



Istituto Nazionale di Fisica Nucleare



Quy Nhon, 19th August 2025

Flavour Physics 2025

21st Rencontres du Vietnam 2025

Challenges in Rare Kaon decays

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Predictions for the rare kaon decays
from QCD in the limit of a large number of colour
With M. Knecht and S.Neshatpour
e-Print: 2409.08568 [hep-ph] MDPI

THANKS to Collaborations Nazila Iyer Neshatpour

• [2311.04878](#) [2404.03643](#)

THANKS to NA48 HIKE LHCb KOTO

T. Kitahara

CP violation in $K \rightarrow \mu^+ \mu^-$ with and without time dependence through a tagged analysis

[Giancarlo D'Ambrosio](#) (INFN, Naples), [Avital Dery](#) (CERN), [Yuval Grossman](#) (Cornell U., LEPP), [Teppei Kitahara](#) (Chiba U. and KMI, Nagoya and Nagoya U.), [Radoslav Marchevski](#) (EPFL, Lausanne, LPPC) et al. (Jul 17, 2025)

e-Print: [2507.13445](#) [hep-ph]



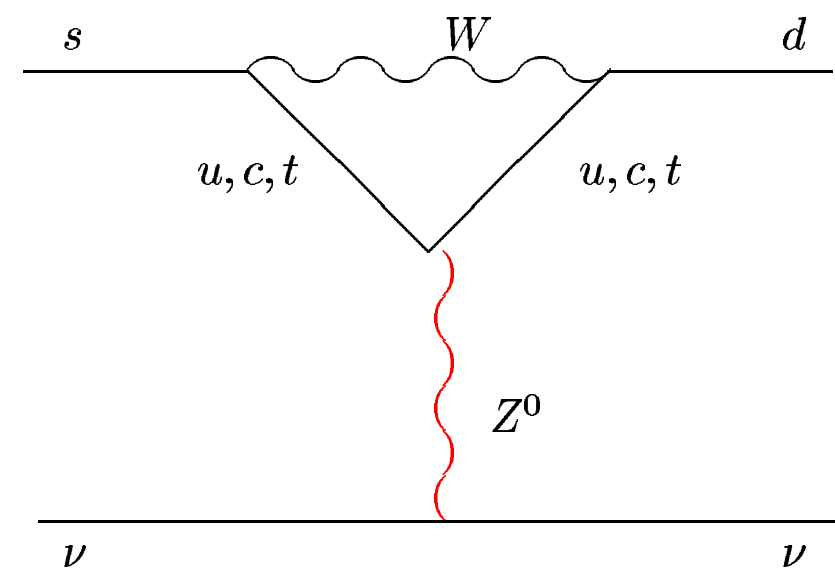
Outline

- Rare Kaon decays crucial in flavour physics
- Measuring $K_S K_L$ interference at LHCb
- Large N_c : may be useful to predict kaon decays $K^+ \rightarrow \pi^+ l^+ l^-$ $K_S \rightarrow \pi^0 l^+ l^-$

$$K \rightarrow \pi \nu \bar{\nu}$$

Why we need KOTO and NA62

$$A(s \rightarrow d \nu \bar{\nu})_{\text{SM}} \sim \bar{s}_L \gamma_\mu d_L \quad \bar{\nu}_L \gamma^\mu \nu_L \times \left[\sum_{q=c,t} V_{qs}^* V_{qd} m_q^2 \right]$$



$$\sim [A^2 \lambda^5 (1 - \rho - i\eta) m_t^2 + \lambda m_c^2]$$

SM

$$\underbrace{V - A \otimes V - A}_{\Downarrow}$$

Littenberg

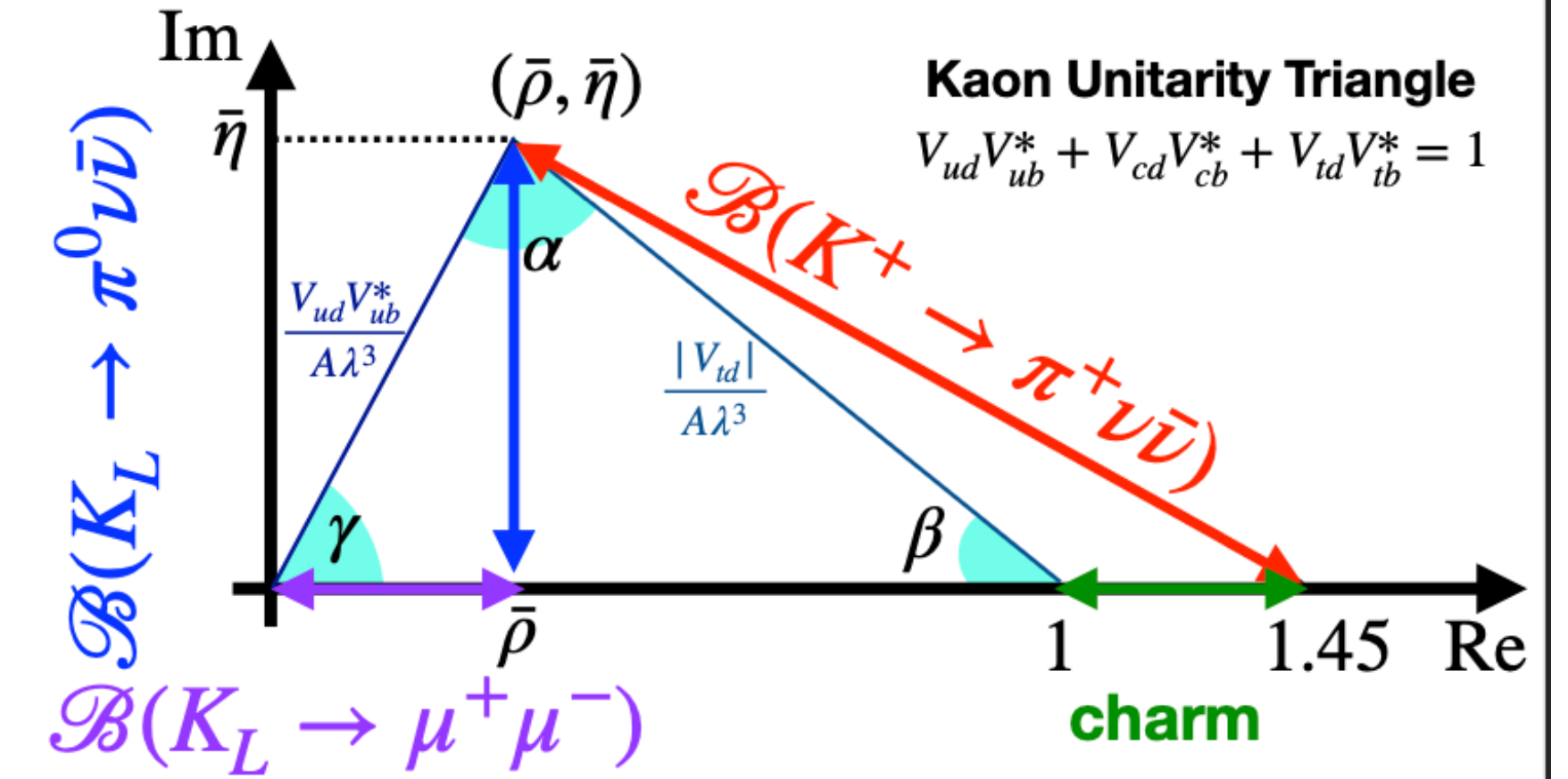
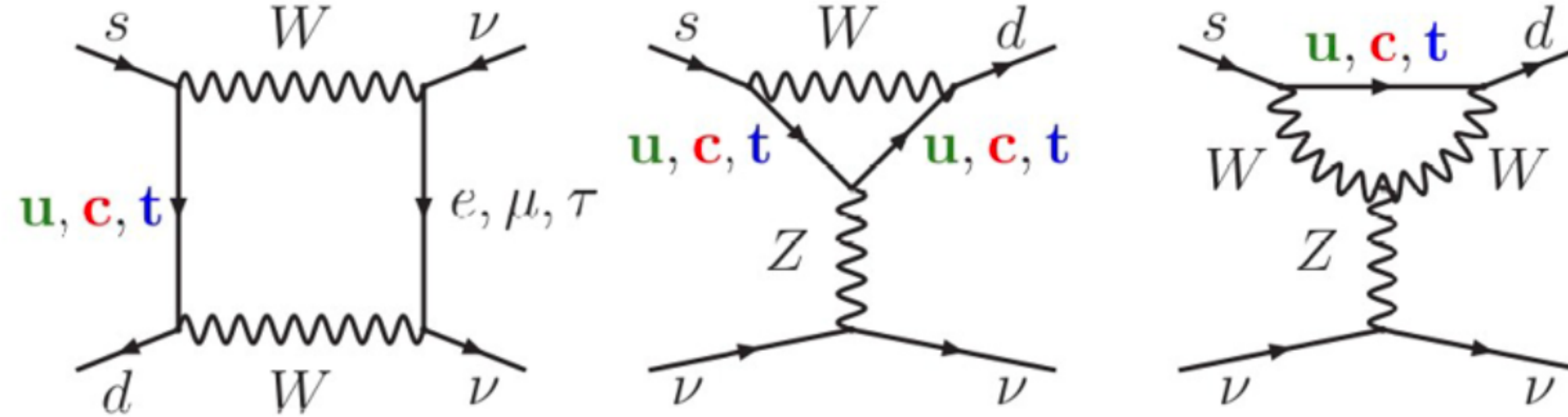
$$\Gamma(K_L \rightarrow \pi^0 \nu \bar{\nu}) \left\{ \begin{array}{l} \text{CP violating} \\ \Rightarrow J = A^2 \lambda^6 \eta \\ \text{Only } \textit{top} \end{array} \right.$$

SM

$K \rightarrow \pi \nu \bar{\nu}$: Precision test of the Standard Model



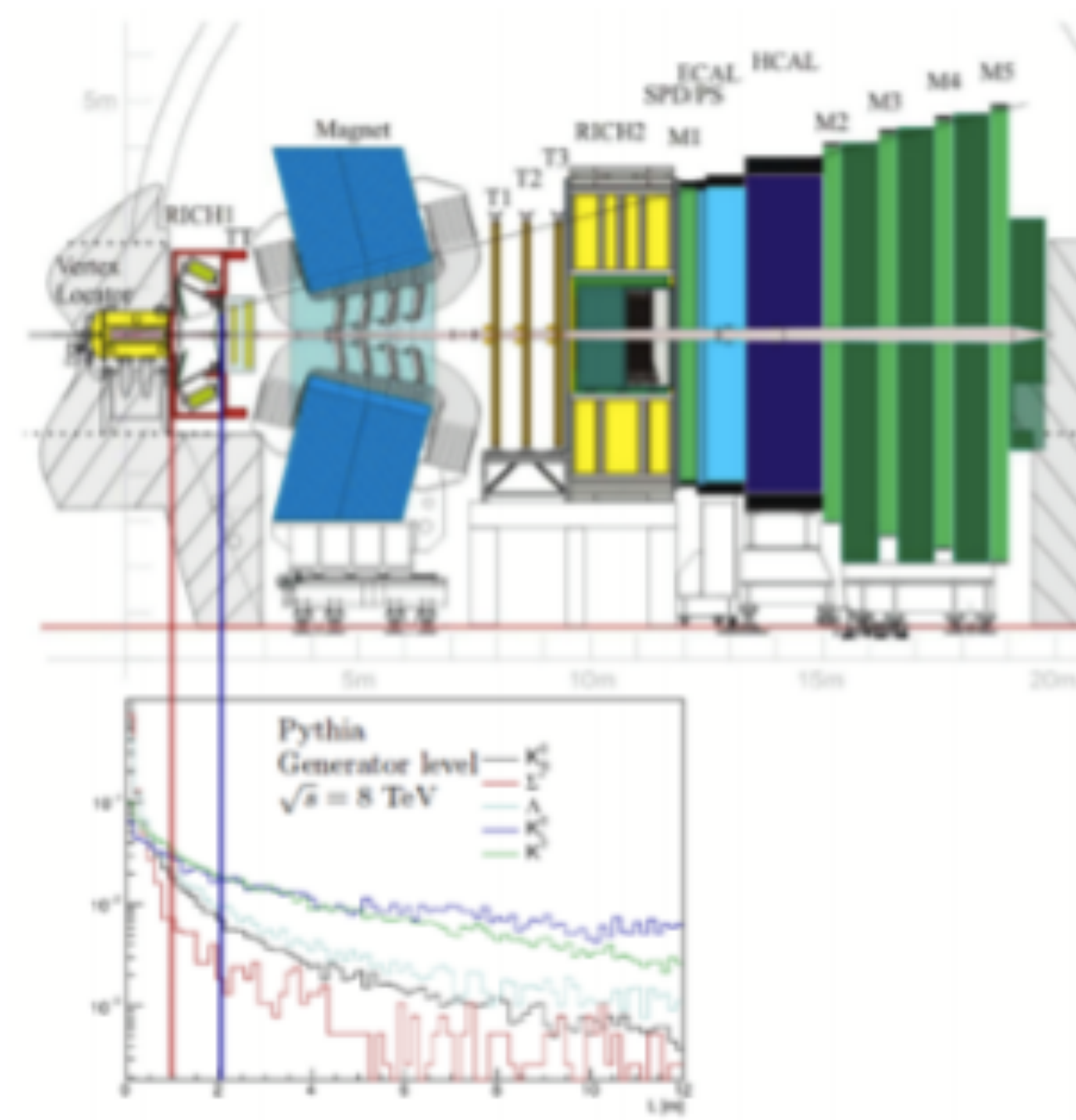
SM: Z-penguin & box diagrams



- $\mathcal{B}(K \rightarrow \pi \nu \bar{\nu})$ highly suppressed in SM
 - GLM mechanism & maximum CKM suppression $s \rightarrow d$ transition: $\sim \frac{m_t}{m_W} \left| V_{ts}^* V_{td} \right|$
- Theoretically clean $\Rightarrow$ high precision SM predictions
 - Dominated by short distance contributions.
 - Hadronic matrix element extracted from $\mathcal{B}(K \rightarrow \pi^0 \ell^+ \nu_\ell)$ decays via isospin rotation.

Mode	SM Branching Ratio [1]	SM Branching Ratio [2]	Experimental Status
$K^+ \rightarrow \pi^+ \nu \bar{\nu}$	$(8.60 \pm 0.42) \times 10^{-11}$	$(7.86 \pm 0.61) \times 10^{-11}$	$(10.6 \pm 4.0) \times 10^{-11}$ NA62 16–18
$K_L \rightarrow \pi^0 \nu \bar{\nu}$	$(2.94 \pm 0.15) \times 10^{-11}$	$(2.68 \pm 0.30) \times 10^{-11}$	$< 2 \times 10^{-9}$ KOTO (2021 data)

Kaon in LHCb



- LHCb experiment has been designed for efficient reconstructions of b and c
- Huge production of strangeness [$O(10^{13})/\text{fb}^{-1} K_S^0$] is suppressed by its trigger efficiency [$\epsilon \sim 1\text{-}2\%$ @LHC Run-I, $\epsilon \sim 18\%$ @LHC Run-II]
- LHCb upgrade (LS2=Phase I upgrade, LS4=Phase II upgrade) could realize high efficiency for K_S^0 [$\epsilon \sim 90\%$ @LHC Run-III] [M. R. Pernas, HL/HE LHC meeting, Fermilab, 2018]
- In LHC Run-III and HL-LHC, we could probe the *ultra* rare decay $\text{Br} \sim O(10^{-11\sim 12})$

Rare Kaon decay program at LHCb

	PDG	Prospects
$K_S \rightarrow \mu\mu$	$< 9 \times 10^{-9}$ at 90% CL	(LD)(5.0 ± 1.5) $\cdot 10^{-12}$ NP $< 10^{-11}$
$K_S \rightarrow \mu\mu\mu\mu$	—	SM LD $\sim 2 \times 10^{-14}$
$K_S \rightarrow ee\mu\mu$	—	$\sim 10^{-11}$
$K_S \rightarrow eeee$	—	$\sim 10^{-10}$
$K_S \rightarrow \pi^0\mu\mu$	$(2.9 \pm 1.3) \cdot 10^{-9}$	$\sim 10^{-9}$
$K_S \rightarrow \pi^+\pi^-e^+e^-$	$(4.79 \pm 0.15) \cdot 10^{-5}$	SM LD $\sim 10^{-5}$
$K_S \rightarrow \pi^+\pi^-\mu^+\mu^-$	—	SM LD $\sim 10^{-14}$

Prospects for Measurements with Strange Hadrons at LHCb

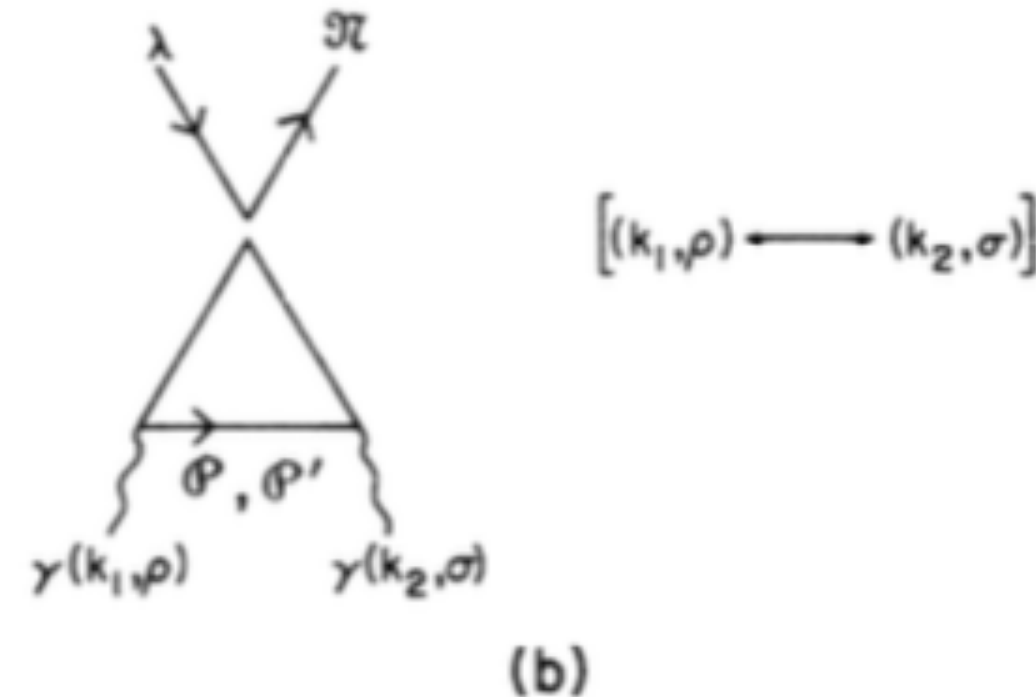
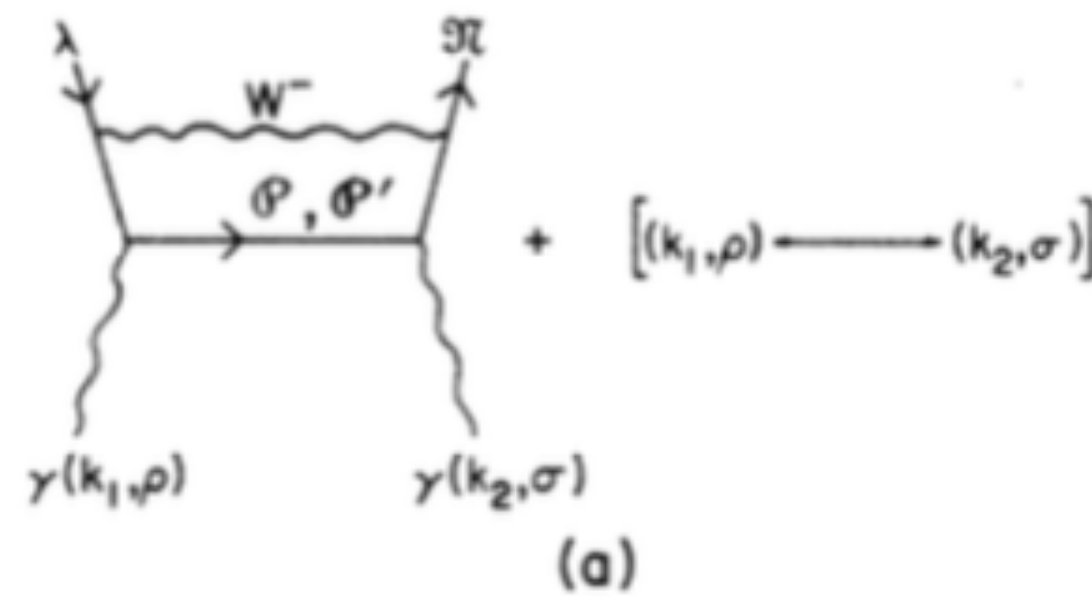
A.A. Alves Junior (Santiago de Compostela U., IGFAE), M.O. Bettler (Cambridge U.), A. Brea Rodríguez (Santiago de Compostela U., IGFAE), A. Casais Vidal (Santiago de Compostela U., IGFAE), V. Chobanova (Santiago de Compostela U., IGFAE) et al. (Aug 10, 2018)

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$$K_{L,S} \rightarrow \mu\mu$$

$K_L \rightarrow \mu\mu$

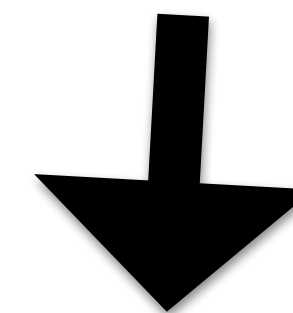
$$\Gamma(K_L^0 \rightarrow \mu^+ \mu^-) / \Gamma(K_L^0 \rightarrow \pi^+ \pi^-)$$



VALUE (10^{-6})	EVTS	DOCUMENT ID	TECN	COMMENT
3.48 ± 0.05	OUR AVERAGE			
3.474 ± 0.057	6210	AMBROSE	2000	B871
3.87 ± 0.30	179	¹ AKAGI	1995	SPEC
3.38 ± 0.17	707	HEINSON	1995	B791
... We do not use the following data for averages, fits, limits, etc. ...				
$3.9 \pm 0.3 \pm 0.1$	178	² AKAGI	1991B	SPEC In AKAGI 1995

$$\mathcal{B}(K_L \rightarrow \mu^+ \mu^-)_{\text{exp}} = (6.84 \pm 0.11) \times 10^{-9}$$

$$K_L \rightarrow \gamma\gamma \mid_{\text{exp}} \text{ known}$$



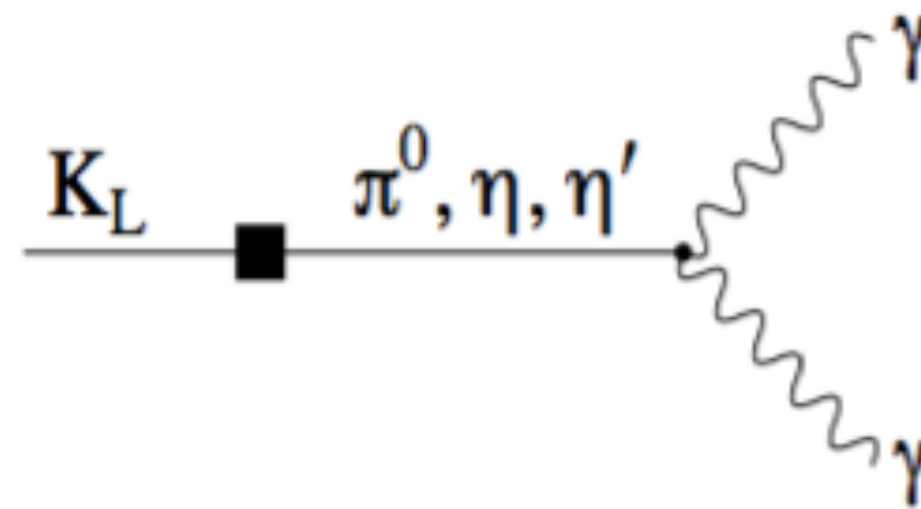
Dispersive calculation: **Re A**, Im A



FIG. 7. Leading contributions to $\lambda + \bar{\pi} \rightarrow \gamma + \gamma$. To leading order in M_W^{-2} , the diagrams in (a) reduce to those of (b).

