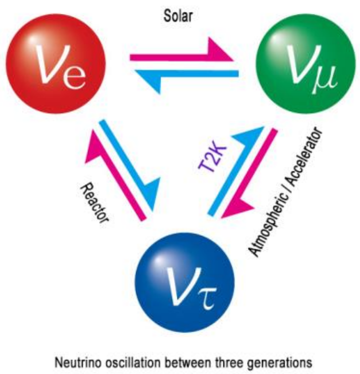
Overview of charged lepton flavor violation

Masato Yamanaka (Hiroshima Institute of Technology)

Why do we search for Lepton Flavor Violation (LFV)?



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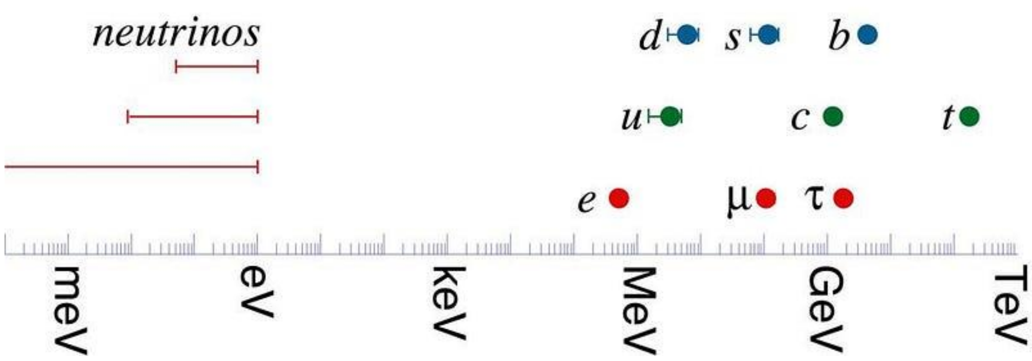


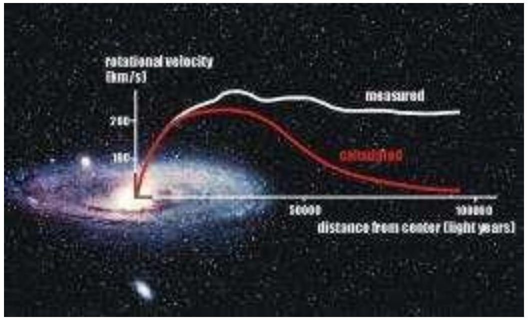
Longstanding puzzles

-- what is origin of neutrino mass?

-- what is dark matter/dark energy?

-- how to generate matter-antimatter asymmetry?

-- why 6 flavors? why are masses of matters hierarchical?



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Standard model ≠ Theory of everything

Why do we search for Lepton Flavor Violation (LFV)?

Lepton flavor number

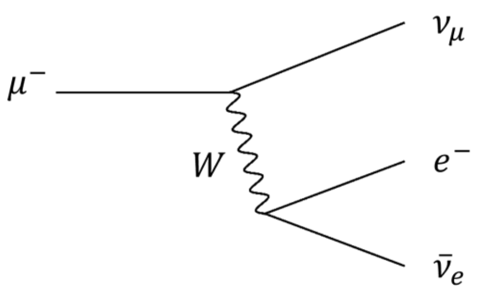
-- **accidental** and **empirical** symmetry of Standard Model (SM)

-- conserved number in all processes, i.e., no LFV in the SM

Particle	Electron flavor number	Muon flavor number	Tau flavor number
e and ν_e	$+1$	0	0
μ and ν_μ	0	$+1$	0
τ and ν_τ	0	0	$+1$

Opposite sign for anti-lepton. Ex. e number $= -1$ for positron

Example: muon decay in the SM



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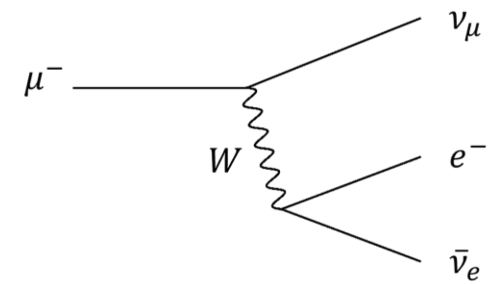

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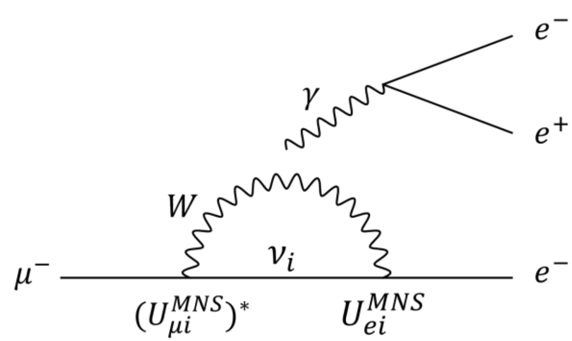
SM + ν oscillation

Will be discovered with 10^{55} μ ☺



$BR \simeq 7.4 \times 10^{-55}$

S. Petcov, Sov. J. Nucl. Phys. (1977)

G. Hernandez-Tome, et al, EPJC (2019)

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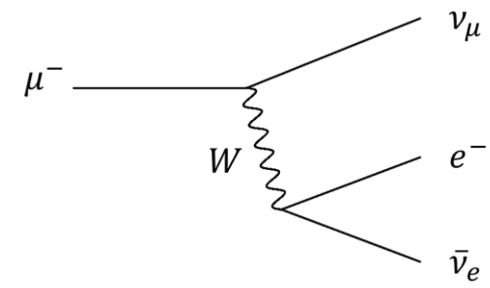

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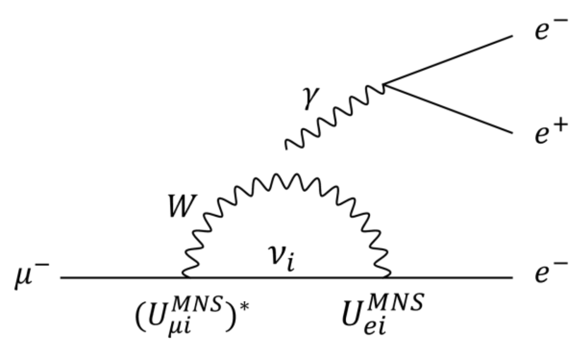
Will be discovered with 10^{55} μ

Impossible!!



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Why do we search for Lepton Flavor Violation (LFV)?

Why do we search for LFV?

Lepton flavor number

-- **accidental** and **empirical** symmetry of standard model (SM)

-- always conserved in each process, i.e., no lepton flavor violation (LFV) in the SM

Particle	Electron flavor number	Muon flavor number	Tau flavor number
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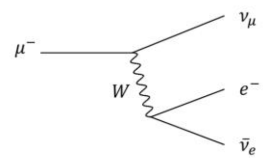

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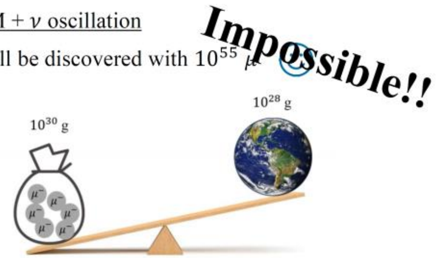

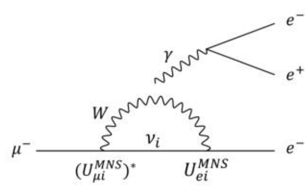

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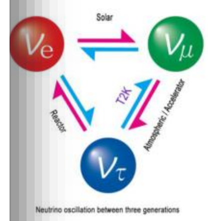

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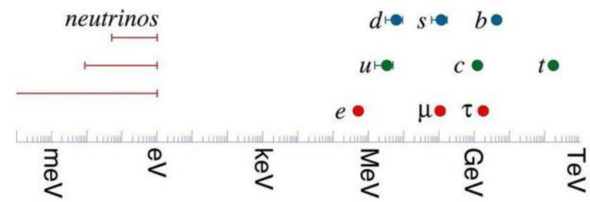


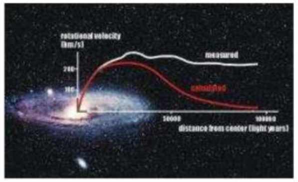
Mysteries

-- what is origin of neutrino mass?

-- what is dark matter/dark energy?

-- how to generate matter-antimatter asymmetry?

-- why 6 flavors? why are masses of matters hierarchical?



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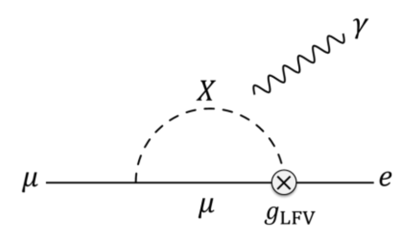
Standard model ≠ Theory of everything

In the physics beyond the SM

-- accessible LFV rate by extra degrees of freedom
(new particles, additional space dimension, …)

-- not only evidence, but also sensitive to the extra degrees

Ex.: muon decay in physics beyond SM



μ number $+1 \rightarrow 0$

e number $0 \rightarrow +1$

Where does LFV come from? (ex. 2 generations)

Transformation from interaction eigenbasis to mass eigenbasis

$$\ell_L^I \to U_L \ell_L = \begin{pmatrix} \cos\theta_L & \sin\theta_L \\ -\sin\theta_L & \cos\theta_L \end{pmatrix} \ell_L \ , \quad e_R^I \to U_R e_R = \begin{pmatrix} \cos\theta_R & \sin\theta_R \\ -\sin\theta_R & \cos\theta_R \end{pmatrix} e_R$$

<u>In interaction eigenbasis</u>

2×2 matrix, in general, **both diagonal and off-diagonal** entries

$$\mathcal{L}^I = -\boldsymbol{y}_\ell^I \bar{\ell}_{Li}^I \, \phi \, e_{Rj}^I + h.c.$$

<u>In mass eigenbasis</u>

2×2 matrix, **diagonal** entries only

$$\mathcal{L} = -\boldsymbol{y}_\ell \bar{\ell}_{Li} \, \phi \, e_{Rj} + h.c.$$

$$\boldsymbol{y}_\ell \langle \phi \rangle = \begin{pmatrix} m_e & 0 \\ 0 & m_\mu \end{pmatrix}$$

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In interaction eigenbasis

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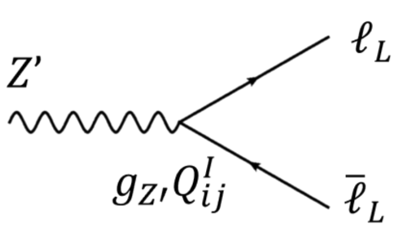
$$\mathcal{L}^I = -\,\boldsymbol{y}_\ell^I \bar{\ell}_{Li}^I \,\phi\, e_{Rj}^I + h.c. + g_{Z'} Q_{ij}^I \bar{\ell}_{Li}^I \, Z'_\mu \gamma^\mu \ell_{Lj}^I$$

$$Q_{ij}^I = \begin{pmatrix} q_1 & 0 \\ 0 & q_2 \end{pmatrix}$$



In mass eigenbasis

2×2 matrix, **diagonal** entries only

$$\mathcal{L} = -\,\boldsymbol{y}_\ell \bar{\ell}_{Li} \,\phi\, e_{Rj} + h.c. + g_{Z'} Q_{ij} \bar{\ell}_{Li} \, Z'_\mu \gamma^\mu \ell_{Lj}$$

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<u>In interaction eigenbasis</u>

2×2 matrix, in general, **<u>both</u>** **<u>diagonal and off-diagonal</u>** entries

$$\mathcal{L}^I = -\, \boldsymbol{y}_\ell^I \, \bar{\ell}_{Li}^I \, \phi \, e_{Rj}^I + h.c.$$

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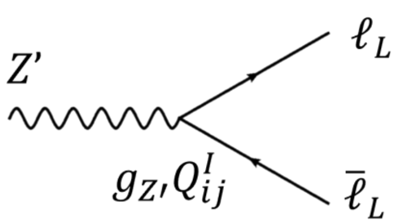


