Greg Landsberg - 21st Rencontres du Vietnam
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Why Do We Care?

- A number of the B meson branching fractions are claimed to be precisely determined (≈2% precision):

Semileptonic and leptonic modes					
Γ_1	$\ell^+\nu_\ell X$	[1]	$(10.99 \pm 0.28)\%$		
D, D^*, or D_s modes					
Γ_{54}	$\overline{D}^0\pi^+$		$(4.61 \pm 0.10) \times 10^{-3}$	2308	
Γ_{295}	$J/\psi(1S)K^+$		$(1.020 \pm 0.019) \times 10^{-3}$	1684	
K or K^* modes					
Γ_{288}	$K^+\pi^-$		$(2.00 \pm 0.04) \times 10^{-5}$	2615	

- However, this is not quite true:
  - A significant number of measurements made at B factories assumed isospin universality in ϒ(4S) decays, namely that

$$R^{\pm,0} \equiv f_{+-}/f_{00} \equiv \frac{\Gamma[\Upsilon(4S) \to B^+B^-]}{\Gamma[\Upsilon(4S) \to B^0\overline{B}^0]} = 1$$

  - Other take into account a handful of existing measurements of $R^{\pm,0}$, which may be statistically different from unity:
    - HFLAV 2024: $R^{\pm,0} = 1.052 \pm 0.031$ dominated by the 20-year-old BaBar measurement using single vs. double tags: PRL **94** (2005) 042001

Why Do We Still Care?

- The problem is not new [pointed out nearly a decade ago, e.g., Jung, PLB **753** (2016) 187] but often ignored or swept under the rug
  - PDG often averages branching fractions calculated with different $R^{\pm,0}$ assumptions without taking into account the difference - it's a mess!
- There are two immediate implications of $R^{\pm,0}$ not equal to unity:
  - Many of the B^+, B^0 branching fractions (measured assuming $R^{\pm,0}$ = 1) may differ from one another by as much as 5%
  - For those that explicitly use the HFLAV $R^{\pm,0}$ number, there is an additional 3% uncertainty related to the current uncertainty in $R^{\pm,0}$
- In fact, with the experimental community paying more and more attention to this issue, many of the new results coming from Belle II and LHCb often have the $R^{\pm,0}$ uncertainty as the leading one
- Consequently, one would like to have a significantly more precise determination of $R^{\pm,0}$ with the uncertainty of about 1% or less

Can One Calculate $R^{\pm,0}$?

- Naively it's not hard to do, as ϒ(4S) → BB decay is non-relativistic
  - The main effect is expected from Coulomb interaction in the B^+B^- system, which is absent in the neutral meson case
    - It significantly enhances the B^+B^- fraction (the overlap $|\psi(0)|^2$ is increased)
  - However, the $R^{\pm,0}$ value obtained this way is about 1.2, clearly inconsistent with the experimental determination
    - Atwood, Marciano, PRD **41** (1990) 1736
    - Lepage, PRD **42** (1990) 3251
  - A decade later, there have been several attempts to lower this value by considering various additional effects, such as the ϒ(4S) lineshape, but all of them can't really go below 1.1, and have large uncertainties
    - Kaiser, Manohar, Mehen, PRL **90** (2003) 142001
    - Voloshin, Phys. Atom. Nucl. **63** (2005) 771
    - Dolinsky et al, PRD **75** (2007) 113001
    - Milstein, Salnikov, PRD **104** (2021) 014007

