Inclusive vs exclusive $b \to s\ell^+\ell^-$ decays: a path around irreducible non-perturbative uncertainties

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Based mostly on:
Huber, Hurth, Jenkins, EL, Qin, Vos; 2404.03517 [*JHEP* 11 (2024) 130]

Operators

- SM operator basis ($q = d, s$):

$$\mathcal{L}_{eff} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{tq}^* \left[\sum_{i=1}^{10} C_i Q_i + \underbrace{\frac{V_{ub} V_{uq}^*}{V_{tb} V_{tq}^*}}_{\equiv \lambda_q} \sum_{i=1}^{2} C_i (Q_i - Q_i^u) + \underbrace{\sum_{i=3}^{6} C_{iQ} Q_{iQ} + C_b Q_b}_{\text{for QED corrections}} \right]$$

- Semileptonic

$$Q_9 = (\bar{q}_L \gamma_\mu b_L) \sum (\bar{\ell} \gamma_\mu \ell_L)$$
$$Q_{10} = (\bar{q}_L \gamma_\mu b_L) \sum (\bar{\ell} \gamma_\mu \gamma_5 \ell_L)$$

- Magnetic & chromo-magnetic

$$Q_7 = \frac{e}{16\pi^2} m_b (\bar{q}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}$$
$$Q_8 = \frac{g_s}{16\pi^2} m_b (\bar{q}_L \sigma^{\mu\nu} T^a b_R) G^a_{\mu\nu}$$

- Current-current

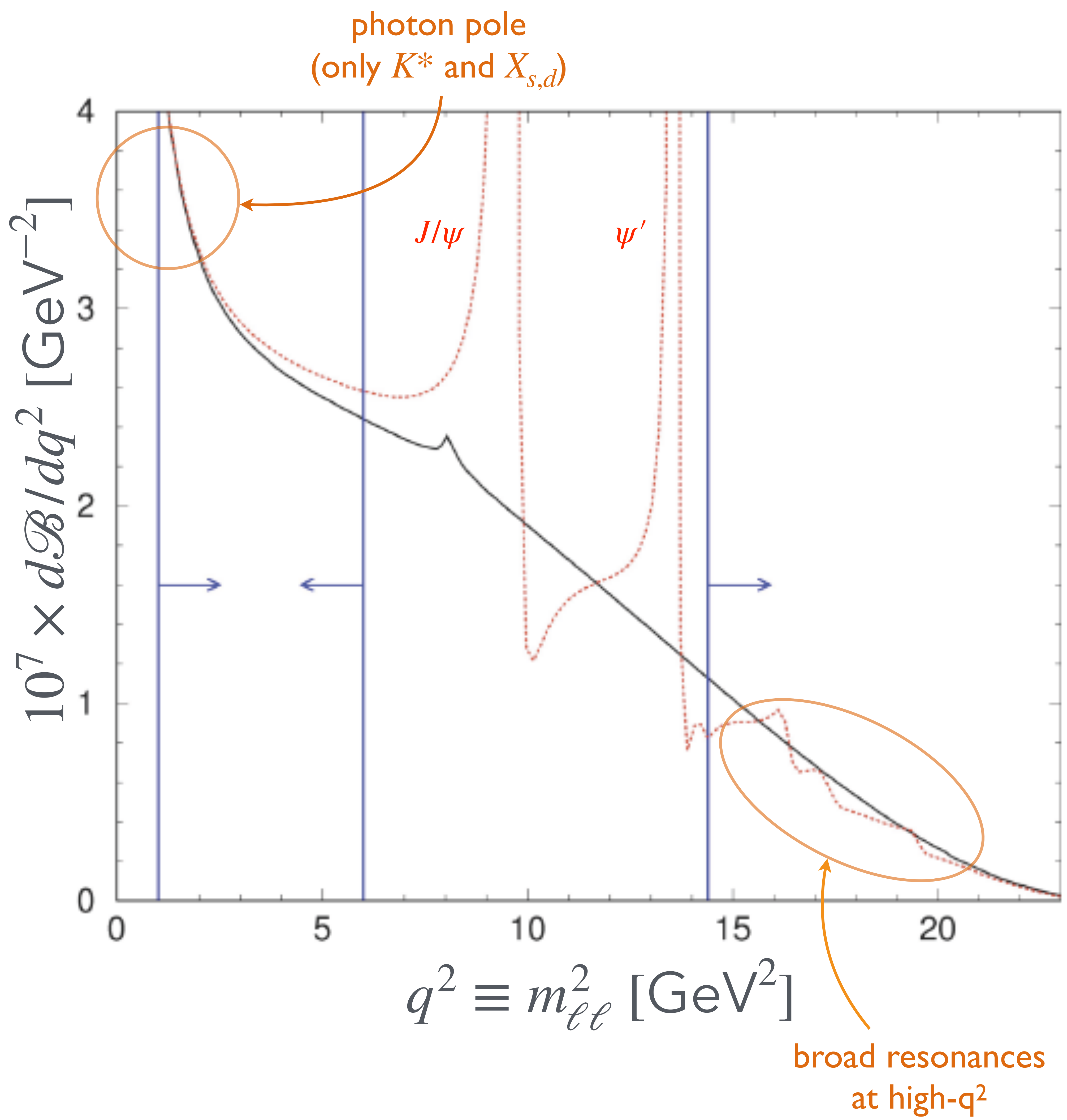
$$Q_1 = (\bar{q}_L \gamma_\mu T^a c_L)(\bar{c}_L \gamma_\mu T^a b_L)$$
$$Q_2 = (\bar{q}_L \gamma_\mu c_L)(\bar{c}_L \gamma_\mu b_L)$$

$$Q_1^u = (\bar{q}_L \gamma_\mu T^a u_L)(\bar{u}_L \gamma_\mu T^a b_L)$$
$$Q_2^u = (\bar{q}_L \gamma_\mu u_L)(\bar{u}_L \gamma_\mu b_L)$$

The $V_{ub} V_{uq}^*$ contribution is small for $b \to s\ell\ell$ but important for $b \to d\ell\ell$ ($\lambda_s \simeq 0.02\ e^{-2.0i}, \lambda_d \simeq 0.4\ e^{-1.5i}$)

$b \to s\ell\ell$: typical spectrum

- Typical branching ratios are of order 10^{-6}
- Intermediate charmonium resonances contribute via: $B \to (K, K^*, X_s)\psi_{c\bar{c}} \to (K, K^*, X_s)\ell^+\ell^-$
- Contributions of J/ψ and ψ' have to be dropped:
  - Low-q^2 $\Rightarrow$ 1 GeV^2 < q^2 < 6 GeV^2
  - High-q^2 $\Rightarrow$ q^2 > (14.4 or 15) GeV^2
- Theory at low-q^2 and high-q^2 presents different challenges

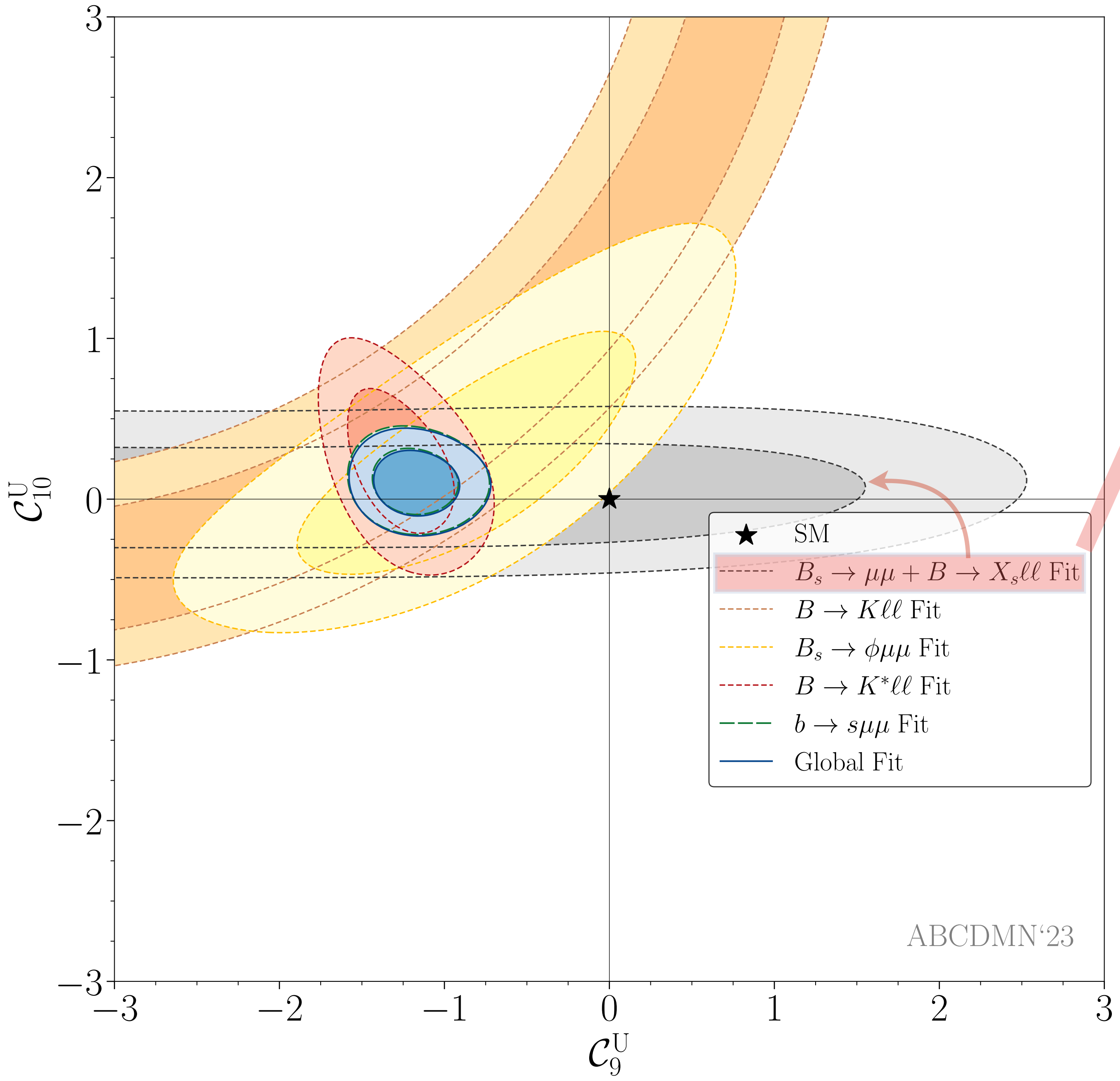


Exclusive: global fits

Scenario		Best-fit point	1σ	Pull_{SM}	p-value
$b \to s\ell^+\ell^-$	$\mathcal{C}_9^{\text{U}}$	-1.17	$[-1.33, -1.00]$	5.8	39.9 %
$b \to s\ell^+\ell^-$	$\mathcal{C}_9^{\text{U}}$	-1.18	$[-1.35, -1.00]$	5.5	39.1 %
	$\mathcal{C}_{10}^{\text{U}}$	$+0.10$	$[-0.04, +0.23]$		

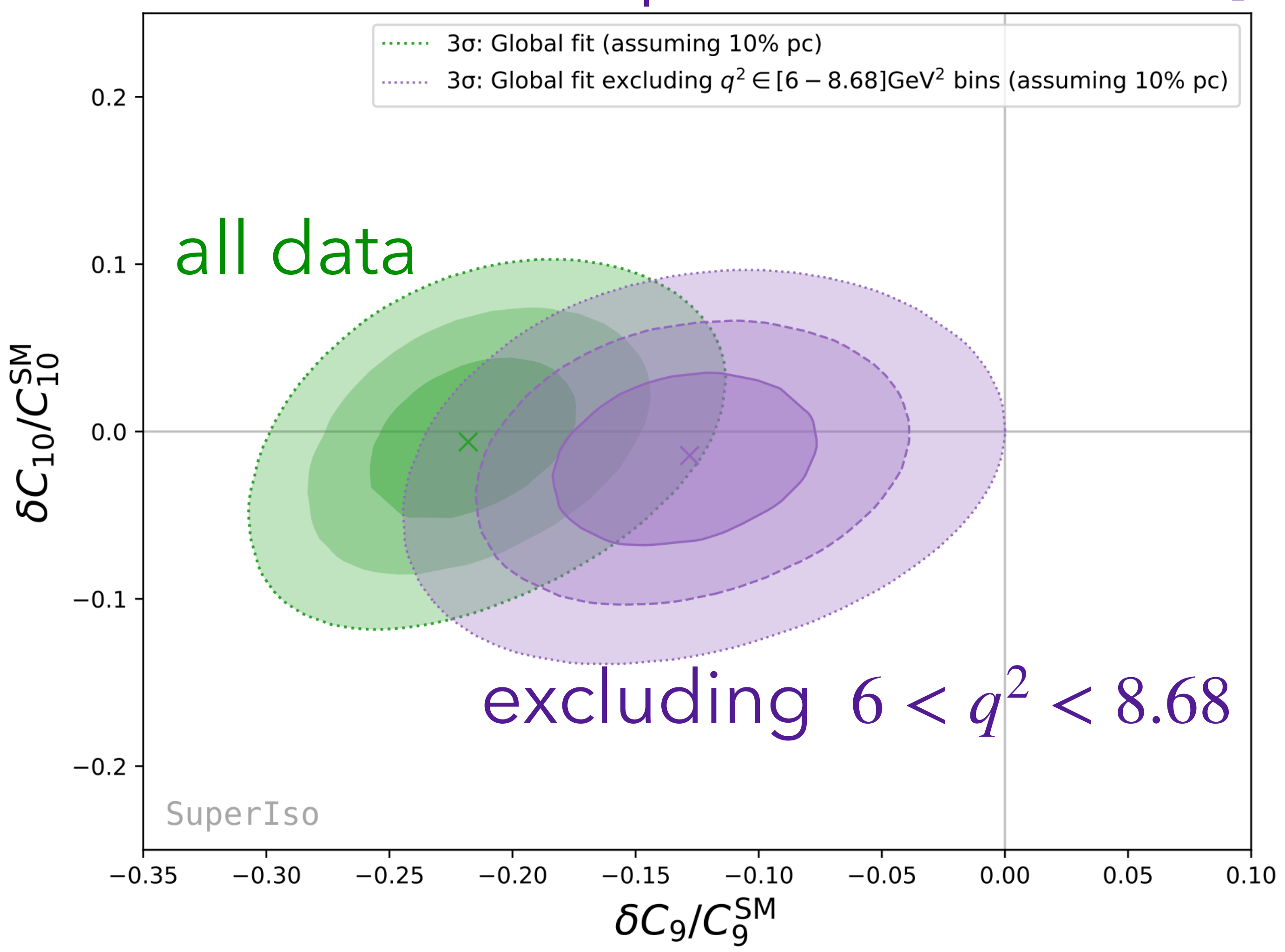
[Capdevila et al, 2309.01311]

$$C_9^U \simeq -1.2 \sim -\frac{1}{4} C_9^{\text{SM}}$$



- Low-q^2 $B \to X_s\ell\ell$ inclusive constraints are consistent with the SM (more on this later)
- Many global fitters:
  - ABCDMN [Algueró et al, 2304.07330]
  - AS/GSSS [Altmannshofer et al, 2212.10497]
  - CFFPSV [Ciuchini et al, 2212.10516]
  - HMMN [Hurth et al, 2104.10058]
  - GRvDV [Gubernari et al, 2206.03797]

[Hurth, Mahmoudi, Monceaux, Neshatpour 2508.09986]

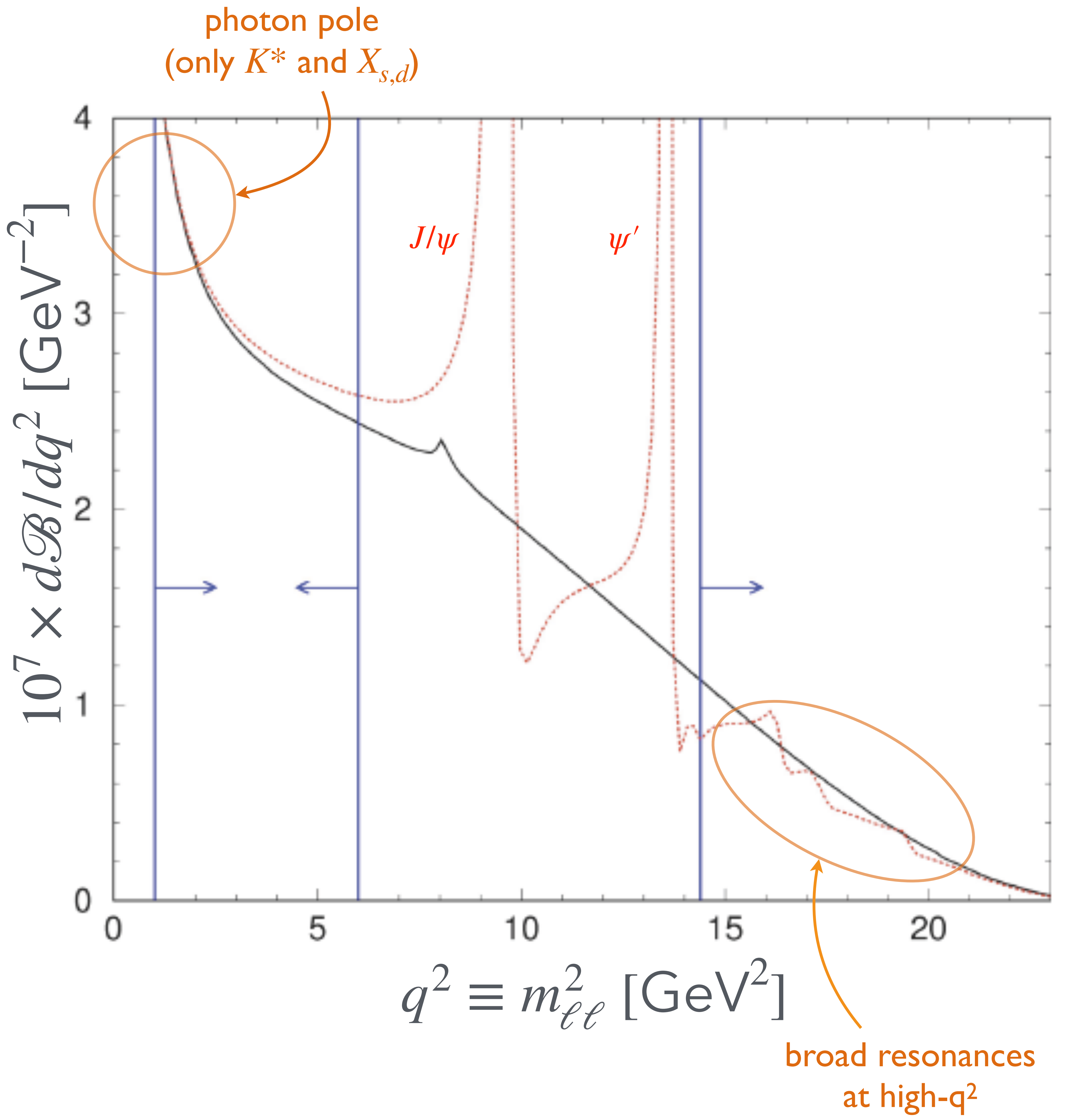


$b \to s\ell\ell$: theoretical frameworks

Legend:
SCET = Soft-Collinear Effective Theory
FF=Form Factors
LCDA=Light-Cone Distribution Amplitudes
PC = Power Corrections
OPE = Operator Product Expansion

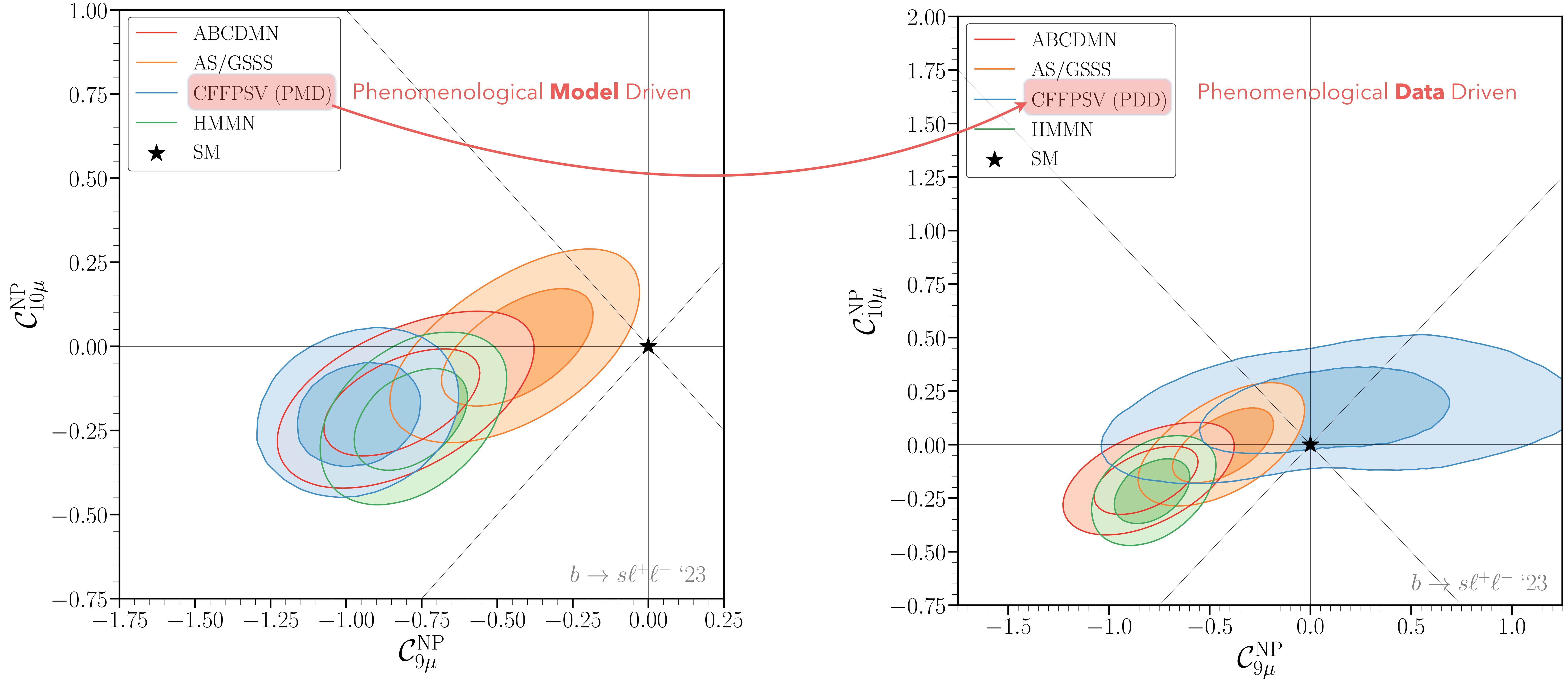
- Intermediate charmonium resonances contribute via: $B \to (K, K^*, X_s)\psi_{c\bar{c}} \to (K, K^*, X_s)\ell^+\ell^-$
- Contributions of J/ψ and ψ' have to be dropped:
  - Low-q^2 $\Rightarrow$ 1 $GeV^2 < q^2 <$ 6 GeV^2
  - High-q^2 $\Rightarrow$ $q^2 >$ (14.4 or 15) GeV^2

	Theory	Experiment
Excl, Low-q^2	SCET (**FF+LCDA**) + **non-local PC**	**High statistics**
Excl, High-q^2	OPE in q^2 (FF) + local PC	Low statistics
Incl, Low-q^2	**OPE in M_X + local PC**	**High X_s multiplicity** (difficult at LHCb)
Incl, High-q^2	**OPE in M_X breaks down** (large local PC)	**Low X_s multiplicity** (possible at LHCb)



Exclusive: global fits

- Good agreement between global fitters if (1) same sets of inputs are used and (2) unkown non-local power corrections are estimated [Capdevila et al, 2309.01311]:



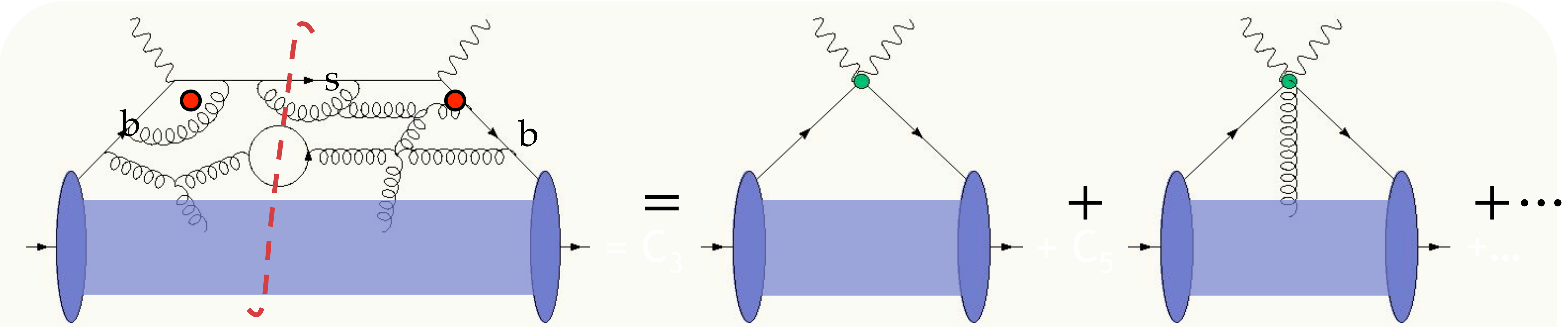
- Main difference between the fits on the left is whether the [6,8] GeV^2 bin is used or not
- In the **P**hen. **D**ata **D**riven approach, non-local power corrections are fitted to data

- The problem is that non-local power corrections **with a mild q² dependence** look identical to new physics contributions to C_9

Inclusive: theory

- Up to power corrections the inclusive rate is free of hadronic uncertainties:

$$\Gamma[B \to X_s \ell\ell] = \Gamma[b \to X_s \ell\ell] + O\left(\frac{\Lambda_{QCD}^2}{m_b^2}, \frac{\Lambda_{QCD}^3}{m_b^3}, \frac{\Lambda_{QCD}^2}{m_c^2} \cdots\right)$$

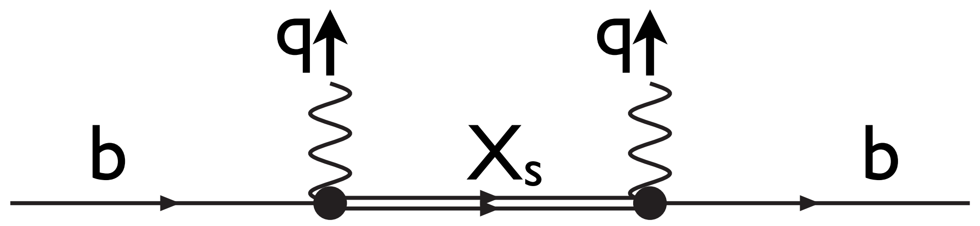


$\langle B | \bar{b} b | B \rangle = 1$

$\langle B | \bar{b} D^2 b | B \rangle \sim \lambda_1$ $\quad \langle B | \bar{b} \sigma_{\mu\nu} G^{\mu\nu} b | B \rangle \sim \lambda_2$

The leading power contribution is most expressed as a series in α_s and $\kappa = \alpha_{em}/\alpha_s$ and is known (almost) up to and including $\alpha_s^3 \kappa^3$

- The OPE breaks down at high-q²:



$$p_{X_s}^2 = (p_b - q)^2 = m_b^2 + q^2 - 2 m_b q_0$$

$$< m_b^2 + q^2 - 2 m_b \sqrt{q^2} = \left(m_b - \sqrt{q^2}\right)^2$$

expansion in $\dfrac{\Lambda_{QCD}}{m_b - \sqrt{q^2}}$

This breakdown manifests as very large power corrections

Inclusive: OPE breakdown

- Power corrections proportional to $|C_{9,10}|^2$ are identical to those which appear in $\bar{B}^0 \to X_u \ell \nu$ and they can be removed by normalizing the rate to the semileptonic rate with the same q^2 cut [Lee, Ligeti, Stewart, Tackmann]:

$$\mathcal{R}(q_0^2) = \frac{\int_{q_0^2}^{m_b^2} dq^2 \frac{d\Gamma(\bar{B} \to X_s \ell^+ \ell^-)}{dq^2}}{\int_{q_0^2}^{m_b^2} dq^2 \frac{d\Gamma(\bar{B}^0 \to X_u \ell \nu)}{dq^2}}$$

[Note that we need the neutral B^0 semileptonic rate to avoid contributions from certain weak annihilation matrix elements]

