B and *D* physics: selected topics

Enrico Lunghi
Indiana University

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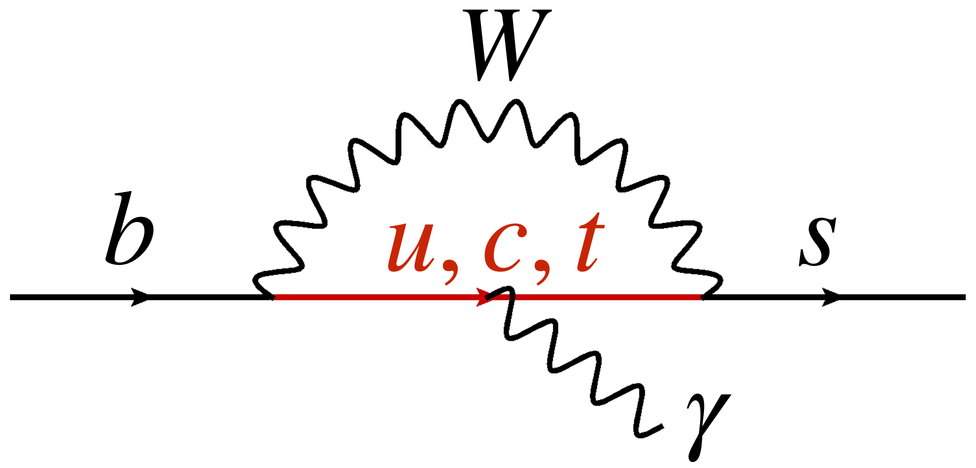
B, D and K physics

- The structure of the SM prohibits Flavor Changing Neutral Currents (FCNC) at tree level:

$$Y_U = U_L^\dagger Y_U^{\rm diag} U_R \qquad \bar{u}_L^0\, Z\, u_L^0 \Rightarrow \bar{u}_L\, Z\, U_L U_L^\dagger u_L = \bar{u}_L\, Z\, u_L$$

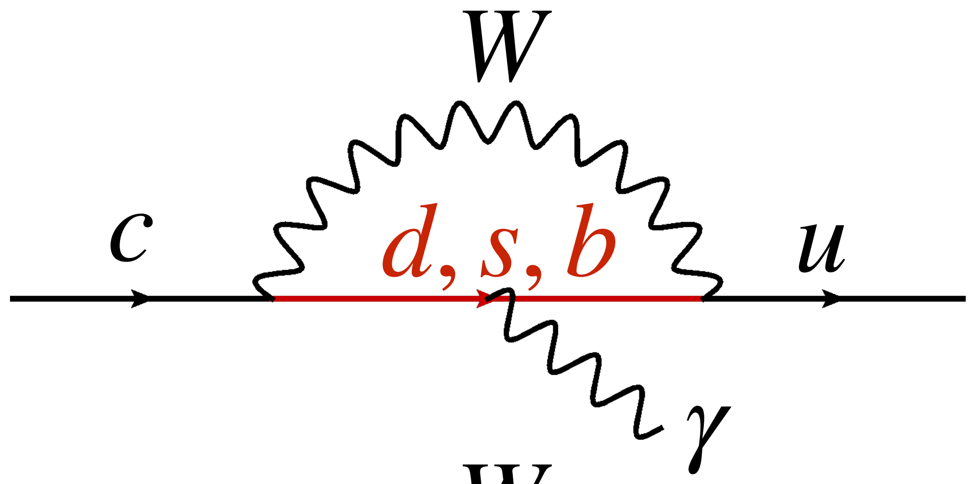
$$Y_D = D_L^\dagger Y_D^{\rm diag} D_R \qquad \bar{u}_L^0\, \not{W}\, d_L^0 \Rightarrow \bar{u}_L\, \not{W}\, U_L D_L^\dagger d_L = \bar{u}_L\, \not{W}\, V_{\rm CKM}\, d_L$$

- FCNC are generated at the 1-loop level but are still subject to the GIM mechanism
  - Bottom:



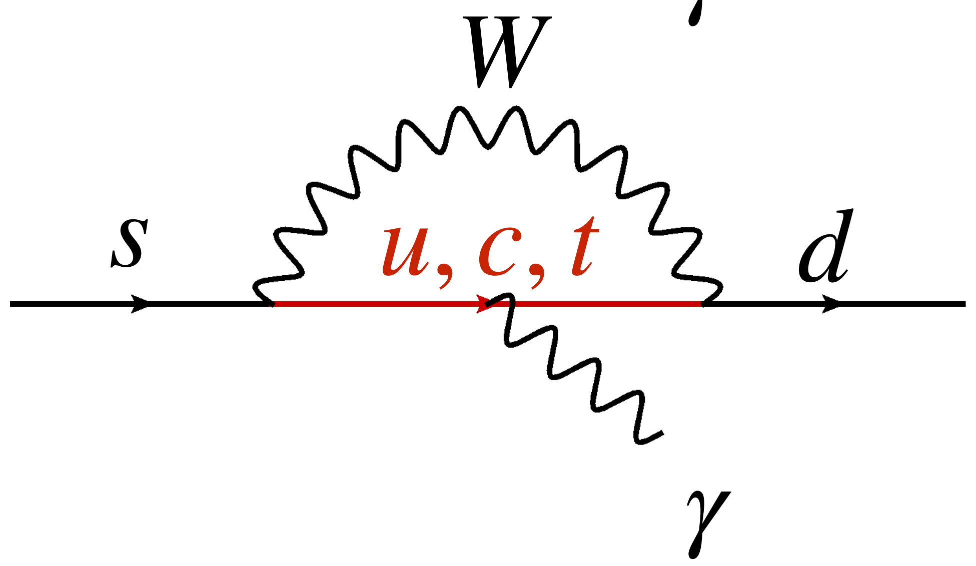
$$\propto V_{tb}^* V_{ts} \frac{m_t^2}{m_W^2} \sim (4 \times 10^{-2}) \times 4$$

  - Charm:



$$\propto V_{ub}^* V_{cb} \frac{m_b^2}{m_W^2} \sim (2 \times 10^{-4}) \times (3 \times 10^{-3})$$

  - Strange:

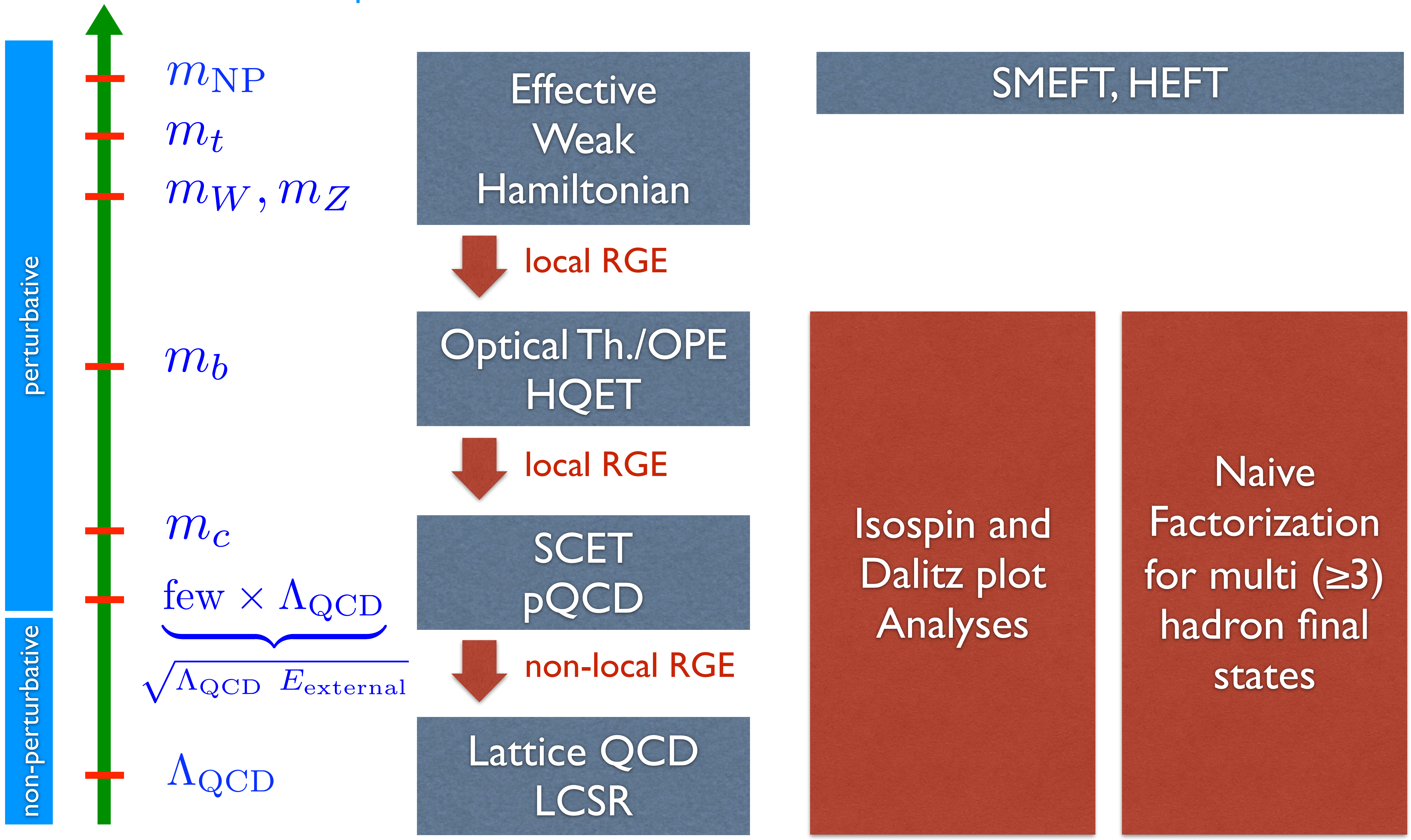


$$\propto V_{ts}^* V_{td} \frac{m_t^2}{m_W^2} \sim (4 \times 10^{-4}) \times 4$$

B, D and K physics

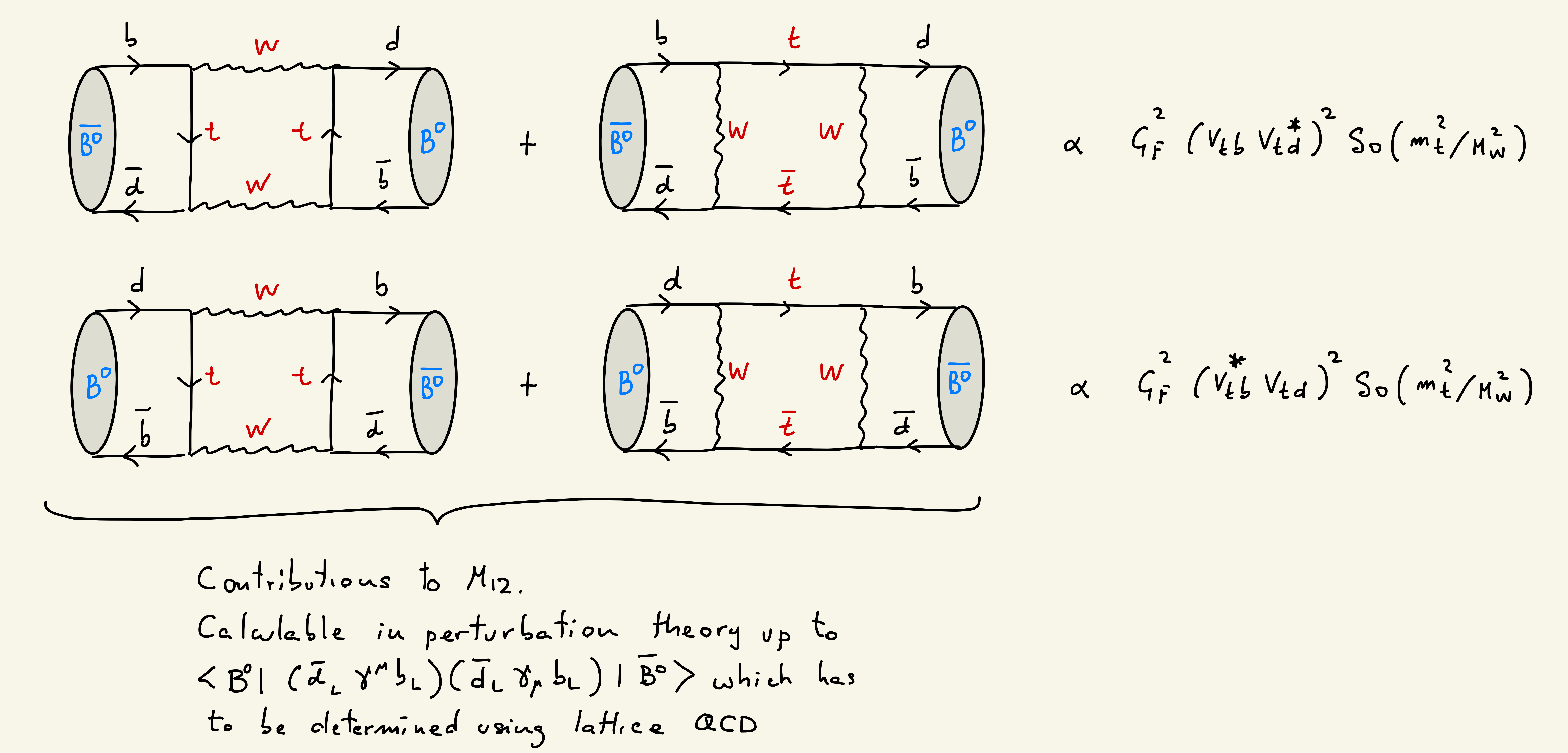
	Size (CKM x loop factor)	**Phase Space**	**Theory uncertainties**
B	+++ Large amplitudes allow study of spectra and asymmetries	+++ Large number of decay modes	+++ O(5%) accuracy is attainable in most observables
D	+ Small SM amplitudes are a challenge for differential studies but an opportunity for BSM searches	++	+ Major problem with non-perturbative effects: charm is neither light nor heavy
K	++	+ Limited number of modes	++ Some processes are very clean ($K_L, K^+ \to \pi\nu\bar{\nu}$), others suffer from large uncertainties ($\Delta M_K, \varepsilon'/\varepsilon$)

A tale of multiple scales



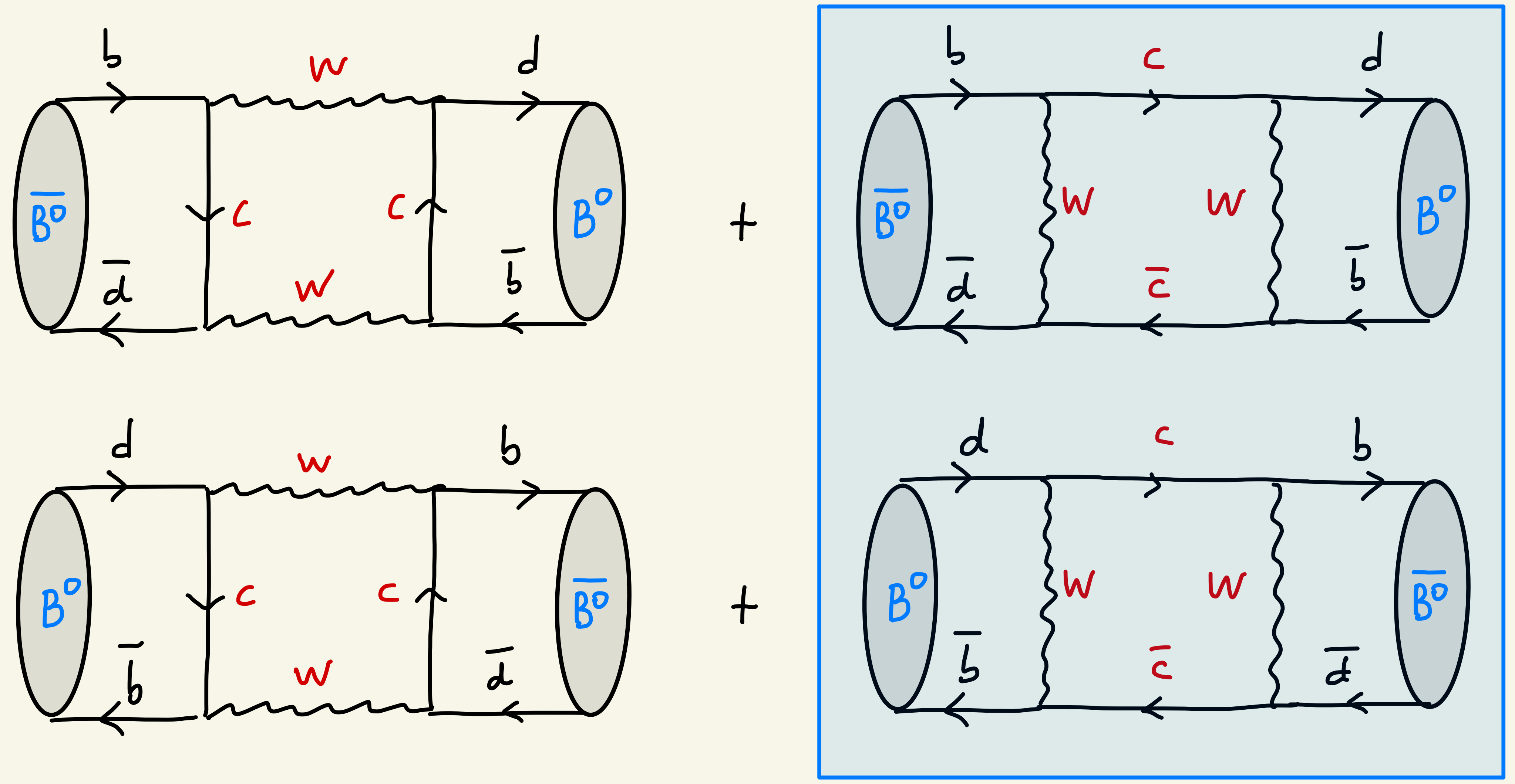
B mixing

- Dispersive contributions are perturbative (M_{12}):



B mixing

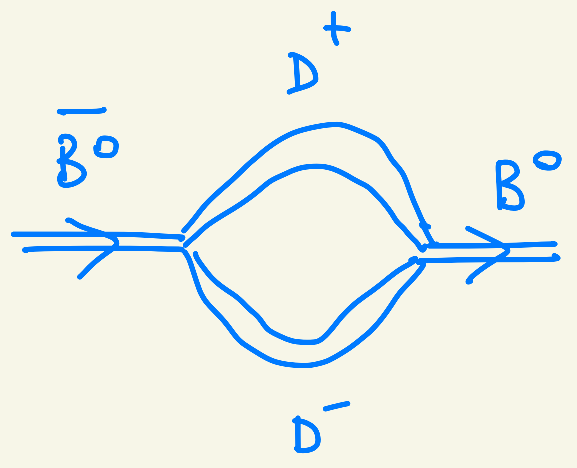
- Absorptive contributions are highly non-perturbative (Γ_{12}):



