Istituto Nazionale di Fisica Nucleare

Quy Nhon, 19th August 2025

Flavour Physics 2025

21st Rencontres du Vietnam 2025

Challenges in Rare Kaon decays

Giancarlo D'Ambrosio

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Predictions for the rare kaon decays
from QCD in the limit of a large number of colour
With M. Knecht and S.Neshatpour

e-Print: 2409.08568 [hep-ph] MDPI

THANKS to Collaborations Nazila Iyer Neshatpour

- 2311.04878 2404.03643

THANKS to NA48 HIKE LHCB KOTO
T. Kitahara

CP violation in $K \to \mu^+\mu^-$ with and without time dependence through a tagged analysis

Giancarlo D'Ambrosio (INFN, Naples), Avital Dery (CERN), Yuval Grossman (Cornell U., LEPP), Teppei Kitahara (Chiba U. and KMI, Nagoya and Nagoya U.), Radoslav Marchevski (EPFL, Lausanne, LPPC) et al. (Jul 17, 2025)

e-Print: 2507.13445 [hep-ph]

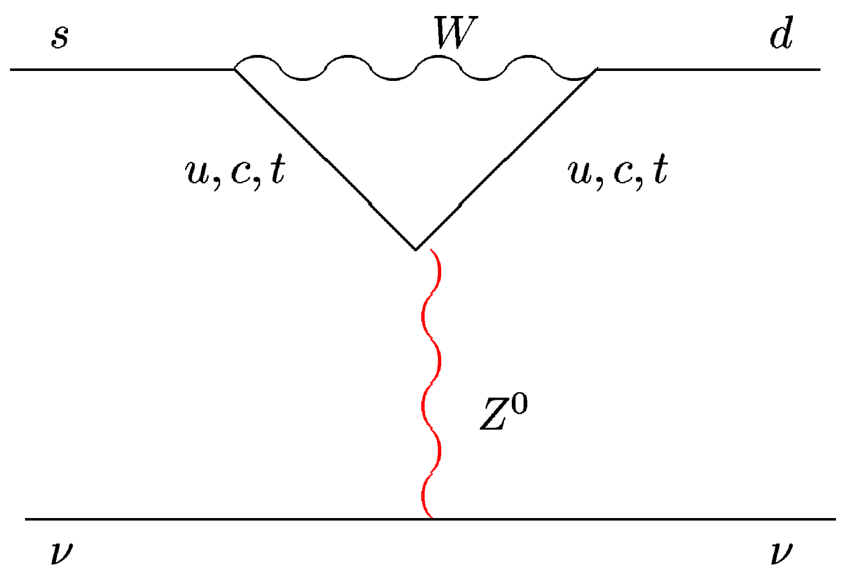
Outline

- Rare Kaon decays crucial in flavour physics
- Measuring $K_S K_L$ interference at LHCB
- Large Nc : may be useful to predict kaon decays $K^+ \to \pi^+ l^+ l^-$ $K_S \to \pi^0 l^+ l^-$

$K \to \pi \nu \overline{\nu}$

Why we need KOTO and NA62 → NEED KOTO AND HIKE

$$A(s \to d\nu\overline{\nu})_{\rm SM} \sim \overline{s}_L \gamma_\mu d_L \quad \overline{\nu}_L \gamma^\mu \nu_L \quad \times \left[\sum_{q=c,t} V^*_{qs} V_{qd}\, m_q^2\right]$$



$$\sim \quad \left[A^2\lambda^5\ (1-\rho-i\eta) m_t^2 + \lambda m_c^2\right]$$

SM $\underbrace{V-A \otimes V-A}$ Littenberg

$\Downarrow$

$$\Gamma(K_L \to \pi^0 \nu\overline{\nu}) \quad \begin{cases} \text{CP violating} \\ \Rightarrow\ J = A^2\lambda^6\eta \\ \text{Only } top \end{cases}$$

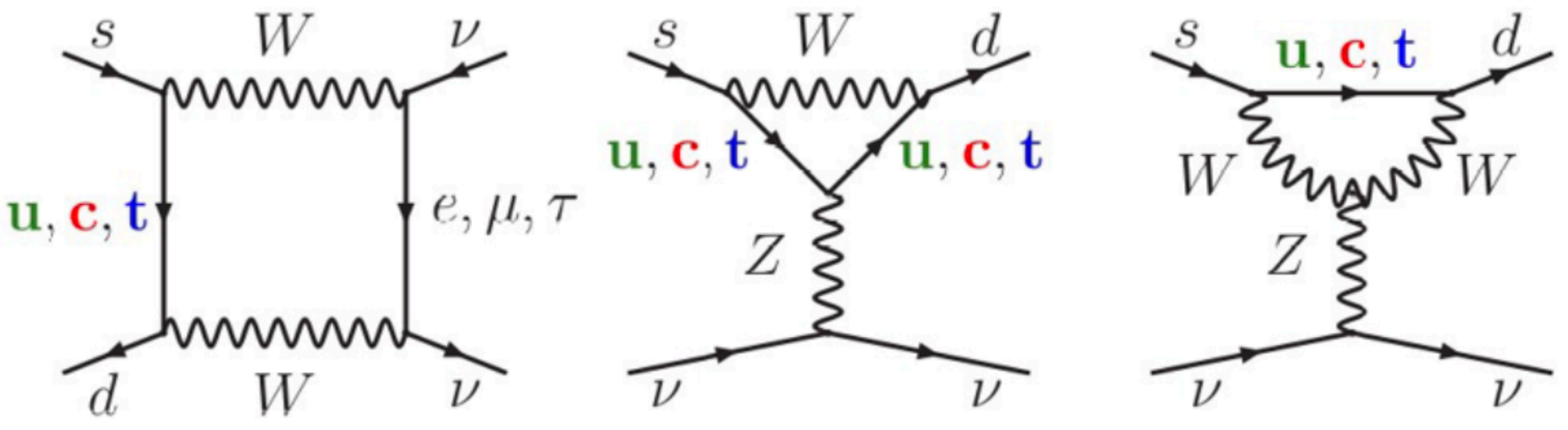
SM

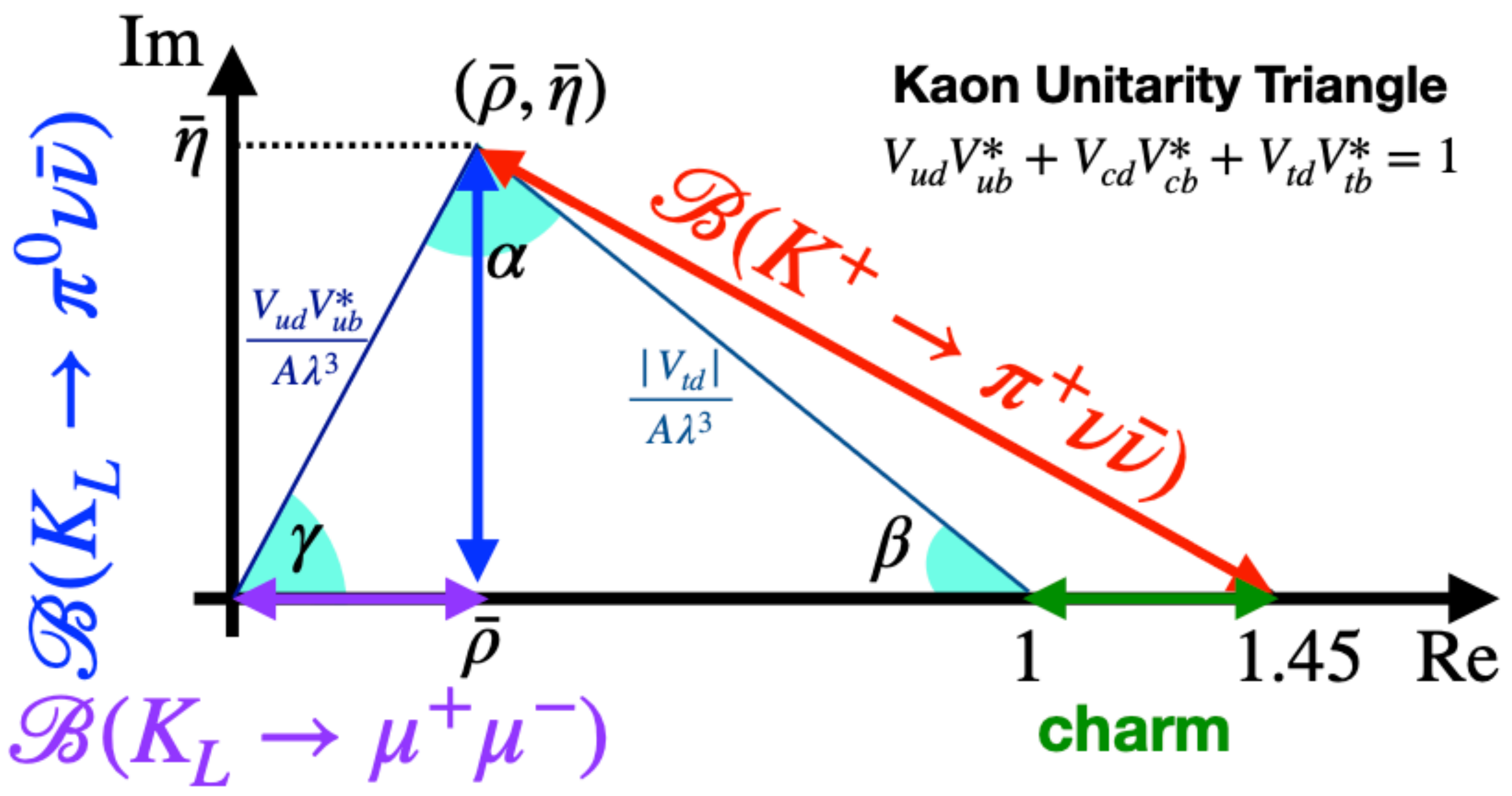
Buchalla and Buras, hep-ph/9308272, Buras et al, 1503.02693.

$K \to \pi\nu\bar{\nu}$: Precision test of the Standard Model

NA62

SM: Z-penguin & box diagrams



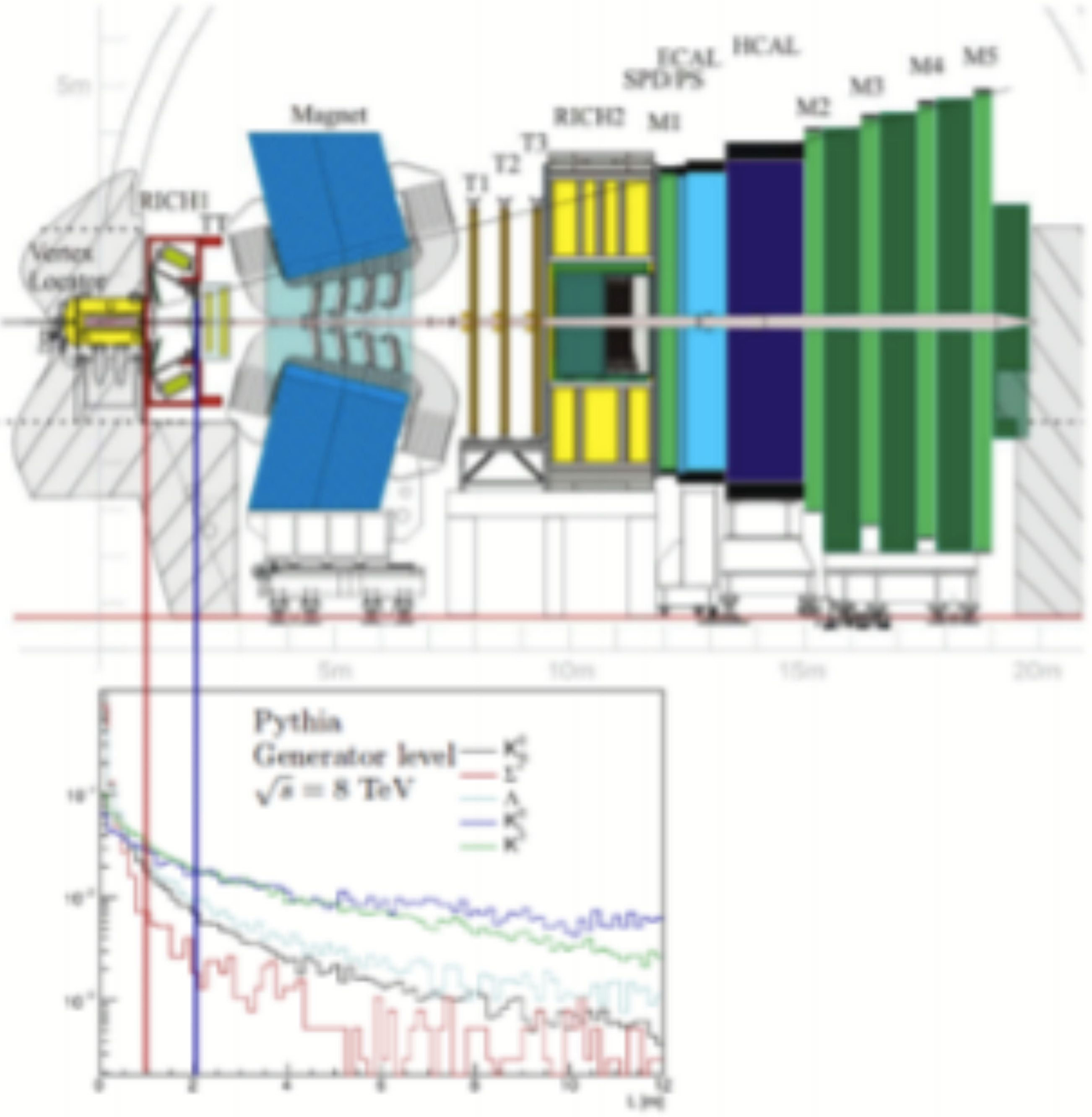


- $\mathscr{B}(K \to \pi\nu\bar{\nu})$ **highly suppressed** in SM
  - GIM mechanism & maximum CKM suppression $s \to d$ transition: $\sim \frac{m_t}{m_W} \left| V^*_{ts} V_{td} \right|$
- Theoretically clean ⇒ high precision SM predictions
  - Dominated by short distance contributions.
  - Hadronic matrix element extracted from $\mathscr{B}(K \to \pi^0 \ell^+ \nu_\ell)$ decays via isospin rotation.

Mode	SM Branching Ratio [1]	SM Branching Ratio [2]	Experimental Status
$K^+ \to \pi^+\nu\bar{\nu}$	$(8.60 \pm 0.42) \times 10^{-11}$	$(7.86 \pm 0.61) \times 10^{-11}$	$(10.6 \pm 4.0) \times 10^{-11}$ NA62 16−18
$K_L \to \pi^0\nu\bar{\nu}$	$(2.94 \pm 0.15) \times 10^{-11}$	$(2.68 \pm 0.30) \times 10^{-11}$	$< 2 \times 10^{-9}$ KOTO (2021 data)

^Recent SM calculations [1:Buras et al. EPJC 82 (2022) 7, 615][2:D'Ambrosio et al. JHEP 09 (2022) 148]
(Differences in SM calculations from choice of CKM parameters: see [Eur.Phys.J.C 84 (2024) 4, 377])

Kaon in LHCb



- LHCb experiment has been designed for efficient reconstructions of ***b*** and ***c***
- Huge production of strangeness [$O(10^{13})/\mathrm{fb}^{-1}$ K^0_S] is suppressed by its trigger efficiency [ε~1-2%@LHC Run-I, ε~18%@LHC Run-II]
- LHCb upgrade (LS2=Phase I upgrade, LS4=Phase II upgrade) could realize high efficiency for K^0_S [ε~90%@LHC Run-III] [M. R. Pernas, HL/HE LHC meeting, Fermilab, 2018]
- In LHC Run-III and HL-LHC, we could probe the *ultra* rare decay $\mathrm{Br}\sim O(10^{-11\sim-12})$

Rare Kaon decay program at LHCB

	PDG	Prospects
$K_S \to \mu\mu$	$< 9 \times 10^{-9}$ at 90% CL	$(\text{LD})(5.0 \pm 1.5) \cdot 10^{-12}$ NP $< 10^{-11}$
$K_S \to \mu\mu\mu\mu$	–	SM LD $\sim 2 \times 10^{-14}$
$K_S \to ee\mu\mu$	–	$\sim 10^{-11}$
$K_S \to eeee$	–	$\sim 10^{-10}$
$K_S \to \pi^0\mu\mu$	$(2.9 \pm 1.3) \cdot 10^{-9}$	$\sim 10^{-9}$
$K_S \to \pi^+\pi^-e^+e^-$	$(4.79 \pm 0.15) \cdot 10^{-5}$	SM LD $\sim 10^{-5}$
$K_S \to \pi^+\pi^-\mu^+\mu^-$	–	SM LD $\sim 10^{-14}$

Prospects for Measurements with Strange Hadrons at LHCb

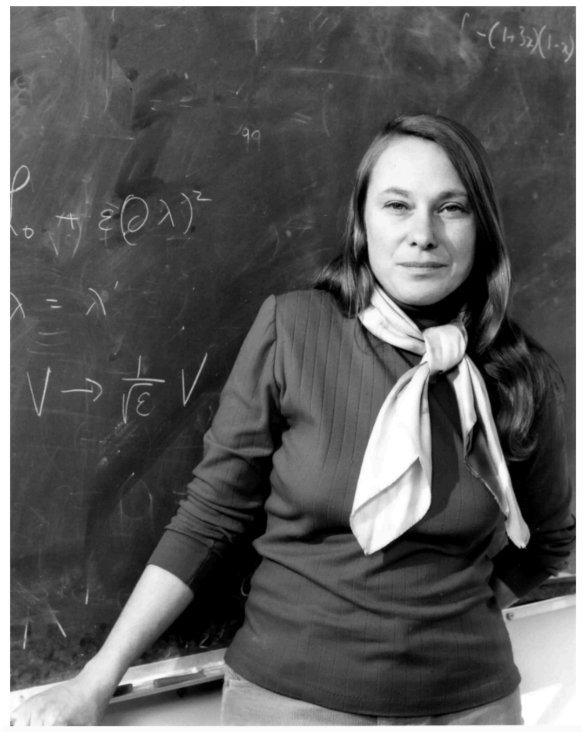
A.A. Alves Junior (Santiago de Compostela U., IGFAE), M.O. Bettler (Cambridge U.), A. Brea Rodríguez (Santiago de Compostela U., IGFAE), A. Casais Vidal (Santiago de Compostela U., IGFAE), V. Chobanova (Santiago de Compostela U., IGFAE) et al. (Aug 10, 2018)

Published in: *JHEP* 05 (2019) 048 • e-Print: 1808.03477 [hep-ex]

Rare n Strange 2017: strange physics at LHCb

$$K_{L,S} \to \mu\mu$$

$K_L \to \mu\mu$


Mary K. Gaillard in 1985, four years after she became the first woman hired by the physics department at the University of California, Berkeley. University of California, Berkeley

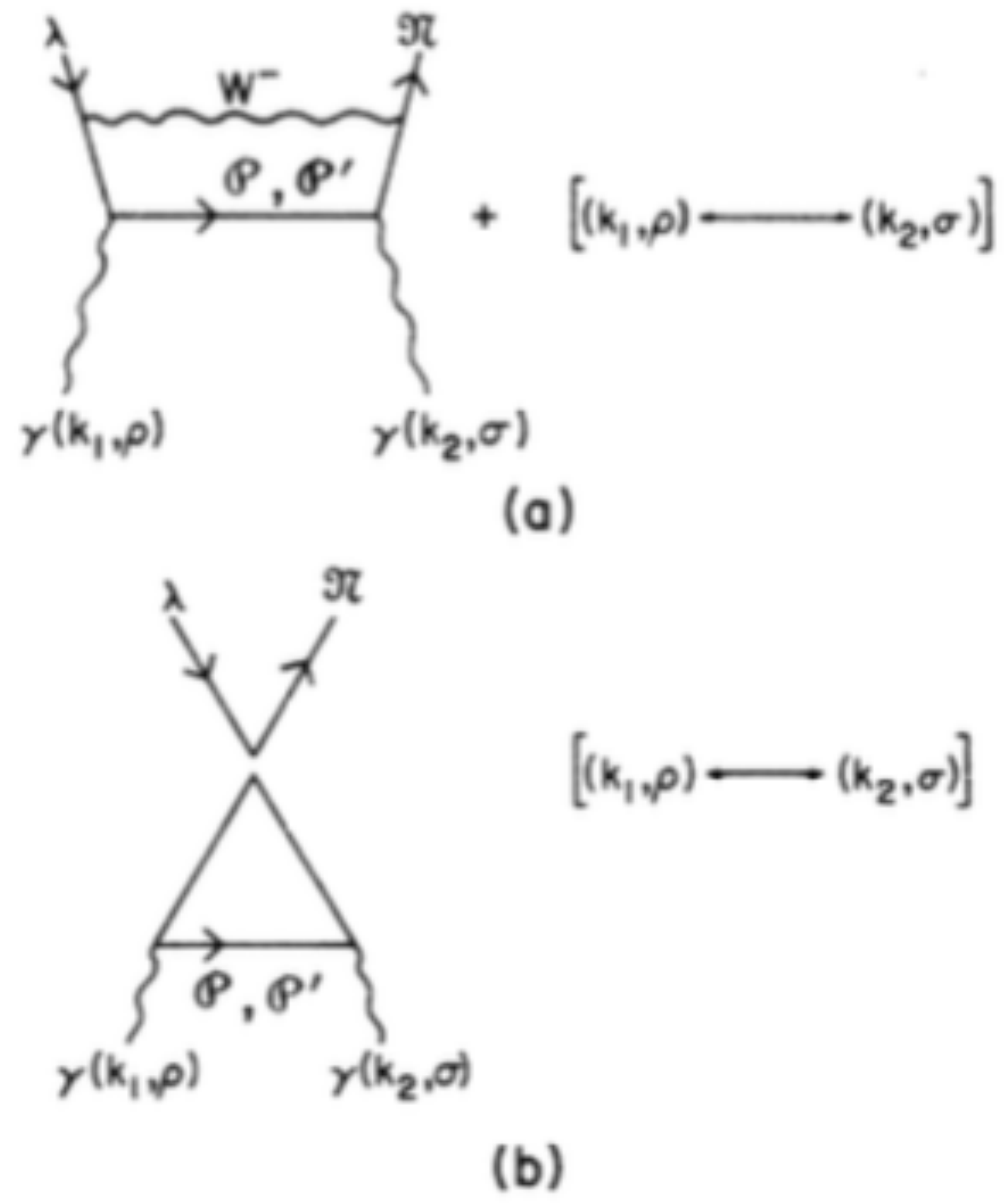


FIG. 7. Leading contributions to $\lambda + \bar{\mathfrak{N}} \to \gamma + \gamma$. To leading order in M_W^{-2}, the diagrams in (a) reduce to those of (b).