<u>In mass eigenbasis</u>

2×2 matrix, **<u>diagonal</u>** entries only

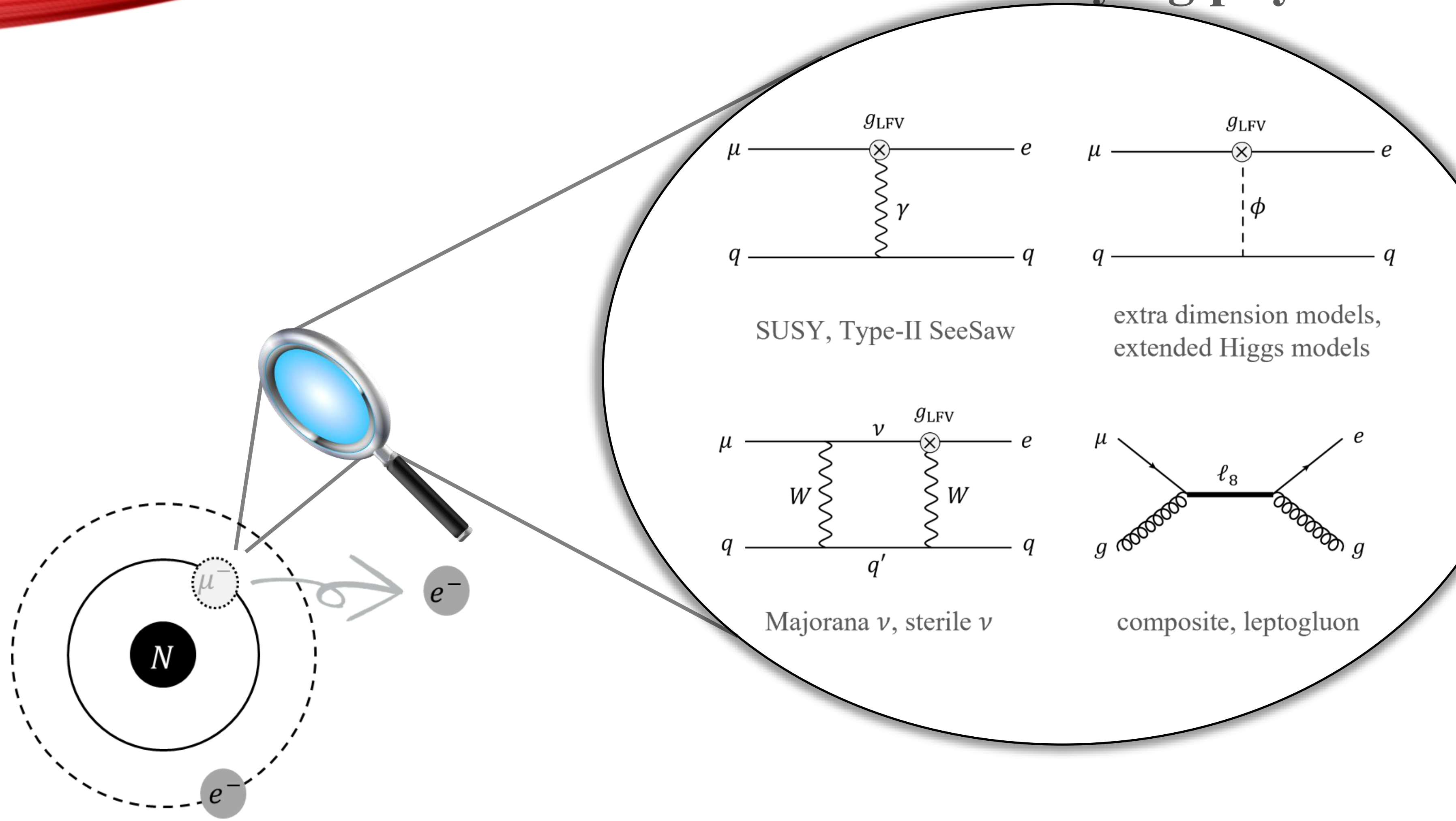
$$\mathcal{L} = -\, \boldsymbol{y}_\ell \, \bar{\ell}_{Li} \, \phi \, e_{Rj} + h.c.$$

$$+\, g_{Z'} Q_{ij} \bar{\ell}_{Li} \, Z'_\mu \gamma^\mu \ell_{Lj}$$

$$U_L^\dagger Q_{ij}^I U_L = \begin{pmatrix} q_1 \cos^2\theta_L + q_2 \sin^2\theta_L & -(q_1 - q_2)\cos\theta_L \sin\theta_L \\ -(q_1 - q_2)\cos\theta_L \sin\theta_L & q_1 \sin^2\theta_L + q_2 \cos^2\theta_L \end{pmatrix}$$

Flavor violating interaction! Information of Z' charge!!

Which is the underlying physics?



Unknown particle **indirectly** appears in LFV reactions

Not appeared in direct observables

How to unravel the physics behind the LFV

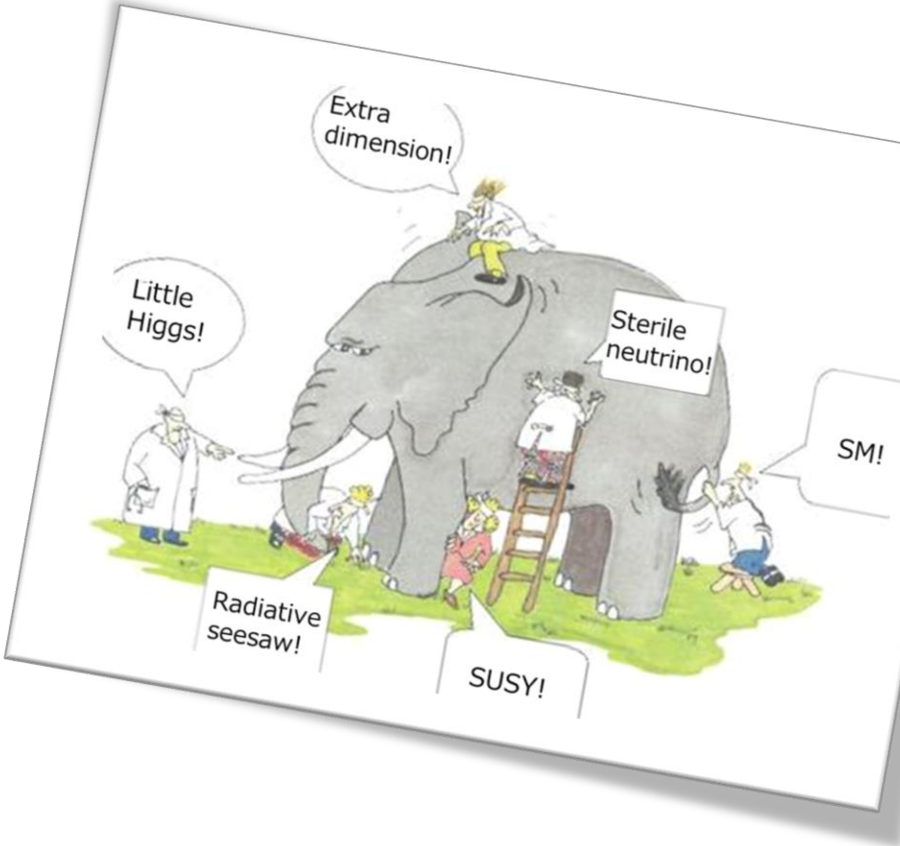
<u>Many LFV observables as possible</u> to draw "unknown" from various angles

- $\mu \to e\gamma,\ \mu \to 3e$
- $\mu^- \to e^-$ conversion in nuclei
- $\mu^- e^- \to e^- e^-$ in muonic atom
- $\tau \to 3\mu, \tau \to e\pi\pi, \tau \to \mu\gamma, \ldots$, etc

<u>Accurate connection</u> between LFV parameters and observables

- Characteristic signatures for each LFV operator
- Dependences on experimental environmental
- Ratios and correlations of BRs etc

The more LFV processes, the "elephant" is more clearly illustrated!



Outline

1. Introduction

2. <u>**Many LFV observables as possible**</u> to draw “unknown” from various angles

   2.1 LFV particle decay

   2.2 LFV processes in muonic atom

   2.3 LFV deep inelastic scattering (LFV DIS)

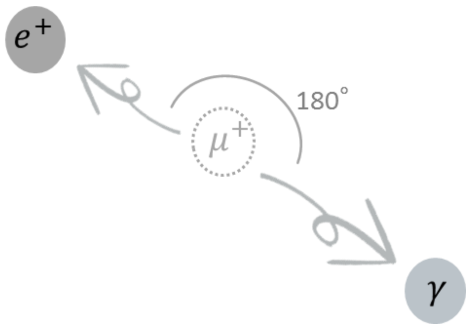
3. How to unravel the physics behind LFV

4. Summary

2. **Many LFV observables as possible** to draw “unknown” from various angles

2.1 LFV particle decay

Radiative decay $\ell_i \to \ell_j\gamma$



Signal

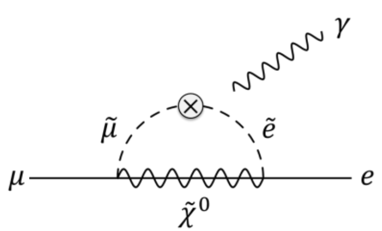
-- back-to-back $e(\mu) + \gamma$,

-- $\sum E = m_\mu$ (for $\mu \to e\gamma$) , $\sum E = m_\tau$ (for $\tau \to \ell_j\gamma$)

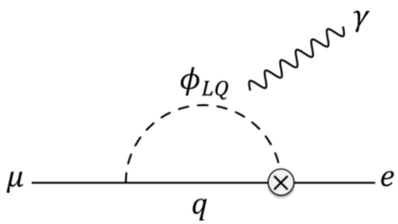
Experimental status

Process	BR limit	Future
$\mu \to e\gamma$	1.5×10^{-13} MEGII (2025)	6.0×10^{-14} MEG II
$\tau \to e\gamma$	3.3×10^{-8} BABAR (2010)	9.0×10^{-9} Belle II
$\tau \to \mu\gamma$	4.2×10^{-8} Belle (2021)	6.9×10^{-9} Belle II

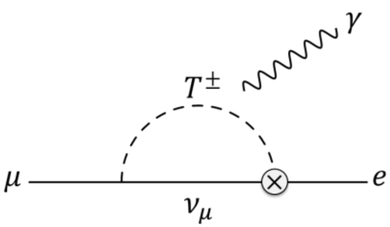
Probe to various models beyond the SM



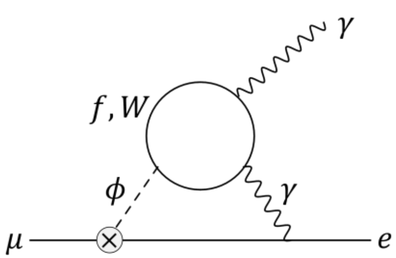
SUSY (+ Type-I SeeSaw)



Leptoquark, GUT models



Type-II SeeSaw, extended Higgs models



Higgs LFV, extended Higgs models

Radiative decay $\ell_i \to \ell_j \gamma$

Interaction

$g_{\rm LFV}$ × (loop factor)

$$\mathcal{L}_{\rm dipole} = \frac{C_{\rm LFV}}{M_X^2} m_\mu \overline{\mu} \sigma^{\mu\nu} e F_{\mu\nu}$$

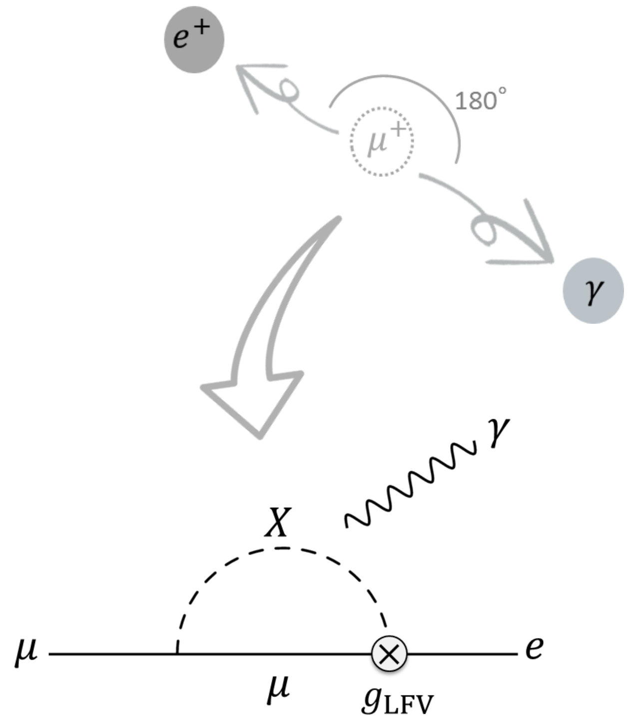
mass of particle X

electromagnetic field strength

Branching ratio

$$\mathrm{BR}(\mu \to e\gamma) \equiv \frac{\Gamma(\mu \to e\gamma)}{\Gamma(\mu \to e\nu\overline{\nu})} = \frac{48\pi^2}{G_F^2} \frac{C_{\rm LFV}^2}{M_X^4}$$

$$= 3.5 \times 10^{-14} \left(\frac{C_{\rm LFV}}{10^{-3}}\right)^2 \left(\frac{100\,\mathrm{TeV}}{M_X}\right)^4$$



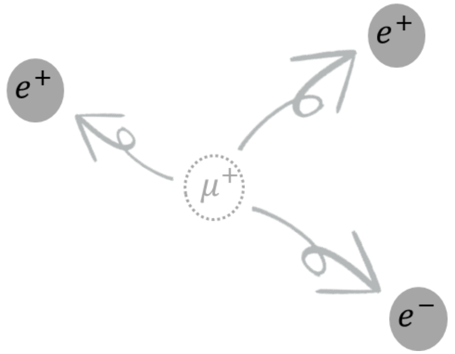
Exploring higher scale beyond the collider energies!