$\propto \quad G_F^2 \left(V_{cb} V_{cd}^*\right)^2 \approx G_F^2 \, |V_{cb} V_{cd}|^2$

$\propto \quad G_F^2 \left(V_{cb}^* V_{cd}\right) \approx G_F^2 \, |V_{cb} V_{cd}|^2$

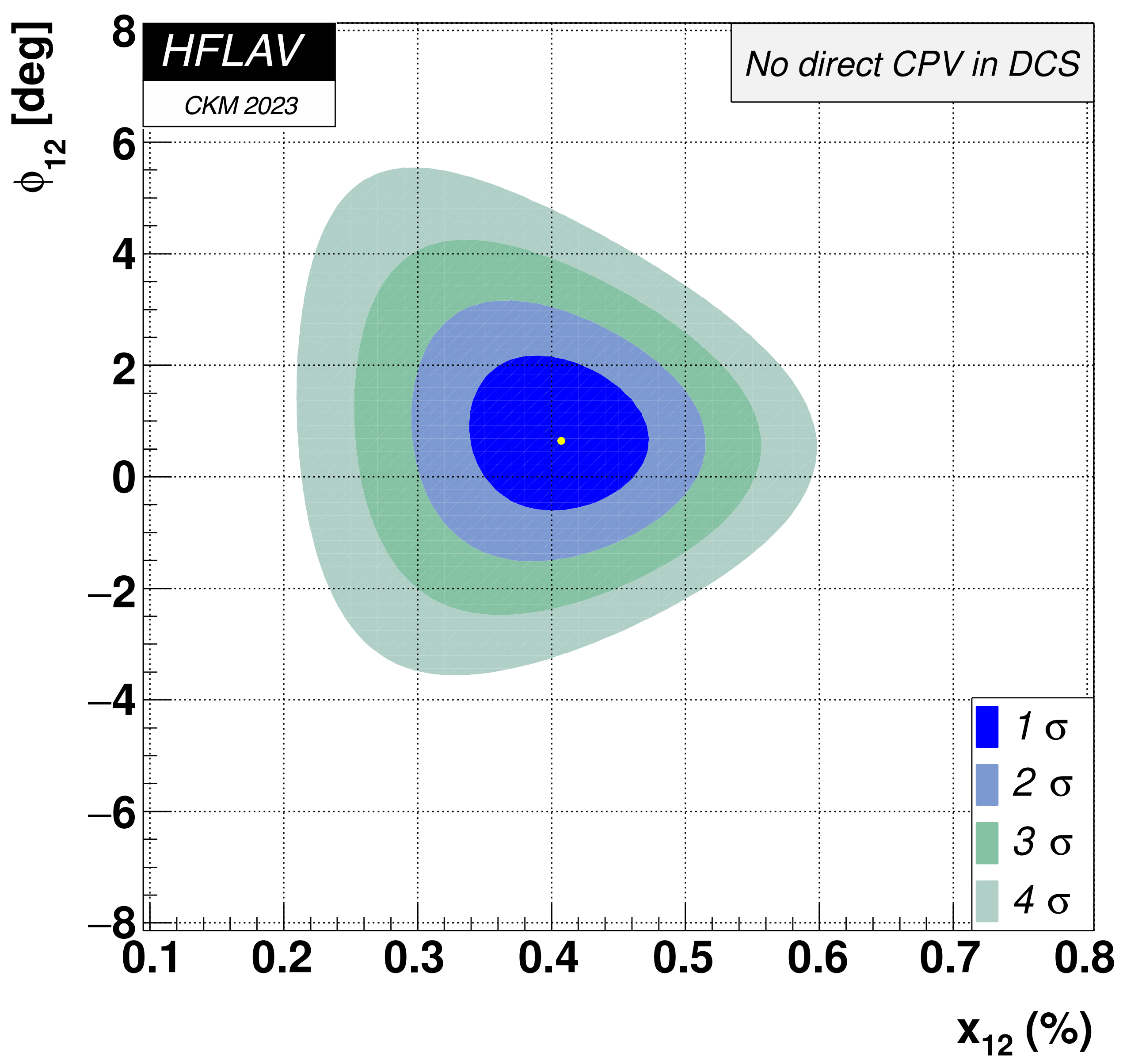
develops an absorptive phase due to intermediate propagation of charmed mesons. ⇒ e.g.

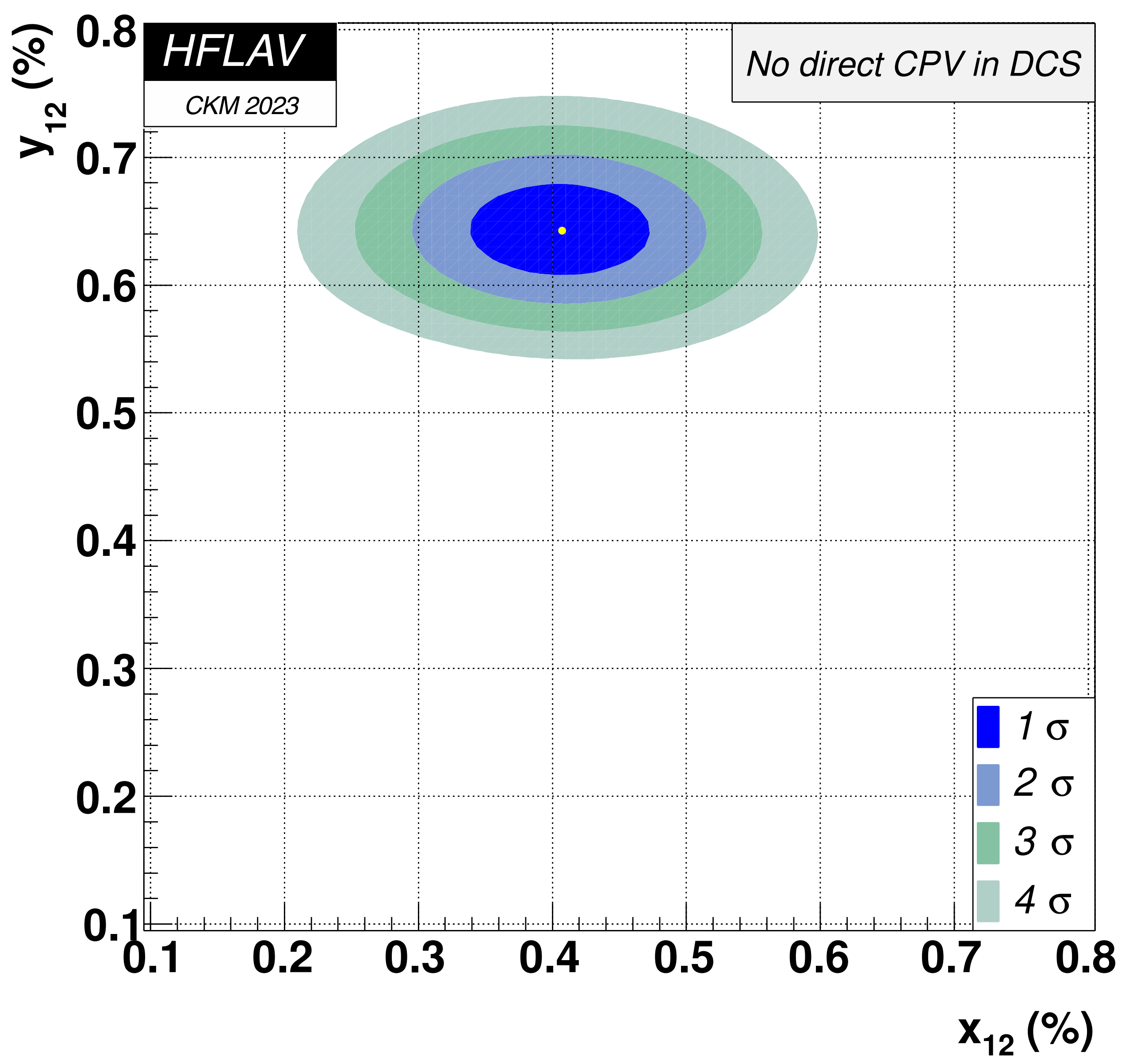


These contributions (and hence Γ_{12}) are small because branching ratios into decays common to B^0 and $\bar{B}^0$ are small ($\sim 10^{-2}$)

D mixing

- It is common to introduce: $x_{12} = 2\frac{|M_{12}|}{\Gamma_D}$, $y_{12} = \frac{|\Gamma_{12}|}{\Gamma_D}$, $\phi_{12} = \arg\left(\frac{M_{12}}{\Gamma_{12}}\right)$





- M_{12} and Γ_{12} receive contributions from matrix elements involving a single insertion of a $\Delta C = 2$ operator (short distance) and two insertions of $\Delta C = 1$ operators (long distance)

- Long distance contributions dominate the D-mixing amplitude (by several orders of magnitude)

Non perturbative matrix elements: $\Delta B = 2$

- B-mixing ($\Delta B = 2$ matrix elements):

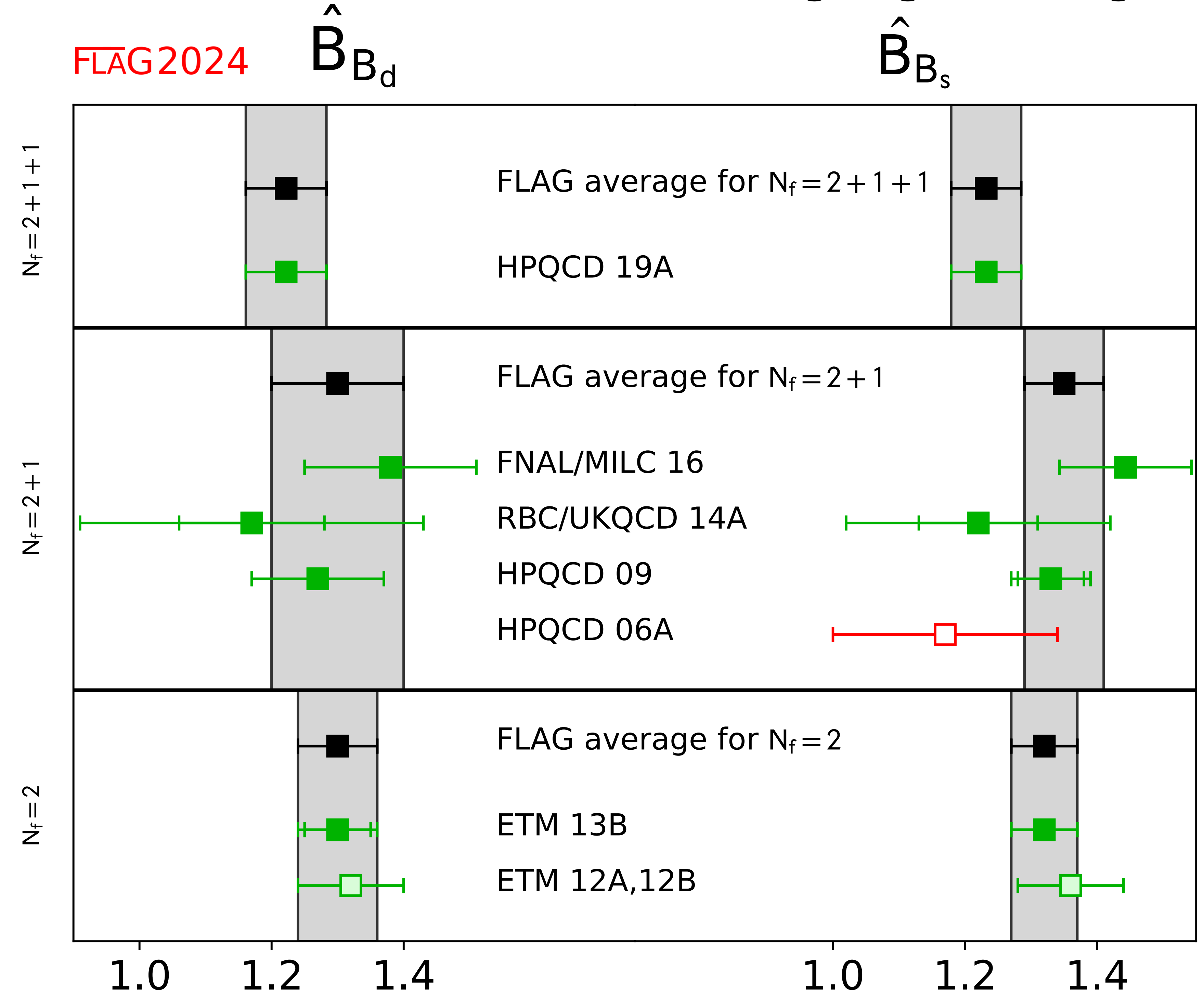
$$H_{\rm eff} = \frac{G_F m_W^2}{4\pi^2}(V_{tb}^* V_{tq})^2 \sum_{i=1}^{8} C_i(\mu)\, O_i(\mu)$$

$$\begin{aligned}
O^{VLL} &= (\bar b_L \gamma_\mu q_L)(\bar b_L \gamma^\mu q_L) \quad \longleftarrow \text{ Dominant operator in the SM}\\
O^{VRR} &= (\bar b_R \gamma_\mu q_R)(\bar b_R \gamma^\mu q_R)\\
O_1^{LR} &= (\bar b_L \gamma_\mu q_L)(\bar b_R \gamma^\mu q_R)\\
O_2^{LR} &= (\bar b_R q_L)(\bar b_L q_R)\\
O_1^{SLL} &= (\bar b_R q_L)(\bar b_R q_L)\\
O_1^{SRR} &= (\bar b_L q_R)(\bar b_L q_R)\\
O_2^{SLL} &= (\bar b_R \sigma_{\mu\nu} q_L)(\bar b_R \sigma^{\mu\nu} q_L)\\
O_2^{SRR} &= (\bar b_L \sigma_{\mu\nu} q_R)(\bar b_L \sigma^{\mu\nu} q_R)
\end{aligned}$$

- We need the matrix elements of the $\Delta B = 2$ operators $O_i(\mu)$
- It is common to introduce Renormalization Group invariant matrix elements:

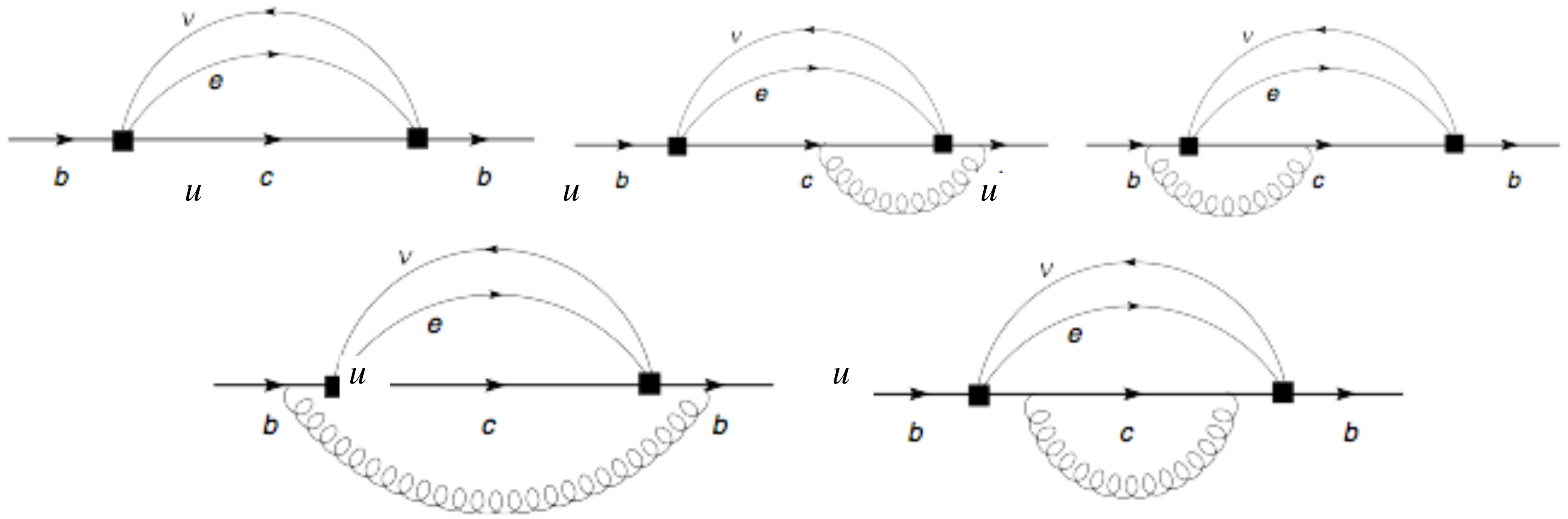
$$b_{B_q}(\mu)\langle B^0 | O^{VLL}(\mu) | \bar B^0\rangle \equiv \frac{2}{3} f_{B_q}^2 \hat B_{B_q} \quad \Longrightarrow \quad [M_{12}^q]_{\rm SM} = \frac{G_F^2 m_W^2}{12\pi^2}\, \eta_B\, m_{B_q}\, f_{B_q}^2 \hat B_q\, S_0(m_t^2/m_W^2)\, (V_{tb}^* V_{tq})^2$$

- The matrix elements for $\Delta B = 2$ relevant for M_{12} have been calculated long ago using lattice-QCD:

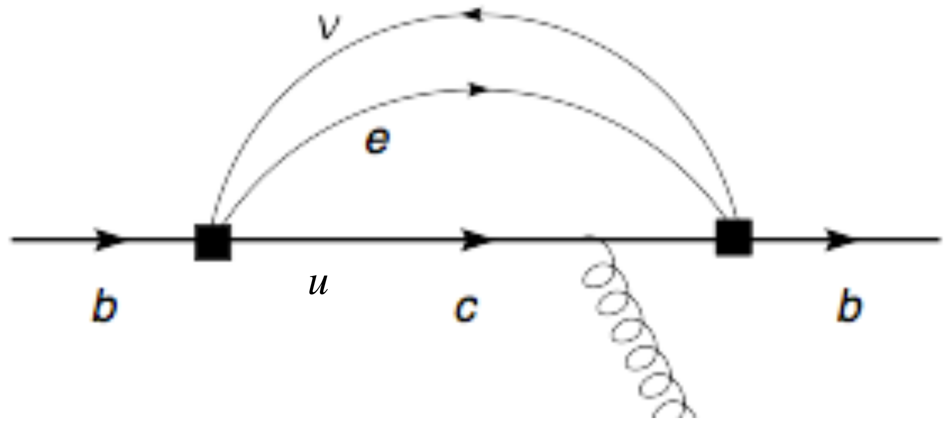


Non perturbative matrix elements: $\Delta B = 0$ (Weak Annihilation)