Gaillard Lee

$\Gamma(K_L^0 \to \mu^+\mu^-)/\Gamma(K_L^0 \to \pi^+\pi^-)$

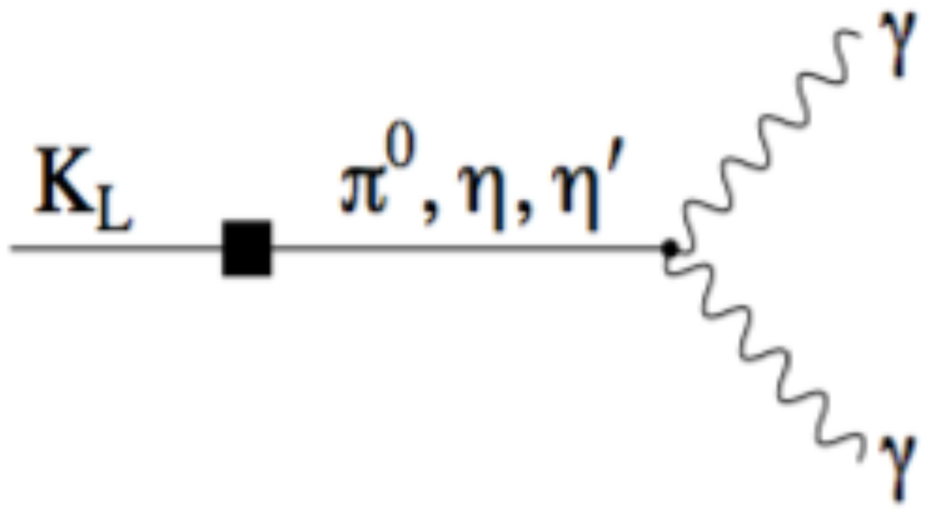
VALUE (10^{-6})	EVTS	DOCUMENT ID		TECN	COMMENT
3.48 ± 0.05	**OUR AVERAGE**				
3.474 ±0.057	6210	AMBROSE	2000	B871	
3.87 ±0.30	179	[1] AKAGI	1995	SPEC	
3.38 ±0.17	707	HEINSON	1995	B791	
• • • We do not use the following data for averages, fits, limits, etc. • • •					
3.9 ±0.3 ±0.1	178	[2] AKAGI	1991B	SPEC	In AKAGI 1995

$$\mathcal{B}(K_L \to \mu^+\mu^-)_{\rm exp} = (6.84 \pm 0.11) \times 10^{-9}$$

$K_L \to \gamma\gamma\,|_{\rm exp}$ known

Dispersive calculation: Re A, Im A

We do not know the sign of $A(K_L \to \gamma\gamma)$

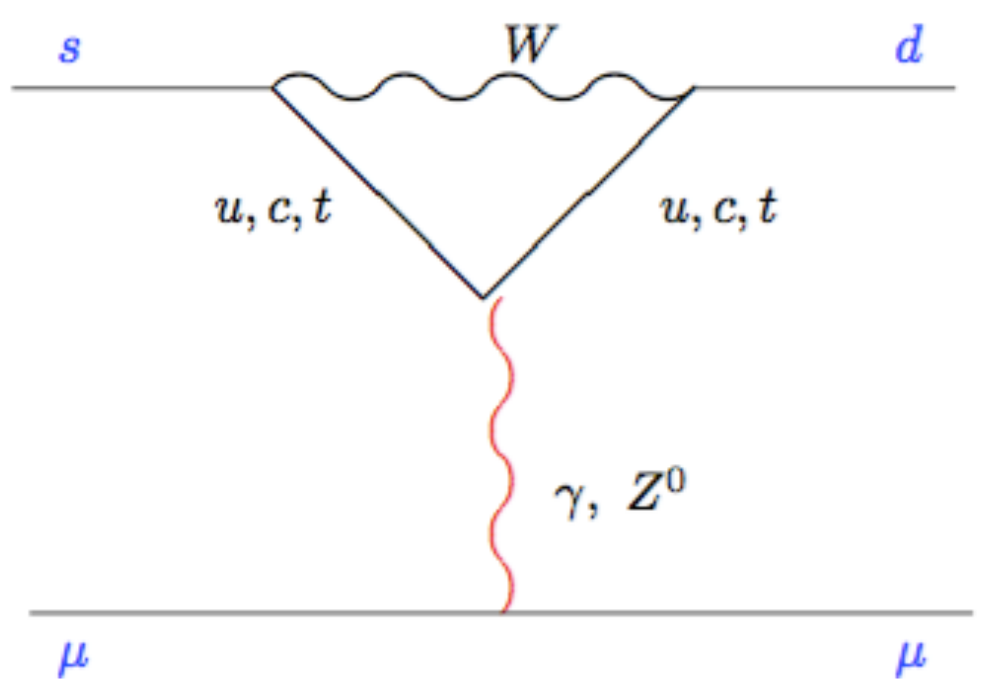


$$A(K_L \to 2\gamma_\perp)_{O(p^4)} = A(K_L \to \pi^0 \to 2\gamma_\perp) + A(K_L \to \eta_8 \to 2\gamma_\perp)$$
$$= A(K_L \to \pi^0)A(\pi^0 \to 2\gamma_\perp)\left[\frac{1}{M_K^2 - M_\pi^2} + \frac{1}{3}\cdot\frac{1}{M_K^2 - M_8^2}\right] \simeq 0$$

Kaon Decays in the Standard Model
Vincenzo Cirigliano (Los Alamos), Gerhard Ecker, Helmut Neufeld (Vienna U.), Antonio Pich, Jorge Portoles, refs therein

Isidori Unterdorfer
+many refs…

$K_L \to \mu\mu$



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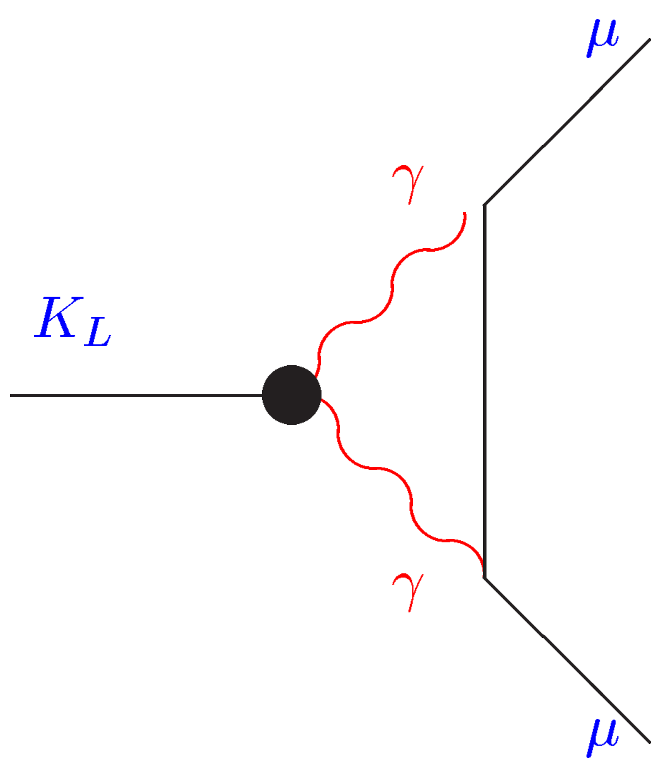


$$\frac{\Gamma(K_L \to \mu\bar{\mu})}{\Gamma(K_L \to \gamma\gamma)} \sim \quad |ReA|^2 + |ImA|^2$$

Absorptive calculation
model independent

27.14

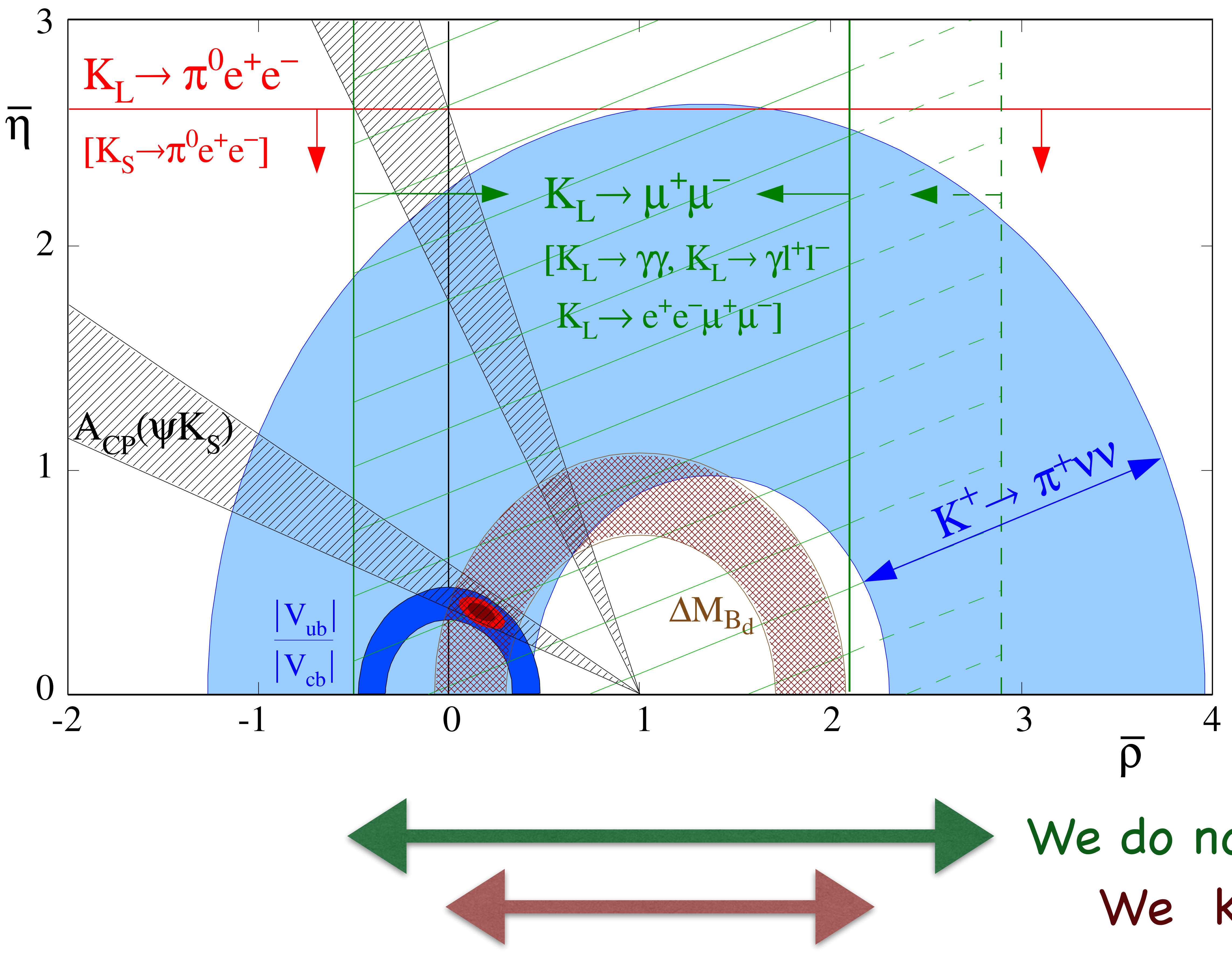
Subtracting from expt. the Absorptive contribution

$$0.98 \pm 0.55 = |ReA|^2 = (\chi_{\gamma\gamma}(M_\rho) + \chi_{\rm short} - 5.12)^2$$

$$|\chi_{\rm short}^{\rm SM}| = 1.96(1.11 - 0.92\bar{\rho})$$

Isidori Unterdorfer

K_L->$\mu\mu$: our sign ignorance



Magically comes LHCB
measuring $K_S \to \mu\mu$

Mostly K_L decays outside fiducial volume

K_S->$\mu\mu$

Dr. Gaillard at Berkeley in the early 1980s. AIP Emilio Segrè Visual Archives, Physics Today Collection

PHYSICAL REVIEW D VOLUME 10, NUMBER 3 1 AUGUST 1974

Rare decay modes of the K mesons in gauge theories

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National Accelerator Laboratory, Batavia, Illinois 60510‡

(Received 4 March 1974)

Rare decay modes of the kaons such as $K\to\mu\bar{\mu}$, $K\to\pi\nu\bar{\nu}$, $K\to\gamma\gamma$, $K\to\pi\gamma\gamma$, and $K\to\pi e\bar{e}$ are of theoretical interest since here we are observing higher-order weak and electromagnetic interactions. Recent advances in unified gauge theories of weak and electromagnetic interactions allow in principle unambiguous and finite predictions for these processes. The above processes, which are "induced" $|\Delta S|=1$ transitions, are a good testing ground for the cancellation mechanism first invented by Glashow, Iliopoulos, and Maiani (GIM) in order to banish $|\Delta S|=1$ neutral currents. The experimental suppression of $K_L\to\mu\bar{\mu}$ and nonsuppression of $K_L\to\gamma\gamma$ must find a natural explanation in the GIM mechanism which makes use of extra quark(s). The procedure we follow is the following: We deduce the effective interaction Lagrangian for $\lambda+\mathfrak{N}\to l+\bar{l}$ and $\lambda+\bar{\mathfrak{N}}\to\gamma+\gamma$ in the free-quark model; then the appropriate matrix elements of these operators between hadronic states are evaluated with the aid of the principles of conserved vector current and partially conserved axial-vector current. We focus our attention on the Weinberg-Salam model. In this model, $K\to\mu\bar{\mu}$ is suppressed due to a fortuitous cancellation. To explain the small K_L-K_S mass difference and nonsuppression of $K_L\to\gamma\gamma$, it is found necessary to assume $m_\varphi/m_{\varphi'}\ll 1$, where m_φ is the mass of the proton quark and $m_{\varphi'}$ the mass of the charmed quark, and $m_{\varphi'}<5$ GeV. We present a phenomenological argument which indicates that the average mass of charmed pseudoscalar
sti-
sup-
very
has the
upper
tion

$K_S^0 \to \mu^+\mu^-$

INSPIRE search

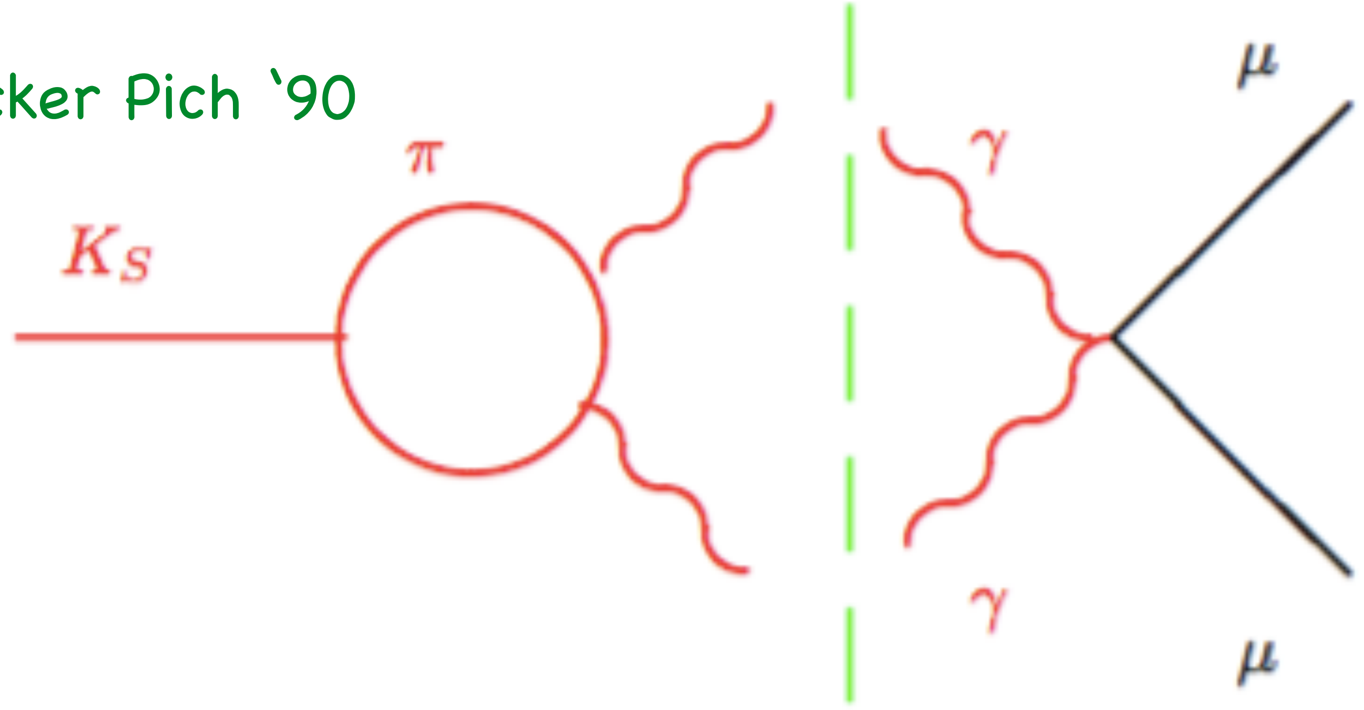
$\Gamma(K_S^0 \to \mu^+\mu^-)/\Gamma_{total}$ Γ_{11}/Γ

Test for ΔS = 1 weak neutral current. Allowed by first-order weak interaction combined with electromagnetic interaction.

VALUE	*CL%*	*DOCUMENT ID*		*TECN*
$< 2.1 \times 10^{-10}$	90	[1] AAIJ	2020AE	LHCB
• • • We do not use the following data for averages, fits, limits, etc. • • •				
$< 8 \times 10^{-10}$	90	[2] AAIJ	2017BQ	LHCB
$< 9 \times 10^{-9}$	90	[3] AAIJ	2013G	LHCB
$< 3.2 \times 10^{-7}$	90	GJESDAL	1973	ASPK

$K_S \to \mu\mu$

Ecker Pich ‘90



No CP conserving Short Distance due to Furry Theorem

Gaillard Lee

LD 5×10^{-12} 20% TH err

Dispersive treatment of $K_S \to \gamma\gamma$ and $K_S \to \gamma l^+ l^-$

Gilberto Colangelo, Ramon Stucki, and Lewis C. Tunstall

Short Distance

SM $10^{-5} |\Im(V_{ts}^* V_{td})|^2 \sim 10^{-13}$

NP few 10^{-11} allowed

Summarizing

$$\mathcal{B}\left(\mathrm{K}_S^0 \to \mu^+\mu^-\right)_{\mathrm{SM}}$$
$$= (5.18 \pm 1.50_{\mathrm{LD}} \pm 0.02_{\mathrm{SD}}) \times 10^{-12}$$
$$\parallel$$
$$(4.99_{\mathrm{LD}} + 0.19_{\mathrm{SD}})$$

[Ecker, Pich ’91; Isidori, Unterdorfer ’04; Chobanova, D’Ambrosio, TK, Martínez, Santos, Fernández, Yamamoto ’18]

SD??