3 lepton decay $\ell_i \to \ell_j \ell_k \ell_k$

Signal

-- $\sum E_e = m_\mu$ (for $\mu \to eee$) , $\sum E_\ell = m_\tau$ (for $\tau \to \ell_j \ell_k \ell_k$)

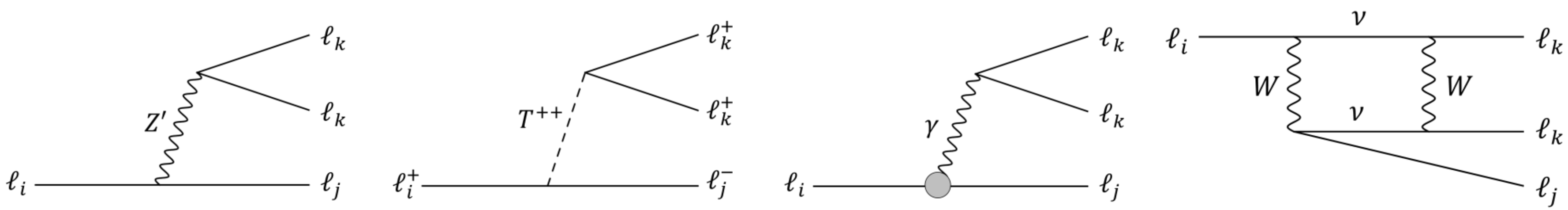
-- spatial momenta $\sum \overrightarrow{p_e} = \vec{0}$, time coincidence $\Delta t_{eee} = 0$



Experimental status

Process	BR limit		Future	
$\mu \to eee$	1.0×10^{-12}	SINDRUM (1988)	1.0×10^{-16}	Mu3e
$\tau \to eee$	2.7×10^{-8}	Belle (2010)	4.7×10^{-10}	Belle II
$\tau \to \mu\mu\mu$	1.9×10^{-8}	Belle (2024)	2.9×10^{-10}	Belle II
$\tau \to \mu ee$	1.5×10^{-8}	Belle (2010)	2.3×10^{-10}	Belle II

Probe to various models beyond the SM



GUT models, Little Higgs, extra dimension models

Type-II SeeSaw, extended Higgs models

SUSY (+ Type-I SeeSaw)

νMSM, sterile ν

Probing LFV operators from differential decay rate

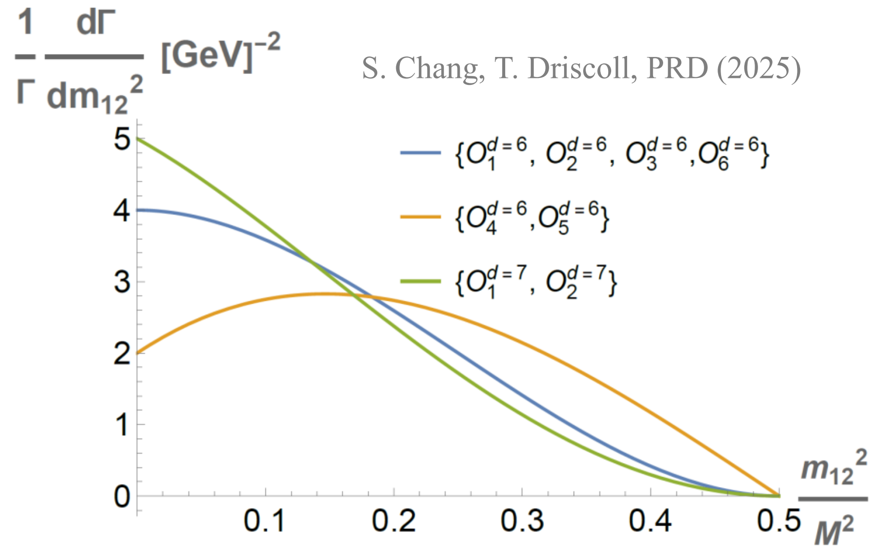
3 lepton decay $\ell_i \to \ell_j \ell_k \ell_k$

$$\mathcal{O}_1^{(6)} = (\bar{f} P_R f')(\bar{f} P_R F)$$
$$\mathcal{O}_2^{(6)} = (\bar{f} P_L f')(\bar{f} P_L F)$$
$$\mathcal{O}_3^{(6)} = (\bar{f} \gamma^\alpha P_L f')(\bar{f} \gamma_\alpha P_L F)$$
$$\mathcal{O}_4^{(6)} = (\bar{f} \gamma^\alpha P_R f')(\bar{f} \gamma_\alpha P_L F)$$
$$\mathcal{O}_5^{(6)} = (\bar{f} \gamma^\alpha P_L f')(\bar{f} \gamma_\alpha P_R F)$$
$$\mathcal{O}_6^{(6)} = (\bar{f} \gamma^\alpha P_R f')(\bar{f} \gamma_\alpha P_R F)$$
$$\mathcal{O}_1^{(7)} = ([\partial_\alpha \bar{f}] P_L f')(\bar{f} \gamma^\alpha P_L F)$$
$$\mathcal{O}_2^{(7)} = ([\partial_\alpha \bar{f}] P_R f')(\bar{f} \gamma^\alpha P_R F)$$

LFV Decay $\quad F(p) \to \bar{f}'(p_1) f(p_2) f(p_3)$

Invariant mass $\quad m_{ij}^2 = (p_i + p_j)^2$

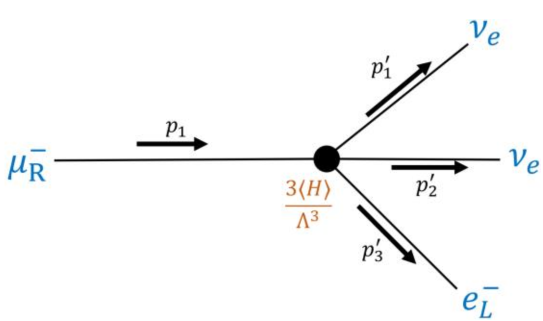
S. Chang, T. Driscoll, PRD (2025)



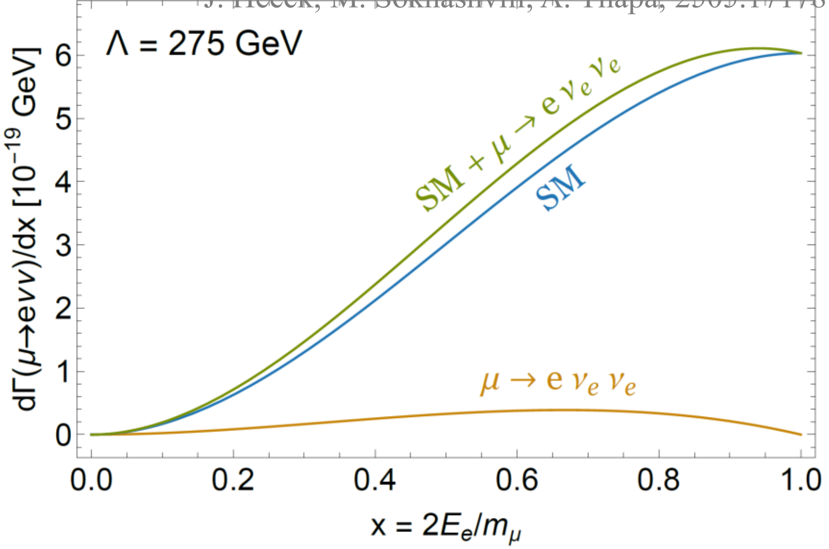
Operator violating lepton flavor by three units

$$\mathcal{L}^{D=7}_{\Delta L_e = 3} = \frac{1}{\Lambda^3} \bar{\mu}_R L_e L_e L_e H$$

LFV decay



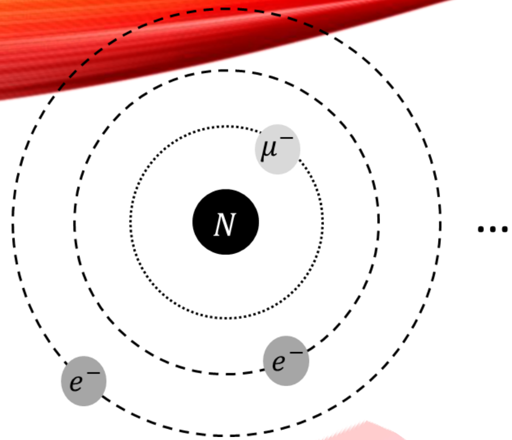
J. Heeck, M. Sokhashvili, A. Thapa, 2505.17178



2. **Many LFV observables as possible** to draw “unknown” from various angles

2.2 LFV processes in muonic atom

LFV processes in muonic atom



Muonic atom

-- bound state of muon μ^- and nucleus N

-- fate of muonic atom in the SM

(1) muon decay in orbit

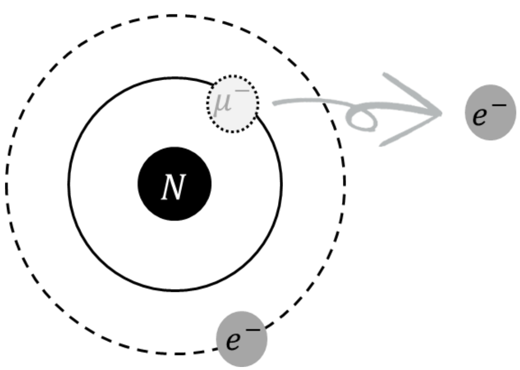
$$(\mu^- N(A,Z)) \to e^- + \bar{\nu}_e + \nu_\mu + N(A,Z)$$

(2) muon capture

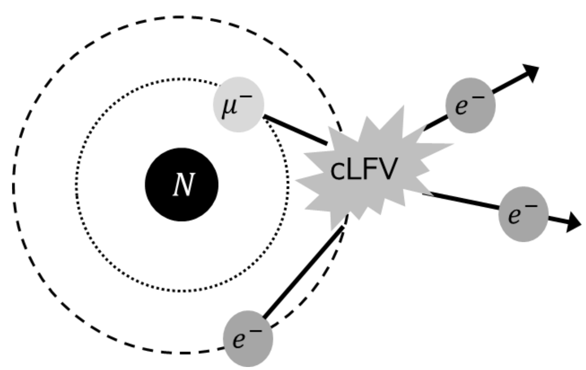
$$(\mu^- N(A,Z)) \to \nu_\mu + N'(A,Z-1)$$

If LFV mediator couples with nucleon

If LFV mediator couples mainly with leptons



$\mu^- \to e^-$ conversion



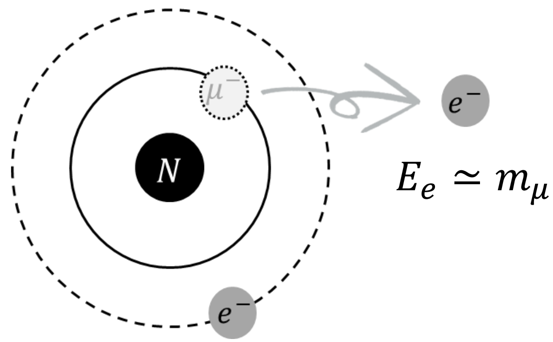
$\mu^- e^- \to e^- e^-$

And also

- $\mu^- \to e^+$ conversion
  [(review)]
- $\mu^- \to e^-\gamma$ in muonic atom
  [Y. Kuno, Y. Uesaka, MY, PRD (2025)]
- $\mu^- \to eX$ in muonic atom
  [X. Tormo, D. Bryman, A. Czarnecki, and M. Dowling PRD (2011)]

$\mu \to e$ conversion in muonic atom

J. Steinberger and H. Wolfe, Phys. Rev. (1955)



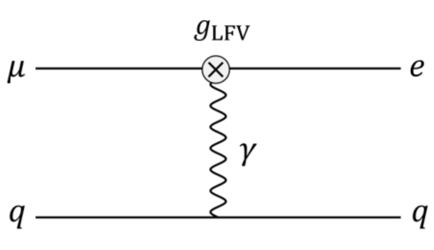
Signal

-- Monoenergetic electron $E_e \simeq 105$ MeV

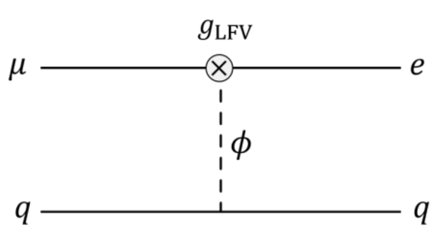
Experimental status

Process	BR limit	Future
$\mu^- \mathrm{Au} \to e^- \mathrm{Au}$	7.0×10^{-13} SINDRUM (2006)	N/A
$\mu^- \mathrm{Ti} \to e^- \mathrm{Ti}$	4.3×10^{-12} SINDRUM (2006)	N/A
$\mu^- \mathrm{Al} \to e^- \mathrm{Al}$	N/A	$a\ few \times 10^{-17}$ COMET, Mu2e