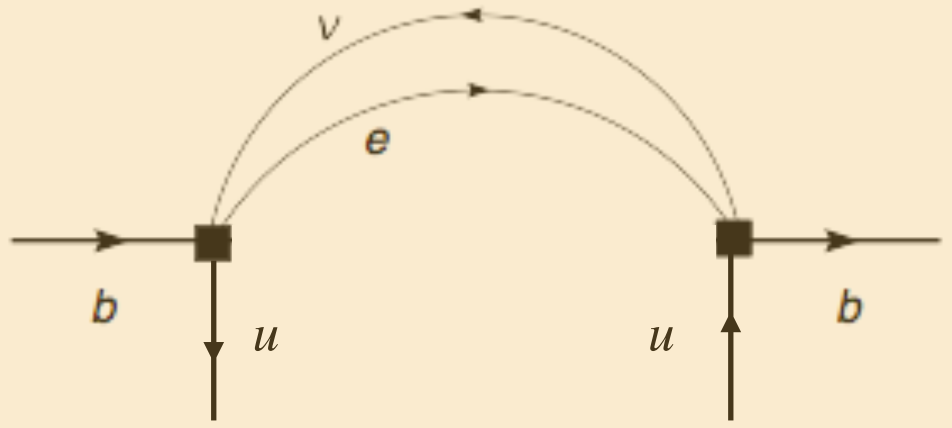
- Inclusive decays and lifetimes ($\Delta B = 0$ matrix elements)
- For instance, for $B \to X_u \ell \nu$:



$C_3\, \bar{b}\, b$



$C_5\, \bar{b}\, \sigma_{\mu\nu} G^{\mu\nu} b$

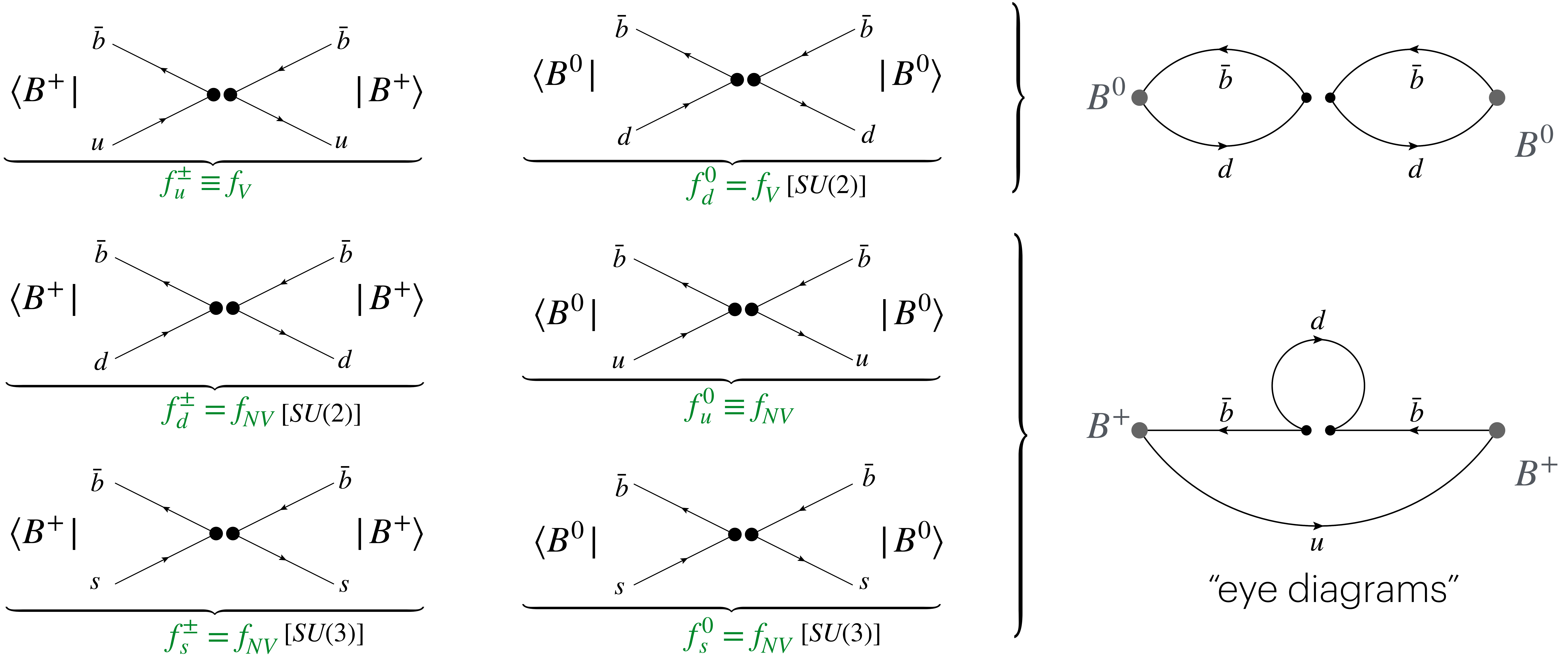


$C_6\, \bar{b}_L \gamma_\mu u_L\, \bar{u}_L \gamma^\mu b_L$

- The matrix elements for $\Delta B = 0$ (**Weak annihilation**) are much more challenging:
  - renormalization is complicated by mixing with operators of lower mass dimension
  - they involve disconnected and "eye" diagrams for which statistical noise is much larger
- Recently, preliminary results for the calculation of $\Delta C = 0$ operators in the D_s system have appeared using the Gradient Flow Renormalization [Black, Harlander, Lange, Rago, Shindler, Witzel 2310.18059 and 2409.18891]

Non perturbative matrix elements: $\Delta B = 0$ (Weak Annihilation)

- Note the difference between valence and non-valence matrix elements



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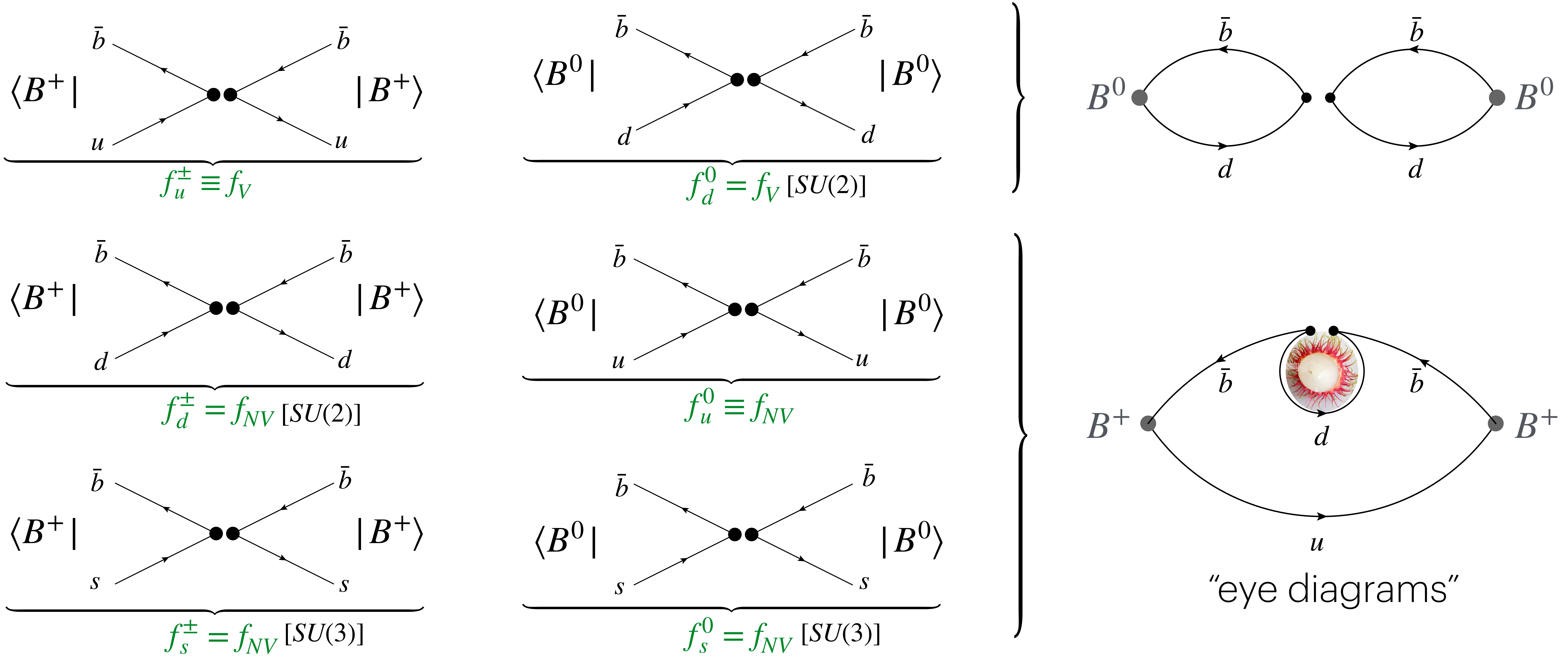

Non perturbative matrix elements: $\Delta B = 0$ (Weak Annihilation)

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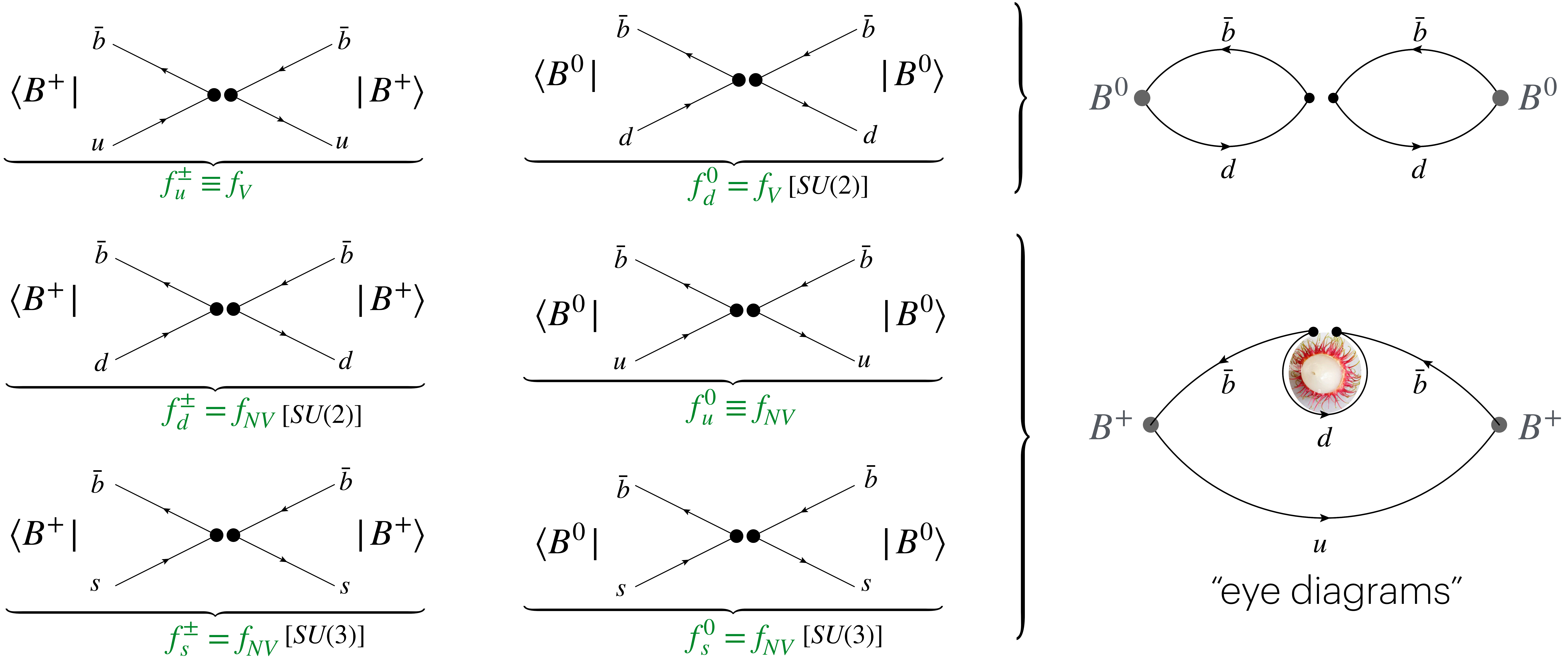


- Currently, the best one can do is to adopt upper limits extracted from $D^{0,\pm}$ and D_s decays rescaled by a factor $m_B f_B^2/(m_D f_D^2)$ [Gambino, Kamenik 1004.0114; Shao, Huang, Qin 2502.05901]

Rare charm decays

- Strong GIM suppression
  Up-quark sector FNCN $\Bigg\}$ $\Longrightarrow$ room for large BSM contributions not excluded by bottom and kaon physics

- Rare charm decays, when not dominated by tree-level W- exchange diagrams, provide powerful null-tests of the SM

- For instance, focusing on $c \to u\ell\ell$ and $c \to u\nu\bar{\nu}$ transitions:

$$\mathrm{BR}(D \to \pi\nu\bar{\nu}) < 2.1 \times 10^{-4} \quad \Longrightarrow |C_\nu| \lesssim 150$$ [BESSIII]

$$\mathrm{BR}(D \to \mu\mu) < 2.2 \times 10^{-9} \quad \Longrightarrow |C_{10}| \lesssim 0.52$$ [CMS, LHCb]

$$\mathrm{BR}(D^+ \to \pi^+\mu\mu) < 2.5 \times 10^{-8} \quad \Longrightarrow |C_{10}| \lesssim 0.7$$ [LHCb]

$$\mathrm{BR}(\Lambda_c \to p\mu\mu) < 7.7 \times 10^{-8} \quad \Longrightarrow |C_7| \lesssim 0.2$$ [LHCb]

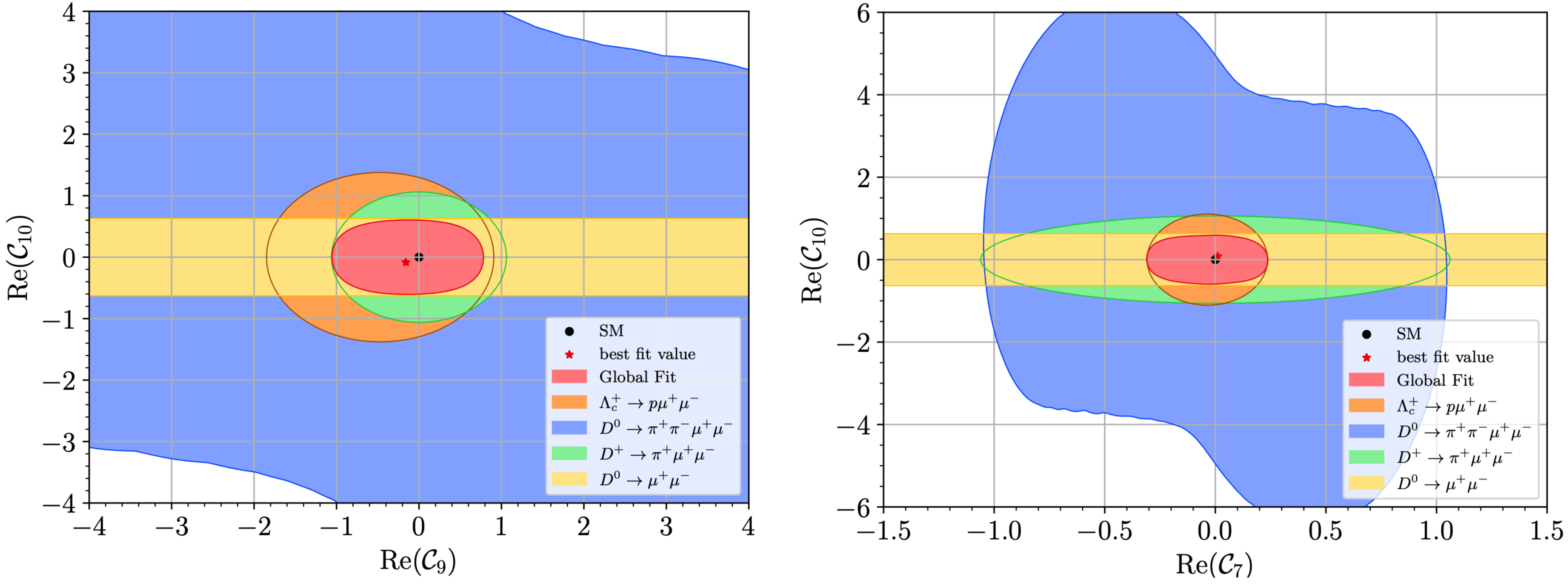
Many orders of magnitude from the SM predictions!