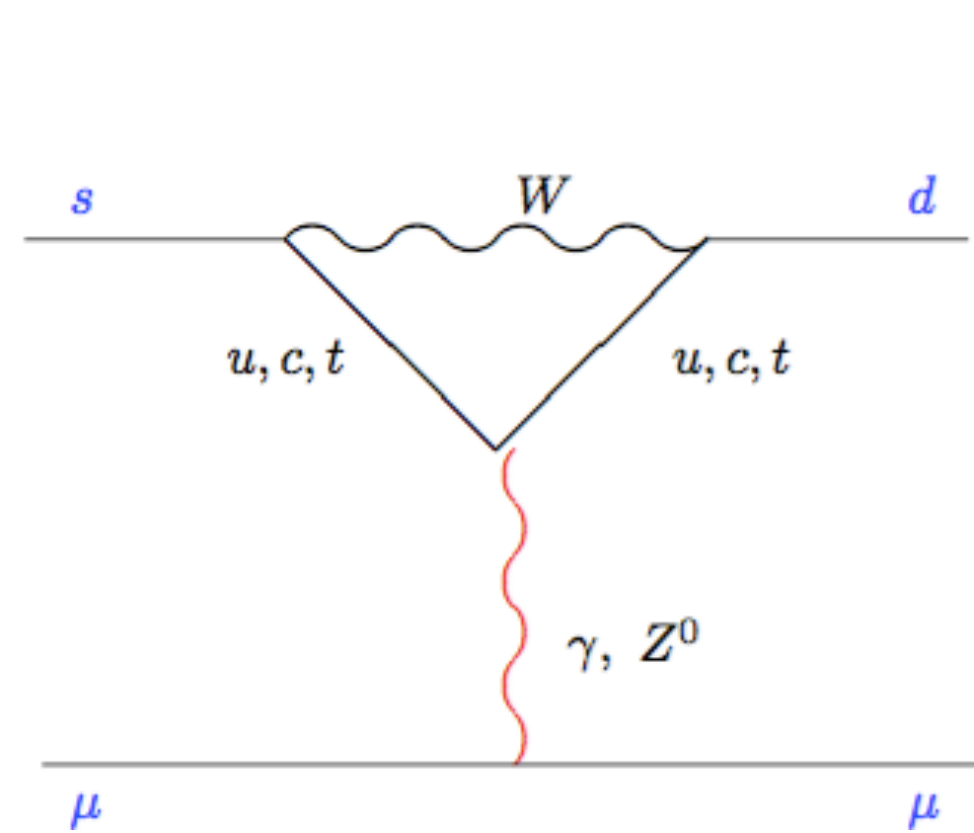
Gaillard Lee

We do not know the sign of $A(K_L \rightarrow \gamma\gamma)$

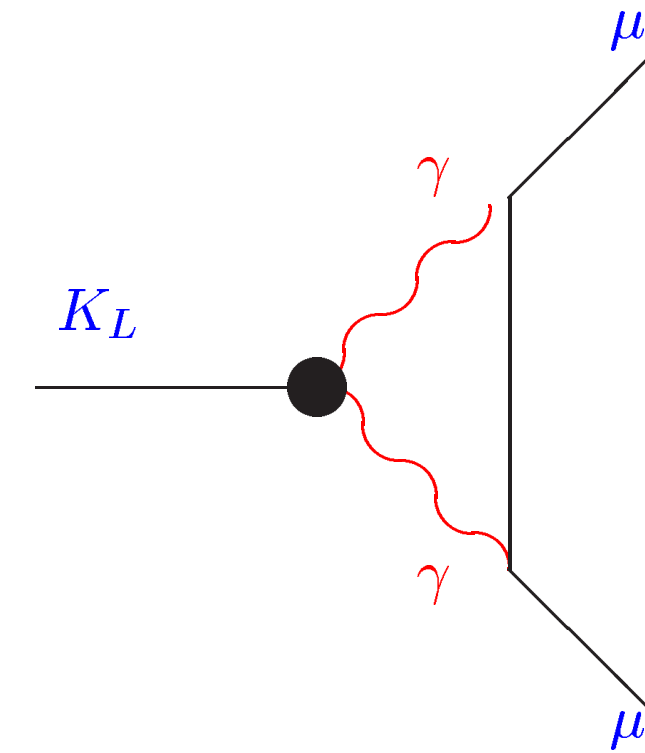


$$\begin{aligned}
 A(K_L \rightarrow 2\gamma_\perp)_{O(p^4)} &= A(K_L \rightarrow \pi^0 \rightarrow 2\gamma_\perp) + A(K_L \rightarrow \eta_8 \rightarrow 2\gamma_\perp) \\
 &= A(K_L \rightarrow \pi^0)A(\pi^0 \rightarrow 2\gamma_\perp) \left[\frac{1}{M_K^2 - M_\pi^2} + \frac{1}{3} \cdot \frac{1}{M_K^2 - M_8^2} \right] \simeq 0
 \end{aligned}$$

$$K_L \rightarrow \mu\mu$$



$\ll$



$$\frac{\Gamma(K_L \rightarrow \mu\bar{\mu})}{\Gamma(K_L \rightarrow \gamma\gamma)} \sim$$

$$|ReA|^2 + |ImA|^2$$

Absorptive calculation
model independent

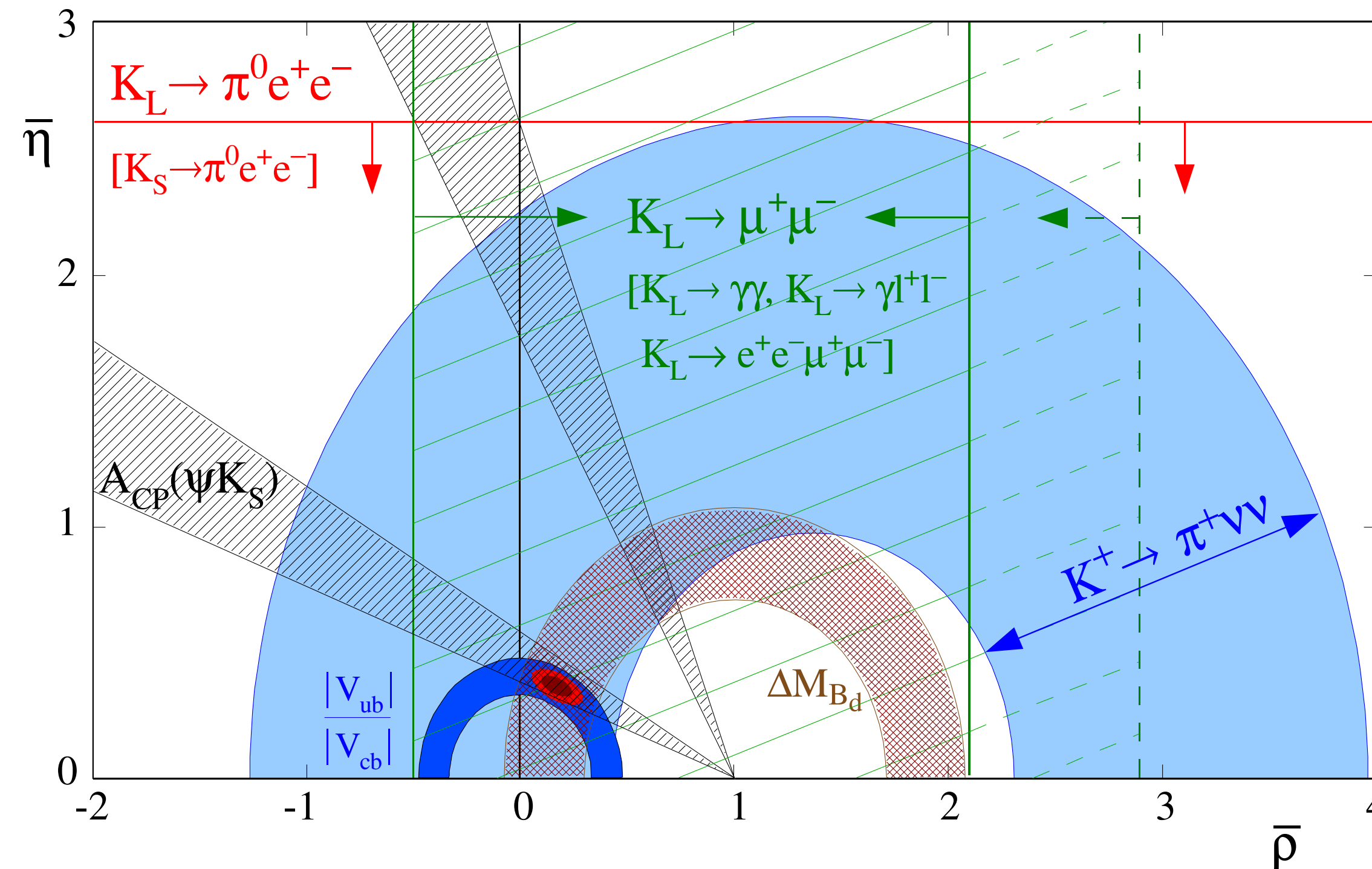
Subtracting from expt. the Absorptive contribution

27.14

$$0.98 \pm 0.55 = |ReA|^2 = (\chi_{\gamma\gamma}(M_\rho) + \chi_{\text{short}} - 5.12)^2$$

$$|\chi_{\text{short}}^{\text{SM}}| = 1.96(1.11 - 0.92\bar{\rho})$$

$K_L \rightarrow \mu\mu$: our sign ignorance



We do not know the sign

We know the sign

Magically comes LHCb

measuring $K_S \rightarrow \mu\mu$

Mostly K_L decays outside fiducial volume

$K_S \rightarrow \mu\mu$

Rare decay modes of the K mesons in gauge theories

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National Accelerator Laboratory, Batavia, Illinois 60510†
(Received 4 March 1974)

Rare decay modes of the kaons such as $K \rightarrow \mu\bar{\mu}$, $K \rightarrow \pi\nu\bar{\nu}$, $K \rightarrow \gamma\gamma$, $K \rightarrow \pi\gamma\gamma$, and $K \rightarrow \pi e\bar{e}$ are of theoretical interest since here we are observing higher-order weak and electromagnetic interactions. Recent advances in unified gauge theories of weak and electromagnetic interactions allow in principle unambiguous and finite predictions for these processes. The above processes, which are "induced" $|\Delta S|=1$ transitions, are a good testing ground for the cancellation mechanism first invented by Glashow, Iliopoulos, and Maiani (GIM) in order to banish $|\Delta S|=1$ neutral currents. The experimental suppression of $K_L \rightarrow \mu\bar{\mu}$ and nonsuppression of $K_L \rightarrow \gamma\gamma$ must find a natural explanation in the GIM mechanism which makes use of extra quark(s). The procedure we follow is the following: We deduce the effective interaction Lagrangian for $\lambda + \bar{\pi} \rightarrow l + \bar{l}$ and $\lambda + \bar{\pi} \rightarrow \gamma + \gamma$ in the free-quark model; then the appropriate matrix elements of these operators between hadronic states are evaluated with the aid of the principles of conserved vector current and partially conserved axial-vector current. We focus our attention on the Weinberg-Salam model. In this model, $K \rightarrow \mu\bar{\mu}$ is suppressed due to a fortuitous cancellation. To explain the small K_L - K_S mass difference and nonsuppression of $K_L \rightarrow \gamma\gamma$, it is found necessary to assume $m_{\phi}/m_{\phi'} \ll 1$, where m_{ϕ} is the mass of the proton quark and $m_{\phi'}$ the mass of the charmed quark, and $m_{\phi'} < 5$ GeV. We present a phenomenological argument which indicates that the average mass of charmed pseudoscalar



Dr. Gaillard at Berkeley in the early 1980s. AIP Emilio Segrè Visual Archives, Physics Today Collection

$K_S^0 \rightarrow \mu^+ \mu^-$

INSPIRE search

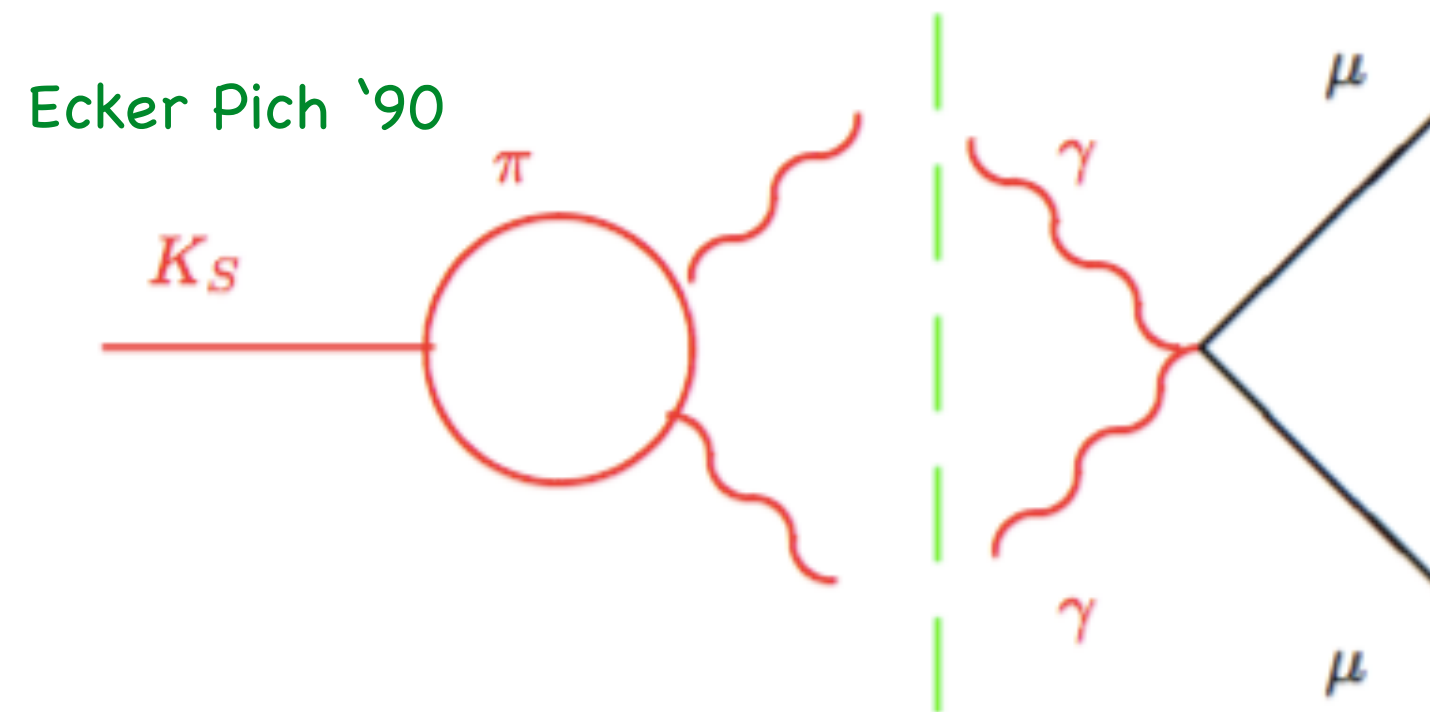
$\Gamma(K_S^0 \rightarrow \mu^+ \mu^-) / \Gamma_{\text{total}}$

Γ_{11} / Γ

Test for $\Delta S = 1$ weak neutral current. Allowed by first-order weak interaction combined with electromagnetic interaction.

VALUE	CL%	DOCUMENT ID	TECN
$< 2.1 \times 10^{-10}$	90	1 AAIJ 2020AE	LHCB
... We do not use the following data for averages, fits, limits, etc. ...			
$< 8 \times 10^{-10}$	90	2 AAIJ 2017BQ	LHCB
$< 9 \times 10^{-9}$	90	3 AAIJ 2013G	LHCB
$< 3.2 \times 10^{-7}$	90	GJESDAL 1973	ASPK

$K_S \rightarrow \mu\mu$



No CP conserving Short Distance due to Furry Theorem

Gaillard Lee

LD 5×10^{-12} 20% TH err

Dispersive treatment of $K_S \rightarrow \gamma\gamma$ and $K_S \rightarrow \gamma l^+ l^-$

Gilberto Colangelo, Ramon Stucki, and Lewis C. Tunstall

Short Distance

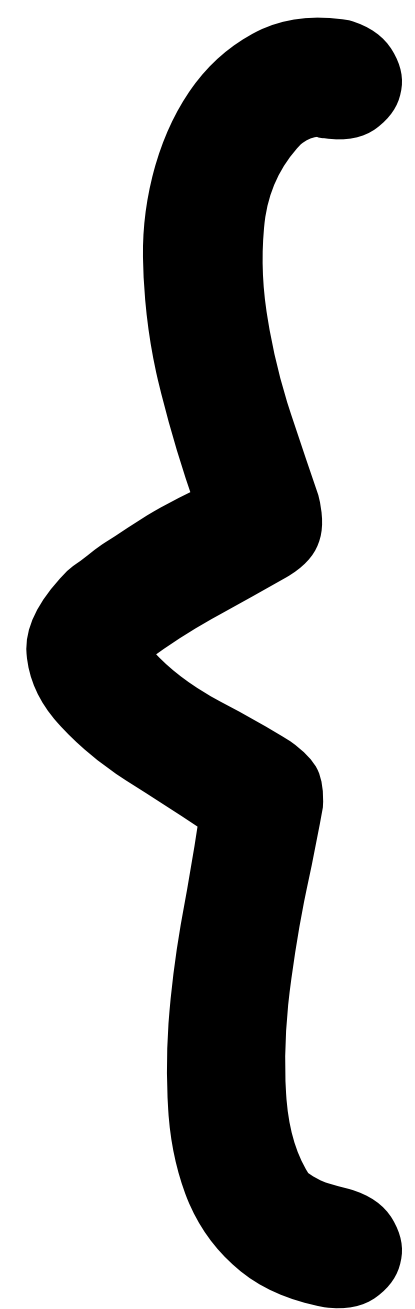
SM	$10^{-5} \Im(V_{ts}^* V_{td}) ^2 \sim 10^{-13}$
NP	few 10^{-11} allowed

Summarizing

$$\begin{aligned}
 &\mathcal{B}(K_S^0 \rightarrow \mu^+ \mu^-)_{\text{SM}} \\
 &= (5.18 \pm 1.50_{\text{LD}} \pm 0.02_{\text{SD}}) \times 10^{-12} \\
 &\quad \parallel \\
 &\quad (4.99_{\text{LD}} + 0.19_{\text{SD}})
 \end{aligned}$$

[Ecker, Pich '91; Isidori, Unterdorfer '04; Chobanova, D'Ambrosio, TK, Martínez, Santos, Fernández, Yamamoto '18]

SD??

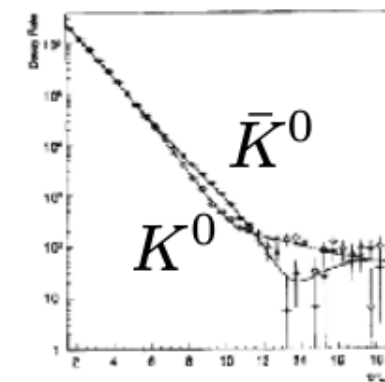


CPLEAR Flavor tagging

Such an interference has been discussed from '67 [Sehgal and Wolfenstein], and has been observed/
utilized in many processes: e.g., $K^0 \rightarrow \pi\pi$, $K^0 \rightarrow 3\pi^0$, $K^0 \rightarrow \pi^+\pi^-\pi^0$, and $K^0 \rightarrow \pi^0 e^+ e^-$