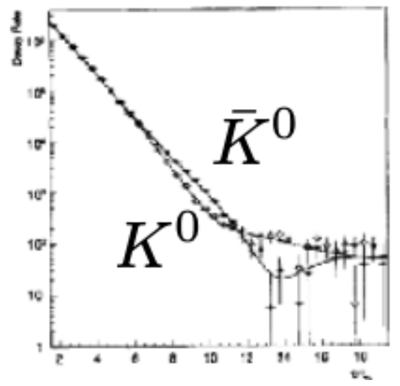
{

CPLEAR Flavor tagging

Such an interference has been discussed from ’67 [Sehgal and Wolfenstein], and has been observed/ utilized in many processes: e.g., $K^0 \to \pi\pi$, $K^0 \to 3\pi^0$, $K^0 \to \pi^+\pi^-\pi^0$, and $K^0 \to \pi^0 e^+ e^-$

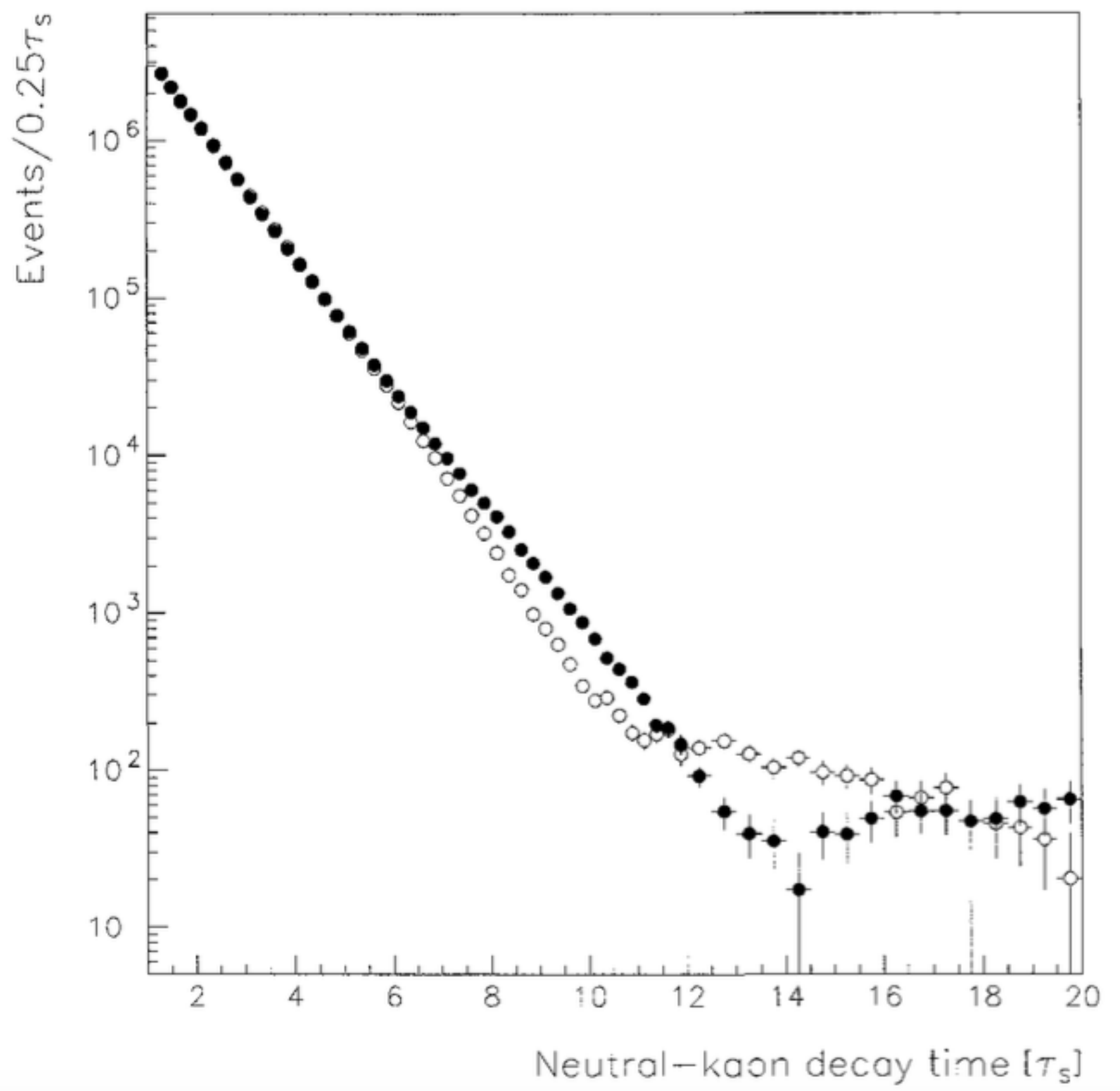
cf. CPLEAR experiment (1990-99@CERN)

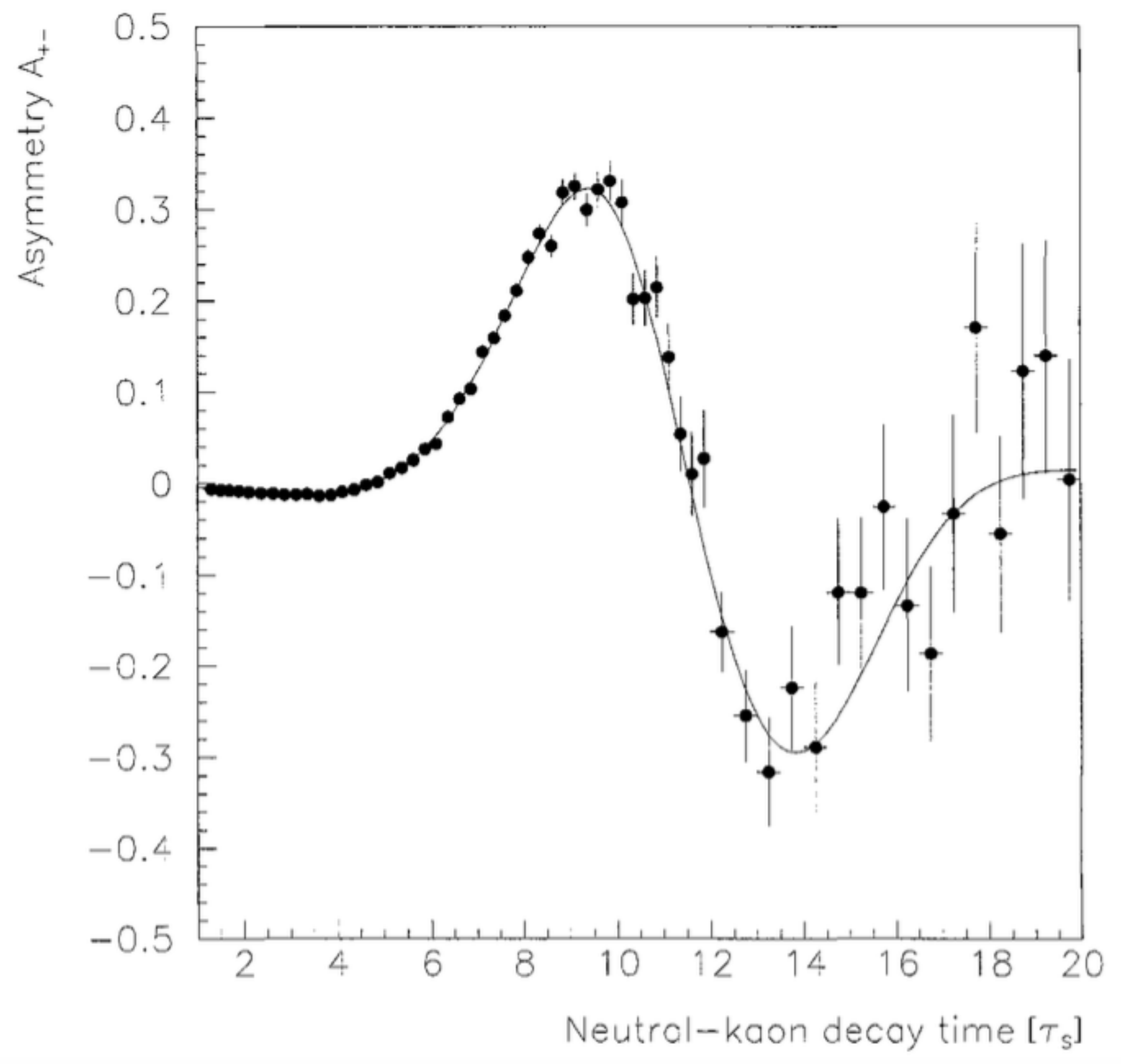
$$p\bar{p} \to \begin{bmatrix} K^0 K^- \pi^+ \\ \bar{K}^0 K^+ \pi^- \end{bmatrix}$$



$\{K_S, K_L\} \to \pi^+\pi^-$

measured the interference between K_L and K_S [CPLEAR collaboration ’95]





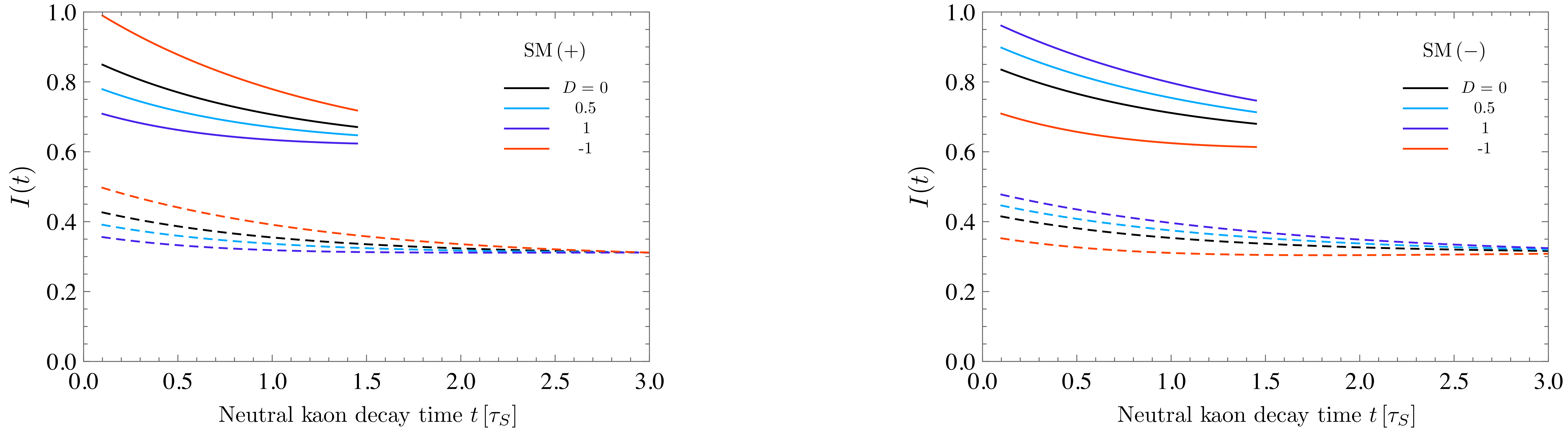
}

Can we study $K^0(t)$?

GD , Kitahara
1707.06999 PRL

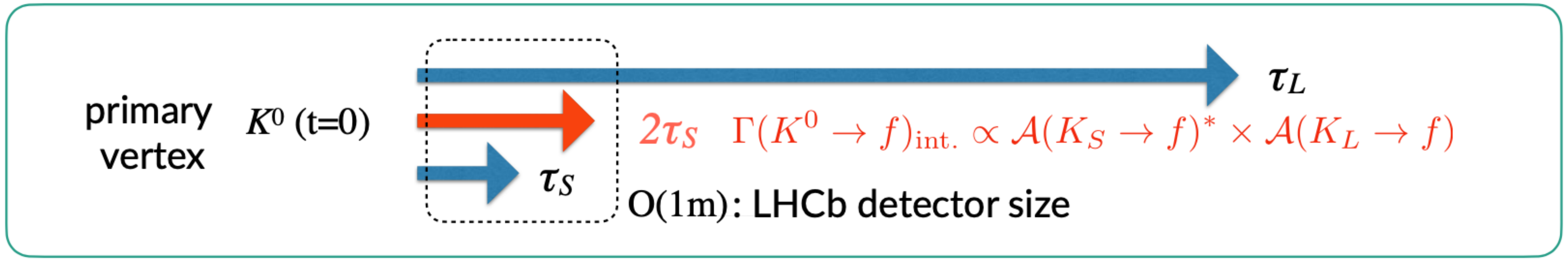
$$pp \to K^0 K^- X$$

$$pp \to K^{*+} X \to K^0 \pi^+ X$$

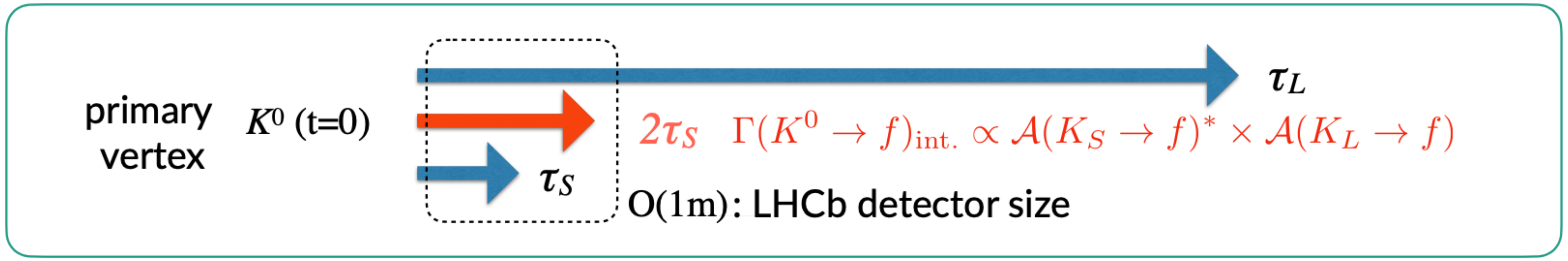


$$|\overset{(-)}{K}{}^0(t)\rangle = \frac{1}{\sqrt{2}(1 \pm \bar{\epsilon})} \left[e^{-iH_S t} \left(|K_1\rangle + \bar{\epsilon}|K_2\rangle \right) \pm e^{-iH_L t} \left(|K_2\rangle + \bar{\epsilon}|K_1\rangle \right) \right]$$

$$D = \frac{K^0 - \overline{K}^0}{K^0 + \overline{K}^0}$$



$$D = \frac{K^0 - \overline{K}^0}{K^0 + \overline{K}^0}$$



Interference $\quad \tau \sim 2\tau_S$

$$I(t) = \frac{1}{2}\left|\mathcal{A}\left(K_S\right)\right|^2 e^{-\Gamma_S t} + \frac{1}{2}\left|\mathcal{A}\left(K_L\right)\right|^2 e^{-\Gamma_L t} + D\,\mathrm{Re}\left[e^{-i\Delta M_K t}\mathcal{A}\left(K_S\right)^*\mathcal{A}\left(K_L\right)\right] e^{-\frac{\Gamma_B+\Gamma_L}{2}t} + \mathcal{O}(\bar{\epsilon})$$

$$\mathcal{H}_{\rm eff} = \frac{G_F\alpha}{\sqrt{2}}\lambda_t y'_{7A}\left(\bar{s}\gamma_\mu\gamma_5 d\right)\left(\bar{\mu}\gamma^\mu\gamma_5\mu\right) + \mathrm{h.c.}$$

$$\sum_{spin}\mathcal{A}\left(K_1 \to \mu^+\mu^-\right)^*\mathcal{A}\left(K_2 \to \mu^+\mu^-\right)$$

$$= \frac{16 i G_F^4 M_W^4 F_K^2 M_K^2 m_\mu^2 \sin^2\theta_W}{\pi^3}\mathrm{Im}\left[\lambda_t\right] y'_{7A}\left\{\underbrace{A^\mu_{L\gamma\gamma}}_{\gamma\gamma\text{ loop}} - 2\pi\sin^2\theta_W\left(\mathrm{Re}\left[\lambda_t\right] y'_{7A} + \mathrm{Re}\left[\lambda_c\right] y_c\right)\right\}$$

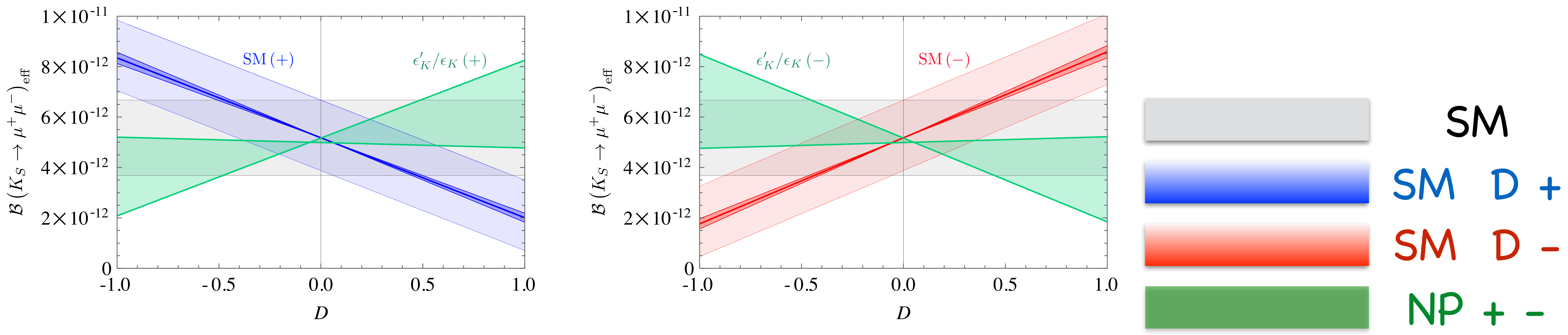
Coefficient is pure imaginary

Leading contribution is proportional to $D \times \mathrm{Im}[\lambda_t] \times \mathrm{Im}[A(K_L \to \gamma\gamma \to \mu\mu)]$

It looks like [weak phase] × [strong phase], namely **the direct CPV in meson decay**

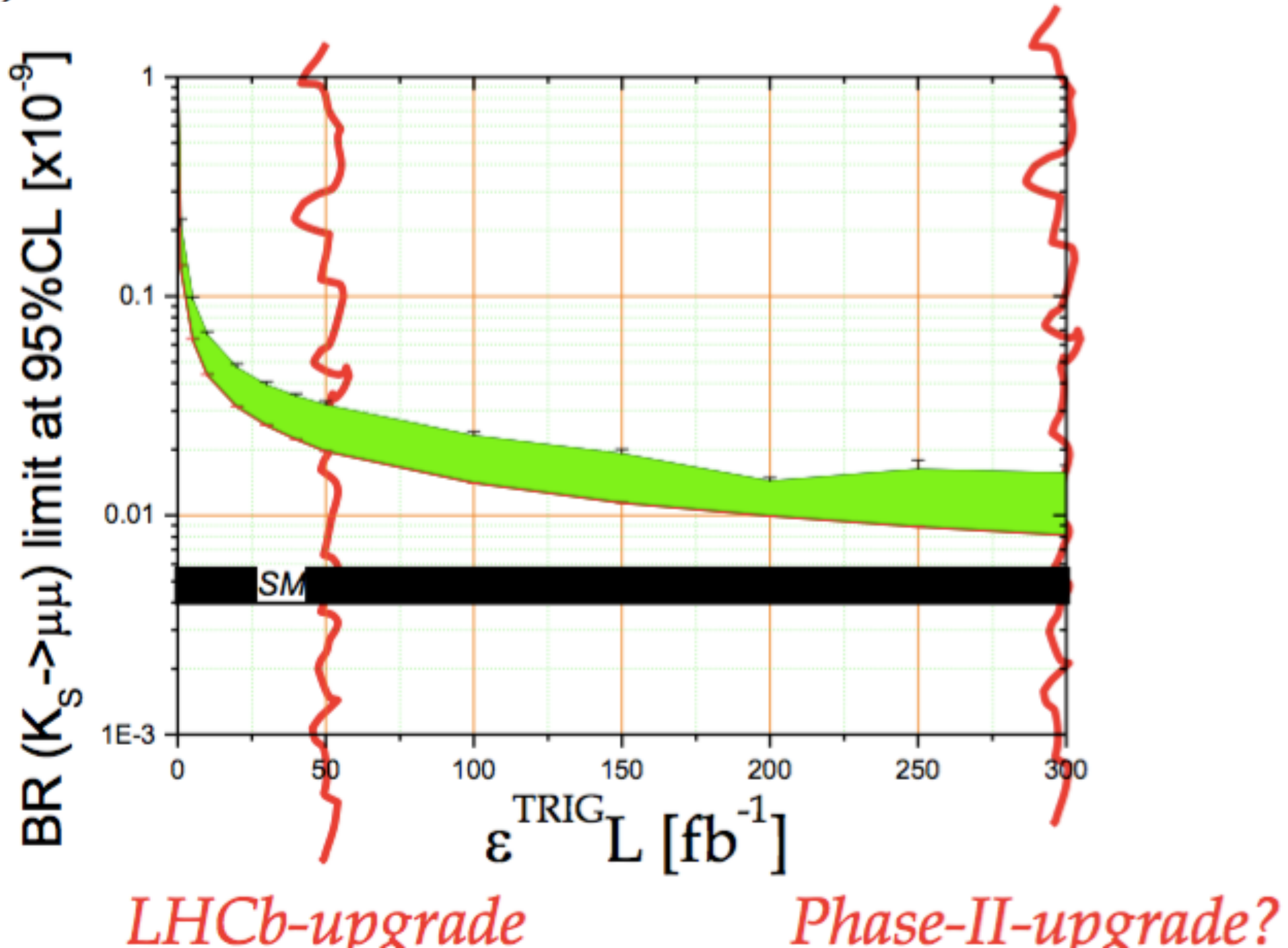
Short distance window

GD , Kitahara
1707.06999 PRL



modified Z-coupling model. ϵ'_K/ϵ

$$\mathcal{B}(K_S \to \mu^+\mu^-)_{\rm eff}$$
$$= \tau_S \left[\int_{t_{min}}^{t_{max}} dt \left(\Gamma(K_1) e^{-\Gamma_S t} + \frac{D}{8\pi M_K} \sqrt{1 - \frac{4m_\mu^2}{M_K^2}} \sum {\rm Re}\left[e^{-i\Delta M_K t} \mathcal{A}(K_1)^* \mathcal{A}(K_2) \right] e^{-\frac{\Gamma_S + \Gamma_L}{2} t} \right) \varepsilon(t) \right]$$
$$\times \left(\int_{t_{min}}^{t_{max}} dt\, e^{-\Gamma_S t} \varepsilon(t) \right)^{-1},$$