Where the operators are $O_\nu = (\bar{u}_L\gamma_\mu c_L)\,(\bar{\nu}_L\gamma^\mu\nu_L)$, $O_7 = m_c/e\,(\bar{u}_L\sigma_{\mu\nu}c_R)\,F^{\mu\nu}$, $O_9 = (\bar{u}_L\gamma_\mu c_L)\,(\bar{\ell}\gamma^\mu\ell)$, $O_{10} = (\bar{u}_L\gamma_\mu c_L)\,(\bar{\ell}\gamma^\mu\gamma_5\ell)$

and the SM expectations are $C_\nu \sim C_{10} \sim 0$, $|C_7^{\rm eff}(q^2)| \lesssim 0.01$ and $|C_9^{\rm eff}(q^2)| \lesssim \mathcal{O}(0.1)$

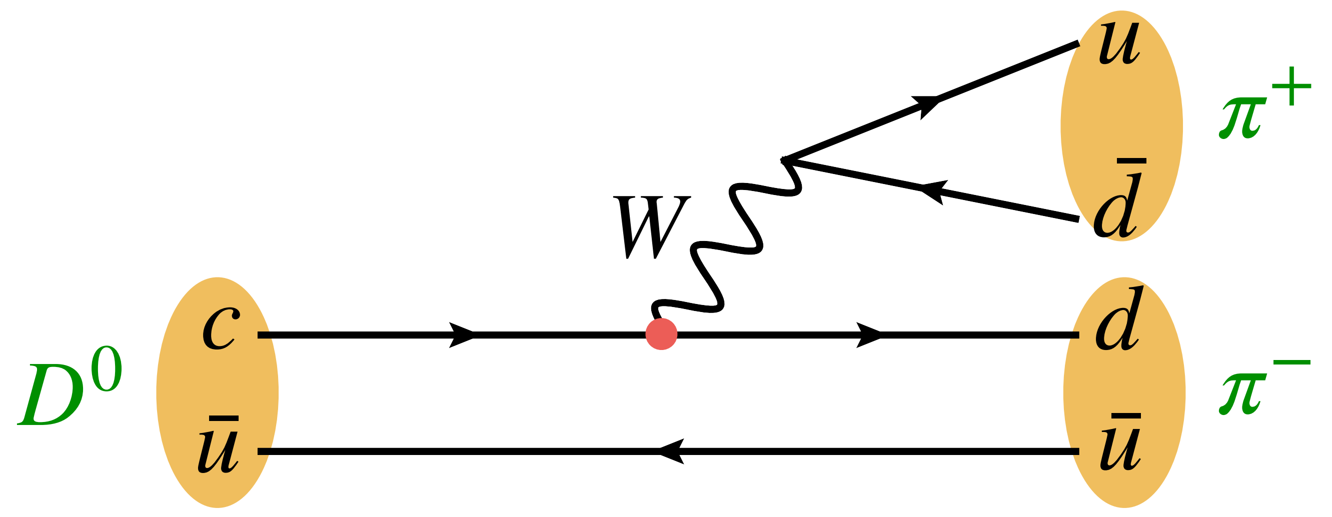
Rare charm decays

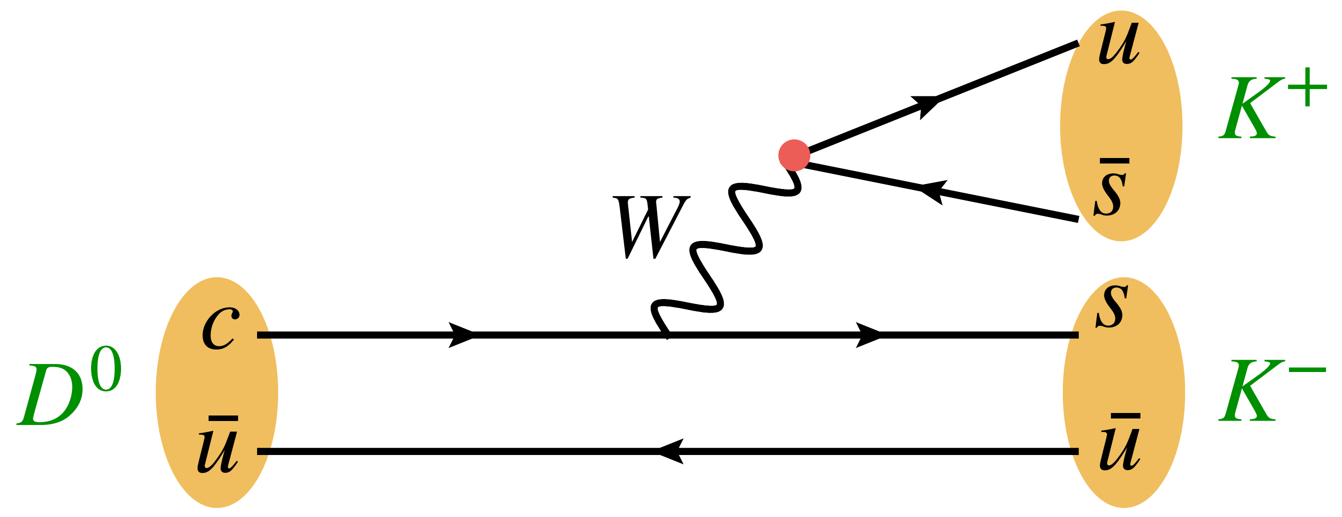
- Global analysis of all leptonic and semileptonic D decays [Gisbert, Hiller, Suelmann 2410.00115]



Rare charm decays: $D^0 \to \pi^+\pi^-$ vs $D^0 \to K^+K^-$

- CP asymmetries in Single Cabibbo Suppressed hadronic decays (for which penguin pollution is expected to be small) can be decomposed in direct, mixing and interference (between the two):





$$\Longrightarrow \quad a_f = a_f^d + a_f^m + a_f^i$$

almost identical for $f = \pi\pi$ and KK

- The difference $\Delta a_{\rm CP} \equiv a_{KK} - a_{\pi\pi} = a^d_{KK} - a^d_{\pi\pi}$ is controlled by direct CP violation

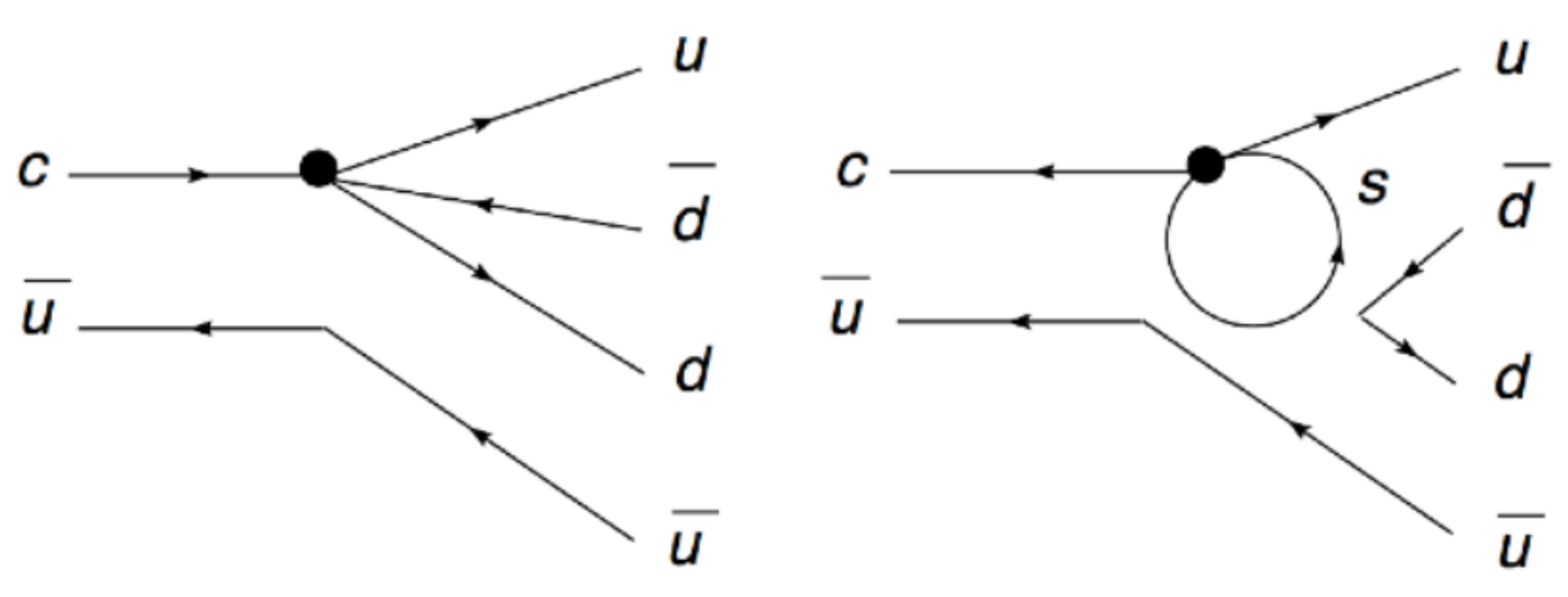
$$\Delta a_{CP}^{\rm exp} = (-15.4 \pm 2.9) \times 10^{-4}$$ [LHCb 1903.08726.]

$$a_{CP}^{\rm exp}(KK) = (7.7 \pm 5.7) \times 10^{-4}$$
$$a_{CP}^{\rm exp}(\pi\pi) = (23.2 \pm 6.1) \times 10^{-4}$$ [LHCb 2209.03179]

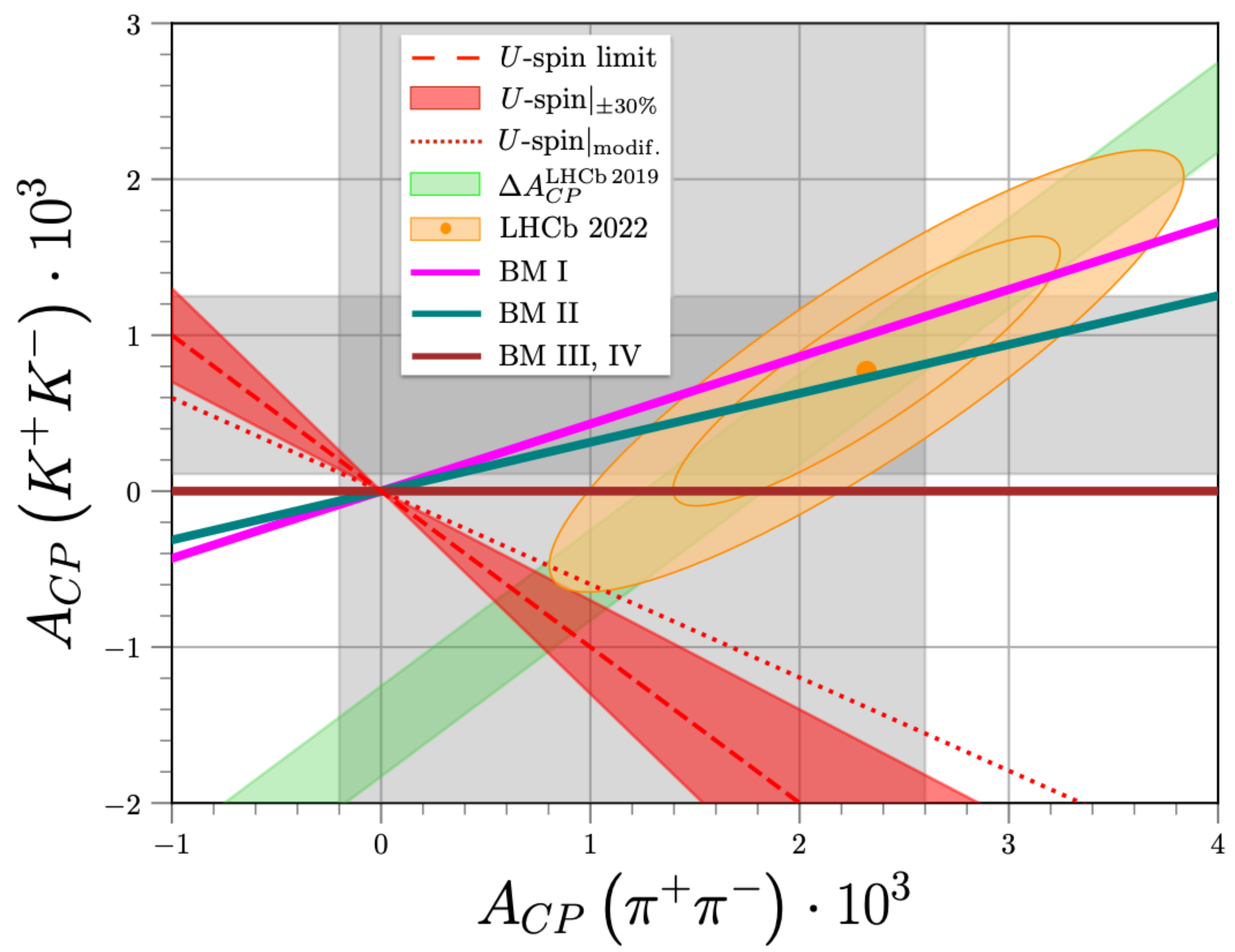
- Unfortunately U-spin (interchange $s \leftrightarrow d$) appears badly broken: $\dfrac{a_{\rm CP}(\pi\pi)}{a_{\rm CP}(KK)} \dfrac{\Gamma(D \to KK)}{\Gamma(D \to \pi\pi)} = \begin{cases} +0.93^{0.62}_{-0.41} & \text{exp} \\ -1 & \text{U-spin} \end{cases}$!!

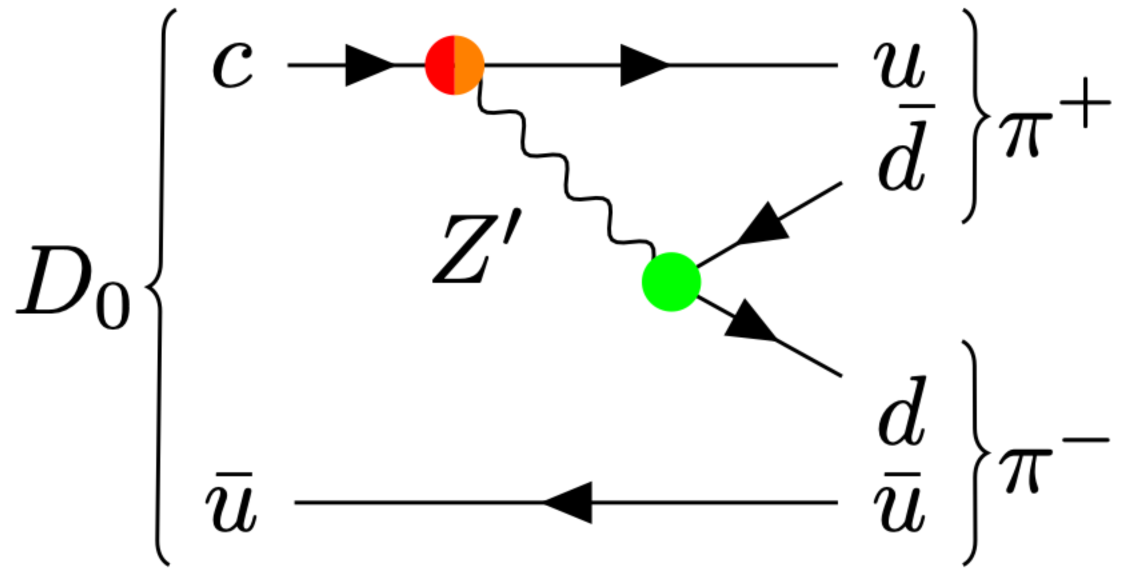
Rare charm decays: $D^0 \to \pi^+\pi^-$ vs $D^0 \to K^+K^-$

- A direct calculation using **Light-Cone Sum Rules** yields $\Delta a_{\rm CP}^{\rm dir} = (2.0 \pm 0.3) \times 10^{-4}$, which is not compatible with the experimental result [Khodjamirian, Petrov 1706.07780]
- A description of hadronic matrix elements which includes a **model-dependent resonant enhancement** due to the nearby f_0 state is able to describe the data [Soni, Schacht 2110.07619]
- Fits based on **flavor SU(3)** are very poor (not a surprise given the large U-spin violation): $\chi^2/\mathrm{dof} = 7477/3\,!$ Inclusion of large SU(3) breaking corrections resolves the tension [Bhattacharya, Datta, Petrov, Waite 2107.13564]
- Perturbative calculations are plagued by the unknown interplay between penguin and tree amplitudes:



- New Physics explanations are also possible: [Bause, Gisbert, Hiller, Höhne, Litim, Steudtner 2210.16330]





Hadronic two-body B(s) decays

- A huge amount of data has been accumulated on hadronic b-decays to two pseudoscalar mesons: $B \to PP$ where $B = B, B_s$ and $P = \pi, K, \eta, \eta'$.

$8 \otimes 8$ $\Delta S = 0$	$8 \otimes 8$ $\Delta S = 1$
$B^+ \to \bar{K}^0 K^+$	$B^+ \to \pi^+ K^0$
$B^+ \to \pi^+ \pi^0$	$B^+ \to \pi^0 K^+$
$B^+ \to \eta_8 \pi^+$	$B^+ \to \eta_8 K^+$
$B^0 \to K^0 \bar{K}^0$	$B_s^0 \to K^0 \bar{K}^0$
$B^0 \to \pi^+ \pi^-$	$B_s^0 \to \pi^+ \pi^-$
$B^0 \to \pi^0 \pi^0$	$B_s^0 \to \pi^0 \pi^0$

$8 \otimes 8$ $\Delta S = 0$	$8 \otimes 8$ $\Delta S = 1$
$B^0 \to K^+ K^-$	$B_s^0 \to K^+ K^-$
$B^0 \to \pi^0 \eta_8$	$B_s^0 \to \pi^0 \eta_8$
$B^0 \to \eta_8 \eta_8$	$B_s^0 \to \eta_8 \eta_8$
$B_s^0 \to \pi^+ K^-$	$B^0 \to \pi^- K^+$
$B_s^0 \to \pi^0 \bar{K}^0$	$B^0 \to \pi^0 K^0$
$B_s^0 \to \eta_8 \bar{K}^0$	$B^0 \to \eta_8 K^0$

$8 \otimes 1$ $\Delta S = 0$	$8 \otimes 1$ $\Delta S = 1$
$B^+ \to \eta_1 \pi^+$	$B^+ \to \eta_1 K^+$
$B^0 \to \pi^0 \eta_1$	$B_s^0 \to \pi^0 \eta_1$
$B^0 \to \eta_8 \eta_1$	$B_s^0 \to \eta_8 \eta_1$
$B_s^0 \to \eta_1 \bar{K}^0$	$B^0 \to \eta_1 K^0$

$1 \otimes 1$ $\Delta S = 0$	$1 \otimes 1$ $\Delta S = 1$
$B^0 \to \eta_1 \eta_1$	$B_s^0 \to \eta_1 \eta_1$

- Historically, $B \to \pi\pi$ data (BR and CP asymmetries) have been used to determine the CKM angle α.
  - This was achieved via an isospin analysis [Gronau, London]
  - The QCD factorization description of these decays receive large power corrections [BBNS]
- Assuming exact flavor SU(3) it is possible to describe all $B \to PP$ measurements in terms of a relatively small set of unknown amplitudes.