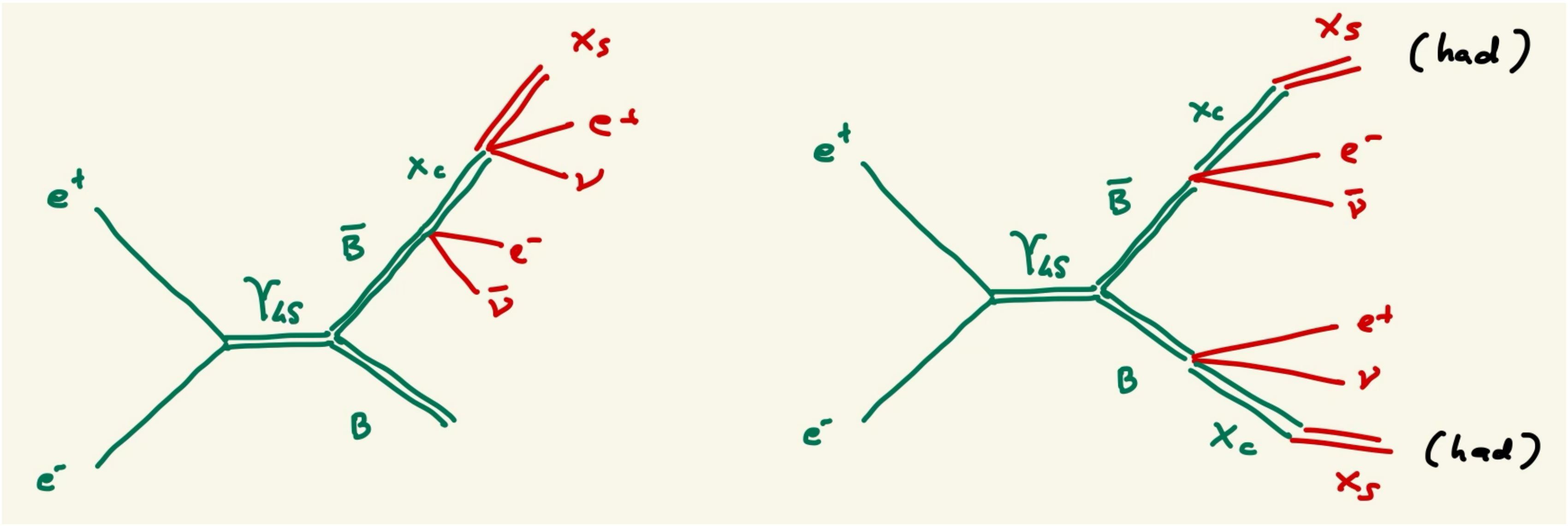
- Non-perturbative effects associated to the breaking of the OPE in the leading $|C_{9,10}|^2$ terms cancel exactly against those in the denominator

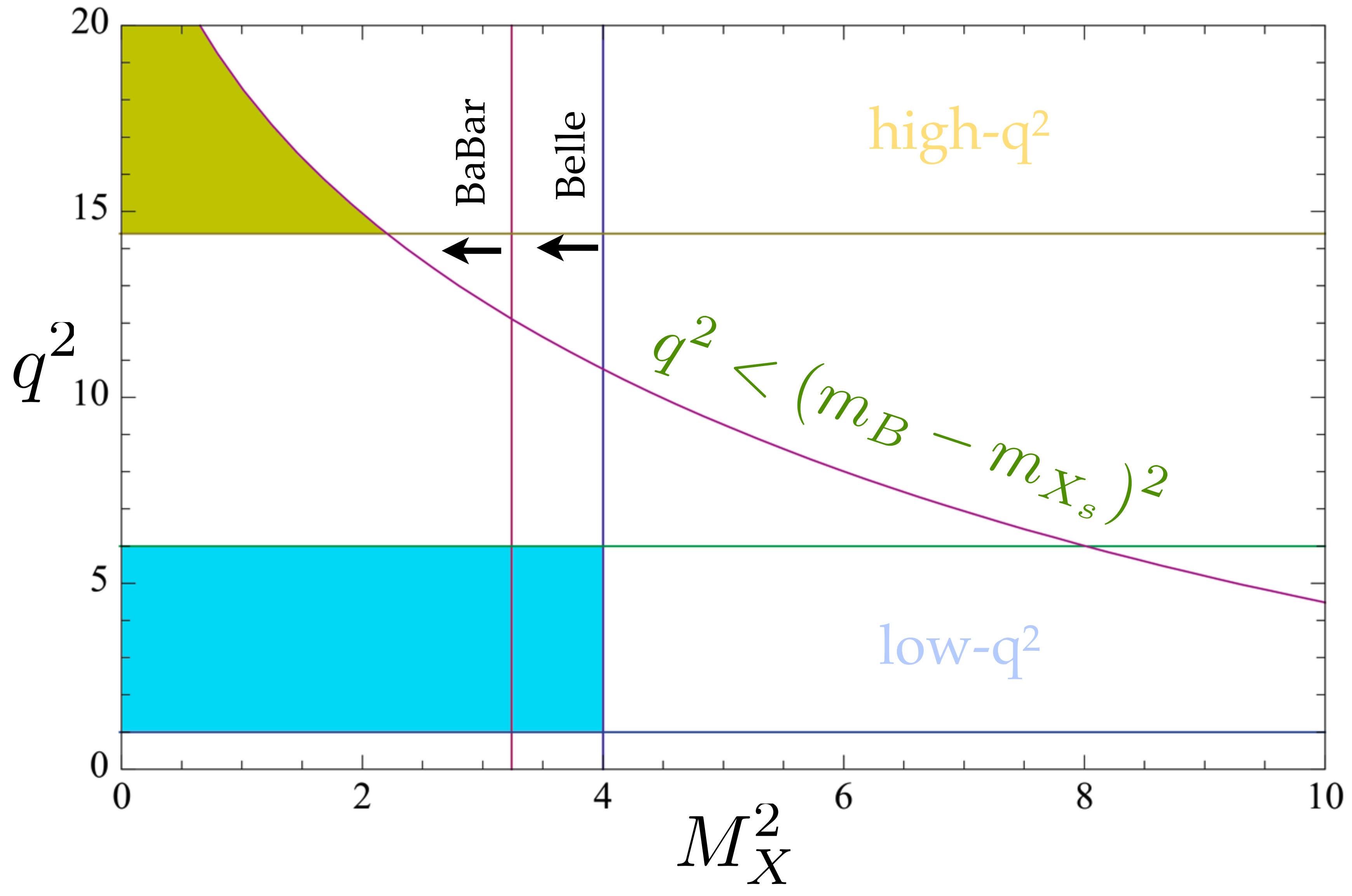
- Non-perturbative effects associated to other operators ($|C_7|^2$, $C_7 C_9$) do not necessarily cancel

Inclusive: hadronic cuts

- m_X cuts are required to suppress background from double semileptonic decays (both same side and opposite side):
  - $B \to (X_c \to X_s \ell^+ \nu)\ell^- \bar{\nu} = X_s \ell \ell$ + missing energy
  - $ee \to (B \to (X_c \to X_s)\ell^- \bar{\nu})(\bar{B} \to (X_c \to X_s)\ell^+ \nu) = X_s \ell \ell$ + missing energy

- These cuts introduce sensitivity to a hard collinear scale (of order 2 GeV) and the rate becomes dependent on the B meson shape function



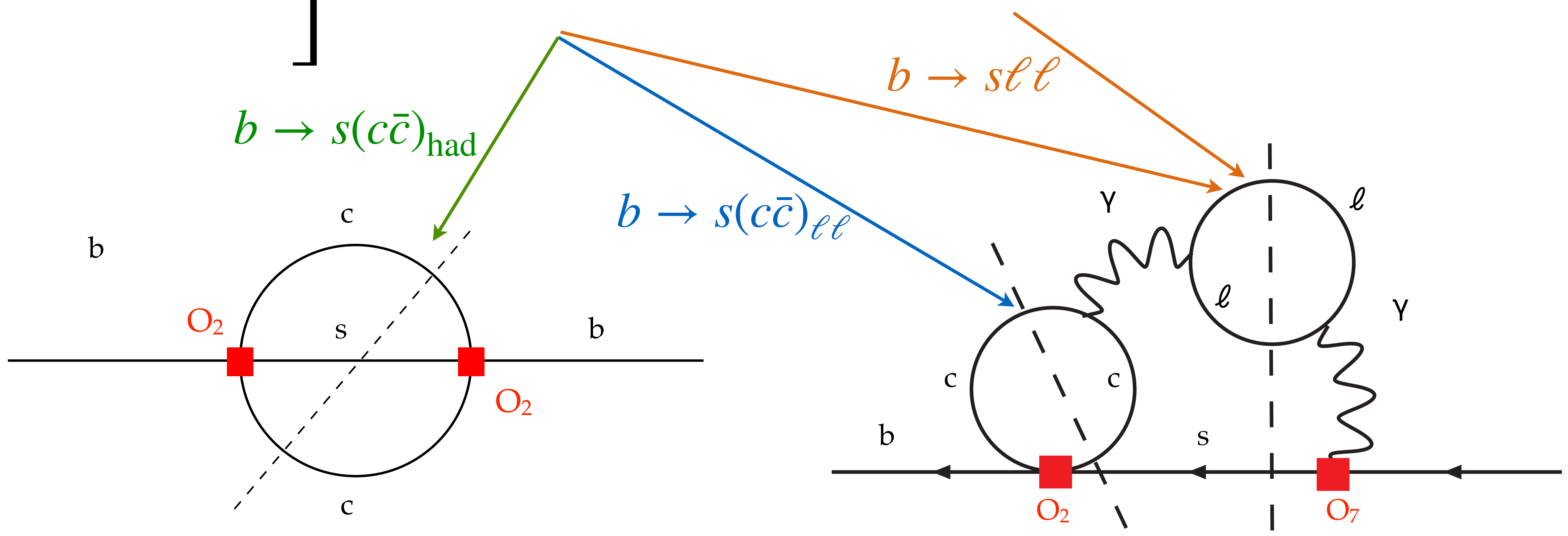


- The high-q² region is unaffected
- Current BaBar and Belle analyses correct using a Fermi motion model
- Better modeling can be achieved within SCET and by using $B \to X_s \gamma$ and $B \to X_u \ell \nu$ data to extract the shape function

Inclusive: resonances

- Optical theorem:

$$\text{Im}\left[\sum_{ij} \langle B\,|\,T\,Q_i(0)\;Q_j(x)\,|\,B\rangle\right] \sim \Gamma(B \to X_s) \neq \Gamma(B \to X_s \ell^+\ell^-)$$



$\text{BR}(B \to X_s) \sim 10^{-2}$

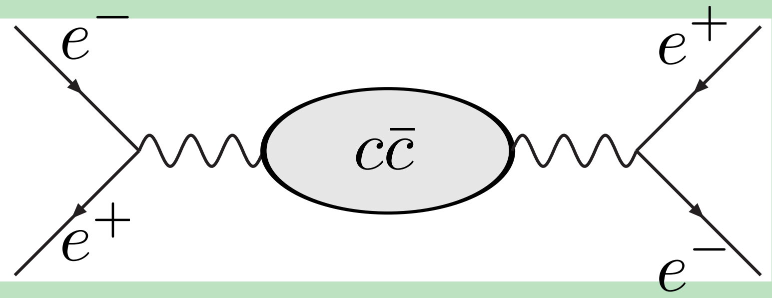
$\text{BR}(B \to X_s\,\psi(1S,2S) \to X_s\ell\ell) \sim 10^{-4}$ ⟶ Experimental cuts (low- and high-q^2)

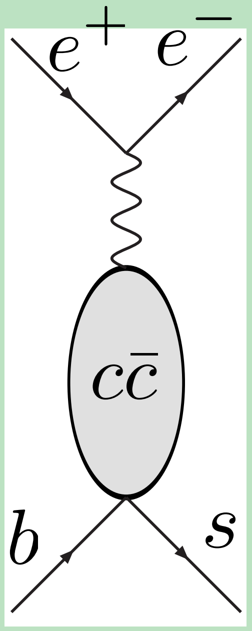
$\text{BR}(B \to X_s\ell\ell) \sim 10^{-6}$ ⟶ Need to control charmonium contamination away from $\psi(1S,2S)$

Inclusive: resonances

- Krüger-Sehgal mechanism:

$$R_{\text{had}}^{c\bar{c}} = \frac{\sigma(e^+e^- \to c\bar{c} \text{ hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$=$ 

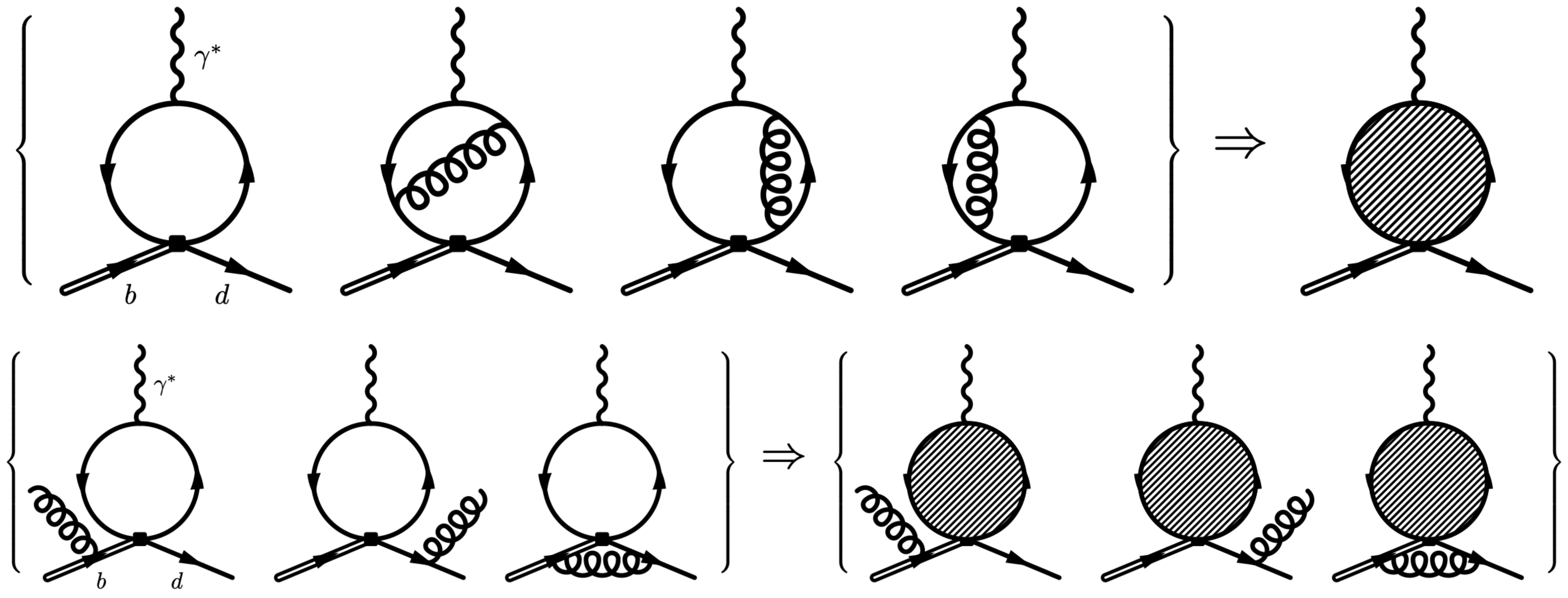
$\langle O_2 \rangle =$ 

$$\text{Im}[h_c] = \frac{\pi}{3} R_{\text{had}}$$

$$\text{Re}[h_c] = \text{Re}[h_c(s_0)] + \frac{s - s_0}{\pi} \int_0^\infty \frac{\text{Im}[h_q(t)]}{(t-s)(t-s_0)} dt$$

perturbative for $s_0 \sim -\mu_b^2$

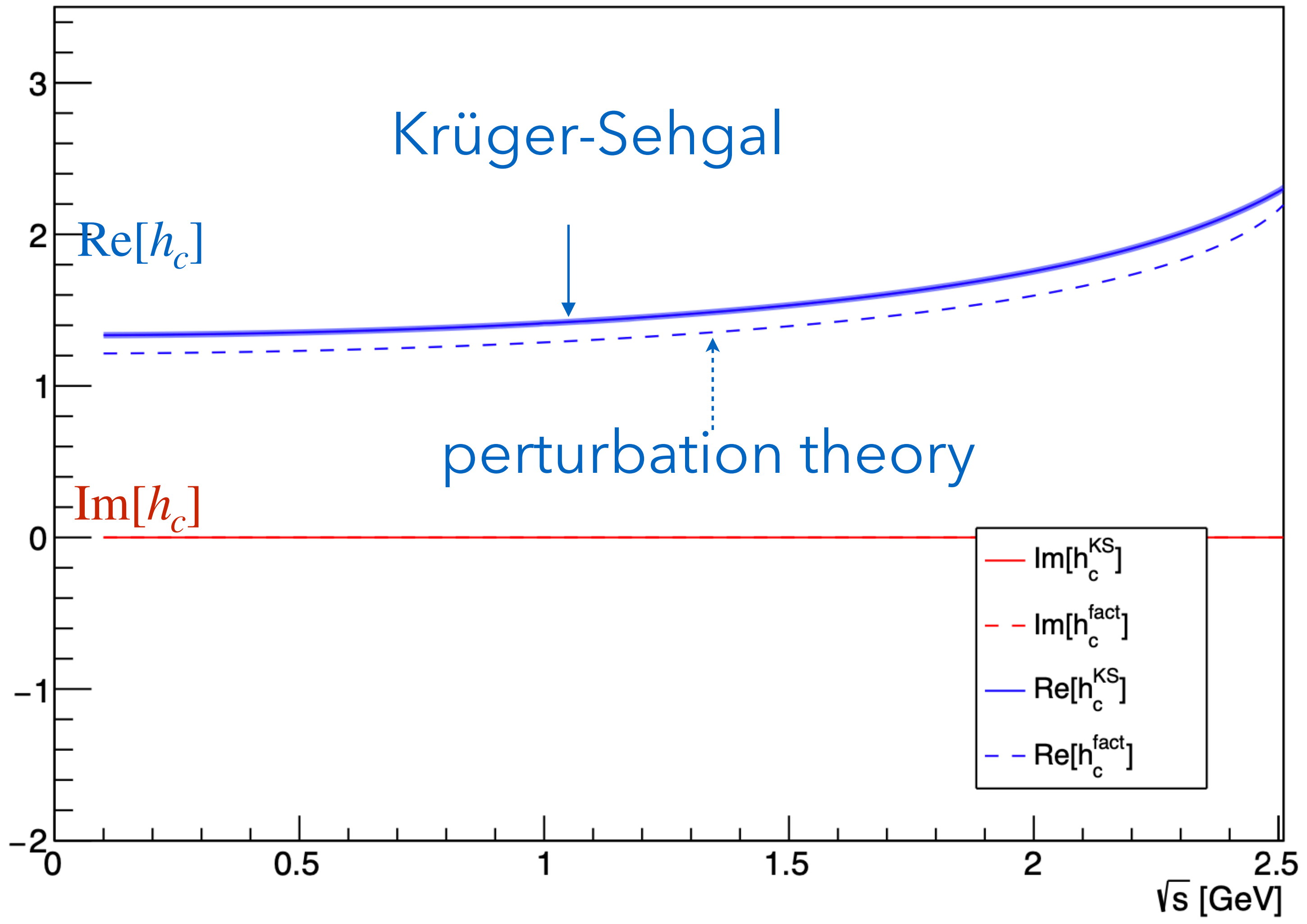
- We can include NLO effects [separation of two-loop perturbative functions provided by de Boer]



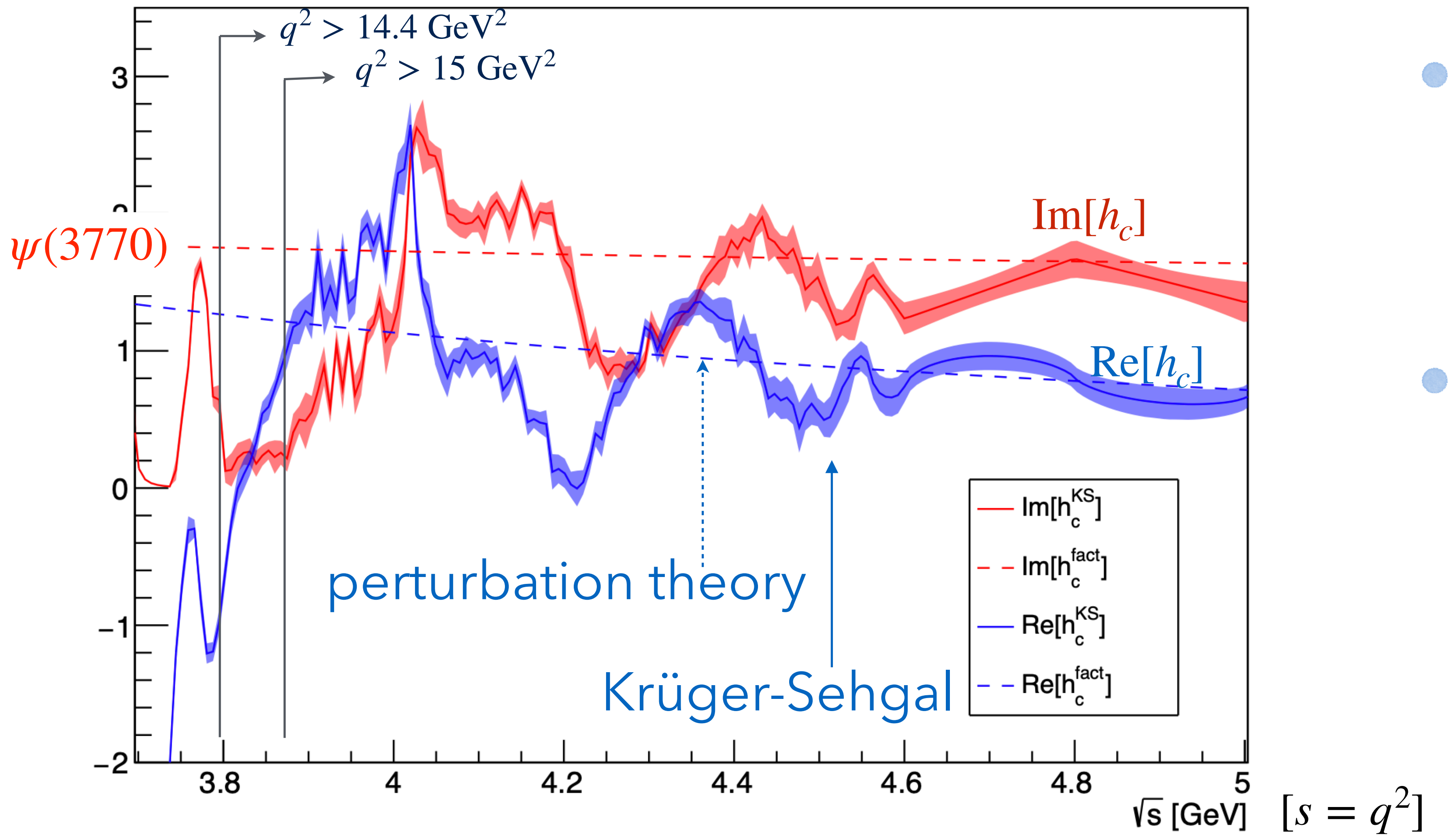
Inclusive: resonances

- We use R_{had} data [BESII, BaBar, ALEPH; Keshavarzi, Nomura, Teubner] and perturbation theory (program `rhad`) for asymptotically large q^2 [Harlander, Steinhauser]

Low-q^2



High-q^2

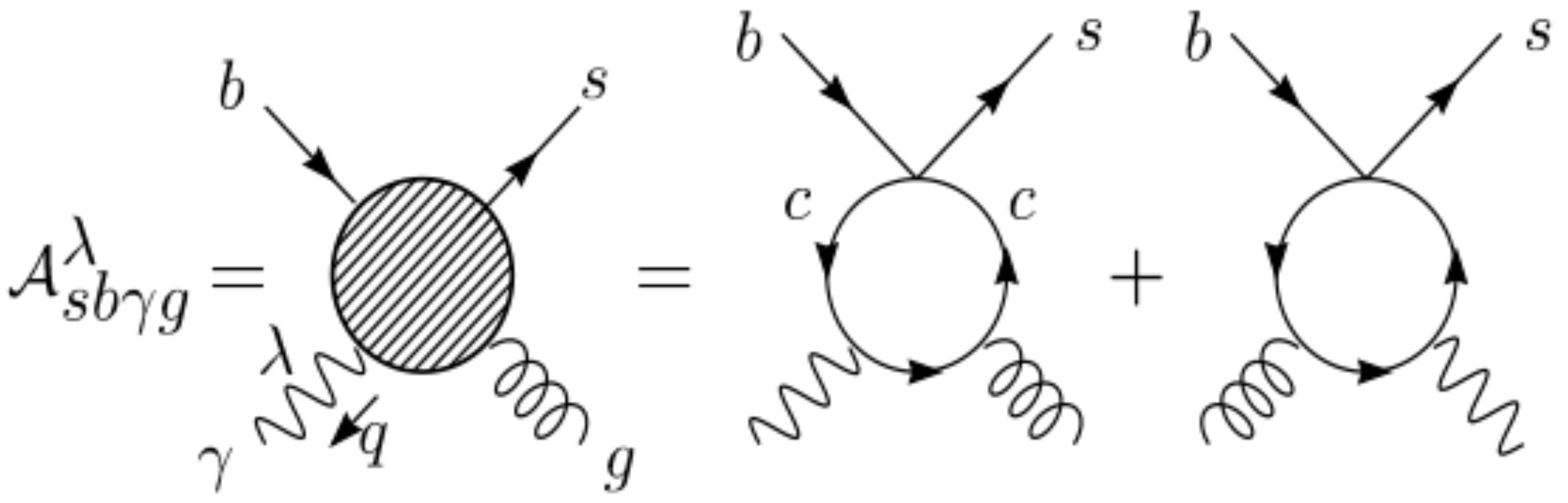


- Exclusive measurements focus on $q^2 > 15\ \mathrm{GeV}^2$ to remove the peak from the $\psi(3770)$
- In inclusive predictions the charmonium spectrum is exactly modeled and we encourage to use the $q^2 > 14.4\ \mathrm{GeV}^2$ cut adopted by BaBar and Belle.

