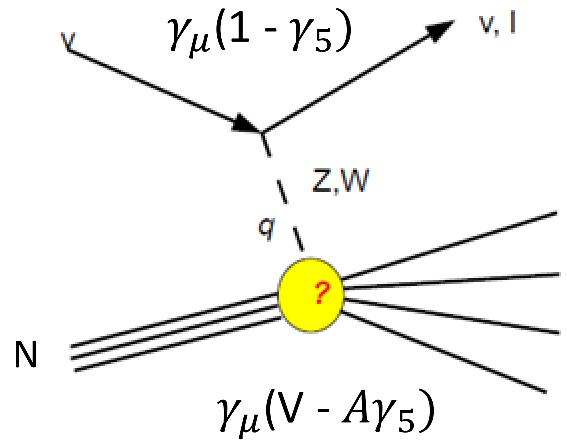
  - **Nuclear Effects:** Scattering occurs within complex multi-particle quantum systems, posing significant **computational challenges**.
- **Uncertainty Quantification:** Estimating model approximations as reliable inputs for precision measurements.

ν-nucleon interaction in the GeV regime

- **Free Nucleons:** Hydrogen target



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Bubble Chambers

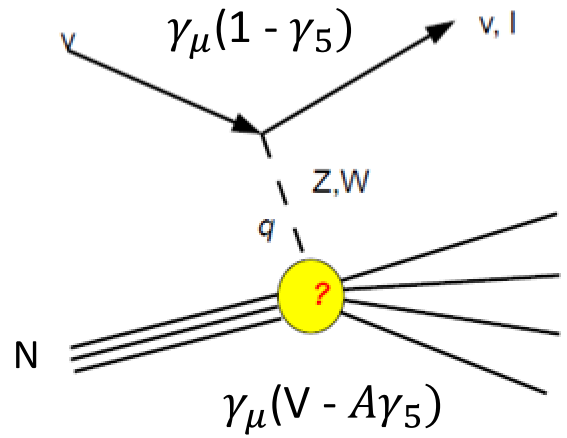
- **Targets:** Hydrogen & deuterium
- **Role:** Pioneered neutrino interaction studies (1960s–1980s)
- **Key Contribution:** Early cross-section measurements
- Now retired, often found in national lab car parks

The 15-foot Bubble Chamber FNAL

Big European Bubble Chamber (BEBC) CERN

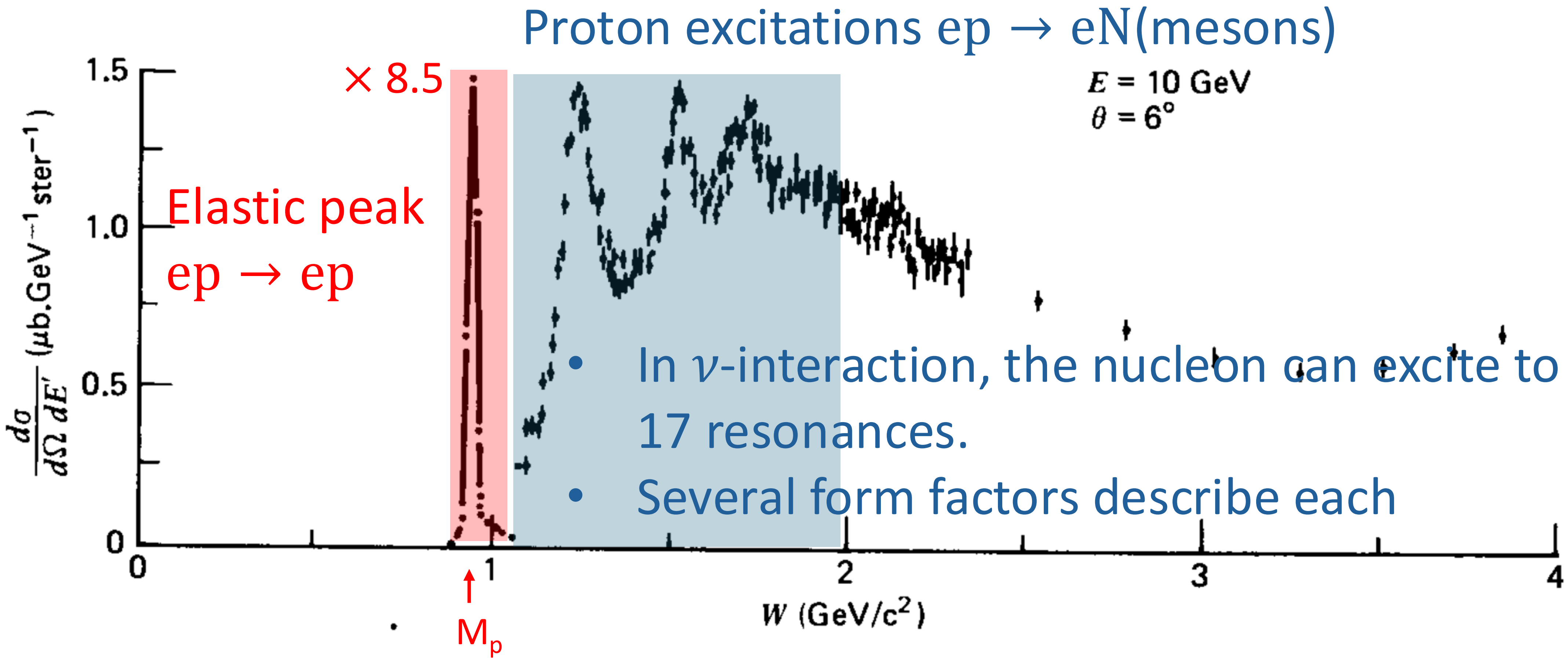
ν-nucleon interaction in the GeV regime

- **Free Nucleons:** Hydrogen target
- **Interaction Types:**
  - **Elastic & Quasi-elastic:**
    $\nu\mathrm{N} \rightarrow \mathrm{l}$(lepton)N(Nucleon)
  - **Inelastic:** $\nu\mathrm{N} \rightarrow \mathrm{lX}$ (hadrons)
- **Data Availability:**
  - Neutrino cross-section data is scarce
  - Electron scattering provides valuable insights



- V (Vector) & A (Axial-Vector) form factors describe how scattering is modified from a point-like nucleon