Our Approach

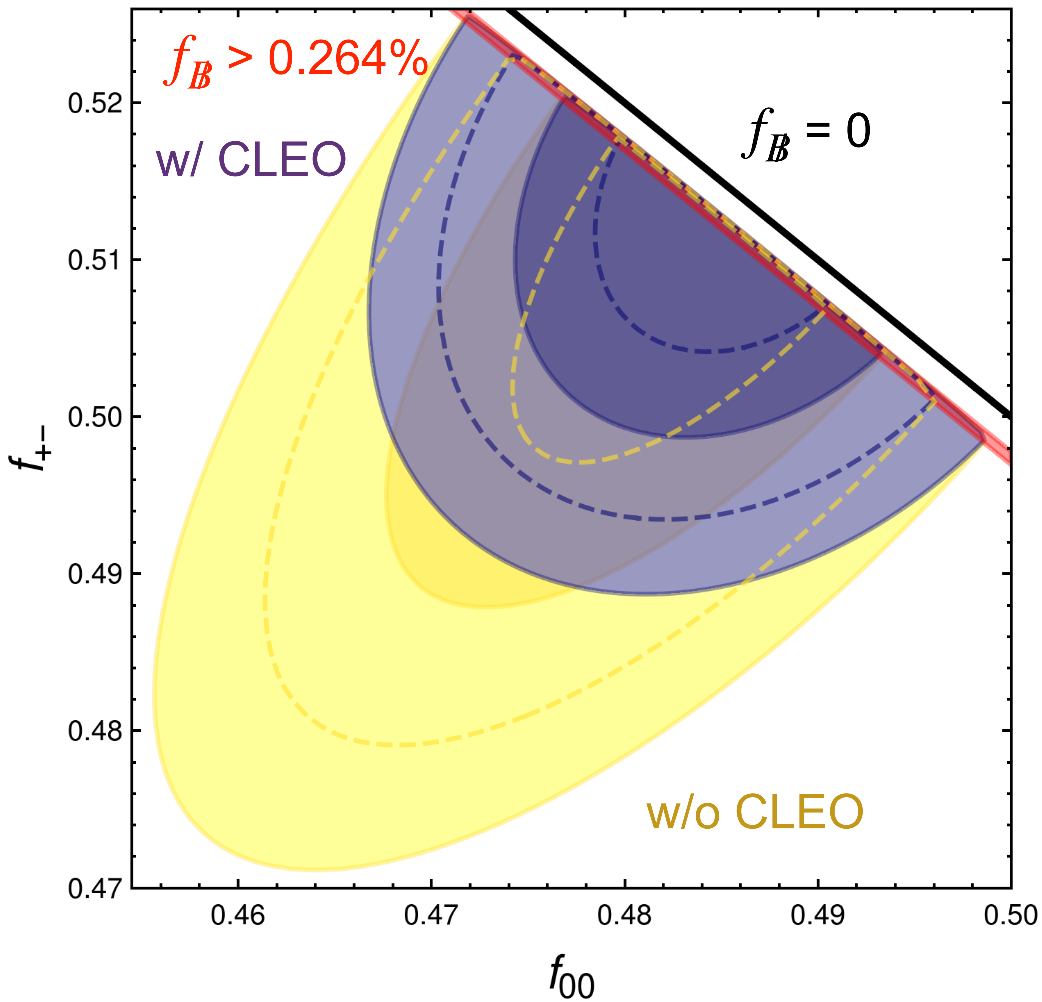
- We decided to systematically review available information on $R^{\pm,0}$ and derive a new average
- We further discuss the dependence of the results on the assumption of the non-BB fraction of the ϒ(4S) decays $f_{\not{B}}$
- With these in mind, we propose an experimental program that could be carried out by Belle II and the (HL-)LHC experiments to determine $R^{\pm,0}$ with the precision of about 1%