KAONS: RICH POTENTIAL

Rare Kaon decay program at LHCB

	PDG	Prospects
$K_S \to \mu\mu$	$< 9 \times 10^{-9}$ at 90% CL	(LD)$(5.0 \pm 1.5) \cdot 10^{-12}$ NP $< 10^{-11}$
$K_S \to \mu\mu\mu\mu$	–	SM LD $\sim 2 \times 10^{-14}$
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$K_S \to \pi^0\mu\mu$	$(2.9 \pm 1.3) \cdot 10^{-9}$	$\sim 10^{-9}$
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$K_S \to \pi^+\pi^-\mu^+\mu^-$	–	SM LD $\sim 10^{-14}$

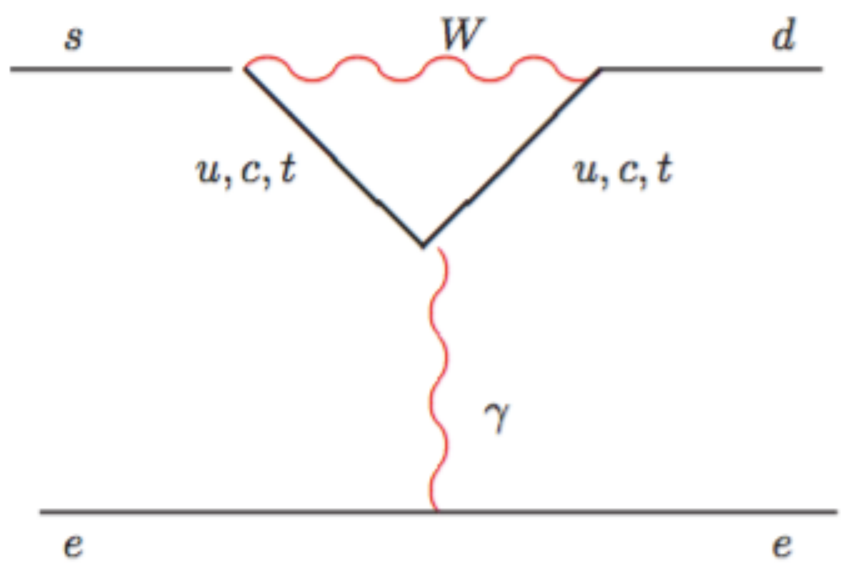
HIKE

$K^+ \to \pi^+\nu\bar{\nu}$	$\sigma_{\mathcal{B}}/\mathcal{B} \sim 5\%$	BSM physics, LFUV
$K^+ \to \pi^+\ell^+\ell^-$	Sub-% precision on form-factors	LFUV
$K^+ \to \pi^-\ell^+\ell^+$, $K^+ \to \pi\mu e$	Sensitivity $\mathcal{O}(10^{-13})$	LFV / LNV
Semileptonic K^+ decays	$\sigma_{\mathcal{B}}/\mathcal{B} \sim 0.1\%$	V_{us}, CKM unitarity
$R_K = \mathcal{B}(K^+ \to e^+\nu)/\mathcal{B}(K^+ \to \mu^+\nu)$	$\sigma(R_K)/R_K \sim \mathcal{O}(0.1\%)$	LFUV
Ancillary K^+ decays (e.g. $K^+ \to \pi^+\gamma\gamma$, $K^+ \to \pi^+\pi^0e^+e^-$)	% – ‰	Chiral parameters (LECs)
$K_L \to \pi^0\ell^+\ell^-$	$\sigma_{\mathcal{B}}/\mathcal{B} < 20\%$	Imλ_t to 20% precision, BSM physics, LFUV
$K_L \to \mu^+\mu^-$	$\sigma_{\mathcal{B}}/\mathcal{B} \sim 1\%$	Ancillary for $K \to \mu\mu$ physics
$K_L \to \pi^0(\pi^0)\mu^\pm e^\mp$	Sensitivity $\mathcal{O}(10^{-12})$	LFV
Semileptonic K_L decays	$\sigma_{\mathcal{B}}/\mathcal{B} \sim 0.1\%$	V_{us}, CKM unitarity
Ancillary K_L decays (e.g. $K_L \to \gamma\gamma$, $K_L \to \pi^0\gamma\gamma$)	% – ‰	Chiral parameters (LECs), SM $K_L \to \mu\mu$, $K_L \to \pi^0\ell^+\ell^-$ rates

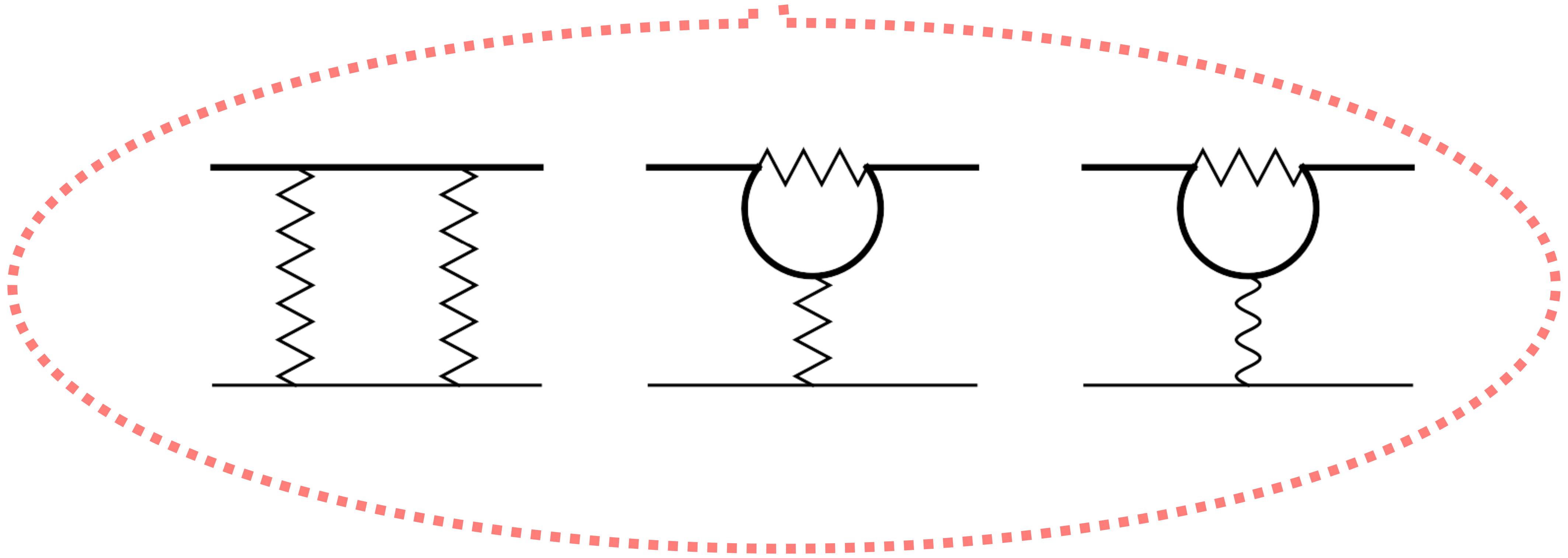
Long Distance enhancement

$$K^{\pm} \to \pi^{\pm} l^+ l^- \quad K_S \to \pi^0 l^+ l^-$$

Gilman Wise 1980



Short distance



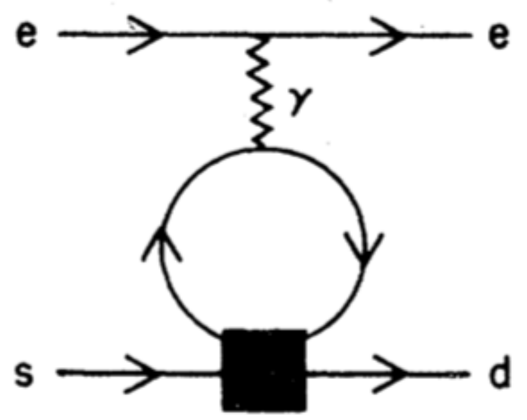


FIG. 1. Diagram contributing to C_7. The black box represents W exchange plus all strong-interaction corrections.

$$\mathcal{H}_{\rm eff} = -\frac{G_F}{\sqrt{2}} s_1 c_1 c_3 (\bar{s}_\alpha u_\alpha)_{V-A} (\bar{u}_\beta d_\beta)_{V-A}$$

$$-\frac{G_F}{\sqrt{2}} \frac{2}{9\pi} \frac{e^2}{4\pi} \left[A_c \ln\left(\frac{m_c^2}{\mu^2}\right) + A_t \ln\left(\frac{m_t^2}{\mu^2}\right) \right]$$

$$\times (\bar{s}_\alpha d_\alpha)_{V-A} (\bar{e} e)_V$$

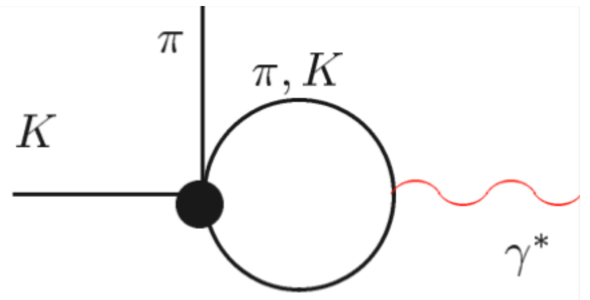
$$+ \text{H.c.}$$

$\mathcal{O}(p^4)$ CHPT

'87 Ecker Pich de Rafael

$K^+ \to \pi^+ l^+ l^-$

$k^2 = M_K^2\ ,\quad p^2 = M_\pi^2\ ,\quad q = k - p\ ,\quad z = q^2/M_K^2\ ,\quad r_\pi = M_\pi/M_K$

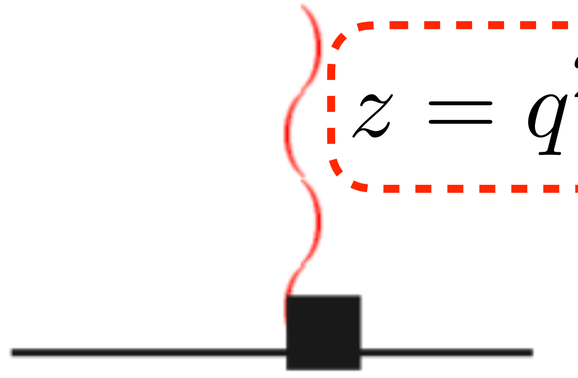


Chiral loop

$W_{\pi\pi}(z)$

Known from K->3pi

+



$z = q^2/M_K^2$

CT

a_+

Gauge and Lorentz invariance

$$\frac{W(z)}{(4\pi)^2}\left[z(k+p)^\mu - (1 - r_\pi^2)q^\mu\right]$$

$$\frac{d\Gamma}{dz} \sim \lambda^{3/2}(1, z, r_\pi^2)\sqrt{1 - 4\frac{r_\ell^2}{z}}\left(1 + 2\frac{r_\ell^2}{z}\right)|W(z)|^2\ ,$$

KS No pion loop

$\mathcal{O}(p^4)\quad a_+$

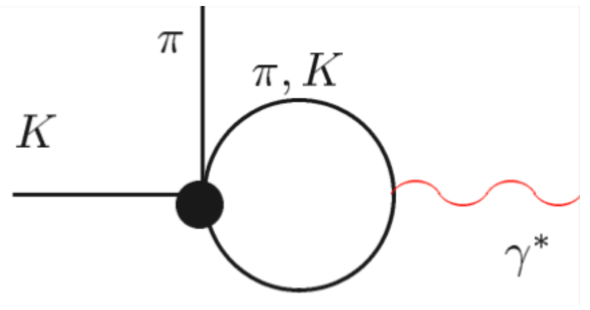
Data: the rate and spectrum not consistent with pheno

$\mathcal{O}(p^4)$ CHPT

'87 Ecker Pich de Rafael

$K_S \to \pi^0 l^+ l^-$

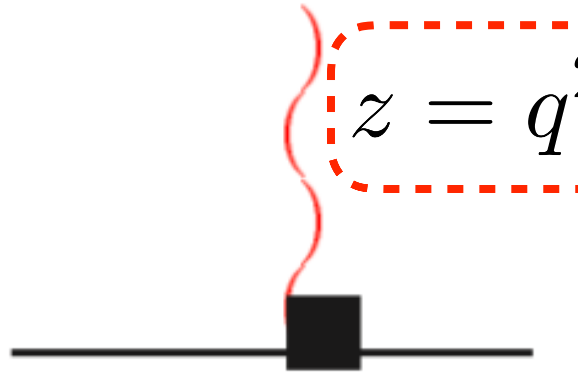
$k^2 = M_K^2\,,\quad p^2 = M_\pi^2\,,\quad q = k - p\,,\quad z = q^2/M_K^2\,,\quad r_\pi = M_\pi/M_K$



Chiral loop

No pion loop

+



CT

a_S

Gauge and Lorentz invariance

$$\frac{W(z)}{(4\pi)^2}\left[z(k+p)^\mu - (1 - r_\pi^2)q^\mu\right]$$

$$\frac{d\Gamma}{dz} \sim \lambda^{3/2}(1, z, r_\pi^2)\sqrt{1 - 4\frac{r_\ell^2}{z}}\left(1 + 2\frac{r_\ell^2}{z}\right)|W(z)|^2\,,$$

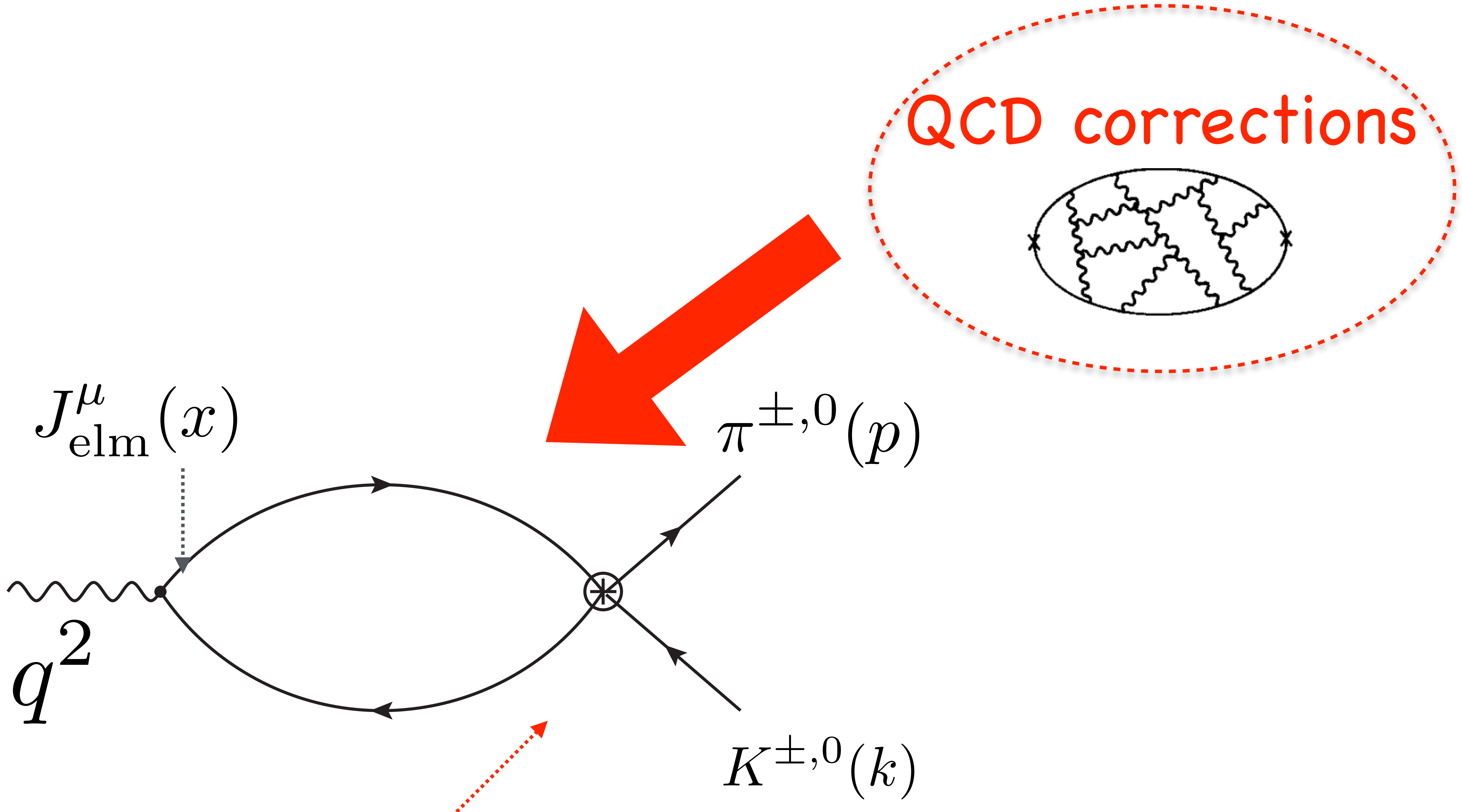
KS No pion loop for KS, CT

$\mathcal{O}(p^4)$ a_S

GD,Ecker,Isidori,Portoles '98

General consideration on the form factor

Lorentz and gauge invariance tell us on the structure of the amplitude and ff



$$z = \frac{q^2}{M_K^2}$$

$$i\int d^4x e^{iqx}\langle\pi(p)|T\left\{J^\mu_{\rm elm}(x)\mathcal{L}_{\Delta S=1}(0)\right\}|K(k)\rangle = \frac{W(z)}{(4\pi)^2}\left[z(k+p)^\mu - (1-r_\pi^2)q^\mu\right]$$

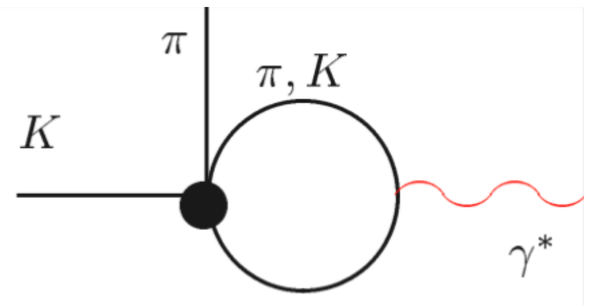
The integral at very short distances QCD quark loop

Pragmatic decision for $O(p^6)$ ff

GD,Ecker,Isidori,Portoles 98

$$W_i(z) = G_F M_K^2 W_i^{\rm pol}(z) + W_i^{\pi\pi}(z)$$

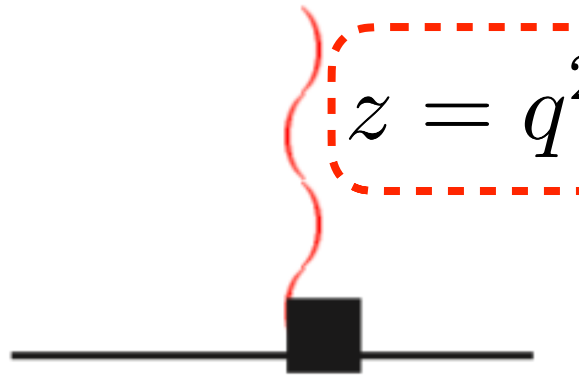
$$W_i^{\rm pol}(z) = a_i + b_i z \qquad (i = +, S)$$



Chiral loop

$W_{\pi\pi}(z)$

Known from K->3pi



$z = q^2/M_K^2$

CT

$a_i + b_i z$

$a_i + b_i z$ Determined expt.