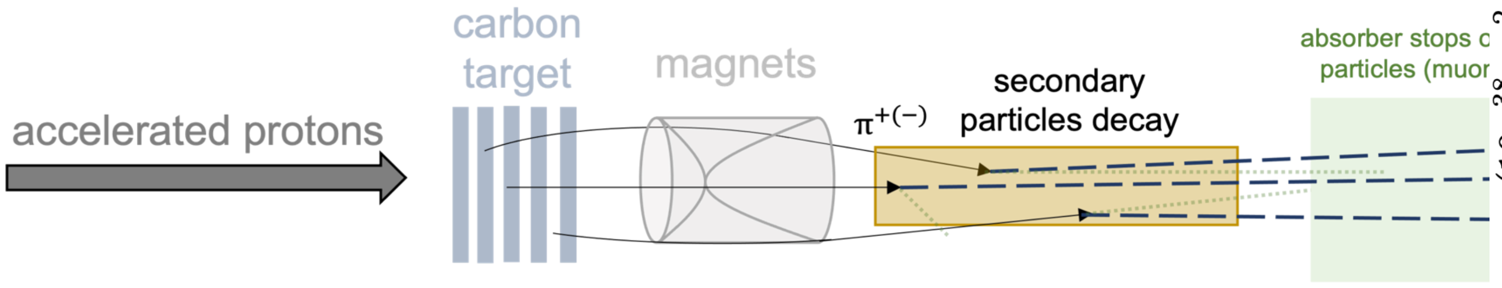
ep → eX cross-section



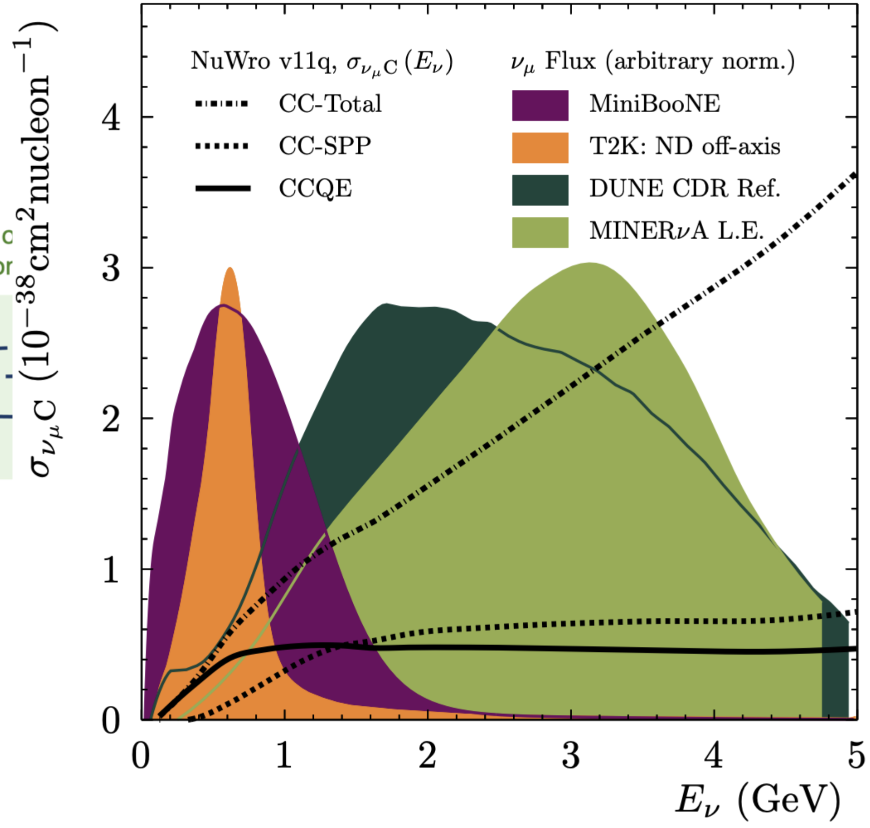
- In ν-interaction, the nucleon can excite to 17 resonances.
- Several form factors describe each

Accelerator-based neutrino experiments

- **ν_μ Beam Source:** Produced from accelerated protons
- **Energy Requirement:** Must be in the few GeV range