Present $R^{\pm,0}$ Average

- Note that $f_{\not{B}} = 1 - f_{00} - f_{+-}$ and given $R^{\pm,0} = f_{+-}/f_{00}$, it's clear that in order to relate $R^{\pm,0}$ to absolute branching fractions, the knowledge of $f_{\not{B}}$ is required
  - The best available measurement to date is a 30 years old measurement from CLEO: $f_{\not{B}} = (-0.11 \pm 1.43 \pm 1.07)\,\%$ [PRL **76** (1996) 1570]
    - The precision is hardly satisfactory; a better measurement is definitely needed!
- The latest HFLAV average [Banerjee et al, arXiv:2411.18639] is $R^{\pm,0} = 1.052 \pm 0.031$
  - Following our paper, this number now includes a fraction of non-BB decays of Y(4S) from CLEO in the fit and also more conservative assumptions about the isospin invariance in the decays they include
    - The previous HFLAV average [PRD **107** (2023) 052008] just used a lower bound $f_{\not{B}} > (0.264 \pm 0.021)\,\%$ that comes from the measurement if Y(4S) decay modes with additional pions, they also used more aggressive uncertainties on isospin violation in decays, resulting in $R^{\pm,0} = 1.057^{+0.024}_{-0.025}$

The $f_{\not B}$ Treatment

- ✦ Accounting for $f_{\not B}$ makes significant difference



N.B. HFLAV does not provide the correlation between the two fractions

Our $R^{\pm,0}$ Analysis

- We considered three classes of measurements w/ different theoretical assumptions and assign uncertainties for these assumptions when deriving the new average

$R^{\pm 0}$	Method	Comment
1.047(44)(36)	Single vs. double-tag	Uses $f_{\not B}$, see text
1.039(31)(50)	$B \to X_c \ell \nu$	Assumes negligible isospin violation
1.068(32)(20)(21)	$B \to X_s \gamma$	Third uncertainty due to resolved photon contributions
1.055(30)		Average categories I and II
1.065(12)(19)(32)	$B \to J/\psi K$	Third uncertainty due to isospin violation in $B \to J/\psi K$
1.013(36)(27)(30)	$B \to J/\psi K$	Third uncertainty due to isospin violation in $B \to J/\psi K$
1.100(35)(35)(33)	$B \to J/\psi(ee)K$	Third uncertainty due to isospin violation in $B \to J/\psi K$
1.066(32)(34)(32)	$B \to J/\psi(\mu\mu)K$	Systematic uncertainties $\sim 100\%$ correlated with ee mode
1.060(18)(32)		Average for $B \to J/\psi K$
1.057(23)		**Average of all categories I–III**

$$R^{\pm,0} = 1.057 \pm 0.023$$

$$R^{\pm,0}_{\mathrm{HFLAV}} = 1.052 \pm 0.031$$

More precise than HFLAV, consistent in the central value, but significantly more robust

Back to $R^{\pm,0}$ from Theory

- Let's examine more closely the original "naive" $R^{\pm,0}$ calculations
- There are two factors that enter $R^{\pm,0}$:
  - Phase space difference between the charged and neutral BB systems, $R_{\rm PS}^{\pm,0} = \dfrac{p_\pm^3}{p_0^3} \approx 1.048$
  - Coulomb enhancement: $R_{\rm CE}^{\pm,0} = \dfrac{2\pi\lambda(1+\lambda^2)}{1-e^{-2\pi\lambda}}$, $\lambda = \dfrac{\alpha}{2v}$, $v = \sqrt{1-(2m_B^+/m_{\Upsilon(4S)})^2}$

Decay Mode	$R_{\rm PS}^{\pm 0}$	$R_{\rm CE}^{\pm 0}$	$R_{\rm PS}^{\pm 0}\, R_{\rm CE}^{\pm 0}$
$\Upsilon(4S) \to B\overline{B}$	1.048	1.20	1.26

- Interestingly enough, our average $R^{\pm,0} = 1.057 \pm 0.023$ is consistent with the phase space only suppression [and so is HFLAV's]
- Why Coulomb interaction does not appear to play any role is still not understood and puzzling

A Way Out (Barolo Lunch)

- Should use a different system with smaller PS/CE effects to ensure that $R^{\pm,0}$ is closer to one!
- Why not use ϒ(5S)? 💡
- Indeed:

Decay Mode	$R_{\rm PS}^{\pm 0}$	$R_{\rm CE}^{\pm 0}$	$R_{\rm PS}^{\pm 0}\, R_{\rm CE}^{\pm 0}$
$\Upsilon(4S) \to B\overline{B}$	1.048	1.20	1.26
$\Upsilon(5S) \to B\overline{B}$	1.003	1.05	1.05
$\Upsilon(5S) \to B^*\overline{B}^*$	1.004	1.06	1.06

- Expect that $R^{\pm,0}$ should be very close to 1 on ϒ(5S)

$$R_{5S}^{\pm,0} = \frac{\Gamma[\Upsilon(5S) \to B^{(*)+}B^{(*)-}]}{\Gamma[\Upsilon(5S) \to B^{(*)0}\overline{B}^{(*)0}]} \approx 1$$

A Way Out (Barolo Dinner)

- Consider the double ratio of the Υ(4S) to Υ(5S) for specific decays:

$$r(f, f') = \left[\frac{N(B^+ \to f)}{N(B^0 \to f')}\right]_{\Upsilon(4S)} \Bigg/ \left[\frac{N(B^+ \to f)}{N(B^0 \to f')}\right]_{\Upsilon(5S)}$$

- All systematic uncertainties related to specific final states (e.g., possible isospin violation in decay) cancels in this double-ratio, giving a direct access to $R^{\pm,0}$ on Υ(4S), assuming it's ≈1 for Υ(5S)!
- Requires dedicated running on Υ(5S) for Belle II and/or reanalysis of the existing BaBar/Belle Υ(5S) data
- Can one reach the desired O(1%) precision?

A Way Out (After Barolo)

- Here are our projections for the few most promising channels we have identified

	Belle	Belle II partial	Belle II full
$\mathcal{L}_{\Upsilon(5S)}$ / $\mathcal{L}_{\Upsilon(4S)}$ [ab^{-1}/ab^{-1}]	0.12 / 0.71	0.5 / 5	5 / 50
$N^{\Upsilon(5S)}_{B^{(*)}B^{(*)}}$ / $N^{\Upsilon(4S)}_{BB}$	2.74×10^7 / 7.72×10^8	1.13×10^8 / 5.55×10^9	1.13×10^9 / 5.55×10^{10}
$f,\ f'$		$\Delta r(f,f')/r(f,f')$	
$J/\psi K^+,\ J/\psi K^0$	7.1%	3.5%	1.1%
$\bar{D}^0\pi^+,\ D^-\pi^+$	2.4%	1.2%	0.4%
$\bar{D}^{*0}\ell^+\nu,\ D^{*-}\ell^+\nu$	4.5%	2.2%	0.7%
$\bar{D}^0\pi^+,\ D^{*-}\ell^+\nu$	1.8%	0.9%	0.3%

- Already could improve the present precision by reanalyzing Belle data
- Even with partial Belle II data (0.5/5 ab^{-1} on Υ(5/4S)), a percent precision can be reached, reaching a fraction of a percent with full Belle II data
- Very encouraging and resonated well with the Belle II management about potential advancement of the Υ(5S) program

What About (HL-)LHC?

- At hadron colliders, measurements of the production fractions f_s, f_u, f_d of B_s^0, B^+, B^0 mesons play similar role to $R^{\pm,0}$ at the B factories
  - The f_s/f_u is now the leading uncertainty in the best $\mathscr{B}(B_s^0 \to \mu^+\mu^-)$ measurement to date
  - Implicitly relies on the assumption of $f_u = f_d$

Summary of the systematic uncertainties for the $B_s^0 \to \mu^+\mu^-$ and $B^0 \to \mu^+\mu^-$ branching fraction measurements.