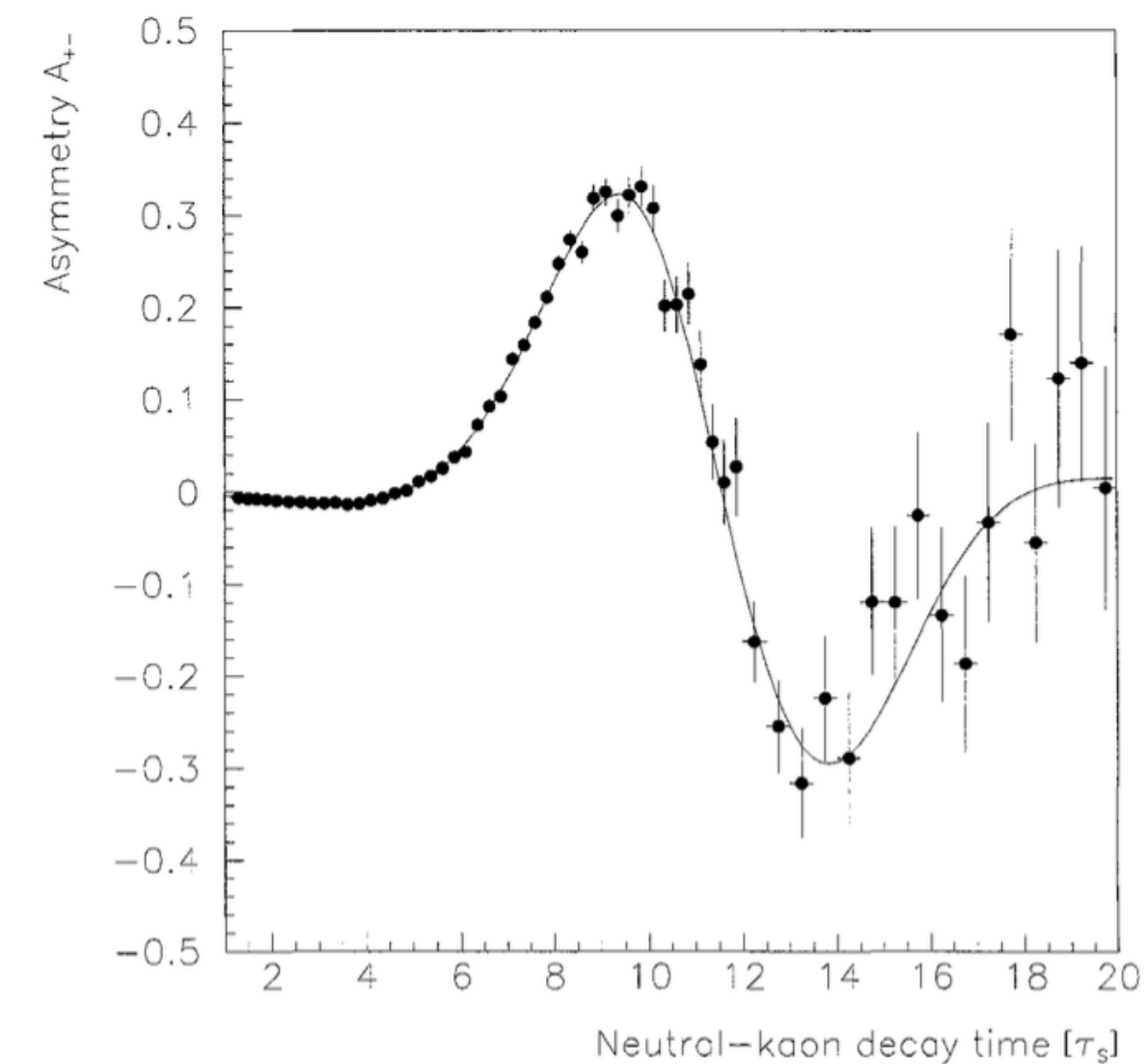
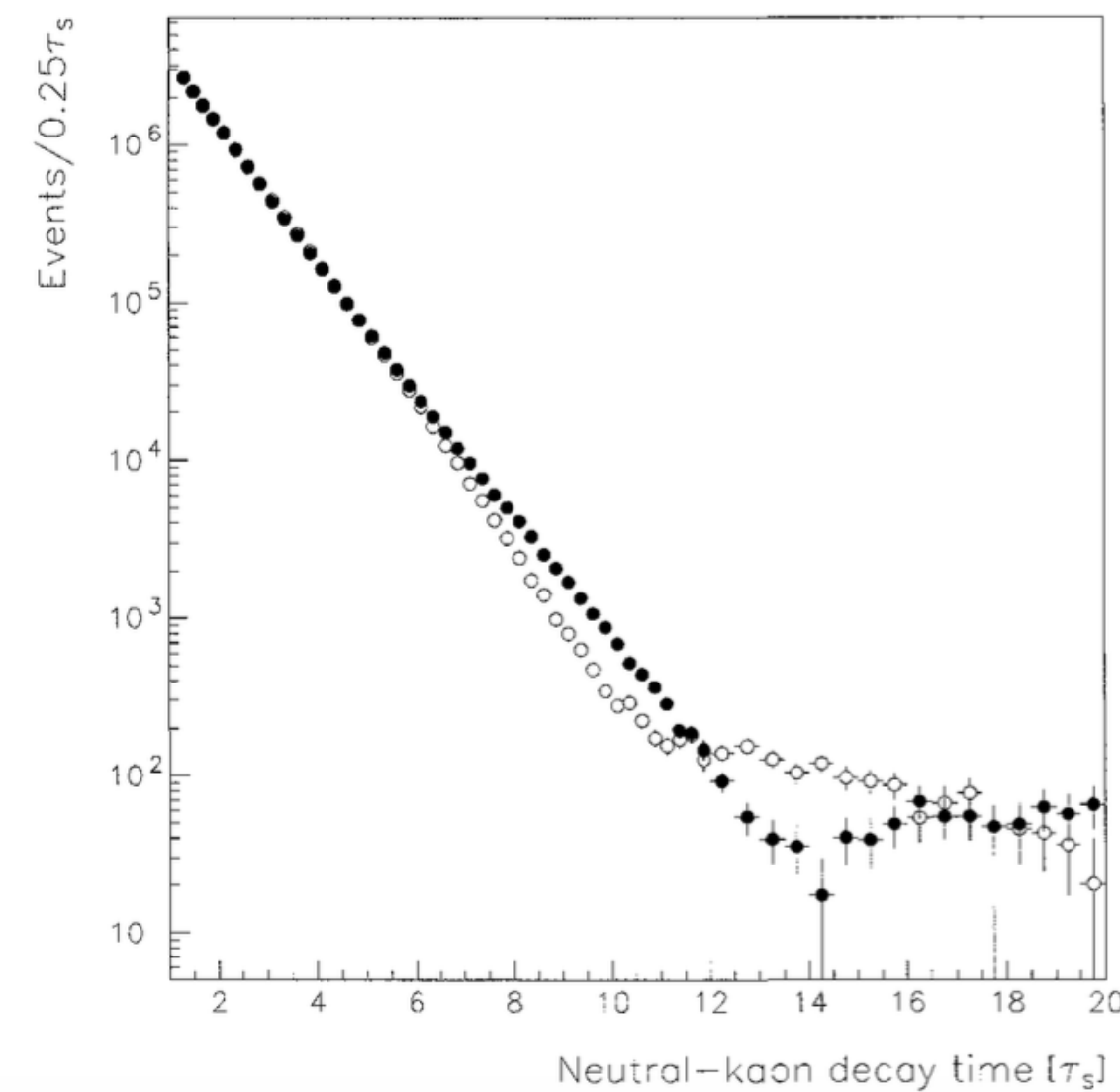
cf. CPLEAR experiment
(1990-99@CERN)

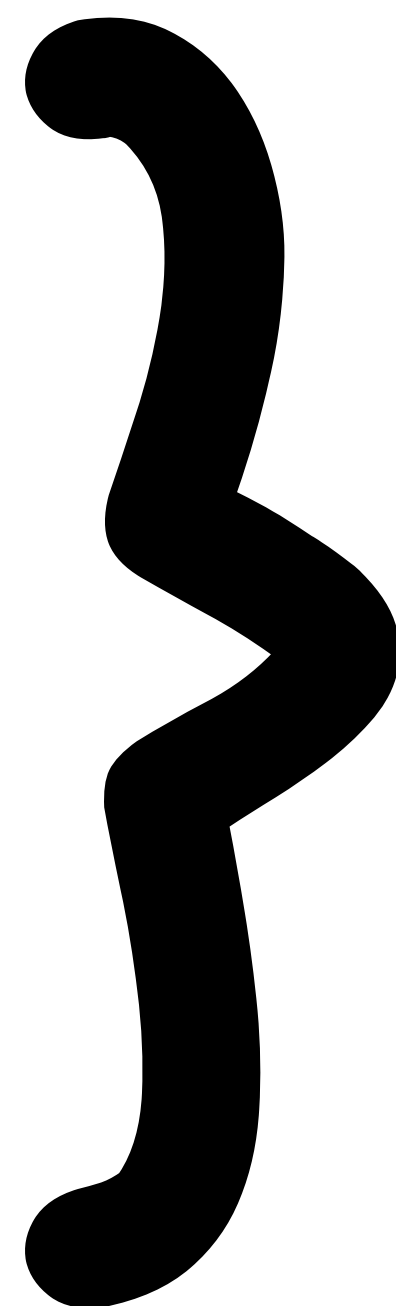
$$p\bar{p} \rightarrow \begin{bmatrix} K^0 K^- \pi^+ \\ \bar{K}^0 K^+ \pi^- \end{bmatrix}$$



$$\{K_S, K_L\} \rightarrow \pi^+\pi^-$$

measured the interference between K_L and K_S
[CPLEAR collaboration '95]



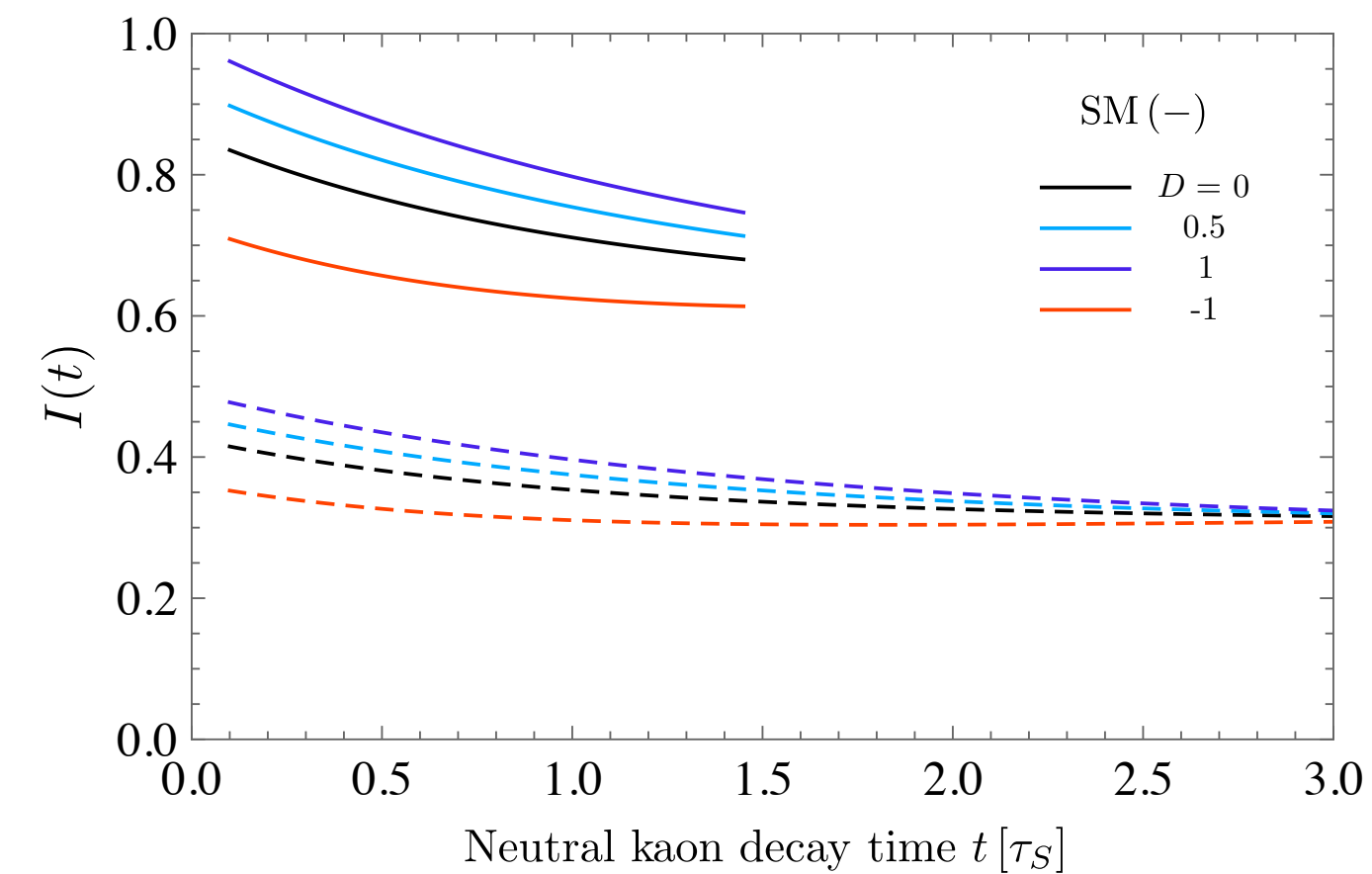
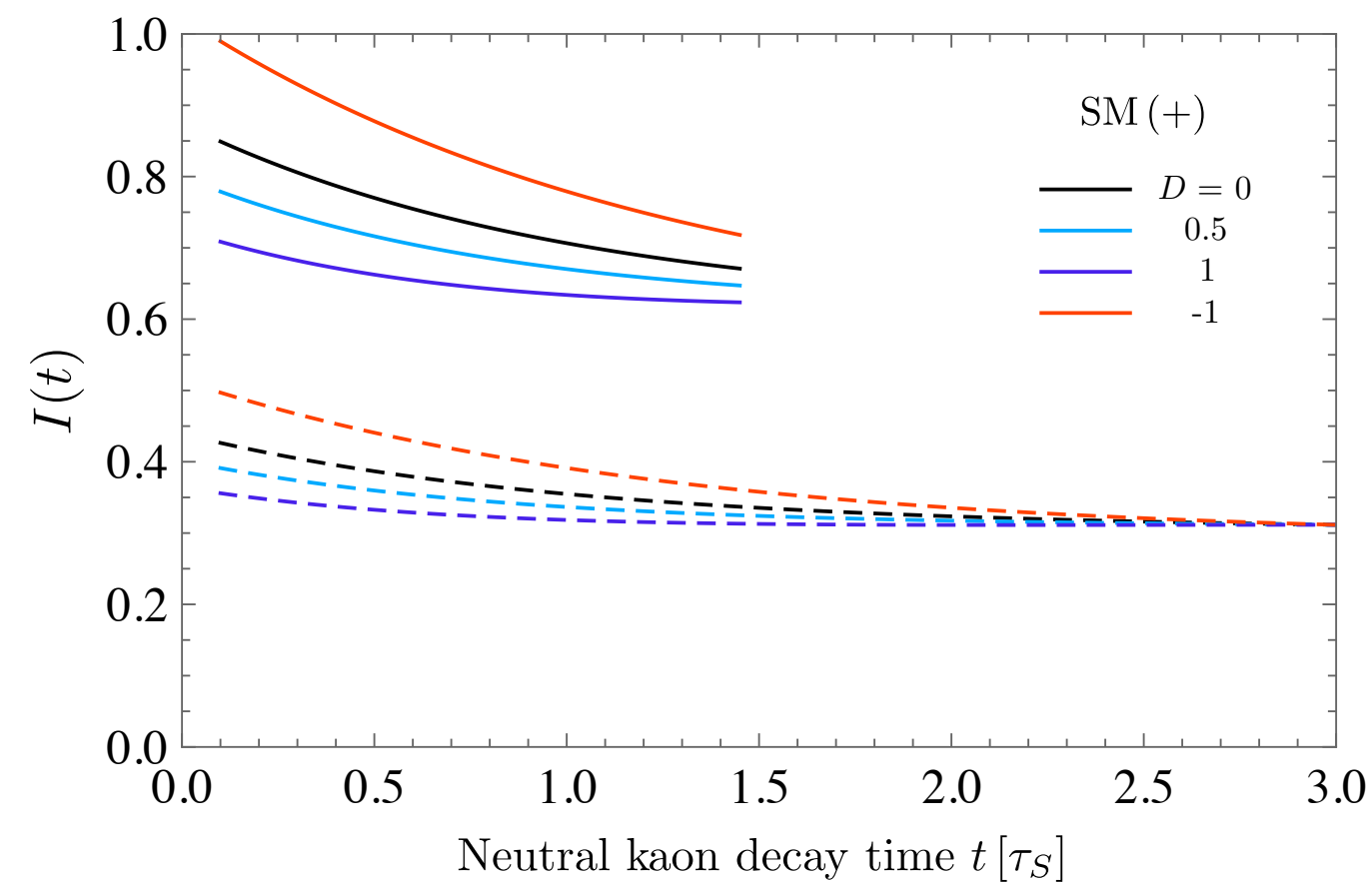


Can we study $K^0(t)$?

GD , Kitahara
1707.06999 PRL

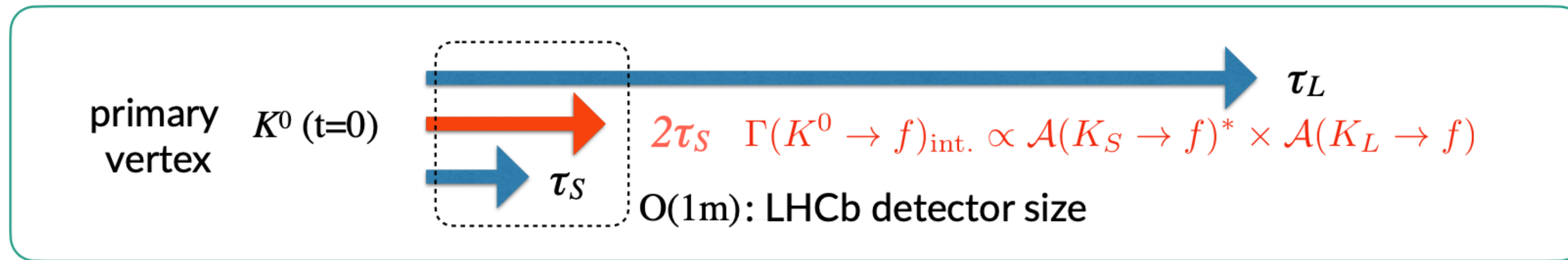
$$pp \rightarrow K^0 \textcolor{red}{K}^- X$$

$$pp \rightarrow K^{*+} X \rightarrow K^0 \textcolor{green}{\pi}^+ X$$

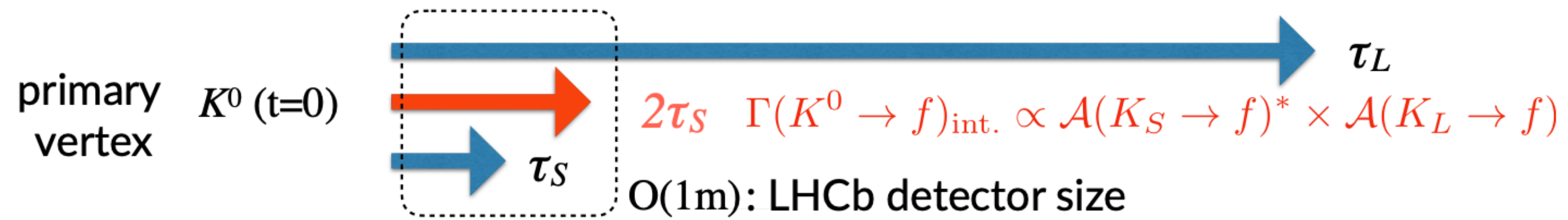


$$|\bar{K}^0(t)\rangle = \frac{1}{\sqrt{2}(1 \pm \bar{\epsilon})} \left[e^{-iH_S t} (|K_1\rangle + \bar{\epsilon}|K_2\rangle) \right. \\ \left. \pm e^{-iH_L t} (|K_2\rangle + \bar{\epsilon}|K_1\rangle) \right]$$

$$D = \frac{K^0 - \bar{K}^0}{K^0 + \bar{K}^0}$$



$$D = \frac{K^0 - \bar{K}^0}{K^0 + \bar{K}^0}$$



$$I(t) = \frac{1}{2} |\mathcal{A}(K_S)|^2 e^{-\Gamma_S t} + \frac{1}{2} |\mathcal{A}(K_L)|^2 e^{-\Gamma_L t} + D \text{Re} [e^{-i\Delta M_K t} \mathcal{A}(K_S)^* \mathcal{A}(K_L)] e^{-\frac{\Gamma_B + \Gamma_L}{2} t} + \mathcal{O}(\bar{\epsilon})$$

Interference $\tau \sim 2\tau_S$

$$\sum_{\text{spin}} \mathcal{A}(K_1 \rightarrow \mu^+ \mu^-)^* \mathcal{A}(K_2 \rightarrow \mu^+ \mu^-)$$

$$\mathcal{H}_{\text{eff}} = \frac{G_F \alpha}{\sqrt{2}} \lambda_t y'_{7A} (\bar{s} \gamma_\mu \gamma_5 d) (\bar{\mu} \gamma^\mu \gamma_5 \mu) + \text{h.c.}$$

$$= \frac{16i G_F^4 M_W^4 F_K^2 M_K^2 m_\mu^2 \sin^2 \theta_W}{\pi^3} \text{Im}[\lambda_t] y'_{7A} \left\{ A_{L\gamma\gamma}^\mu - 2\pi \sin^2 \theta_W (\text{Re}[\lambda_t] y'_{7A} + \text{Re}[\lambda_c] y_c) \right\}$$

$\gamma\gamma$ loop

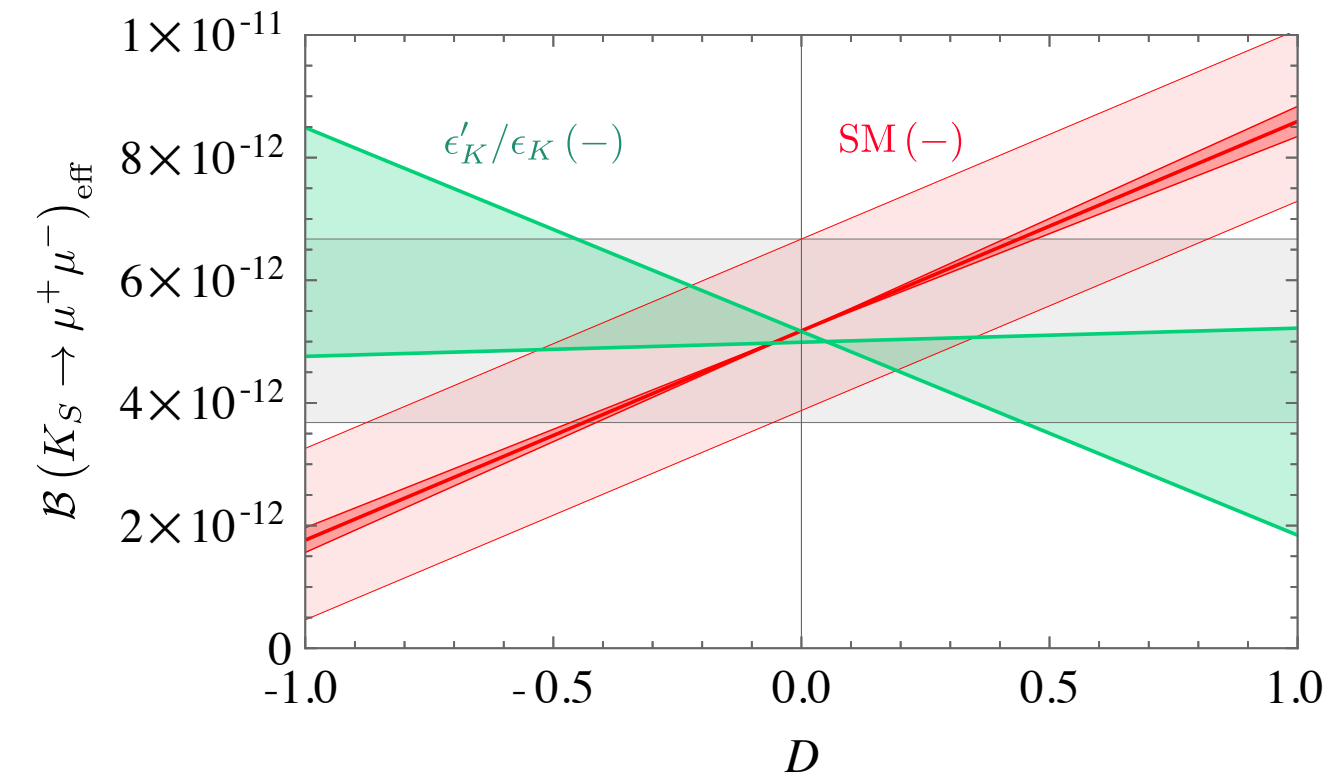
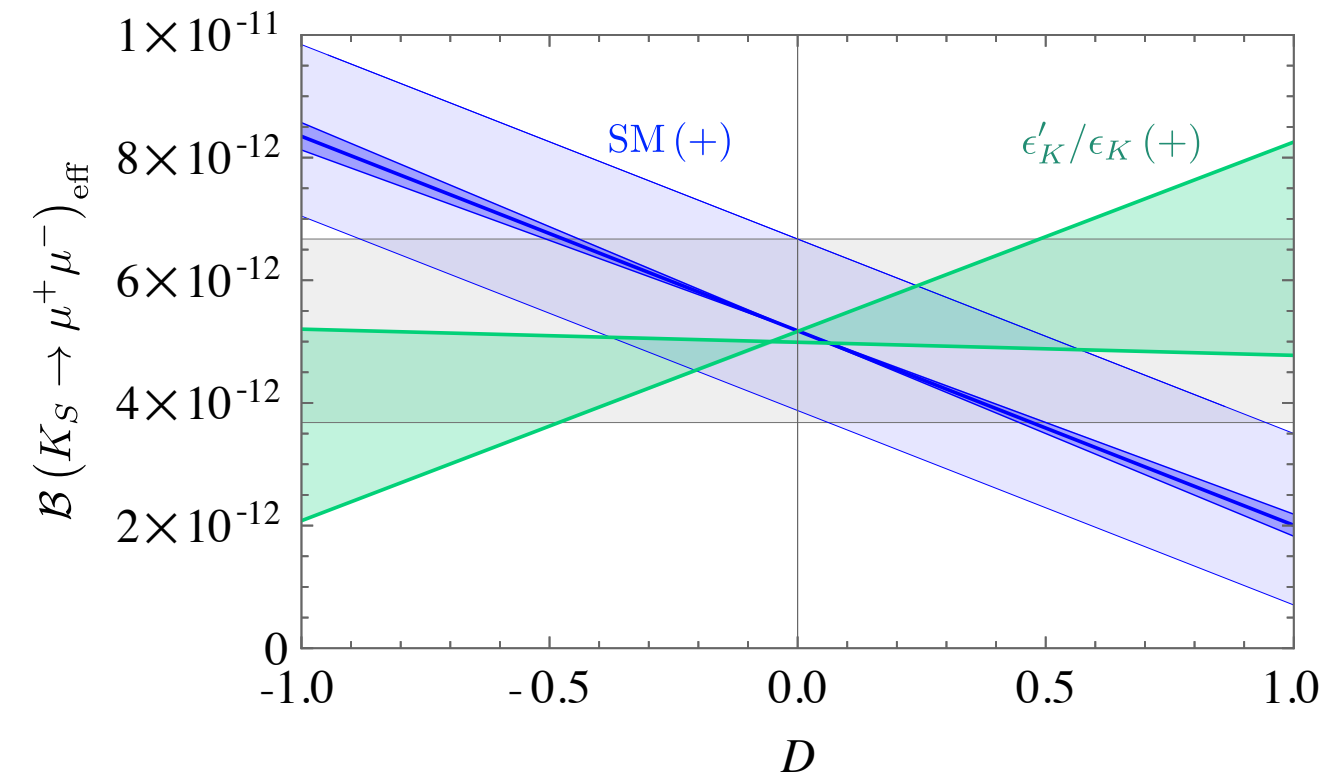
Coefficient is pure imaginary