- Impact at low-q^2 is small ($\simeq 2\,\%$)
  Perturbation theory and dispersive approaches agree because below threshold we are mostly sensitive to the total integral over R_{had} which is well described in perturbation theory
- Impact at high-q^2 region is large ($\simeq -10\,\%$)

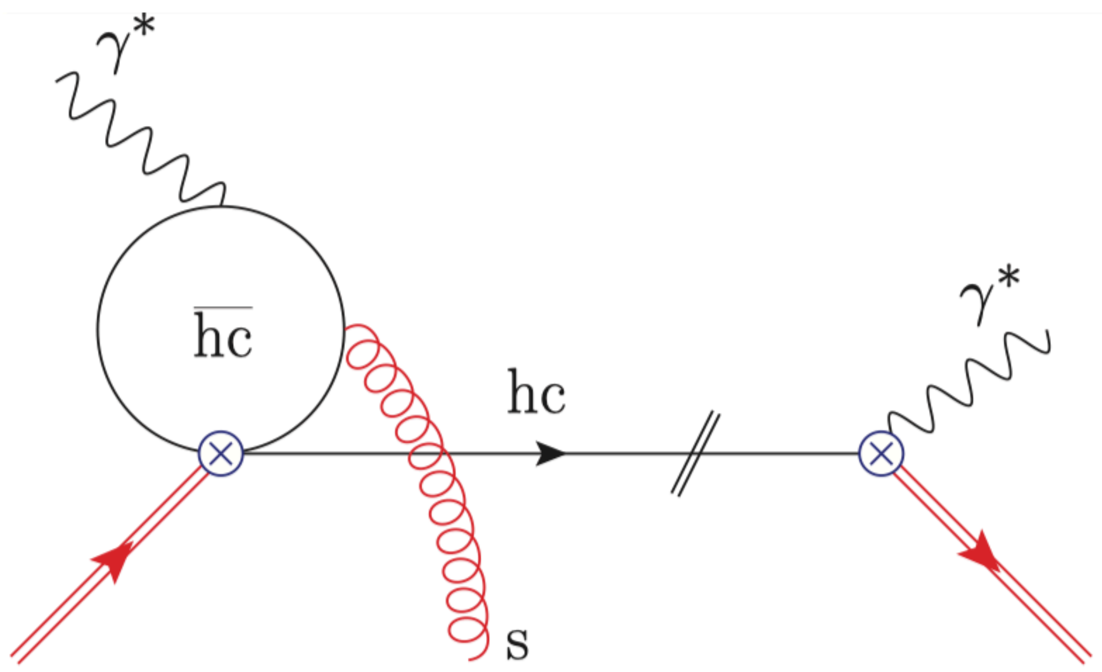
Inclusive: resonances

- Non-resonant color octet effects at high-q² can be calculated in perturbation theory and it scales as $\Lambda^2_{\rm QCD}/q^2$
  [Buchalla, Isidori, Rey]:



$$\Longrightarrow \frac{\langle B|\bar{b}\sigma_{\mu\nu}G^{\mu\nu}b|B\rangle}{m_c^2}\left(-6\frac{m_c^2}{q^2}\right) \sim \frac{\lambda_2}{q^2}$$

- Non-resonant color octet effects at low-q² and with a cut on m_X can treated using SCET
  [Hurth, Benzke, Fickinger, Turczyk]:

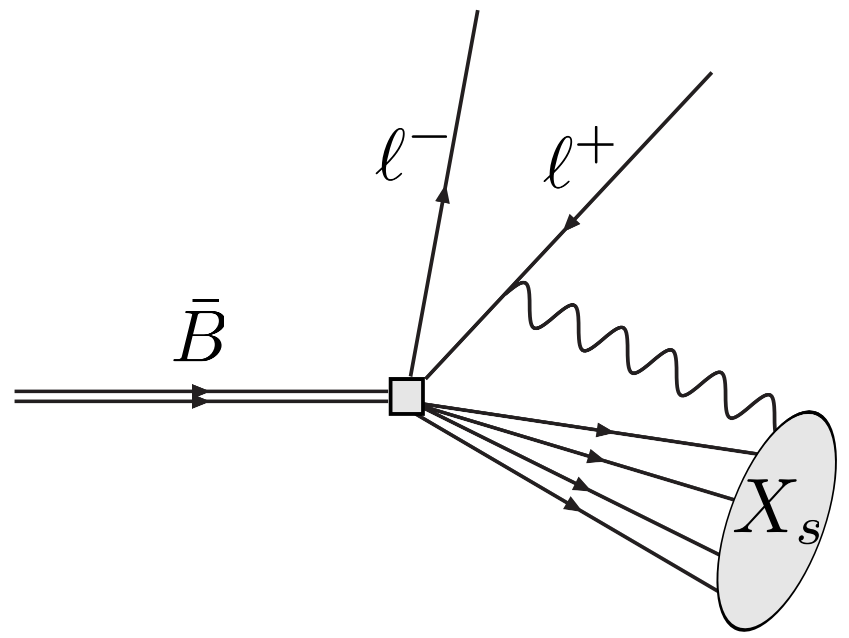


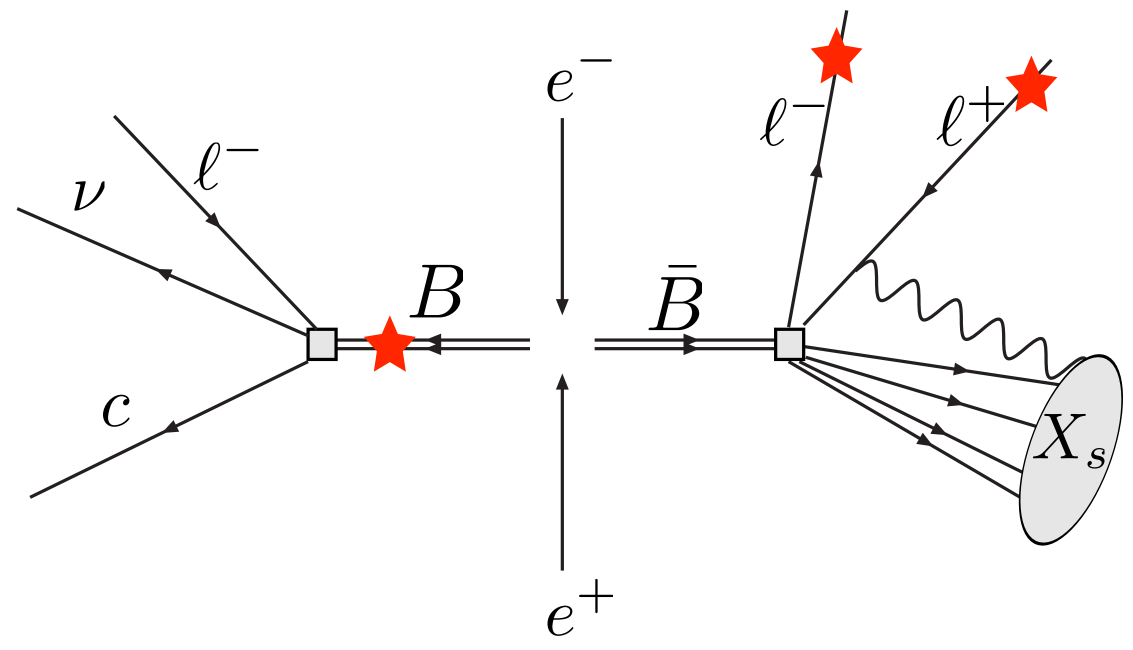
  - Power corrections remain non-local after m_X cut is released
    ⇒ so-called resolved contributions
  - Depend on mostly unknown subleading B shape functions
  - A rough estimate yields an irreducible uncertainty of about 5%

- Effect of resonant non-factorizable charmonium production at low and high q² is being investigated

Inclusive: QED radiation

- The rate is proportional to $\alpha_{\rm em}^2(\mu)$. Without QED corrections the scale μ is undetermined $\rightarrow \pm 4\,\%$ uncertainty
- Focus on corrections enhanced by large logarithms:
  - $\alpha_{\rm em} \log(m_W/m_b) \sim \alpha_{\rm em}/\alpha_s$ [WC, RG running] [Bobeth,Gambino,Gorbahn,Haisch]
  - $\alpha_{\rm em} \log(m_\ell/m_b)$ [Matrix Elements] [EL, Huber, Misiak, Wyler]
- Fate of photons emitted by the final state leptons (especially electrons):





- At B-factories most but not all of these photons are included in the X_s system $\Rightarrow$ some collinear QED logs survive
- At LHCb all photons emitted by the charged leptons are recovered (physically and using PHOTOS) and included in the lepton 4-momentum:
  $\Rightarrow$ all collinear QED logs must not be included

Inclusive: theory summary

issues

- **NNLO$_{QCD}$ + NLO$_{QED}$**
- $c\bar{c}$ **resonances**: included using e^+e^- data via a dispersion relation (Krüger-Sehgal mechansim)
- **QED radiation**: soft/collinear photons treatment at B-factories and LHCb needs to be taken into account
- m_X **cuts**: at low-q² removal of double semileptonic background, $b \to (c \to s\ell\nu)\ell\nu$, requires extrapolation in m_X

errors

- Dominant uncertainties:
  - **Low-q²** ⇒ **resolved photon contributions** (irreducible 5%) **and scale** (N³LO)
  - **High-q² (BR)** ⇒ **power corrections** (OPE breakdown)
  - **High-q² ($b \to s\ell\ell / b \to u\ell\nu$ ratio)** ⇒ **parametric** (CKM) and **power corrections** (5x smaller than in BR!)

results

$$\mathscr{B}[1,6]_{\mu\mu} = 17.29\,(1 \pm 4.4\%_{\rm scale} \pm 1.1\%_{m_t} \pm 2.3\%_{C,m_c} \pm 1.2\%_{m_b} \pm 0.5\%_{\alpha_s} \pm 0.1\%_{\rm CKM} \pm 1.5\%_{\rm BR_{sl}} \underbrace{\pm 0.7\%_{\lambda_2} \pm 5\%_{\rm resolved}}_{5.1\,\%}) \times 10^{-7}$$

$$= (17.29 \pm 1.28) \times 10^{-7} \quad [7.4\%]$$

$$\mathscr{B}[>14.4]_{\rm no\ QED} = 2.59\,(1 \pm 8.1\%_{\rm scale} \pm 1.2\%_{m_t} \pm 1.9\%_{C,m_c} \pm 7.3\%_{m_b} \pm 0.2\%_{\alpha_s} \pm 0.08\%_{\rm CKM} \pm 1.5\%_{\rm BR_{sl}} \underbrace{\pm 3.9\%_{\lambda_2} \pm 10\%_{\rho_1} \pm 21\%_{f_{u,s}}}_{23.5\,\%}) \times 10^{-7}$$

$$= (2.59 \pm 0.68) \times 10^{-7} \quad [26\%]$$

$$\mathscr{R}(14.4)_{\rm no\ QED} = 26.02\,(1 \pm 1.6\%_{\rm scale} \pm 1.2\%_{m_t} \pm 0.4\%_{C,m_c} \pm 0.4\%_{m_b} \pm 0.5\%_{\alpha_s} \pm 4.3\%_{\rm CKM} \underbrace{\pm 0.2\%_{\lambda_2} \pm 1.3\%_{\rho_1} \pm 4.6\%_{f_{u,s}}}_{4.8\,\%}) \times 10^{-4}$$

$$= (26.02 \pm 1.76) \times 10^{-4} \quad [6.8\%]$$

5x reduction

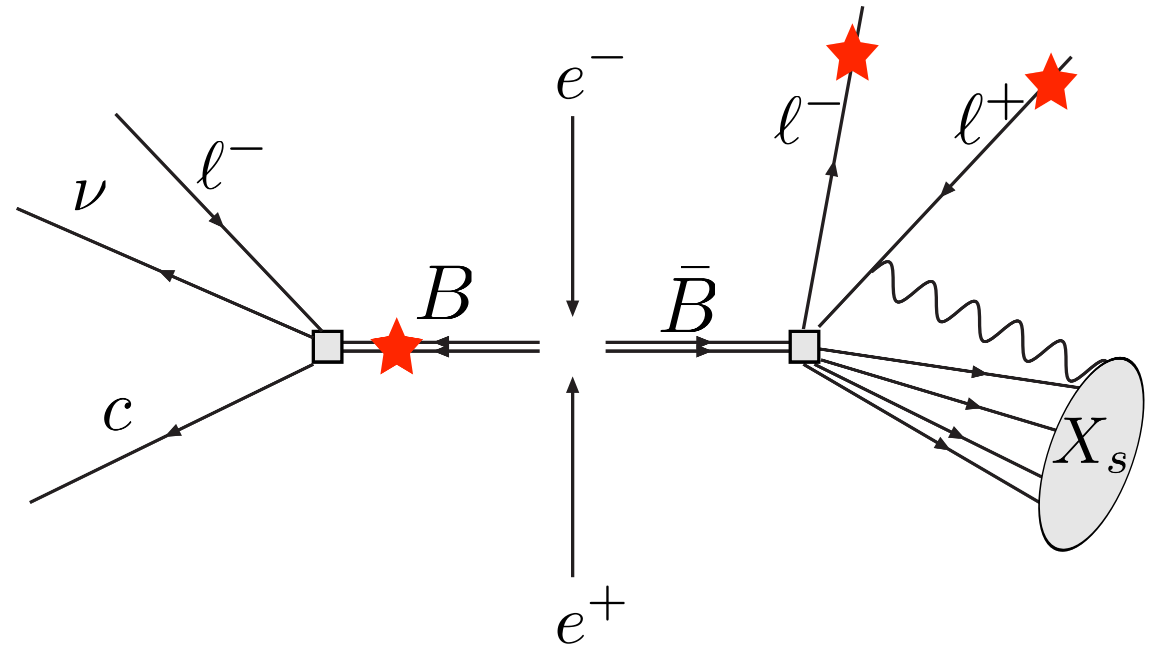
Inputs

$\alpha_s(M_z) = 0.1181(11)$	$m_e = 0.51099895$ MeV
$\alpha_e(M_z) = 1/127.955$	$m_\mu = 105.65837$ MeV
$s_W^2 \equiv \sin^2\theta_W^{\overline{\rm MS}} = 0.2312$	$m_\tau = 1.77686$ GeV
$\lvert V_{ts}^* V_{tb}/V_{cb}\rvert^2 = 0.96403(87)$	$\overline{m}_c(\overline{m}_c) = 1.275(25)$ GeV
$\lvert V_{ts}^* V_{tb}/V_{ub}\rvert^2 = 123.5(5.3)$	$m_b^{1S} = 4.691(37)$ GeV
$\lvert V_{td}^* V_{tb}/V_{cb}\rvert^2 = 0.04195(78)$	$\lvert V_{us}^* V_{ub}/(V_{ts}^* V_{tb})\rvert = 0.02022(44)$
$\lvert V_{td}^* V_{tb}/V_{ub}\rvert^2 = 5.38(26)$	$\arg\left[V_{us}^* V_{ub}/(V_{ts}^* V_{tb})\right] = 115.3(1.3)^\circ$
$\mathcal{B}(\bar B \to X_c e \bar\nu)_{\rm exp} = 0.1065(16)$	$\lvert V_{ud}^* V_{ub}/(V_{td}^* V_{tb})\rvert = 0.420(10)$
$m_B = 5.2794$ GeV	$\arg\left[V_{ud}^* V_{ub}/(V_{td}^* V_{tb})\right] = -88.3(1.4)^\circ$
$M_Z = 91.1876$ GeV	$m_{t,\rm pole} = 173.1(0.9)$ GeV
$M_W = 80.379$ GeV	$C = 0.568(7)(10)$
$\mu_b = 5^{+5}_{-2.5}$ GeV	$\mu_0 = 120^{+120}_{-60}$ GeV
$f_{\rm NV} = (0.02 \pm 0.16)$ GeV^3	$\lambda_2^{\rm eff} = 0.111(18)$ GeV^2
$f_{\rm V} - f_{\rm NV} = (0.041 \pm 0.052)$ GeV^3	$\lambda_1 = -0.314(56)$ GeV^2
$[\delta f]_{\rm SU(3)} = (0 \pm 0.04)$ GeV^3	$\rho_1 = 0.080(31)$ GeV^3
$[\delta f]_{\rm SU(2)} = (0 \pm 0.004)$ GeV^3	

References for all inputs can be found in:
2007.04191
2404.03517 ($\lambda_{1,2}$ and ρ_1)

Dominant uncertainties at high-q^2

Inclusive: experiment



- **B-factories**
  - A fully inclusive measurement is possible (opposite side tag + dilepton)
  - Current measurements use sum over exclusives:
    final states containing up to 4 pions are identified and the rest is reconstructed using Isospin and JetSet
- **LHCb at low-q²**
  - $B^{0,+} \to \mu^+\mu^-K^+ + n\pi^\pm$ (only charged particles) and use isospin to reconstruct the full inclusive rate
    [Koppenburg, CERN-THESIS-2002-010]
  - $X_b \to K^+\mu^+\mu^-X$, use isospin to reconstruct the X_s system and subtract Λ_b and B_s modes
    [Amhis, Owen, 2106.15943]
  - Neural Network search for $B^+ \to \mu^+\mu^-K^+ + \text{tracks}$
    [Graverini]
- **LHCb at high-q²**
  The inclusive rate is dominated by the K, K^*, $K\pi$ and $K\pi\pi$ modes: an inclusive measurement is already at hand!
  [Isidori, Polonsky, Tinari, 2305.03076]
  [Huber, Hurth, Jenkins, EL, Qin, Vos, 2404.03517]

BR at high-q² from LHCb

- For $q^2 > 15\ \mathrm{GeV}^2$ we have $M_{X_s} < 1.41\ \mathrm{GeV}$ the $\mathscr{B}(B \to X_s \mu^+\mu^-)$ and the rate is saturated by the $X_s = K^{(*)}$, $(K\pi)_{\text{S-wave}}$, $K\pi\pi$ and $K(n\pi)_{n>2}$ modes with progressively smaller contributions

- Dominant $K^{(*)}$ modes [LHCb, 1403.8044, 3 fb-1]:

$$\begin{aligned}\mathscr{B}(B^+ \to K^+\mu\mu)[>15] &= (0.85 \pm 0.05) \times 10^{-7}\\ \mathscr{B}(B^0 \to K^0\mu\mu)[>15] &= (0.67 \pm 0.12) \times 10^{-7}\end{aligned} \implies \mathscr{B}(\bar{B} \to K\mu\mu)[>15] = (0.82 \pm 0.05) \times 10^{-7}$$

$$\begin{aligned}\mathscr{B}(B^+ \to K^{*+}\mu\mu)[>15] &= (1.58 \pm 0.32) \times 10^{-7}\\ \mathscr{B}(B^0 \to K^{*0}\mu\mu)[>15] &= (1.74 \pm 0.14) \times 10^{-7}\end{aligned} \implies \mathscr{B}(\bar{B} \to K^*\mu\mu)[>15] = (1.72 \pm 0.13) \times 10^{-7}$$

$$\mathscr{B}(\bar{B} \to K^{(*)}\mu\mu)[>15] = (2.54 \pm 0.14) \times 10^{-7}$$

- $(K\pi)_{\text{S-wave}}$ contribution [LHCb, 1606.04731, 3 fb-1]:

$$F_S = \frac{\mathscr{B}(B \to (K^+\pi^-)_{J=0}\mu\mu)}{\mathscr{B}(B \to (K^+\pi^-)_{J=0}\mu\mu) + \mathscr{B}(B \to (K^+\pi^-)_{J=1}\mu\mu)} \implies F_S\begin{pmatrix} 15 < q^2 < 19 \\ 0.64 < M_X < 1.20 \end{pmatrix} = 0.019^{+0.030}_{-0.025} \pm 0.015$$

Using isospin: $\mathscr{B}(\bar{B} \to (K\pi)_J \ell^+\ell^-) = \mathscr{B}(\bar{B} \to (K^+\pi^-)_J \ell^+\ell^-) \times \begin{cases} \frac{3}{2} & J=0 \\ 1 & J=1 \end{cases}$

$$\implies \mathscr{B}(\bar{B} \to (K\pi)_S\mu\mu)[>15] = \frac{3}{2}\frac{F_S}{1-F_S}\mathscr{B}(\bar{B} \to K^{*0}\mu\mu)[>15] = (0.05 \pm 0.09) \times 10^{-7}$$

[to be compared with the χ_{PT} estimate of the same quantity: $(0.58 \pm 0.25) \times 10^{-7}$]

BR at high-q² from LHCb

- $K\pi\pi$ contribution [LHCb, 1408.1137, 3 fb^{-1}]:

$$\frac{\mathcal{B}(B^+ \to K^+\pi^+\pi^-\mu\mu)[14.18 < q^2 < 19]}{\Delta q^2} = \left(0.10^{+0.08}_{-0.06} \pm 0.01\right) \times 10^{-8}\ \mathrm{GeV}^{-2}$$

  Assuming that the $K\pi\pi$ mode is dominated by $\pi\pi$ in S wave and using isospin we obtain:

$$\mathcal{B}(\bar{B} \to K\pi\pi\,\mu\mu)[>15] \simeq \mathcal{B}(\bar{B} \to K(\pi\pi)_S\,\ell^+\ell^-) = \mathcal{B}(B^+ \to K^+(\pi^+\pi^-)_S\,\ell^+\ell^-) \times \frac{3}{2} = (0.06 \pm 0.04) \times 10^{-7}$$

  [we assume a flat differential rate in the [14.18,19] bin to obtain the q2>15 GeV² branching ratio]

- $K(n\pi)_{n>2}$ contribution (rough guess):

$$\mathcal{B}(\bar{B} \to K(n\pi)_{n>2}\mu\mu)[>15] \simeq (0.00 \pm 0.04) \times 10^{-7}$$

  where the uncertainty is simply lifted from the $K\pi\pi$ mode.

- The complete $K(n\pi)$ contribution is $\mathcal{B}(\bar{B} \to K(n\pi)\mu\mu)[>15] = (0.10 \pm 0.10) \times 10^{-7}$ and accounts for only about 5% of the inclusive rate at high-q²

- Combining $K^{(*)}$ and $K(n\pi)$ modes we finally obtain:

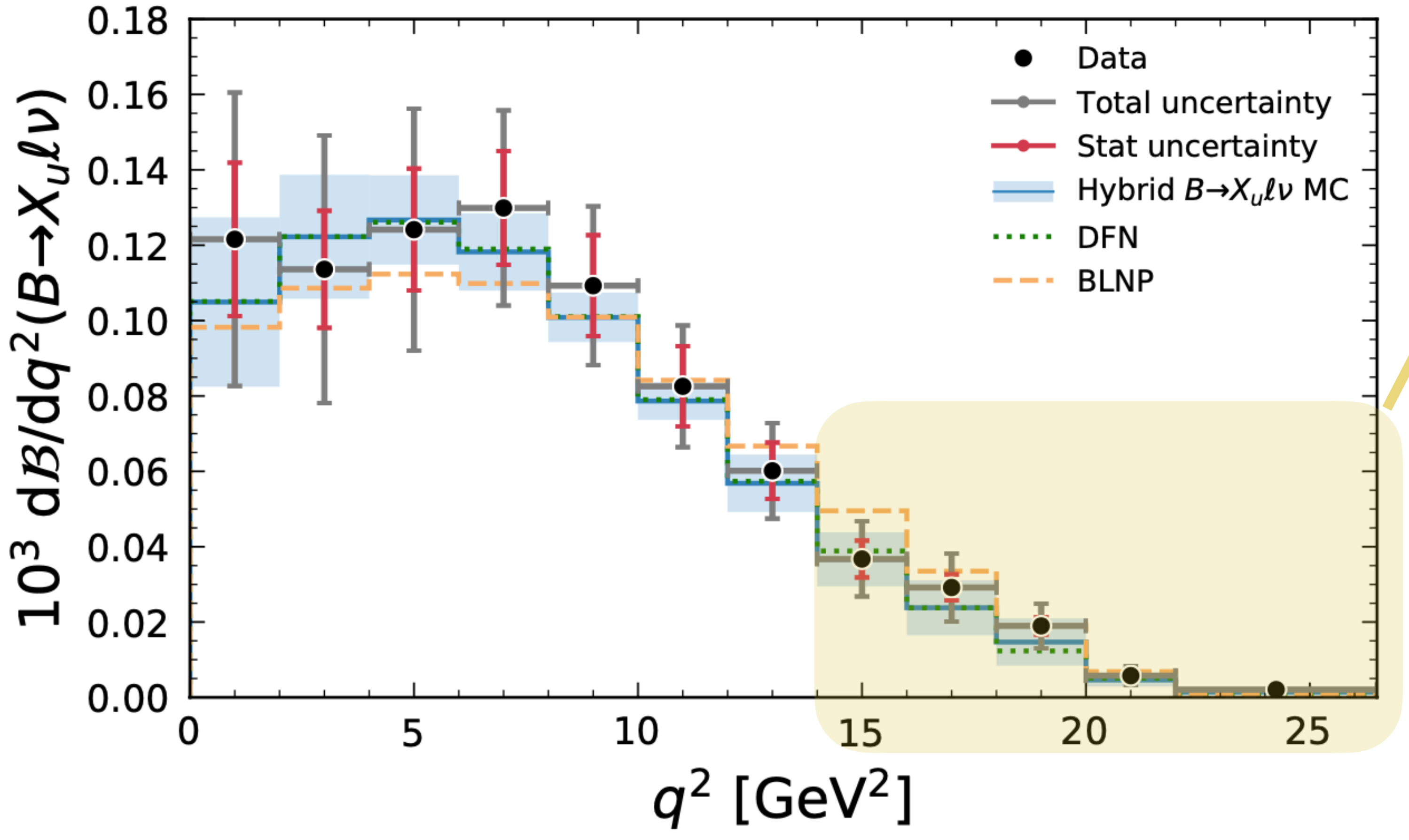
$$\mathcal{B}(\bar{B} \to X_s\mu\mu)[>15] = (2.65 \pm 0.17) \times 10^{-7}$$

Result obtained in collaboration with G. Isidori, Z, Polonsky and A. Tinari

BR at high-q²: SM

- Using the Belle [2107.13855] measurement of the $B \to X_u \ell \nu$ q^2 spectrum we can convert our SM prediction for the ratio $\mathscr{R}(q_0^2)$ into a "experiment assisted prediction" for the high-q² branching ratio.
- This SM prediction can be used to discuss compatibility with the exclusive anomalies under the assumption of no New Physics in $B \to X_u \ell \nu$

Belle, 711 fb^{-1}



$$\mathscr{B}(\bar{B} \to X_u \ell \bar{\nu})[>14.4]_{\rm exp} = (1.76 \pm 0.32) \times 10^{-4} \quad [18.2\%]$$

$$\mathscr{B}(\bar{B} \to X_u \ell \bar{\nu})[>15]_{\rm exp} = (1.52 \pm 0.28) \times 10^{-4} \quad [18.4\%]$$

$$\Downarrow$$

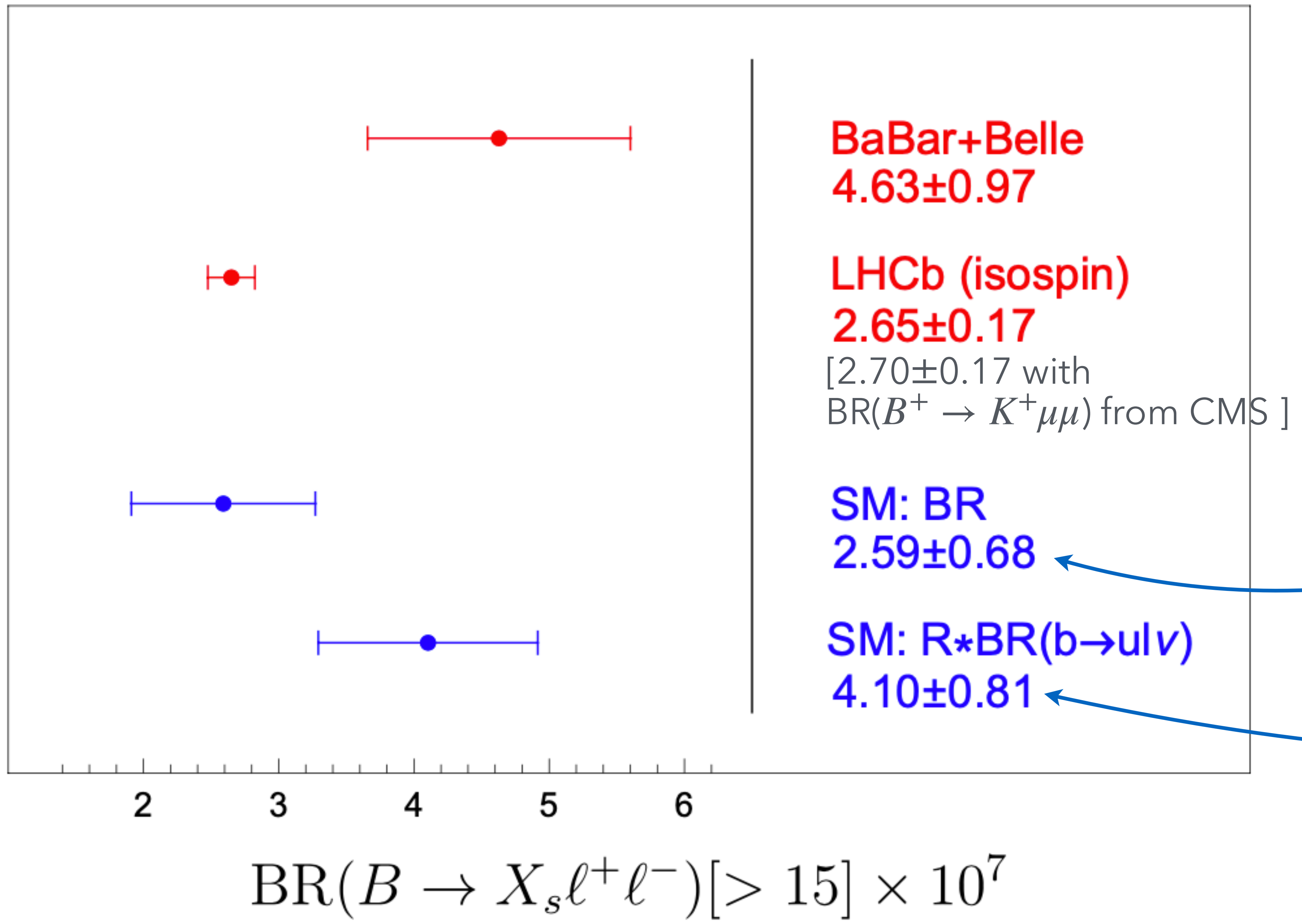
$$\mathscr{B}[>14.4]_{{\rm SM},\mathscr{R}} = \mathscr{R}(14.4) \times \mathscr{B}(B \to X_u \ell \bar{\nu})[>14.4]_{\rm exp} = (4.58 \pm 0.89) \times 10^{-7} \quad [19.4\%]$$

$$\mathscr{B}[>15]_{{\rm SM},\mathscr{R}} = \mathscr{R}(15) \times \mathscr{B}(B \to X_u \ell \bar{\nu})[>15]_{\rm exp} = (4.10 \pm 0.81) \times 10^{-7} \quad [19.8\%]$$

- The total uncertainty is dominated by the $B \to X_u \ell \nu$ partial rate

BR at high-q²: SM vs experiment

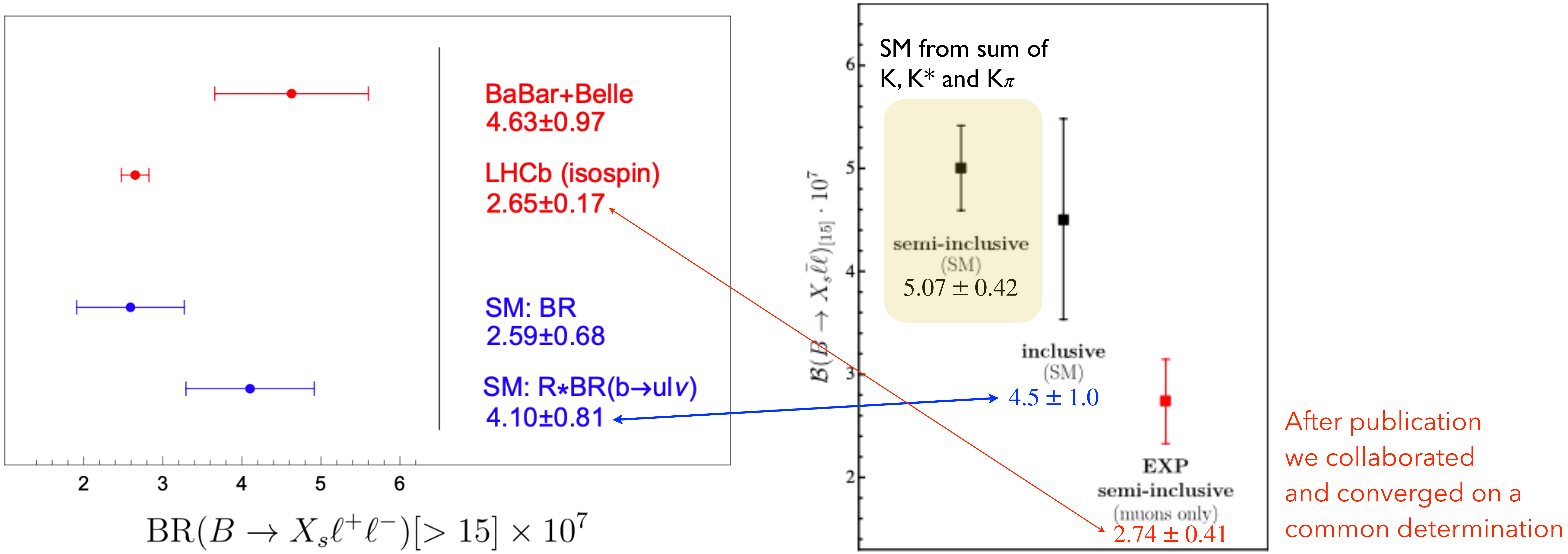
- Let's begin putting all this information together by "rescaling" the BaBar and Belle inclusive measurements to something that can be directly compared to the LHCb one: BaBar (with QED, $q_0^2 = 14.2$), Belle (with QED, $q_0^2 = 14.4$), LHCb (no QED, $q_0^2 = 15$).
- The required rescaling factors are: $\left(\frac{\mathcal{B}[>14.4]_{\text{with QED}}}{\mathcal{B}[>14.2]_{\text{with QED}}}\right)_{\text{SM}} = 0.96$ and $\left(\frac{\mathcal{B}[>15]_{\text{no QED}}}{\mathcal{B}[>14.4]_{\text{with QED}}}\right)_{\text{SM}} = 0.97$



- The picture that emerges is not clear: there are tensions between the two experimental and the two theoretical determinations!
- The two SM predictions are dominated by power corrections and by the experimental $b \to u \ell \nu$ rate, respectively.