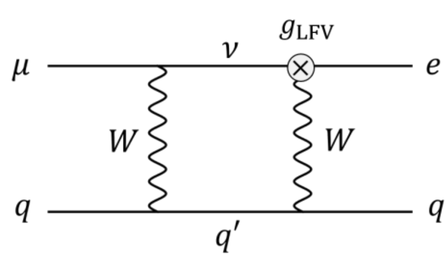
Probe to various LFV operators



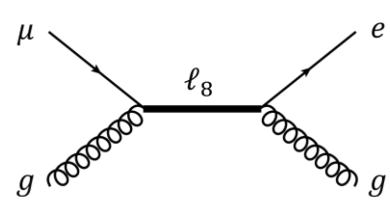
SUSY, Type-II SeeSaw



extra dimension models, extended Higgs models

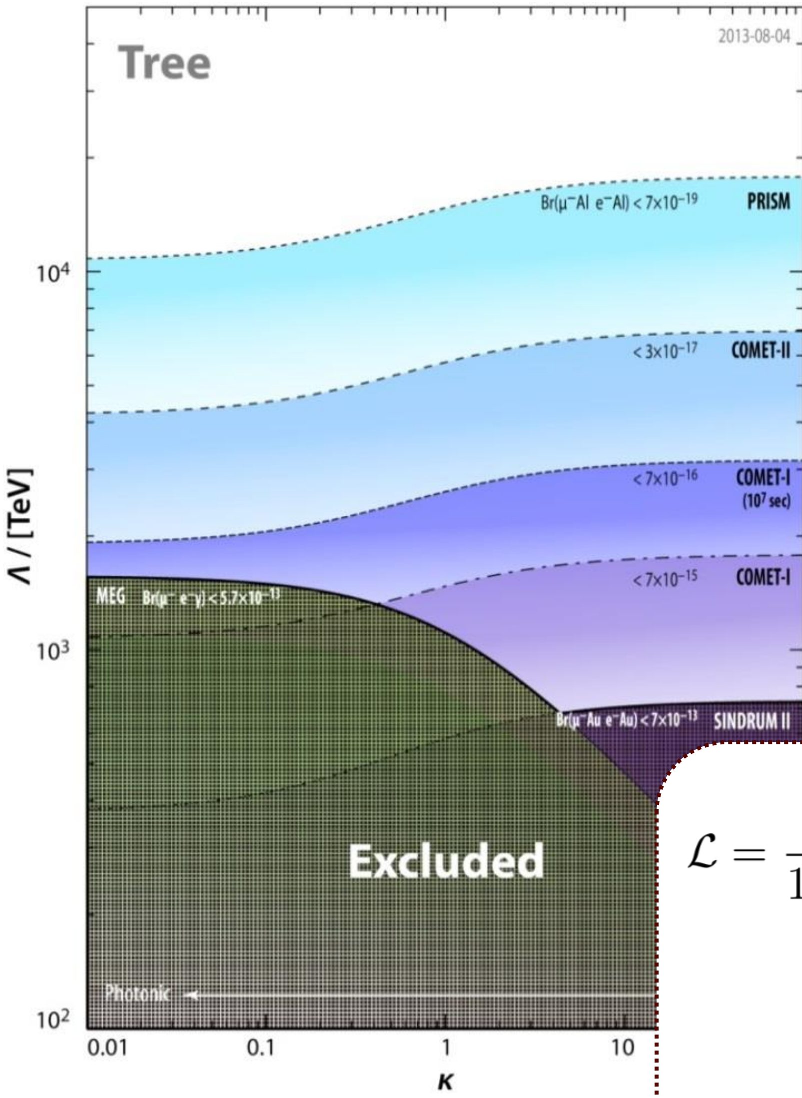


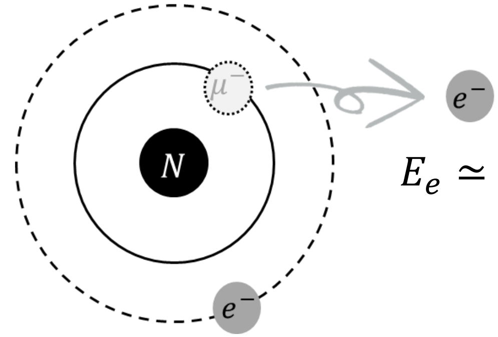
Majorana ν, sterile ν



composite, leptogluon

$\mu \to e$ conversion in muonic atom





$E_e \simeq m_\mu$

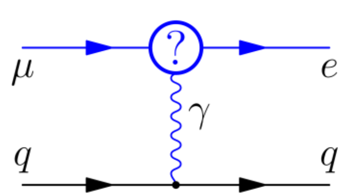
- **Probe to various types of LFV operator**

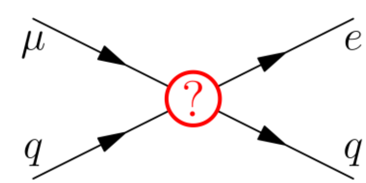
  Photonic dipole, scalar, vector, Rayleigh, etc.

- **Exploring higher scale beyond the collider energies**

  Energy scale $\gtrsim 10^4$ TeV!

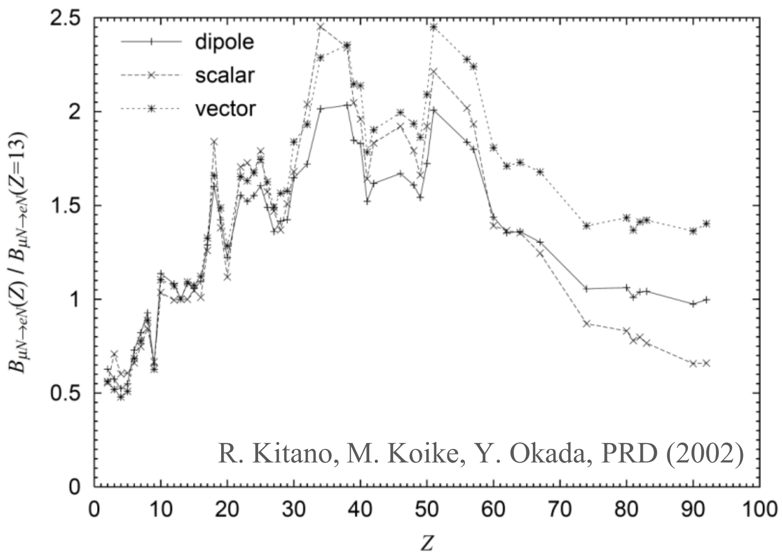
$$\mathcal{L} = \frac{1}{1+\kappa}\frac{m_\mu}{\Lambda^2}\bar{\mu}_{\rm R}\sigma^{\mu\nu}e_{\rm L}F_{\mu\nu} + \frac{\kappa}{1+\kappa}\frac{1}{\Lambda^2}\left(\bar{\mu}_{\rm L}\gamma^\mu e_{\rm L}\right)\left(\bar{q}_{\rm L}\gamma_\mu q_{\rm L}\right)$$

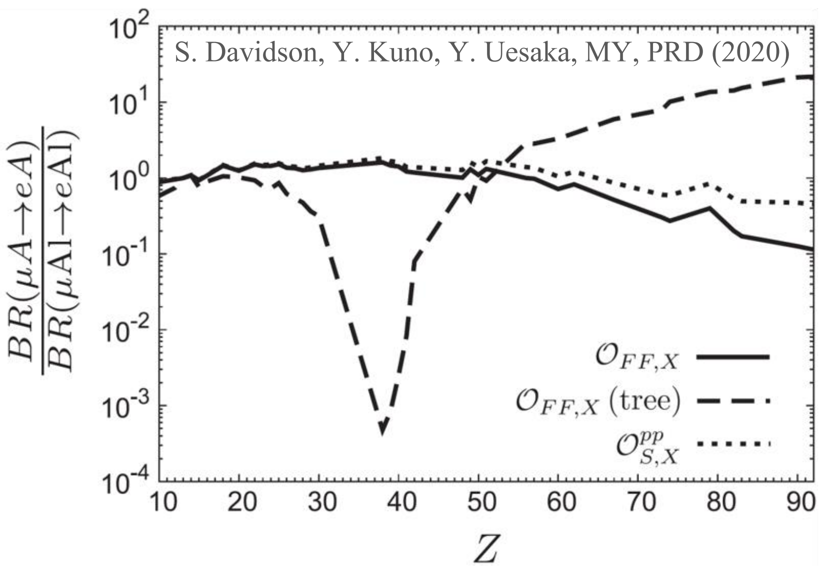




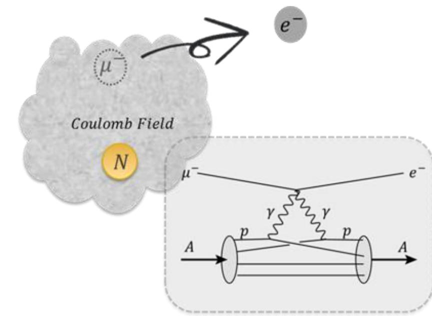
$\mu \to e$ conversion in muonic atom

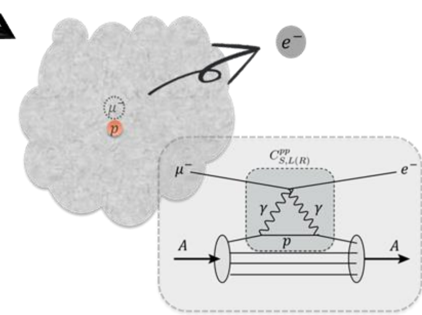
Z (atomic number) vs BR of $\mu \to e$ conversion for various type of operators





Measure the *Z* dependence of conversion rate
-- not only a discovery channel, also has the potential to identify the types of LFV operator





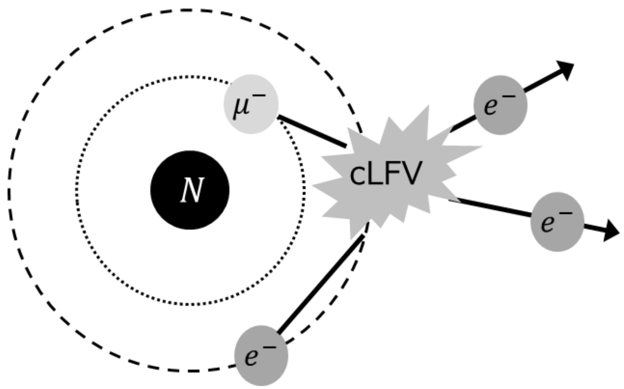
$\mu^- e^- \to e^- e^-$ in muonic atom

M. Koike, Y. Kuno, J. Sato, MY, PRL (2010)

Signal

-- back-to-back di-electron of $E_e \simeq m_\mu/2$

-- time coincidence $\Delta t_{ee} = 0$

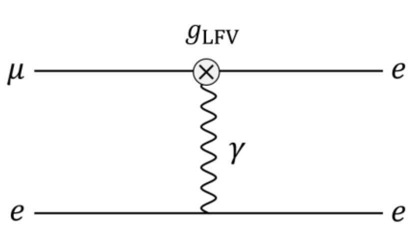


Experimental status

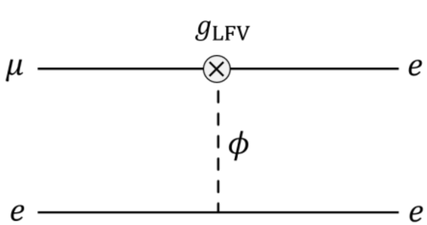
-- Limit: N/A (New process!)

-- included in physics programs of COMET

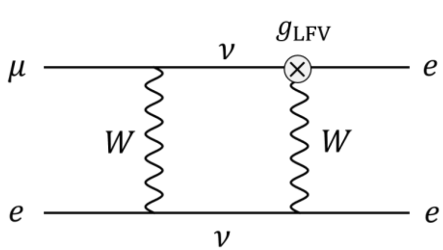
Probe to various LFV operators (pure leptonic operator, unlike $\mu \to e$ conv.)