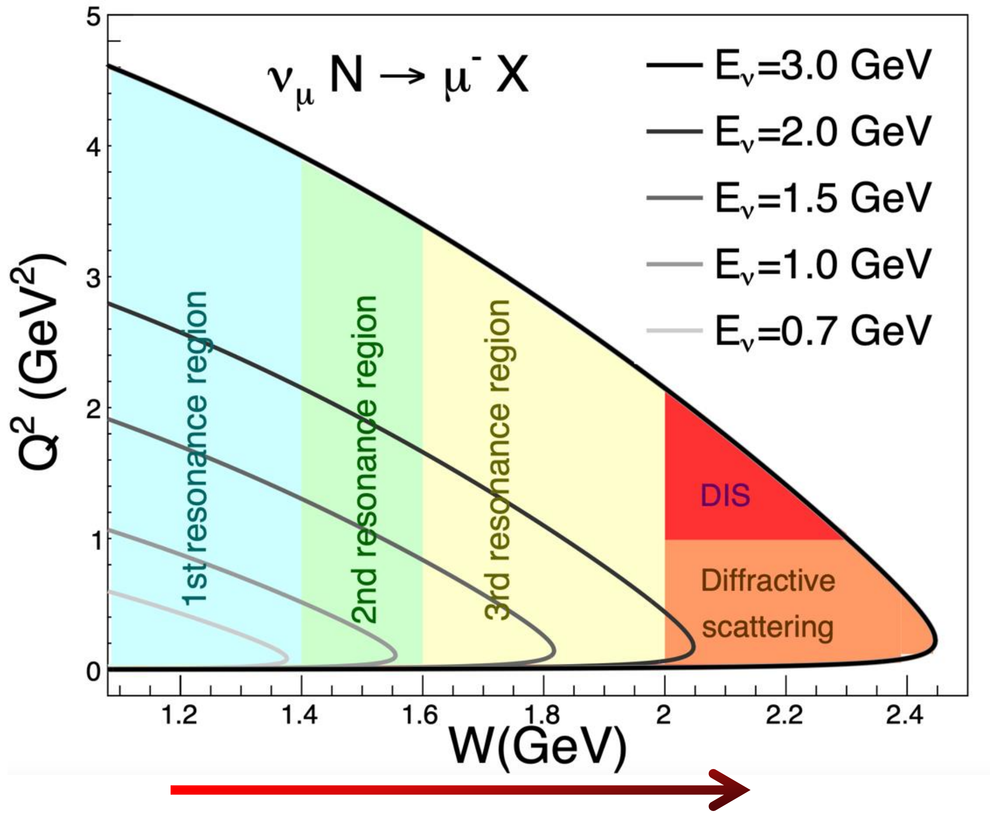


Wide energy neutrino beam

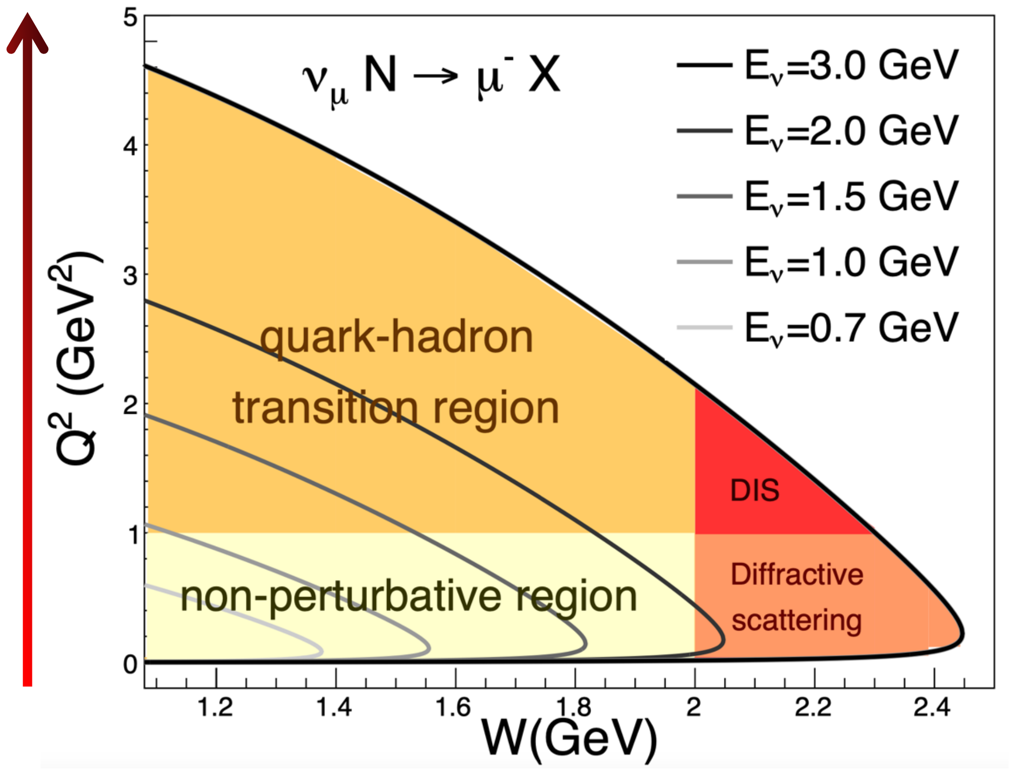


Transition region

W evolution



Q^2 evolution



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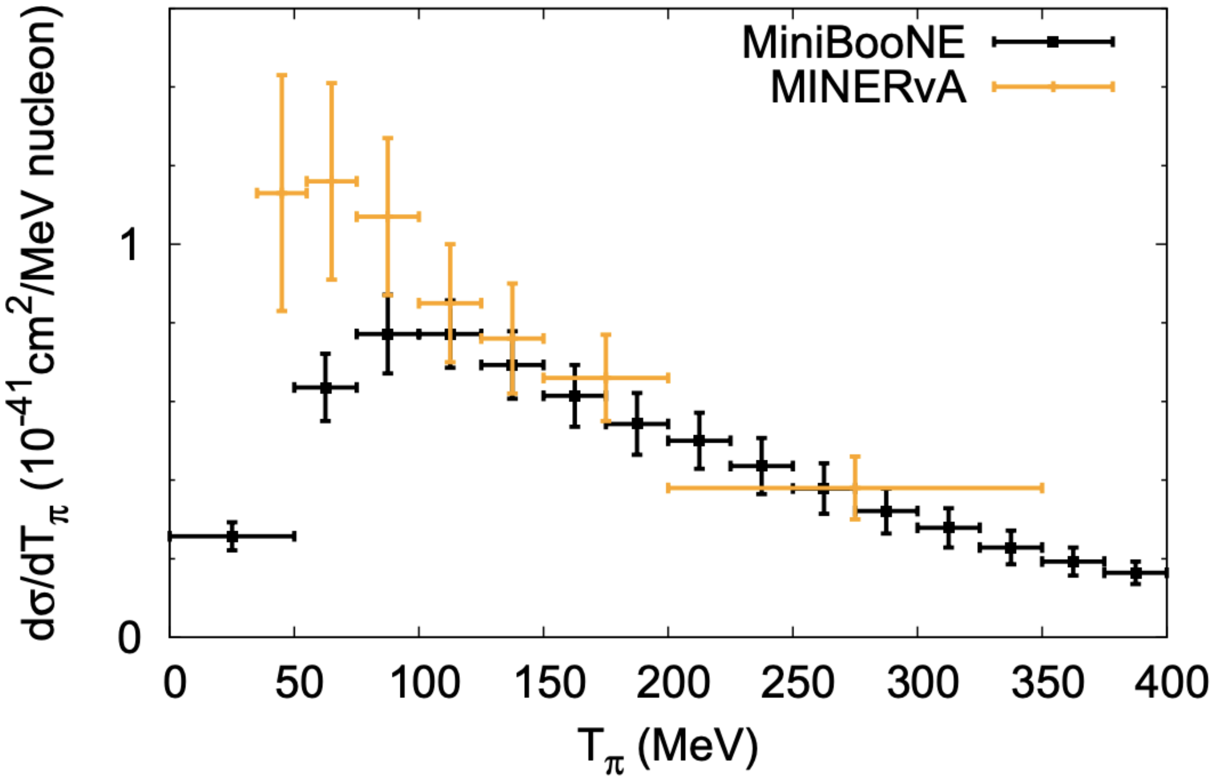
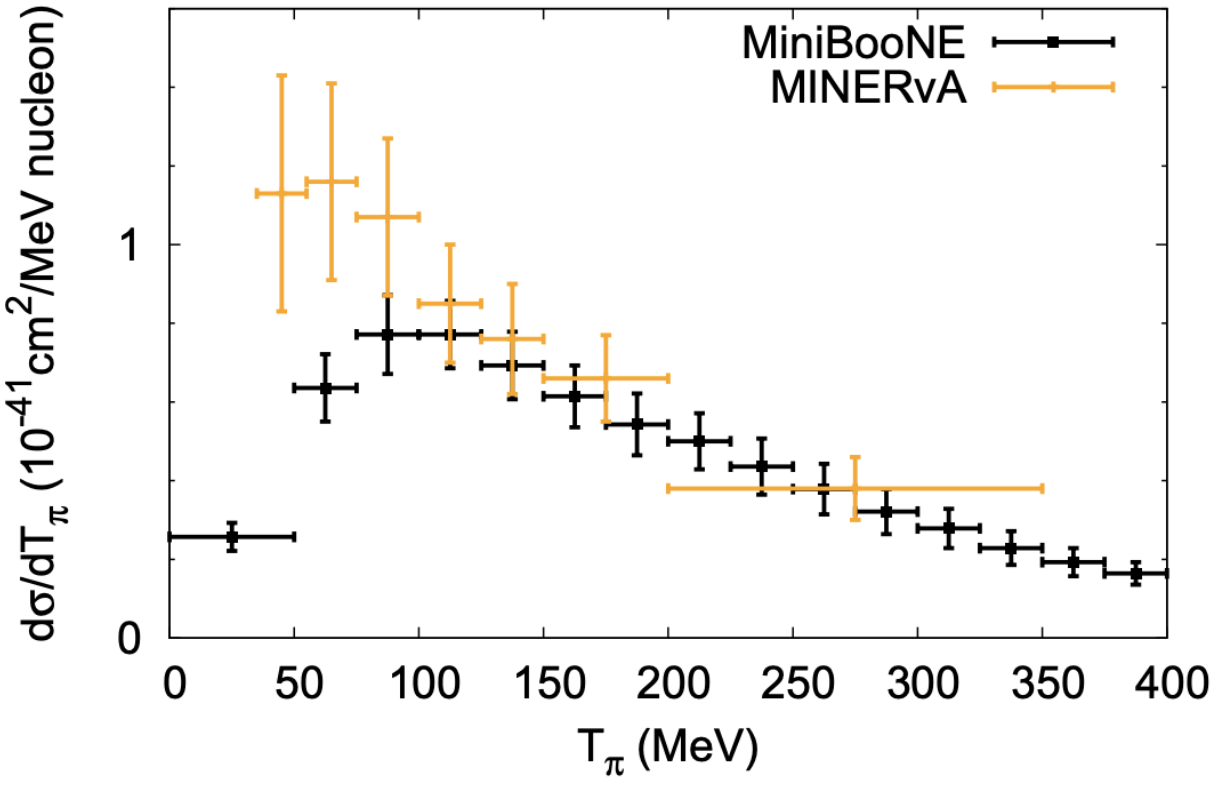
Tensions between **MiniBooNE** and **MINERvA**