Hadronic two-body $B_{(s)}$ decays

- The amplitudes can be described either in terms of **Reduced Matrix Elements** (Wigner-Eckart) or of **Topological Quark Diagrams** (Tree, Color suppressed, Penguin, Annihilation, ...)
- Many groups performed these fits and, using the most recent data, there is a consensus that **SU(3) corrections are required to provide a good description of data**

  [He, Wang 1803.04227]
  [Hsiao, Chang, He 1512.09223]
  [Huber, Tetlalmatzi-Xolocotzi 2111.06418]
  [Berthiaume, Bhattacharya, Boumris, Jean, Kumbhackar, London 2311.18011]
  [Burgos Marcos, Reboud, Vos 2504.05209] $\longrightarrow \chi^2/\mathrm{dof} = 32.3/15,\, p = 5.8 \times 10^{-3}$
  [Bhattacharya, Bouchard, Hudy, Jean, London, MacKenzie 2505.11492] $\longrightarrow \chi^2/\mathrm{dof} = 43.2/17,\, p = 4.5 \times 10^{-4}$

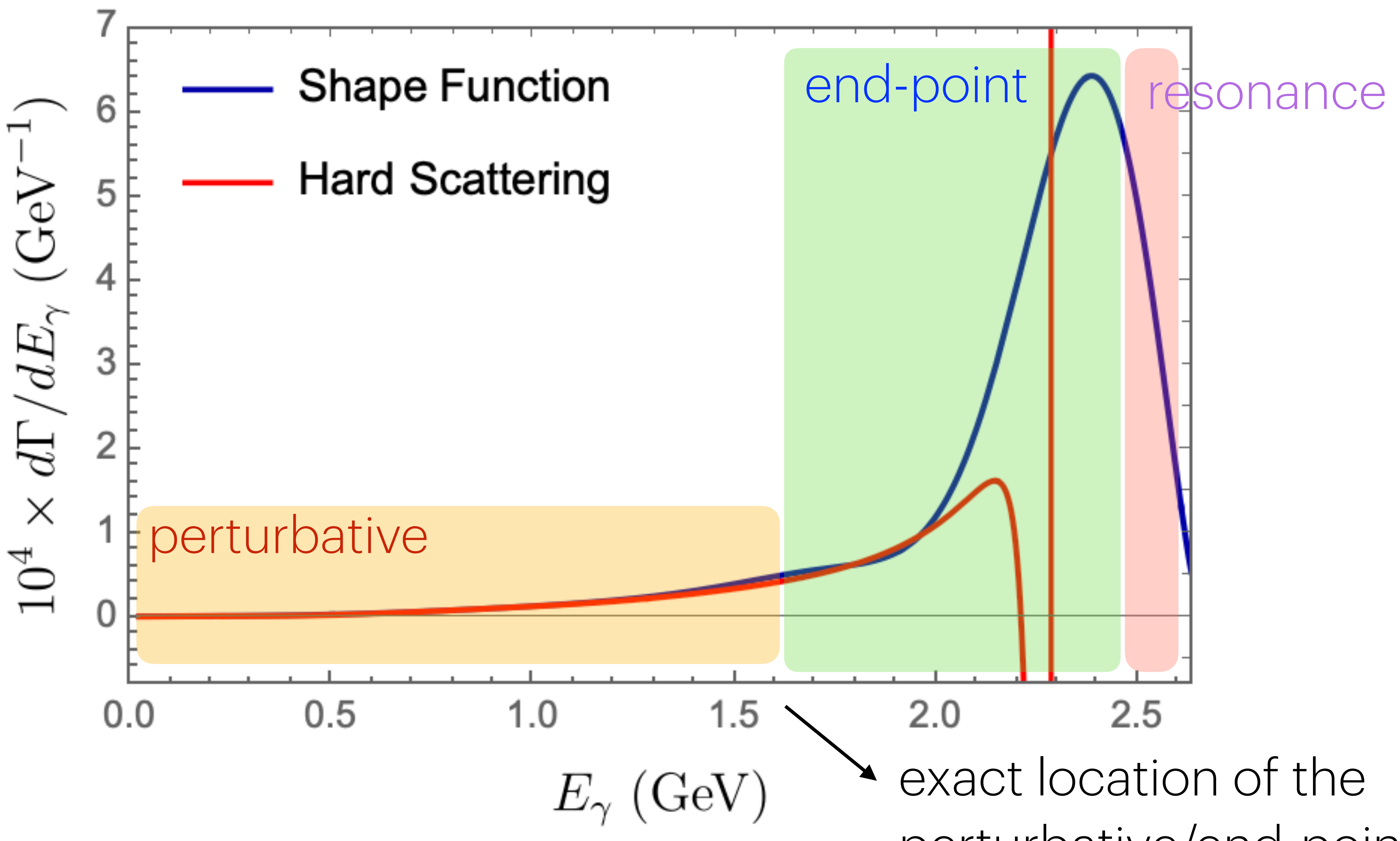
- There is still a strong disagreement, stemming from the **counting of the required independent SU(3) amplitudes**, on whether this can be resolved invoking reasonable (~20%) or unreasonable (~1000%) SU(3) breaking corrections!
- Whether these results are a hint for New Physics is, therefore, an open question
- It is interesting that the SU(3) limit is problematic both in hadronic B and D decays

Inclusive decays

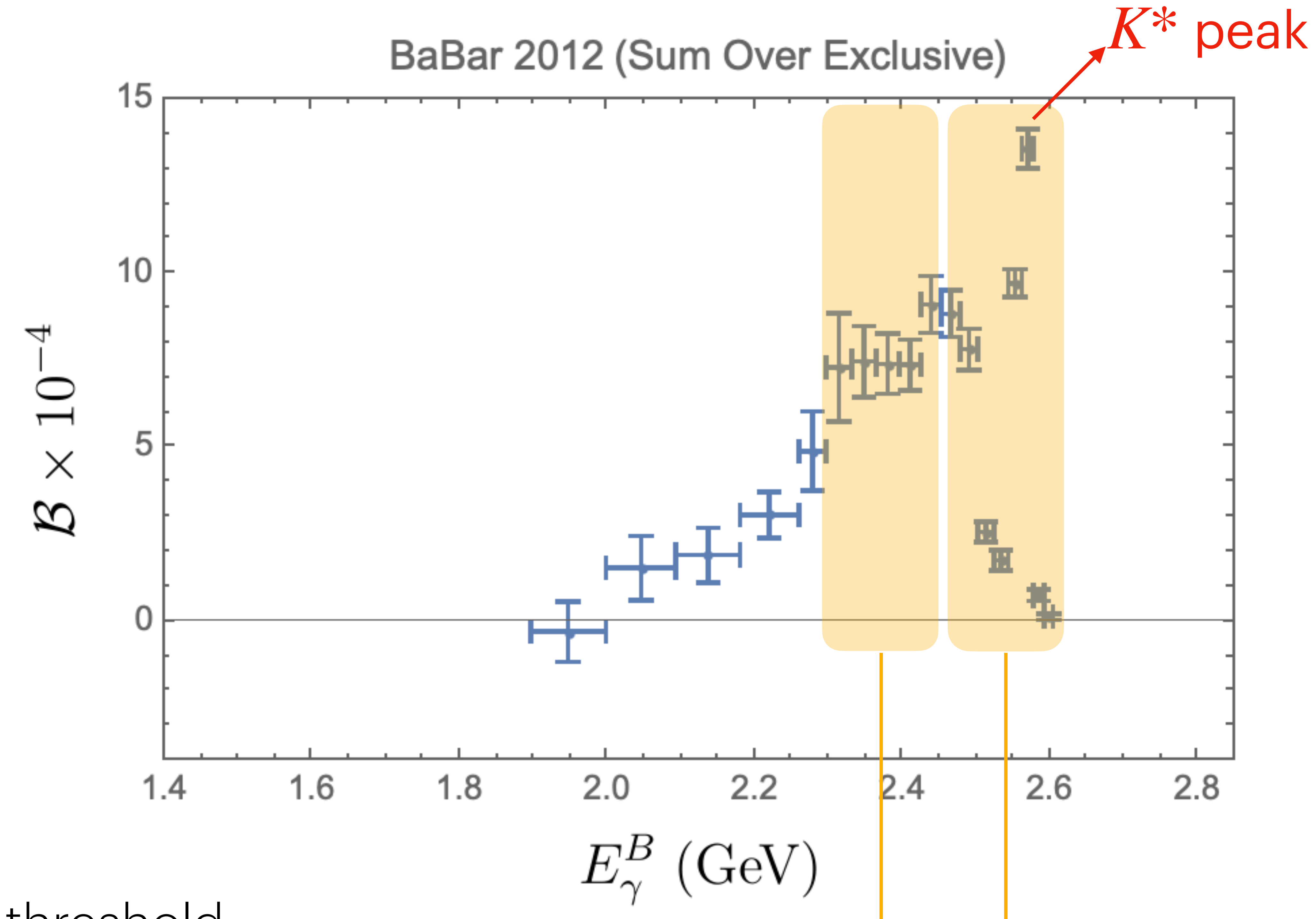
- Inclusive b-decays to light final states $B \to X_u \ell \nu$, $B \to X_s \gamma$, and $B \to X_s \ell \ell$ are extremely important because, in the full OPE limit (no m_X cut), have **theoretical uncertainties which are independent from those that appear in the corresponding exclusive channels** ($B \to \pi \ell \nu$, $B \to K^* \gamma$, $B \to K^{(*)} \ell \ell$)
- Moreover, global fits to $B \to (X_u, X_c) \ell \nu$ are also essential for the determination of the b-quark mass in a short distance scheme (e.g. kinetic) and to extract HQET matrix elements of power suppressed operators
- Unfortunately all these modes can only be measured at low m_X
- The extrapolation to the whole phase space is currently addressed separately in the three modes
    - $B \to X_s \gamma$: the experimental cut at $E_\gamma \gtrsim E_0$ with $E_0 \in [1.7, 2.0]$ GeV is sufficiently close to edge of the perturbative region ($E_0 = 1.6$ GeV) that a simple extrapolation can be used
    - $B \to X_u \ell \nu$: sophisticated fits involving parameterizations of the B-meson shape function
    - $B \to X_s \ell \ell$: extrapolation performed by the experimental collaborations using a Fermi motion model [Ali, Hiller, Handoko, Morozumi hep-ph/9609449] which is hard coded in EVTGEN (Monte Carlo generator)

Inclusive decays

- Shape function vs hard scattering spectra:



exact location of the perturbative/end-point threshold depends on Shape Function



combined to remove sensitivity to the resonance region

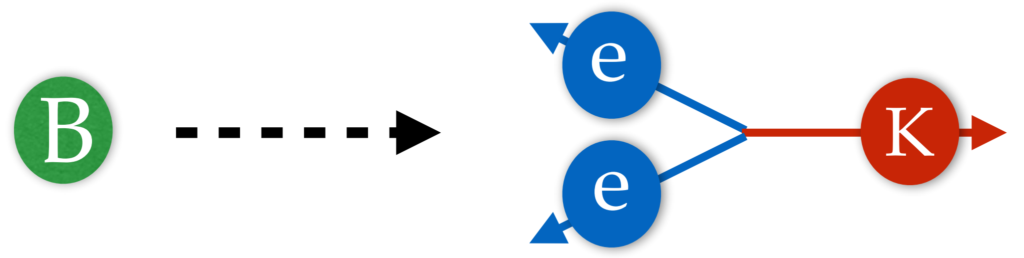
- Note that two equivalent theoretical approaches are most commonly adopted
  - GGOU [Gambino, Giordano, Ossola, Uraltsev]: each channel has its own shape function
  - BLNP [Bosch, Lange, Neubert, Paz]: universal leading power shape function + subleading shape functions

Form Factors for exclusive decays

- The central problem is the calculation of matrix elements of the type: $\langle K^{(*)}\ell\ell \,|\, O(y) \,|\, B\rangle \approx \langle K^{(*)} \,|\, T\, J_\mu^{\rm em}(x)\; O(y) \,|\, B\rangle$

- At low-q^2 the large energy of the K$^{(*)}$ introduces three scales: m_b^2, $\Lambda_{\rm QCD} m_b$ and $\Lambda_{\rm QCD}^2$:

$$\langle K^{(*)} \,|\, T J_\mu^{\rm em}(x) O(y) \,|\, B\rangle \sim C \times [\,\text{Form Factor} + \phi_B \star J \star \phi_{K^{(*)}}] + O\left(\Lambda_{\rm QCD}/m_b\right)$$

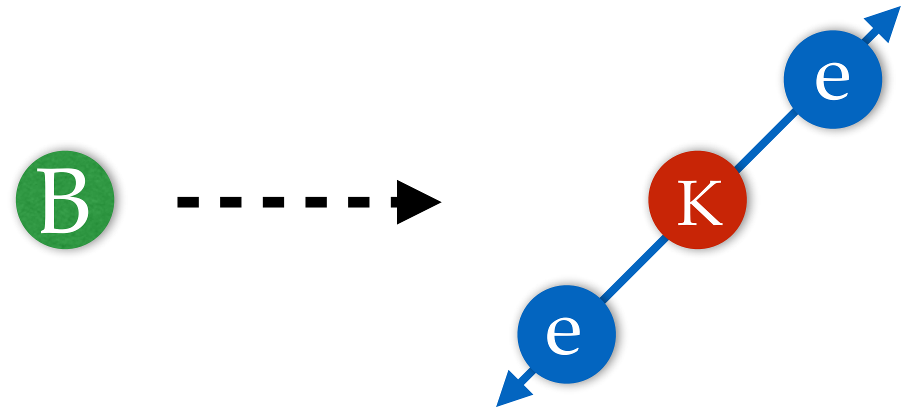


Soft-Collinear Effective Theory

Most problematic especially near q^2 ~ 6 GeV^2

- At high-q^2 the K$^{(*)}$ does not recoil, $(x-y)^2 \sim 1/q^2 \sim 1/m_b^2$:

$$\langle K^{(*)} \,|\, T J_\mu^{\rm em}(x) O(y) \,|\, B\rangle \sim C \times [\,\text{Form Factor}] + O\left(\Lambda/m_b\right)$$

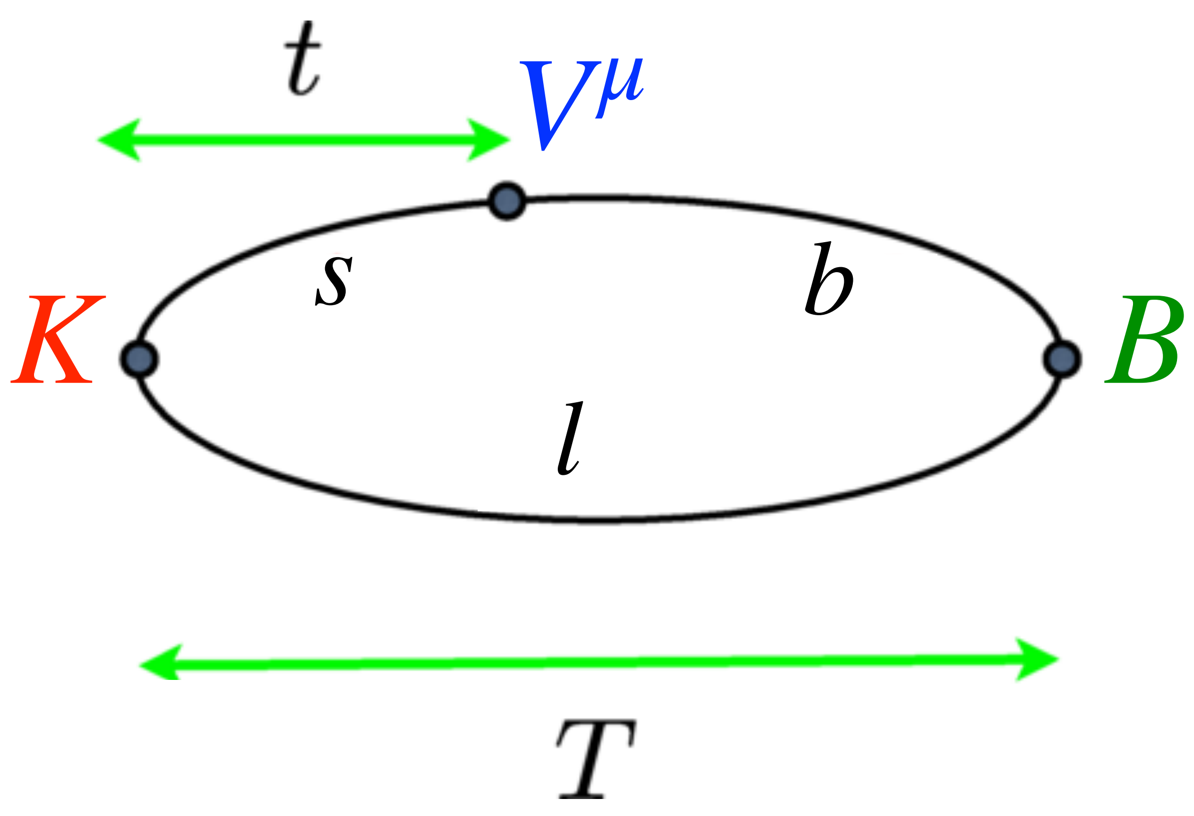


local OPE

Form Factors

- **Light-Cone Sum Rules** (low-q²): some uncertainties have to be ball-parked but gets access to all form factors
- **Lattice QCD** (high-q²): $B \to K$ complete, $B \to K^*$ and $B_s \to \phi$ ongoing
  [see the recently completed FLAG 2024 review, 2411.04268]
  - For stable mesons the Euclidean correlator is dominated at large Euclidean time by the meson ground state:

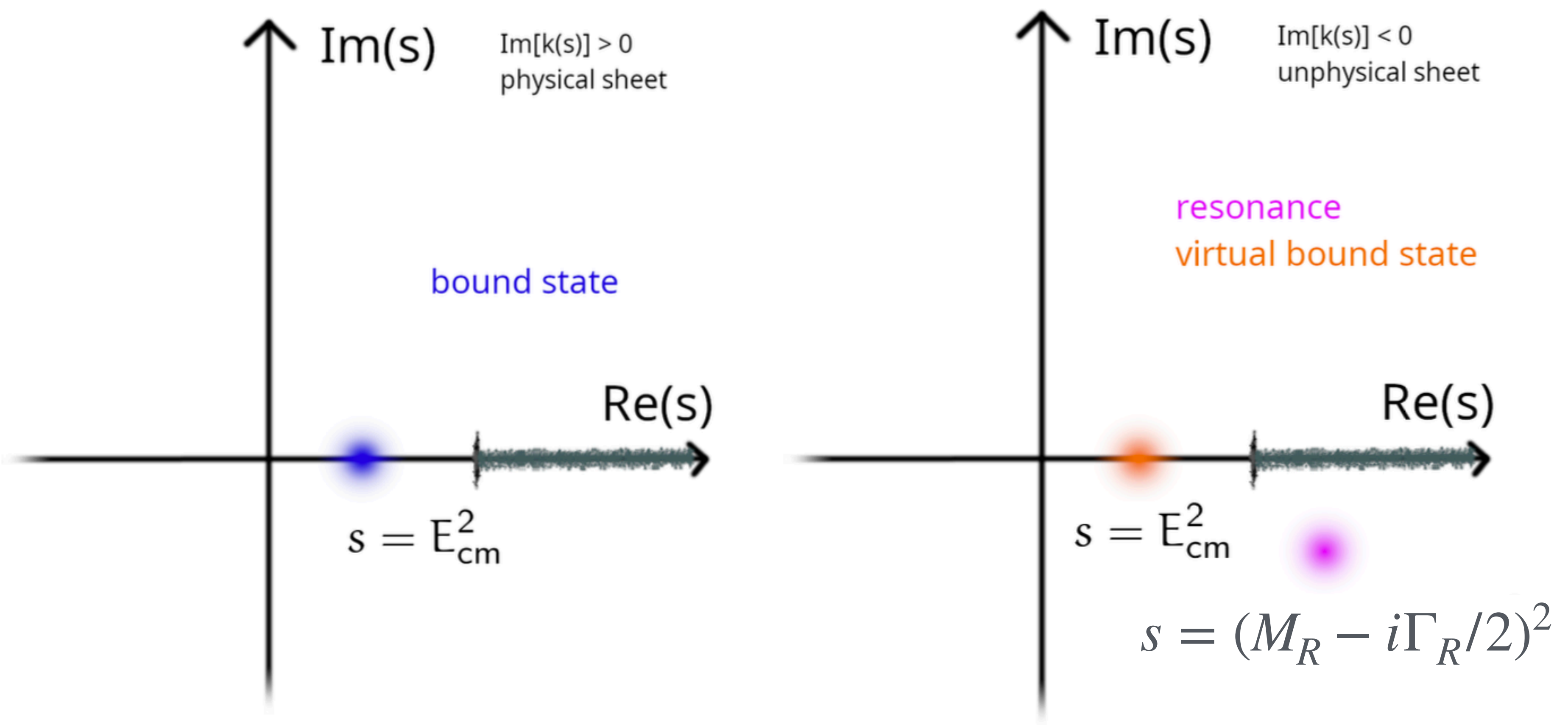
$$C_3^{B\to K}(t,T;\vec{p}_K) = \sum_{\vec{x},\vec{y}} e^{-i\vec{p}_K\cdot\vec{y}} \langle 0 | \mathcal{O}_K(0,\vec{0})\, V^\mu(t,\vec{y})\, \mathcal{O}_B^\dagger(T,\vec{x}) | 0\rangle \sim Z_\pi\, Z_B^*\, \langle K | V^\mu | B\rangle\, e^{-E_K t}\, e^{-M_B(T-t)}$$