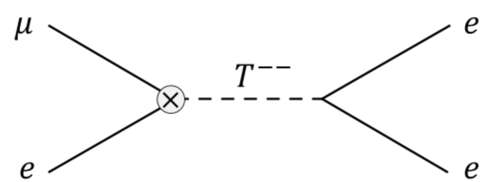
SUSY, Type-II SeeSaw



extra dimension models, little Higgs models

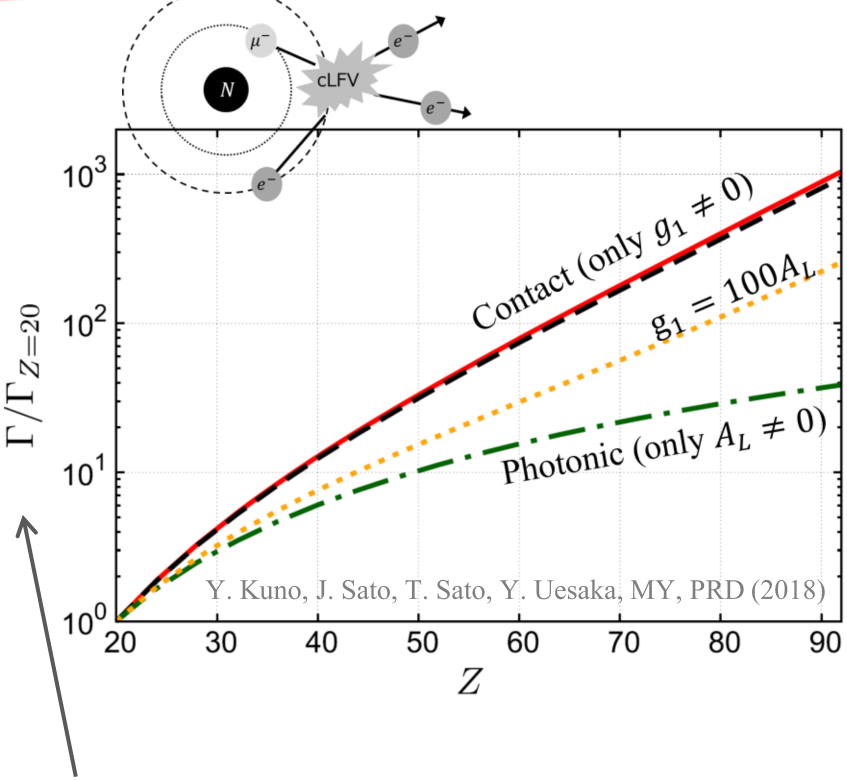


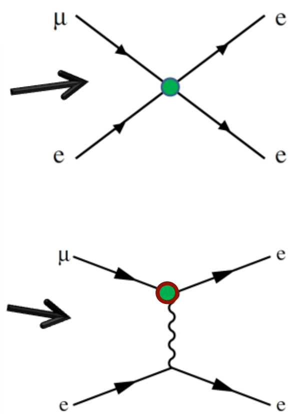
νMSM, sterile ν

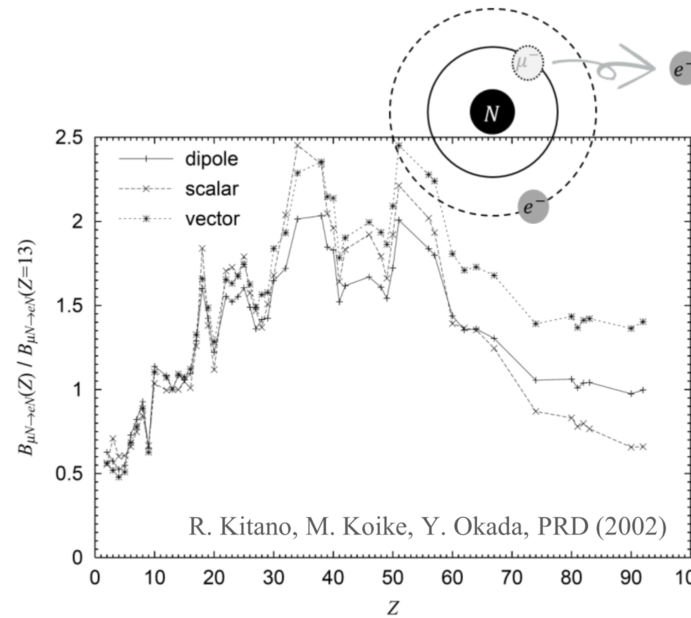


extended Higgs models

$\mu^- e^- \to e^- e^-$ in muonic atom







Reaction rate normalized by that for Z=20

Different *Z* dependence from $\mu \to e$ conv.

-- valuable to discriminate LFV source!

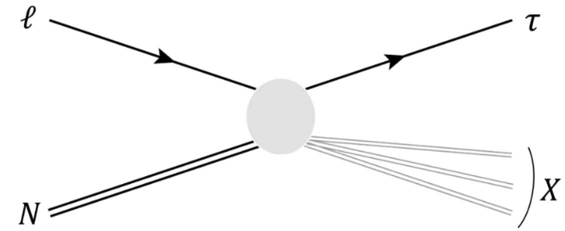
Why heavy nucleus is favored?

Coulomb potential attracts leptons toward the nucleus position, and enhances overlap of their wave functions

2. **Many LFV observables as possible** to draw “unknown” from various angles

2.3 LFV Deep Inelastic Scattering (LFV DIS)

LFV Deep inelastic scattering $e(\mu)N \to \tau X$



<u>Signal</u>

-- τ with large momentum along the beam-axis
(highly depends on types of LFV operator)

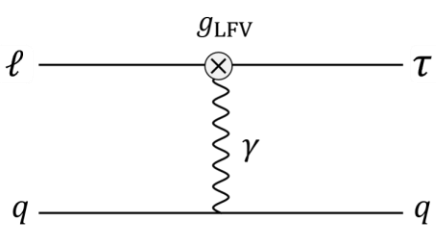
<u>Experimental status</u>

-- Limit: HERA exp.

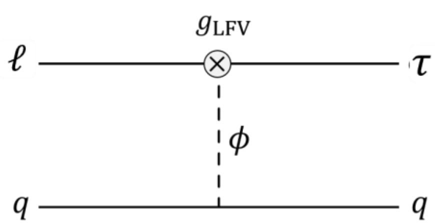
-- Future: EIC, LHeC, ILC, …

$e \to \tau$	ZEUS	$F = 0$					
$\alpha\beta$	$S^L_{1/2}$ e^+u_α	$S^R_{1/2}$ $e^+(u+d)_\alpha$	$\tilde{S}^L_{1/2}$ e^+d_α	V^L_0 e^+d_α	V^R_0 e^+d_α	$\tilde{V}^R_0$ e^+u_α	V^L_1 $e^+(\sqrt{2}u+d)_\alpha$
1 1	$\tau \to \pi e$ 0.4 **3.0**	$\tau \to \pi e$ 0.2 **2.5**	$\tau \to \pi e$ 0.4 **4.6**	G_F 0.2 **3.3**	$\tau \to \pi e$ 0.2 **3.3**	$\tau \to \pi e$ 0.2 **2.4**	G_F 0.2 **1.2**
1 2	$\boxed{3.1}$	$\tau \to Ke$ 5 $\boxed{2.5}$	$K \to \pi\nu\bar{\nu}$ 10^{-3} **4.7**	$\tau \to Ke$ 3 **3.7**	$\tau \to Ke$ 3 **3.7**	$\boxed{2.7}$	$K \to \pi\nu\bar{\nu}$ 2.5×10^{-4} **1.3**
1 3	*	$B \to \tau\bar{e}X$ 8 $\boxed{5.1}$	$B \to \tau\bar{e}X$ 8 $\boxed{5.1}$	$B \to l\nu X$ 2 **4.6**	$B \to \tau\bar{e}X$ 4 **4.6**	*	$B \to l\nu X$ 2 **4.6**
2 1	$\boxed{16}$	$\tau \to Ke$ 5 **9.2**	$K \to \pi\nu\bar{\nu}$ 10^{-3} **12**	$\tau \to Ke$ 3 **4.9**	$\tau \to Ke$ 3 **4.9**	$\boxed{6.2}$	$K \to \pi\nu\bar{\nu}$ 2.5×10^{-4} **2.6**
2 2	$\tau \to ee\bar{e}$ 20 $\boxed{20}$	$\tau \to ee\bar{e}$ 30 $\boxed{11}$	$\tau \to ee\bar{e}$ 66 $\boxed{12}$	$\tau \to ee\bar{e}$ 33 $\boxed{6.2}$	$\tau \to ee\bar{e}$ 33 $\boxed{6.2}$	$\tau \to ee\bar{e}$ 10 **11**	$\tau \to ee\bar{e}$ 6.1 $\boxed{4.3}$
2 3	*	$B \to \tau\bar{e}X$ 8 **16**	$B \to \tau\bar{e}X$ 8 **16**	$B \to l\nu X$ 2 **12**	$B \to \tau\bar{e}X$ 4 **12**	*	$B \to l\nu X$ 2 **12**
3 1	*	$B \to \tau\bar{e}X$ 8 **17**	$B \to \tau\bar{e}X$ 8 **17**	V_{ub} 0.2 **5.4**	$B \to \tau\bar{e}X$ 4 **5.4**	*	V_{ub} 0.2 **5.4**
3 2	*	$B \to \tau\bar{e}X$ 8 **22**	$B \to \tau\bar{e}X$ 8 **22**	$B \to l\nu X$ 2 **7.6**	$B \to \tau\bar{e}X$ 4 **7.6**	*	$B \to l\nu X$ 2 **7.6**
3 3	*	$\tau \to ee\bar{e}$ 30 $\boxed{30}$	$\tau \to ee\bar{e}$ 66 $\boxed{30}$	$\tau \to ee\bar{e}$ 33 $\boxed{15}$	$\tau \to ee\bar{e}$ 33 $\boxed{15}$	*	$\tau \to ee\bar{e}$ 6.1 **15**

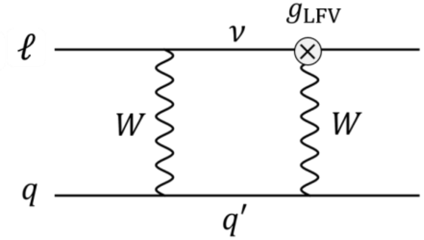
<u>Probe to various models beyond the SM</u>



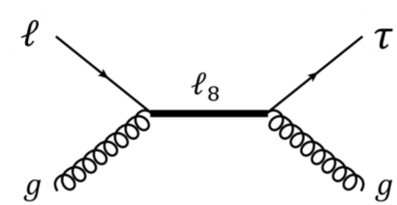
SUSY, Type-II SeeSaw



extra dimension models,
extended Higgs models



Majorana ν, sterile ν



composite, leptogluon

LFV Deep inelastic scattering $e(\mu)N \to \tau X$

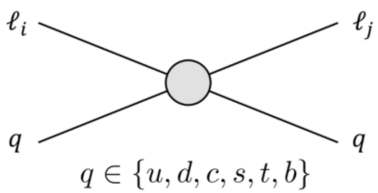
Cross section for $e(\mu)N \to \tau X$

$$\sigma_{\ell_i N \to \tau X} = \sum_{\hat{p}=g,q} \int dx dy \int_0^1 d\xi_{\hat{p}} \frac{d^2\hat{\sigma}_{\ell_i \hat{p} \to \tau \hat{X}}}{dxdy} f_{\hat{p}}(\xi_{\hat{p}}, Q^2)$$

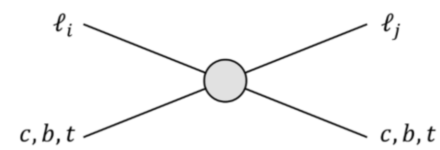
Parton distribution function (PDF)

Subprocess cross section

LFV mediator couples universally with all quarks (e.g., GUT Z', dipole LFV, etc)

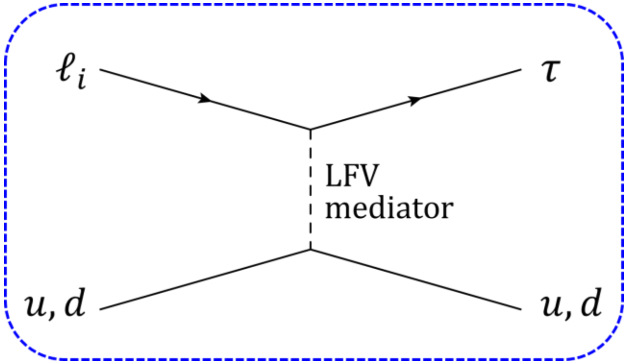


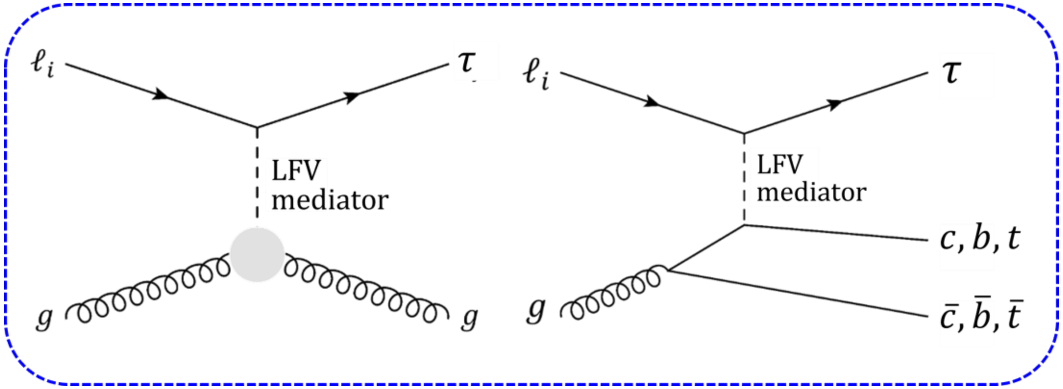
LFV mediator couples dominantly with heavy quarks (e.g., Higgs, KK bosons, etc)