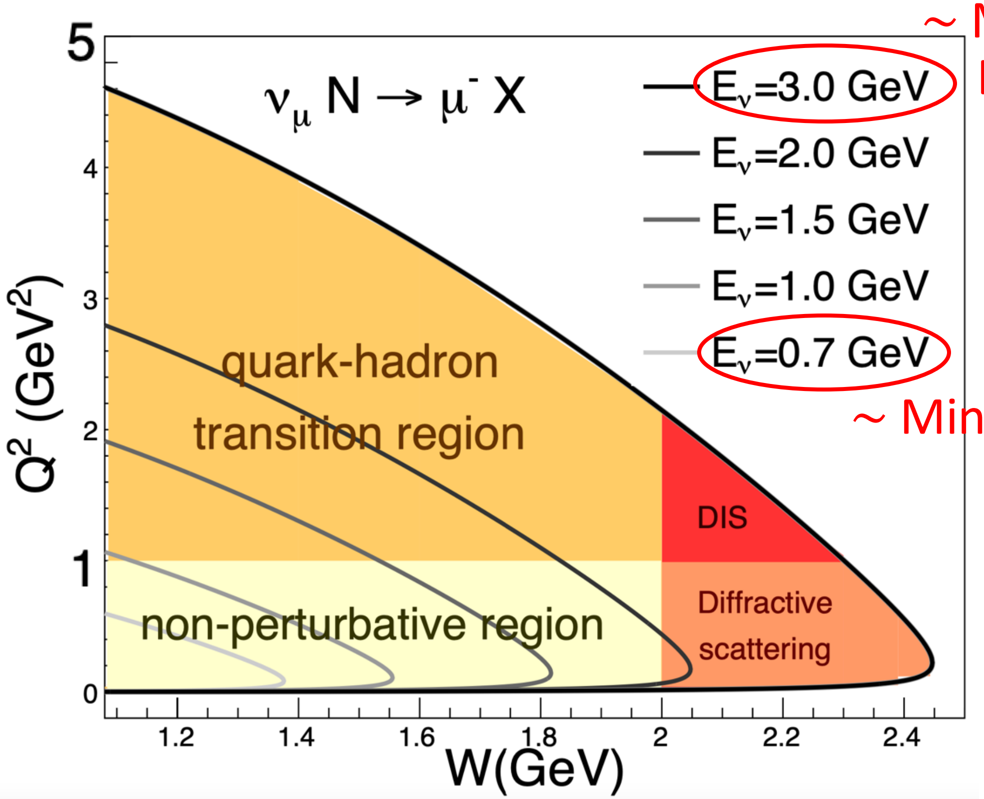
J. Sobczyk and J. Zmuda
Phys. Rev. C **91** (2015)

- Tensions between MiniBooNE and MINERvA for single pion production measurements on CH_2 and CH targets in the **first resonance region**.

Tensions between MiniBooNE and MINERvA



J. Sobczyk and J. Zmuda
Phys. Rev. C **91** (2015)



~ MINERvA, DUNE

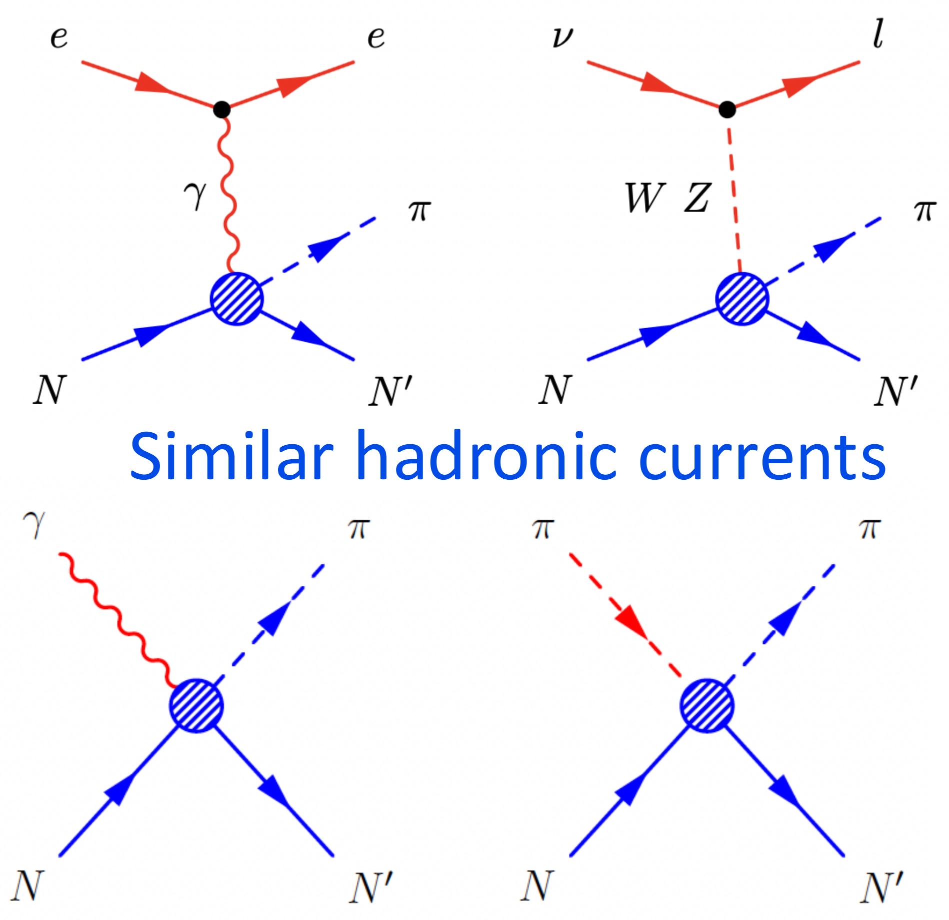
~ MiniBooNE

Challenges and Uncertainties in the complex GeV Region

- **Multiple Processes**: The GeV energy range involves a complex interplay of interaction mechanisms on the nucleon, requiring precise knowledge of form factors for each process.
- **Quark-Hadron Transition**: Begins as early as Q^2, requiring careful treatment.
- **Parametrisation**: Essential for incorporating unknown physics and handling interfering processes.
- **Uncertainty Estimation**: Accurate assessment of theoretical uncertainties is a crucial input for neutrino measurements.