Leading contribution is proportional to $D \times \text{Im}[\lambda_t] \times \text{Im}[A(K_L \rightarrow \gamma\gamma \rightarrow \mu\mu)]$

It looks like [weak phase] $\times$ [strong phase], namely the direct CPV in meson decay

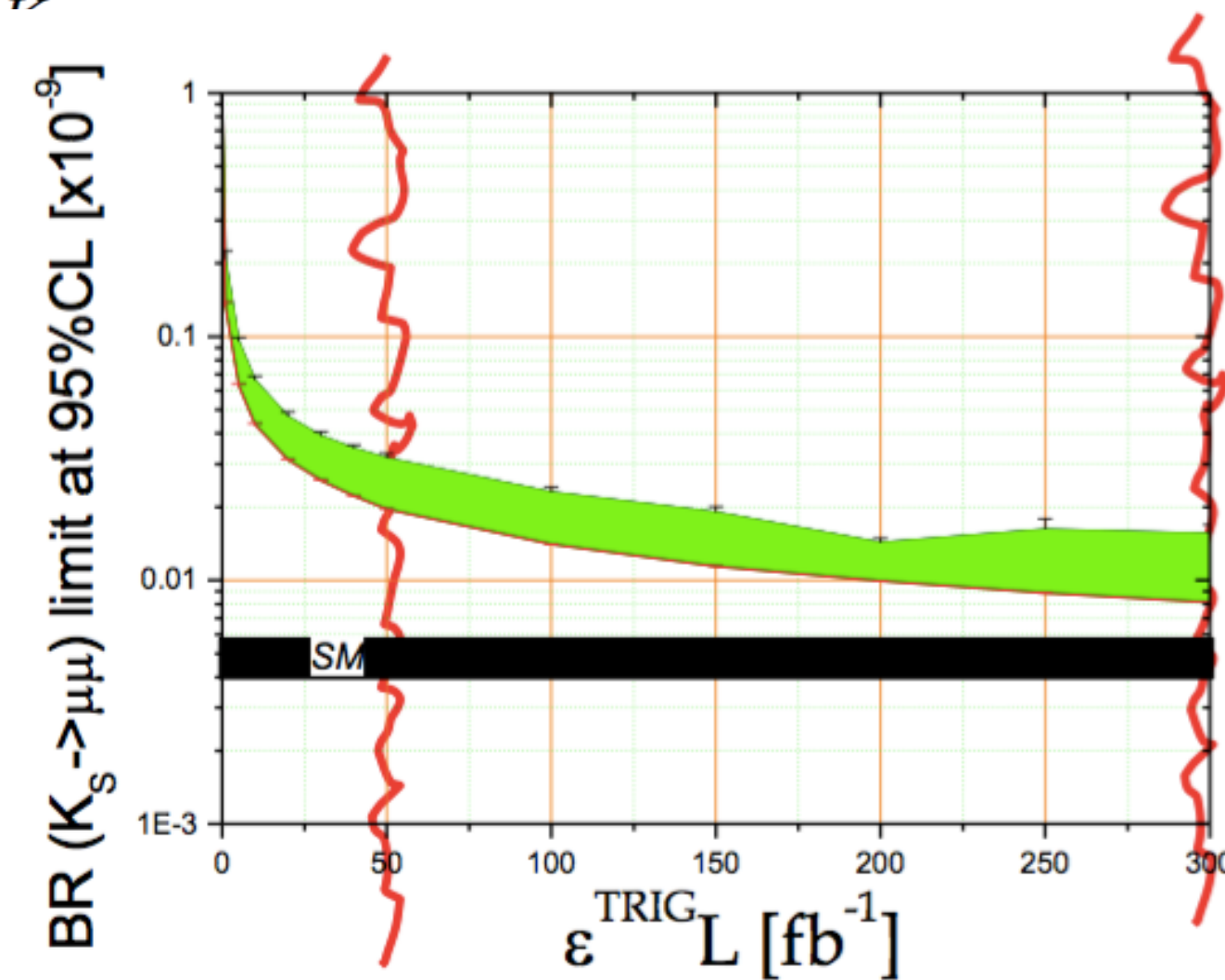
Short distance window

GD , Kitahara
1707.06999 PRL



modified Z-coupling model. ϵ'_K/ϵ

$$\mathcal{B}(K_S \rightarrow \mu^+ \mu^-)_{\text{eff}} = \tau_S \left[\int_{t_{\min}}^{t_{\max}} dt \left(\Gamma(K_1) e^{-\Gamma_S t} + \frac{D}{8\pi M_K} \sqrt{1 - \frac{4m_\mu^2}{M_K^2}} \sum \text{Re} \left[e^{-i\Delta M_K t} \mathcal{A}(K_1)^* \mathcal{A}(K_2) \right] e^{-\frac{\Gamma_S + \Gamma_L}{2} t} \right) \varepsilon(t) \right] \times \left(\int_{t_{\min}}^{t_{\max}} dt e^{-\Gamma_S t} \varepsilon(t) \right)^{-1},$$



LHCb-upgrade

Phase-II-upgrade?

KAONS: RICH POTENTIAL

Rare Kaon decay program at LHCb

	PDG	Prospects
$K_S \rightarrow \mu\mu$	$< 9 \times 10^{-9}$ at 90% CL	(LD) $(5.0 \pm 1.5) \cdot 10^{-12}$ NP $< 10^{-11}$
$K_S \rightarrow \mu\mu\mu\mu$	—	SM LD $\sim 2 \times 10^{-14}$
$K_S \rightarrow ee\mu\mu$	—	$\sim 10^{-11}$
$K_S \rightarrow eeee$	—	$\sim 10^{-10}$
$K_S \rightarrow \pi^0\mu\mu$	$(2.9 \pm 1.3) \cdot 10^{-9}$	$\sim 10^{-9}$
$K_S \rightarrow \pi^+\pi^-\ell^+\ell^-$	$(4.79 \pm 0.15) \cdot 10^{-5}$	SM LD $\sim 10^{-5}$
$K_S \rightarrow \pi^+\pi^-\mu^+\mu^-$	—	SM LD $\sim 10^{-14}$

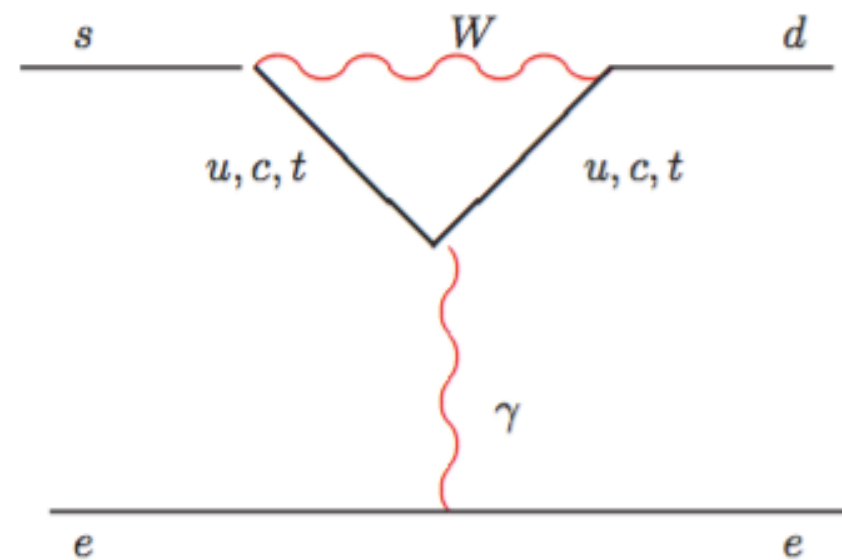
HIKE

$K^+ \rightarrow \pi^+\nu\bar{\nu}$ $K^+ \rightarrow \pi^+\ell^+\ell^-$ $K^+ \rightarrow \pi^-\ell^+\ell^+, K^+ \rightarrow \pi\mu e$ Semileptonic K^+ decays $R_K = \mathcal{B}(K^+ \rightarrow e^+\nu)/\mathcal{B}(K^+ \rightarrow \mu^+\nu)$ Ancillary K^+ decays (e.g. $K^+ \rightarrow \pi^+\gamma\gamma, K^+ \rightarrow \pi^+\pi^0 e^+ e^-$)	$\sigma_{\mathcal{B}}/\mathcal{B} \sim 5\%$ Sub-‰ precision on form-factors Sensitivity $\mathcal{O}(10^{-13})$ $\sigma_{\mathcal{B}}/\mathcal{B} \sim 0.1\%$ $\sigma(R_K)/R_K \sim \mathcal{O}(0.1\%)$ ‰ – ‰ ₀₀	BSM physics, LFUV LFUV LFV / LNV V_{us} , CKM unitarity LFUV Chiral parameters (LECs)
$K_L \rightarrow \pi^0\ell^+\ell^-$ $K_L \rightarrow \mu^+\mu^-$ $K_L \rightarrow \pi^0(\pi^0)\mu^\pm e^\mp$ Semileptonic K_L decays Ancillary K_L decays (e.g. $K_L \rightarrow \gamma\gamma, K_L \rightarrow \pi^0\gamma\gamma$)	$\sigma_{\mathcal{B}}/\mathcal{B} < 20\%$ $\sigma_{\mathcal{B}}/\mathcal{B} \sim 1\%$ Sensitivity $\mathcal{O}(10^{-12})$ $\sigma_{\mathcal{B}}/\mathcal{B} \sim 0.1\%$ ‰ – ‰ ₀₀	Im λ_t to 20% precision, BSM physics, LFUV Ancillary for $K \rightarrow \mu\mu$ physics LFV V_{us} , CKM unitarity Chiral parameters (LECs), SM $K_L \rightarrow \mu\mu, K_L \rightarrow \pi^0\ell^+\ell^-$ rates

Long Distance enhancement

$$K^\pm \rightarrow \pi^\pm l^+ l^- \quad K_S \rightarrow \pi^0 l^+ l^-$$

Gilman Wise 1980



Short distance

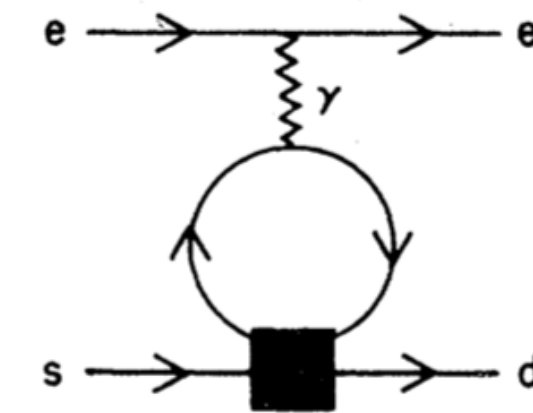
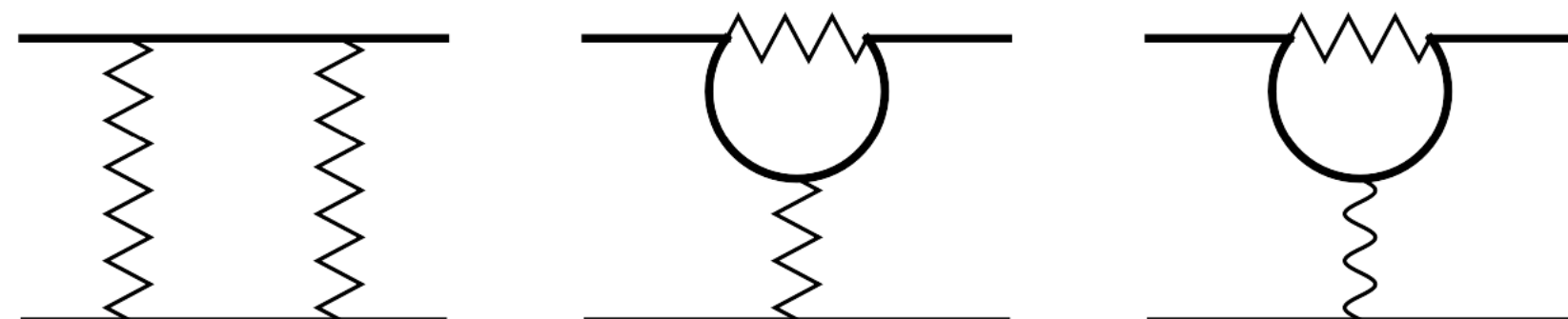


FIG. 1. Diagram contributing to C_7 . The black box represents W exchange plus all strong-interaction corrections.

$$\begin{aligned} \mathcal{H}_{\text{eff}} = & -\frac{G_F}{\sqrt{2}} s_1 c_1 c_3 (\bar{s}_\alpha u_\alpha)_{V-A} (\bar{u}_\beta d_\beta)_{V-A} \\ & -\frac{G_F}{\sqrt{2}} \frac{2}{9\pi} \frac{e^2}{4\pi} \left[A_c \ln\left(\frac{m_c^2}{\mu^2}\right) + A_t \ln\left(\frac{m_t^2}{\mu^2}\right) \right] \\ & \times (\bar{s}_\alpha d_\alpha)_{V-A} (\bar{e}e)_V \\ & + \text{H.c.} \end{aligned}$$

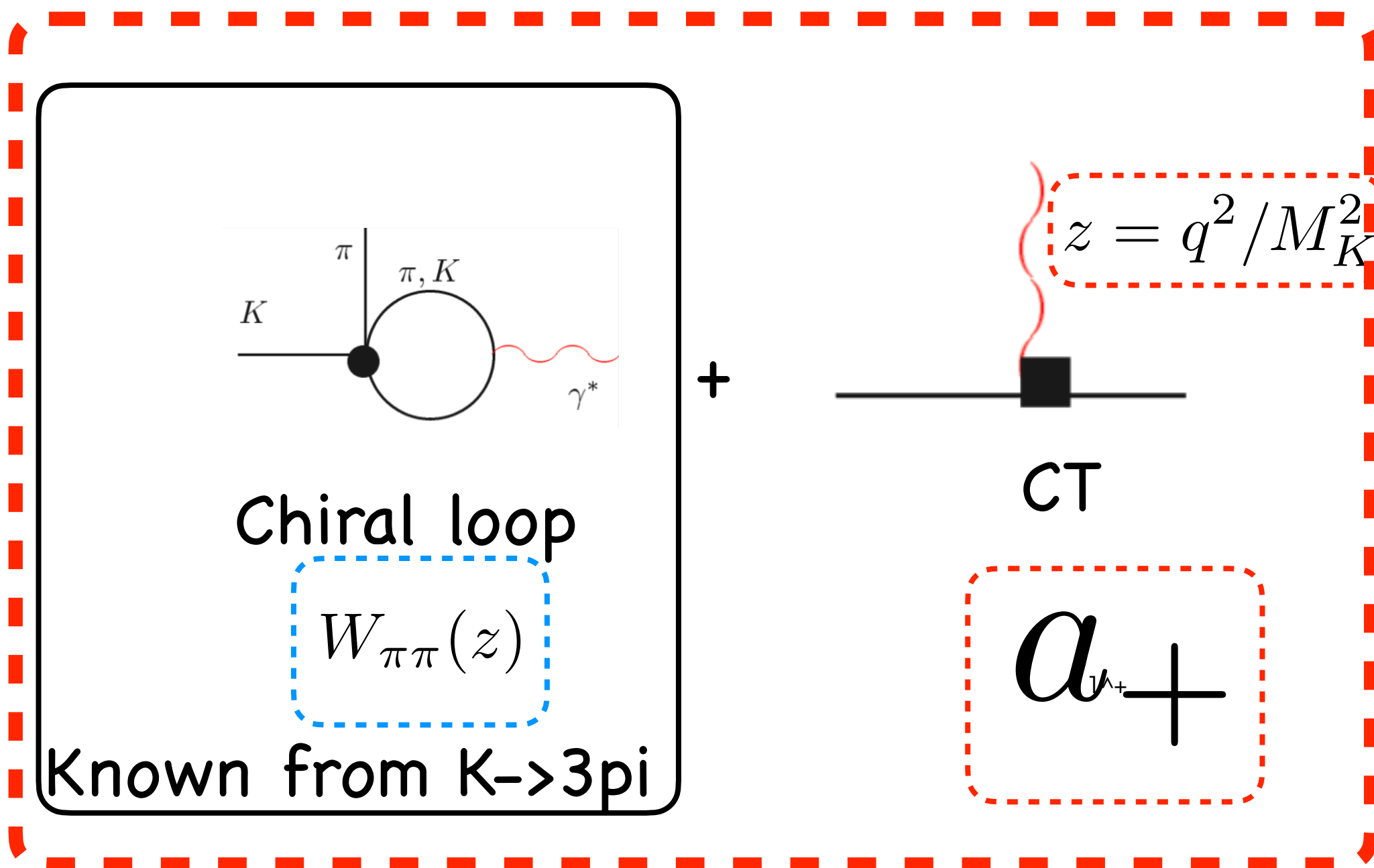


$\mathcal{O}(p^4)$ CHPT

'87 Ecker Pich de Rafael

$$K^+ \rightarrow \pi^+ l^+ l^-$$

$$k^2 = M_K^2, \quad p^2 = M_\pi^2, \quad q = k - p, \quad z = q^2/M_K^2, \quad r_\pi = M_\pi/M_K$$



KS No pion loop

$$\mathcal{O}(p^4) \quad a_+$$

Gauge and Lorentz invariance

$$\frac{W(z)}{(4\pi)^2} \left[z(k + p)^\mu - (1 - r_\pi^2) q^\mu \right]$$

$$\frac{d\Gamma}{dz} \sim \lambda^{3/2}(1, z, r_\pi^2) \sqrt{1 - 4\frac{r_\ell^2}{z}} \left(1 + 2\frac{r_\ell^2}{z} \right) |W(z)|^2$$

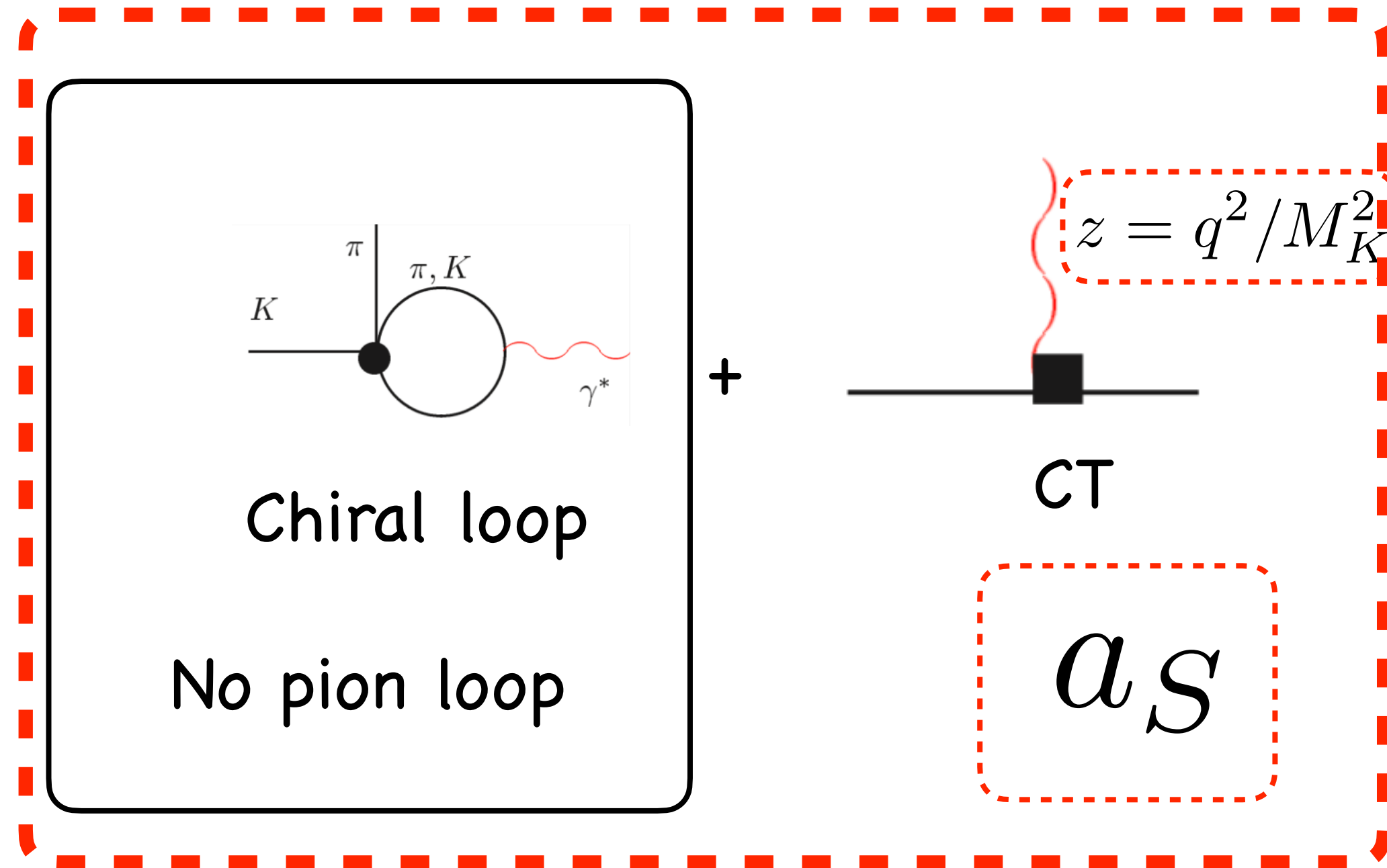
Data: the rate and spectrum not consistent with pheno

$\mathcal{O}(p^4)$ CHPT

'87 Ecker Pich de Rafael

$$K_S \rightarrow \pi^0 l^+ l^-$$

$$k^2 = M_K^2, \quad p^2 = M_\pi^2, \quad q = k - p, \quad z = q^2/M_K^2, \quad r_\pi = M_\pi/M_K$$