Form Factors

- **Light-Cone Sum Rules** (low-q^2): some uncertainties have to be ball-parked but gets access to all form factors

- **Lattice QCD** (high-q^2): $B \to K$ complete, $B \to K^*$ and $B_s \to \phi$ ongoing
  [see the recently completed FLAG 2024 review, 2411.04268]
  - For resonances (states unstable in QCD, e.g. $K^* \to K\pi$) one relies on the Lüscher formalism:
    1) Resonances appear as poles on the unphysical Riemann sheet ($\mathrm{Im}[k(s)] < 0$) with respect to the scattering amplitude ($k(s)$ is the momentum transfer)

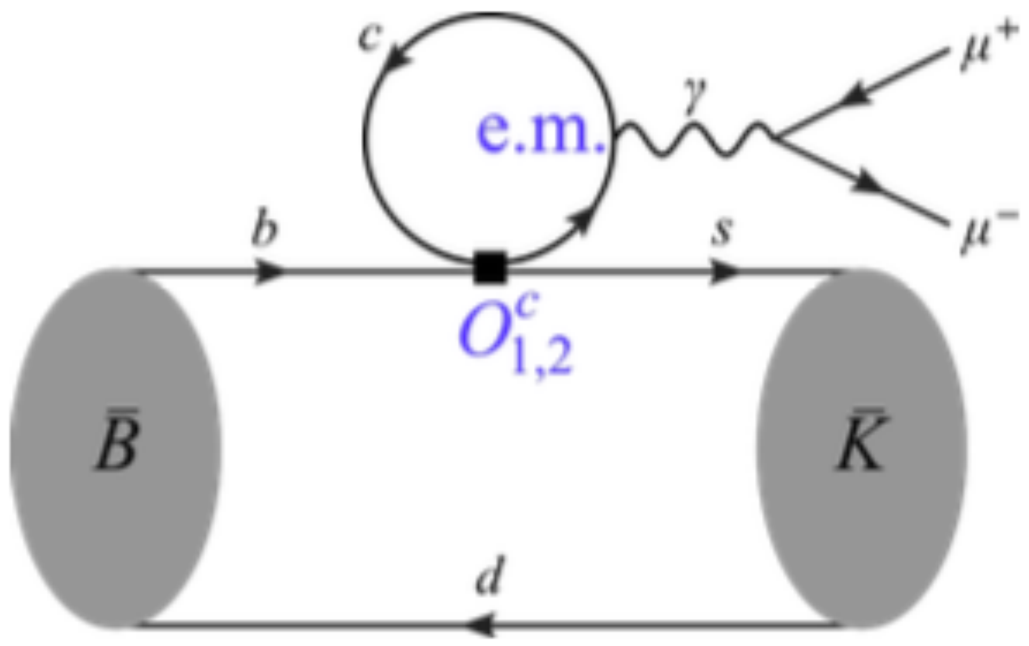


2) Interactions (which are responsible for the appearance of resonances) shifts the finite volume discrete energy levels

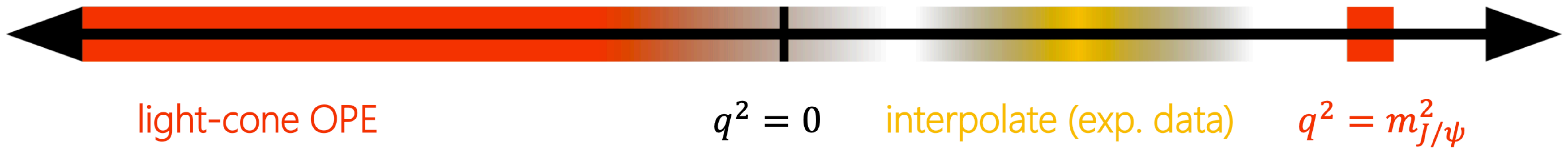
3) From the Volume dependence of the energy levels one can extract the infinite volume phase shifts and thus the resonance parameters [Boyle et al. 2406.19194]

4) The form factors can be calculated once the solution of the Generalized EigenValue Problem (GEVP) is at hand
[Di Carlo, Erben, Tsang et al, ongoing]

Non-local Power Corrections



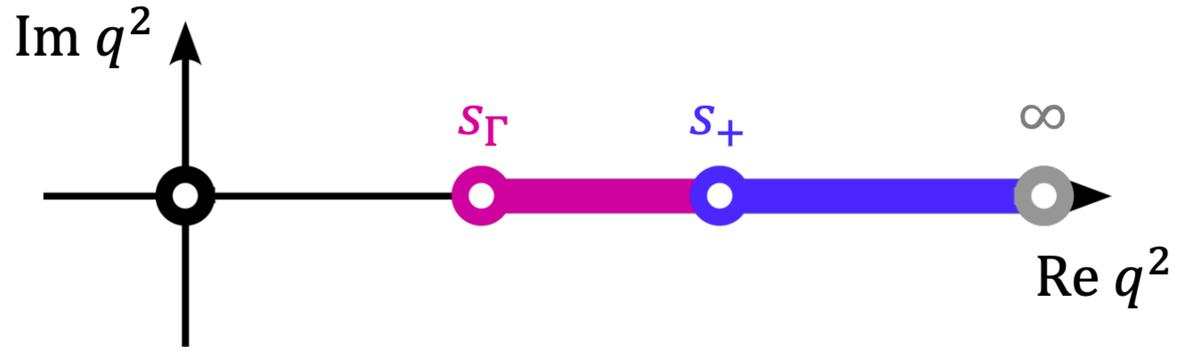
- **Power corrections at low-q^2**
  - Within SCET, nonlocal matrix elements, like $\langle K^{(*)} | T J_\mu^{\rm em} O_2 | B \rangle$, are expressed in terms of form factors and light cone wave functions up to unknown subleading non-perturbative functions
  - The power suppression ($\sim \Lambda_{\rm QCD}/m_b$) of these contributions arises dynamically and is very difficult to model
    - Parameterize and assume a typical $O(10-20\%)$ size (**with respect to the factorization limit**)
    - Parameterize and use information at $q^2 < 0$ and $q^2 = m_{J/\psi}^2$ to interpolate
      [Mutke, Hoferichter, Kubis 2406.14608. Gubernari, Reboud, van Dyk, Virto 2305.06301. Gopal, Gubernari 2412.04388.]

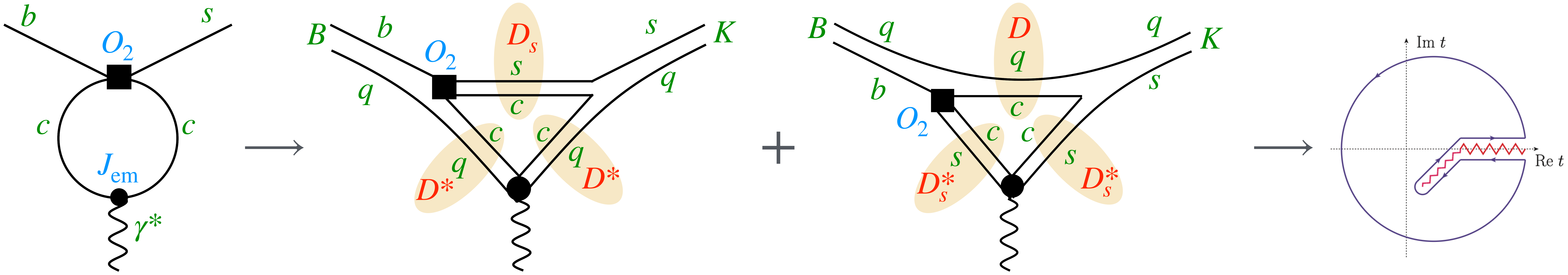


    - Direct calculation using lattice-QCD (in O(5) years)
      [Frezzotti, Gagliardi, Lubicz, Martinelli, Sachrajda, Sanfilippo, Simula, Tantalo and Silvestrini, in progress]

Non-local Power Corrections

- The parameterization needs to respect the analytic structure of the non-local form factor.
- Besides poles, we have (I use $B \to K$ for reference)
  - regular thresholds: $s_+ = (m_B + m_K)^2$
  - subthresholds: $s_\Gamma = (m_{B_s} + m_\pi)^2 < s_+$
  - anomalous thresholds associated to triangle diagrams

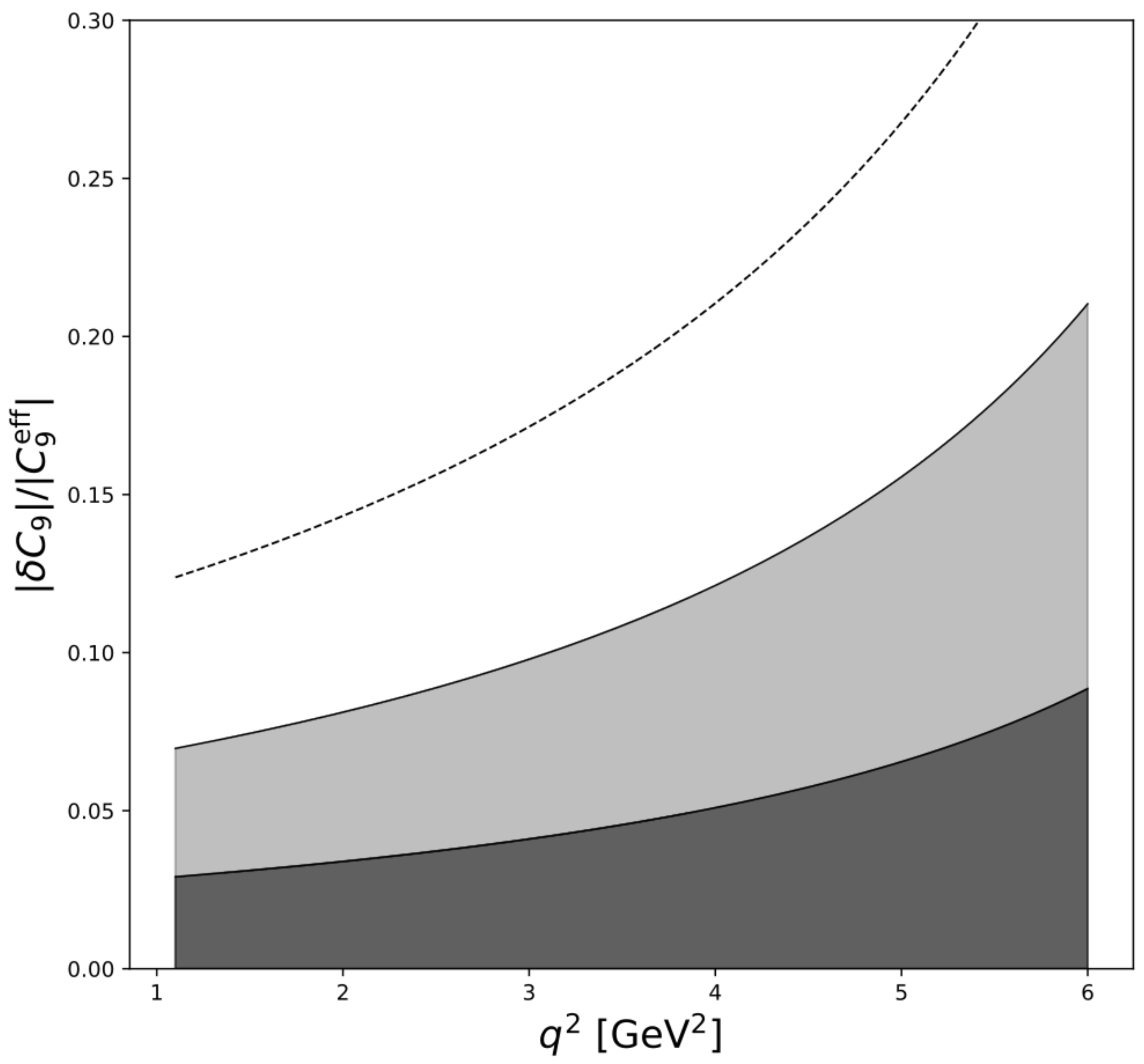


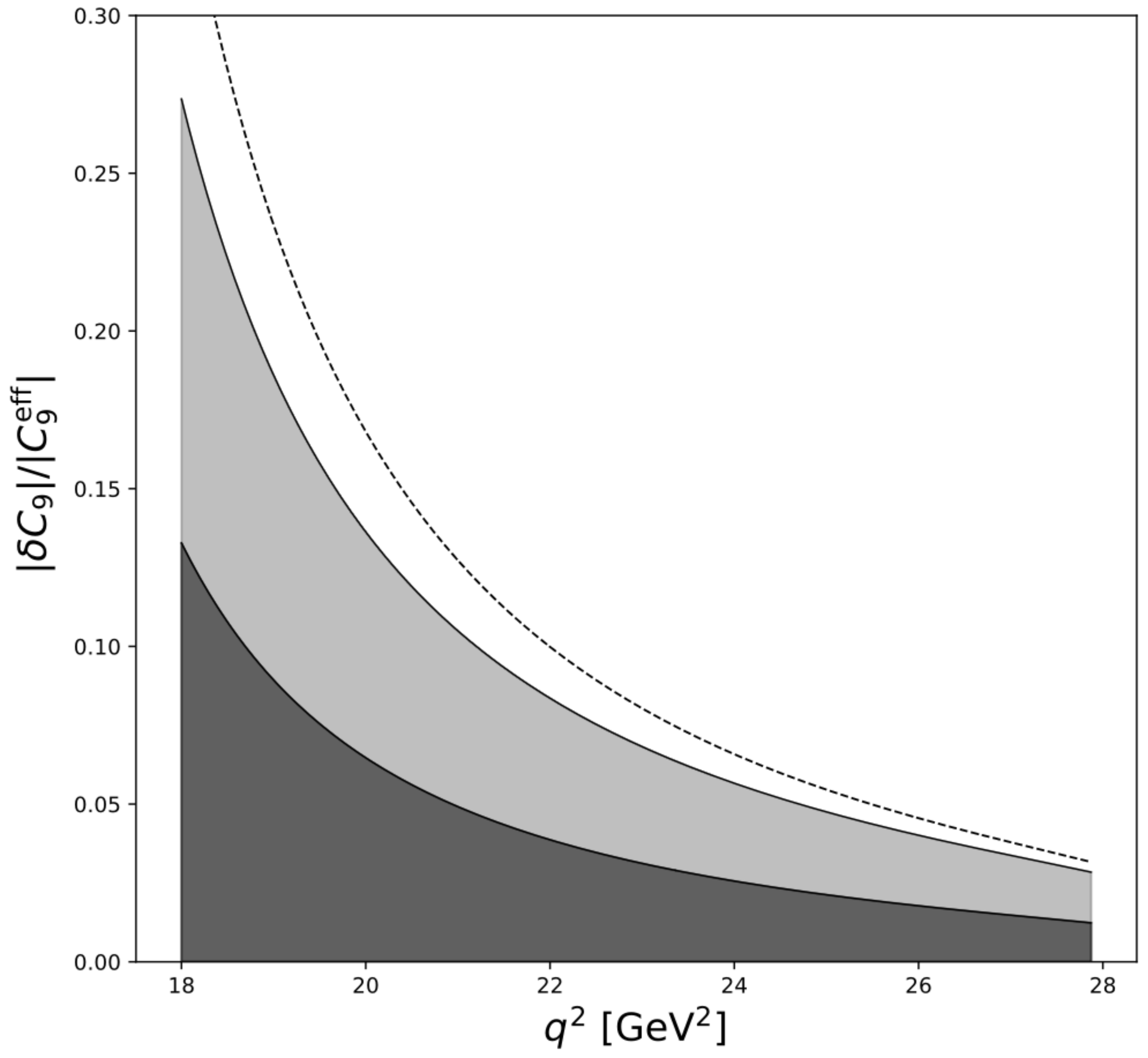


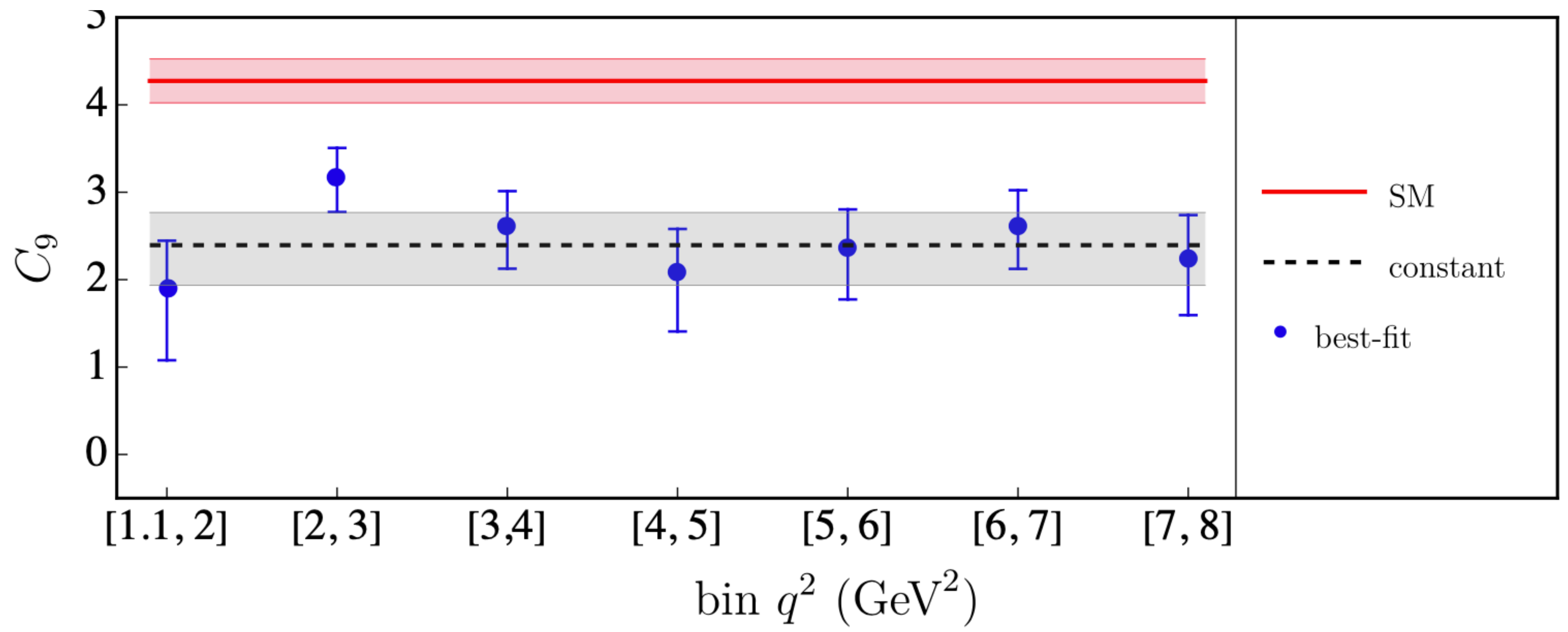
- How large can these rescattering effects be?
  - **As large as they need to be** to explain observations: **20% of the total amplitude** [Ciuchini et al 2022]
  - A **direct estimate** using Heavy Hadron Chiral Perturbation Theory (for the $D^{(*)}D_s^{(*)}K$ and $D^*_{(s)}D^*_{(s)}\gamma$ vertices) and data (for the $BD^{(*)}D_s^{(*)}$ vertex) finds effects **up to 10% with a strong q^2 dependence**
    [Isidori, Polonsky, Tinari 2405.17551]