Solution for single pion production: MK model

- The MK model comprehensively describes single-pion production in interactions involving **photons, electrons, and neutrinos** with nucleons.
- Phenomenological models in this region must account for numerous processes and parameters.
- A unified model is essential for interpreting all interactions and **maximising data utilisation**.



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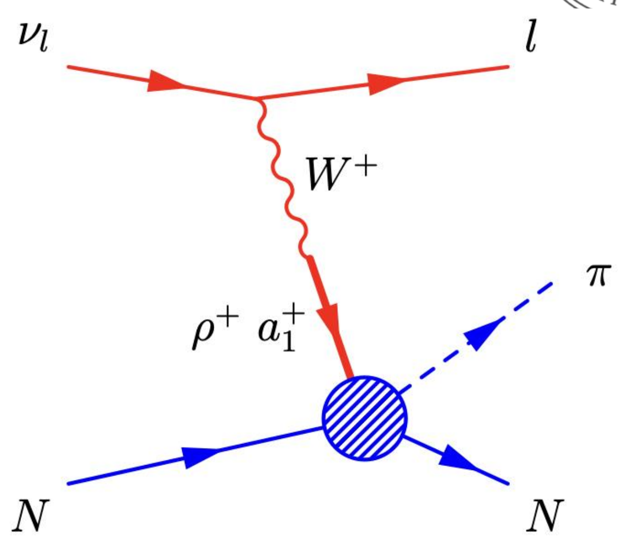
Form Factors in the model



- The Meson dominance model is rooted in the effective Lagrangian of quantum field theory.

1. J. J. Sakurai, Annals Phys.11, 1 (1960)
2. M. Gell-Mann and F. Zachariasen, Phys. Rev. 124, 953 (1961)

- This framework explains the interaction between neutrinos and nucleons through meson exchange.
- The number of parameters in several form factors is reduced by imposing all symmetries and unitarity.
- At large Q2, resonance form factors must align with the perturbative QCD constraints.

Data used in the Joint analysis

# data point	Photon, electron, pion, Neutrino Channels	Q^2 Range $(GeV/C)^2$	W Range GeV	Form Factors	
≈ 9800	$\gamma p \to n + \pi^+$, $\gamma p \to p + \pi^0$	0	1.08 – 2.0	Proton	Vector
≈ 31000	$ep \to en + \pi^+$, $ep \to ep + \pi^0$	0.16 – 6.0	1.08 – 2.0		
≈ 2500	$\gamma n \to p + \pi^-$	0	1.08 – 2.0	Neutron	
≈ 700	NEW $en \to ep + \pi^-$	0.4 – 1.0	1.08 – 1.8		
≈ 400	$\pi^+ p \to p + \pi^+$, $\pi^- p \to p + \pi^-$	0	1.08 – 2.0	Axial-Vector	
<100	$\nu N \to l^- N + \pi$, $\bar{\nu} N \to l^+ N + \pi$	Q^2>0 Integrated	1.08 – 2.0 Integrated		

Intermediate Energy: Non-Perturbative QCD

- **QCD Breakdown at Low Q^2:**
  - As the momentum transfer Q^2 decreases, the coupling strength of QCD increases, causing the perturbative expansion to fail.
- **Lattice QCD (LQCD)**:
  - A numerical approach formulated with quark and gluon degrees of freedom to describe the non-perturbative regime.
  - powerful computational tool for calculating weak matrix elements in neutrino physics.
- **Complementary to Experiment**: LQCD provides essential insights into axial form factors and other observables, particularly when experimental neutrino data is scarce.

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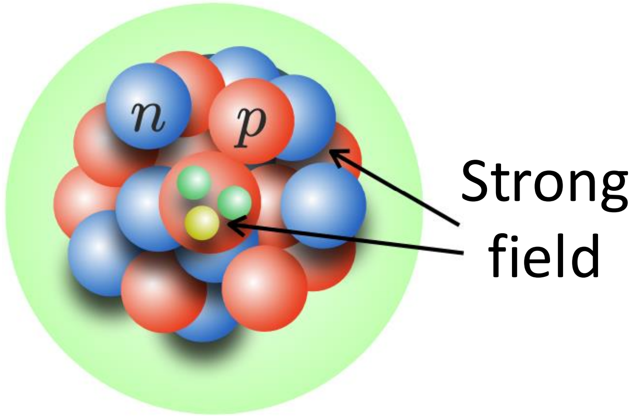
Nuclear Structure in the GeV regime

- Nuclei are strongly interacting many-body quantum systems.
- Simulating strongly correlated quantum systems, such as argon targets in neutrino experiments, remains a challenge for classical computers.



Picture from N. Rocco

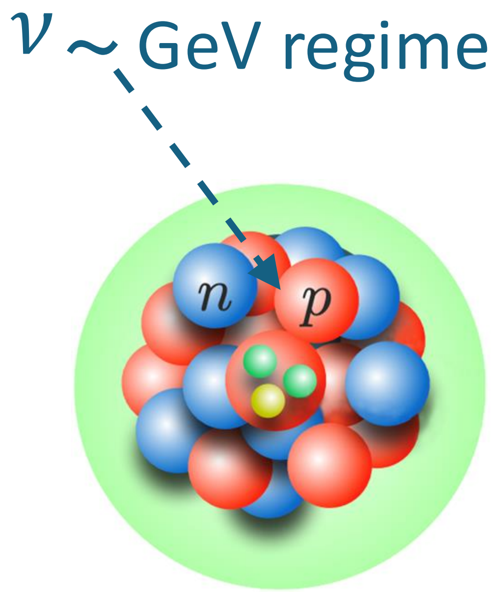
- **Modelling assumptions** are crucial for describing medium-to-heavy nuclei in neutrino interactions.
- These modelling assumptions, along with how we study nuclear structure, vary depending on the energy regime under investigation

How We Study Nuclei Across Energy Scales

- **High Energy (TeV)**
  - **Perturbative domain:** The strong force can be treated using perturbative methods.
  - **Quantum Chromodynamics (QCD):** Describes interactions between quarks via gluon exchange.
- **Low Energy (MeV)**
  - **Non-perturbative domain:** The strong force requires effective theoretical approaches.
  - **Nuclear physics:** Describes interactions between protons and neutrons through pion exchange.
- **Intermediate energy (GeV) ?**

How We Study Nuclei Across Energy Scales

- **High Energy (TeV): Perturbative QCD**
- **Low Energy (MeV): Nuclear Physics**
- **Intermediate energy (GeV)**
  - **Transition region:** Neither fully perturbative nor fully non-perturbative.
  - **Hadronisation & Resonances:** Quark interactions give rise to bound states (hadrons) and excited nuclear resonances.
  - **Final-State Interactions (FSI):** Produced hadrons undergo re-scattering within the nuclear medium before detection.