Gauge and Lorentz invariance

$$\frac{W(z)}{(4\pi)^2} \left[z(k+p)^\mu - (1 - r_\pi^2)q^\mu \right]$$

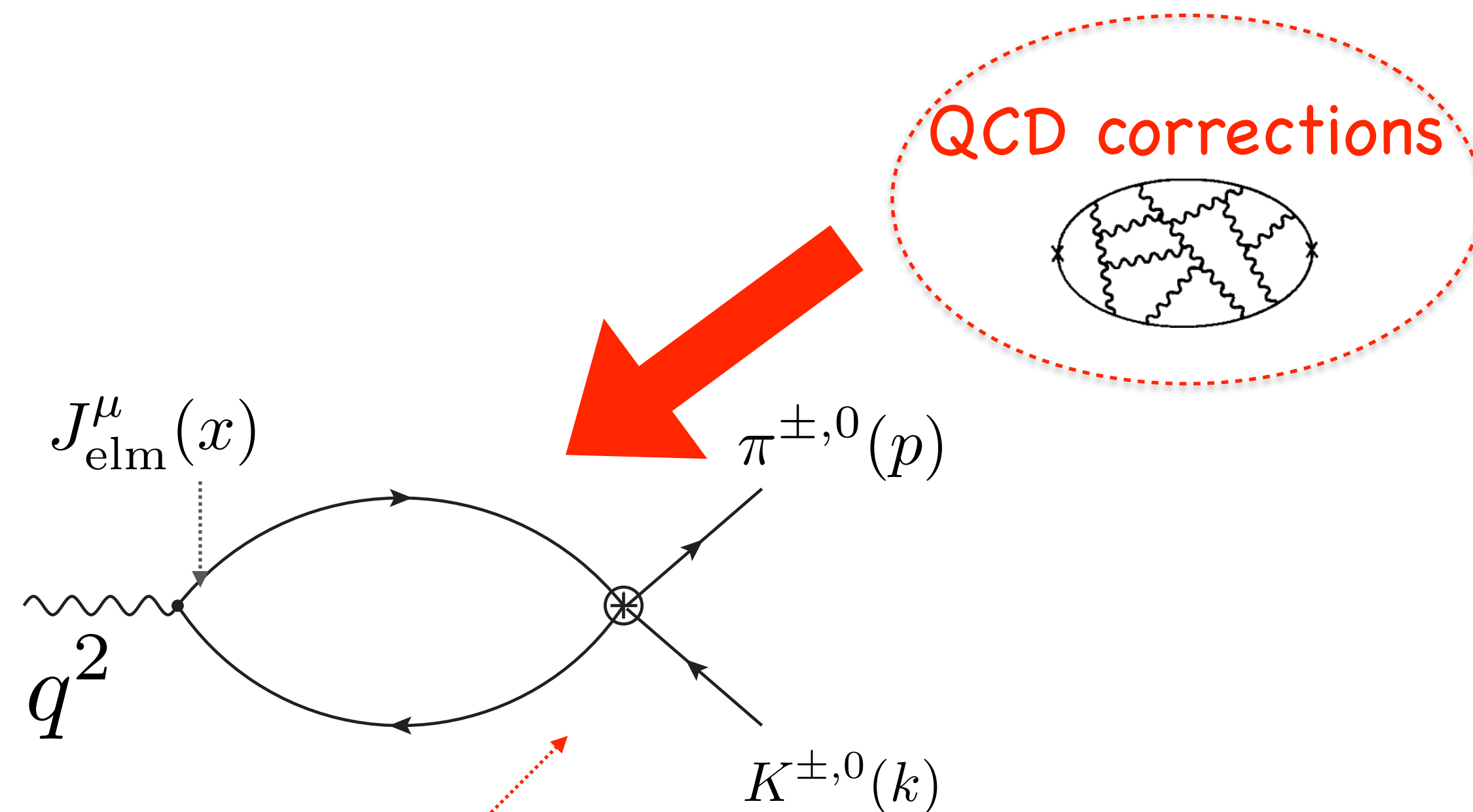
$$\frac{d\Gamma}{dz} \sim \lambda^{3/2}(1, z, r_\pi^2) \sqrt{1 - 4\frac{r_\ell^2}{z}} \left(1 + 2\frac{r_\ell^2}{z} \right) |W(z)|^2$$

KS No pion loop for KS, CT

$$\mathcal{O}(p^4) \quad a_S$$

General consideration on the form factor

Lorentz and gauge invariance tell us on the structure of the amplitude and ff



$$z = \frac{q^2}{M_K^2}$$

$$i \int d^4x e^{iqx} \langle \pi(p) | T \{ J_{\text{elm}}^\mu(x) \mathcal{L}_{\Delta S=1}(0) \} | K(k) \rangle = \frac{W(z)}{(4\pi)^2} [z(k+p)^\mu - (1 - r_\pi^2)q^\mu]$$

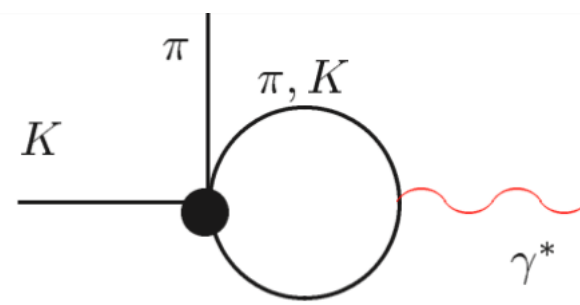
The integral at very short distances QCD quark loop

Pragmatic decision for $O(p^6)$ ff

GD,Ecker,Isidori,Portoles 98

$$W_i(z) = G_F M_K^2 W_i^{\text{pol}}(z) + W_i^{\pi\pi}(z)$$

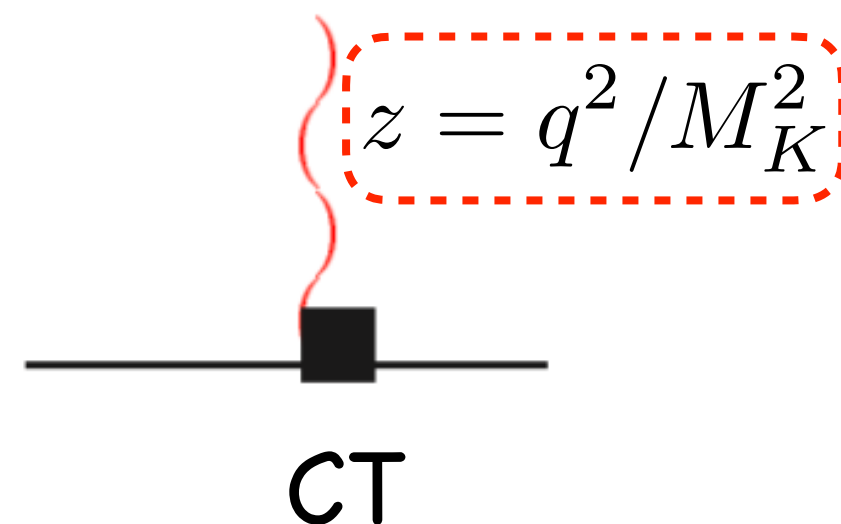
$$W_i^{\text{pol}}(z) = a_i + b_i z \quad (i = +, S)$$



Chiral loop

$$W_{\pi\pi}(z)$$

Known from $K \rightarrow 3\pi$



$$a_i + b_i z$$

$$a_i + b_i z$$

Determined expt.

Extremely good agreement with data few %

$$\text{LFUV test } a_+^{\mu\mu} - a_+^{ee}$$

Crivellin et al '16, Nazila et al '22

$K \rightarrow \pi\gamma^*$: Experimental situation

exp.	mode	number of events	a_+	b_+
BNL-E865	$K^+ \rightarrow \pi^+ e^+ e^-$	10 300	$-0.587(10)$	$-0.655(44)$
NA48/2	$K^\pm \rightarrow \pi^\pm e^+ e^-$	7 253	$-0.578(16)$	$-0.779(66)$
NA48/2	$K^\pm \rightarrow \pi^\pm \mu^+ \mu^-$	3120	$-0.575(39)$	$-0.813(145)$
NA62	$K^+ \rightarrow \pi^+ \mu^+ \mu^-$	27679	$-0.575(13)$	$-0.722(43)$

E. Cortina Gil et al. [NA62 Collaboration], JHEP 11, 011 (2022)

exp.	mode	number of events
NA48/1	$K_S \rightarrow \pi^0 e^+ e^-$	7
NA48/1	$K_S \rightarrow \pi^0 \mu^+ \mu^-$	6

$$a_S = -1.29(3.15) \quad b_S = +17.8(10.6)$$

or

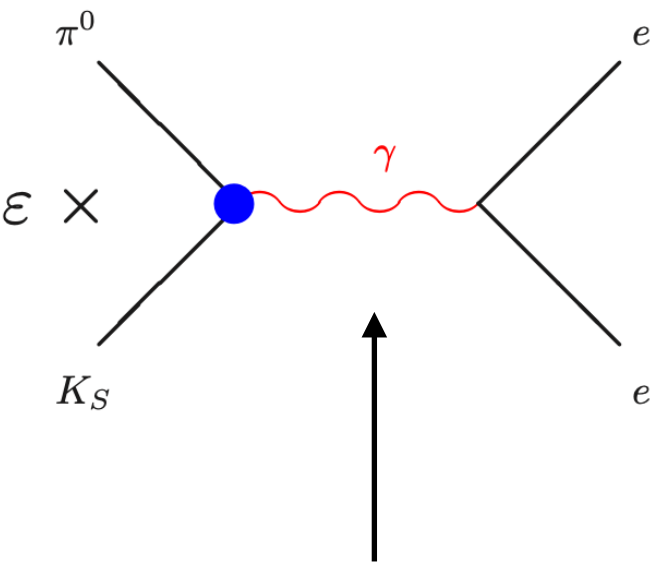
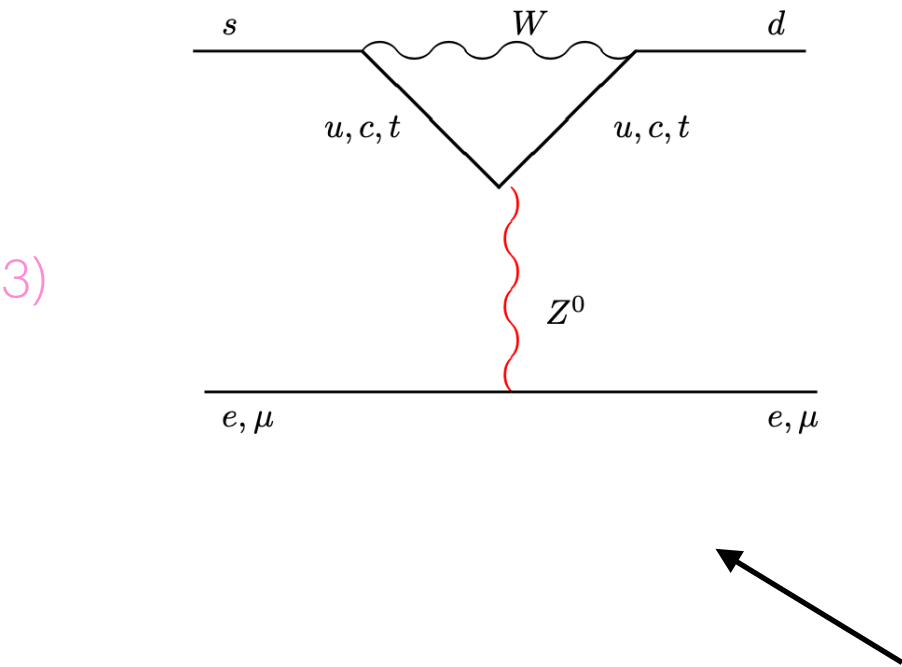
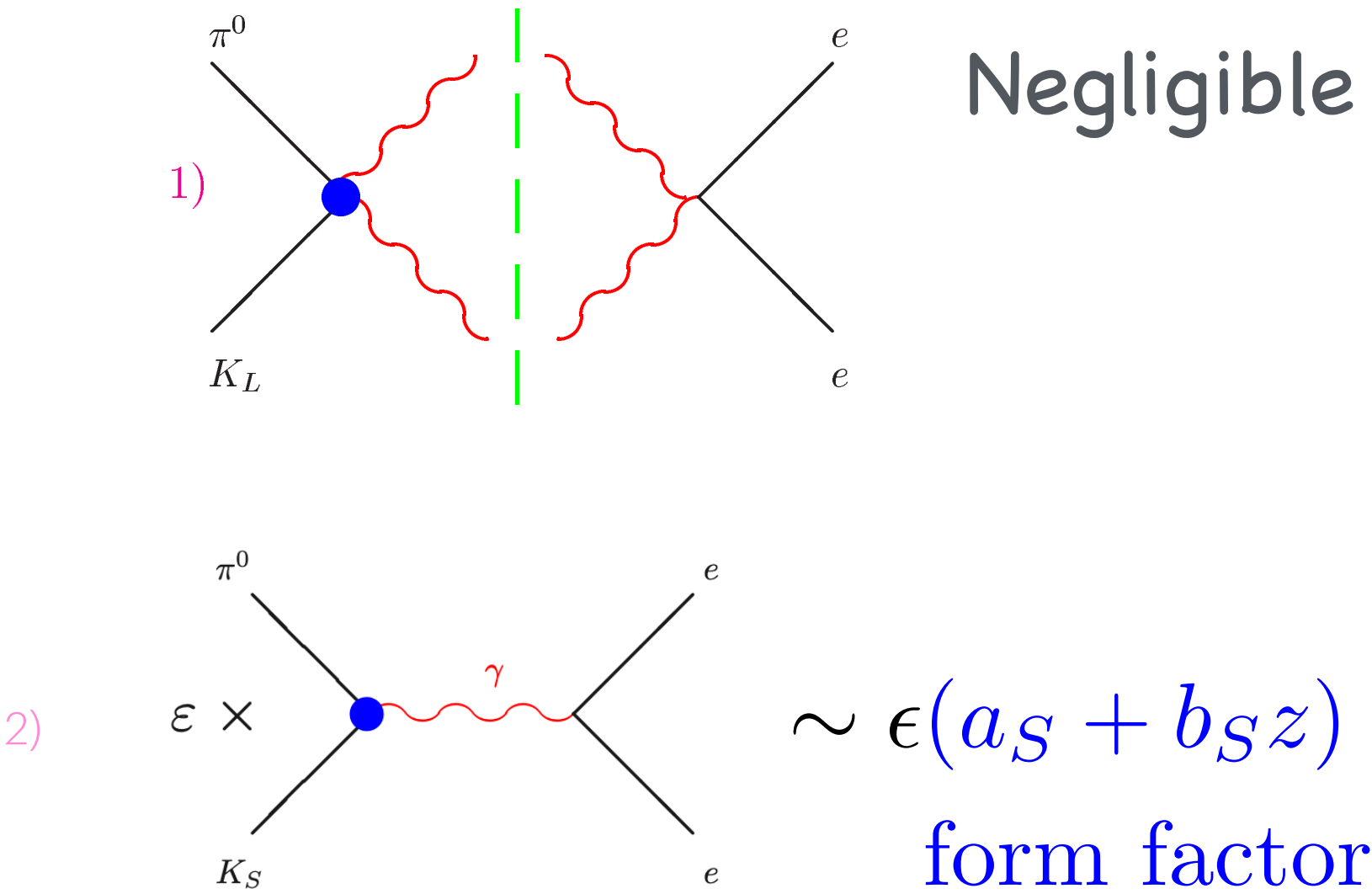
$$a_S = +1.28(3.16) \quad b_S = -17.6(10.6)$$

G. D'Ambrosio, D. Greynat, MK, JHEP 02, 049 (2019)

- Neither the sign of a_S nor the sign of a_S/b_S are fixed by data

Why interesting $K_L \rightarrow \pi^0 l^+ l^-$?

KOTO II , european strategy group



$$\text{BR}(K_L \rightarrow \pi^0 \ell \bar{\ell}) = (C_{\text{dir}}^\ell \pm C_{\text{int}}^\ell |a_S| + C_{\text{mix}}^\ell |a_S|^2 + C_{\gamma\gamma}^\ell) \cdot 10^{-12}$$

$$|a_S| = 1.20 \pm 0.20,$$

$$\text{BR}^{\text{SM}}(K_L \rightarrow \pi^0 e \bar{e}) = 3.46^{+0.92}_{-0.80} (1.55^{+0.60}_{-0.48}) \times 10^{-11}$$

$$\text{BR}^{\text{SM}}(K_L \rightarrow \pi^0 \mu \bar{\mu}) = 1.38^{+0.27}_{-0.25} (0.94^{+0.21}_{-0.20}) \times 10^{-11}$$

	C_{dir}^ℓ	C_{int}^ℓ	C_{mix}^ℓ	$C_{\gamma\gamma}^\ell$
$\ell = e$	$(4.62 \pm 0.24)(w_{7V}^2 + w_{7A}^2)$	$(11.3 \pm 0.3)w_{7V}$	14.5 ± 0.5	≈ 0
$\ell = \mu$	$(1.09 \pm 0.05)(w_{7V}^2 + 2.32w_{7A}^2)$	$(2.63 \pm 0.06)w_{7V}$	3.36 ± 0.20	5.2 ± 1.6

Buchalla et al 03, Isidori et al 06

Nazila et al 22, Knecht GD 24 , Hoferichter et al.24

$$w_{7A,7V} = \text{Im}(\lambda_t y_{7A,7V}) / \text{Im} \lambda_t$$

$$\text{BR}^{\text{exp}}(K_L \rightarrow \pi^0 e \bar{e}) < 28 \times 10^{-11} \quad \text{at 90\% CL.}$$

$$\text{BR}^{\text{exp}}(K_L \rightarrow \pi^0 \mu \bar{\mu}) < 38 \times 10^{-11} \quad \text{at 90\% CL.}$$

LARGE N QCD

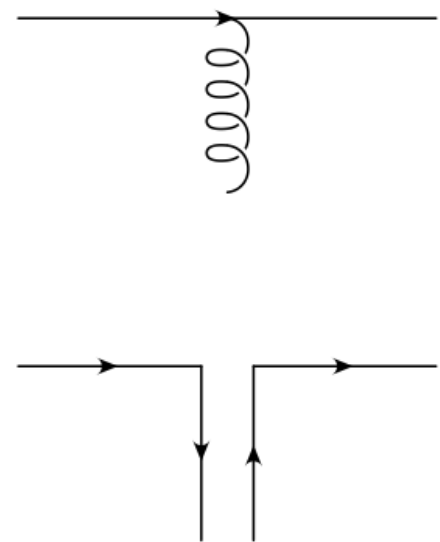
't Hooft, Witten, Coleman

$$\tilde{\mathcal{L}}_{QCD} = \sum_{q=u,d,s} \bar{q} \gamma^\mu \left(i \partial_\mu - g_s \frac{\lambda_a}{2} G_\mu^a \right) q - \frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu}$$

QCD properties OK: asympt. Freedom

$$\mu \frac{dg}{d\mu} = -b_0 \frac{g^3}{16\pi^2} + \mathcal{O}(g^5), \quad b_0 = \frac{11}{3}N - \frac{2}{3}N_F$$

Confinement assumed



$$8 + 1 = 3 + \bar{3}$$

Large N => correct expansion parameter? Geometric interpretation of this expansion: planar diagrams

Question in Large N: The leading order term in N of your amplitude

't Hooft model QCD₂

Exactly solved

Gross Neveu model

SSB+ mass Gap

Successes:

Zweig's rule (suppression of gluon exchange decays)

VMD, computability..

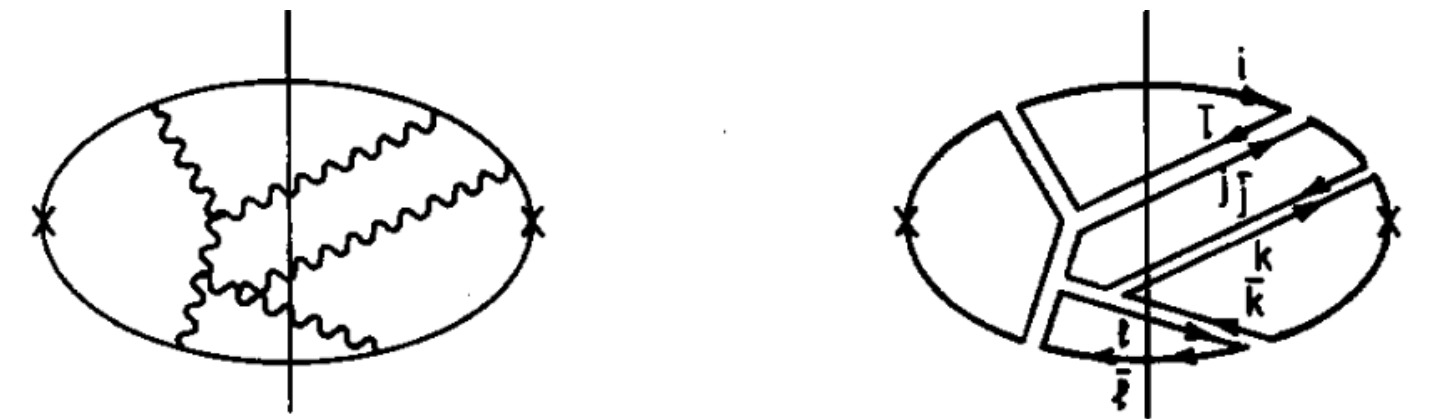
Witten '79 Large N QCD

Section 3

- Mesons are free, stable, non-interacting, the number of states infinite
- Meson decay amplitudes $\mathcal{O}(1/\sqrt{N})$
- One meson exchange leading (see two point function with J bilinear)

$$\langle JJ \rangle = \sum_n \begin{array}{c} a_n \quad \quad a_n \\ \times \text{---} \times \\ \quad \uparrow \\ \quad \frac{1}{k^2 - m_n^2} \end{array}$$

fig. 21. $\langle JJ \rangle$ represented as a sum of one-meson poles.



$$\langle J(q)J(-q) \rangle = \sum_n \frac{a_n^2}{q^2 - m_n^2}.$$

$$\langle J(q)J(-q) \rangle \underset{q^2 \rightarrow \infty}{\sim} \log q^2.$$

$K_S \rightarrow \pi^0 l^+ l^-$: determination of the form factor

$$i \int d^4x e^{iqx} \langle \pi(p) | T \{ J_{\text{elm}}^\mu(x) \mathcal{L}_{\Delta S=1}(0) \} | K(k) \rangle = \frac{W(z)}{(4\pi)^2} \left[z(k+p)^\mu - (1 - r_\pi^2) q^\mu \right]$$

$$k^2 = M_K^2, \quad p^2 = M_\pi^2, \quad q = k - p, \quad z = q^2/M_K^2, \quad r_\pi = M_\pi/M_K,$$