Cross section, kinematics, and so on highly depends on types of LFV mediator

M. Takeuchi, Y. Uesaka, MY, PLB (2017)





LFV Deep inelastic scattering $e(\mu)N \to \tau X$

<u>Cross section for $e(\mu)N \to \tau X$</u>

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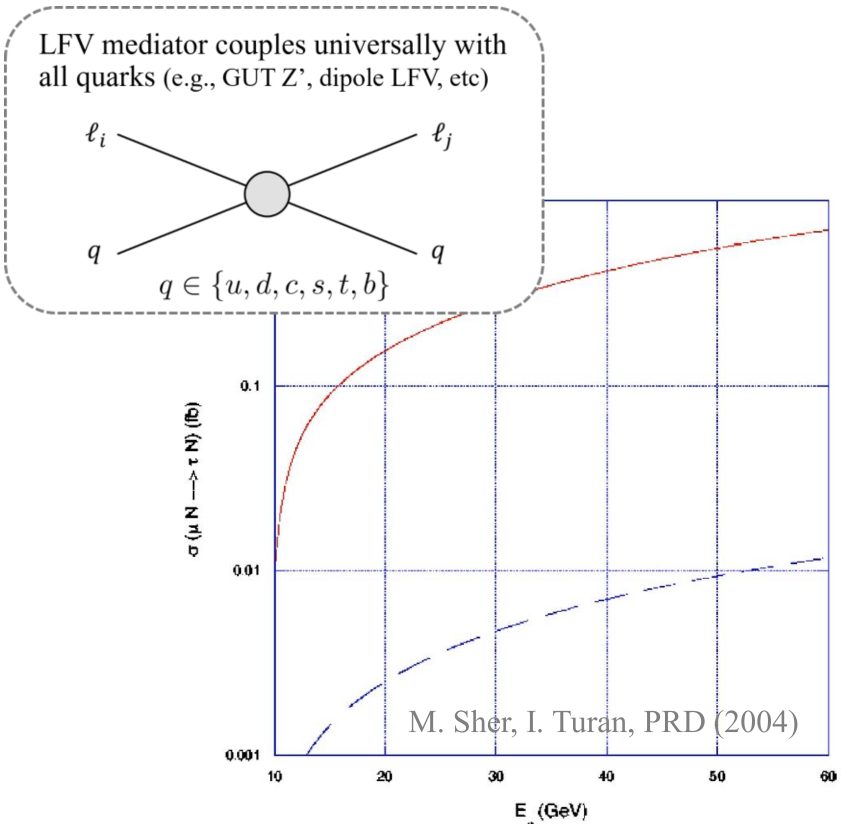
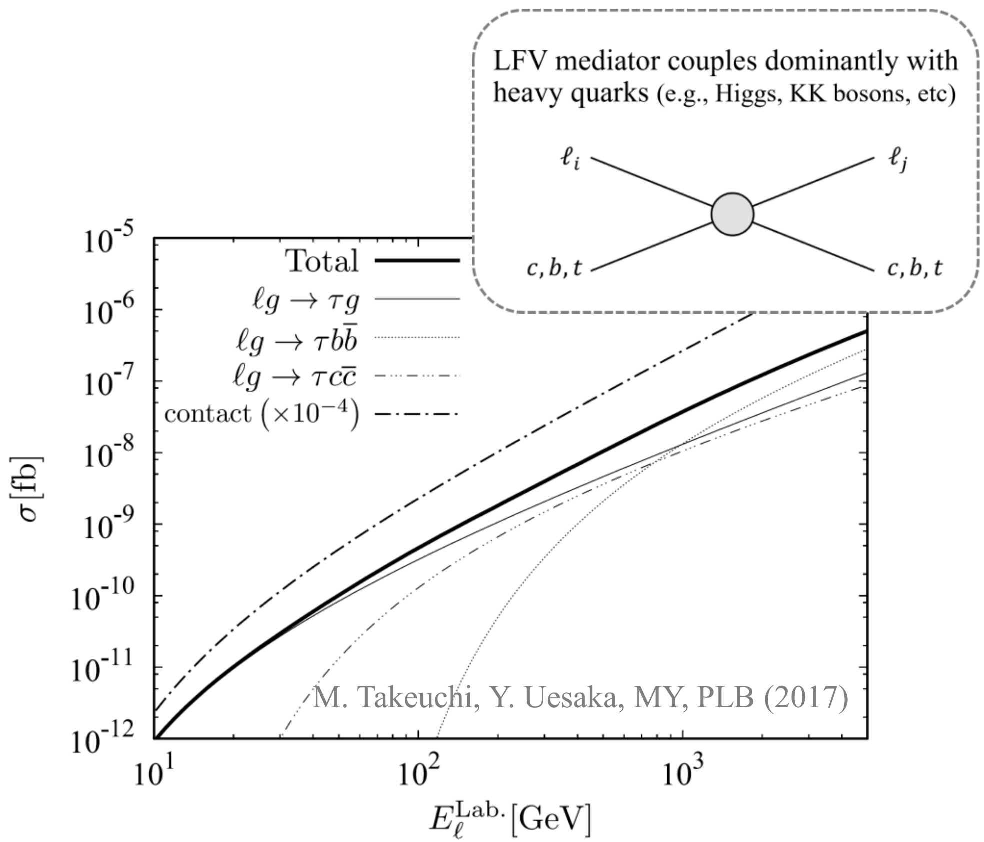
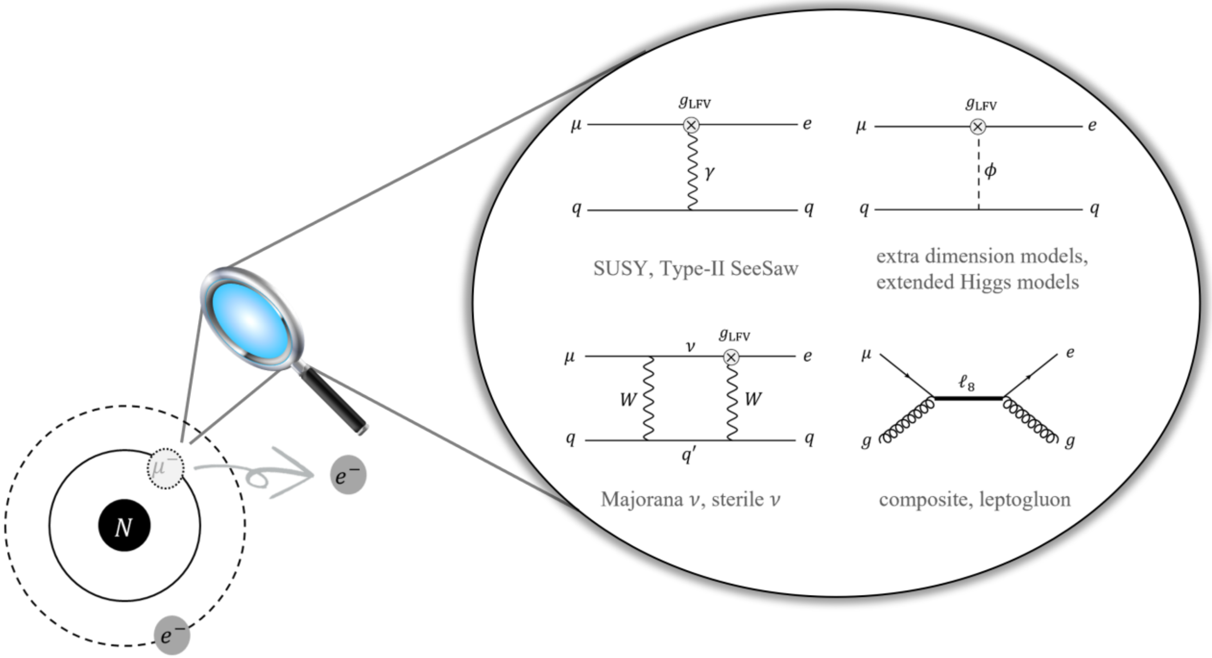
Parton distribution function (PDF)

Subprocess cross section





3. How to unravel the physics behind LFV



Unknown particle **indirectly** appears in LFV reactions

Not appeared in direct observables

Find intrinsic patterns of correlation of observables!

How to unravel the physics behind LFV

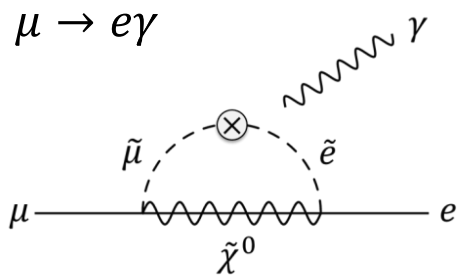
Measure the ratio of each branching ratio

Source of suppression 0.006

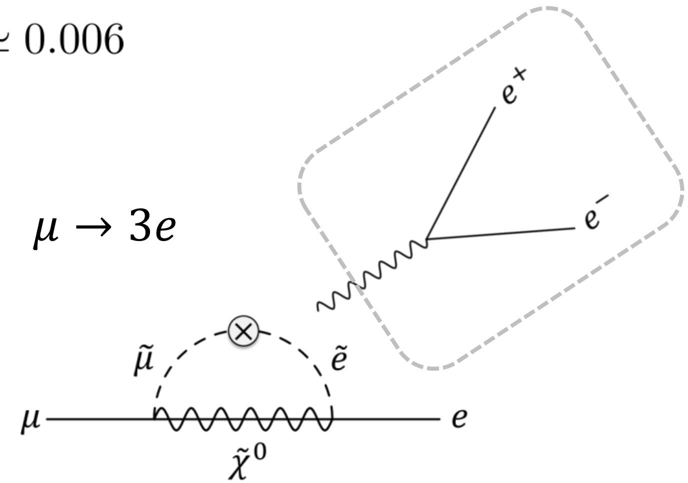
$$\frac{\mathrm{BR}(\mu \to eee)}{\mathrm{BR}(\mu \to e\gamma)} \simeq \frac{\alpha_{\mathrm{em}}}{3\pi}\left(\mathrm{Log}\left[\frac{m_\mu^2}{m_e^2}\right] - \frac{11}{4}\right) \simeq 0.006$$

Yes

$\mu \to e\gamma$



$\mu \to 3e$



Expected LFV mediator

-- Flavored massive particle (e.g., slepton)

-- Charged particle (electric charge of ± 1)

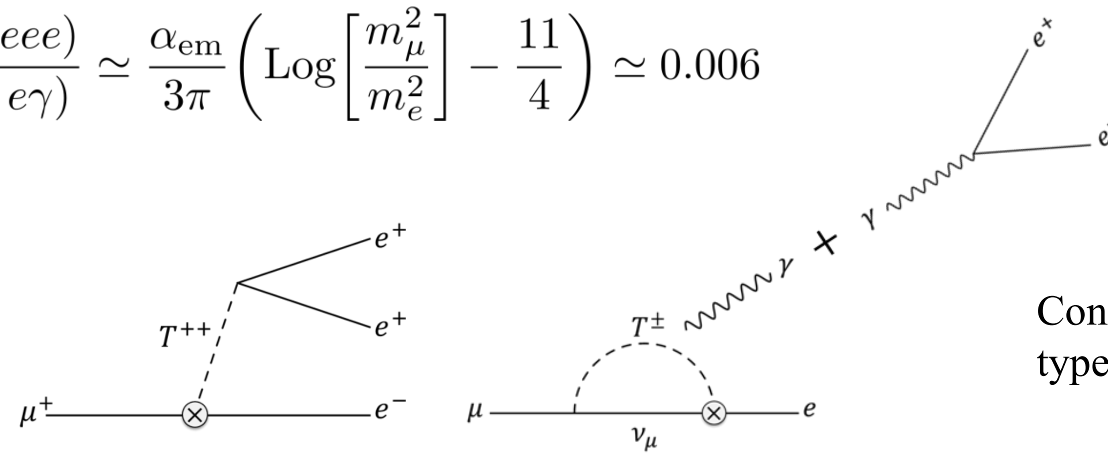
To confirm, check the similar relation for $(\tau \to 3e)/(\tau \to e\gamma)$!

How to unravel the physics behind LFV

Measure the ratio of each branching ratio

$$\frac{\mathrm{BR}(\mu \to eee)}{\mathrm{BR}(\mu \to e\gamma)} \simeq \frac{\alpha_{\mathrm{em}}}{3\pi}\left(\mathrm{Log}\left[\frac{m_\mu^2}{m_e^2}\right] - \frac{11}{4}\right) \simeq 0.006$$

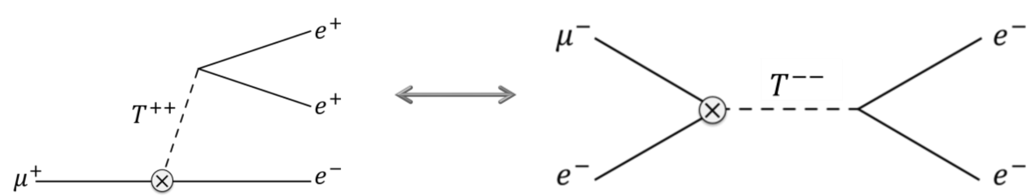
No



Contributions of two types of LFV mediators?