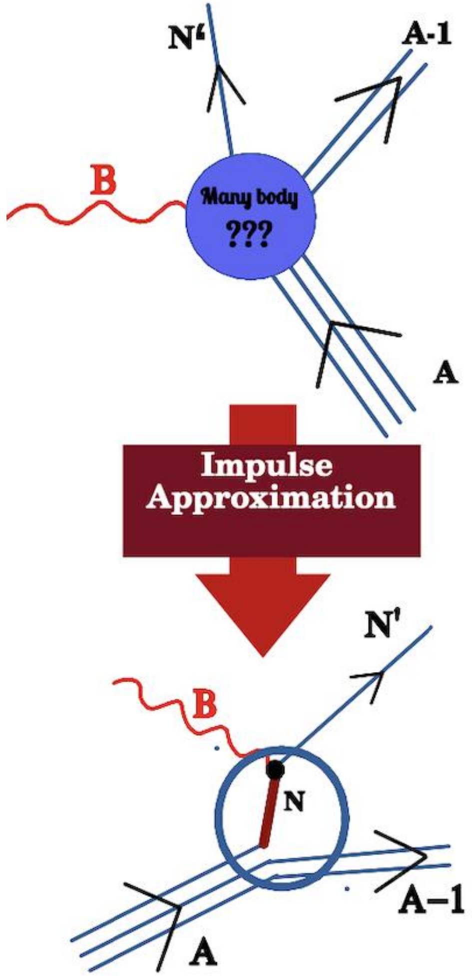
Intermediate Energy: Nuclear Physics

- **Independent-Particle Models:** Effective at hundreds of MeV but limited at higher energies.
  - **Leading order:** Nucleons behave independently.
  - **Next-to-leading order:** Includes nucleon-nucleon correlations.
  - **Energy Dependence:** The importance of nuclear correlations varies with energy.
- **Long- to Short-Range Correlations**:
- Includes multi-pion exchange or contributions from heavy mesons.
- Characteristic range:

Long-range: $\frac{1}{m_\pi} \sim 1.4\ fm$ **Short-range**: $< \frac{1}{2m_\pi} \sim 0.7\ \text{fm}$

ν-Nucleus scattering (Impulse approximation)

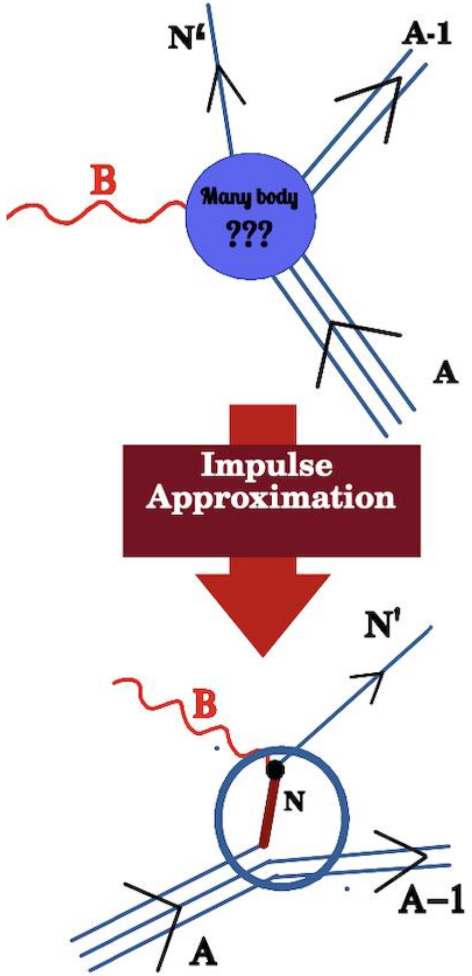
- The neutrino interacts with a single nucleon moving in an average potential created by surrounding nucleons.
- A bound nucleon is described by a wave function (Dirac spinor), which is a solution of the Schrödinger or Dirac equation.
- The nuclear many-body current reduces to a sum of one-body operators.



Picture from R. González Jiménez

Scattered Nucleon (Pion) Wavefunction

- The wavefunction of the scattered nucleon (or pion) is a solution to the Dirac or Schrödinger (Klein-Gordon) equation in the presence of a nuclear potential.
- The choice of nuclear potential significantly affects the scattered nucleon wavefunctions, influencing predictions.
- **Factorisation Approximation:** Outgoing hadrons are described by plane waves, reducing computational cost.



Picture from R. González Jiménez

Conclusion and prospect

- Understanding neutrino interactions is essential for precision measurements in current and future neutrino experiments.
- A major challenge arises from the limitations of classical computing and existing data in handling strongly correlated systems, such as argon nuclei in the GeV region.
- Advancing nuclear models requires the integration of cutting-edge computational techniques and a rigorous approach to uncertainty estimation.
- Progress in this field demands interdisciplinary collaboration across nuclear physics, quantum many-body theory, experimental techniques, and advanced computational and statistical methods.

Backup

M. Kabirnezhad
Phys. Rev. D **97** (2018)
Phys. Rev. D **102** (2020)
Phys.Rev.C **107** (2023)

MK model

The MK model comprehensively describes single-pion production in interactions involving **photons, electrons, and neutrinos** with nucleons.

- Meson Dominance (MD) form factor: Maintains **unitarity** and integrates **QCD principles** for both resonant and non-resonant interactions.
- **CVC and PCAC** fulfilment: Ensures model consistency at low Q^2.
- Q^2 evolution: Utilises QCD calculations and **quark-hadron duality**.
- W evolution: Applies **Regge trajectory** and the Hybrid model.