BR at high-q^2: SM vs experiment

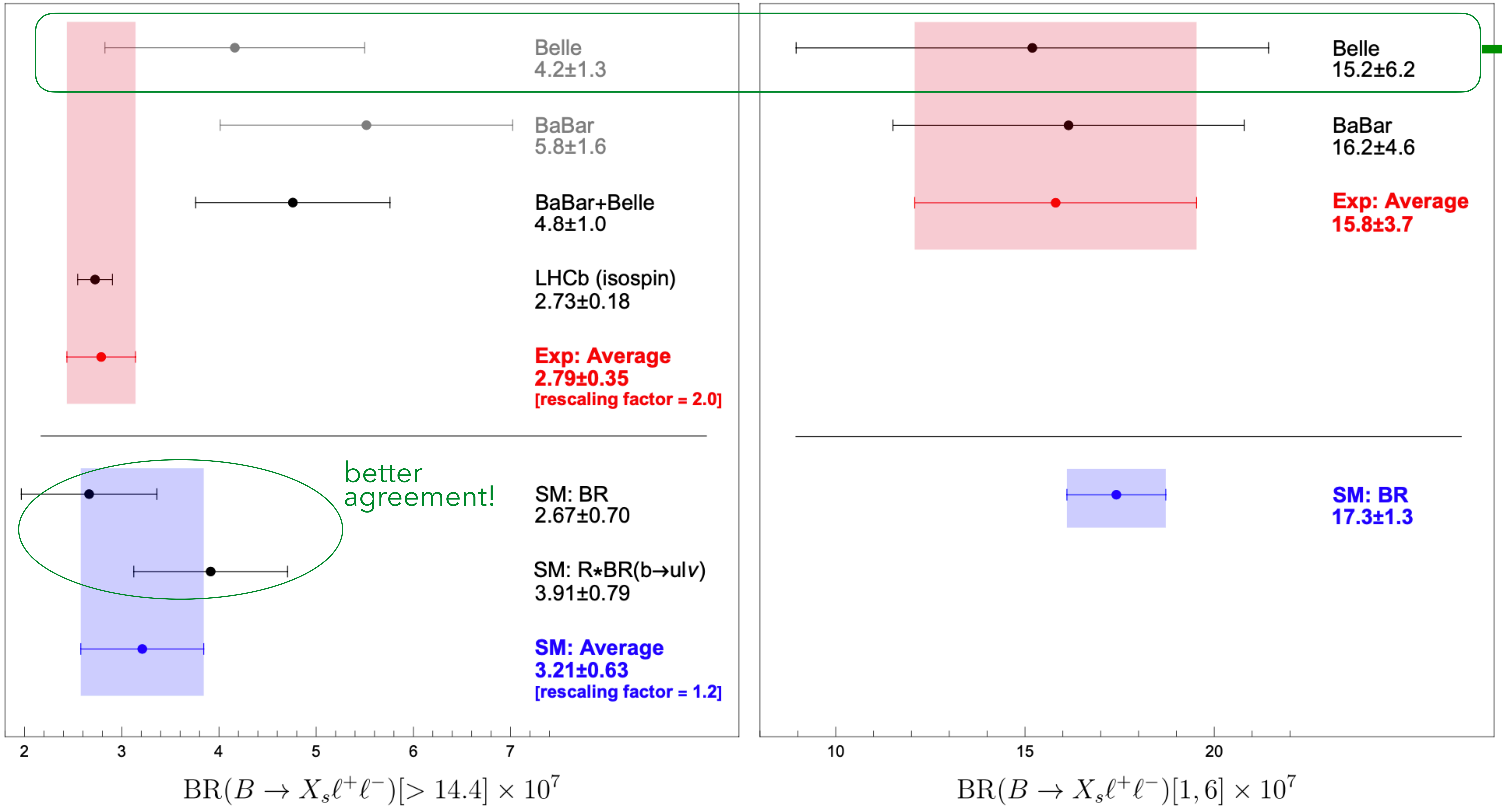
- Comparison with the Isidori, Polonsky, Tinari analysis [2305.03076]



- Both analyses find a tension between the LHCb "measurement" and the SM from $\mathcal{R} \times \mathcal{B}_{b\to u\ell\nu}$
- The tension between the semi-inclusive (SM) and LHCb is a restatement of the anomalies
- The difference between the $\mathcal{R} \times \mathcal{B}_{b\to u\ell\nu}$ determinations originates from NLO $Q_{1,2} - Q_{7,9}$ interference ($-9\,\%$) contributions and from long distance $c\bar{c}$ effects ($-4\,\%$).

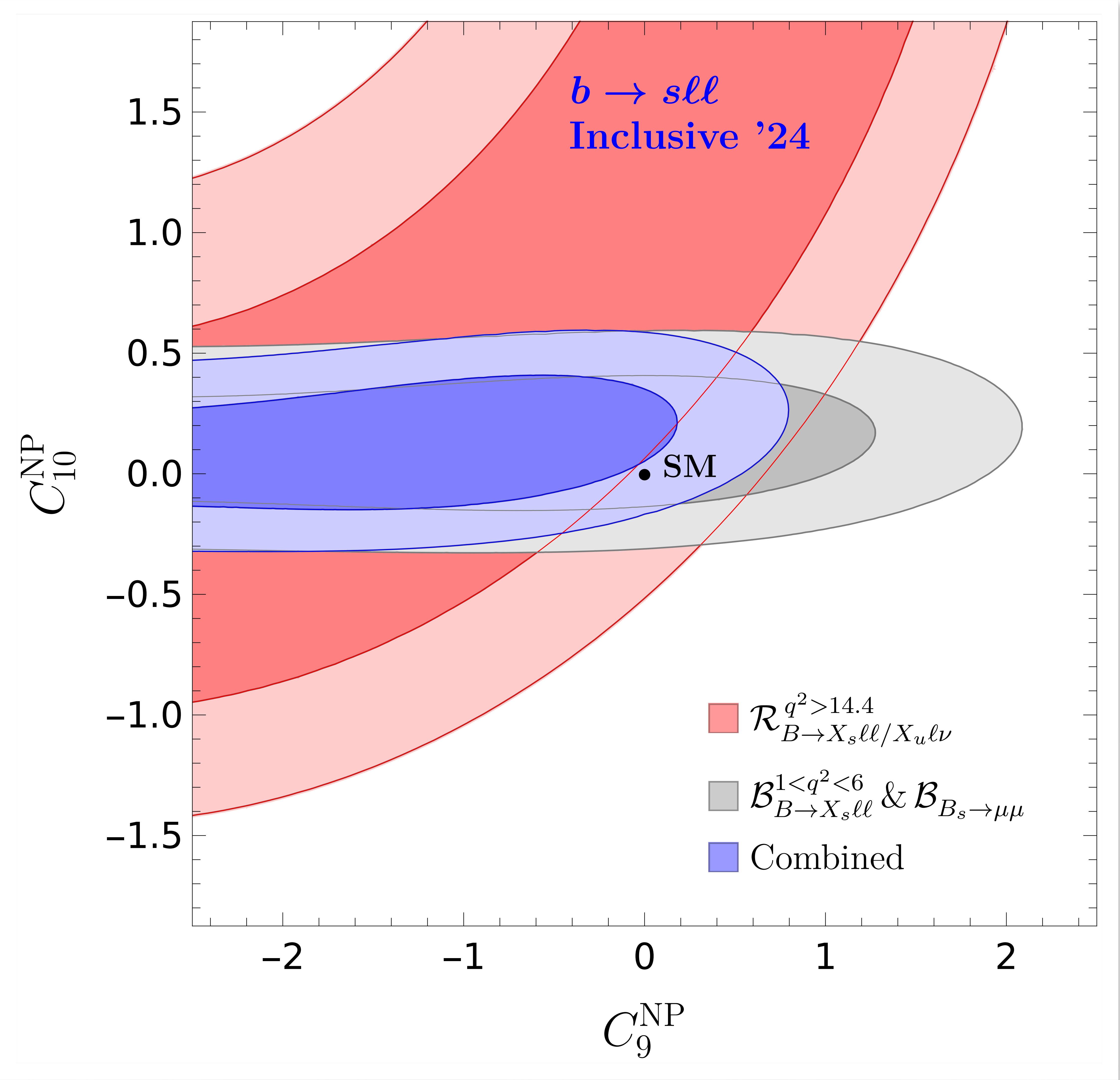
BR at high-q^2: SM vs experiment

- All results corresponding to a cut at 14.4 GeV^2 (the LHCb "measurement" has been rescaled)
- A lower q^2 cut corresponds to a larger hadronic phase space (implying better OPE behavior)



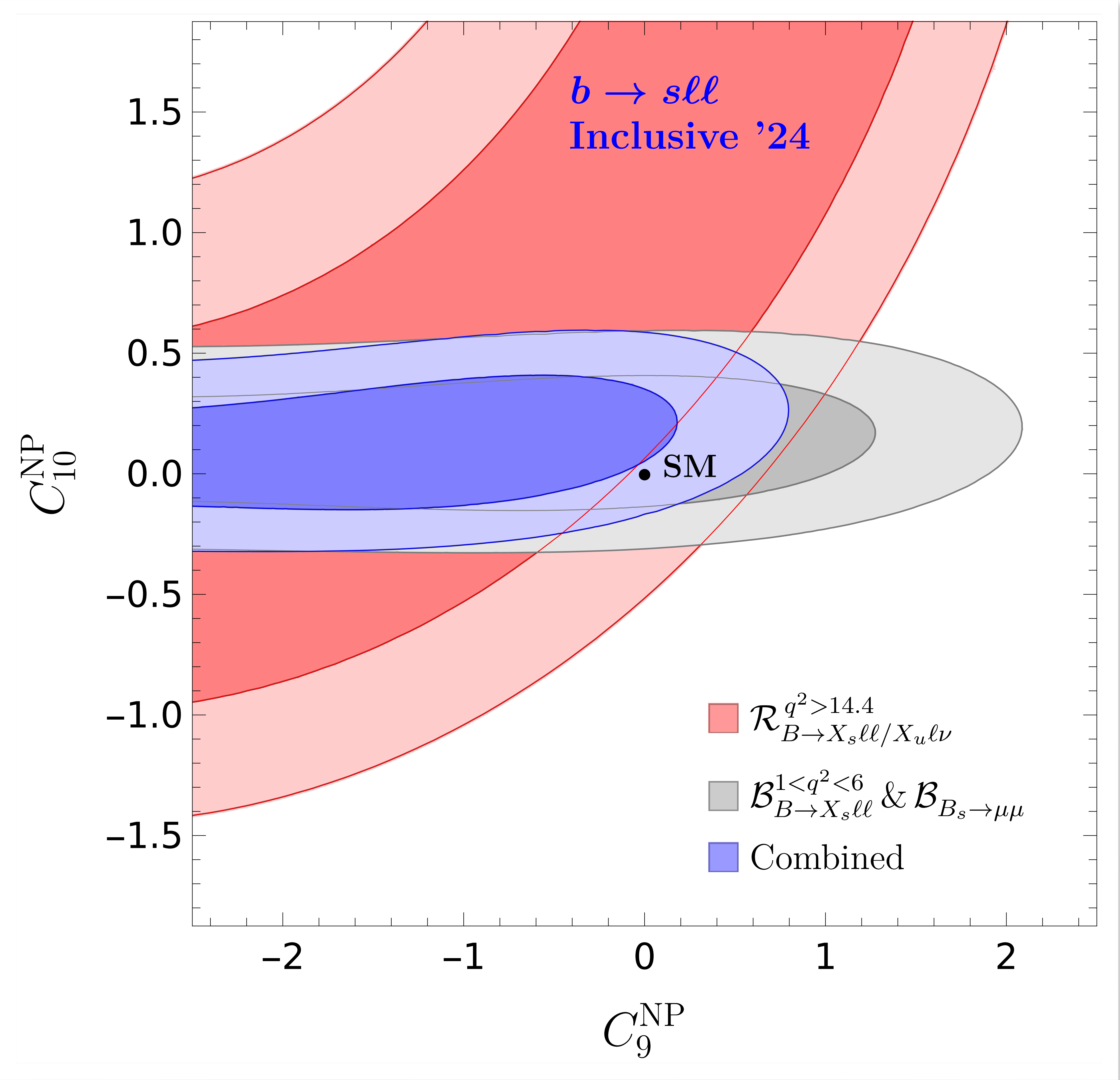
- Based on 140M $B\bar{B}$ pairs but about **800M** have been collected! Imperative to use the whole Belle dataset in Belle+Belle II analyses
- The experimental average requires a PDG rescaling factor of 2!
- The SM average includes a $20\,\%$ correlation: the $\mathscr{R} \times$ BR error is dominated by the $b \to u \ell \nu$ experimental rate

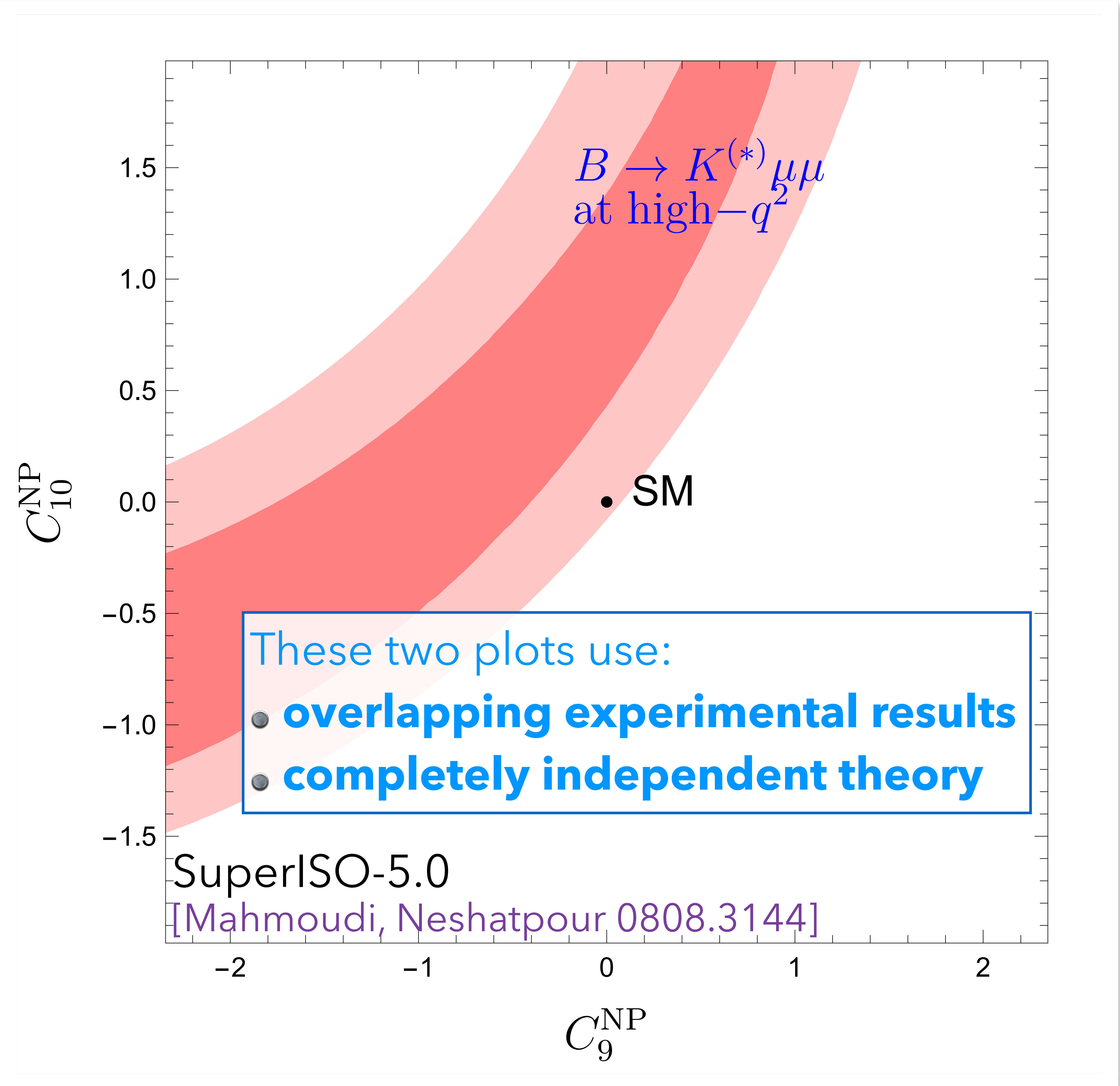
Current constraints



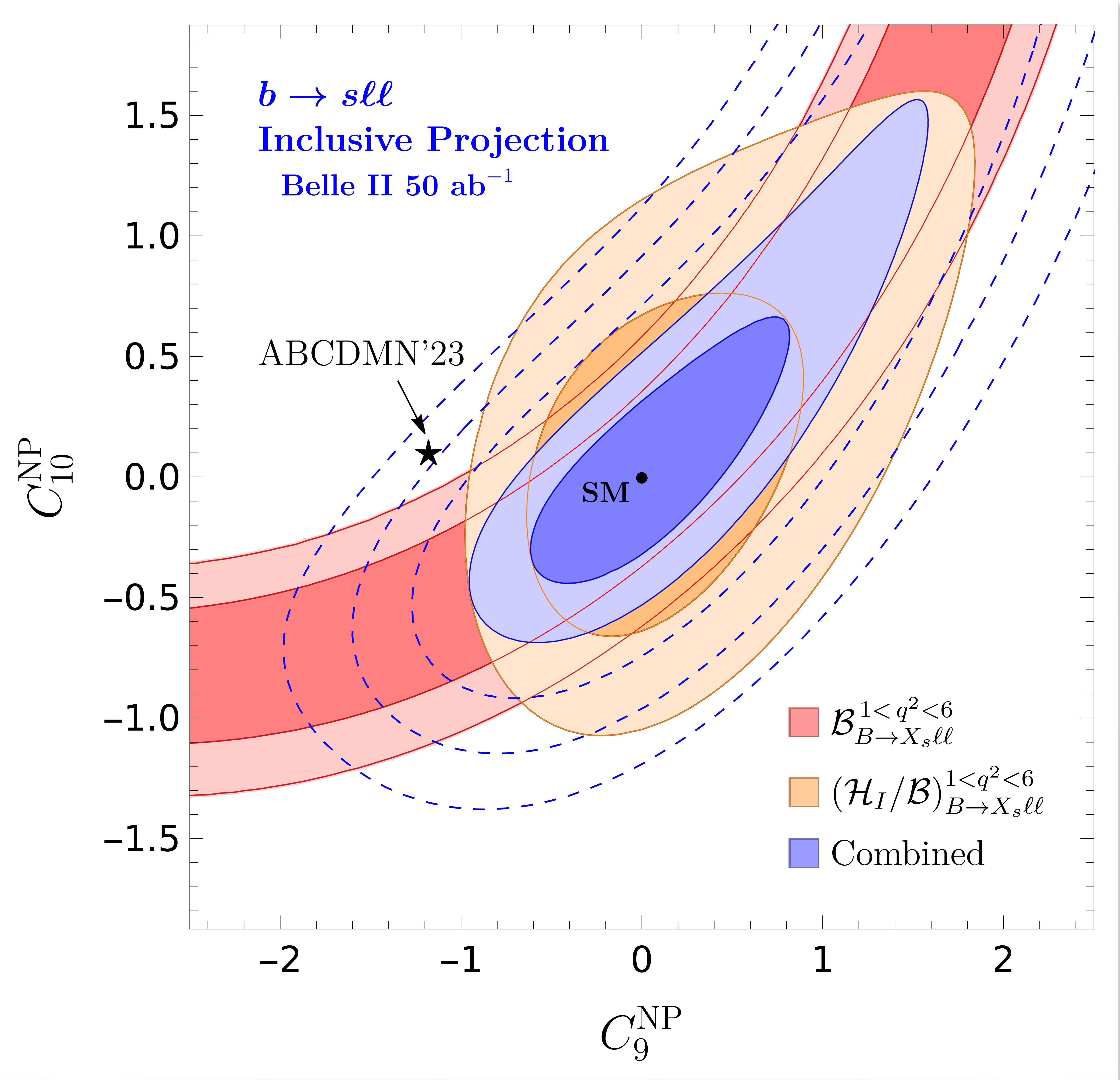
- Inputs:
  - $\mathrm{BR}(B_s \to \mu\mu)^{\mathrm{LHC}}$
  - $\mathrm{BR}(B \to X_s\ell\ell)^{\mathrm{B-factories}}_{\mathrm{low}}$
  - $\mathrm{BR}(B \to X_s\ell\ell)^{\mathrm{LHCb}}_{\mathrm{high}}$
  - $\mathrm{BR}(B \to X_u\ell\nu)^{\mathrm{Belle}}_{\mathrm{high}}$

Current constraints: inclusive vs exclusive



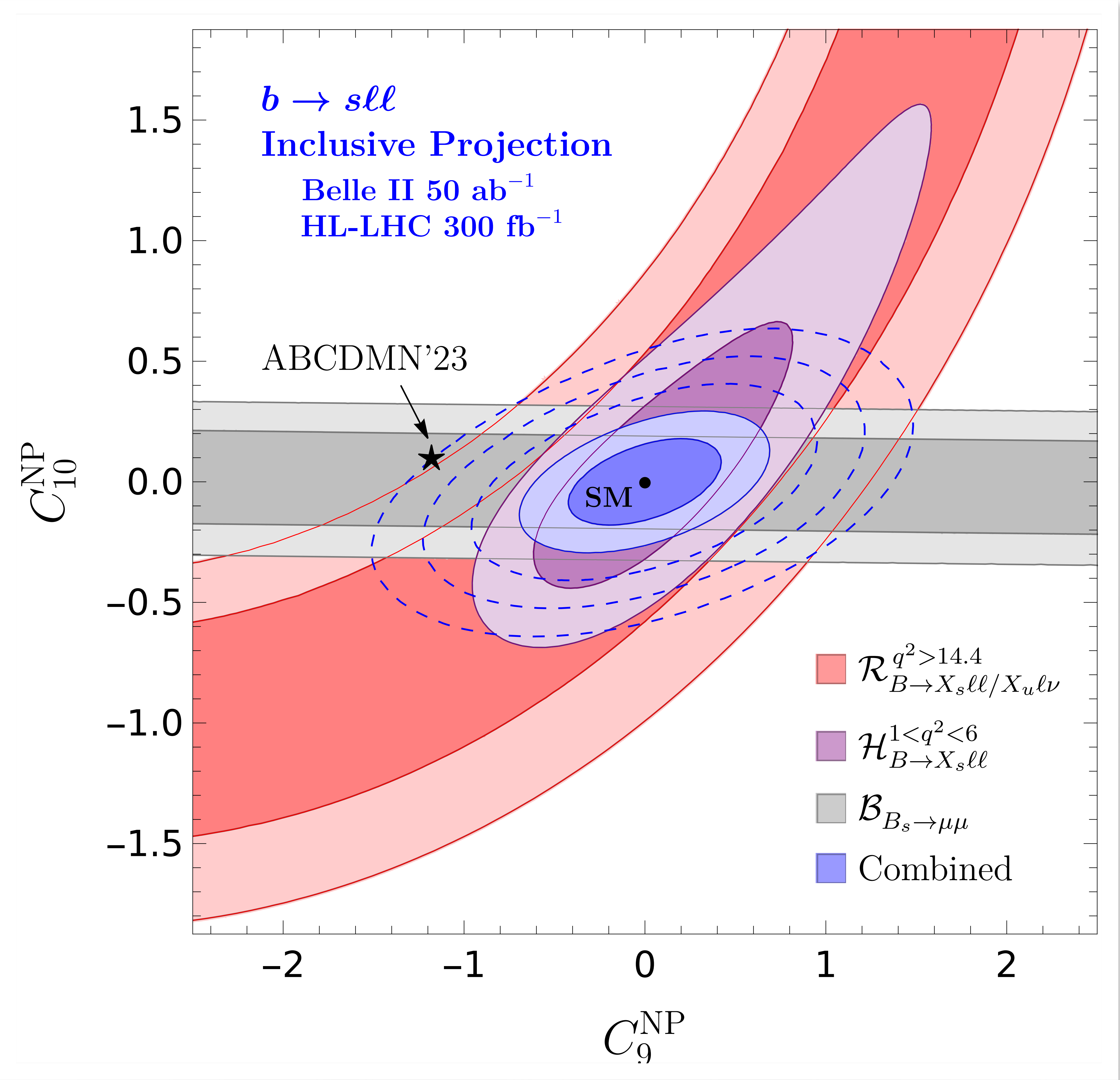


Projected constraints: Belle II



- Focus on low-q^2 where the inclusive OPE is better
- Use of normalized angular observables ($H_I/\mathscr{B}$), lowers impact of the M_X cut in the low-q^2 region
- Dashed contours correspond to 3σ, 4σ and 5σ
- ★ is the exclusive best fit from ABCDMN'23
- Low-q^2 observables at Belle-II can confirm current anomalies at 4σ