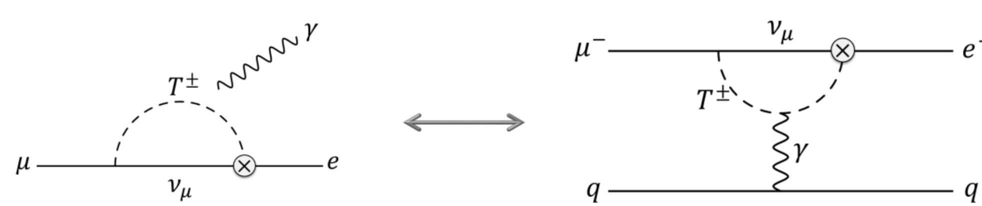
By charged particles ($\pm 2e$)?

Cross check by $\mu^- e^- \to e^- e^-$

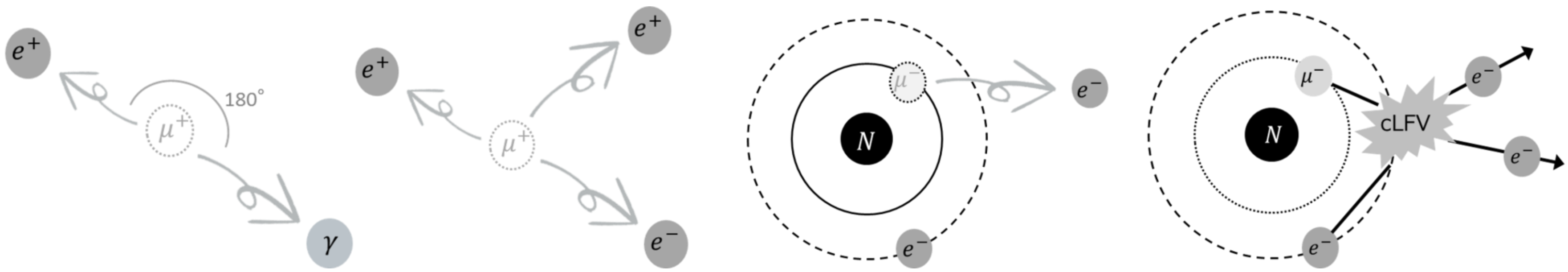


By charged particles ($\pm 1e$)?

Cross check by $\mu^- \to e^-$ conversion

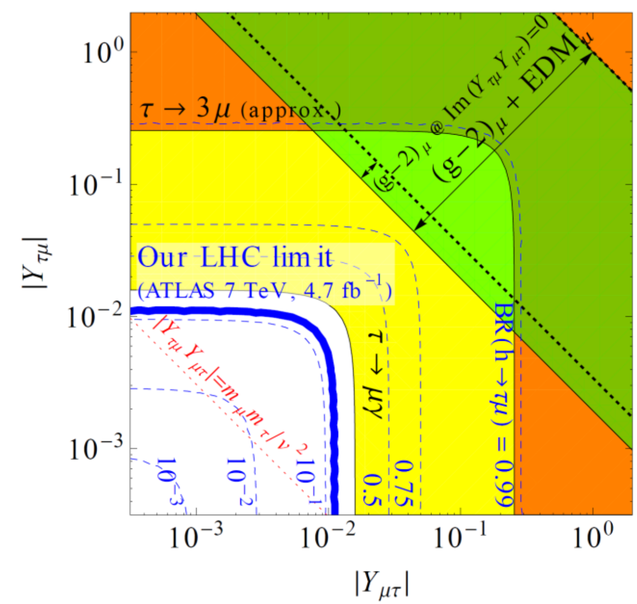


How to unravel the physics behind LFV



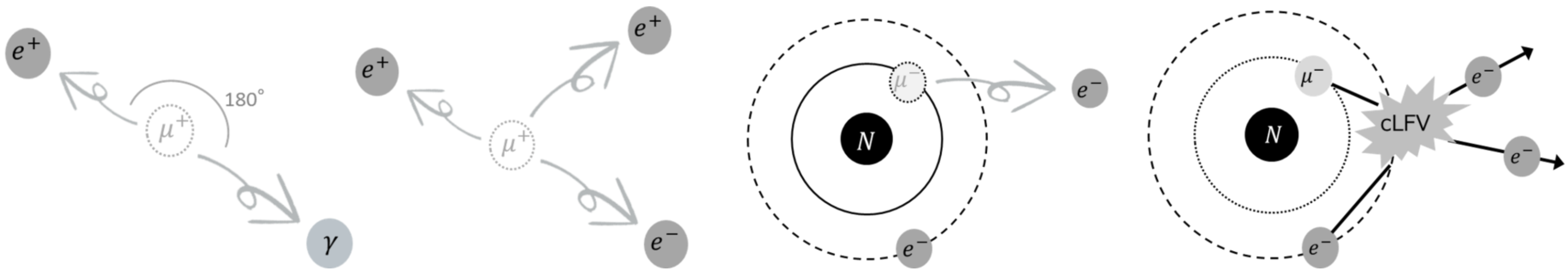
Various LFV processes are observed → Left-handed LFV ope.? $Y_{\mu e}\bar{\mu}P_L e\phi_{LFV}$ or Right-handed LFV ope.? $Y_{e\mu}\bar{\mu}P_R e\phi_{LFV}$

Chirality of LFV operators (directly) reflects the structure of physics behind the LFV



R. Harnik, J. Kopp, J. Zupan, JHEP (2013)

How to unravel the physics behind LFV

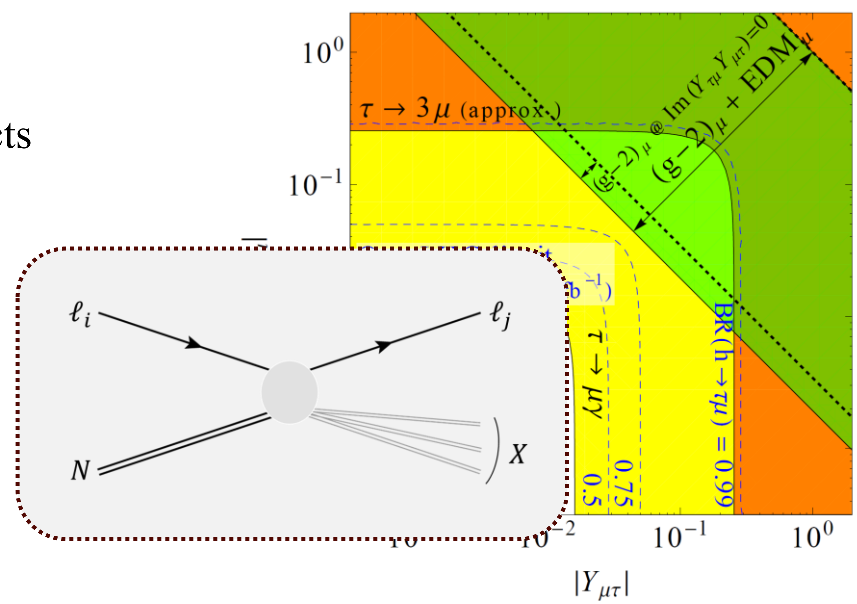


Various LFV processes are observed → Left-handed LFV ope.? $Y_{\mu e}\bar{\mu}P_L e\phi_{LFV}$ or Right-handed LFV ope.? $Y_{e\mu}\bar{\mu}P_R e\phi_{LFV}$

Chirality of LFV operators (directly) reflects the structure of physics behind the LFV

LFV DIS with polarized beam!!

e.g., for $Y_{\mu e}\bar{\mu}P_L e\phi_{LFV}$, LFV-DIS takes place with e_L-beam only



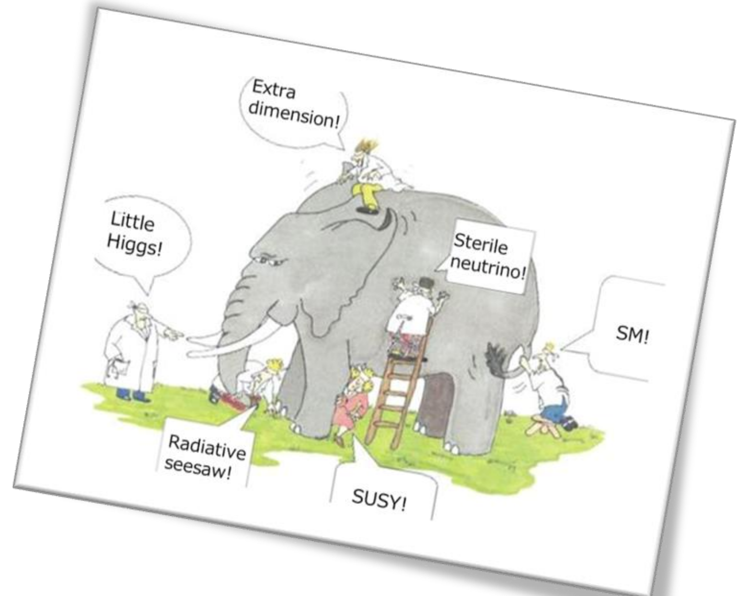
R. Harnik, J. Kopp, J. Zupan, JHEP (2013)

Summary

Summary

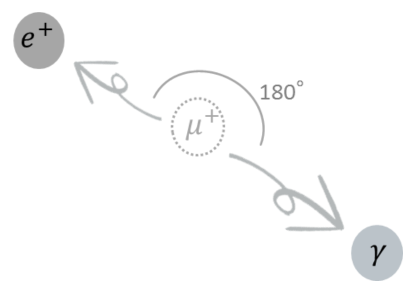
Lepton Flavor Violation (LFV)

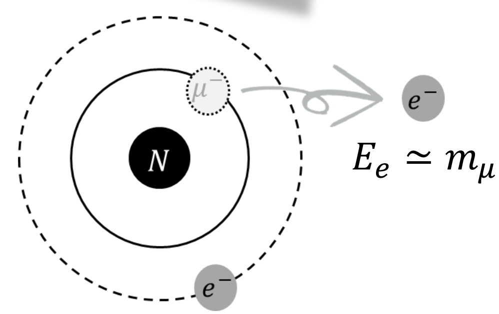
Not only an evidence, but also versatile probe to new physics!



Many LFV observables as possible to draw "unknown" from various angles

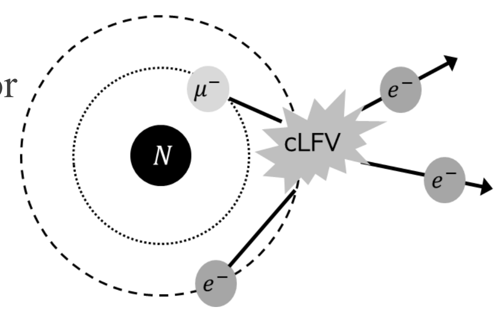
- $\mu \to e\gamma,\ \mu \to 3e$
- $\mu^- \to e^-$ conversion in nuclei
- $\mu^- e^- \to e^- e^-$ in muonic atom
- $\tau \to 3\mu, \tau \to e\pi\pi, \tau \to \mu\gamma, \dots$, etc

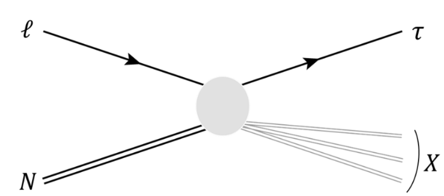




Accurate connection between LFV parameters and observables

- Characteristic signatures for each LFV operator
- Dependences on experimental stage
- Ratios and correlations of BRs etc





Backup slides

$\mu \to e$ conversion $\mu^- + N(A,Z) \to e^- + N(A,Z)$

Overlap of wave functions of μ, e, and nucleon

$$\mathrm{BR}(\mu^- \to e^- \text{ conv.}) \propto \left|\vec{v_Z} \cdot \vec{C}\right|^2$$