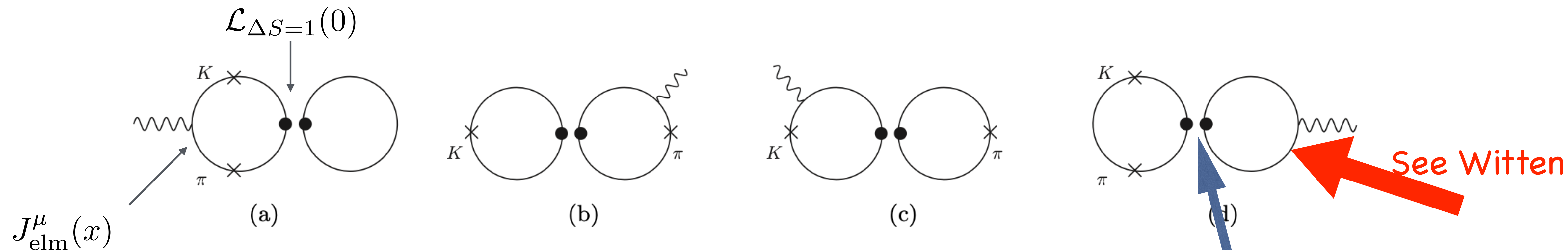


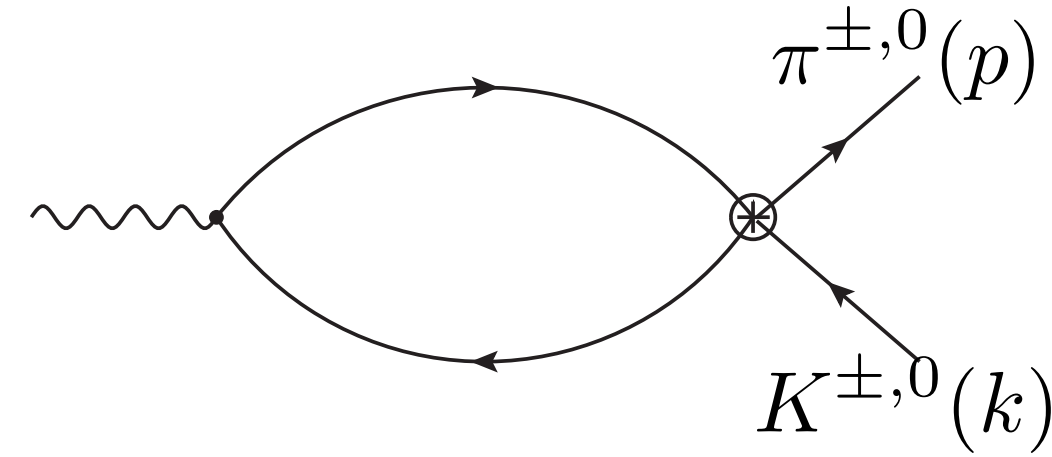
Figure 1: Potential diagrams contributing at leading order in large N_c (all at the order $(N_c)^2/(\sqrt{N_c})^2 = N_c$). The dots represent the effective $\Delta S = 1$ operators.

$$\mathcal{L}_{\text{non-lept}}^{|\Delta S|=1} = -\frac{G_F}{\sqrt{2}} V_{us} V_{ud} \sum_{I=1}^6 C_I(\nu) Q_I(\nu) + \text{H.c.}$$

$$Q_1 = (\bar{s}^i u_j)_{V-A} (\bar{u}^j d_i)_{V-A}, \quad Q_2 = (\bar{s}^i u_i)_{V-A} (\bar{u}^j d_j)_{V-A}$$

Large Nc results for $K_S \rightarrow \pi^0 l^+ l^-$

GD Knecht, Neshatpour '24



$$W_i(z) = G_F M_K^2 W_i^{\text{pol}}(z) + W_i^{\pi\pi}(z)$$

$$W_i^{\text{pol}}(z) = a_i + b_i z \quad (i = +, S)$$

TH $\text{Br}(K_S \rightarrow \pi^0 e^+ e^-)|_{m_{ee} > 165 \text{ MeV}} = 2.9(1.0) \cdot 10^{-9}$

NA48/1 $\text{Br}(K_S \rightarrow \pi^0 e^+ e^-)|_{m_{ee} > 165 \text{ MeV}} = (3.0_{-1.2}^{+1.5} \pm 0.2) \cdot 10^{-9}$

$$\text{Br}(K_S \rightarrow \pi^0 e^+ e^-) = 5.1(1.7) \cdot 10^{-9}$$

Large Nc predictions => Wilson coefficients + hadr. parameters

TH

$$\text{Br}(K_S \rightarrow \pi^0 \mu^+ \mu^-) = 1.3(0.4) \cdot 10^{-9}$$

Conclusions

- We have focused maybe too much on

$$\Delta I = 1/2 \text{ rule}$$

$$\mathcal{L}_{\Delta S=1} = \mathcal{L}_{\Delta S=1}^2 + \mathcal{L}_{\Delta S=1}^4 + \cdots = G_8 F^4 \underbrace{\langle \lambda_6 D_\mu U^\dagger D^\mu U \rangle}_{K \rightarrow 2\pi/3\pi} + G_8 F^2 \underbrace{\sum_i N_i W_i}_{K^+ \rightarrow \pi^+ \gamma \gamma, K \rightarrow \pi l^+ l^-} + \cdots$$

$$K^+ \rightarrow \pi^+ l^+ l^-$$

π	2π	3π	N_i
$\pi^+\gamma^*$ $\pi^0\gamma^*(S)$ $\pi^+\gamma\gamma$	$\pi^+\pi^0\gamma^*$ $\pi^0\pi^0\gamma^*(L)$ $\pi^+\pi^0\gamma\gamma$ $\pi^+\pi^-\gamma\gamma(S)$ $\pi^+\pi^0\gamma$ $\pi^+\pi^-\gamma(S)$		$N_{14}^r - N_{15}^r$ $K^+ \rightarrow \pi^+ l^+ l^-$ $2N_{14}^r + N_{15}^r$ $K_S \rightarrow \pi^0 l^+ l^-$ $N_{14} - N_{15} - 2N_{18}$ "
	$\pi^+\pi^-\gamma^*(L)$ $\pi^+\pi^-\gamma^*(S)$ $\pi^+\pi^0\gamma^*$	$\pi^+\pi^+\pi^-\gamma$ $\pi^+\pi^0\pi^0\gamma$ $\pi^+\pi^-\pi^0\gamma(L)$ $\pi^+\pi^-\pi^0\gamma(S)$	$N_{14} - N_{15} - N_{16} - N_{17}$ "
	$\pi^+\pi^-\gamma(L)$		$7(N_{14}^r - N_{16}^r) + 5(N_{15}^r + N_{17})$ $N_{14}^r - N_{15}^r - 3(N_{16}^r - N_{17})$ $N_{14}^r - N_{15}^r - 3(N_{16}^r + N_{17})$ $N_{14}^r + 2N_{15}^r - 3(N_{16}^r - N_{17})$
	$\pi^+\pi^0\gamma$	$\pi^+\pi^-\pi^0\gamma(S)$ $\pi^+\pi^+\pi^-\gamma$ $\pi^+\pi^0\pi^0\gamma$ $\pi^+\pi^-\pi^0\gamma(S)$ $\pi^+\pi^-\pi^0\gamma(L)$	$N_{29} + N_{31}$ "
			$3N_{29} - N_{30}$ $5N_{29} - N_{30} + 2N_{31}$ $6N_{28} + 3N_{29} - 5N_{30}$

- $K_{S,L} \rightarrow \mu^+ \mu^- (e^+ e^-)$ 3-point function

LHCB

Conclusions II

$$K^+ \rightarrow \pi^+ l^+ l^-$$

3-point function
 $K_{S,L} \rightarrow \mu^+ \mu^- (e^+ e^-)$

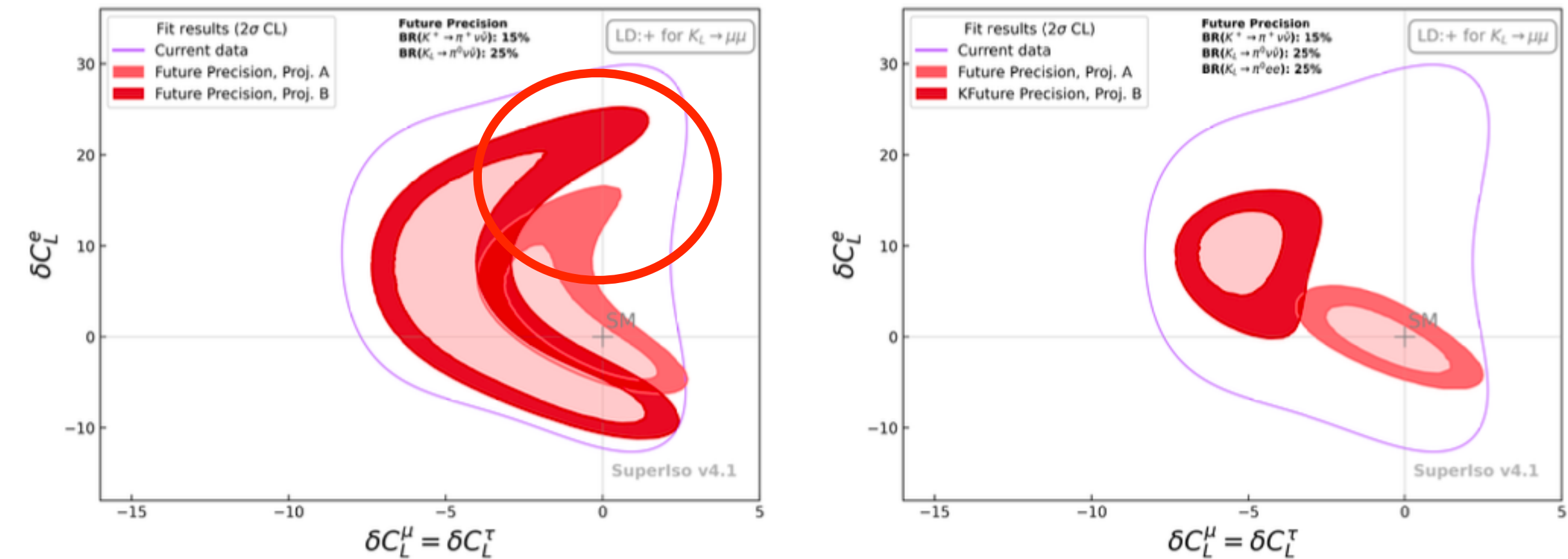
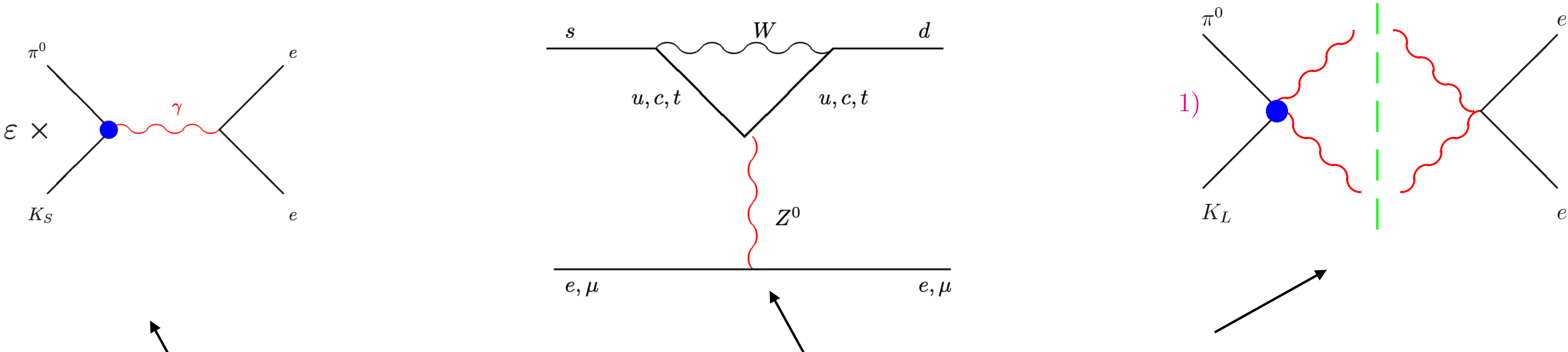


Figure 5: BSM parameter space for Wilson coefficients in scenarios with LFU violation where the NP effects for electrons are different from the those for muons and taus [18, 19]. Left: Impact on allowed parameter space from measurements of the golden channels $K \rightarrow \pi \nu \bar{\nu}$ from NA62 and KOTO II with the expected final precision. Right: Impact on the parameter space by the inclusion in the fits of a measurement of the $K_L \rightarrow \pi^0 e^+ e^-$ branching ratio with 25% precision.

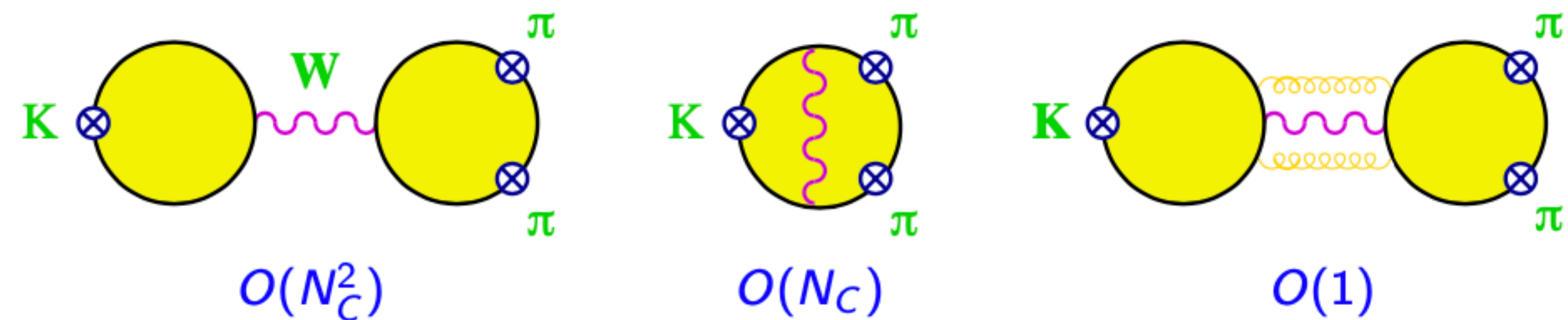


$$\text{Br}(K_L \rightarrow \pi^0 \ell^+ \ell^-) = 10^{-12} \left[C_{\text{mix}}^{(\ell)} + C_{\text{int}}^{(\ell)} \frac{\text{Im } \lambda_t}{10^{-4}} + C_{\text{dir}}^{(\ell)} \left(\frac{\text{Im } \lambda_t}{10^{-4}} \right)^2 + C_{\gamma^* \gamma^*}^{(\ell)} \right]$$

$$C_{\text{int}}^{(e)} = +7.8(2.6) \frac{y_{7V}}{\alpha}, \quad C_{\text{int}}^{(\mu)} = +1.9(0.6) \frac{y_{7V}}{\alpha}$$

$$K \rightarrow 2\pi$$

Weak Currents Factorize at Large N_c



- Large sub

- Failing understanding $\Delta I = 1/2$ rule

- Lattice

Outline

- Motivations for $K \rightarrow \pi l l$ $K^\pm \rightarrow \pi^\pm l^+ l^-$ $K_S \rightarrow \pi^0 l^+ l^-$
- $K \rightarrow \pi l l$ phenomenology
- Large N QCD: the dream '70s '80s '90s 't Hooft, Witten, Coleman
- Ideas, Successes, some undelivered messages $K \rightarrow \pi\pi : \Delta I = \frac{1}{2}$ rule ϵ'
—
- $K \rightarrow \pi l l$ phenomenology and large N
- conclusions

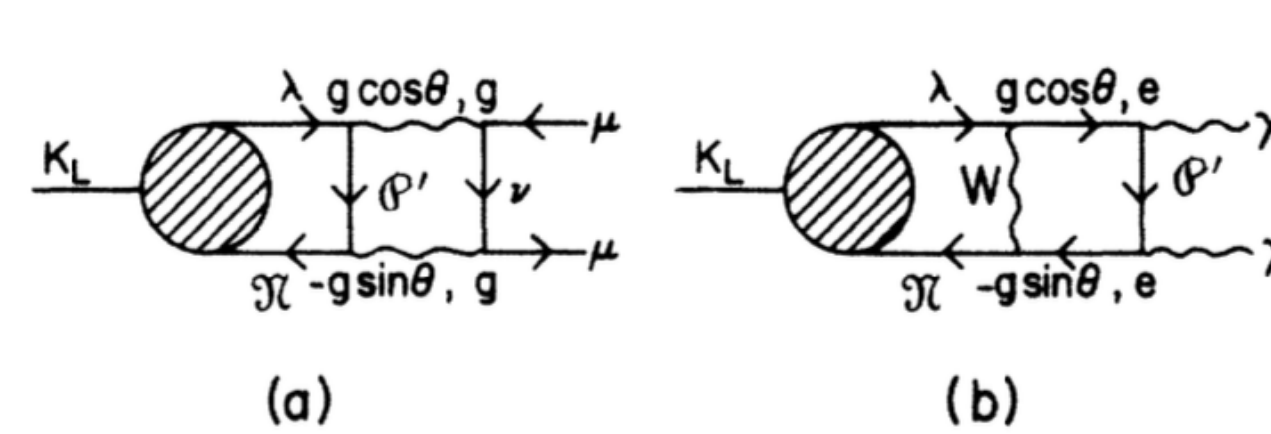


FIG. 3. Diagrams with the θ' -quark line which suppress the contributions of the diagrams in Fig. 1.

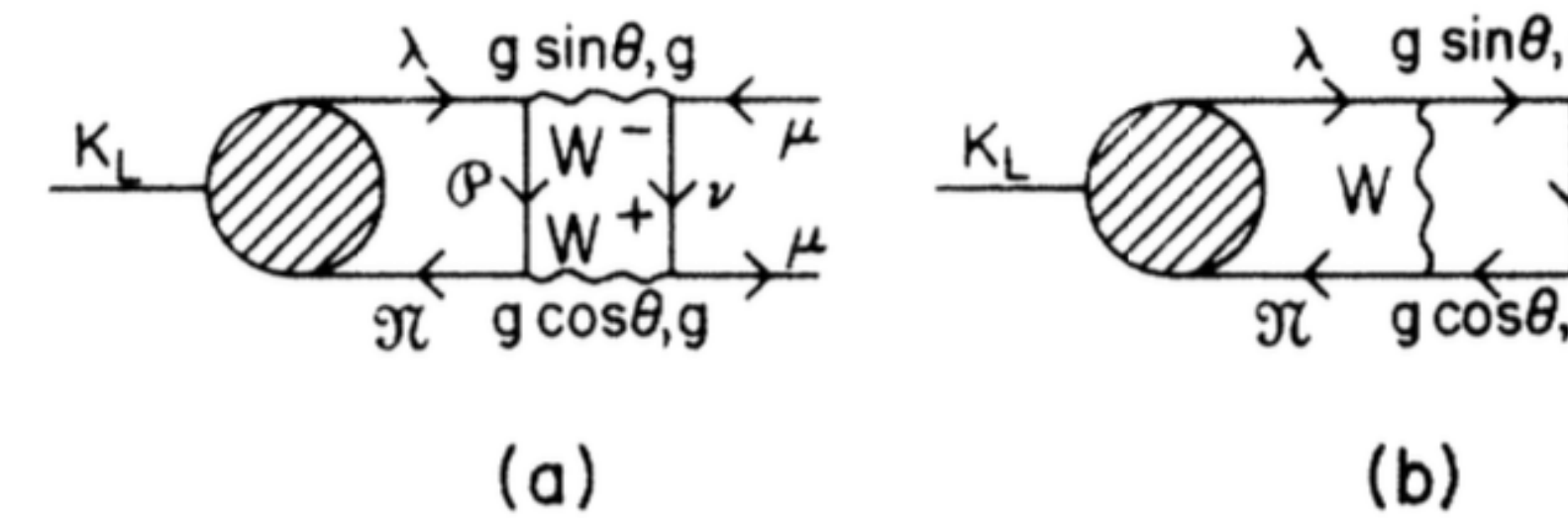
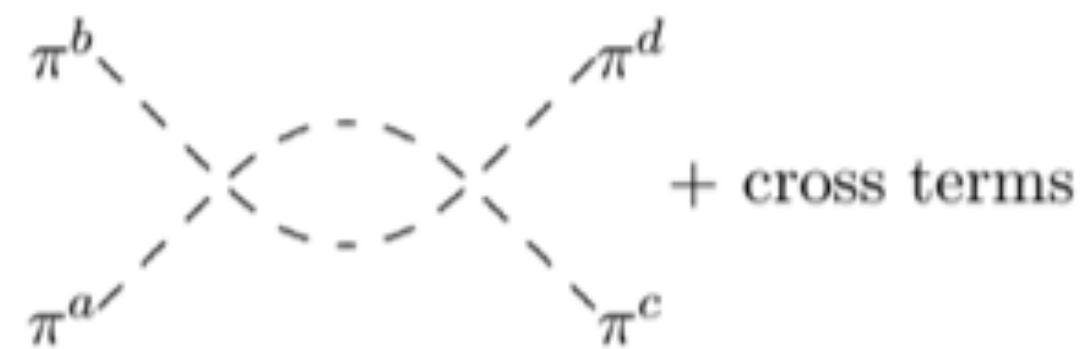


FIG. 1. Important diagrams for (a) $K_L \rightarrow \mu\bar{\mu}$ and $K_L \rightarrow \gamma\gamma$.

- GIM Gaillard Lee “3 months to understand the calculation, one day to do it
- Weinberg Effective field theories , chiral loops ‘76



•

The kaon community requests to

- protect and amplify the European kaon-physics programme, exploring opportunities for
 - $K^+ \rightarrow \pi^+ \nu \bar{\nu}$
 - $K_{S,L} \rightarrow \mu^+ \mu^-$ decay and interference,
- enable European contributions for KOTO II for $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \ell^+ \ell^-$, spanning both analysis and hardware development,
.....
- maintain the European leadership on theory computations for kaon physics (phenomenology, dispersion theory, effective theory and lattice QCD, including high-performance computing).

EU strategy group
Lazzeroni

The contribution "Kaon Physics: A Cornerstone for Future Discoveries" has been drafted by:
M. Bordone, A. Ceccucci, A. Dery, M. Gorbahn, E. Goudzovski, M. Hoferichter, A. Jüttner, C. Lazzeroni,
Z. Ligeti, D. Martinez, F. Mahmoudi, R. Marchevski, M. Moulson, G. Ruggiero.

Year	Main object
1	Beam line survey
2	Construction of the rest of the detector
3-6	Phase I: Physics run for mainly $K_L \rightarrow \pi^0 \nu \bar{\nu}$
7	Single event sensitivity will reach 8.5×10^{-13} for the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ search
8	Detector upgrade
9-12	Phase II: Physics run mainly for $K_L \rightarrow \pi^0 \ell^+ \ell^-$ with an optimized setup
13	End of Phase II

Moulson

QCD theoretical tools

- analytic calculation 't Hooft, large N_c (it explains basic phenomenological facts of QCD, i.e. Zweig's rule) many implications: Skyrme model, VMD, Maldacena
- G. Parisi, '80s lattice: can we predict from QCD the proton mass at 10% level?
- Precise calculation of low energy QCD?