Extremely good agreement with data few %

LFUV test $a_+^{\mu\mu} - a_+^{ee}$

Crivellin et al '16, Nazila et al '22

$K \to \pi\gamma^*$: Experimental situation

exp.	mode	number of events	a_+	b_+
BNL-E865	$K^+ \to \pi^+ e^+ e^-$	$10\,300$	$-0.587(10)$	$-0.655(44)$
NA48/2	$K^\pm \to \pi^\pm e^+ e^-$	$7\,253$	$-0.578(16)$	$-0.779(66)$
NA48/2	$K^\pm \to \pi^\pm \mu^+ \mu^-$	3120	$-0.575(39)$	$-0.813(145)$
NA62	$K^+ \to \pi^+ \mu^+ \mu^-$	27679	$-0.575(13)$	$-0.722(43)$

E. Cortina Gil et al. [NA62 Collaboration], JHEP 11, 011 (2022)

exp.	mode	number of events
NA48/1	$K_S \to \pi^0 e^+ e^-$	7
NA48/1	$K_S \to \pi^0 \mu^+ \mu^-$	6

$a_S = -1.29(3.15) \quad b_S = +17.8(10.6)$

or

$a_S = +1.28(3.16) \quad b_S = -17.6(10.6)$

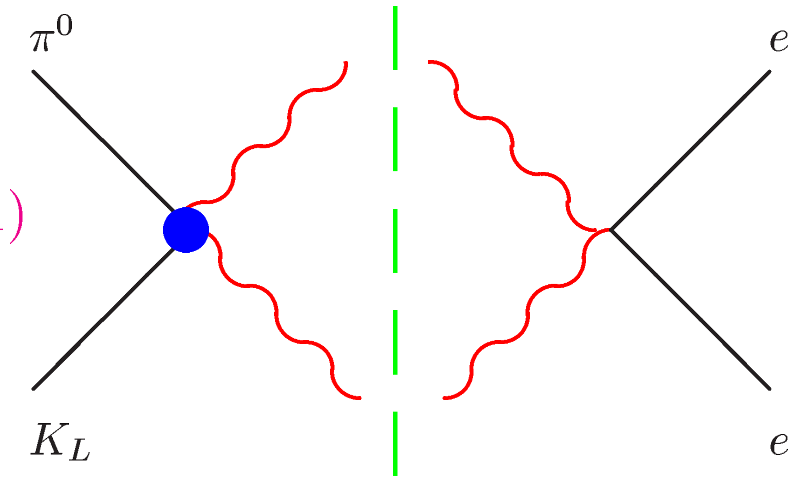
G. D'Ambrosio, D. Greynat, MK, JHEP 02, 049 (2019)

- Neither the sign of a_S nor the sign of a_S/b_S are fixed by data

Why interesting $K_L \to \pi^0 l^+ l^-$?

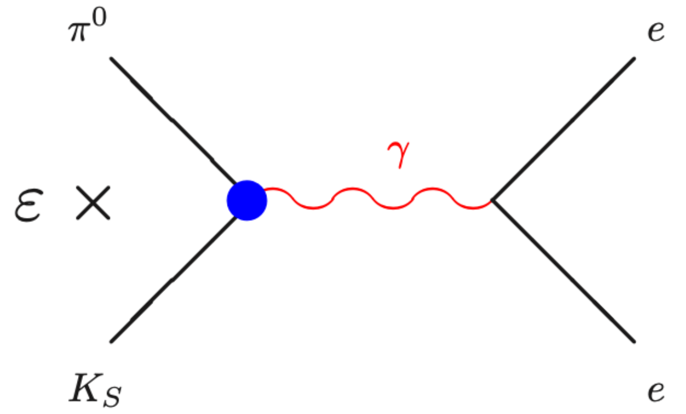
KOTO II , european strategy group

1)



Negligible for e, calculable for μ

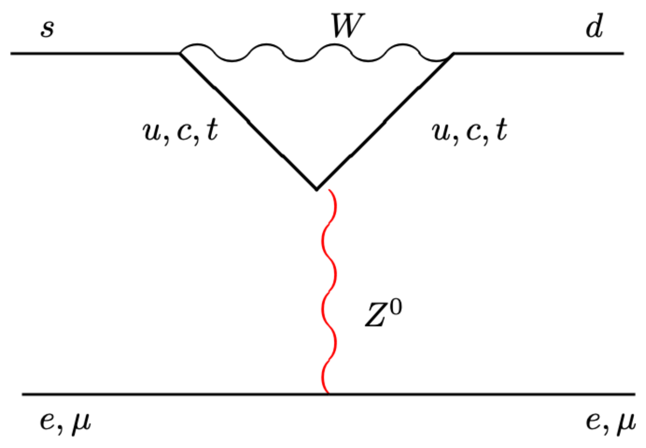
2)

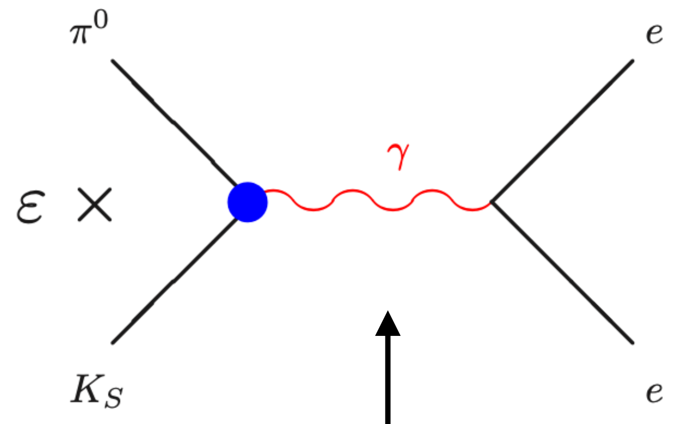


$\sim \epsilon(a_S + b_S z)$

form factor

3)





$$\mathrm{BR}(K_L \to \pi^0 \ell\bar{\ell}) = \left(C^\ell_{\rm dir} \pm C^\ell_{\rm int}|a_S| + C^\ell_{\rm mix}|a_S|^2 + C^\ell_{\gamma\gamma}\right)\cdot 10^{-12}$$

$$|a_S| = 1.20 \pm 0.20,$$

	$C^\ell_{\rm dir}$	$C^\ell_{\rm int}$	$C^\ell_{\rm mix}$	$C^\ell_{\gamma\gamma}$
$\ell = e$	$(4.62 \pm 0.24)(w^2_{7V} + w^2_{7A})$	$(11.3 \pm 0.3)w_{7V}$	14.5 ± 0.5	≈ 0
$\ell = \mu$	$(1.09 \pm 0.05)(w^2_{7V} + 2.32w^2_{7A})$	$(2.63 \pm 0.06)w_{7V}$	3.36 ± 0.20	5.2 ± 1.6

$$\mathrm{BR}^{\rm SM}(K_L \to \pi^0 e\bar{e}) = 3.46^{+0.92}_{-0.80}\left(1.55^{+0.60}_{-0.48}\right)\times 10^{-11}$$

$$\mathrm{BR}^{\rm SM}(K_L \to \pi^0 \mu\bar{\mu}) = 1.38^{+0.27}_{-0.25}\left(0.94^{+0.21}_{-0.20}\right)\times 10^{-11}$$

$$\mathrm{BR}^{\rm exp}(K_L \to \pi^0 e\bar{e}) < 28 \times 10^{-11} \quad \text{at 90\% CL}$$

$$\mathrm{BR}^{\rm exp}(K_L \to \pi^0 \mu\bar{\mu}) < 38 \times 10^{-11} \quad \text{at 90\% CL}$$

Buchalla et al 03, Isidori et al 06

Nazila et al 22, Knecht GD 24 , Hoferichter et al.24

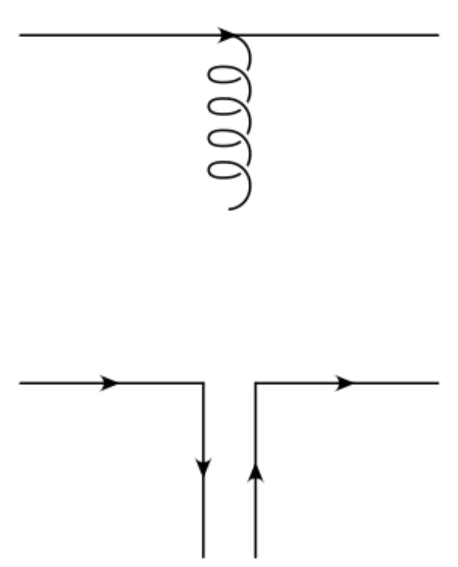
$$w_{7A,7V} = \mathrm{Im}\left(\lambda_t y_{7A,7V}\right) / \,\mathrm{Im}\,\lambda_t$$

LARGE N QCD

't Hooft, Witten, Coleman

$$\widetilde{\mathcal{L}}_{QCD} = \sum_{q=u,d,s} \bar{q}\gamma^{\mu}\left(i\partial_{\mu} - g_s \frac{\lambda_a}{2} G^a_{\mu}\right) q - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$$

QCD properties OK: asympt. Freedom $\mu \frac{dg}{d\mu} = -b_0 \frac{g^3}{16\pi^2} + \mathcal{O}(g^5)\,, \qquad b_0 = \frac{11}{3}N - \frac{2}{3}N_F$ Confinement assumed



$$8 + 1 = 3 + \bar{3}$$

Large N => correct expansion parameter? Geometric interpretation of this expansion: planar diagrams

Question in Large N: The leading order term in N of your amplitude

't Hooft model QCD_2	Exactly solved
Gross Neveu model	SSB+ mass Gap

Successes:

Zweig's rule (suppression of gluon exchange decays)

VMD, computability..

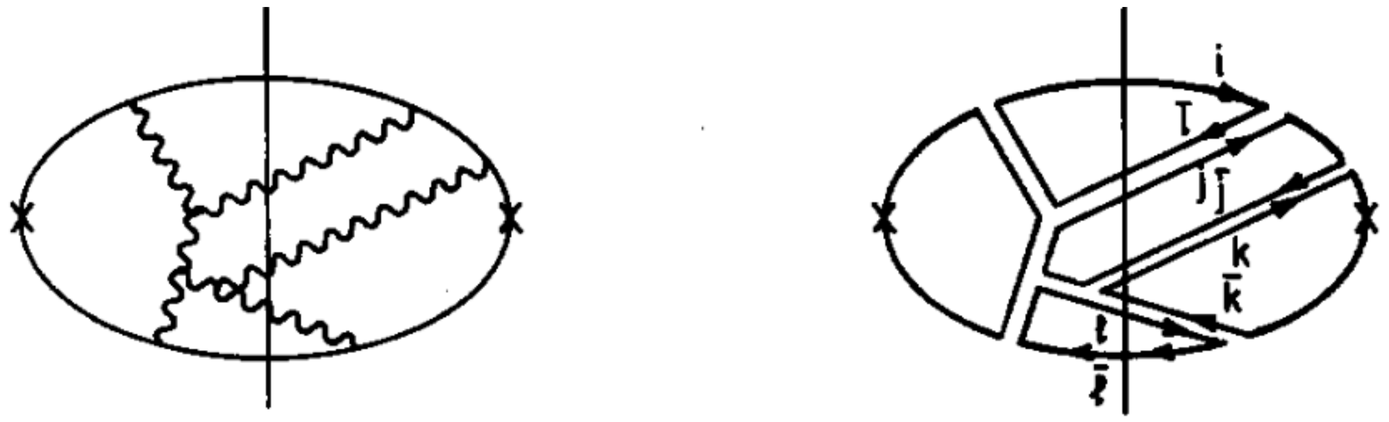
Witten '79 Large N QCD

Section 3

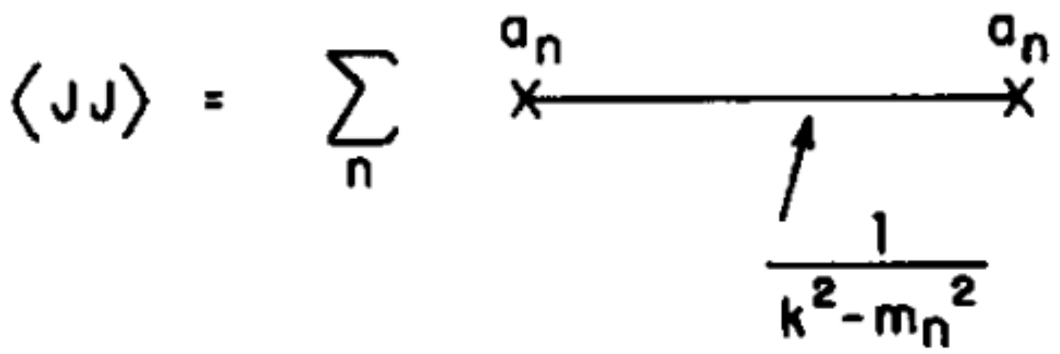
- Meson are free, stable, non-interacting, the number of states infinite

- Meson decay amplitudes $\mathcal{O}(1/\sqrt{N})$

- One meson exchange leading (see two point function with J bilinear)



$$\langle J(q)J(-q)\rangle = \sum_n \frac{a_n^2}{q^2 - m_n^2} \, .$$



ig. 21. ⟨JJ⟩ represented as a sum of one-meson poles.

$$\langle J(q)J(-q)\rangle \underset{q_2 \to \infty}{\sim} \log q^2 .$$

$K_S \to \pi^0 l^+ l^-$: determination of the form factor

$$i \int d^4x e^{iqx} \langle \pi(p) | T\left\{ J^\mu_{\rm elm}(x) \mathcal{L}_{\Delta S=1}(0) \right\} | K(k) \rangle = \frac{W(z)}{(4\pi)^2} \left[z(k+p)^\mu - (1-r_\pi^2) q^\mu \right]$$

$$k^2 = M_K^2 \ , \quad p^2 = M_\pi^2 \ , \quad q = k - p \ , \quad z = q^2/M_K^2 \ , \quad r_\pi = M_\pi / M_K \ ,$$

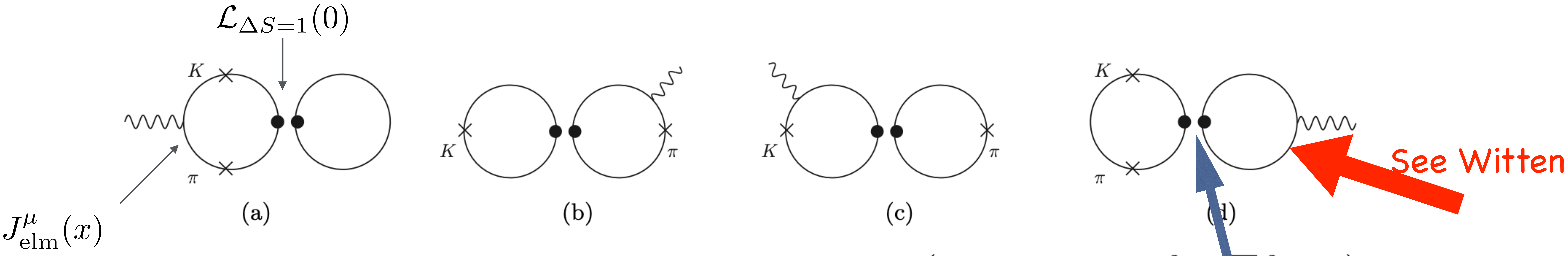


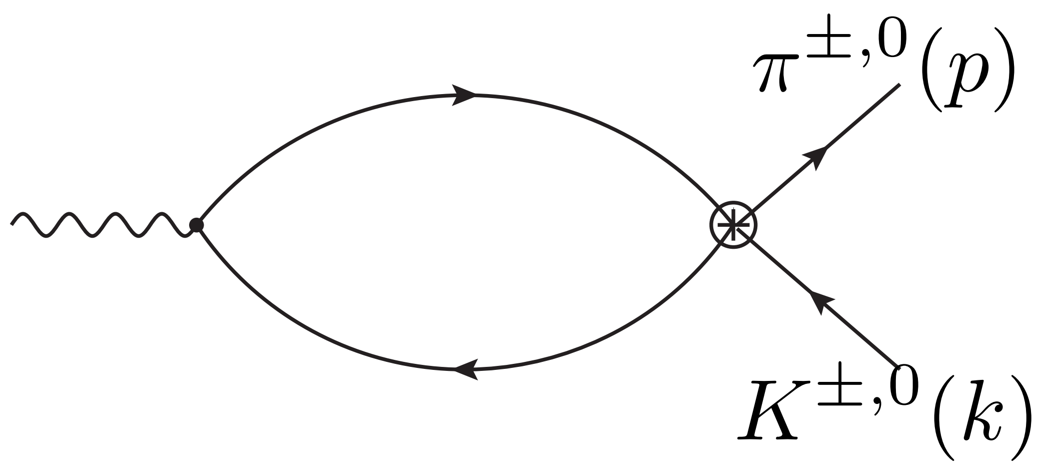
Figure 1: Potential diagrams contributing at leading order in large N_c (all at the order $(N_c)^2/(\sqrt{N_c})^2 = N_c$). The dots represent the effective $\Delta S = 1$ operators.

$$\mathcal{L}^{|\Delta S|=1}_{\rm non-lept} = -\frac{G_F}{\sqrt{2}} V_{us} V_{ud} \sum_{I=1}^{6} C_I(\nu) Q_I(\nu) + {\rm H.c}$$

$$Q_1 = (\bar{s}^i u_j)_{V-A} (\bar{u}^j d_i)_{V-A}, \; Q_2 = (\bar{s}^i u_i)_{V-A} (\bar{u}^j d_j)_{V-A}$$

Large Nc results for $K_S \to \pi^0 l^+ l^-$

GD Knecht, Neshatpour '24



$$W_i(z) = G_F M_K^2 W_i^{\rm pol}(z) + W_i^{\pi\pi}(z)$$

$$W_i^{\rm pol}(z) = a_i + b_i z \qquad (i = +, S)$$

$$\mathrm{Br}(K_S \to \pi^0 e^+ e^-) = 5.1(1.7) \cdot 10^{-9}$$

TH $$\mathrm{Br}(K_S \to \pi^0 e^+ e^-)|_{m_{ee} > 165 \text{ MeV}} = 2.9(1.0) \cdot 10^{-9}$$

NA48/1 $$\mathrm{Br}(K_S \to \pi^0 e^+ e^-)|_{m_{ee} > 165 \text{ MeV}} = (3.0^{+1.5}_{-1.2} \pm 0.2) \cdot 10^{-9}$$

Large Nc predictions =>Wilson coefficients +hadr. parameters

TH

$$\mathrm{Br}(K_S \to \pi^0 \mu^+ \mu^-) = 1.3(0.4) \cdot 10^{-9}$$

Conclusions

- We have focused maybe too much on

$\Delta I = 1/2$ rule

$K^+ \to \pi^+ l^+ l^-$

$$\mathcal{L}_{\Delta S=1} = \mathcal{L}^2_{\Delta S=1} + \mathcal{L}^4_{\Delta S=1} + \cdots = G_8 F^4 \underbrace{\langle \lambda_6 D_\mu U^\dagger D^\mu U \rangle}_{K\to 2\pi/3\pi} + \underbrace{G_8 F^2 \sum_i N_i W_i}_{K^+\to\pi^+\gamma\gamma,\, K\to\pi l^+ l^-} + \cdots$$

π	2π	3π	N_i
$\pi^+\gamma^*$	$\pi^+\pi^0\gamma^*$		$N^r_{14} - N^r_{15}$ $K^+ \to \pi^+ l^+ l^-$
$\pi^0\gamma^*$ (S)	$\pi^0\pi^0\gamma^*$ (L)		$2N^r_{14} + N^r_{15}$ $K_S \to \pi^0 l^+ l^-$
$\pi^+\gamma\gamma$	$\pi^+\pi^0\gamma\gamma$		$N_{14} - N_{15} - 2N_{18}$
	$\pi^+\pi^-\gamma\gamma$ (S)		"
	$\pi^+\pi^0\gamma$	$\pi^+\pi^+\pi^-\gamma$	$N_{14} - N_{15} - N_{16} - N_{17}$
	$\pi^+\pi^-\gamma$ (S)	$\pi^+\pi^0\pi^0\gamma$	"
		$\pi^+\pi^-\pi^0\gamma$ (L)	"
		$\pi^+\pi^-\pi^0\gamma$ (S)	$7(N^r_{14} - N^r_{16}) + 5(N^r_{15} + N_{17})$
	$\pi^+\pi^-\gamma^*$ (L)		$N^r_{14} - N^r_{15} - 3(N^r_{16} - N_{17})$
	$\pi^+\pi^-\gamma^*$ (S)		$N^r_{14} - N^r_{15} - 3(N^r_{16} + N_{17})$
	$\pi^+\pi^0\gamma^*$		$N^r_{14} + 2N^r_{15} - 3(N^r_{16} - N_{17})$
	$\pi^+\pi^-\gamma$ (L)	$\pi^+\pi^-\pi^0\gamma$ (S)	$N_{29} + N_{31}$
		$\pi^+\pi^+\pi^-\gamma$	"
	$\pi^+\pi^0\gamma$	$\pi^+\pi^0\pi^0\gamma$	$3N_{29} - N_{30}$
		$\pi^+\pi^-\pi^0\gamma$ (S)	$5N_{29} - N_{30} + 2N_{31}$
		$\pi^+\pi^-\pi^0\gamma$ (L)	$6N_{28} + 3N_{29} - 5N_{30}$