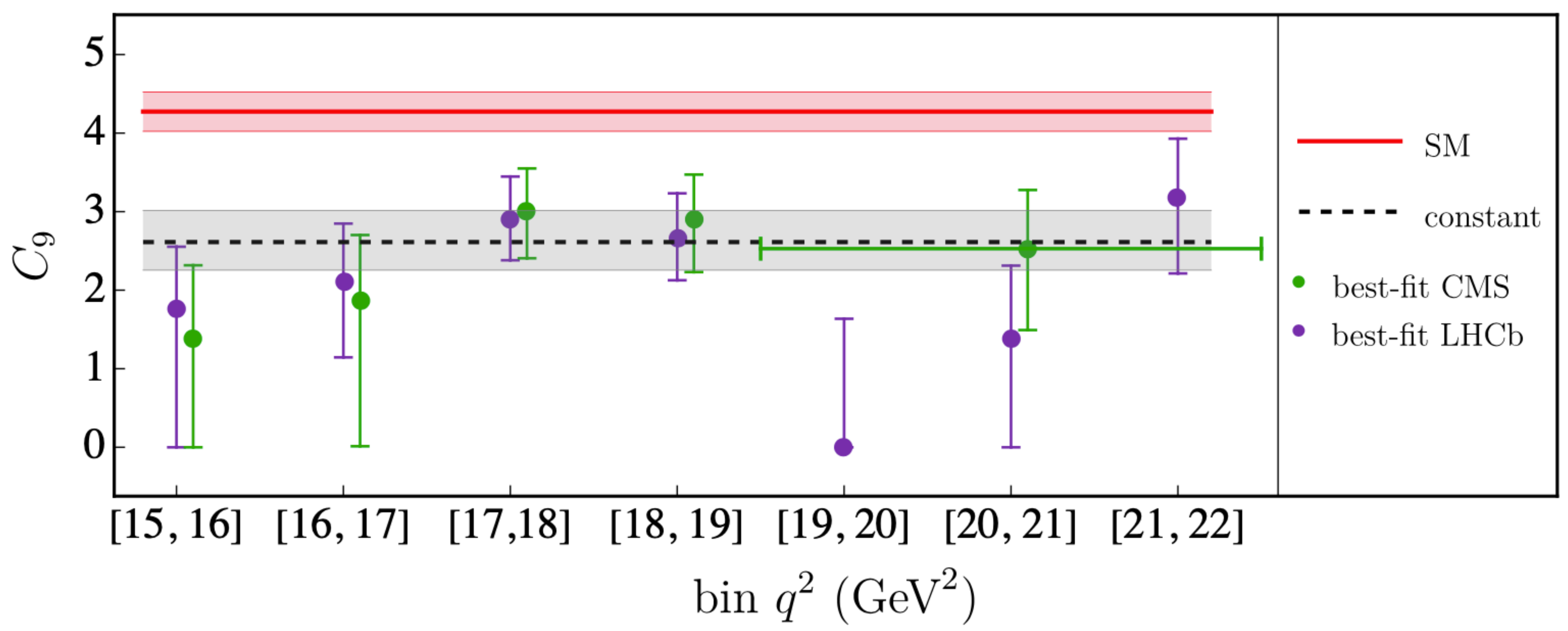
Non-local Power Corrections

- $B \to K\mu\mu$: q^2 dependence of DD_s^* and D_sD^* rescattering [Isidori, Polonsky, Tinari 2405.17551] vs fit results (assuming NP in C_9) [Bordone, Isidori, Mächler, Tinari 2401.18007]









Non-local Power Corrections

- **Lattice-QCD** breakthrough via **Spectral Density reconstuction**
- Sketch of the mechanism:

$$H^\mu(q_0,\vec{q}) = i\int d^4x\, e^{iqx}\, \langle K(\vec{p}_K)\,|\,T J^\mu_{\rm em}(x) O^c_{1,2}(0)\,|\,B(\vec{0})\rangle$$

$$\overset{t>0}{=} i\int_0^\infty dt\, e^{iq_0 t} \underbrace{\int \frac{dE}{2\pi} e^{-i(E-E_K)t}\, \rho^\mu(E,\vec{q})}_{\equiv C^\mu(t,\vec{q})}$$

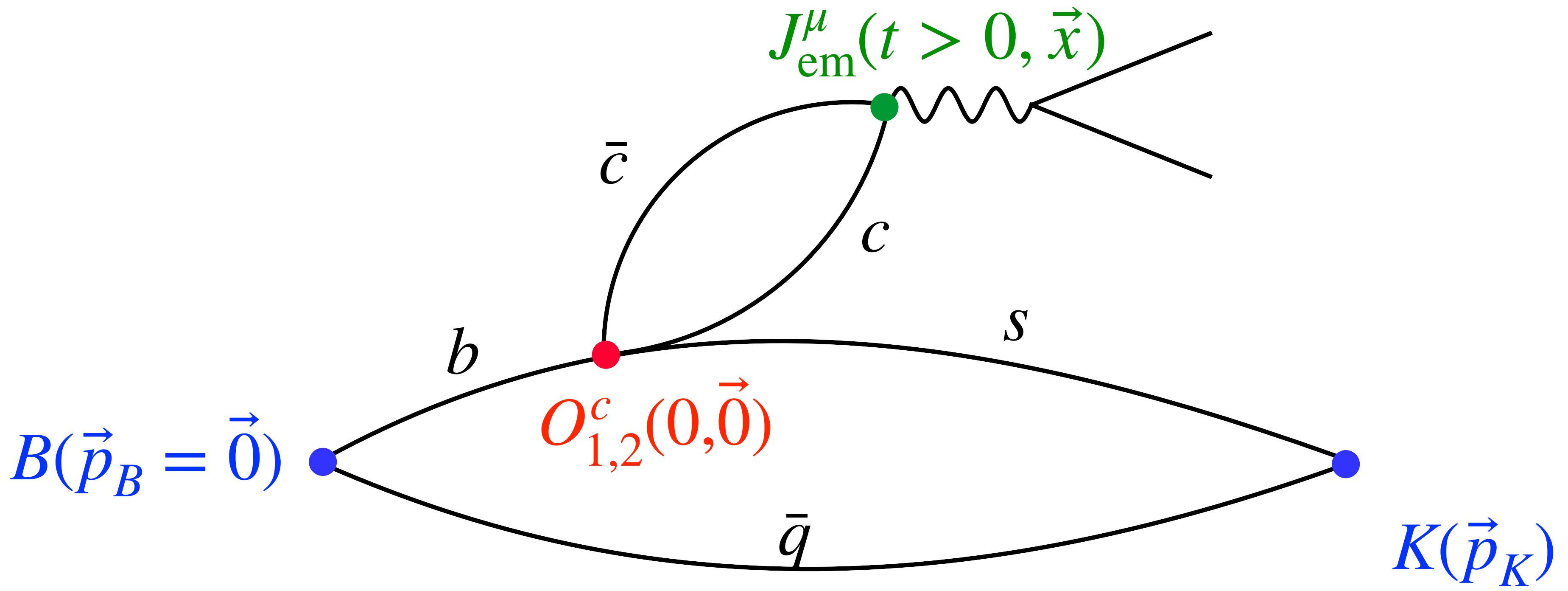
$$= \lim_{\epsilon->0} \int_{E^*}^\infty \frac{dE}{2\pi} \underbrace{\frac{1}{E-m_B-i\epsilon}}_{\simeq \sum_{n=1}^N g_n(\epsilon) e^{-anE}} \rho^\mu(E,\vec{q})$$

$\frac{1}{L} \ll \epsilon \ll \Delta(E)$ typical size over which the amplitude varies

$$= \lim_{\epsilon->0} \sum_{n=1}^N g_n(\epsilon)\, e^{-anE_K} \underbrace{\int_{E^*}^\infty \frac{dE}{2\pi} e^{-an(E-E_K)}\, \rho^\mu(E,\vec{q})}_{C^\mu_{\rm euclid}(na,\vec{q})}$$

$$= \lim_{\epsilon->0} \sum_{n=1}^N g_n(\epsilon)\, e^{-anE_K}\, C^\mu_{\rm euclid}(na,\vec{q})$$

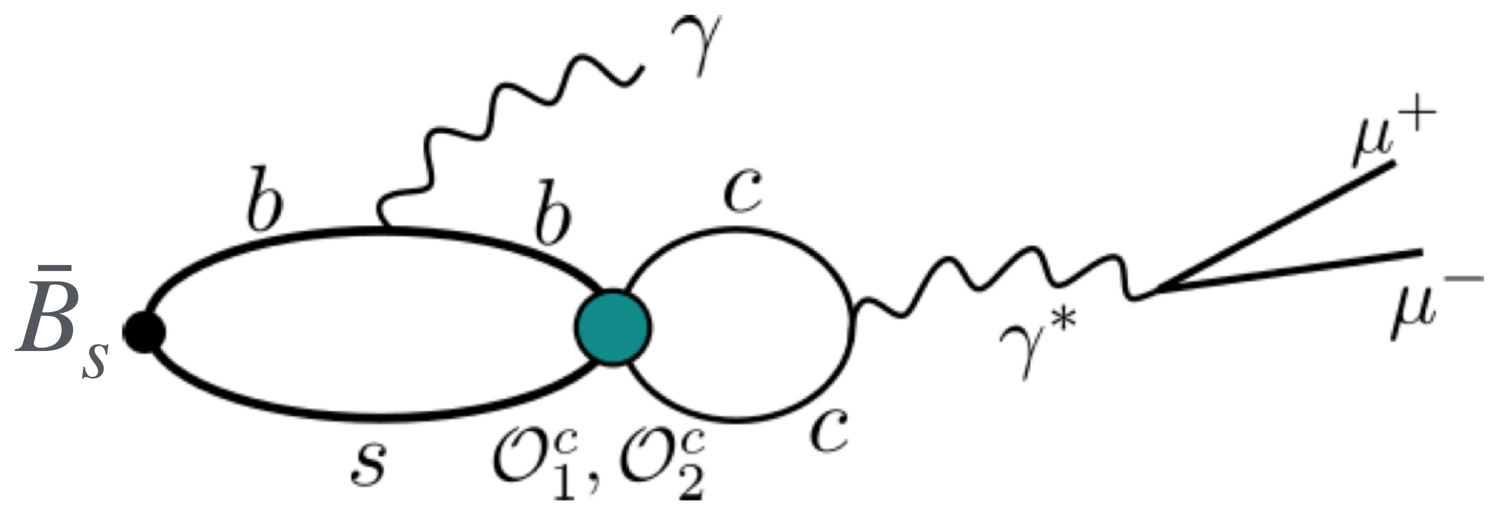
from Lattice QCD



[Hansen, Meyer, Robaina 1704.08993]
[Hansen, Lupo, Tantalo 1903.06476]
[Bailas, Hashimoto, Ishikawa 2001.11779]
[Frezzotti, .Gagliardi, Lubicz, Sanfilippo, Simula, Tantalo 2306.07228]
[Frezzotti, Gagliardi, Lubicz, Martinelli, Sachrajda, Sanfilippo, Simula, Tantalo 2402.03262]
[Frezzotti, Gagliardi, Lubicz, Martinelli, Sachrajda, Sanfilippo, Simula, Silvestrini 2508.03655]

Other applications of the Spectral Function technique

- Calculation of charming penguin contributions to $\bar{B}_s \to \gamma\mu^+\mu^-$
  [Frezzotti, Gagliardi, Lubicz, Martinelli, Sachrajda, Sanfilippo, Simula, Tantalo 2402.03262]



$$\langle 0|\, T\left[J^{\mu}_{\gamma}(t,\vec{x})\, J^{\nu}_{\gamma^*}(0,\vec{y})\, O^{(c)}_{1,2}(t_W,\vec{0})\right] |\bar{B}_s(\vec{0})\rangle$$

- Long-distance contributions to D mixing: x_{12} and y_{12}
  [di Carlo, Erben, Hansen 2504.16189]

$$\mathcal{M}^{\text{LD}}_{D^0\to\bar{D}^0} = \frac{\mathrm{i}}{2}\int \mathrm{d}^4x\, \langle \bar{D}^0, \boldsymbol{p}_D|\mathrm{T}\{\mathcal{H}_{\mathrm{w}}(x)\mathcal{H}_{\mathrm{w}}(0)\}|D^0,\boldsymbol{p}_D\rangle = \lim_{\epsilon\to 0}\int \frac{\mathrm{d}\omega}{2\pi}\frac{\rho(\omega)}{\omega - E_D - \mathrm{i}\epsilon}$$

$$\rho(\omega) = \langle \bar{D}^0, \boldsymbol{p}_D|\, \mathcal{H}_{\mathrm{w}}(0)\, (2\pi)^4\delta(\hat{H}-\omega)\delta^3(\hat{\mathbf{P}}-\boldsymbol{p}_D)\, \mathcal{H}_{\mathrm{w}}(0)\, |D^0,\boldsymbol{p}_D\rangle$$

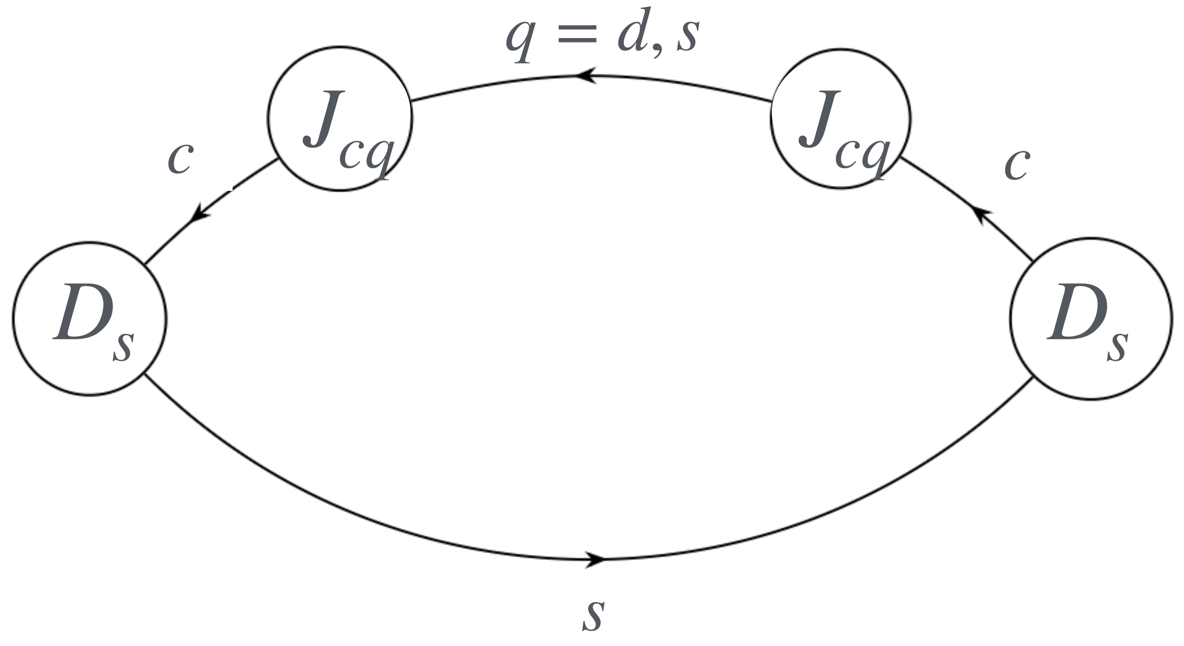
$$\Gamma_{12} = \frac{(\lambda_s-\lambda_d)^2}{4}\Gamma_2 + \frac{(\lambda_d-\lambda_s)\lambda_b}{2}\Gamma_1 + \frac{\lambda_b^2}{4}\Gamma_0$$

$$\Gamma_2 = \Gamma_{dd} - 2\Gamma_{ds} + \Gamma_{ss}$$
$$\Gamma_1 = \Gamma_{dd} - \Gamma_{ss}$$
$$\Gamma_0 = \Gamma_{dd} + 2\Gamma_{ds} + \Gamma_{ss}$$

- Decomposition in U-spin multiplets: $\Delta U = 2,1,0$
- CKM suppression: $|\lambda_b|/|\lambda_s - \lambda_d| \simeq 10^{-4}$