π	2π	3π	N_i
$\pi^+\gamma^*$ $\pi^0\gamma^* (S)$ $\pi^+\gamma\gamma$	$\pi^+\pi^0\gamma^*$ $\pi^0\pi^0\gamma^* (L)$ $\pi^+\pi^0\gamma\gamma$ $\pi^+\pi^-\gamma\gamma (S)$ $\pi^+\pi^0\gamma$ $\pi^+\pi^-\gamma (S)$ $\pi^+\pi^-\gamma^* (L)$ $\pi^+\pi^-\gamma^* (S)$ $\pi^+\pi^0\gamma^*$	 $\pi^+\pi^+\pi^-\gamma$ $\pi^+\pi^0\pi^0\gamma$ $\pi^+\pi^-\pi^0\gamma (L)$ $\pi^+\pi^-\pi^0\gamma (S)$	$N_{14}^r - N_{15}^r$ $K^+ \rightarrow \pi^+ l^+ l^-$ $2N_{14}^r + N_{15}^r$ $K_S \rightarrow \pi^0 l^+ l^-$ $\underline{N_{14} - N_{15} - 2N_{18}}$,, $N_{14} - N_{15} - N_{16} - N_{17}$,, ,, $7(N_{14}^r - N_{16}^r) + 5(N_{15}^r + N_{17})$ $N_{14}^r - N_{15}^r - 3(N_{16}^r - N_{17})$ $N_{14}^r - N_{15}^r - 3(N_{16}^r + N_{17})$ $N_{14}^r + 2N_{15}^r - 3(N_{16}^r - N_{17})$
	$\pi^+\pi^-\gamma (L)$ $\pi^+\pi^0\gamma$	$\pi^+\pi^-\pi^0\gamma (S)$ $\pi^+\pi^+\pi^-\gamma$ $\pi^+\pi^0\pi^0\gamma$ $\pi^+\pi^-\pi^0\gamma (S)$ $\pi^+\pi^-\pi^0\gamma (L)$	$N_{29} + N_{31}$,, $3N_{29} - N_{30}$ $5N_{29} - N_{30} + 2N_{31}$ $6N_{28} + 3N_{29} - 5N_{30}$

$$\mathcal{L}_{\Delta S=1} = \mathcal{L}_{\Delta S=1}^2 + \mathcal{L}_{\Delta S=1}^4 + \cdots = G_8 F^4 \underbrace{\langle \lambda_6 D_\mu U^\dagger D^\mu U \rangle}_{K \rightarrow 2\pi/3\pi} + \underbrace{G_8 F^2 \sum_i N_i W_i}_{K^+ \rightarrow \pi^+ \gamma\gamma, K \rightarrow \pi l^+ l^-} + \cdots$$

$$K_L(p) \rightarrow \pi^0(p_3)\gamma(q_1)\gamma(q_2)$$

$$\text{Lorentz + gauge invariance} \Rightarrow M \sim \begin{matrix} A(y, z) & B(y, z) \\ \gamma\gamma & \gamma\gamma \\ J=0 & \text{D-wave too} \\ F^{\mu\nu}F_{\mu\nu} & F^{\mu\nu}F_{\mu\lambda}\partial_\nu K_L\partial^\lambda\pi^0 \end{matrix}$$

$$y=p\cdot(q_1-q_2)/m_K^2, \quad z=(q_1+q_2)^2/m_K^2$$

$$r_\pi=m_\pi/m_K$$

- $\frac{d^2\Gamma}{dydz} \sim z^2 |A + B|^2 + \left(y^2 - \left(\frac{(1+r_\pi^2-z)^2}{4} - r_\pi^2 \right) \right)^2 |B|^2 \quad S, B$
- Different gauge structure $\Rightarrow B \neq 0$ at $z \rightarrow 0$ (collinear photons).

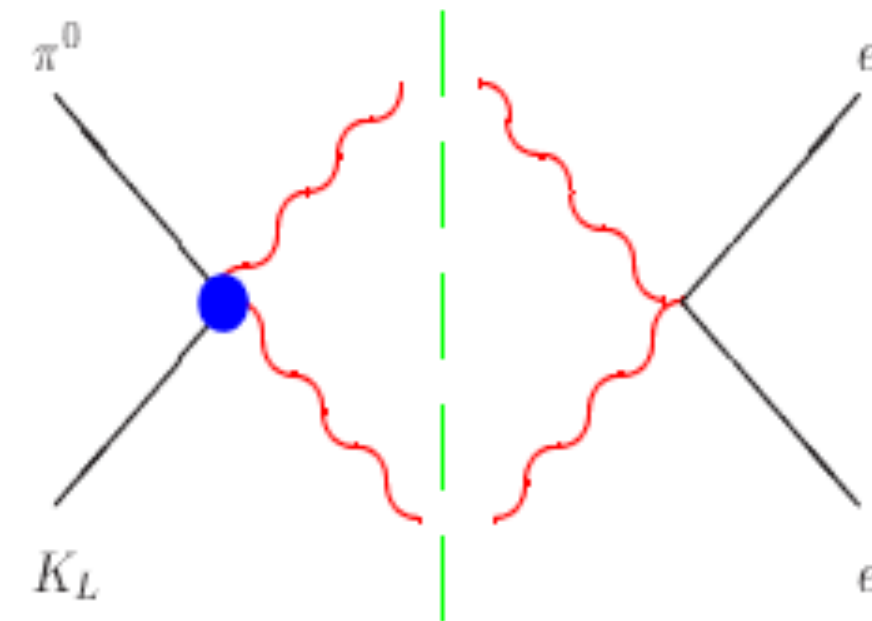
Crucial role in $K_L \rightarrow \pi^0 e^+ e^-$

A suppressed by m_e/m_K

B is not

Morozumi et al, Flynn Randall

Sehgal Heiliger, Ecker et al., Donoghue et al.



KTeV and NA48 **not only** ϵ'

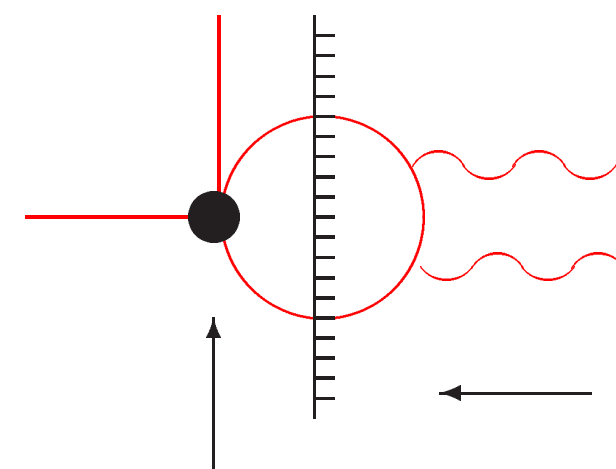
Actually the study of unit. cut was crucial to i) to bring **agreement** expt vs Theory in $K_L \rightarrow \pi^0 \gamma \gamma$ and ii) show that $K_L \rightarrow \pi^0 e e$ CP conserving was negligible

3 CT's

$$F_{\mu\nu} F^{\mu\alpha} \partial_\alpha K_L \partial^\nu \pi^0$$

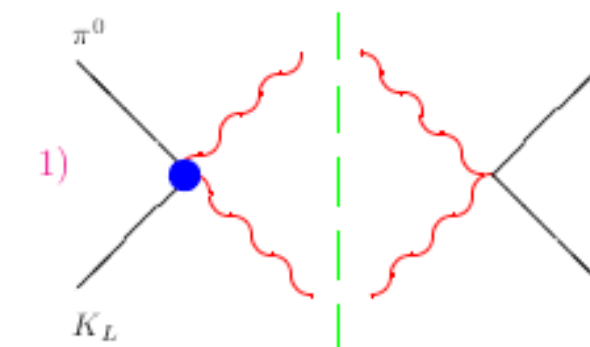
$$F^2 \partial K_L \partial \pi^0$$

$$F^2 m_K^2 K_L \pi^0$$



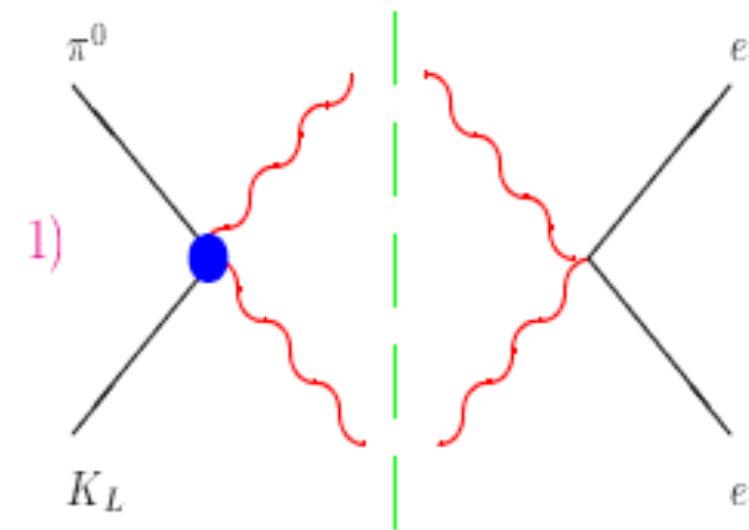
Full description of unitarity cut

$$A(K \rightarrow 3\pi) = a + b Y + c Y^2 + d X^2$$



$K_L \rightarrow \pi^0 e^+ e^-$: summary

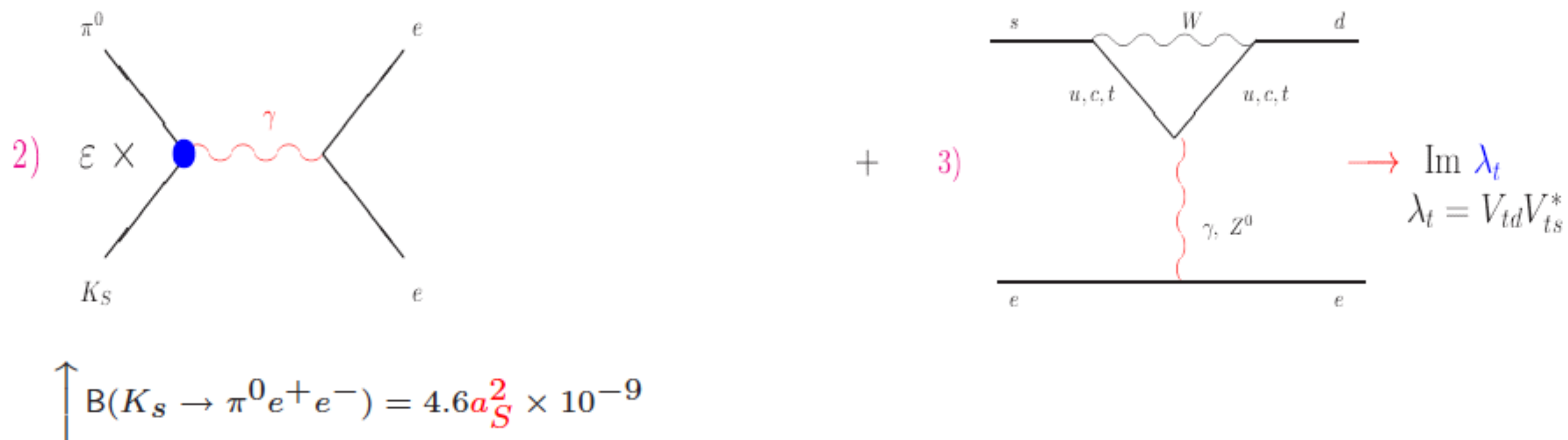
$$\text{Br}(K_L \rightarrow \pi^0 e^+ e^-) \leq 2.8 \cdot 10^{-10} \text{ at 90\% CL} \quad \text{KTeV}$$



CP conserving NA48

$$\text{Br}(K_L \rightarrow \pi^0 e^+ e^-) < 3 \cdot 10^{-12}$$

$$V-A \otimes V-A \Rightarrow \langle \pi^0 e^+ e^- | (\bar{s}d)_{V-A} (\bar{e}e)_{V-A} | K_L \rangle \text{ violates CP}$$



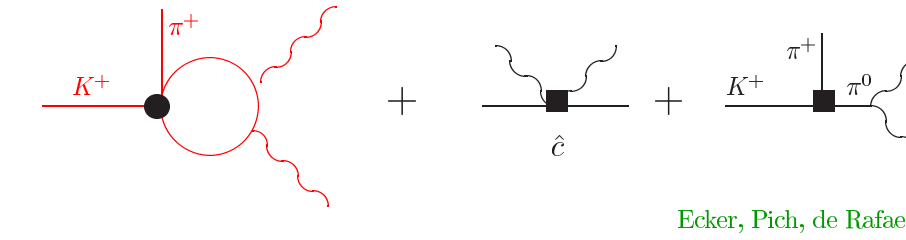
Possible large interference: $a_S < -0.5$ or $a_S > 1$; short distance probe even for a_S large

$$|2) + 3)|^2 = \left[15.3 a_S^2 - 6.8 \frac{\text{Im} \lambda_t}{10^{-4}} a_S + 2.8 \left(\frac{\text{Im} \lambda_t}{10^{-4}} \right)^2 \right] \cdot 10^{-12}$$

$$[17.7 \pm 9.5 + 4.7] \cdot 10^{-12}$$

$K^+ \rightarrow \pi^+ \gamma \gamma$ NA48/2 + NA62 ('14)

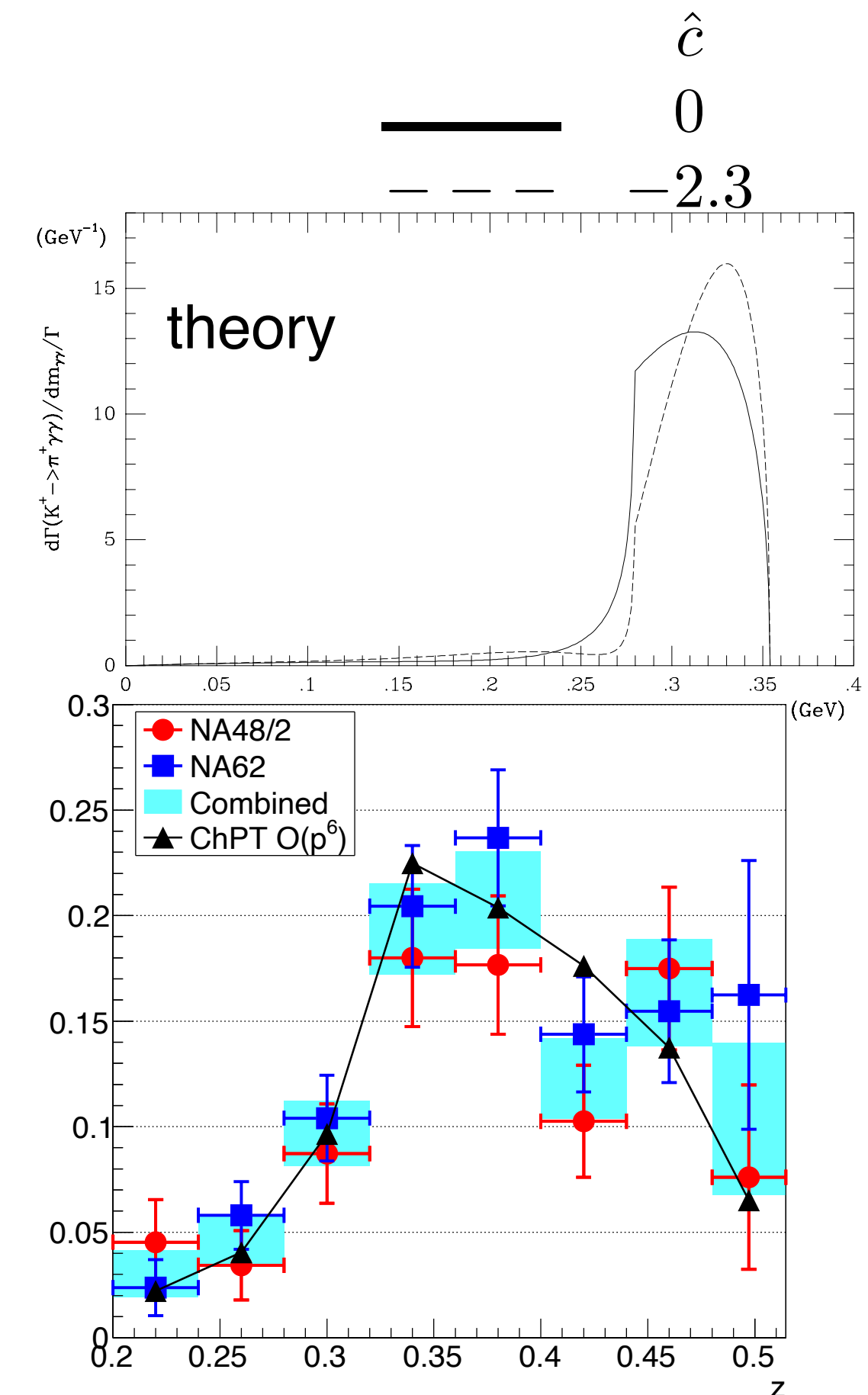
Auxiliary channel useful to assess the CP conserving contribution to $K_L \rightarrow \pi^0 ee$



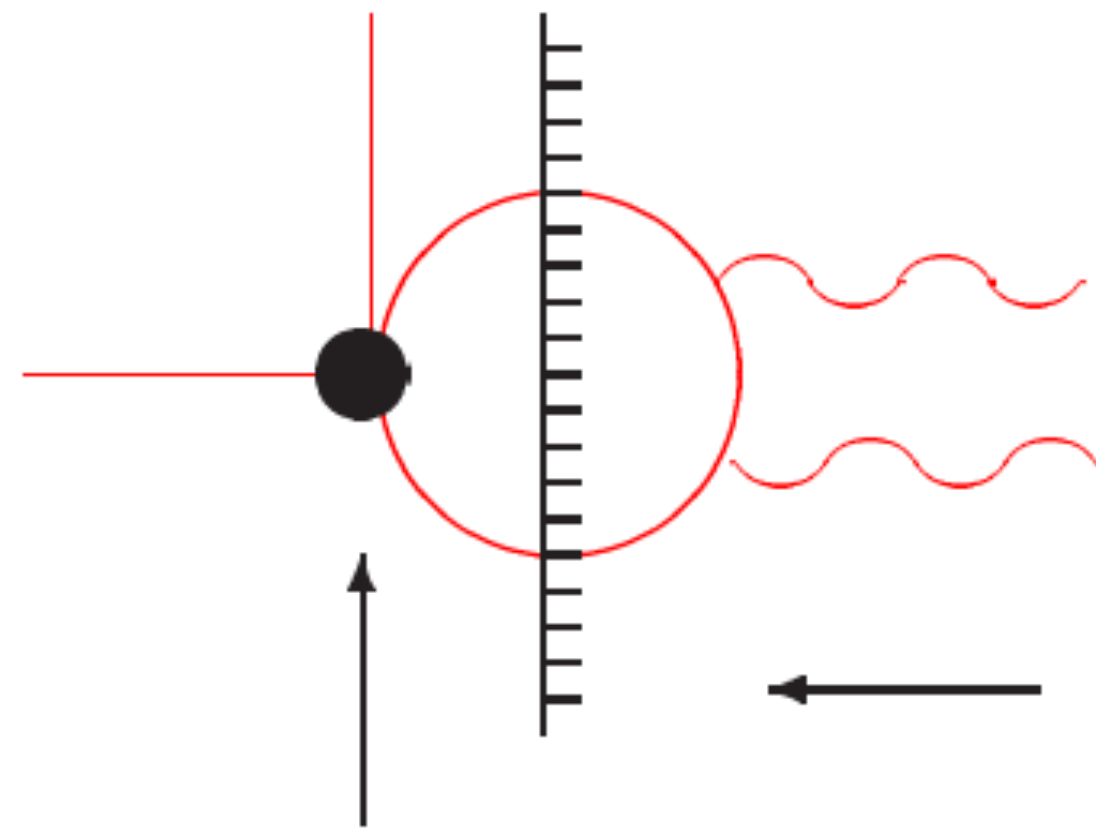
Final 381 evts NA48/2 + NA62
during a 3-day special NA48/2 run in
2004 and a 3-month NA62 run in 2007

$$B = (1.003 \pm 0.051_{\text{stat}} \pm 0.024_{\text{syst}}) \cdot 10^{-6}$$

$$\hat{c} = 1.86 \pm 0.26$$



$K^+ \rightarrow \pi^+ \gamma \gamma$ NA62 sensitivity



Full description of unitarity cut

$$A(K \rightarrow 3\pi) = a + b Y + c Y^2 + d X^2$$

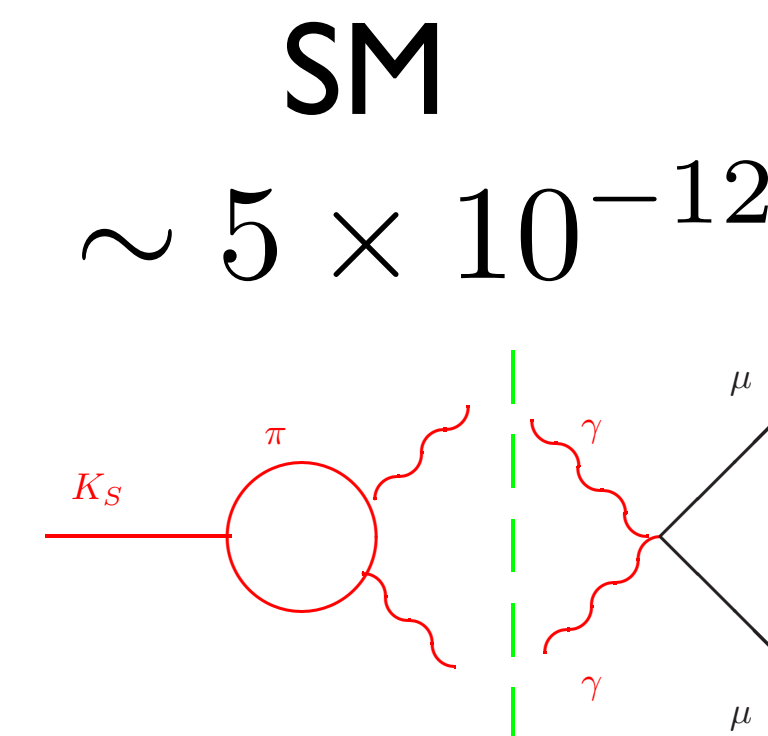
This decay $K^+ \rightarrow \pi^+ \gamma \gamma$: The error obtained in the form factor ($\hat{c}$) is dominated by the expt K-
> 3pi error in the quadratic slope !