LHCB

- $K_{S,L} \to \mu^+\mu^-(e^+e^-)$ 3-point function

Conclusions II

$K^+ \to \pi^+ l^+ l^-$

3-point function

$K_{S,L} \to \mu^+\mu^-(e^+e^-)$

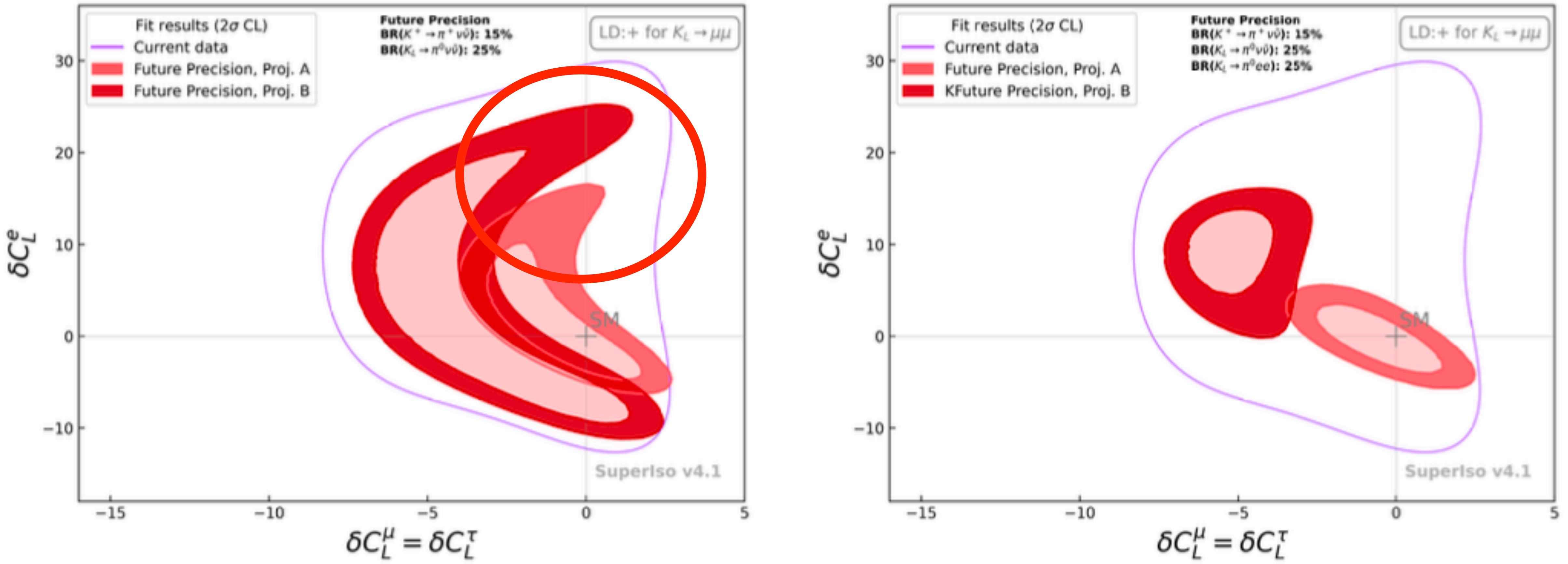
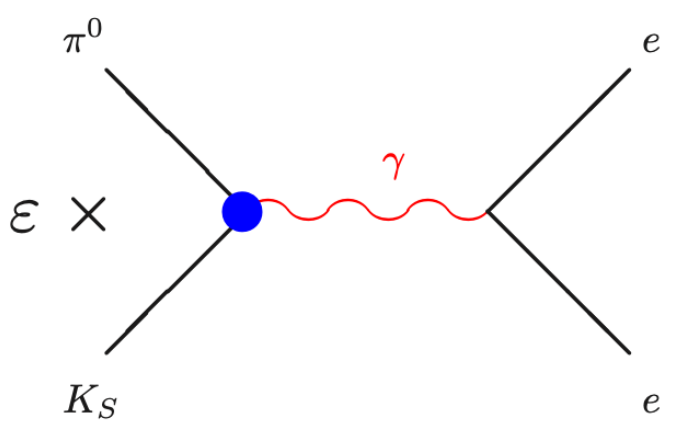
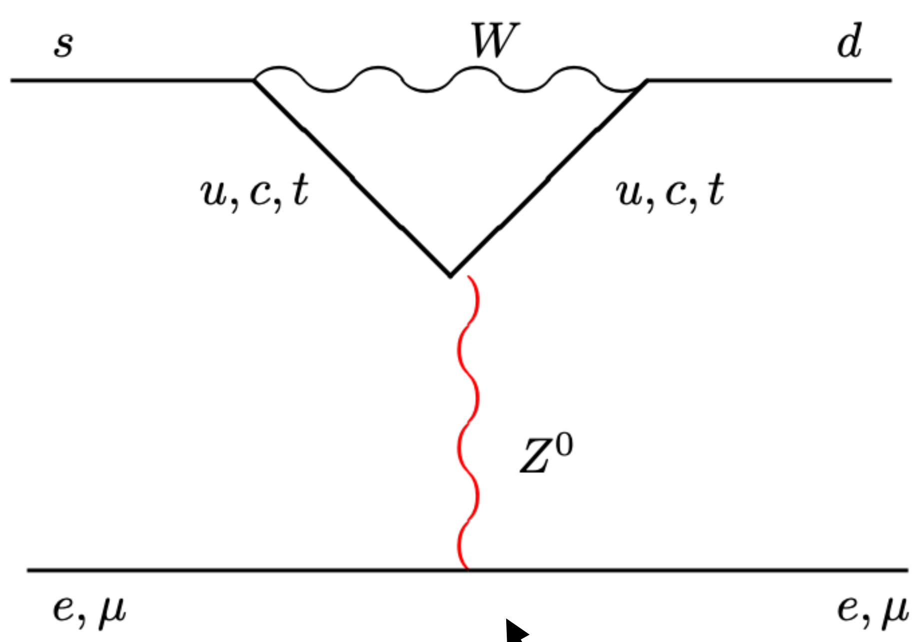
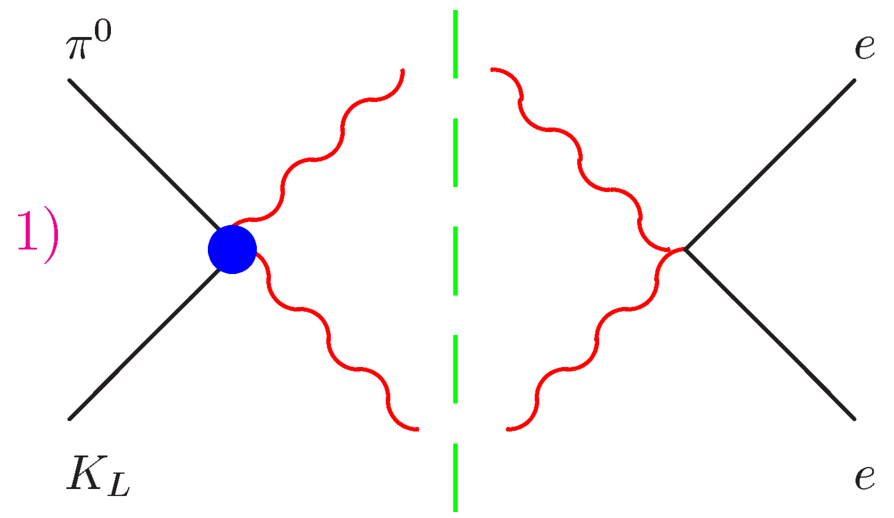


Figure 5: BSM parameter space for Wilson coefficients in scenarios with LFU violation where the NP effects for electrons are different from the those for muons and taus [18, 19]. Left: Impact on allowed parameter space from measurements of the golden channels $K \to \pi\nu\bar{\nu}$ from NA62 and KOTO II with the expected final precision. Right: Impact on the parameter space by the inclusion in the fits of a measurement of the $K_L \to \pi^0 e^+ e^-$ branching ratio with 25% precision.

GD Knecht, Neshatpour '24







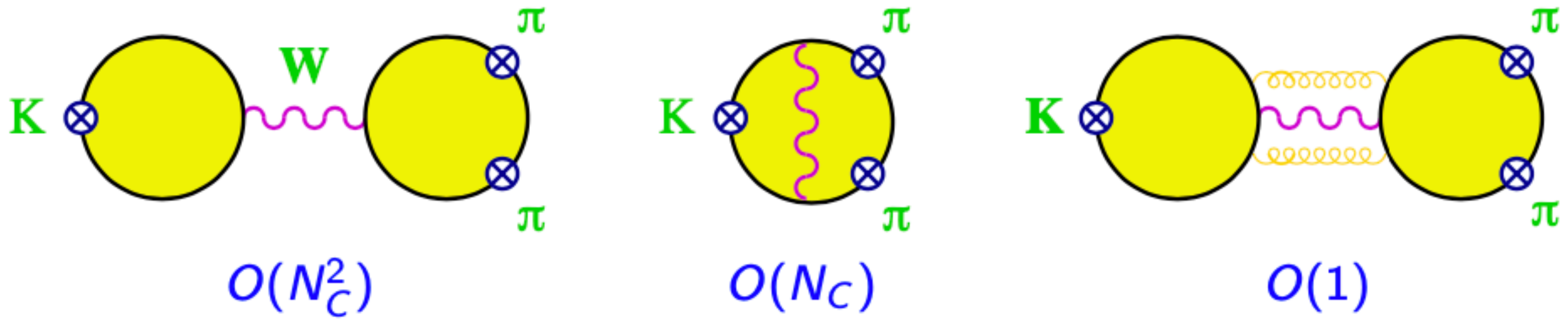
$$\mathrm{Br}(K_L \to \pi^0 \ell^+ \ell^-) = 10^{-12}\left[C_{\mathrm{mix}}^{(\ell)} + C_{\mathrm{int}}^{(\ell)} \frac{\mathrm{Im}\,\lambda_t}{10^{-4}} + C_{\mathrm{dir}}^{(\ell)} \left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}}\right)^2 + C_{\gamma^*\gamma^*}^{(\ell)}\right]$$

+

$$C_{\mathrm{int}}^{(e)} = +7.8(2.6)\,\frac{y_{7V}}{\alpha},\ C_{\mathrm{int}}^{(\mu)} = +1.9(0.6)\,\frac{y_{7V}}{\alpha}$$

$K \to 2\pi$

Weak Currents Factorize at Large N_C



- Large sub
- Failing understanding $\quad \Delta I = 1/2$ rule
- Lattice

Outline

- Motivations for K->πll $\quad K^{\pm} \to \pi^{\pm} l^{+} l^{-} \quad K_S \to \pi^{0} l^{+} l^{-}$
- K->πll phenomenology
- Large N QCD: the dream '70s '80s '90s 't Hooft, Witten, Coleman
- Ideas, Successes, some undelivered messages $\quad K \to \pi\pi : \ \Delta I = \frac{1}{2}$ rule $\quad \epsilon'$
- K->πll phenomenology and large N
- conclusions

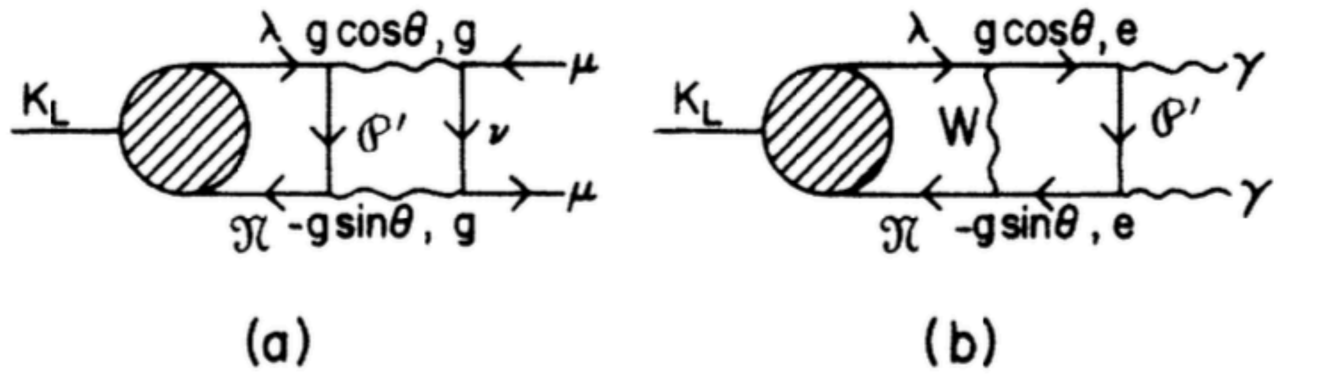


FIG. 3. Diagrams with the $\mathcal{P}'$–quark line which suppress the contributions of the diagrams in Fig. 1.

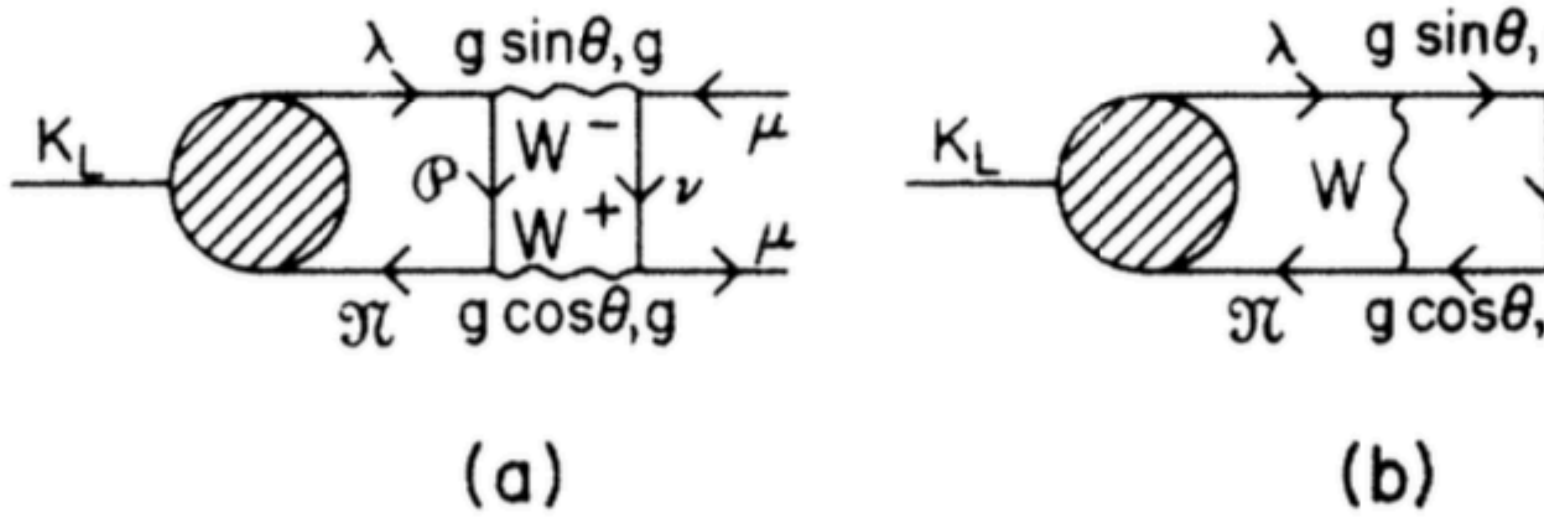
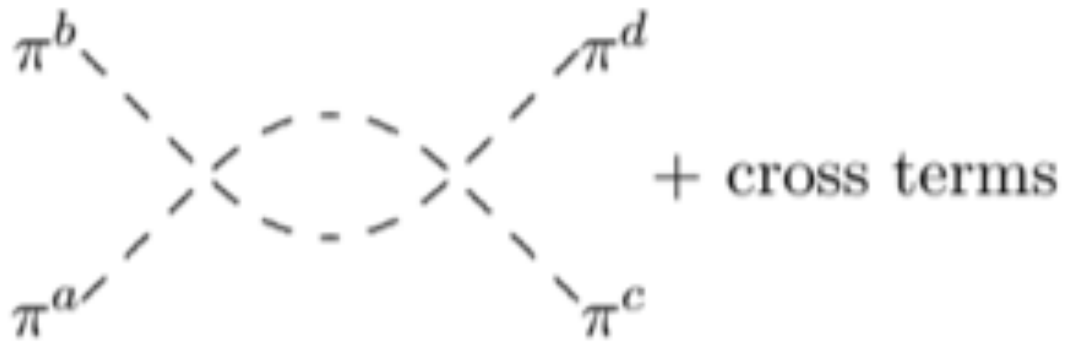


FIG. 1. Important diagrams for (a) $K_L \to \mu\bar{\mu}$ ar
$K_L \to \gamma\gamma$.

- GIM Gaillard Lee "3 months to understand the calculation, one day to do it
- Weinberg Effective field theories , chiral loops '76



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European Strategy for Particle Physics 2026: Kaon physics input document

The kaon community requests to

- protect and amplify the European kaon-physics programme, explori[ng] opportunities for
  - $K^+ \to \pi^+ \nu\bar{\nu}$
  - $K_{S,L} \to \mu^+\mu^-$ decay and interference,
- enable European contributions for KOTO II for $K_L \to \pi^0 \nu\bar{\nu}$ and $K_L \to \pi^0 \ell^+\ell^-$, spanning both analysis and hardware development,
- maintain the European leadership on theory computations for kaon physics (phenomenology, dispersion theory, effective theory and lattice QCD, including high-performance computing).

EU strategy group

Lazzeroni

The contribution "Kaon Physics: A Cornerstone for Future Discoveries" has been drafted by:
M. Bordone, A. Ceccucci, A. Dery, M. Gorbahn, E. Goudzovski, M. Hoferichter, A. Jüttner, C. Lazzeroni, Z. Ligeti, D. Martinez, F. Mahmoudi, R. Marchevski, M. Moulson, G. Ruggiero.

Year	Main object
1	Beam line survey
2	Construction of the rest of the detector
3-6	Phase I: Physics run for mainly $K_L \to \pi^0 \nu\bar{\nu}$
7	Single event sensitivity will reach 8.5×10^{-13} for the $K_L \to \pi^0 \nu\bar{\nu}$ search
8	Detector upgrade
9-12	Phase II: Physics run mainly for $K_L \to \pi^0 \ell^+\ell^-$ with an optimized setup
13	End of Phase II

Moulson

QCD theoretical tools

- analytic calculation 't Hooft, large Nc (it explains basic phenomenological facts of QCD, i.e. Zweig's rule) many implications: Skyrme model, VMD, Maldacena
- G. Parisi, '80s lattice: can we predict from QCD the proton mass at 10% level?
- Precise calculation of low energy QCD?

π	2π	3π	N_i	
$\pi^+\gamma^*$	$\pi^+\pi^0\gamma^*$		$N_{14}^r - N_{15}^r$	$K^+ \to \pi^+ l^+ l^-$
$\pi^0\gamma^*$ (S)	$\pi^0\pi^0\gamma^*$ (L)		$2N_{14}^r + N_{15}^r$	$K_S \to \pi^0 l^+ l^-$
$\pi^+\gamma\gamma$	$\pi^+\pi^0\gamma\gamma$		$N_{14} - N_{15} - 2N_{18}$	
	$\pi^+\pi^-\gamma\gamma$ (S)		"	
	$\pi^+\pi^0\gamma$	$\pi^+\pi^+\pi^-\gamma$	$N_{14} - N_{15} - N_{16} - N_{17}$	
	$\pi^+\pi^-\gamma$ (S)	$\pi^+\pi^0\pi^0\gamma$	"	
		$\pi^+\pi^-\pi^0\gamma$ (L)	"	
		$\pi^+\pi^-\pi^0\gamma$ (S)	$7(N_{14}^r - N_{16}^r) + 5(N_{15}^r + N_{17})$	
	$\pi^+\pi^-\gamma^*$ (L)		$N_{14}^r - N_{15}^r - 3(N_{16}^r - N_{17})$	
	$\pi^+\pi^-\gamma^*$ (S)		$N_{14}^r - N_{15}^r - 3(N_{16}^r + N_{17})$	
	$\pi^+\pi^0\gamma^*$		$N_{14}^r + 2N_{15}^r - 3(N_{16}^r - N_{17})$	
	$\pi^+\pi^-\gamma$ (L)	$\pi^+\pi^-\pi^0\gamma$ (S)	$N_{29} + N_{31}$	
		$\pi^+\pi^+\pi^-\gamma$	"	
	$\pi^+\pi^0\gamma$	$\pi^+\pi^0\pi^0\gamma$	$3N_{29} - N_{30}$	
		$\pi^+\pi^-\pi^0\gamma$ (S)	$5N_{29} - N_{30} + 2N_{31}$	
		$\pi^+\pi^-\pi^0\gamma$ (L)	$6N_{28} + 3N_{29} - 5N_{30}$	

$$\mathcal{L}_{\Delta S=1} = \mathcal{L}^2_{\Delta S=1} + \mathcal{L}^4_{\Delta S=1} + \cdots = G_8 F^4 \underbrace{\langle \lambda_6 D_\mu U^\dagger D^\mu U \rangle}_{K\to 2\pi/3\pi} + \underbrace{G_8 F^2 \sum_i N_i W_i}_{K^+\to\pi^+\gamma\gamma, K\to\pi l^+ l^-} + \cdots$$

$$K_L(p) \to \pi^0(p_3)\gamma(q_1)\gamma(q_2)$$

Lorentz + gauge invariance $\Rightarrow$ $M \sim$ $A(y,z)$ $B(y,z)$

	$A(y,z)$	$B(y,z)$
	$\gamma\gamma$	$\gamma\gamma$
	$J=0$	$\mathrm{D}-\mathrm{wave\ too}$
	$F^{\mu\nu}F_{\mu\nu}$	$F^{\mu\nu}F_{\mu\lambda}\partial_\nu K_L \partial^\lambda \pi^0$

$y = p\cdot(q_1-q_2)/m_K^2$, $z = (q_1+q_2)^2/m_K^2$

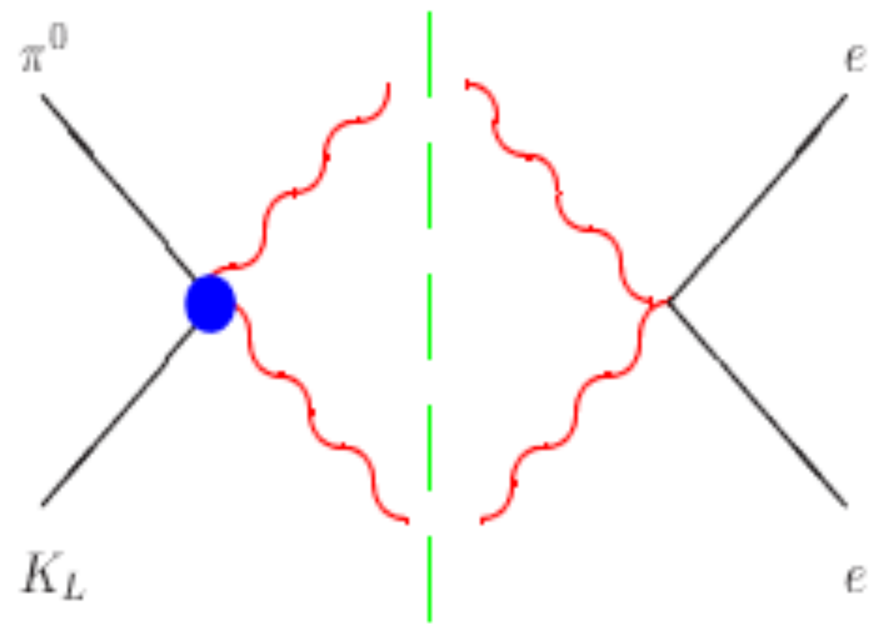
$r_\pi = m_\pi/m_K$

- $\frac{d^2\Gamma}{dydz} \sim z^2|A+B|^2 + \left(y^2 - \left(\frac{(1+r_\pi^2-z)^2}{4} - r_\pi^2\right)\right)^2 |B|^2$ S, B
- Different gauge structure $\Rightarrow$ B$\neq$0 at $z \to 0$ (collinear photons).