Projected constraints: LHCb & Belle II



- We assume $\delta(B_s \to \mu\mu) = 4.8\,\%$ corresponding to 300 fb^{-1} at the HL-LHC

- Projected uncertainty on $\mathscr{R}(14.4)$ is obtained combining:

$$\delta\mathscr{B}_{bs\ell\ell}[>14.4] = \sqrt{(2.6\%_{\rm stat})^2 + (3.9\%_{\rm syst})^2} = 4.7\,\%$$

$$\delta\mathscr{B}_{bu\ell\nu}[>14.4] = 5.2\,\%$$

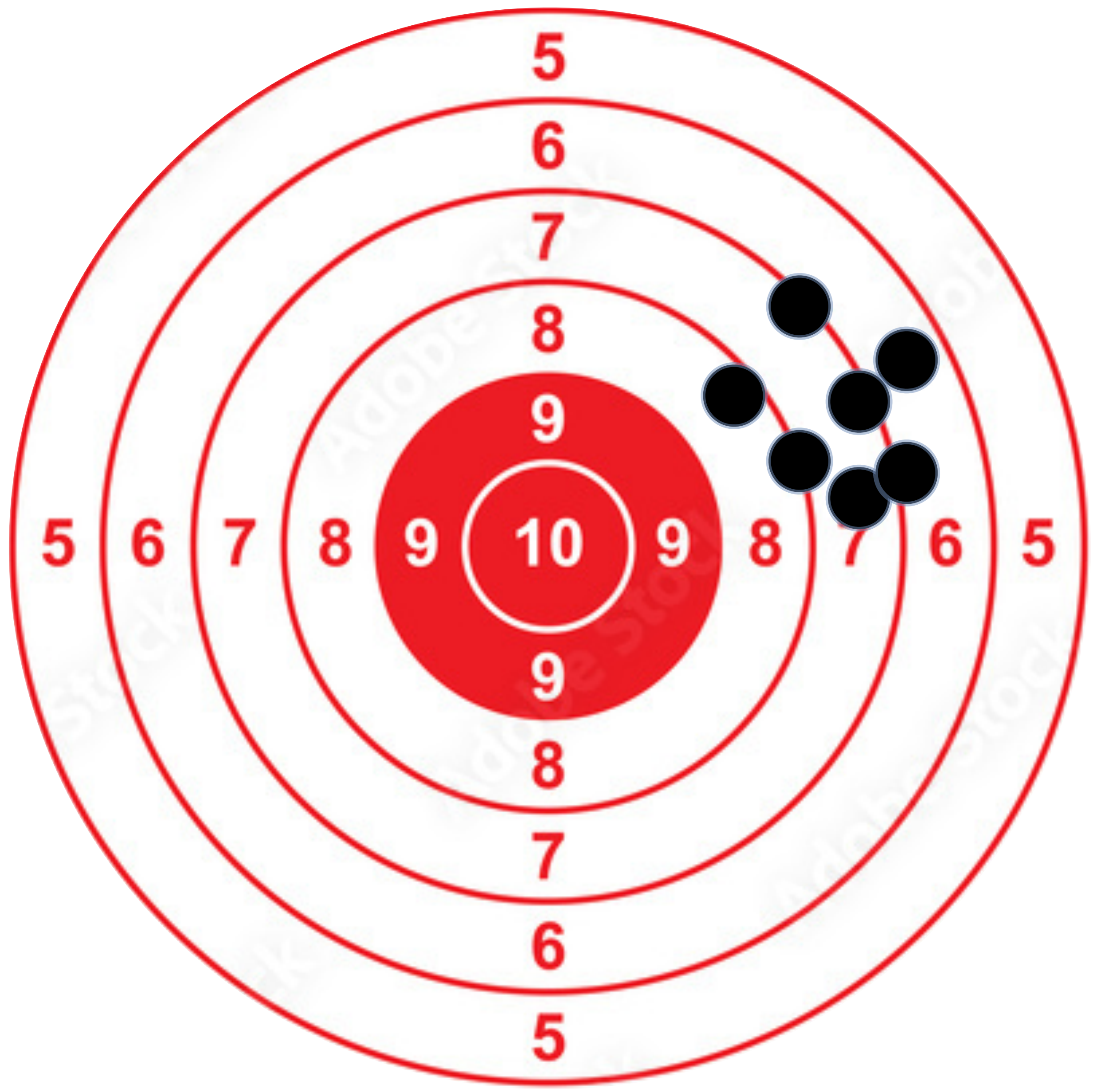
$$\Longrightarrow \delta\mathscr{R}(14.4) = 7.0\,\%$$

- Inclusion of high-q^2 observables allows to confirm the exclusive anomalies at the 5σ level

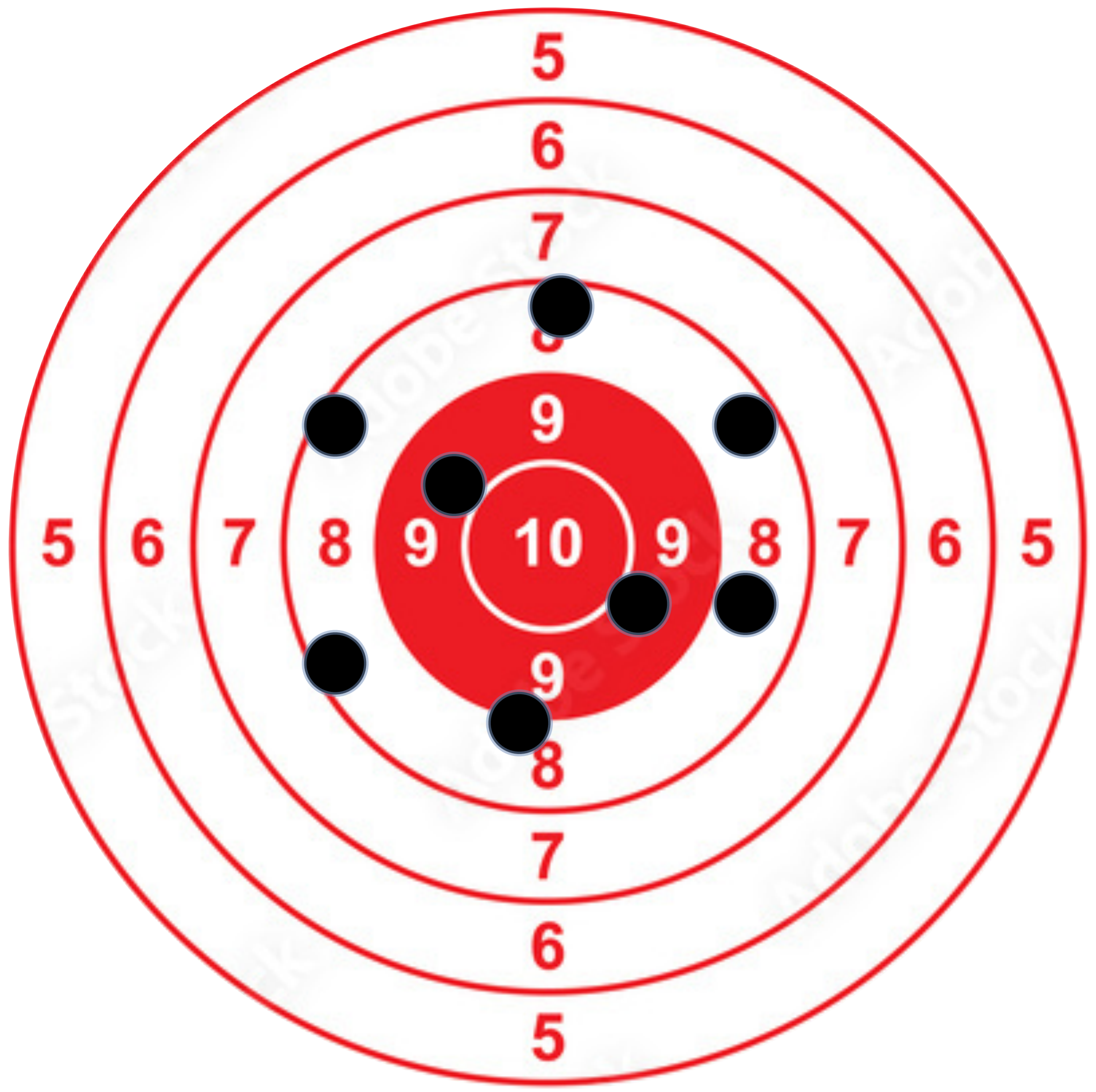
Summary

- Exclusive $b \to s\mu\mu$ modes point to a $> 5\sigma$ deviation from SM barring anomalously large non-local power corrections at low-q^2
- Inclusive modes probe the exact **same New Physics**, have **"orthogonal" theoretical issues** but are experimentally **harder to measure**
- A precise measurement of the inclusive branching ratio at high-q^2 is already at hand at LHCb
- **Steps towards the resolution of the anomalies (< 5 years time horizon):**
  - **Inclusive measurements at Belle** (800M BB pairs) **+ Belle-II** (currently 400M BB pairs)
  - **Inclusive measurement at high-q^2 at LHCB**:
    - Improved measurements of exclusive $B \to (K, K^*, K\pi, K\pi\pi)\mu\mu$ branching ratios at high-q^2 at LHCb.
    - Improved $B \to K^{(*)} J/\psi$ measurements at Belle-II are also needed (LHCb can only measure relative rates)
  - **Lattice-QCD** determination of charming penguins (even an upper limit would be useful)
  - Possible **breakthrough in understanding anomalous thresholds** and their impact on non-local power corrections
  - **Inclusive measurement at low-q^2 at LHCB**

Summary



$b \to s\ell\ell$ exclusive



$b \to s\ell\ell$ inclusive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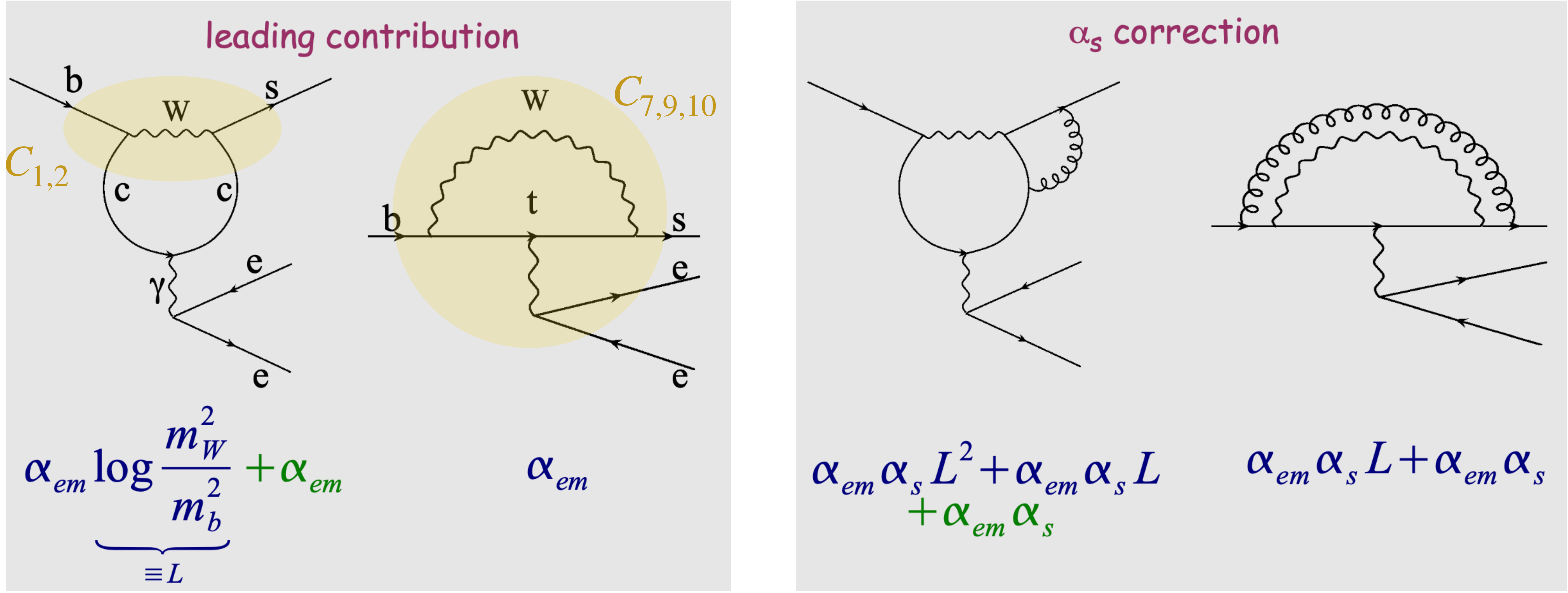
Inclusive: theory

- The perturbative expansion has two peculiar features:
  - the amplitude is proportional to $\alpha_e(\mu)$
  - the one-loop matrix element of $O_{1,2}$ is "super-leading"



$C_1\langle O_1\rangle + C_2\langle O_2\rangle \qquad C_7\langle O_7\rangle + C_9\langle O_9\rangle + C_{10}\langle O_{10}\rangle$

$$\eta \equiv \frac{\alpha_s(\mu_0)}{\alpha_s(\mu_b)} = 1 + \beta_s^{(00)}\frac{\alpha_s(\mu_0)}{4\pi}\log\frac{\mu_b^2}{\mu_0^2} \sim O(1) \Longrightarrow \log\frac{\mu_b^2}{\mu_0^2} \sim \frac{1}{\alpha_s(\mu_0)}$$

$$\frac{\alpha_e(\mu_0)}{\alpha_e(\mu_b)} = 1 + \beta_e^{(00)}\frac{\alpha_e(\mu_0)}{4\pi}\log\frac{\mu_b^2}{\mu_0^2} \sim 1 + \frac{\alpha_e(\mu_0)}{\alpha_s(\mu_0)}$$

Expansion in α_s and $\kappa = \alpha_e/\alpha_s$

Inclusive: theory

- Structure of the amplitude ($\kappa = \alpha_{em}/\alpha_s$ and $\tilde{\alpha}_s = \alpha_s/4\pi$):

$$A = \kappa\left[A_{LO} + \tilde{\alpha}_s A_{NLO} + \tilde{\alpha}_s^2 A_{NNLO} + \tilde{\alpha}_s^3 A_{N^3LO} + O(\tilde{\alpha}_s^4)\right]$$
$$+\kappa^2\left[A_{LO}^{em} + \tilde{\alpha}_s A_{NLO}^{em} + \tilde{\alpha}_s^2 A_{NNLO}^{em} + \tilde{\alpha}_s^3 A_{N^3LO}^{em} + O(\tilde{\alpha}_s^4)\right] + O(\kappa^3)$$

with $A_{LO}^{em} \lesssim A_{LO} \sim 0.03$ and $A_{NLO} \sim 4$

include only terms enhanced by $m_t^2/(M_W^2 \sin^2\theta_W)$ and $\log(m_b^2/m_\ell^2)$

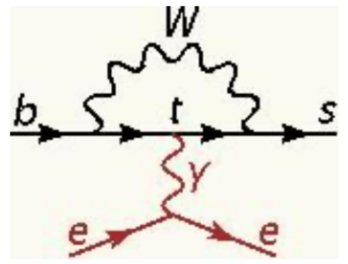
- Decay width:

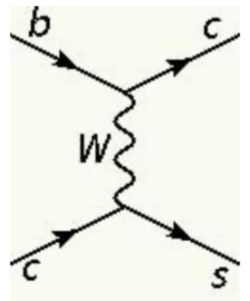
$$|A|^2 = \kappa^2\left[A_{LO}^2 + \tilde{\alpha}_s 2A_{LO}A_{NLO} + \tilde{\alpha}_s^2 A_{NLO}^2\right]$$
$$+\kappa^2\left[\tilde{\alpha}_s^2 A_{LO}A_{NNLO} + \alpha_s^3\left(2A_{NLO}A_{NNLO} + 2A_{LO}A_{N^3LO}\right)\right]$$
$$+\kappa^3\Big[2A_{LO}A_{LO}^{em} + \tilde{\alpha}_s\left(2A_{NLO}A_{LO}^{em} + 2A_{LO}A_{NLO}^{em}\right)$$
$$+\tilde{\alpha}_s^2\left(2A_{NLO}A_{NLO}^{em} + 2A_{NNLO}A_{LO}^{em} + A_{LO}A_{NNLO}^{em}\right)$$
$$+\tilde{\alpha}_s^3\left(2A_{NLO}A_{NNLO}^{em} + 2A_{NNLO}A_{NLO}^{em} + 2A_{N^3LO}A_{LO}^{em} + 2A_{LO}A_{N^3LO}^{em}\right)\Big] + O(\kappa^4)$$

Inclusive: theory

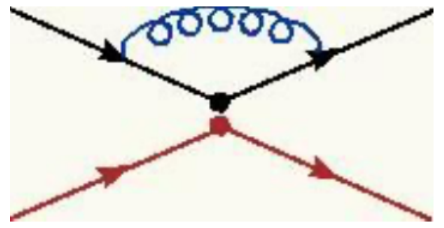
- QCD at NLO ($A_{\rm LO}$, $A_{\rm NLO}$)

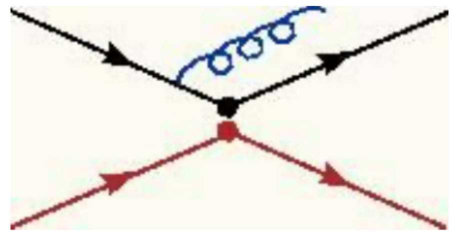
WCs:

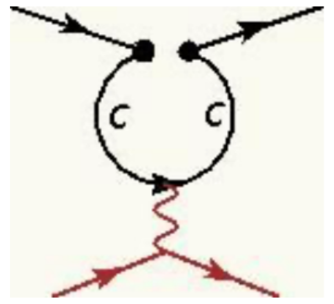




MEs:



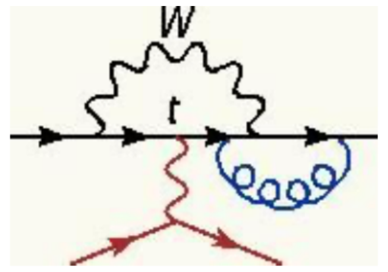


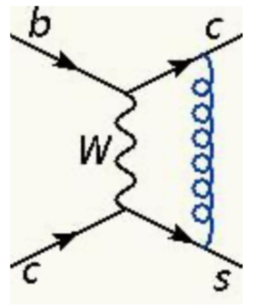


Misiak
Buras, Münz

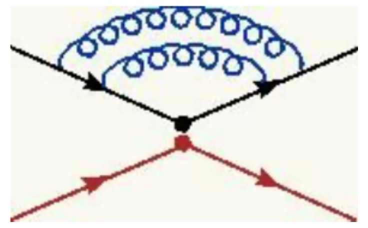
- QCD at NNLO ($A_{\rm NNLO}$)

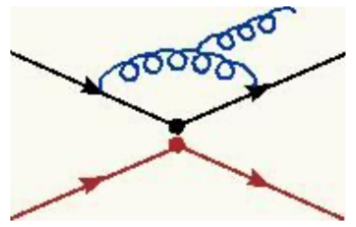
WCs:

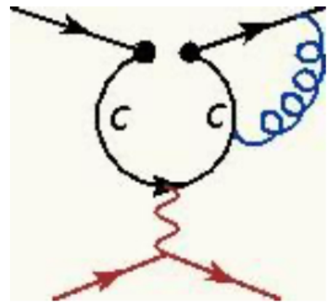




MEs:





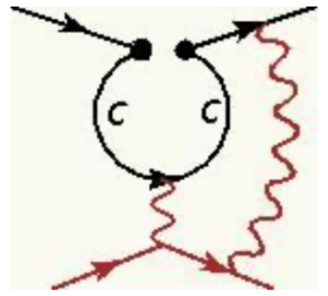


Bobeth, Misiak, Urban

Asatrian, Asatryan, Greub Walker
Ghinculov, Hurth, Isidori, Yao
Bobeth, Gambino, Gorbahn, Haisch
de Boer

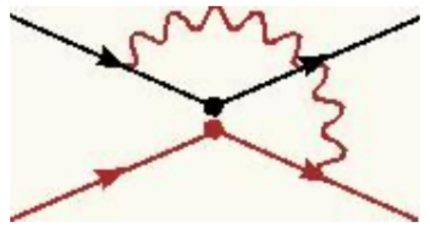
- QED at NLO ($A^{\rm em}_{\rm LO}$, $A^{\rm em}_{\rm NLO}$)

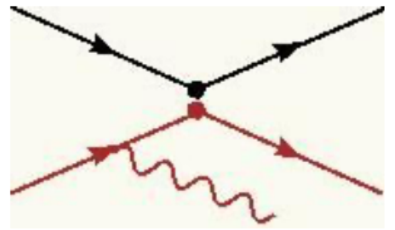
WCs:



Bobeth, Gambino, Gorbahn, Haisch

MEs:





Huber, Lunghi, Misiak, Wyler

Inclusive: : m_b scheme and normalization

- b quark mass scheme
  - $\Gamma(b \to X_s \ell\ell)$ is a renormalon free observable but $m_b^{\rm pole}$ is not
    [see e.g.: Beneke - Renormalons]
  - These spurious renormalon ambiguities can be removed by analytically converting $m_b^{\rm pole}$ to a short distance scheme (e.g. $m_b^{\rm 1S}$ or $m_b^{\rm kin}$)
  - We adopt the $1S$ scheme using the Upsilon expansion

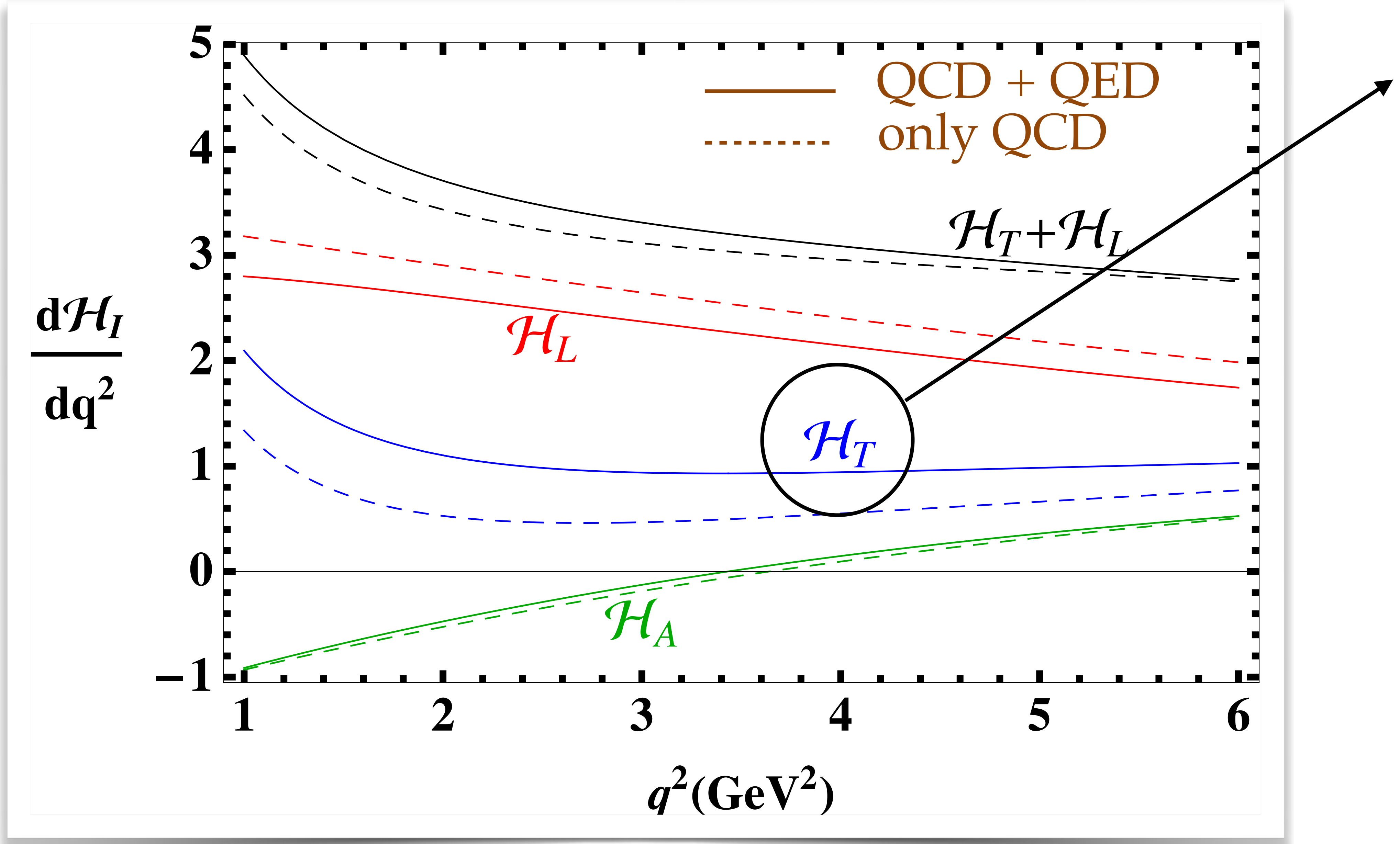
    [Hoan, Ligeti, Manohar]
- Choice of normalization
  - In order to remove an overall m_b^5 prefactor the rate is usually normalized to either the total $B \to X_u \ell \nu$ or $B \to X_c \ell \nu$ rate.
  - We adopt the former:

$$\Gamma(B \to X_s \ell\ell) = {\rm BR}(B \to X_c \ell \nu) \left| \frac{V_{ts}^* V_{tb}}{V_{cb}} \right|^2 \frac{1}{C} \frac{\Phi_{\ell\ell}}{\Phi_u}$$

where $C = \left| \frac{V_{ub}}{V_{cb}} \right|^2 \frac{\Gamma(B \to X_c \ell \nu)}{\Gamma(B \to X_u \ell \nu)}$ and $\Phi_{\ell\ell,u}$ are free of CKM angles.

Inclusive: QED radiation

- Impact of collinear photon radiation is huge on some observables
- Cross check with Monte Carlo study (EVTGEN + PHOTOS)



Shift on H_T is $\sim 70\,\%$!