<table>
<tr><th>Effect</th><th>$B_s^0 \to \mu^+\mu^-$</th><th>$B^0 \to \mu^+\mu^-$</th></tr>
<tr><td>f_s/f_u ratio of the B meson production fractions</td><td>3.5%</td><td>–</td></tr>
<tr><td>d_{MVA} correction</td><td colspan="2">2–3%</td></tr>
<tr><td>Tracking efficiency (per kaon)</td><td colspan="2">2.3%</td></tr>
<tr><td>Trigger efficiency</td><td colspan="2">2.4–3.7%</td></tr>
<tr><td>Fit bias</td><td>2.2%</td><td>4.5%</td></tr>
<tr><td>Pileup</td><td colspan="2">1%</td></tr>
<tr><td>Vertex quality requirement</td><td colspan="2">1%</td></tr>
<tr><td>$B^+ \to J/\psi K^+$ shape uncertainty</td><td colspan="2">1%</td></tr>
<tr><td>$B^+ \to J/\psi K^+$ branching fraction</td><td colspan="2">1.9%</td></tr>
</table>

CMS, PLB **842** (2023) 137955

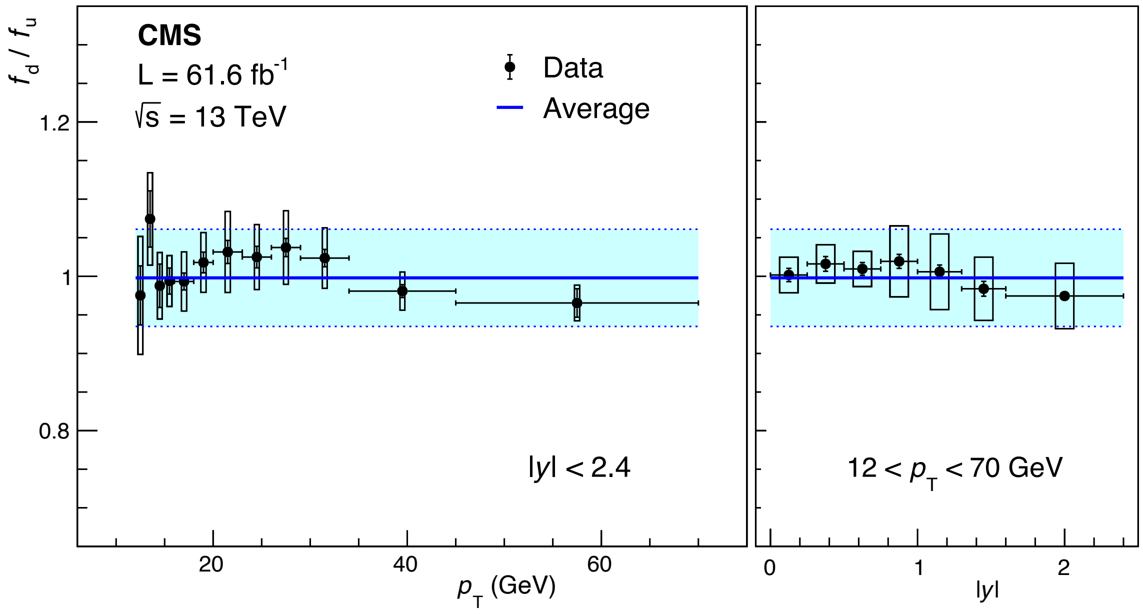
Is f_u Equal to f_d?

- This is essentially an isospin invariance assumption
- However, unlike e^+e^-, the pp collisions do not correspond to an isospin eigenstate, so the isospin conservation in the final state is questionable
  - Furthermore, there are additional particles always present, so the produced system is not a pure BB final state
- Since b and $\overline{b}$ quarks fragment independently, measurements of Type I (single vs. double tag) are not possible at the (HL-)LHC
- Hence, there is always a mixture of potential isospin violation in the initial and final state, with the latter needed to be disentangled

First Measurement of f_d/f_u

- The first Type-II measurement of the f_d/f_u ratio has been recently performed by CMS
  - Based on the ratio of $B^0 \to K^{*0} J/\psi$ and $B^+ \to K^+ J/\psi$ decay
  - Explicitly uses HFLAV $R^{\pm,0}$ as an external input, thus bringing in an 3% extra uncertainty
  - $f_d/f_u = 0.998 \pm 0.063$