Crucial role in $K_L \to \pi^0 e^+ e^-$

A suppressed by m_e/m_K

B is not



Morozumi et al, Flynn Randall

Sehgal Heiliger, Ecker et al., Donoghue et al.

KTeV and NA48 not only ϵ'

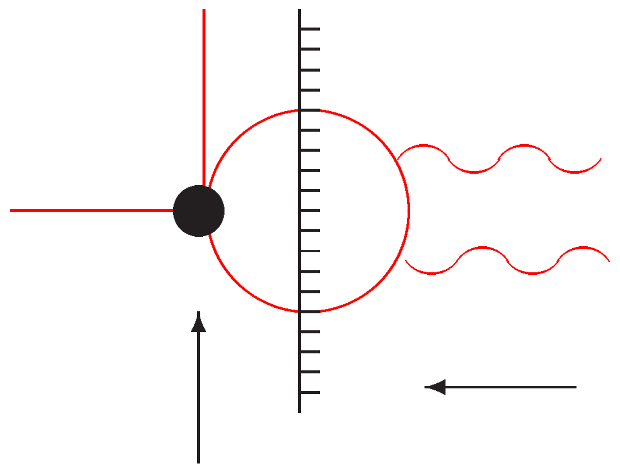
Actually the study of unit. cut was crucial to i) to bring agreement expt vs Theory in $K_L \to \pi^0 \gamma\gamma$ and ii) show that $K_L \to \pi^0 ee$ CP conserving was negligible

3 CT's

$F_{\mu\nu} F^{\mu\alpha} \partial_\alpha K_L \partial^\nu \pi^0$

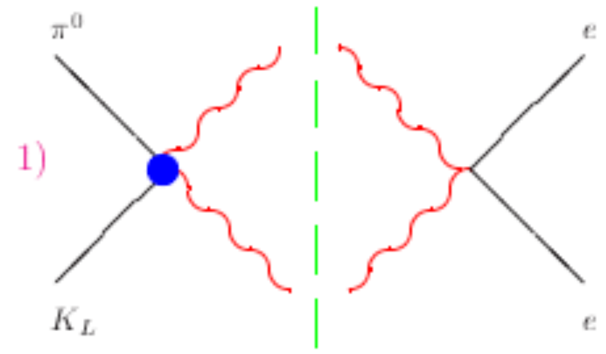
$F^2 \; \partial K_L \partial \pi^0$

$F^2 \; m_K^2 K_L \pi^0$



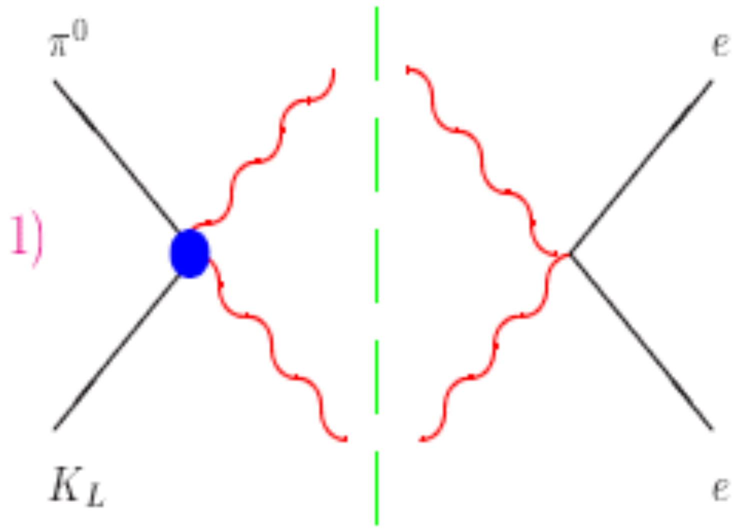
Full description of unitarity cut

$A(K \to 3\pi) = a + b\, Y + c\, Y^2 + d\, X^2$



$K_L \to \pi^0 e^+ e^-$: summary

$\mathrm{Br}(K_L \to \pi^0 e^+ e^-) \leq 2.8 \cdot 10^{-10}$ at 90% CL KTeV

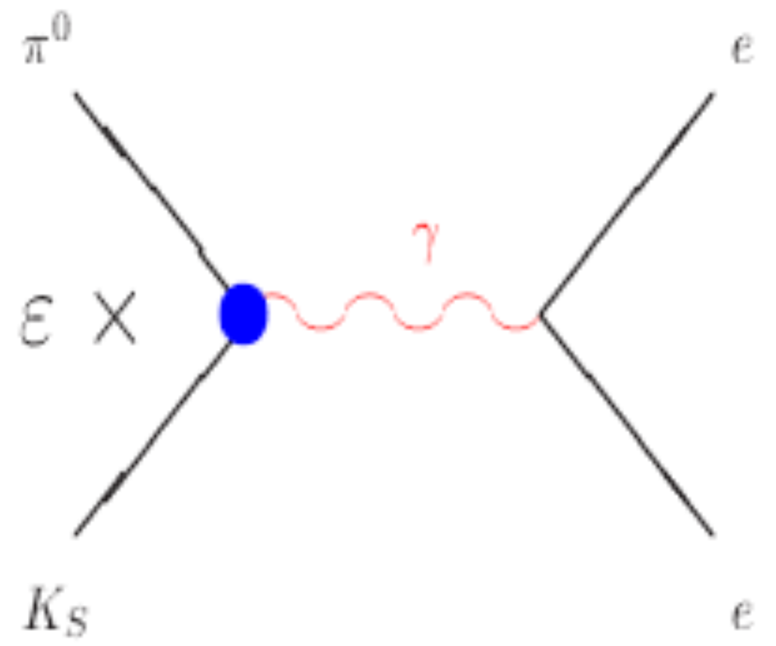


CP conserving NA48

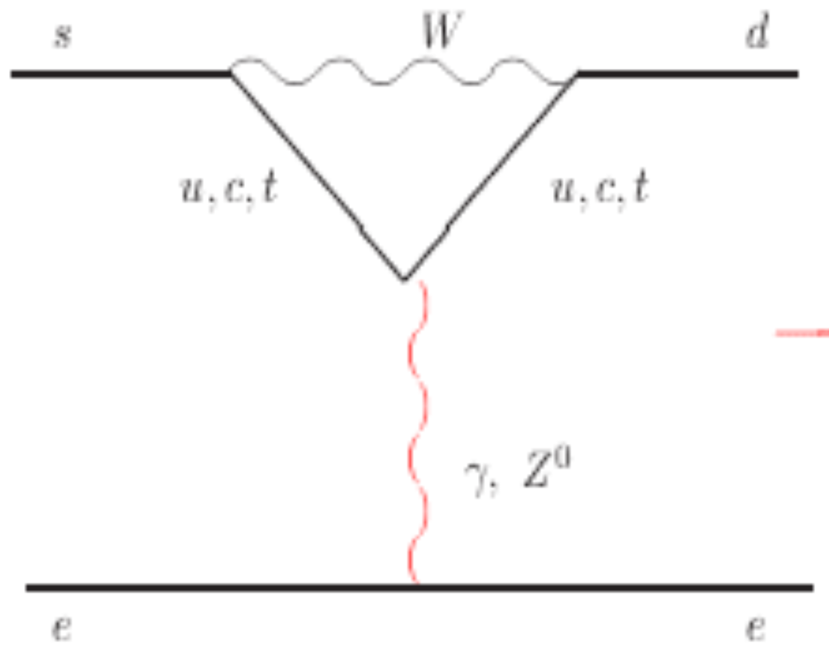
$\mathrm{Br}(K_L \to \pi^0 e^+ e^-) < 3 \cdot 10^{-12}$

V-A$\otimes$ V-A $\Rightarrow \langle \pi^0 e^+ e^- | (\bar{s} d)_{V-A} (\bar{e} e)_{V-A} | K_L \rangle$ violates CP

2) $\varepsilon \times$



+ 3)



$\longrightarrow$ Im λ_t

$\lambda_t = V_{td}V_{ts}^*$

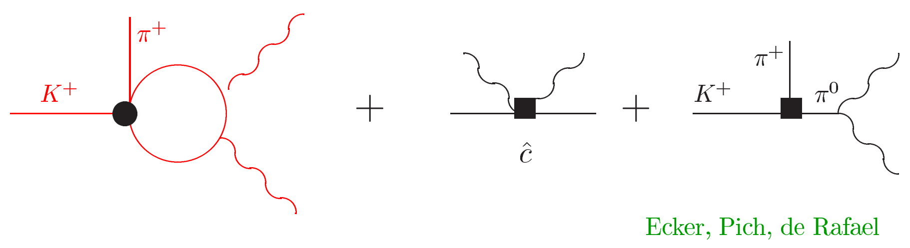
$\uparrow$ $\mathrm{B}(K_S \to \pi^0 e^+ e^-) = 4.6 a_S^2 \times 10^{-9}$

Possible large interference: $a_S < -0.5$ or $a_S > 1$; short distance probe even for a_S large

$$|2) + 3)|^2 = \left[15.3\, a_S^2 - 6.8 \frac{Im\lambda_t}{10^{-4}}\, a_S + 2.8 \left(\frac{Im\lambda_t}{10^{-4}}\right)^2\right] \cdot 10^{-12}$$

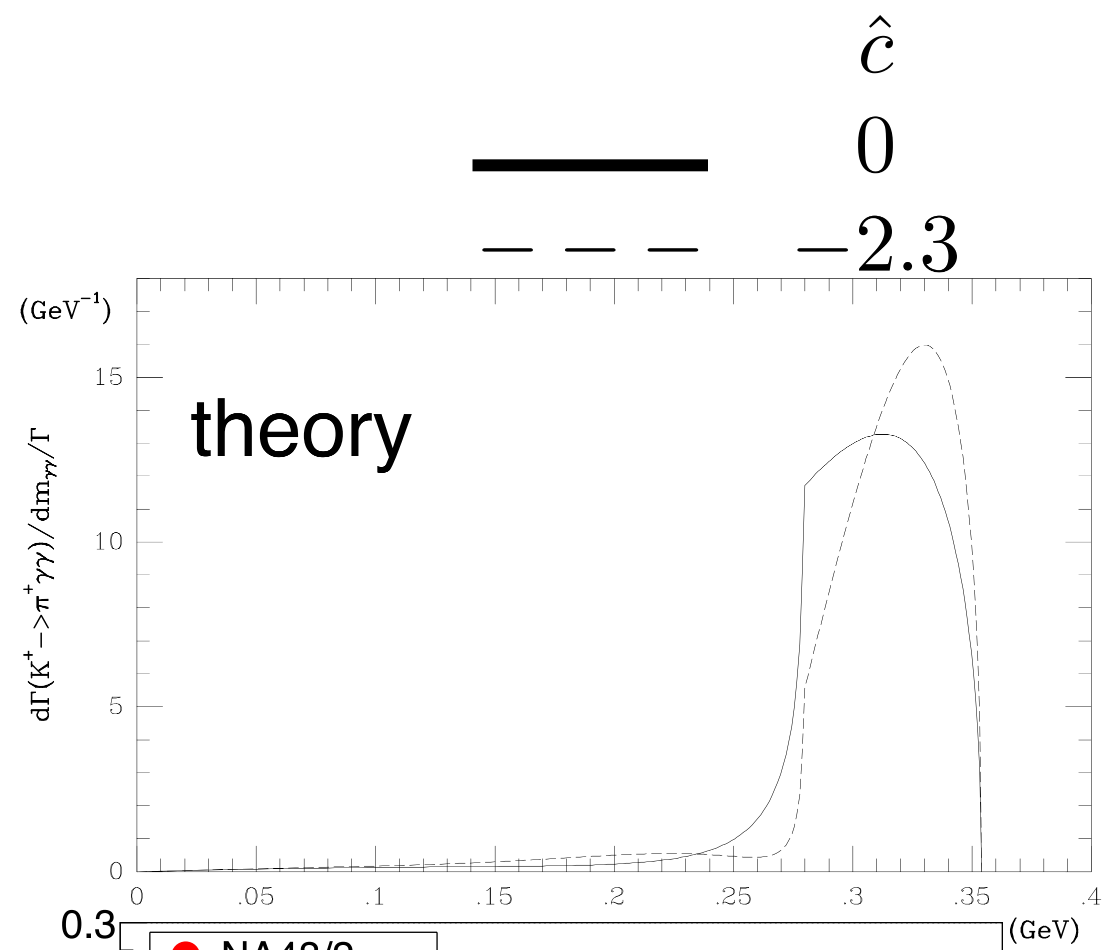
$$[17.7 \pm \quad 9.5 + \quad 4.7] \quad \cdot 10^{-12}$$

$K^+ \to \pi^+\gamma\gamma$ NA48/2 + NA62 ('14)



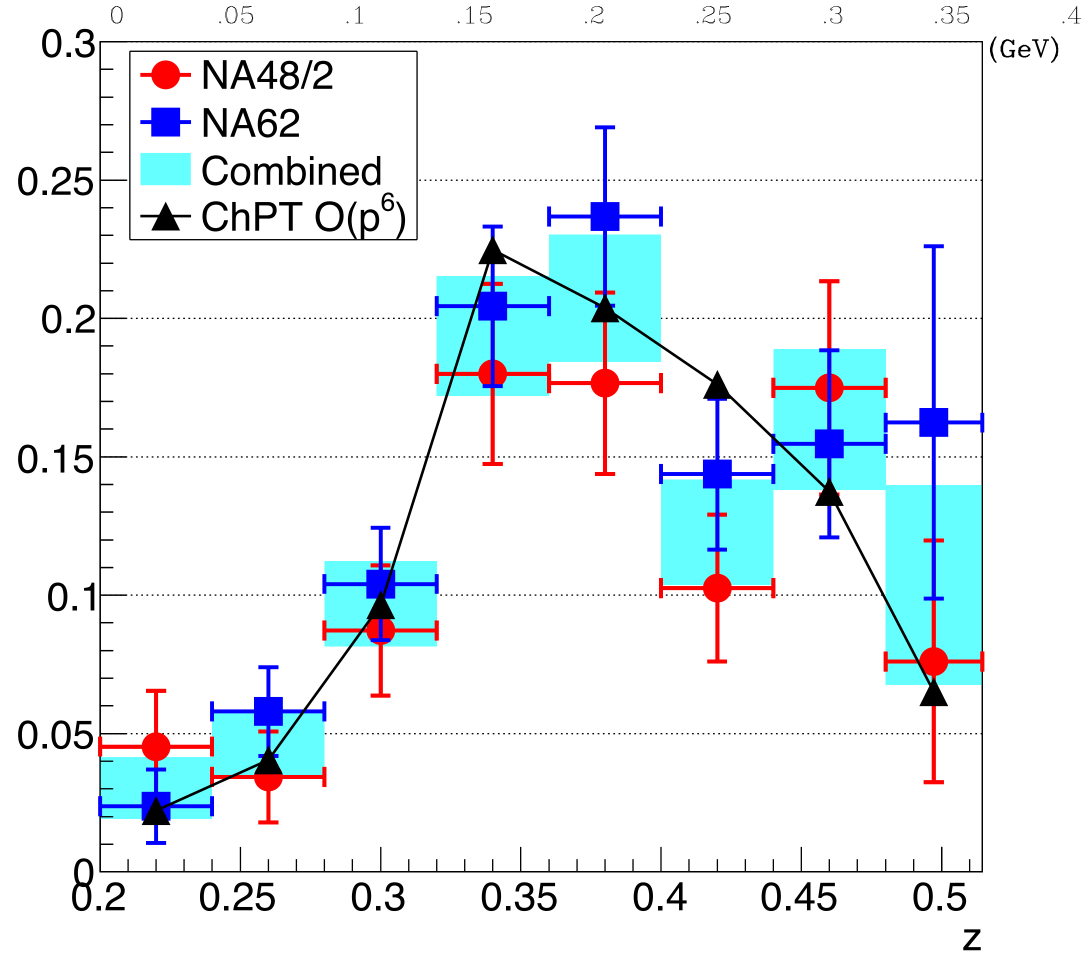
Auxiliary channel useful to assess the CP conserving contribution to $K_L \to \pi^0 ee$

Final 381 evts NA48/2 + NA62 during a 3-day special NA48/2 run in 2004 and a 3-month NA62 run in 2007

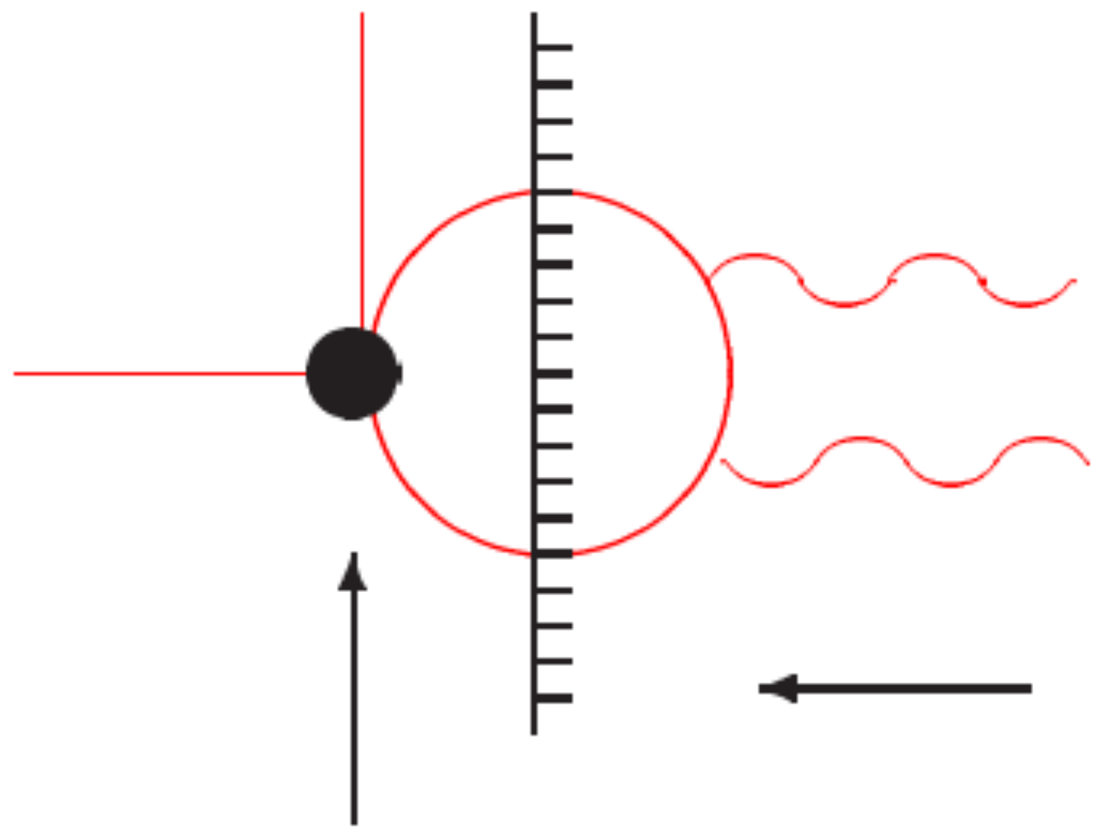


$$B = (1.003 \pm 0.051_{\text{stat}} \pm 0.024_{\text{syst}}) \cdot 10^{-6}$$

$$\hat{c} = 1.86 \pm 0.26$$



$K^+ \to \pi^+\gamma\gamma$ NA62 sensitivity



Full description of unitarity cut

$$A(K \to 3\pi) = a + b\,Y + c\,Y^2 + d\,X^2$$

This decay $K^+ \to \pi^+\gamma\gamma$: The error obtained in the form factor ($\hat{c}$) is dominated by the expt K->3pi error in the quadratic slope !