H_T is smaller than H_L ($\hat{s} < 0.3$ and $C_7 < 0$):

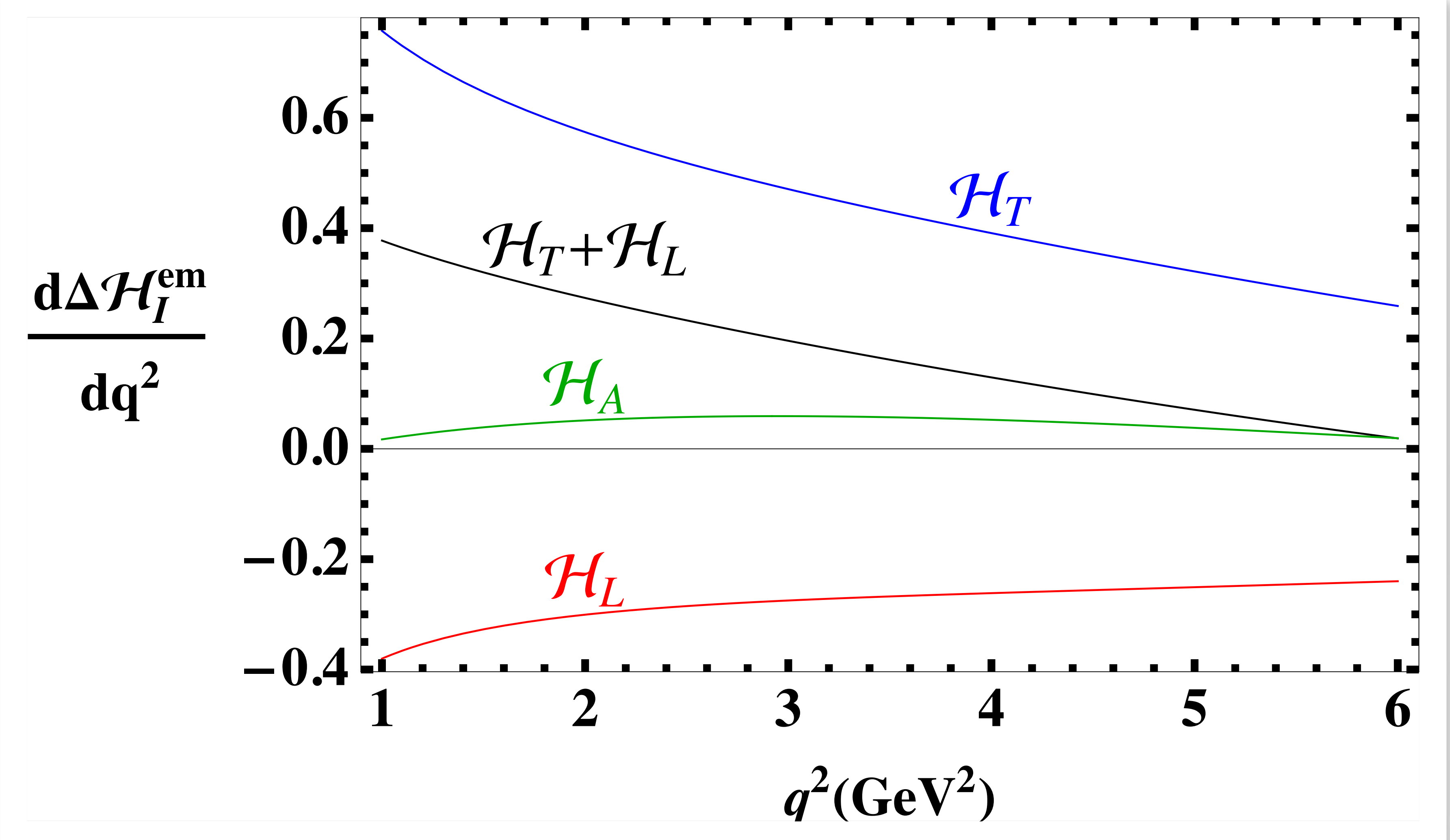
$$H_T \sim 2\hat{s}(1-\hat{s})^2 \left[\left|C_9 + \frac{2}{\hat{s}} C_7\right|^2 + |C_{10}|^2 \right]$$

$$H_L \sim (1-\hat{s})^2 \left[|C_9 + 2C_7|^2 + |C_{10}|^2 \right]$$

	$q^2 \in [1,6]$ GeV^2			$q^2 \in [1,3.5]$ GeV^2			$q^2 \in [3.5,6]$ GeV^2		
	$\frac{O_{[1,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,6]}}{O_{[1,6]}}$	$\frac{O_{[1,3.5]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,3.5]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,3.5]}}{O_{[1,3.5]}}$	$\frac{O_{[3.5,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[3.5,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[3.5,6]}}{O_{[3.5,6]}}$
$\mathcal{B}$	100	5.1	5.1	54.6	3.7	6.8	45.4	1.4	3.1
$\mathcal{H}_T$	19.5	14.1	72.5	9.5	8.8	92.1	10.0	5.4	53.6
$\mathcal{H}_L$	80.0	-8.7	-10.9	44.7	-4.7	-10.6	35.3	-4.0	-11.3
$\mathcal{H}_A$	-3.3	1.4	-43.6	-7.2	0.8	-10.7	4.0	0.6	16.2

Inclusive: QED radiation

- We calculated the effect of collinear photon radiation and found large effects on some observables



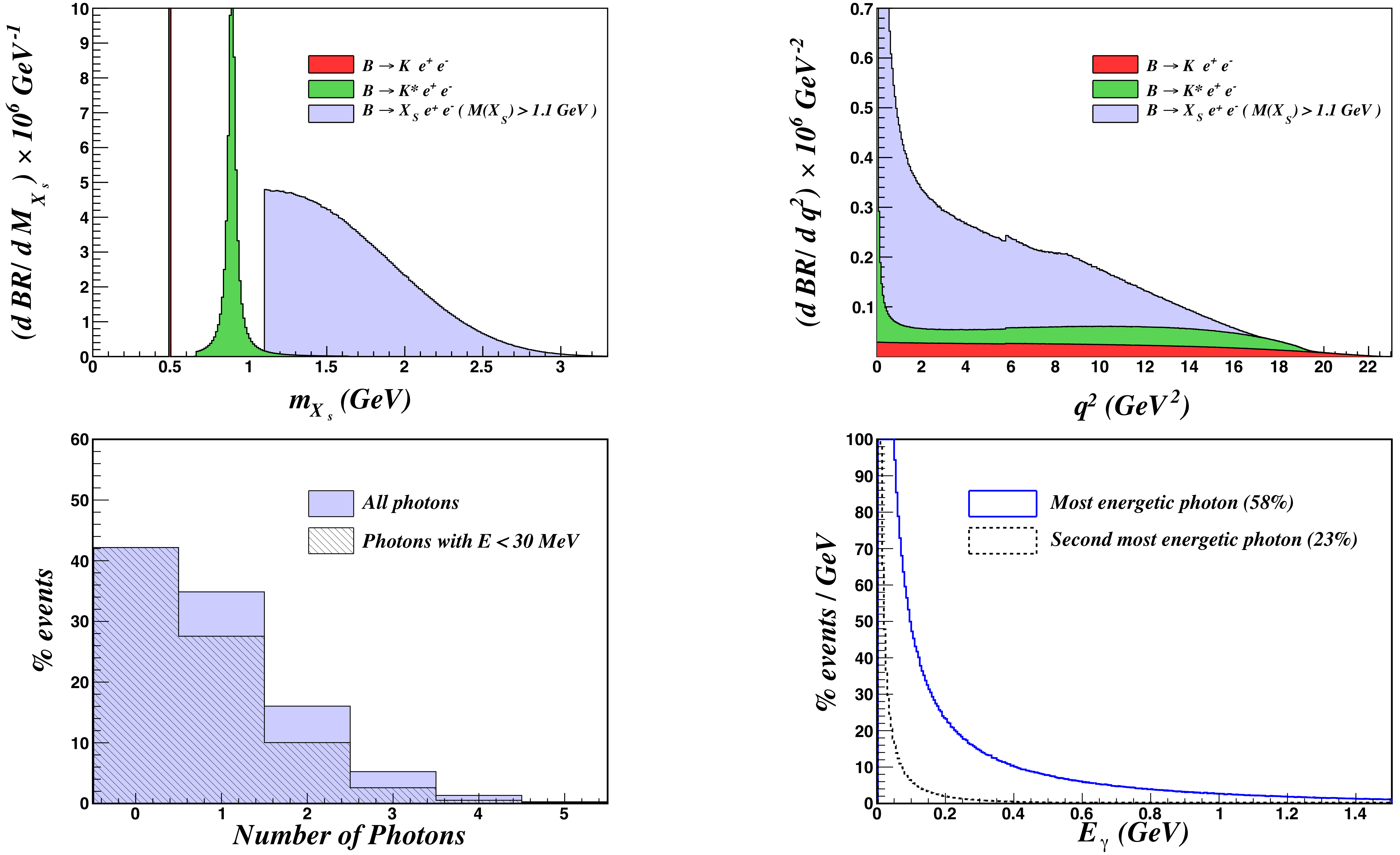
Size of QED contributions to the H_T and H_L is similar

	$q^2 \in [1,6]$ GeV^2			$q^2 \in [1,3.5]$ GeV^2			$q^2 \in [3.5,6]$ GeV^2		
	$\frac{O_{[1,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,6]}}{O_{[1,6]}}$	$\frac{O_{[1,3.5]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,3.5]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,3.5]}}{O_{[1,3.5]}}$	$\frac{O_{[3.5,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[3.5,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[3.5,6]}}{O_{[3.5,6]}}$
$\mathcal{B}$	100	5.1	5.1	54.6	3.7	6.8	45.4	1.4	3.1
$\mathcal{H}_T$	19.5	14.1	72.5	9.5	8.8	92.1	10.0	5.4	53.6
$\mathcal{H}_L$	80.0	-8.7	-10.9	44.7	-4.7	-10.6	35.3	-4.0	-11.3
$\mathcal{H}_A$	-3.3	1.4	-43.6	-7.2	0.8	-10.7	4.0	0.6	16.2

Inclusive: QED radiation (Monte Carlo)

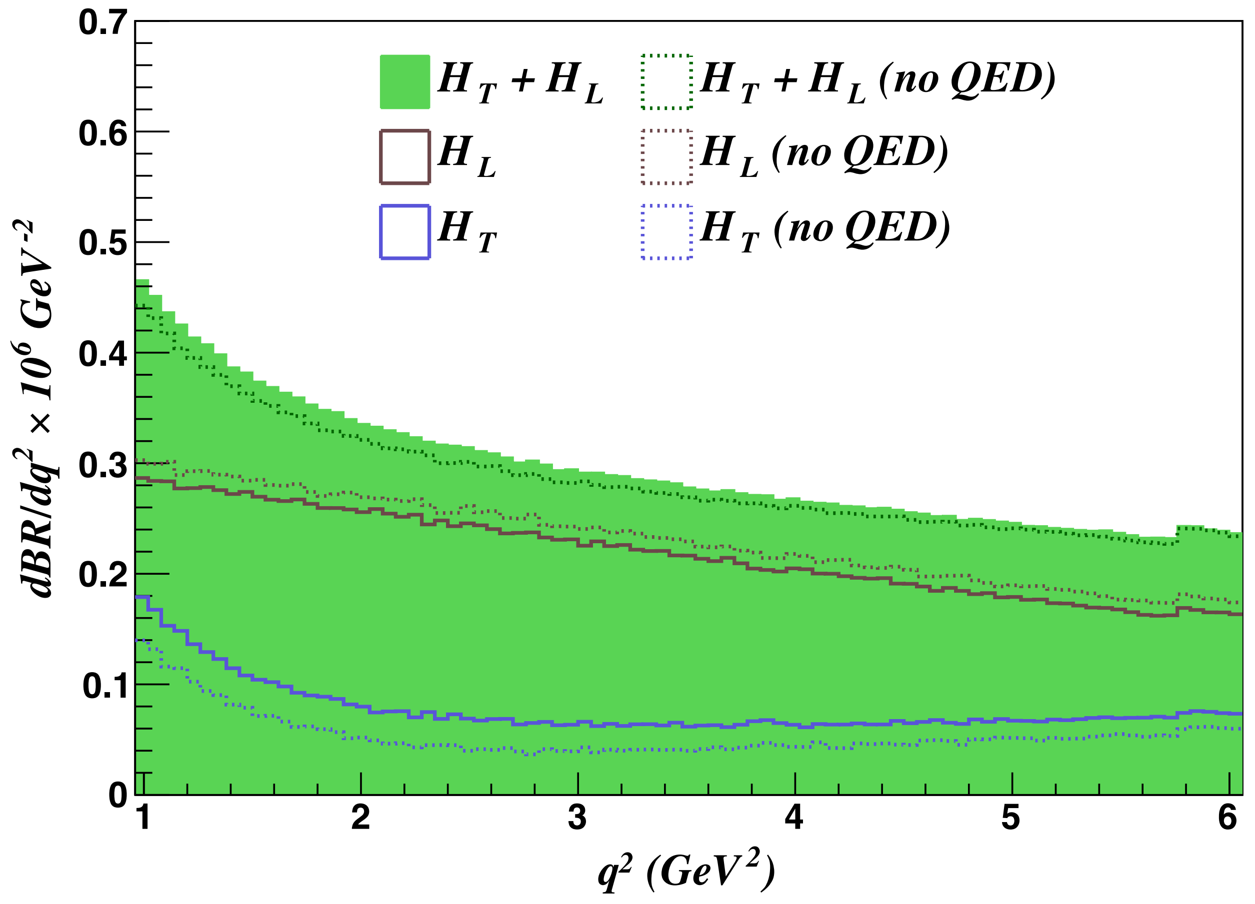
- EM effects have been calculated analytically and cross checked against Monte Carlo generated events (EVTGEN + PHOTOS)

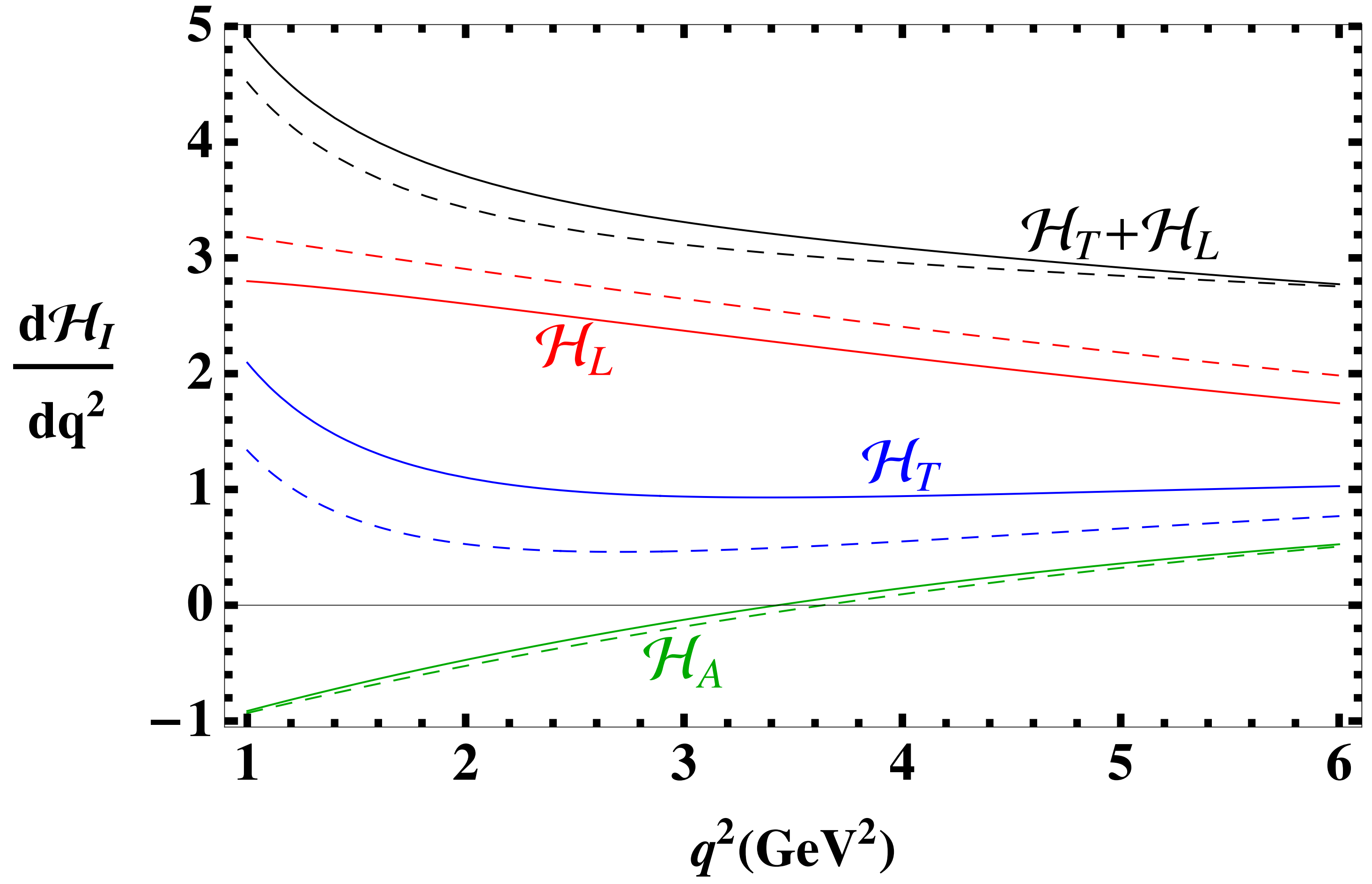
[Many thanks to K. Flood, O. Long and C. Schilling]

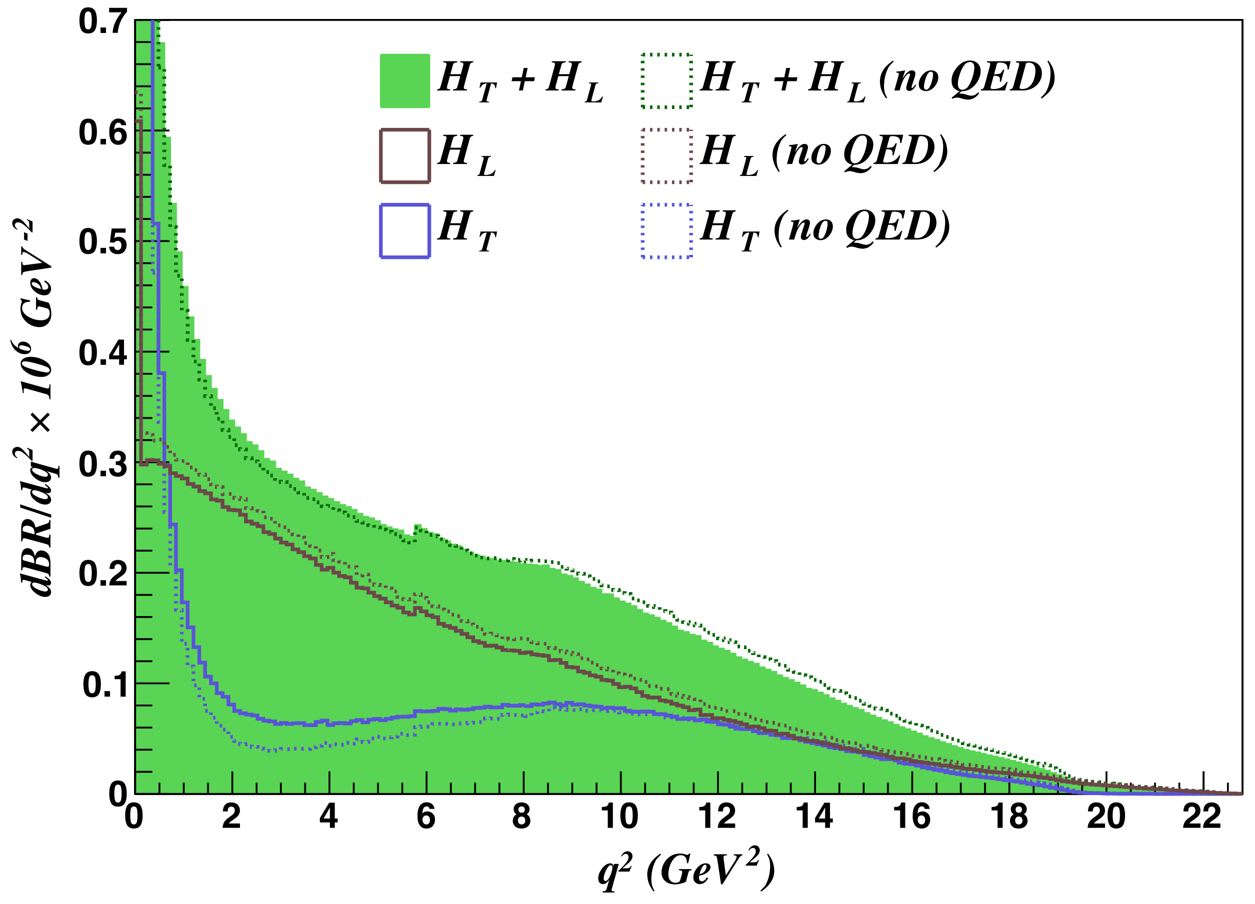


Inclusive: QED radiation (Monte Carlo)

- The Monte Carlo study reproduces the main features of the analytical results







Monte Carlo:

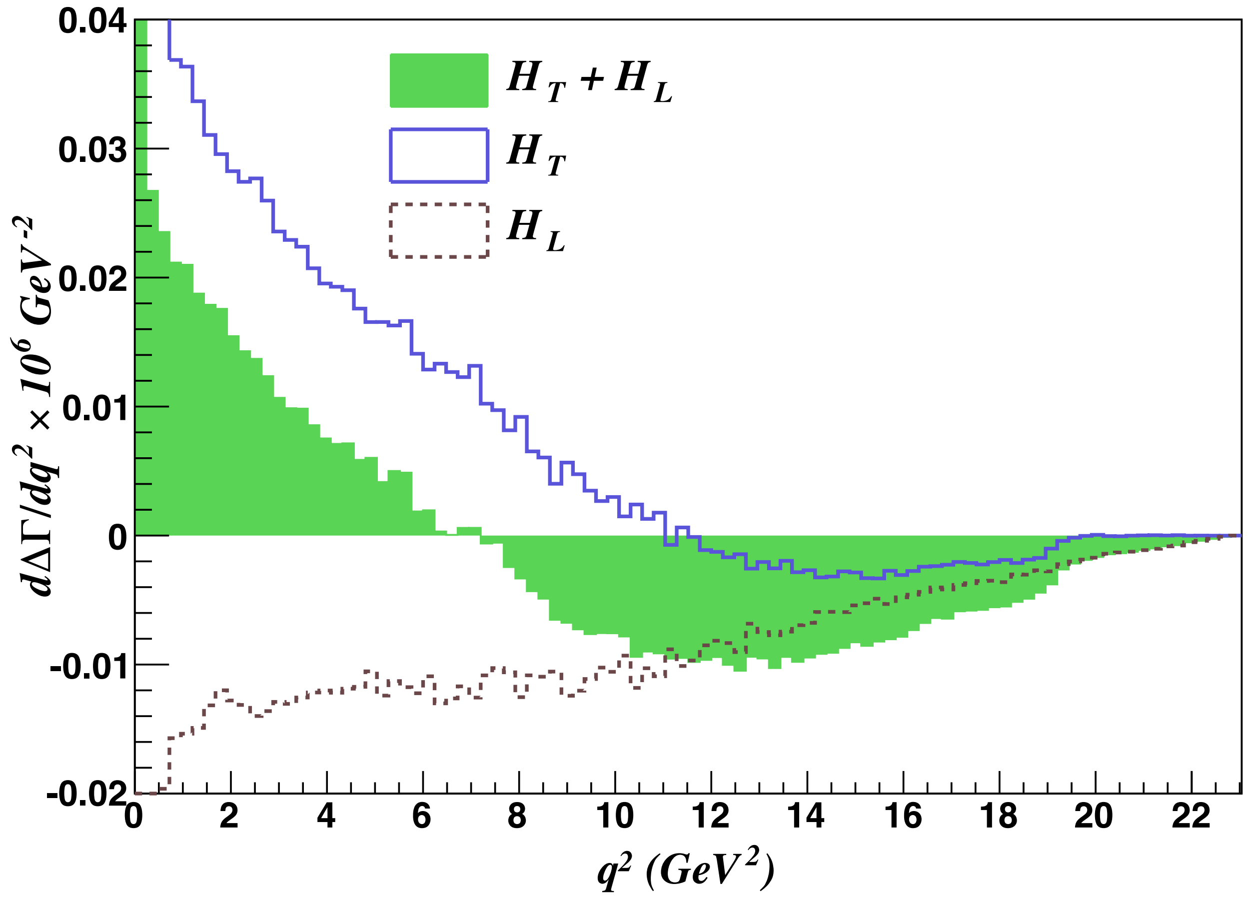
	$q^2 \in [1,6]$ GeV^2		
	$\frac{O_{[1,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,6]}}{O_{[1,6]}}$
$\mathcal{B}$	100	3.5	3.5
$\mathcal{H}_T$	19.0	8.0	43.0
$\mathcal{H}_L$	81.0	-4.5	-5.5

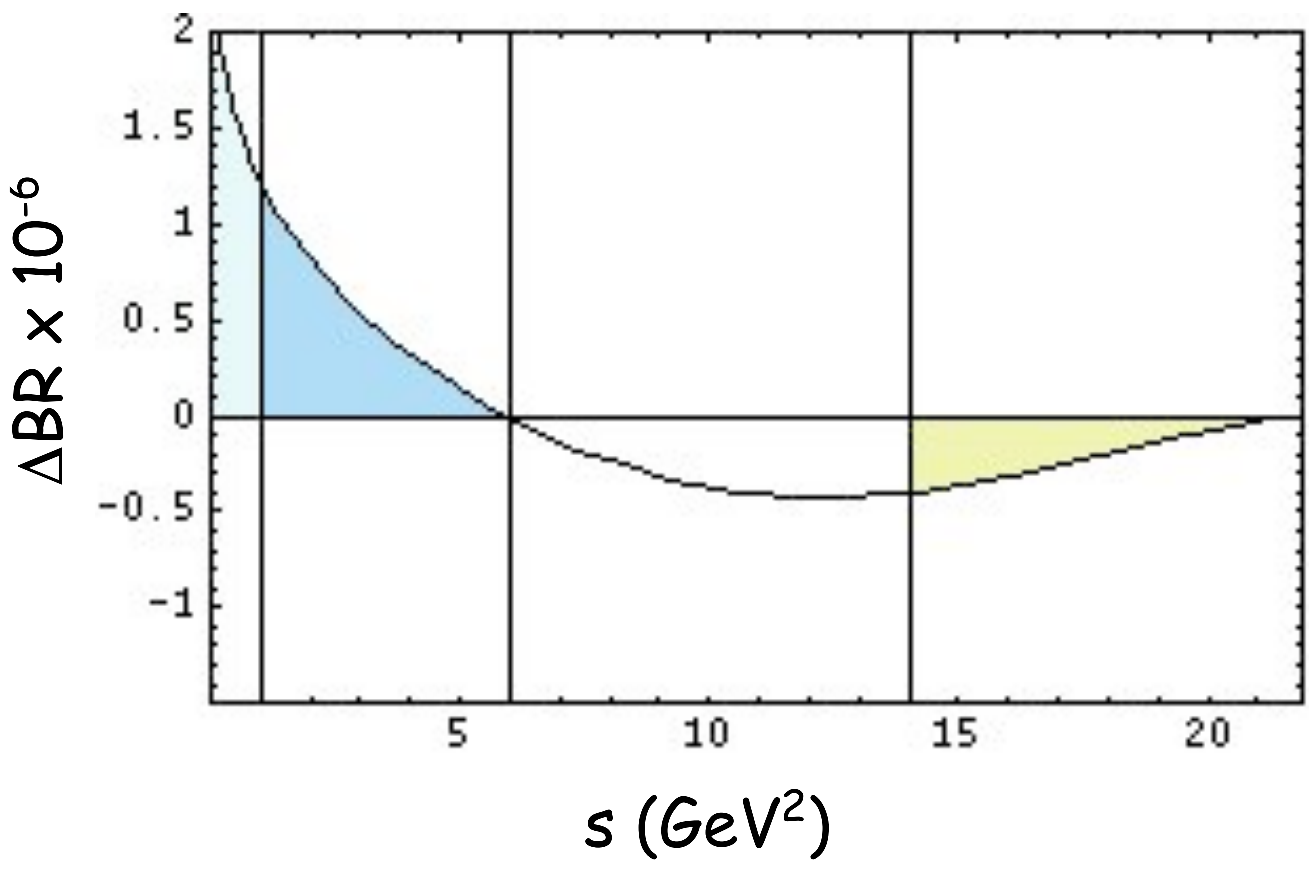
Analytical:

	$q^2 \in [1,6]$ GeV^2		
	$\frac{O_{[1,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,6]}}{O_{[1,6]}}$
$\mathcal{B}$	100	5.1	5.1
$\mathcal{H}_T$	19.5	14.1	72.5
$\mathcal{H}_L$	80.0	-8.7	-10.9

Inclusive: QED radiation (Monte Carlo)

- The Monte Carlo study reproduces the main features of the analytical results:





- Take home points on QED radiation and treatment of photons:
  - Large impact (up to 70 % for H_T)
  - Strong dependence on the observable (e.g. H_T) and on the shape of the spectrum (as shown by the comparison between theory and EVTGEN+PHOTOS)

- Experimental strategies:
  - be as inclusive as possible (i.e. include photons in X_s system)
  - "remove" collinear photons effects with PHOTOS (be wary of dependence on the shape of the EVTGEN generated spectrum)