$$K_S \rightarrow \mu \bar{\mu} \quad \text{LHCB}$$

After 40 years improvement by 3
orders of magnitudes from LHCB

$$B(K_S \rightarrow \mu \bar{\mu}) < 11 \times 10^{-9} \\ \text{95\% CL}$$

Isidori Underdorfer



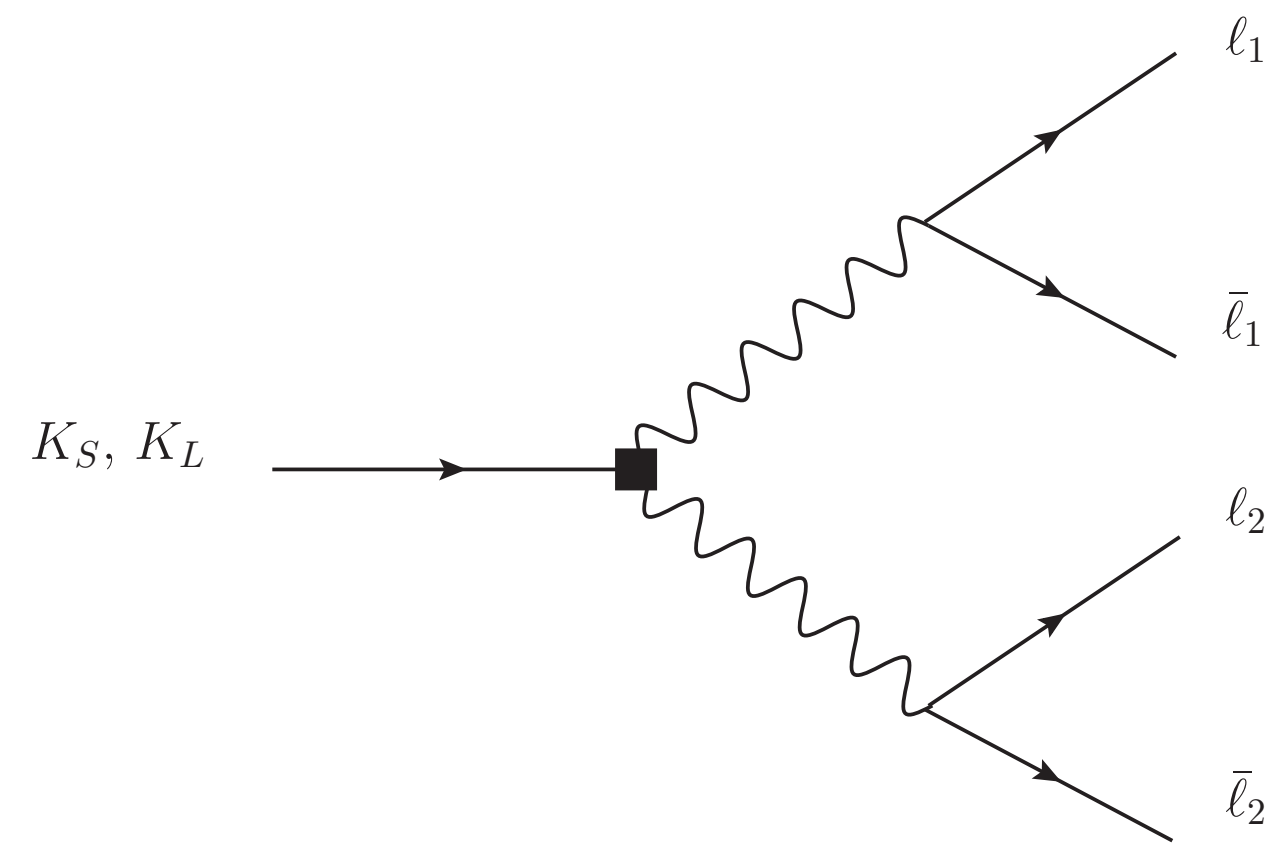
SD $1.5 \cdot 10^{-12}$

NP $1.5 \cdot 10^{-11}$
Allowed

NP Limits from
CPviol in $K_L \rightarrow \mu \mu$

Other interesting channels

$$\begin{array}{lll} K_S \rightarrow \mu\mu\mu\mu & - & \text{SM LD} \sim 2 \times 10^{-14} \\ K_S \rightarrow ee\mu\mu & - & \sim 10^{-11} \\ K_S \rightarrow eeee & - & \sim 10^{-10} \end{array}$$



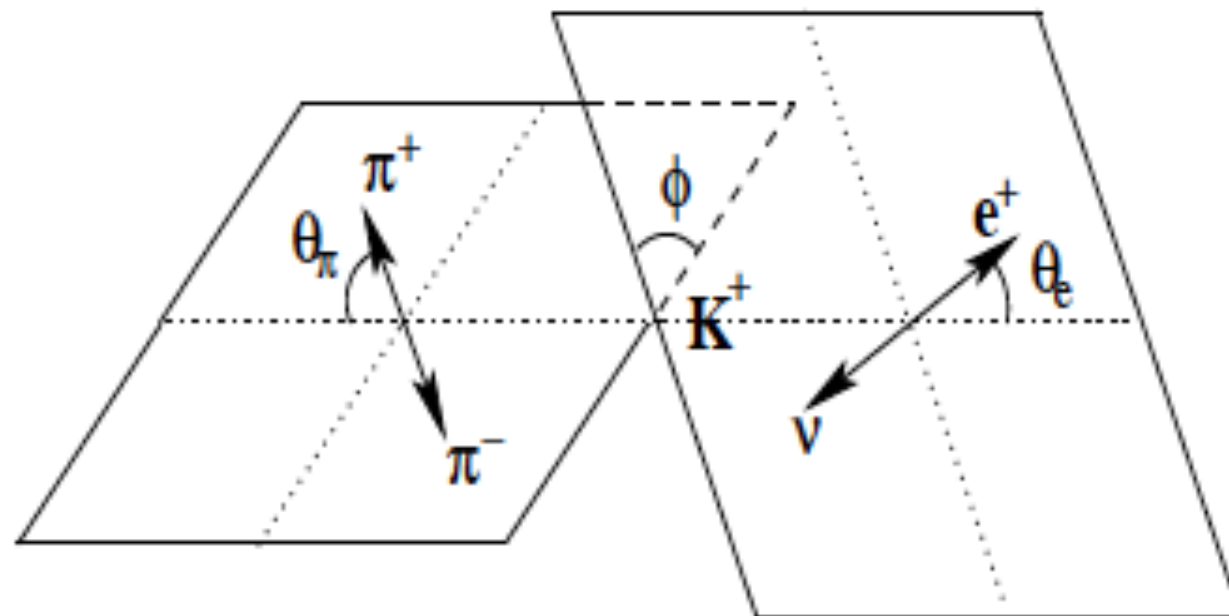
GD, Greynat, Vulvert

K_{l4} and $\pi\pi$ strong phases $\delta_I^l(s)$

Cabibbo Maksymowicz

$$\frac{G_F}{\sqrt{2}} V_{us} \bar{e} \gamma^\mu (1 - \gamma^5) \nu H_\mu(p_1, p_2, q)$$

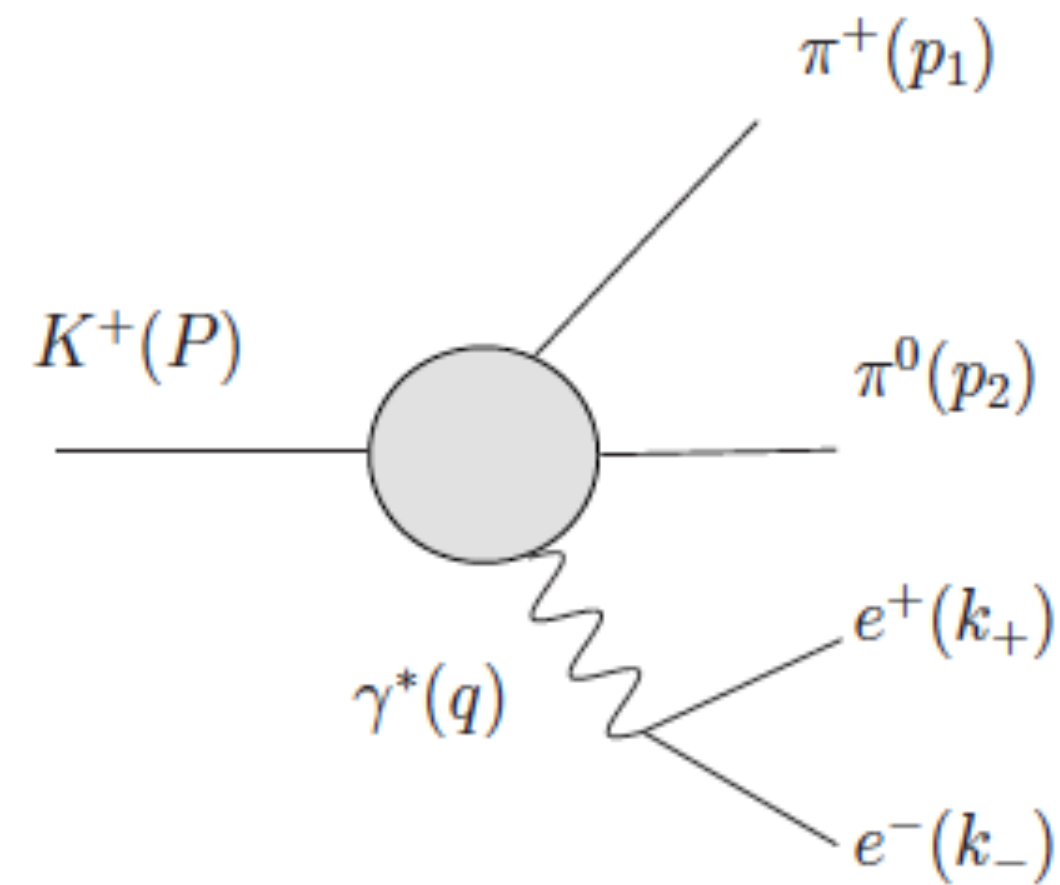
$$H^\mu = F_1 p_1^\mu + F_2 p_2^\mu + F_3 \varepsilon^{\mu\nu\alpha\beta} p_{1\nu} p_{2\alpha} q_\beta. \quad F_i(s) = f_i(s) e^{i\delta_0^0(s)} + ..$$



- crucial to measure $\sin \delta \implies$ interf F_3
- Look angular plane asymmetry

$$K_L \rightarrow \pi^+ \pi^- \gamma^* \rightarrow \pi^+ \pi^- e^+ e^-$$

Sehgal et al; Savage, Wise et al



- $\mathcal{M}_{LD} = \frac{e}{q^2} \bar{e} \gamma^\mu (1 - \gamma^5) e H_\mu$
- $H^\mu = F_1 p_1^\mu + F_2 p_2^\mu + F_3 \varepsilon^{\mu\nu\alpha\beta} p_{1\nu} p_{2\alpha} q_\beta$
- $F_{1,2} \sim E \quad F_3 \sim M$

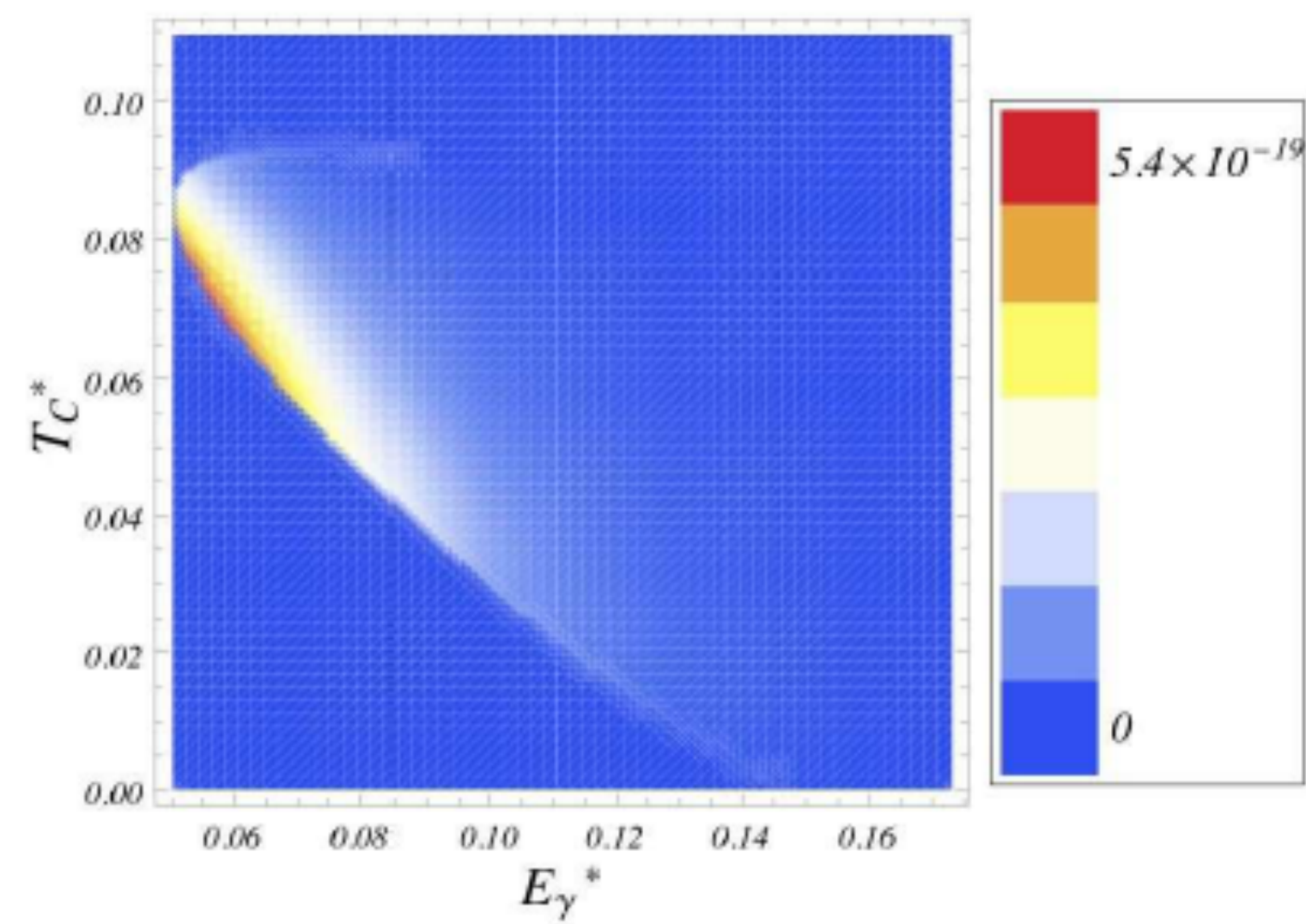
- Interference $E \quad M$ novel compared to $K_L \rightarrow \pi^+ \pi^- \gamma$
- $E \quad M$ known from $K_L \rightarrow \pi^+ \pi^- \gamma$ (IB and DE)

$$K^+ \rightarrow \pi^+ \pi^0 \gamma^* \rightarrow \pi^+ \pi^0 e^+ e^-$$

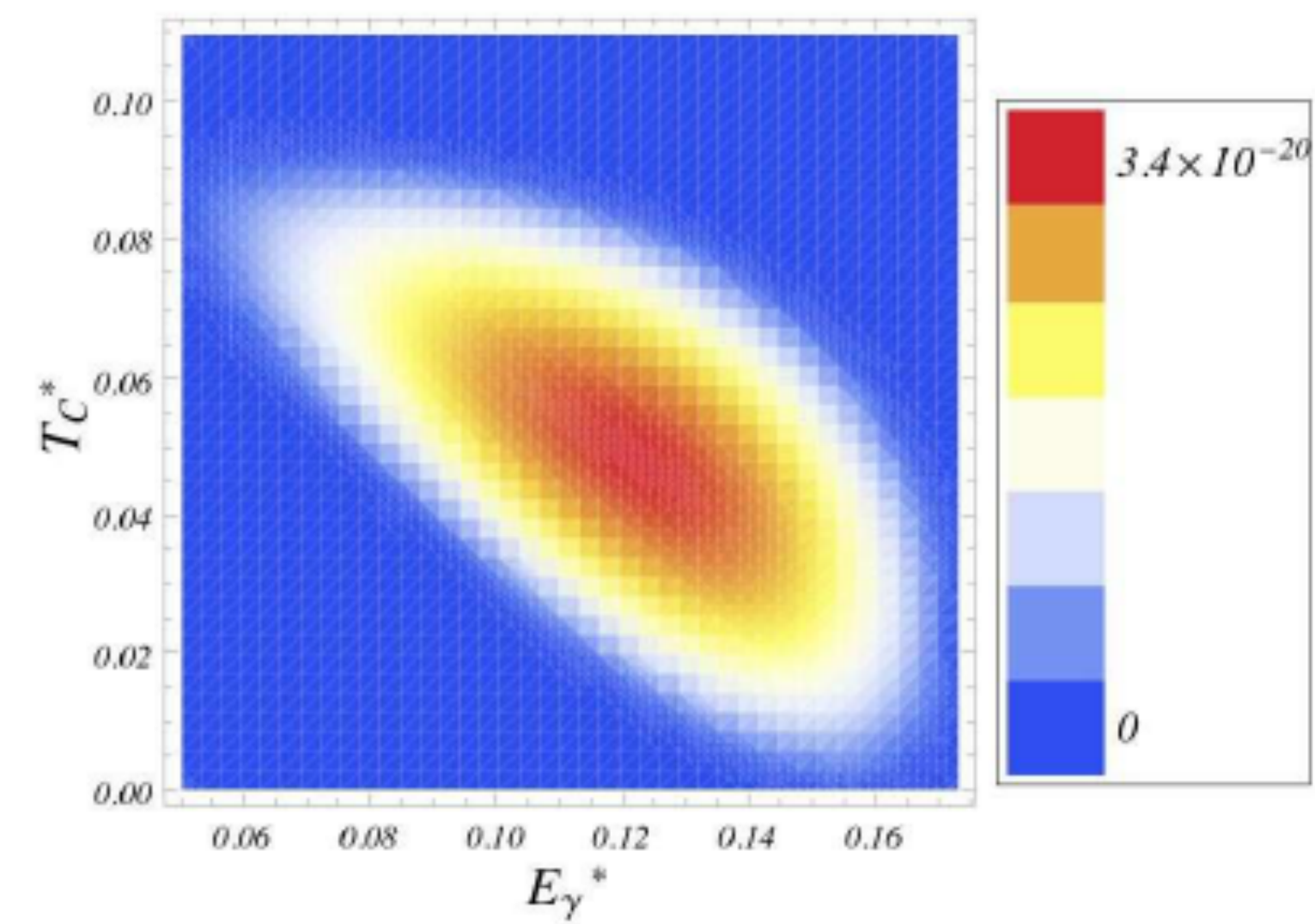
Cappiello, Cata, G.D. and Gao,

- the asymm. , $\frac{\Re(E_B M^*)}{|E_B|^2 + |M|^2}$, not as lucky $E_B \gg M$:
- $B(K^+)_{IB} \sim 3.3 \times 10^{-6} \sim 50 B(K^+)_{DE}$
- Short distance info without having simultaneously K^+ and K^- , asymm. in phase space, (P-violation) interesting! No ϵ -contamination
- interesting Dalitz plots (at fixed q^2) to disentangle M from E_B
- at $q^2 = 50\text{MeV}$ IB only 10 times larger than DE

q_c (MeV)	B [10^{-8}]	B/M	B/E	B/BE	B/BM
$2m_l$	418.27	71	4405	128	208
55	5.62	12	118	38	44
100	0.67	8	30	71	36
180	0.003	12	5	-19	44



IB



DE

Outline

- Motivations for $K \rightarrow \pi l l$ $K^\pm \rightarrow \pi^\pm l^+ l^-$ $K_S \rightarrow \pi^0 l^+ l^-$
- $K \rightarrow \pi l l$ phenomenology
- Large N QCD: the dream '70s '80s '90s 't Hooft, Witten, Coleman
- Ideas, Successes, some undelivered messages $K \rightarrow \pi\pi : \Delta I = \frac{1}{2}$ rule ϵ'
- $K \rightarrow \pi l l$ phenomenology and large N
- conclusions

The kaon community requests to

- protect and amplify the European kaon-physics programme, exploring opportunities for
 - $K^+ \rightarrow \pi^+ \nu \bar{\nu}$
 - $K_{S,L} \rightarrow \mu^+ \mu^-$ decay and interference,
- enable European contributions for KOTO II for $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \ell^+ \ell^-$, spanning both analysis and hardware development,
.....
- maintain the European leadership on theory computations for kaon physics (phenomenology, dispersion theory, effective theory and lattice QCD, including high-performance computing).

EU strategy group
Lazzeroni

The contribution "Kaon Physics: A Cornerstone for Future Discoveries" has been drafted by:
M. Bordone, A. Ceccucci, A. Dery, M. Gorbahn, E. Goudzovski, M. Hoferichter, A. Jüttner, C. Lazzeroni,
Z. Ligeti, D. Martinez, F. Mahmoudi, R. Marchevski, M. Moulson, G. Ruggiero.

Year	Main object
1	Beam line survey
2	Construction of the rest of the detector
3-6	Phase I: Physics run for mainly $K_L \rightarrow \pi^0 \nu \bar{\nu}$
7	Single event sensitivity will reach 8.5×10^{-13} for the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ search
8	Detector upgrade
9-12	Phase II: Physics run mainly for $K_L \rightarrow \pi^0 \ell^+ \ell^-$ with an optimized setup
13	End of Phase II

Moulson

Conclusions

Interesting TH CP violating and CP conserving

$$K_{S,L} \rightarrow \pi^0 l^+ l^-$$

Crucial to study

$$K^+ \rightarrow \pi^+ l^+ l^-$$

3-point function

$$K_{S,L} \rightarrow \mu^+ \mu^- (e^+ e^-)$$

EU strategy Group '25, Nazila et al '22

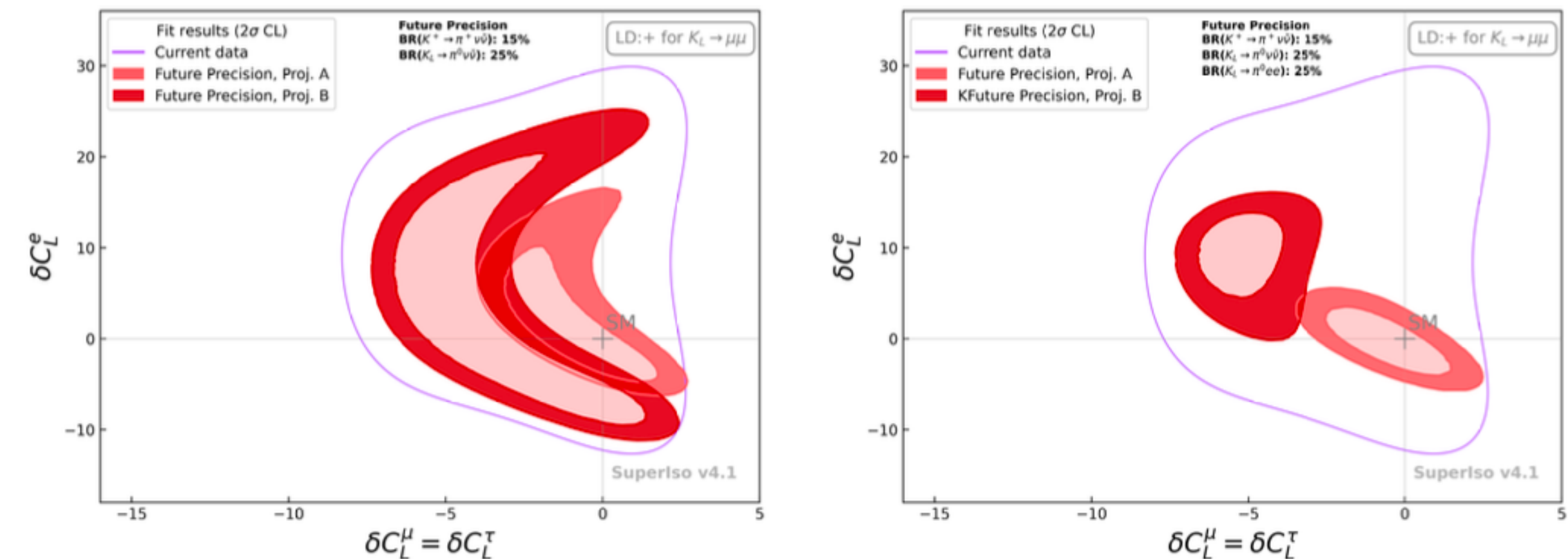


Figure 5: BSM parameter space for Wilson coefficients in scenarios with LFU violation where the NP effects for electrons are different from the those for muons and taus [18, 19]. Left: Impact on allowed parameter space from measurements of the golden channels $K \rightarrow \pi \nu \bar{\nu}$ from NA62 and KOTO II with the expected final precision. Right: Impact on the parameter space by the inclusion in the fits of a measurement of the $K_L \rightarrow \pi^0 e^+ e^-$ branching ratio with 25% precision.