R. González-Jiménez, *et al*
Phys. Rev. D **95** (2017)

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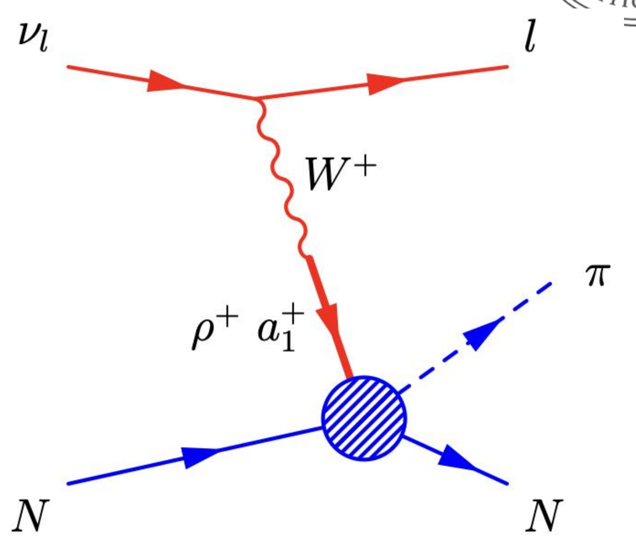
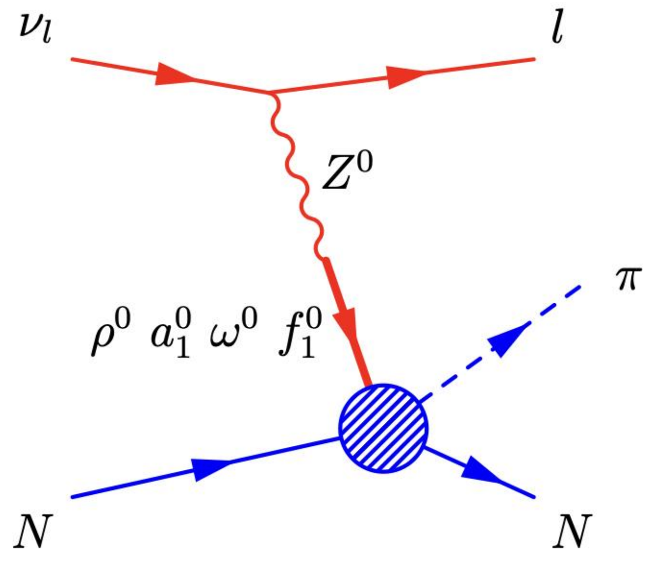
Meson Dominance (MD) model

- The MD model is rooted in the effective Lagrangian of quantum field theory.

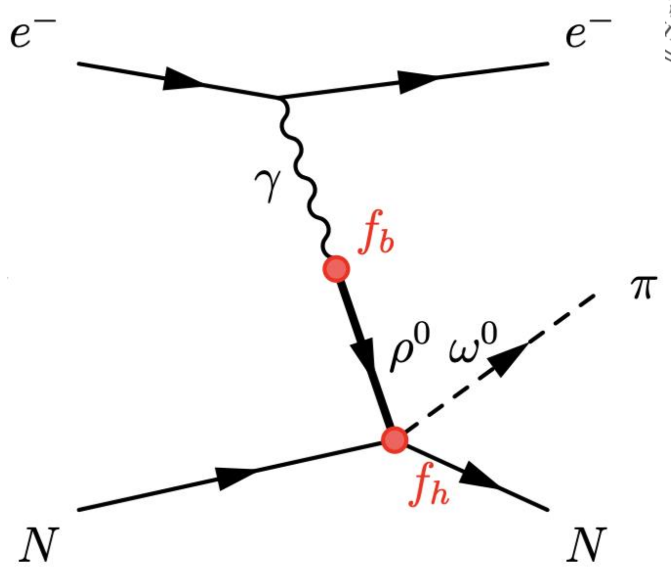
1. J. J. Sakurai, Annals Phys.11, 1 (1960)
2. M. Gell-Mann and F. Zachariasen, Phys. Rev. 124, 953 (1961)

- It establishes connections between vector and axial currents and corresponding meson fields with analogous quantum properties.
- This framework explains the interaction between neutrinos and nucleons through meson exchange.





Meson Dominance (MD) model



- MD form factors can be expressed in terms of the meson masses and the coupling strengths, summing over all possible mesons:

$$\mathrm{F_N}(Q^2) = \sum_{j=1}^{n} \frac{m_j^2}{m_j^2 - Q^2} \left(\frac{f_h}{f_b}\right)$$

- Although they do not inherently comply to the unitarity condition (analytic model) or accurately predict behaviour at high Q^2, they can be **imposed**!

C. Adamuscin *et al*. Eur. Phys. J. C 28, 115 (2003)

k	ρ -group	$m_{(\rho)k}$[GeV]	ω -group	$m_{(\omega)k}$[GeV]
1	$\rho(770)$	0.77526	$\omega(782)$	0.78265
2	$\rho(1450)$	1.465	$\omega(1420)$	1.410
3	$\rho(1700)$	1.720	$\omega(1650)$	1.670
4	$\rho(1900)$	1.885	$\omega(1960)$	1.960
5	$\rho(2150)$	2.150	$\omega(2205)$	2.205
k	a_1 -group	$m_{(a_1)k}$[GeV]	f_1 -group	$m_{(f_1)k}$[GeV]
1	$a_1(1260)$	1.230	$f_1(1285)$	1.2819
2	$a_1(1420)$	1.411	$f_1(1420)$	1.4263
3	$a_1(1640)$	1.655	$f_1(1510)$	1.518
4	$a_1(2095)$	2.096	$f_1(1970)$	1.1971

Asymptotic behaviour of form factor

- At large Q2, resonance form factors must align with the perturbative QCD constraints.
- For spin 3/2 resonance:

G. Vereshkov and N. Volchanskiy (PRD 2007)

$$F_\alpha(Q^2) \cong \left(\frac{4M_N^2}{Q^2}\right)^{p_\alpha} \frac{f_\alpha}{\ln^{n_\alpha}\left(Q^2/\Lambda_{QCD}^2\right)}, \qquad (\alpha = 1-3)$$

$$p_1 = 3, p_2 = p_3 = 4,$$
$$n_3 > n_1 > n_2, \qquad n_1 \cong 3$$

MD form factors used in the model

- For spin 3/2 resonance:

$$F_\alpha(Q^2) = \frac{f_\alpha}{L_\alpha(Q^2)} \sum_{k=1}^{K} \frac{a_{\alpha k}\, m_k^2}{m_k^2 + Q^2}, \qquad (\alpha = 1-3)$$