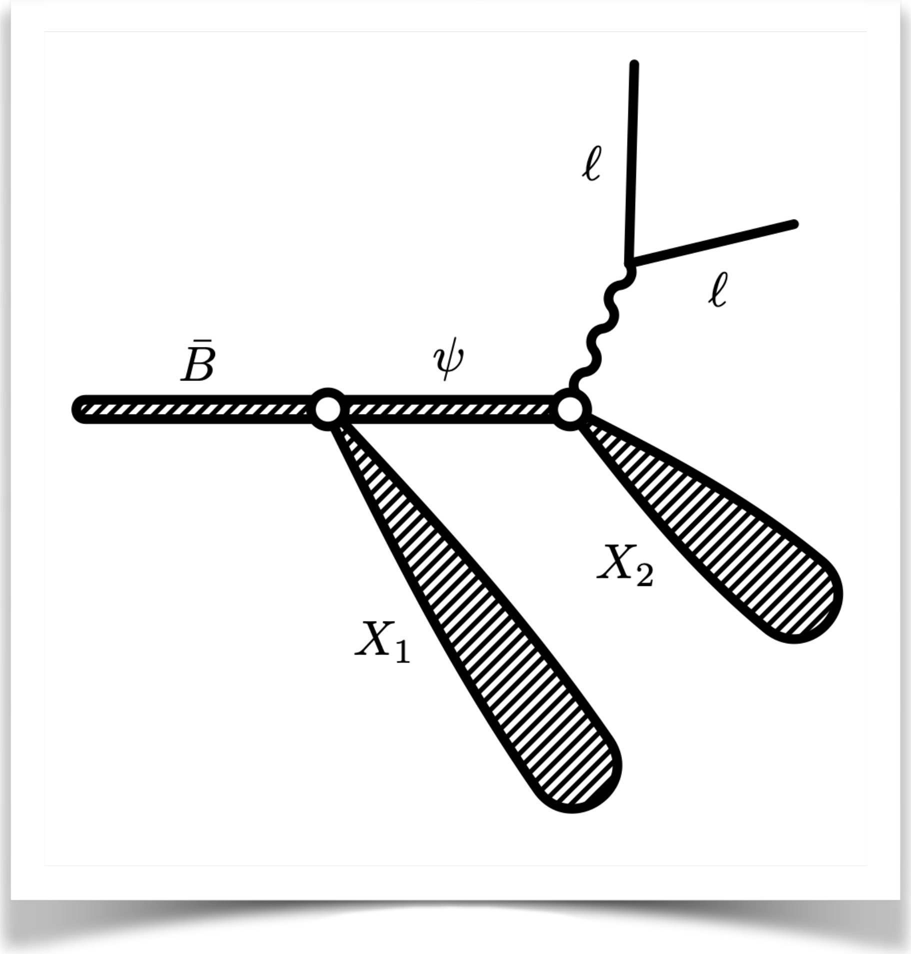
Inclusive: Cascades

- Cascade decays $B \to X_1(\psi \to X_2 \ell\ell)$ constitute another long distance effect [Buchalla, Isidori, Rey; Beneke, Buchalla, Neubert, Sachrajda]

- Effects are potentially very large:

	$\mathcal{B} \times 10^3$
$\bar{B} \to X_s \psi$	7.8 ± 0.4
$\bar{B} \to X_s \psi'$	3.07 ± 0.21
$\bar{B} \to X_s \chi_{c1}$	3.09 ± 0.22
$\bar{B} \to X_s \chi_{c2}$	0.75 ± 0.11
$\bar{B} \to X_s \eta_c$	4.88 ± 0.97 [111]
$\bar{B} \to X_s \chi_{c0}$	3.0 ± 1.0 [112]
$\bar{B} \to X_s h_c$	$2.4 \pm 1.0^\dagger$ [53]
$\bar{B} \to X_s \eta_c'$	$0.12 \pm 0.22^\dagger$ [113]

	$\mathcal{B} \times 10^5$
$\psi \to \eta \ell^+ \ell^-$	1.43 ± 0.07
$\psi \to \eta' \ell^+ \ell^-$	6.59 ± 0.18
$\psi \to \pi^0 \ell^+ \ell^-$	0.076 ± 0.014
$\psi' \to \eta' \ell^+ \ell^-$	0.196 ± 0.026



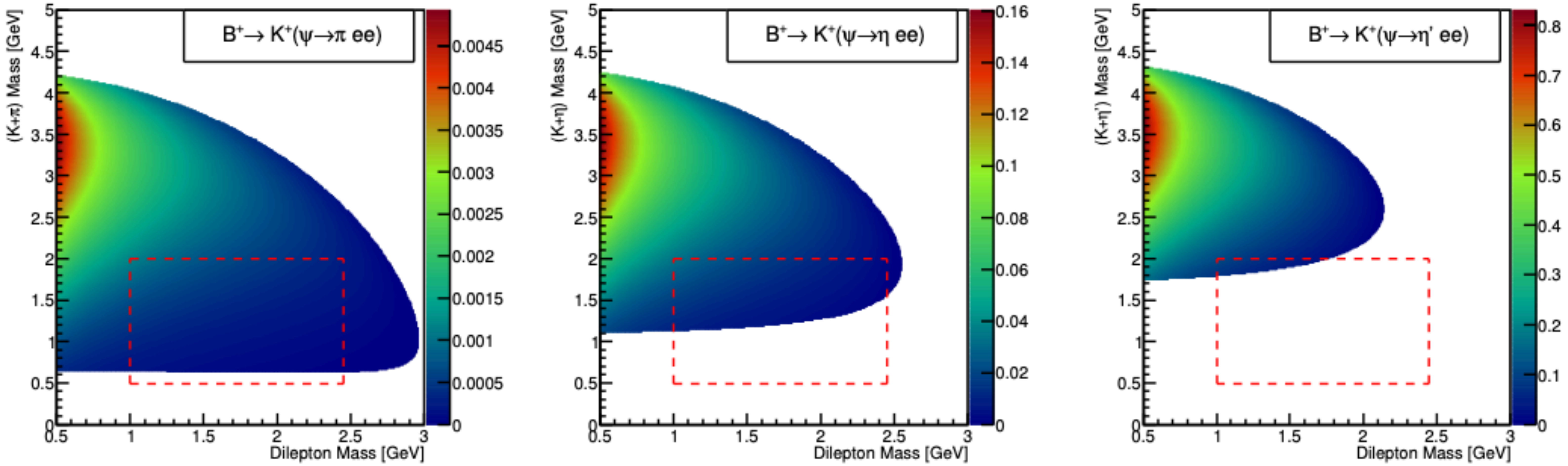
- For instance, the η' contribution alone yields a contribution which is of the same order as the short distance $b \to s\ell\ell$:

$$\mathrm{BR}(B \to X_s J/\psi)\mathrm{BR}(J/\psi \to \eta'\ell\ell) = 5.1 \times 10^{-7}$$

Inclusive: Cascades

- Even though the inclusive process has not be studied yet, we can study cascade effects as sum over exclusive

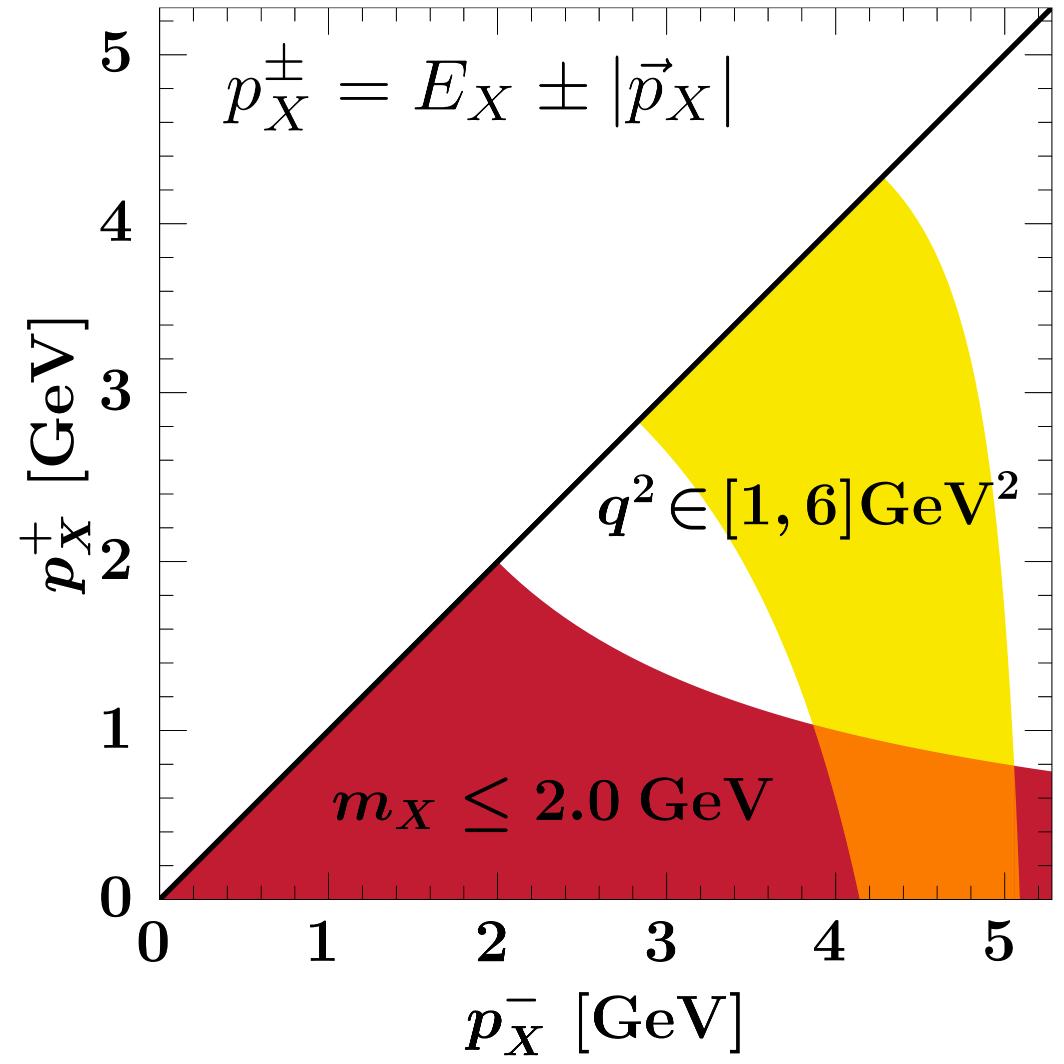
- This background is concentrated at low-q2:



- After imposing $m_X < 2\ \mathrm{GeV}$ this background becomes $\ll 1\,\%$!

Inclusive: m_X cuts

- Kinematics:

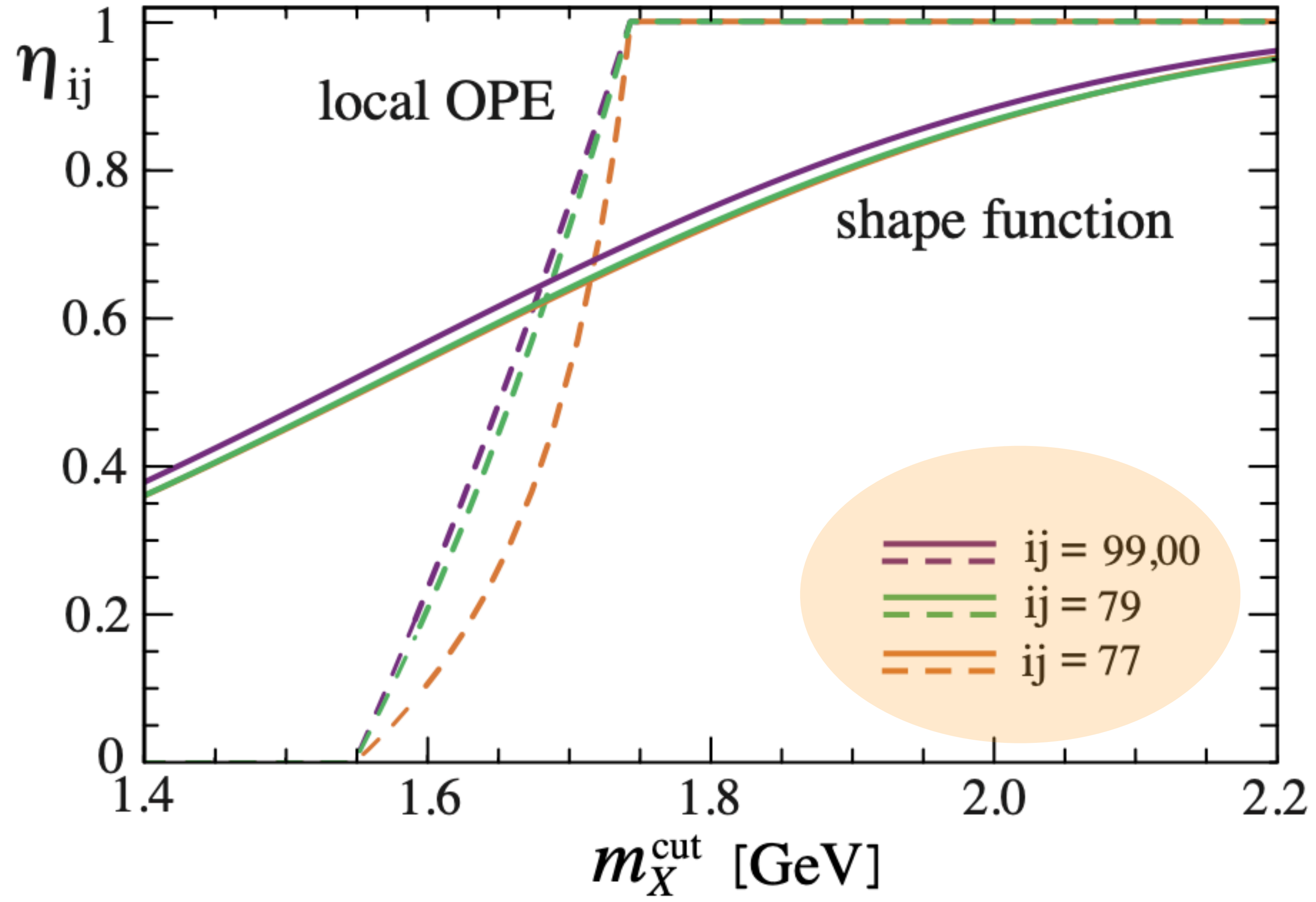


$$p_X^+ \ll p_X^- \Longrightarrow m_X^2 \ll E_X^2$$

X is hard-collinear:

$$\Lambda^2 \ll m_X^2 \sim \Lambda m_b \ll m_b^2$$

- The impact of the cuts is universal ($\eta = \Gamma_{\rm cut}/\Gamma$):
  [Lee, Ligeti, Stewart, Tackmann]



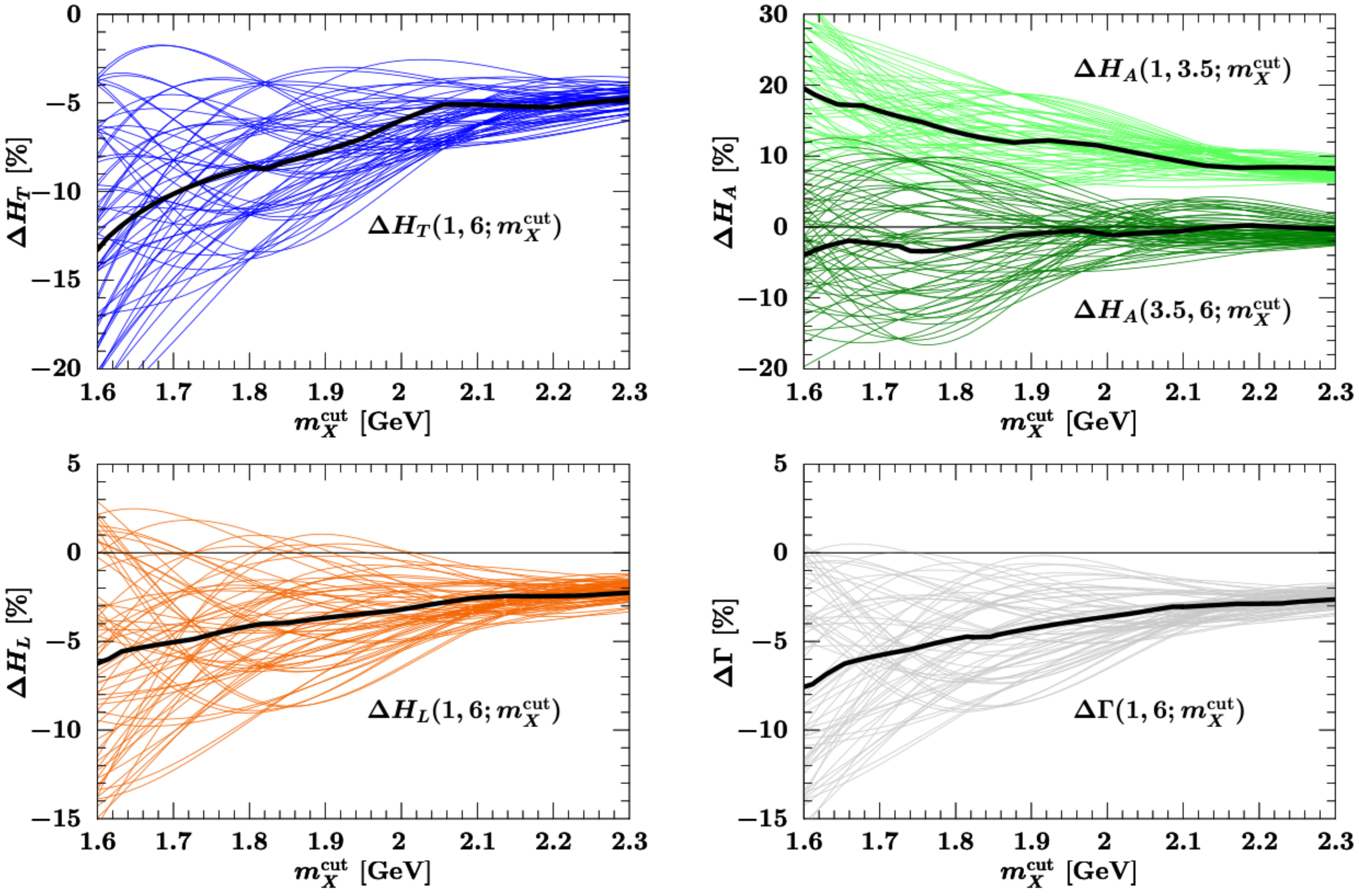
- Since the universality of the cuts extends to $B \to X_u \ell \nu$, the following ratio is minimally sensitive to the shape function modeling:

$$\frac{\Gamma(B \to X_s \ell\ell)_{\rm cut}}{\Gamma(B \to X_u \ell\nu)_{\rm cut}}$$

[same m_X cut]

Inclusive: m_X cuts (shape function)

- Current status of shape function modeling:
  [Lee, Ligeti, Stewart, Tackmann; Bell, Beneke, Huber, Li]



The same-color curves correspond to a sampling of potential shape functions

Inclusive: Shape function from $B \to X_s\gamma$

- SCET at leading power shows that inclusive $b \to s\ell\ell$ and $b \to s\gamma$ depend on a universal shape function
- Subleading effects introduce dependence on subleading shape functions which destroy this universality (in particular the "effective" shape function that appears in $b \to s\ell\ell$ acquires a q^2 dependence
- As an alternative to SCET (and following the kinetic scheme analysis of $B \to X_c\ell\nu$) we write the $b \to s\gamma$ rate with a Wilsonian cutoff ($\mu \sim 1$ GeV):

$$\frac{d\Gamma}{dE_\gamma} = \int dk_+ \, f(k_+,\mu) \frac{d\Gamma^{pert}}{dE_\gamma}\left(E_\gamma - \frac{k_+}{2}, \mu\right)$$

$$= \Gamma_0 \sum_{i \le j=1}^{8} C_i^{\text{eff}\,*}(\mu_b) C_j^{\text{eff}}(\mu_b) \int_{-\infty}^{\lambda} d\kappa \, F(\kappa,\mu) W_{ij}^{pert}(\xi - \kappa, \mu, \mu_b)$$

Shape Function in the kinetic scheme

where $F(\kappa,\mu) = m_b \, f(m_b\kappa,\mu)$ $\quad \lambda = (m_B - m_b)/m_b$

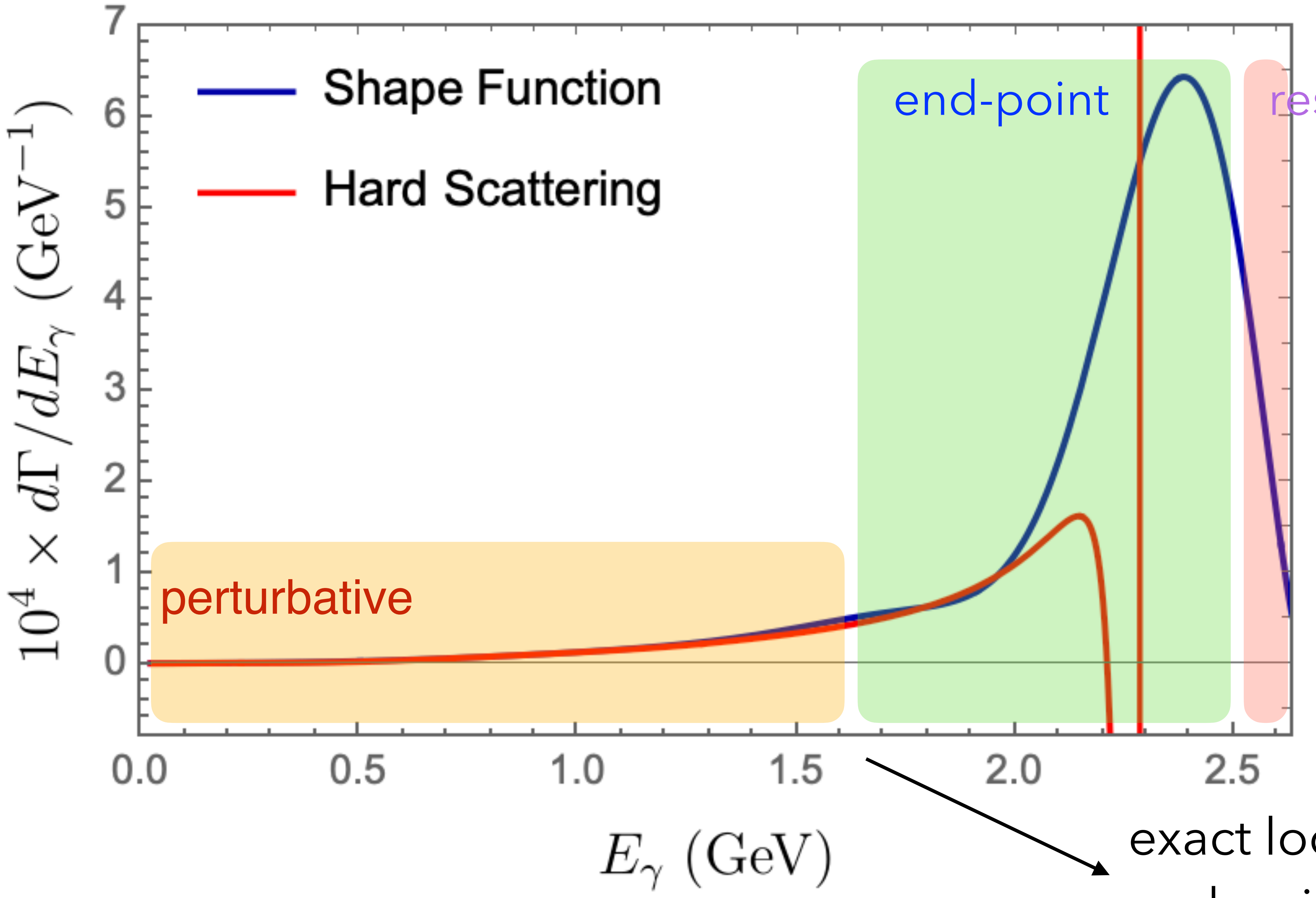
$m_b = m_b^{\text{kin}}(\mu)$ $\quad \Gamma_0 = \frac{G_F^2 \alpha m_b^2 m_b^{\overline{\text{MS}}}(\mu_b)^2}{16\pi^4} |V_{tb}V_{ts}^*|^2$

$\xi = 2E_\gamma/m_b$

Inclusive: Shape function from $B \to X_s\gamma$

[Gambino, EL, Schacht - Work in progress]

- Shape function vs hard scattering spectra:



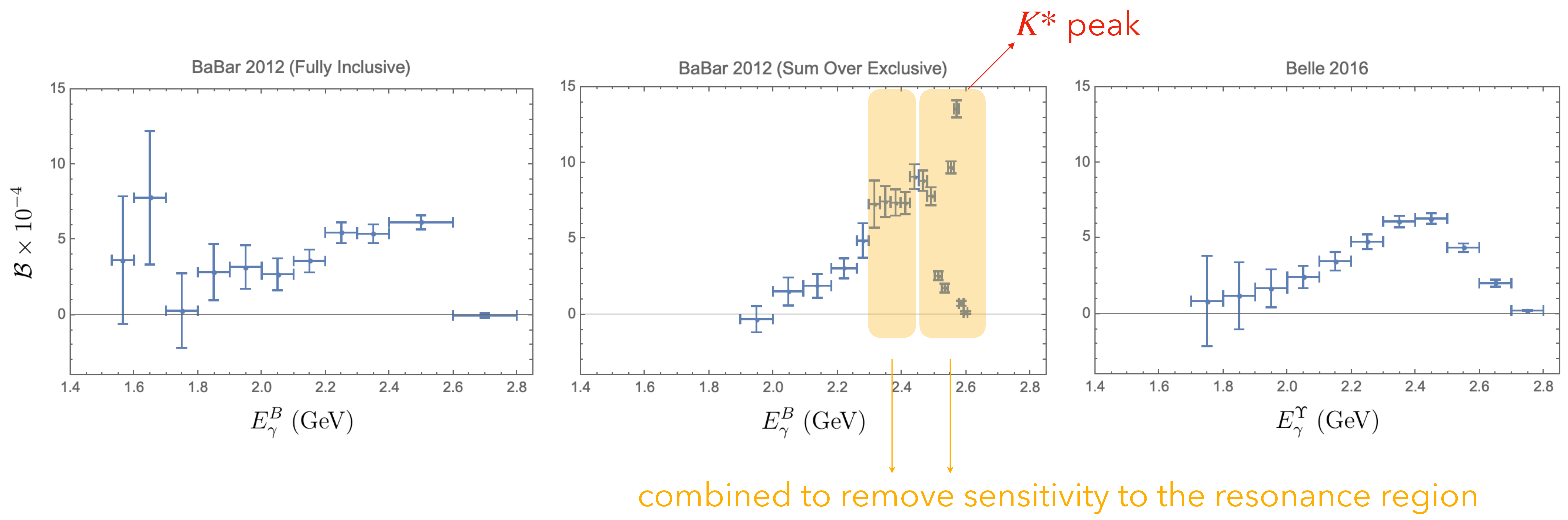
no Shape Function description: need to integrate over large enough E_γ range

exact location of the perturbative/end-point threshold depends on Shape Function

Inclusive: Shape function from $B \to X_s\gamma$

[Gambino, EL, Schacht - Work in progress]

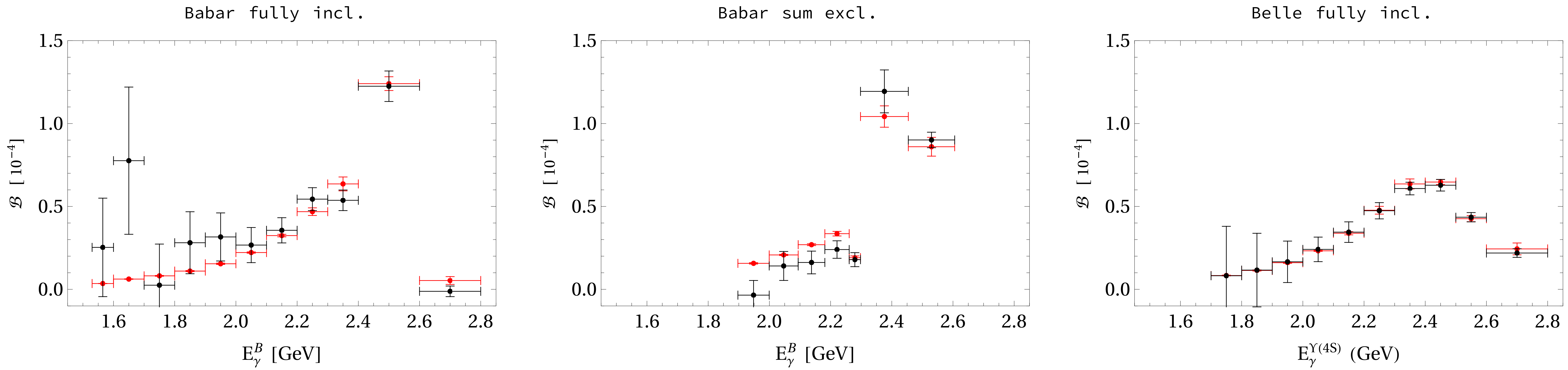
- We considered data from 2012 BaBar fully inclusive and sum over exclusive analyses (in the B rest frame) and 2016 Belle results (in the $\Upsilon(4S)$ rest frame):