$K_S \to \mu\bar{\mu}$ LHCB

After 40 years improvement by 3 orders of magnitudes from LHCB

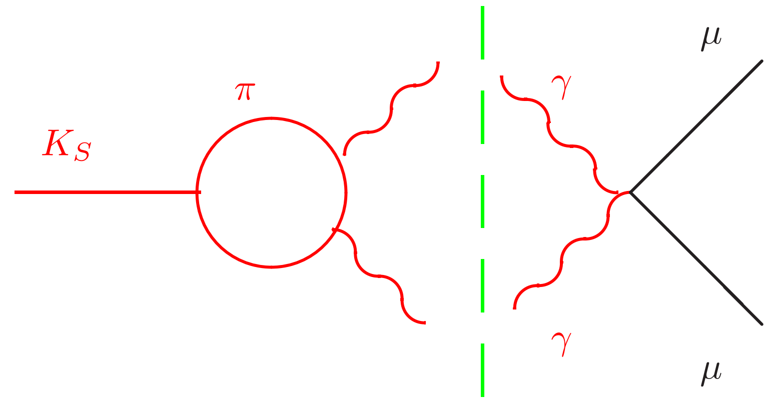
$$B(K_S \to \mu\bar{\mu}) < 11 \times 10^{-9}$$

95% CL

Isidori Underdorfer

SM

$\sim 5 \times 10^{-12}$



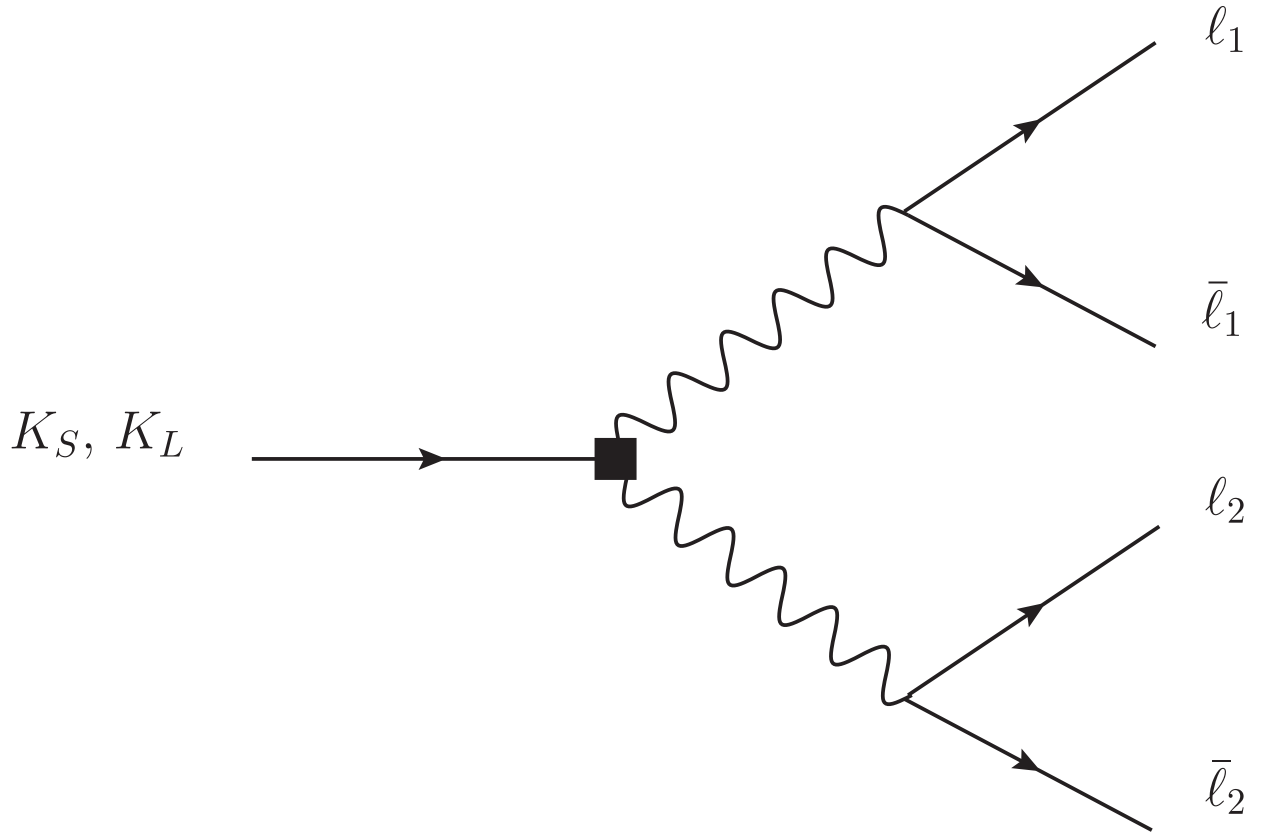
SD 1.5 10^-12

NP 1.5 10^-11 Allowed

NP Limits from CPviol in $K_L \to \mu\mu$

Other interesting channels

$$
\begin{array}{lcr}
K_S \to \mu\mu\mu\mu & - & \text{SM LD} \ \sim 2 \times 10^{-14} \\
K_S \to ee\mu\mu & - & \sim 10^{-11} \\
K_S \to eeee & - & \sim 10^{-10}
\end{array}
$$



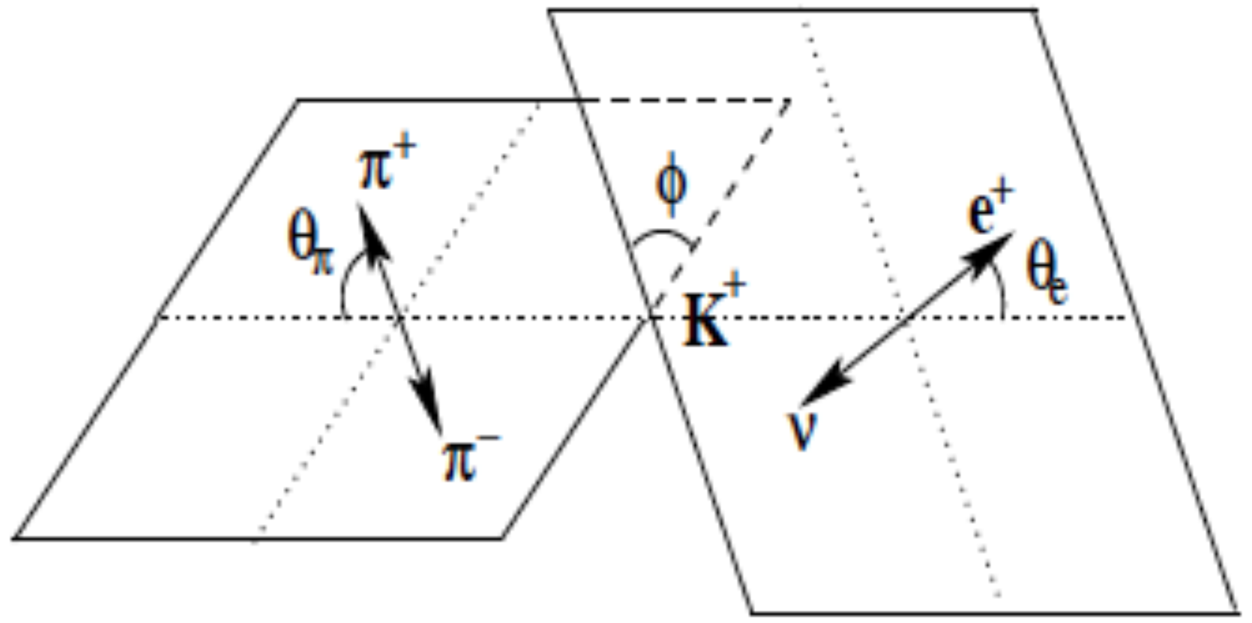
GD,Greynat, Vulvert

K_{l4} and $\pi\pi$ strong phases $\delta_I^l(s)$

Cabibbo Maksymowicz

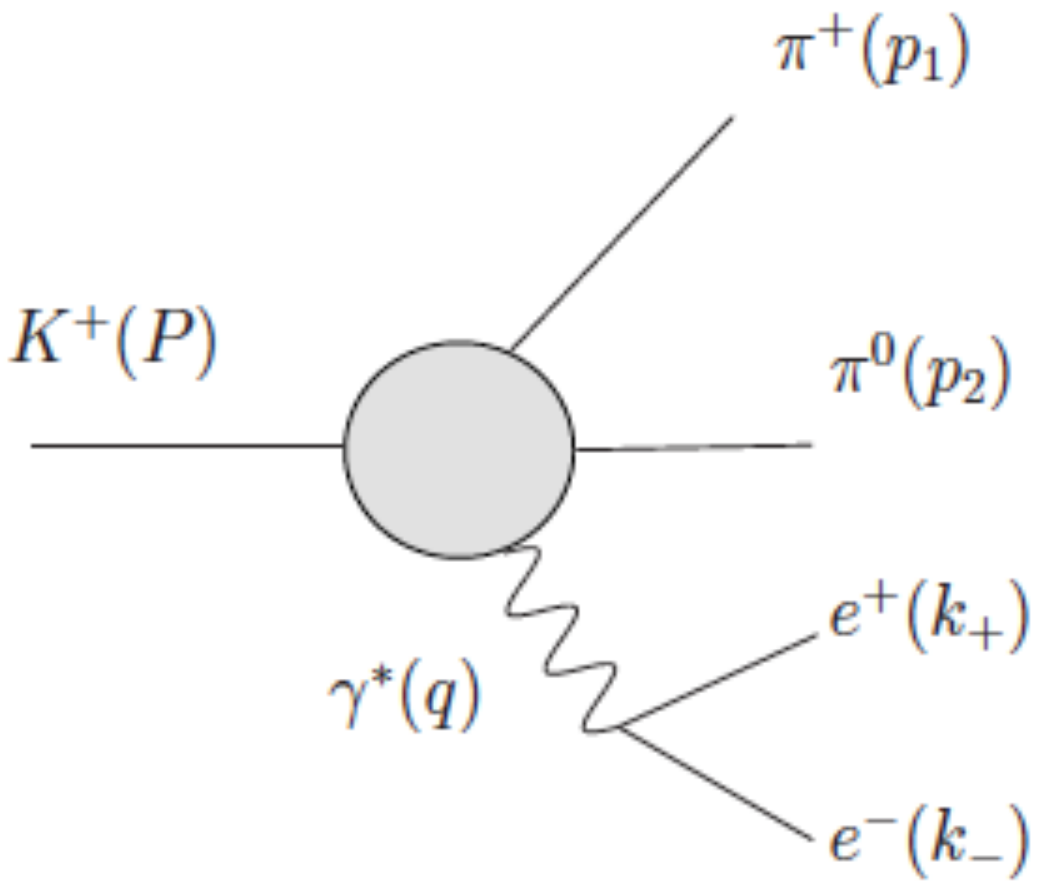
$$\frac{G_F}{\sqrt{2}} V_{us}\, \bar{e}\gamma^\mu(1-\gamma^5)\nu\ H_\mu(p_1, p_2, q)$$

$$H^\mu = F_1 p_1^\mu + F_2 p_2^\mu + F_3\ \varepsilon^{\mu\nu\alpha\beta} p_{1\nu} p_{2\alpha} q_\beta. \qquad F_i(s) = f_i(s) e^{i\delta_0^0(s)} + ..$$



- crucial to measure $\sin\delta \Longrightarrow$ interf F_3
- Look angular plane asymmetry

$K_L \to \pi^+\pi^-\gamma^* \to \pi^+\pi^-e^+e^-$ Sehgal et al; Savage,Wise et al



- $\mathcal{M}_{LD} = \frac{e}{q^2}\, \bar{e}\gamma^\mu(1-\gamma^5)\ e\ H_\mu$
- $H^\mu = F_1 p_1^\mu + F_2 p_2^\mu + F_3\ \varepsilon^{\mu\nu\alpha\beta} p_{1\nu} p_{2\alpha} q_\beta$
- $F_{1,2} \sim E \quad F_3 \sim M$

- Interference $E \quad M$ novel compared to $K_L \to \pi^+\pi^-\gamma$
- $E \quad M$ known from $K_L \to \pi^+\pi^-\gamma$ (IB and DE)

$K^+ \rightarrow \pi^+\pi^0\gamma^* \rightarrow \pi^+\pi^0 e^+ e^-$ Cappiello, Cata, G.D. and Gao,

- the asymm. , $\dfrac{\Re(E_B\, M^*)}{|E_B|^2 + |M|^2}$, not as lucky $E_B >> M$:

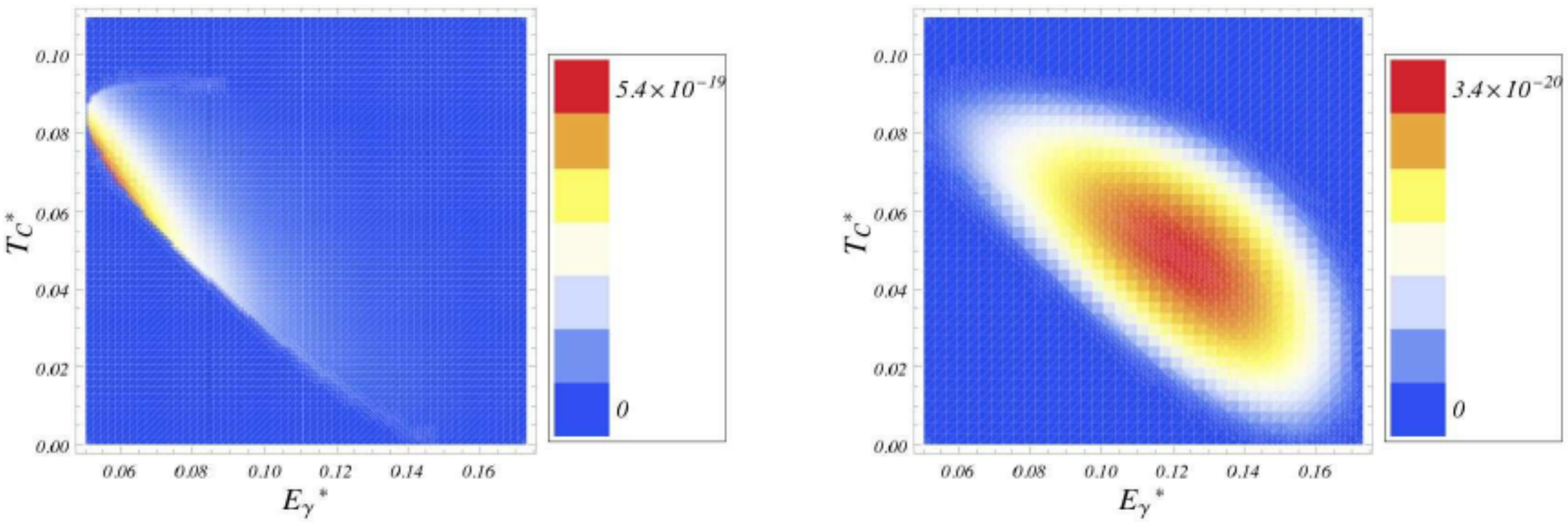
- $B(K^+)_{IB} \sim 3.3 \times 10^{-6} \ \sim \ 50 \ B(K^+)_M$

- Short distance info without having simultaneously K^+ and K^-, asymm. in phase space, (P-violation) interesting! No ϵ-contamination

- interesting Dalitz plots (at fixed q^2) to disentangle M from E_B

- at $q^2 = 50 \mathrm{MeV}$ IB only 10 times larger than DE

q_c (MeV)	B $[10^{-8}]$	B/M	B/E	B/BE	B/BM
$2m_l$	418.27	71	4405	128	208
55	5.62	12	118	38	44
100	0.67	8	30	71	36
180	0.003	12	5	-19	44



IB DE

Outline

- Motivations for K->πll $\quad K^{\pm} \to \pi^{\pm} l^{+} l^{-} \quad K_S \to \pi^0 l^+ l^-$
- K->πll phenomenology
- Large N QCD: the dream '70s '80s '90s 't Hooft, Witten, Coleman
- Ideas, Successes, some undelivered messages $\quad K \to \pi\pi : \ \Delta I = \frac{1}{2}$ rule $\quad \epsilon'$
- K->πll phenomenology and large N
- conclusions

The kaon community requests to

- protect and amplify the European kaon-physics programme, explori[...] opportunities for
  - $K^+ \to \pi^+ \nu\bar{\nu}$
  - $K_{S,L} \to \mu^+\mu^-$ decay and interference,
- enable European contributions for KOTO II for $K_L \to \pi^0\nu\bar{\nu}$ and $K_L \to \pi^0\ell^+\ell^-$, spanning both analysis and hardware development,
- maintain the European leadership on theory computations for kaon physics (phenomenology, dispersion theory, effective theory and lattice QCD, including high-performance computing).

European Strategy for Particle Physics 2026: Kaon physics input document

EU strategy group

Lazzeroni

The contribution "Kaon Physics: A Cornerstone for Future Discoveries" has been drafted by:
M. Bordone, A. Ceccucci, A. Dery, M. Gorbahn, E. Goudzovski, M. Hoferichter, A. Jüttner, C. Lazzeroni, Z. Ligeti, D. Martinez, F. Mahmoudi, R. Marchevski, M. Moulson, G. Ruggiero.

Year	Main object
1	Beam line survey
2	Construction of the rest of the detector
3-6	Phase I: Physics run for mainly $K_L \to \pi^0\nu\bar{\nu}$
7	Single event sensitivity will reach 8.5×10^{-13} for the $K_L \to \pi^0\nu\bar{\nu}$ search
8	Detector upgrade
9-12	Phase II: Physics run mainly for $K_L \to \pi^0\ell^+\ell^-$ with an optimized setup
13	End of Phase II

Moulson

Conclusions

Interesting TH CP violating and CP conserving

$$K_{S,L} \to \pi^0 l^+ l^-$$

Crucial to study

$$K^+ \to \pi^+ l^+ l^-$$

3-point function

$$K_{S,L} \to \mu^+ \mu^- (e^+ e^-)$$

EU strategy Group '25, Nazila et al '22

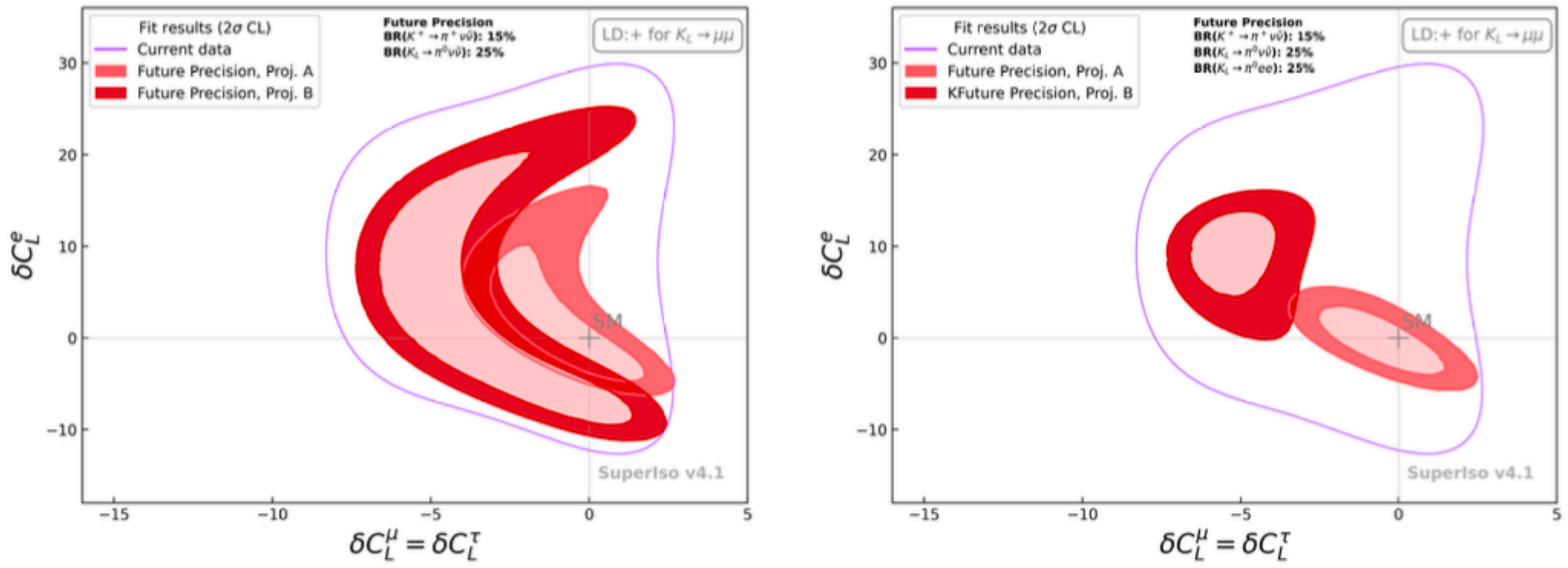


Figure 5: BSM parameter space for Wilson coefficients in scenarios with LFU violation where the NP effects for electrons are different from the those for muons and taus [18, 19]. Left: Impact on allowed parameter space from measurements of the golden channels $K \to \pi\nu\bar{\nu}$ from NA62 and KOTO II with the expected final precision. Right: Impact on the parameter space by the inclusion in the fits of a measurement of the $K_L \to \pi^0 e^+ e^-$ branching ratio with 25% precision.