Other applications of the Spectral Function technique

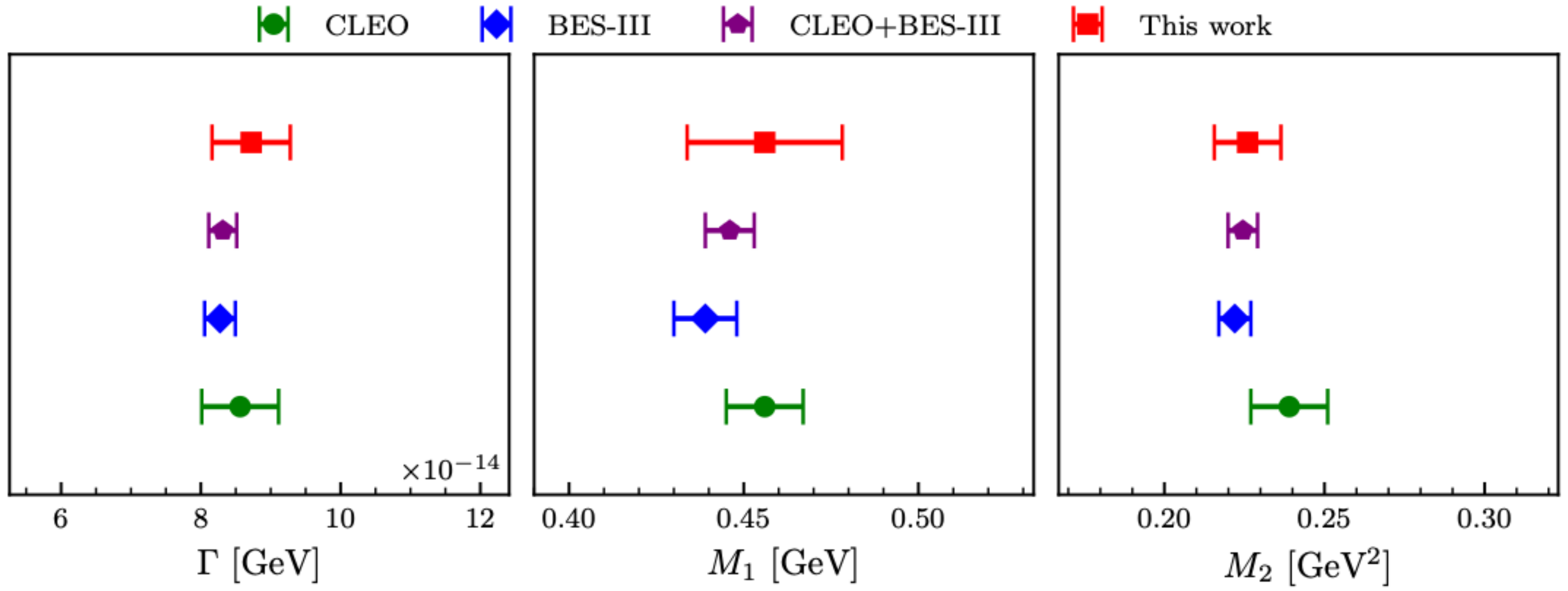
- Inclusive $D_s \to X\ell\bar{\nu}$ [de Santis et al (ETMC + Gambino) 2504.06063, 2504.06064]
  - Not yet competitive with the exclusive channel for the extraction of $|V_{cs}|$
  - Assuming SM provides an important test of the OPE in charm decays and information on hadronic matrix elements that appear at higher power in the HQET inclusive expansion



$$\Gamma^{\text{lat}} = 8.72(56) \times 10^{-14}\ \text{GeV} \qquad \Gamma^{\text{exp}} = 8.31(20) \times 10^{-14}\ \text{GeV}$$

$$M_1^{\text{lat}} = 0.456(22)\ \text{GeV} \qquad M_1^{\text{exp}} = 0.446(7)\ \text{GeV}$$

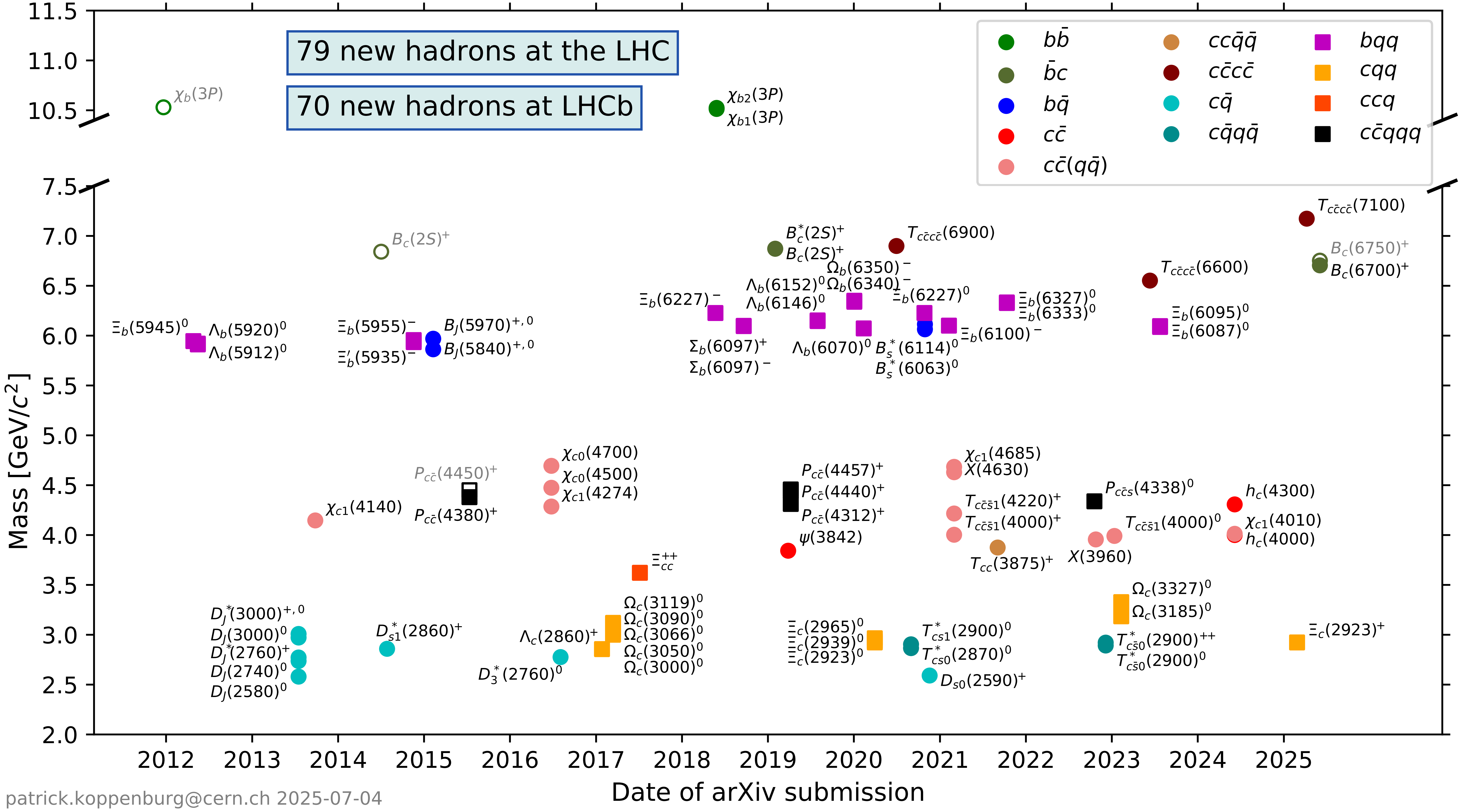
$$M_2^{\text{lat}} = 0.227(10)\ \text{GeV}^2 \qquad M_2^{\text{exp}} = 0.2245(46)\ \text{GeV}^2$$



- Important for cross checking Weak Annihilation matrix elements extracted from experimental data [Gambino Kamenik 1004.0114]

Spectroscopy

- In the last decade more than hundred new exotic states containing charm (mostly) and bottom quarks have been discovered



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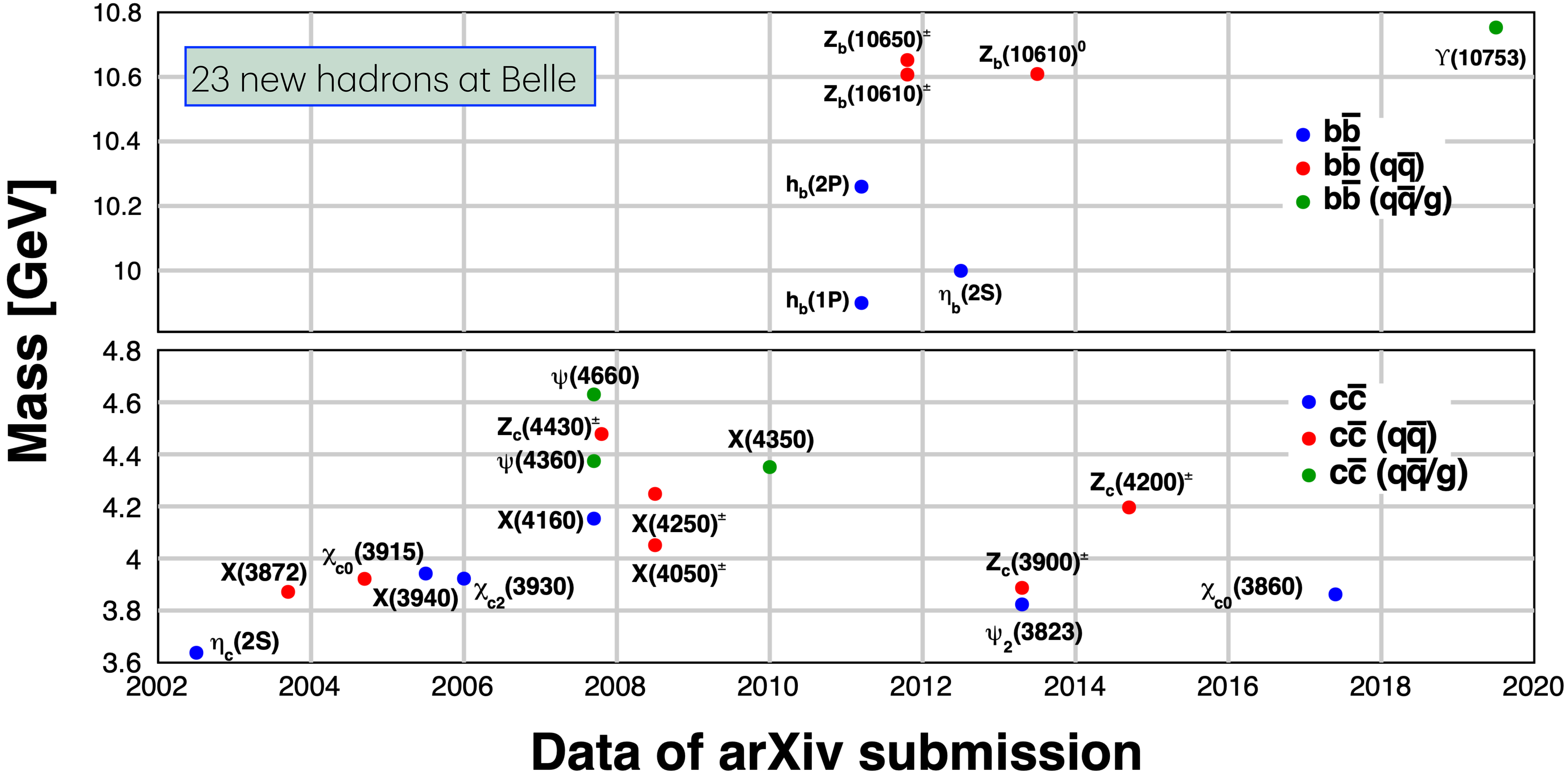

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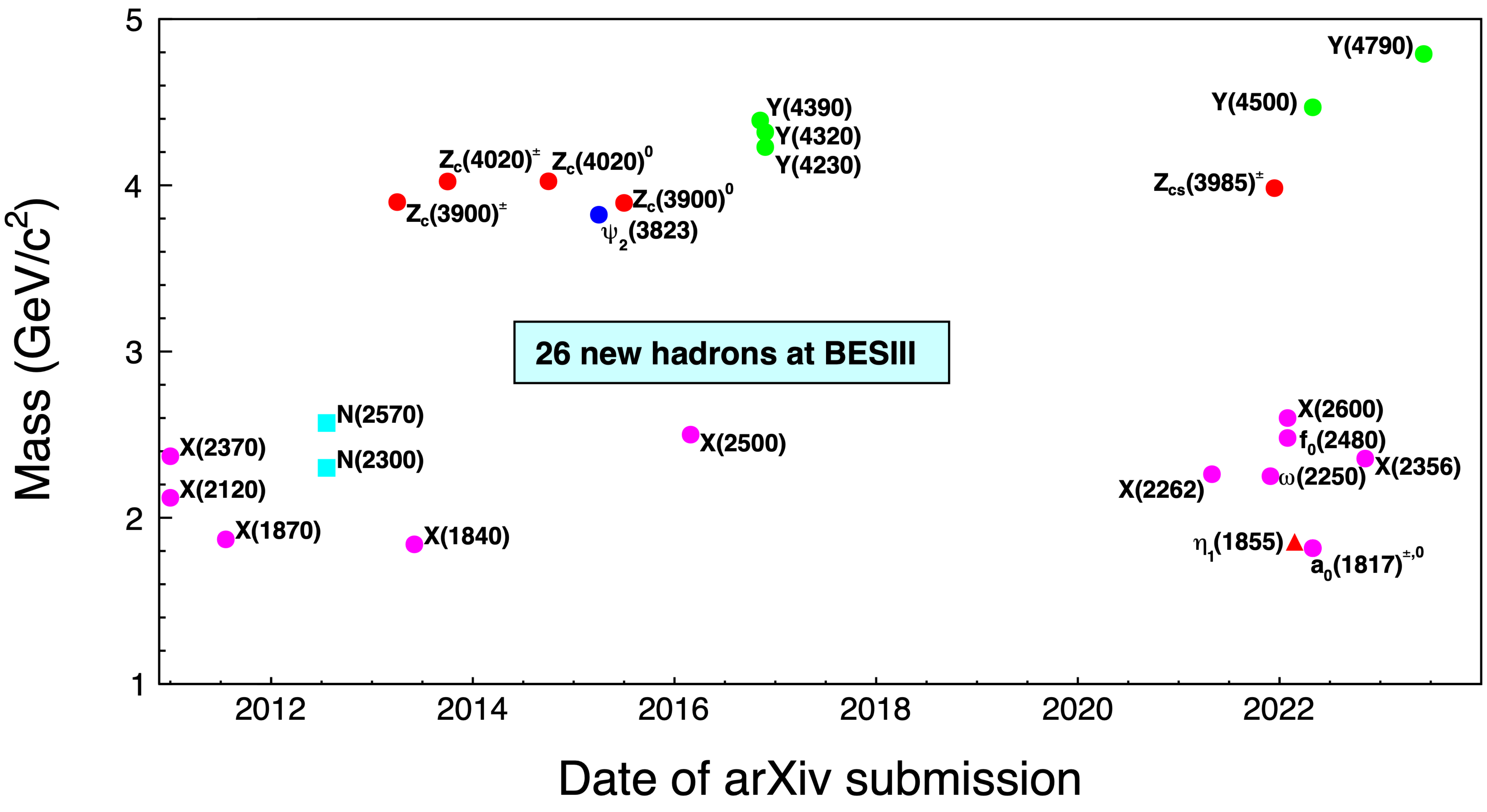
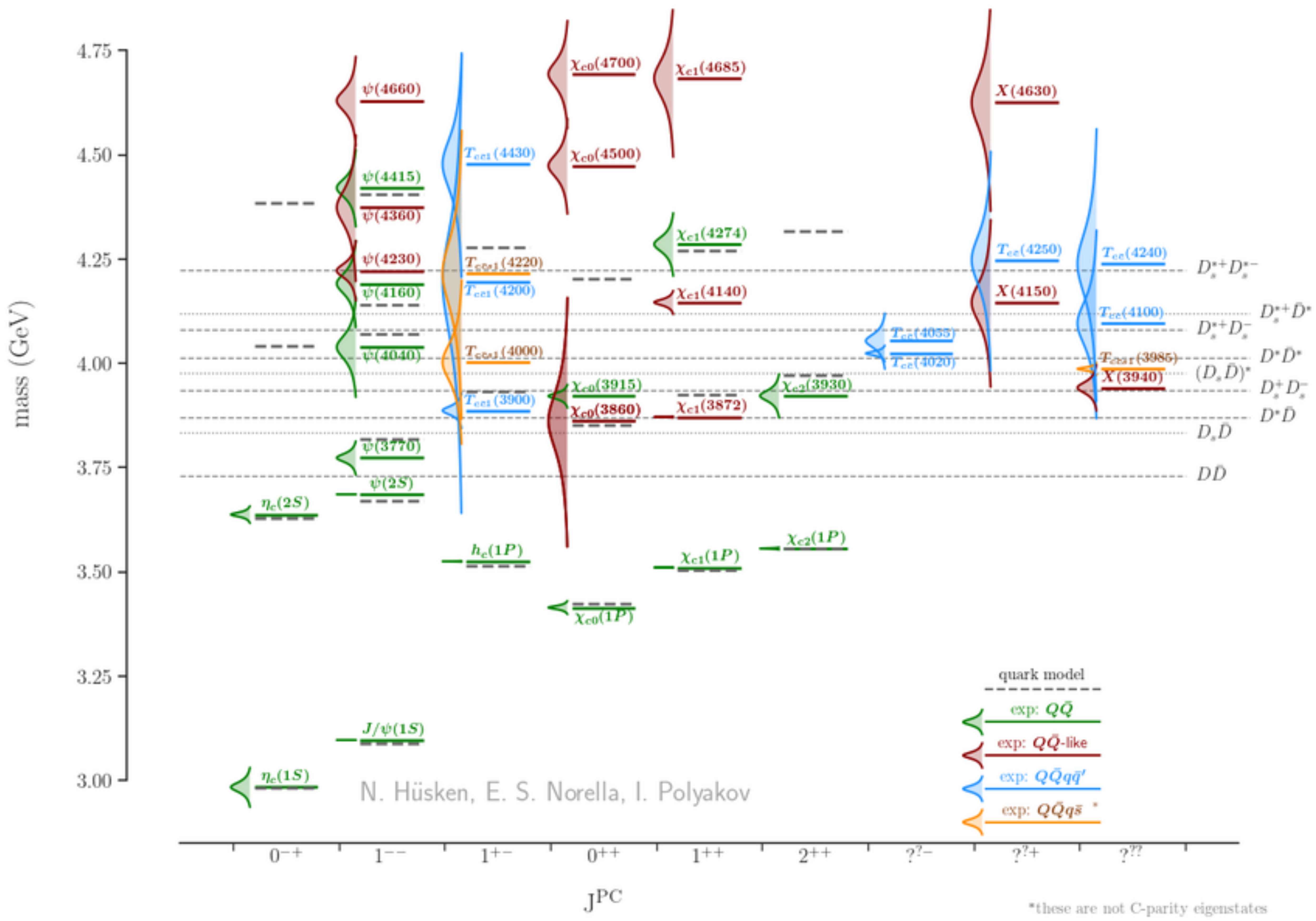


Spectroscopy

- Multi quark systems can be described using **lattice QCD** and **phenomenological approaches**
- Currently these two strategies are seen as complementary
- States with two heavy quarks because non-perturbative and relativistic effects tend to be smaller



Potential model calculations for conventional and exotic charmonium-like states
[Hüsken, Norella, Polyakov 2410.06923]

Spectroscopy: many concepts (not mutually exclusive)

- **Compact multiquark states**
  Two quarks in a $\bar{3}$ state ($3 \otimes 3 = \bar{3} \oplus 6$) attract each other and behave like an antiquark. In this view tetra- and penta-quarks can be seen as $[q_1 q_2][\bar{q}_3 \bar{q}_4]$ and $[q_1 q_2][q_3 q_4]\bar{q}_5$ configurations
- **Molecular picture**
  Exotic states seen as systems of two color-neutral objects (meson-meson, meson-baryon) which interact via nuclear forces (pion exchanges, contact terms, ...)
- **Hadroquarkonium**
  Exotic states with two heavy quarks, are thought as a color-singlet $\bar{c}c$ or $\bar{b}b$ state surrounded by a light quark cloud (like regular heavy mesons). This would explain why some exotics decay preferentially to quarkonium
- **Hybrid mesons and glueballs**
  Focus on $[qqg]$, $[gg]$ and $[ggg]$ structures
- **Fake resonances**
  It has been proposed that two-body thresholds and the presence of triangle diagram singularities (which are production process dependent) might be able to fake a resonant peak