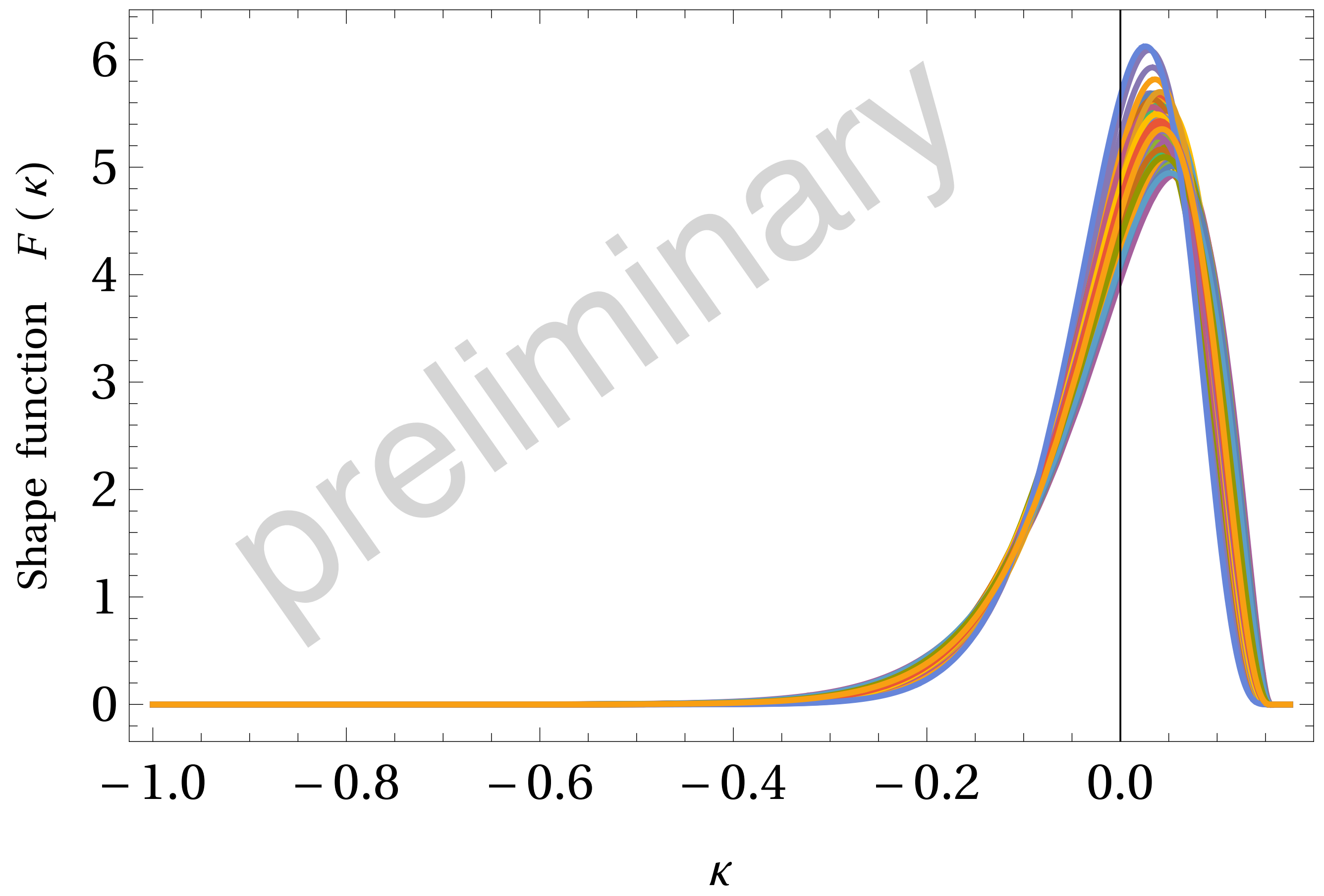


Inclusive: Shape function from $B \to X_s\gamma$

[Gambino, EL, Schacht - Work in progress]

- Some preliminary results:





- Some NNLO corrections missing
- Still working on training
- …

Inclusive: Shape function from $B \to X_s \gamma$

[Gambino, EL, Schacht - Work in progress]

- Implications for $B \to X_s \ell\ell$:
- SF needed for extrapolation in and to improve the EvtGen Monte Carlo event generator which is the heart of Belle, BaBar and Belle II analyses.
  [EvtGen: Ryd, Lange, Kuznetsova, Versille, Rotondo, Kirkby, Wuerthwein, Ishikawa;
  Maintained by J. Back, M. Kreps and T. Latham at University of Warwick]
- Hadronic spectrum is based on the Fermi motion implementation presented in Ali, Hiller, Handoko, Morozumi hep-ph/9609449:

parton level with momentum dependent b mass

$$\frac{d\Gamma_B}{ds\,du\,dp} = \int du' \frac{m_b(p)^2}{m_B} p \left[\frac{4}{\sqrt{\pi p_F^3}} \exp(-p^2/p_F^2)\right] \left(u'^2 + 4 m_b(p)^2 s\right)^{-1/2} \left[\frac{d\Gamma_b}{ds\,du}\right]_{m_b \to m_b(p)}$$

- We need to urgently update the code!
- Work in progress on the complete triple differential rate at $O(\alpha_s)$
  [T. Huber, T. Hurth, J. Jenkins, EL, in preparation]

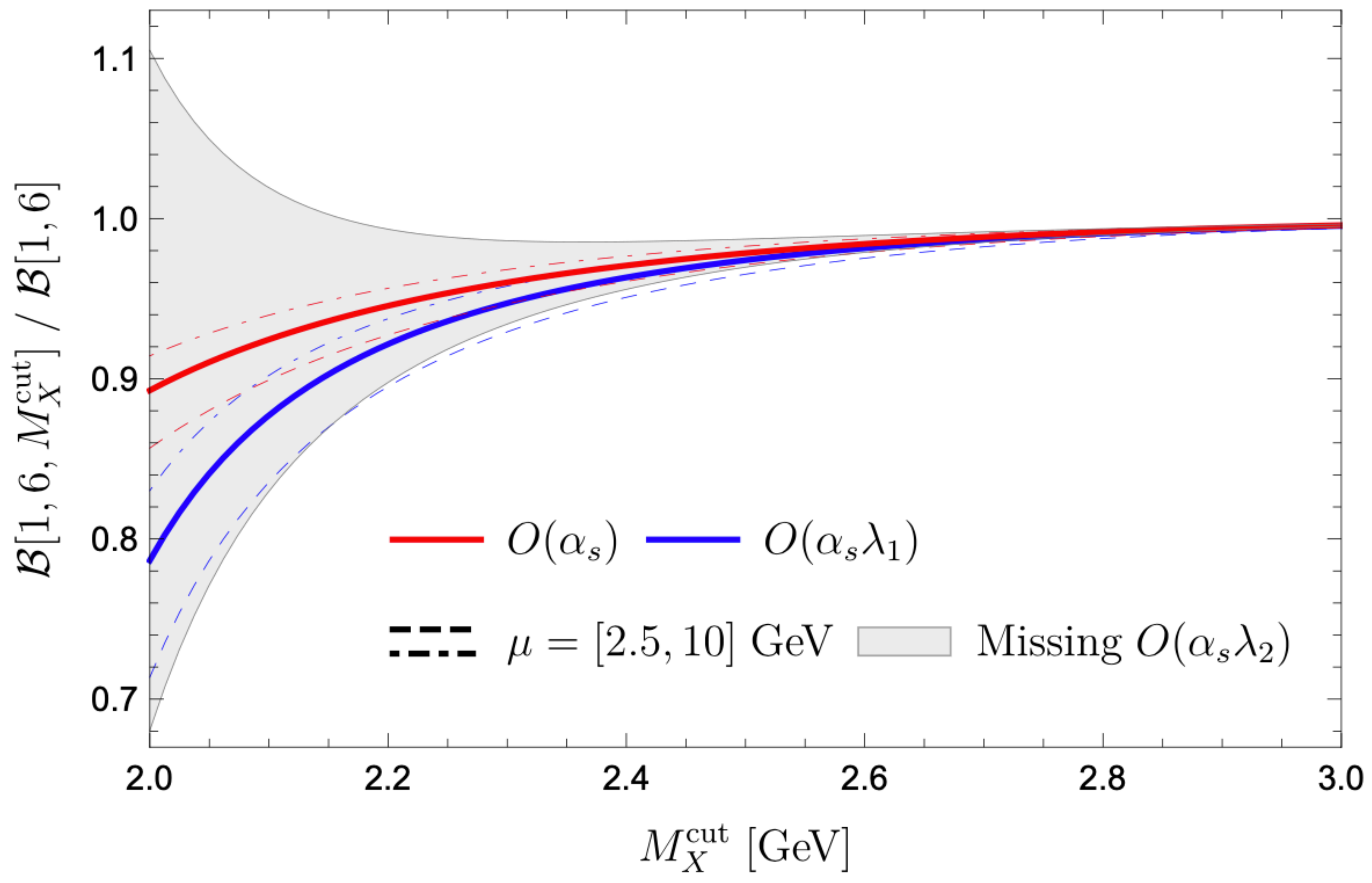
```
{
  pb = _calcprob->FermiMomentum(_pf);

  // effective b-quark mass
  mb = mB*mB + _mq*_mq - 2.0*mB*sqrt(pb*pb + _mq*_mq);
  if ( mb>0. && sqrt(mb)-_ms  < 2.0*ml ) mb= -10.;
}
mb = sqrt(mb);
```

Inclusive: perturbative study of m_X cuts

- We calculated the $B \to X_s \ell\ell$ M_X spectrum in perturbation theory at NLO including α_s and $\alpha_s\ \lambda_1/m_b^2$

  [Huber, Hurth, Jenkins, EL, 2306.03134]

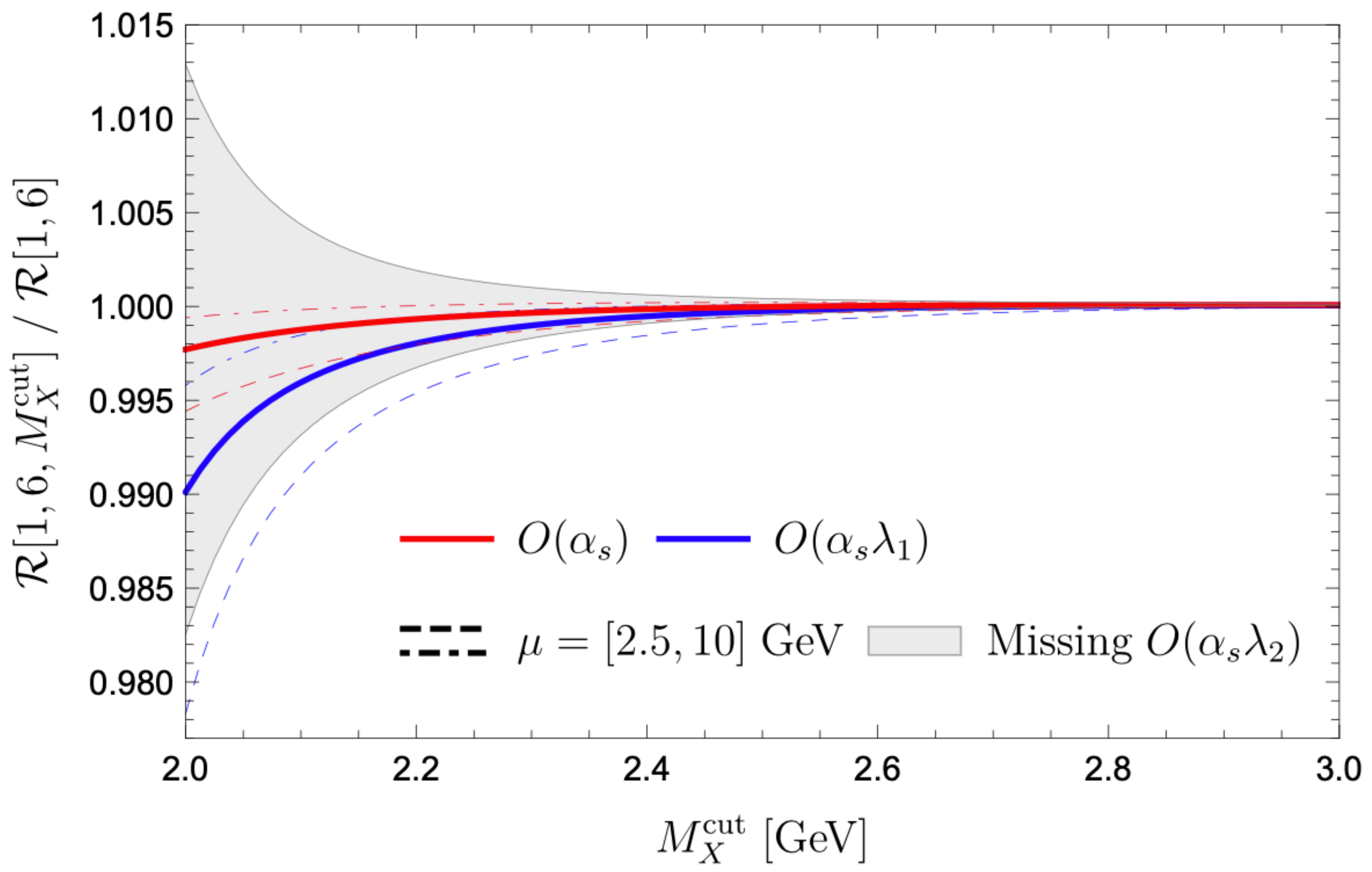
- The spectrum deviates develops a tail in M_X at $O(\alpha_s)$
- The $O(\alpha_s \mu_\pi^2)$ correction is necessary in order to asses the breakdown of the OPE
- The aim is to identify the minimum value of $M_X^{\rm cut}$ for which the perturbative calculation still holds (similar to a similar analysis for the photon energy spectrum in $B \to X_s\gamma$).



- A threshold can be tentatively set at $M_X^{\rm cut} = 2.5$ GeV
- Experimental cuts are at $M_X^{\rm cut} = 2$ GeV and they will require an extrapolation based on a Shape Function approach

Inclusive: perturbative study of m_X cuts

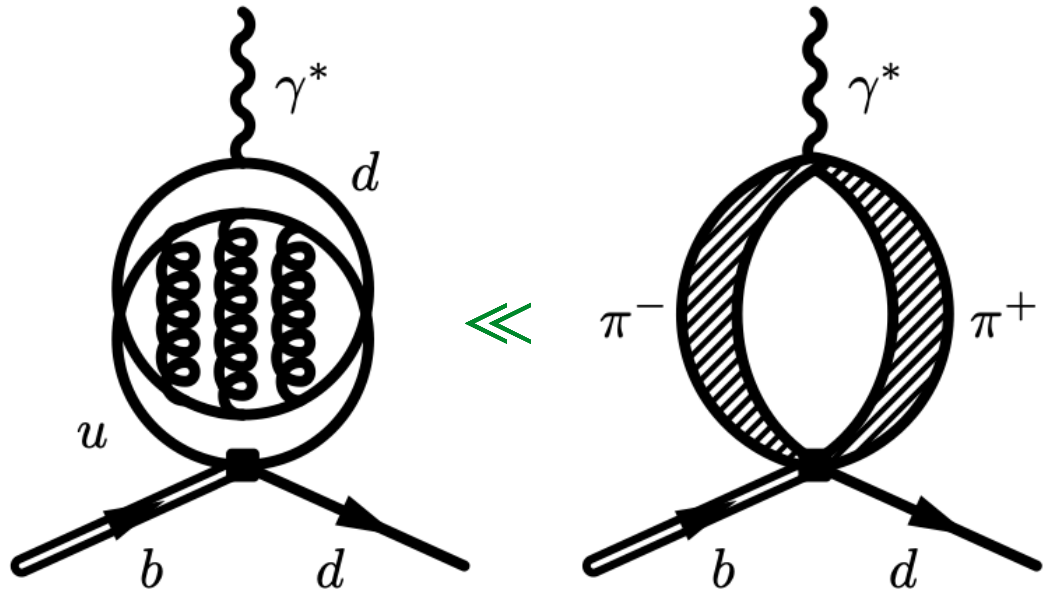
- The ratio of the low-q^2 branching ratio normalized to the $\bar{B} \to X_u \ell \bar{\nu}$ rate measured in the same q^2 range has much smaller power corrections: this suggests that the OPE for this quantity is much better behaved.



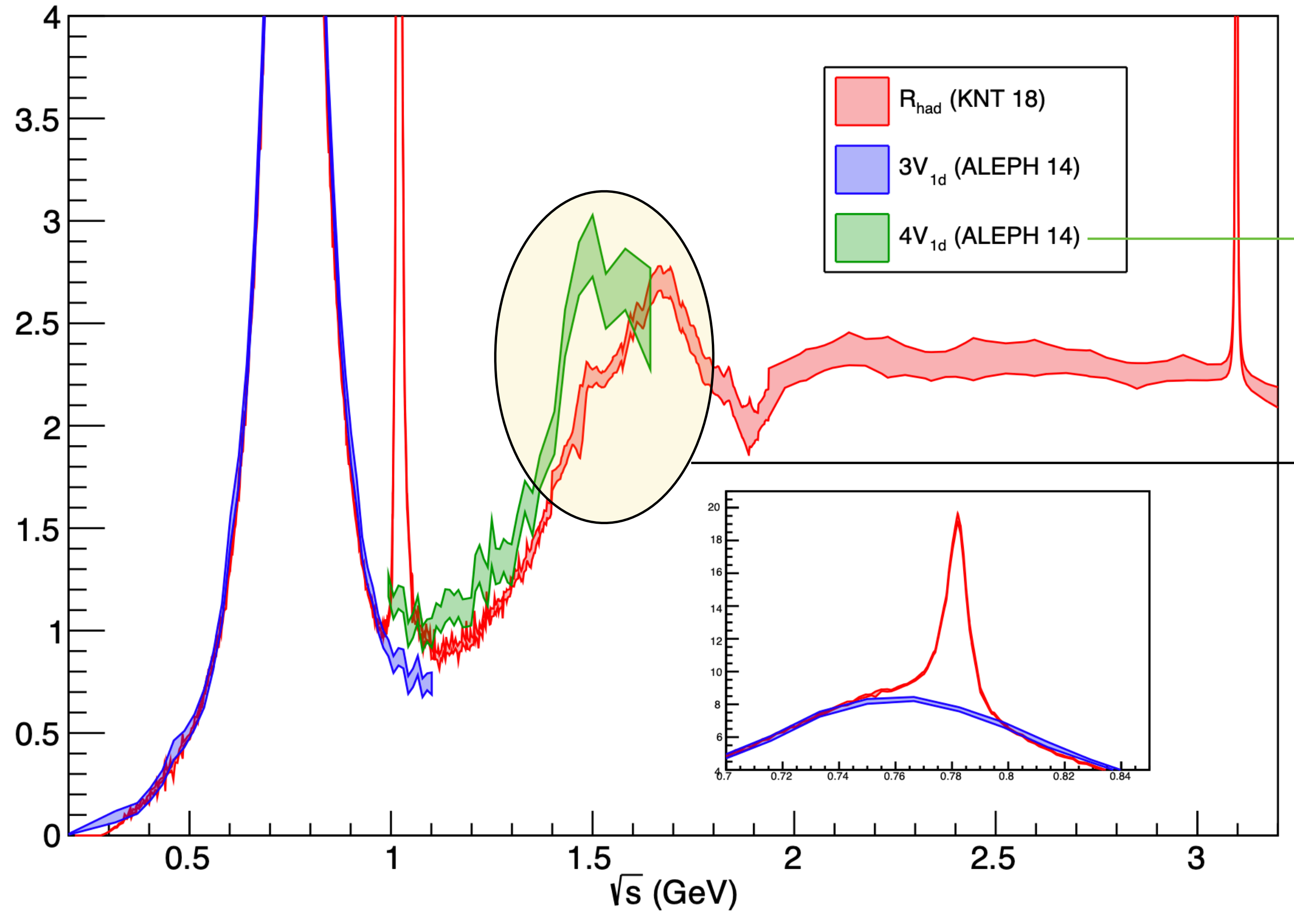
- The next step is to study the interpolation between the Shape Function region at small M_X and the perturbative region for $M_X > 2.5$ GeV

Inclusive: resonant color singlet production

- For $B \to X_d \ell\ell$ we need to include $u\bar{u}$ resonant effects
  - Considerable complications arise because we need to estimate $\langle J_q J_{q'} \rangle$ correlators with $q, q' = u, d, s$ whose relative size at low-q² is not described by perturbation theory at all



  - Using both Isospin SU(2) and SU(3) we were able to express the $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ Krüger-Sehgal functions in terms of R_{had} and τ decay data only

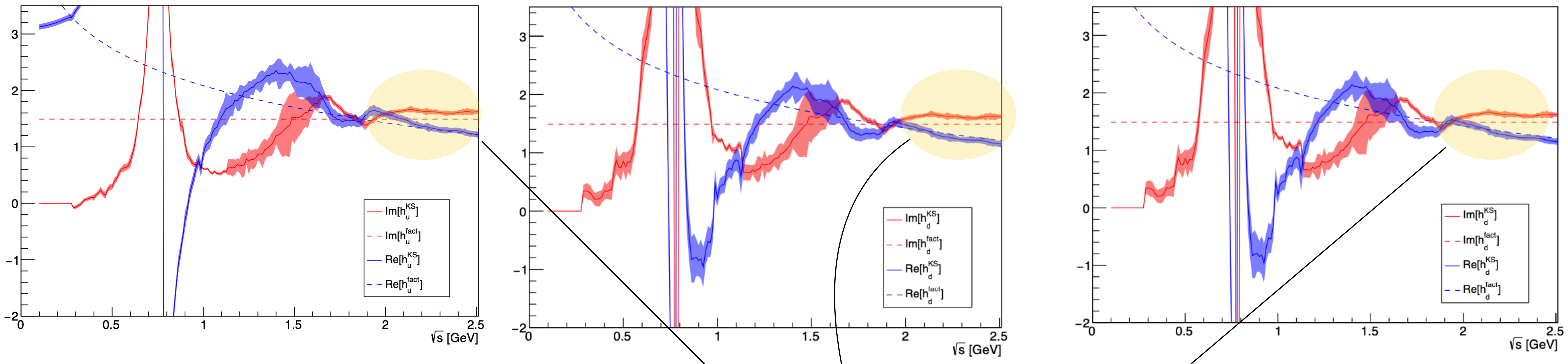


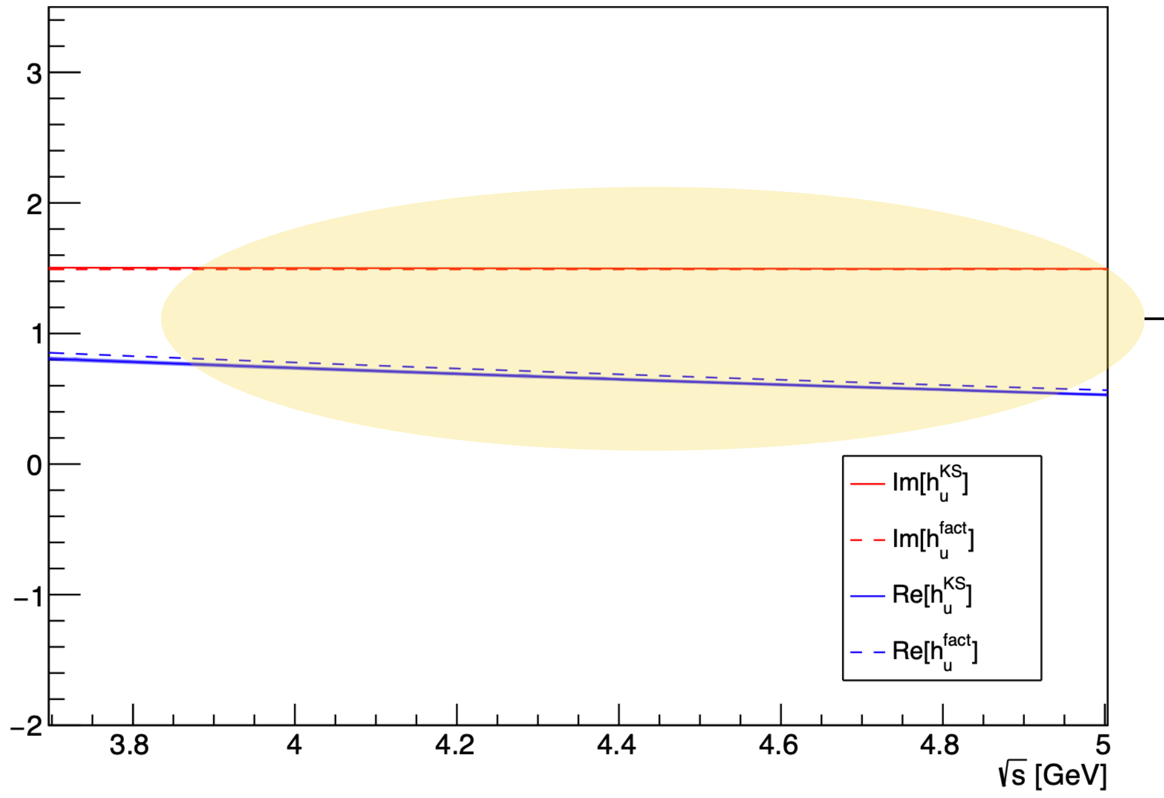
Rhad predicted from τ data using Isospin SU(3)

We use its deviation from the actual Rhad measurement (in red) as an estimate of SU(3) breaking effects

Inclusive: resonant color singlet production

- For $B \to X_d \ell\ell$ we need to include $u\bar{u}$ resonant effects





Very good asymptotic agreement with perturbation theory

Inclusive: $B \to X_d \ell \ell$

- Branching ratios

$$\begin{aligned}\mathscr{B}[1,6]_{ee} &= (7.81 \pm 0.37_{\rm scale} \pm 0.08_{m_t} \pm 0.17_{C,m_c} \pm 0.08_{m_b} \pm 0.04_{\alpha_s} \pm 0.15_{\rm CKM} \\ &\quad \pm 0.12_{{\rm BR}_{\rm sl}} \pm 0.05_{\lambda_2} \pm 0.39_{\rm resolved}) \cdot 10^{-8} \\ &= 7.81\ (1 \pm 7.8\%) \cdot 10^{-8} \\ \mathscr{B}[1,6]_{\mu\mu} &= 7.59\ (1 \pm 7.8\%) \cdot 10^{-8}\end{aligned}$$

$$\begin{aligned}\mathscr{B}[>14.4]_{ee} &= (0.86 \pm 0.12_{\rm scale} \pm 0.01_{m_t} \pm 0.01_{C,m_c} \pm 0.08_{m_b} \pm 0.02_{\rm CKM} \pm 0.02_{{\rm BR}_{\rm sl}} \\ &\quad \pm 0.06_{\lambda_2} \pm 0.25_{\rho_1} \pm 0.25_{f_{u,d}}) \cdot 10^{-8} \\ &= 0.86\ (1 \pm 45\%) \cdot 10^{-8} \\ \mathscr{B}[>14.4]_{\mu\mu} &= 1.00\ (1 \pm 39\%) \cdot 10^{-8}\end{aligned}$$

- Scale and resolved uncertainties dominate at low-q^2 (hard to improve)
- Power corrections and scale uncertainties dominate at high-q^2

Inclusive: $B \to X_d \ell \ell$

- Ratio $\mathscr{R}(s_0)$

$$\mathscr{R}(14.4)_{ee} = (0.93 \pm 0.02_{\rm scale} \pm 0.01_{m_t} \pm 0.01_{C,m_c} \pm 0.002_{m_b} \pm 0.01_{\alpha_s} \pm 0.05_{\rm CKM}$$
$$\pm 0.004_{\lambda_2} \pm 0.06_{\rho_1} \pm 0.05_{f_{u,d}}) \times 10^{-4}$$
$$= 0.93\ (1 \pm 9.7\%) \times 10^{-4}$$
$$\mathscr{R}(14.4)_{\mu\mu} = 1.10\ (1 \pm 6.4\%) \times 10^{-4}$$

- Forward-backward asymmetry and zero-crossing

$$H_A[1,3.5]_{ee} = -0.41\ (1 \pm 9.8\%) \cdot 10^{-8}$$
$$H_A[3.5,6]_{ee} = 0.40\ (1 \pm 18\%) \cdot 10^{-8}$$
$$H_A[1,3.5]_{\mu\mu} = -0.44\ (1 \pm 9.1\%) \cdot 10^{-8}$$
$$H_A[3.5,6]_{\mu\mu} = 0.37\ (1 \pm 19\%) \cdot 10^{-8}$$
$$(q_0^2)_{ee} = 3.28 \pm 0.11_{\rm scale} \pm 0.001_{m_t} \pm 0.02_{C,m_c} \pm 0.05_{m_b}$$
$$\pm 0.03_{\alpha_s} \pm 0.004_{\rm CKM} \pm 0.001_{\lambda_2} \pm 0.06_{\rm resolved} = 3.28 \pm 0.14$$
$$(q_0^2)_{\mu\mu} = 3.39 \pm 0.14$$