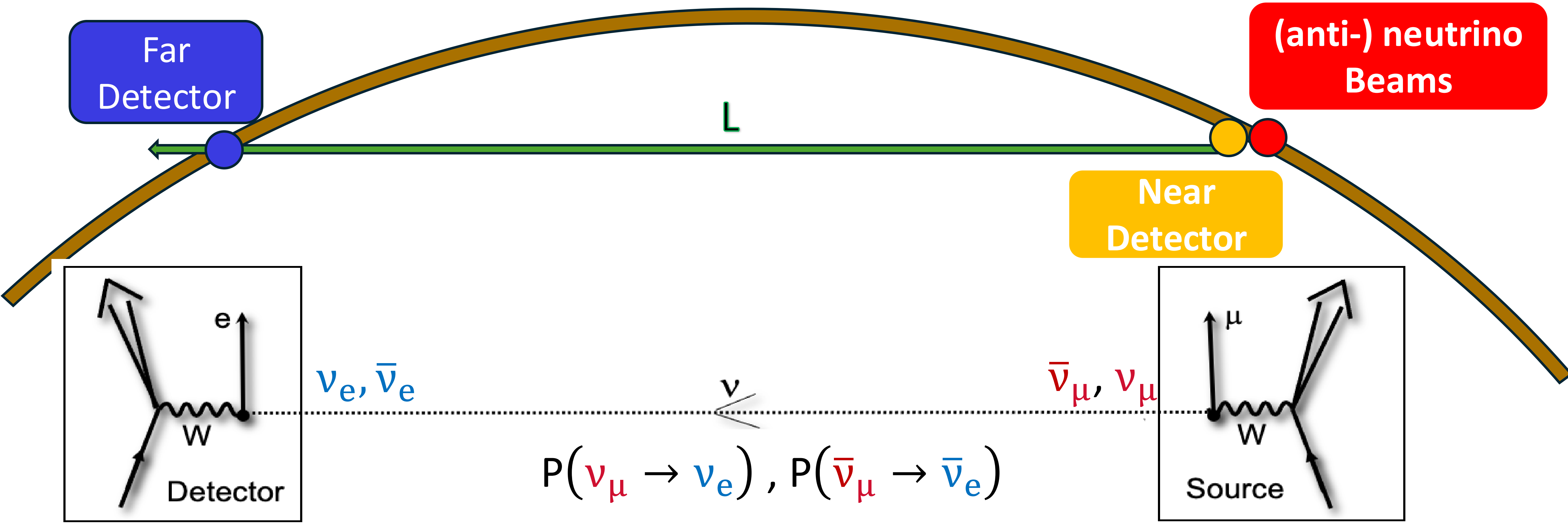
$$L_\alpha(Q^2) = \left[1 + g_\alpha \ln\left(1 + \frac{Q^2}{\Lambda_{QCD}^2}\right) + h_\alpha \ln^2\left(1 + \frac{Q^2}{\Lambda_{QCD}^2}\right)\right]^{n_\alpha}$$

$n_1 = 3,\ n_2 = 2,\ n_3 = 4$

$\Lambda_{QCD} \in [0.19 - 0.24]$ GeV

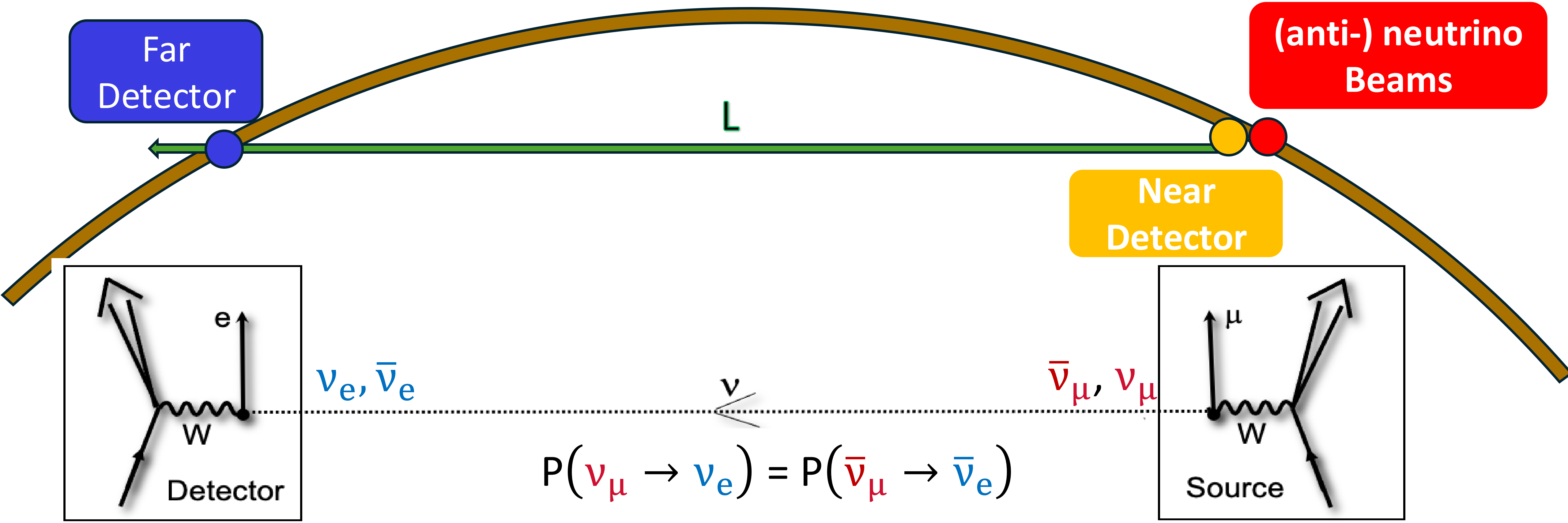
- $a_{\alpha k}$ and $b_{\beta k}$ are constrained by unitarity conditions that also satisfy asymptotic QCD requirements.

Neutrino Oscillation experiments



$P(\nu_\mu \to \nu_e)$, $P(\bar{\nu}_\mu \to \bar{\nu}_e)$

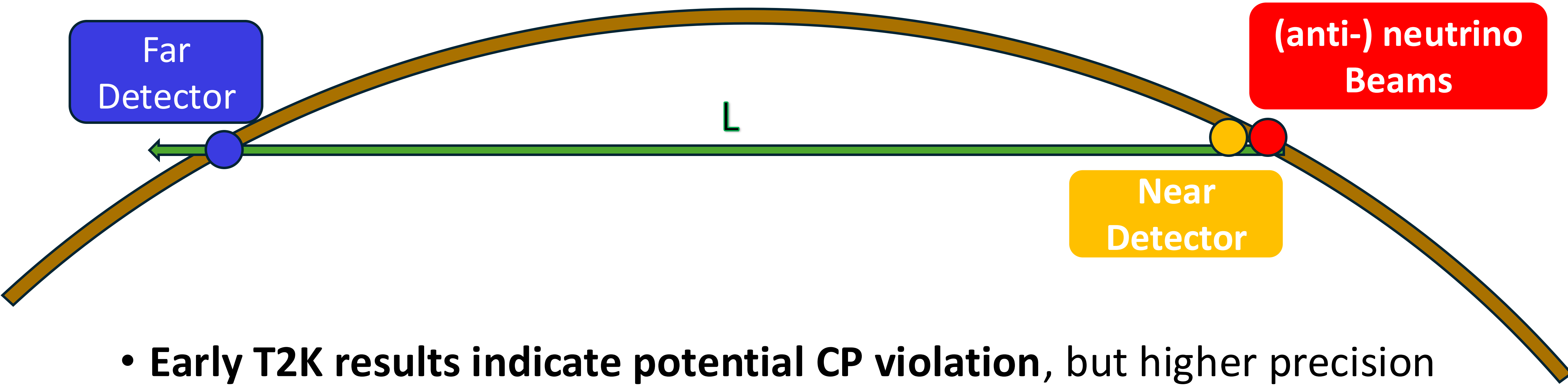
Neutrino Oscillation experiments



$P(\nu_\mu \rightarrow \nu_e) = P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$

CP conservation ($\delta_{CP} = 0$)

Neutrino Oscillation experiments



- **Early T2K results indicate potential CP violation**, but higher precision measurements are required to confirm this.
- **Cross-section uncertainties remain a major challenge**, impacting the precision of oscillation parameter measurements.