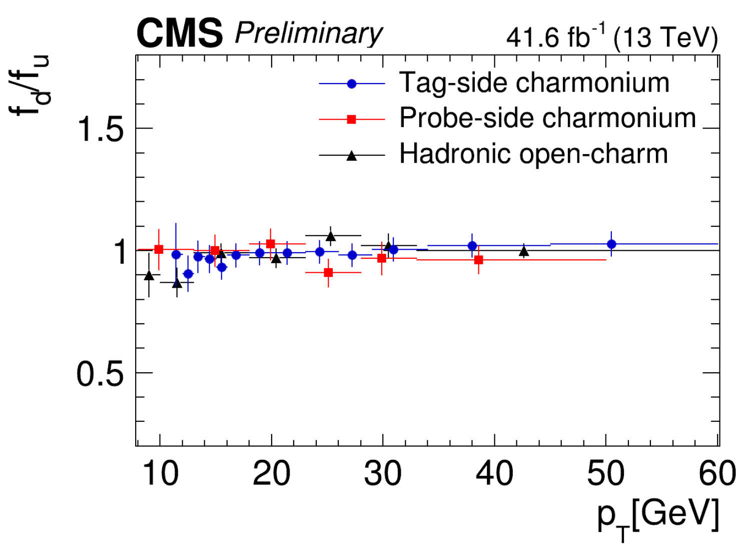
CMS, PRL **131** (2023) 121901

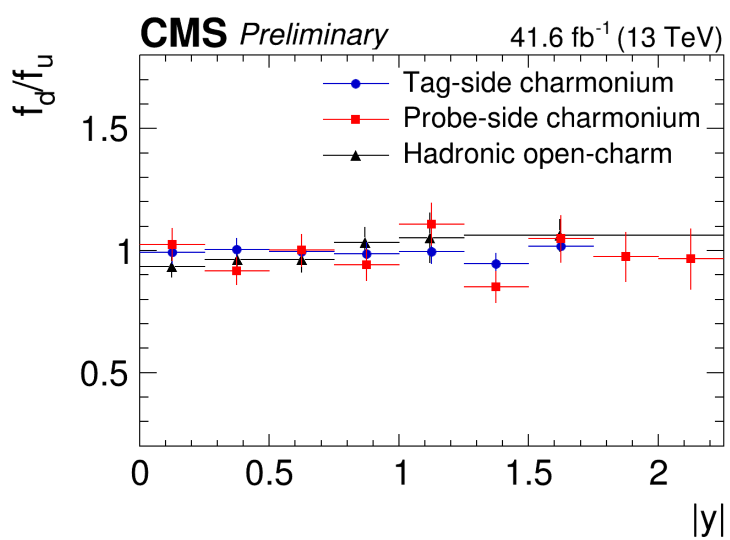


Open Charm Agrees

- Even more recently, CMS made public an improved isospin invariance test based on the charmonium and open-charm B meson decays, made possible by the B parking data program [Phys. Rep. **1115** (2025) 678]
  - $f_d/f_u = 0.98 \pm 0.05$

CMS PAS-BPH-21-007





Conclusions

- Precision determination of $R^{\pm,0}$ is important for a variety of measurements and theoretical calculations in many areas of B physics
  - Current precision makes it inconclusive whether the central value is consistent with unity or not
  - Affects the precision of several important branching fractions determination
- A precision measurement of non-BB fraction in ϒ(4S) decays is an important input to the extraction of $R^{\pm,0}$
  - The only existing measurement by CLEO, which is nearly 30 years old and has ~2% precision is not sufficient to reach a percent precision in $R^{\pm,0}$
- We proposed several novel methods of significantly shrinking the $R^{\pm,0}$ uncertainties (along with the fragmentation fraction ratios), using combination of B factories and (HL-)LHC measurements
- The phenomenological results look promising and we hope that they will serve as a guide for the ultimate experimental resolution of the $R^{\pm,0}$ puzzle!