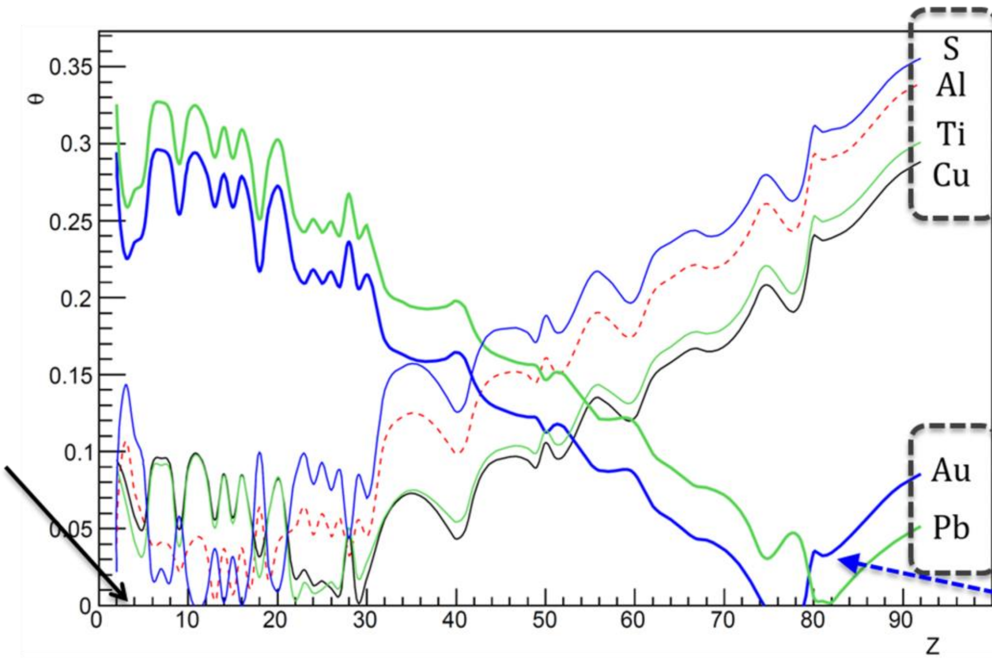
Coupling for each type of LFV int. of μ, e, and nucleon

$\vec{v}_Z$ should be misaligned each other to span $\vec{C}$

<u>Criterion for independent target</u>

$$\theta \gtrsim \begin{cases} 0.1 & \text{(light nuclei)} \\ 0.2 & \text{(heavy nuclei)} \end{cases} \qquad \cos\theta = \frac{\vec{v}_{Z_1} \cdot \vec{v}_{Z_2}}{|\vec{v}_{Z_1}||\vec{v}_{Z_2}|}$$

Target independency



"same" target

"same" target

S. Davidson, Y. Kuno, MY, PLB (2019)

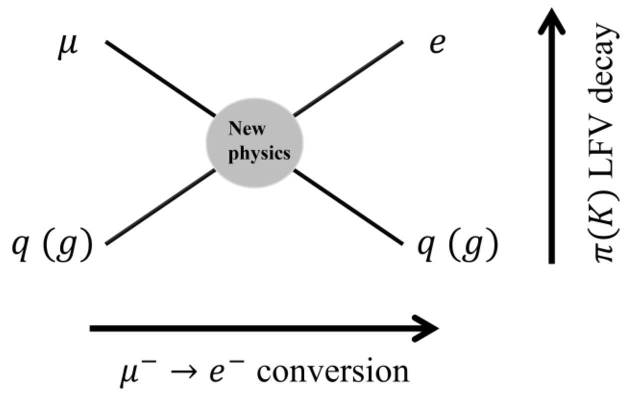
For Au at Z=82 (Pb), $\theta \simeq 0.03 < 0.2$

Complemental probe to $\ell_i \ell_j qq$ ($\ell_i \ell_j gg$)

Channel	Upper limit	Experiment	Reference
$\pi^0 \to \mu^\pm e^\mp$	3.59×10^{-10}	KTeV	75
$\eta \to \mu^\pm e^\mp$	6×10^{-6}	Saturne SPES2	76
$K_L^0 \to \pi^0 \mu^\pm e^\mp$	7.56×10^{-11}	KTeV	75
$K_L^0 \to 2\pi^0 \mu^\pm e^\mp$	1.64×10^{-10}	KTeV	75
$K_L^0 \to \mu^+ e^-$	4.7×10^{-12}	BNL E871	74
$K^+ \to \pi^+ \mu^+ e^-$	1.3×10^{-11}	BNL E865, E777	73
$D^+ \to \pi^+ \mu^\pm e^\mp$	3.4×10^{-5}	Fermilab E791	77
$D^+ \to K^+ \mu^\pm e^\mp$	6.8×10^{-5}	Fermilab E791	77
$D^0 \to \mu^\pm e^\mp$	8.1×10^{-7}	BaBar	78
$D_s^+ \to \pi^+ \mu^\pm e^\mp$	6.1×10^{-4}	Fermilab E791	77
$D_s^+ \to K^+ \mu^\pm e^\mp$	6.3×10^{-4}	Fermilab E791	77
$B^0 \to \mu^\pm e^\mp$	9.2×10^{-8}	BaBar (347 fb^{-1})	79
$B^0 \to \tau^\pm e^\mp$	1.1×10^{-4}	CLEO (9.2 fb^{-1})	80
$B^0 \to \tau^\pm \mu^\mp$	3.8×10^{-5}	CLEO (9.2 fb^{-1})	80
$B^+ \to K^+ e^\pm \mu^\mp$	9.1×10^{-8}	BaBar (208 fb^{-1})	81
$B^+ \to K^+ e^\pm \tau^\mp$	7.7×10^{-5}	BaBar (348 fb^{-1})	82
$B_s^0 \to e^\pm \mu^\mp$	6.1×10^{-6}	CDF (102 pb^{-1})	83

Abbreviations: BNL, Brookhaven National Laboratory; CDF, Collider Detector at Fermilab; KTeV, Kaons at the Tevatron.

W. J. Marciano, T. Mori, J. M. Roney, Ann. Rev. Nucl. Part (2008)

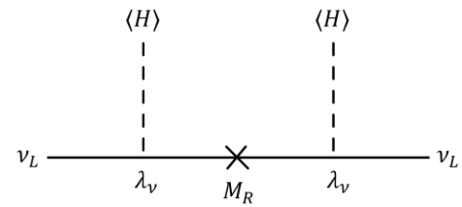


No future projects of search for LFV meson decay at J-PARC???

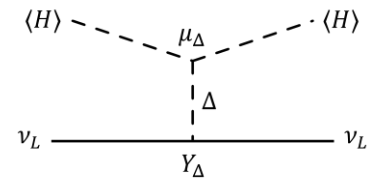
ν mass generation scenarios

(1) Tree-level mass generation

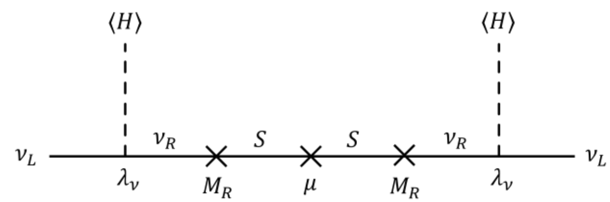
Type-I (-II, -III) Seesaw
Inverse Seesaw
Linear Seesaw
High- (Low-, middle-) scale Seesaw, etc
[with or without SUSY, extra dimension, etc]



Type I Seesaw



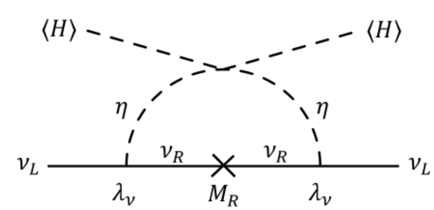
Type II Seesaw



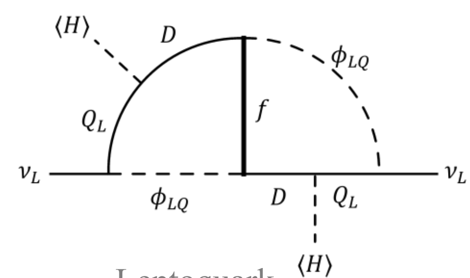
Inverse Seesaw

(2) loop-level mass generation

Scotogenic
Leptoquark
Zee-Babu
R-parity violating SUSY, etc
[1-loop, 2-loop, 3-loop, …]



Scotogenic



Leptoquark

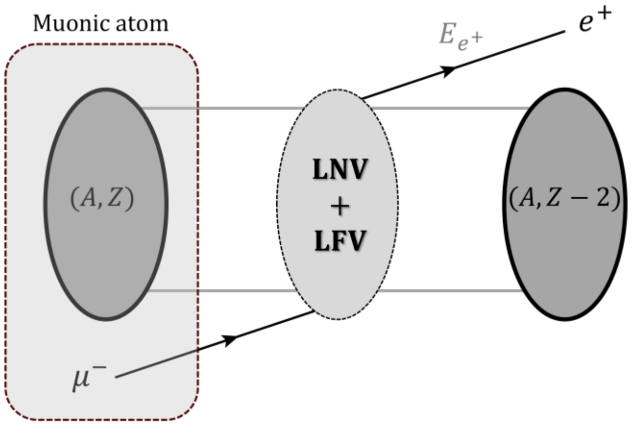
$\boldsymbol{\mu^- \to e^+}$ conversion $\boldsymbol{\mu^- + N(A,Z) \to e^+ + N'(A,Z-2)}$

Signal

Monoenergetic positron $E_{e^+} \simeq (depends\ on\ N)$

$$E_{e^+} = m_\mu - B_\mu - E_{recoil} + [M(A,Z) - M(A,Z-2)]$$

B_μ : binding energy of the muonic atom
E_{recoil} : recoil energy of the outgoing nucleus



Lepton number	+1	→	−1
μ flavor number	+1	→	0
e flavor number	0	→	−1

Bound

SINDRUM

$\mathrm{BR}(\mu^- + \mathrm{Ti} \to e^+ + \mathrm{Ca}) < 1.7 \times 10^{-12}$

$\mathrm{BR}(\mu^- + \mathrm{Ti} \to e^+ + \mathrm{Ca}^*) < 3.6 \times 10^{-11}$

Discovery = new physics scenario beyond the SM!!

-- **neither** lepton number nor flavor number are conserved, while both are always conserved in SM

Particle	Lepton number	Electron flavor number	Muon flavor number	Tau flavor number
e and ν_e	+1	+1	0	0
μ and ν_μ	+1	0	+1	0
τ and ν_τ	+1	0	0	+1

Opposite sign for anti-lepton. Ex. e number $= -1$ for positron

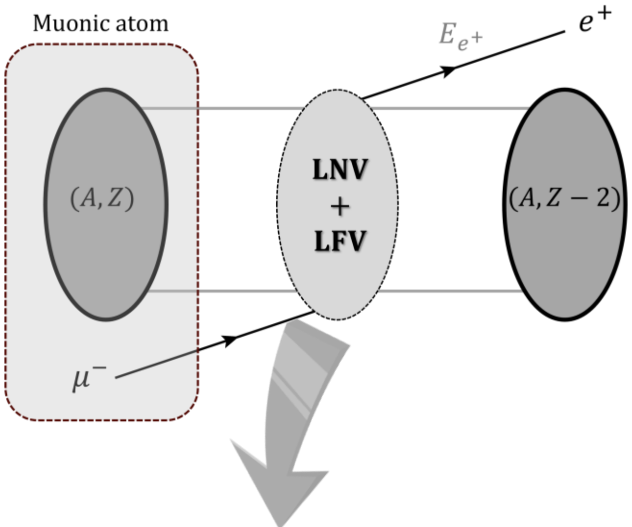
$\mu^- \to e^+$ conversion $\mu^- + N(A,Z) \to e^+ + N'(A,Z-2)$

Signal

Monoenergetic positron $E_{e^+} \simeq (depends\ on\ N)$

$$E_{e^+} = m_\mu - B_\mu - E_{recoil} + [M(A,Z) - M(A,Z-2)]$$

B_μ : binding energy of the muonic atom
E_{recoil} : recoil energy of the outgoing nucleus

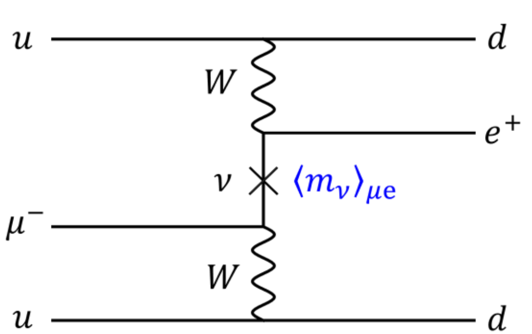


Bound

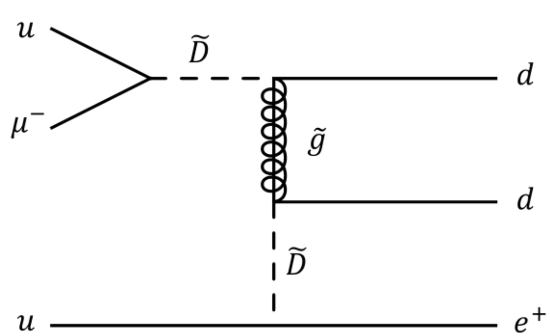
SINDRUM

$$\mathrm{BR}(\mu^- + \mathrm{Ti} \to e^+ + \mathrm{Ca}) < 1.7 \times 10^{-12}$$
$$\mathrm{BR}(\mu^- + \mathrm{Ti} \to e^+ + \mathrm{Ca}^*) < 3.6 \times 10^{-11}$$

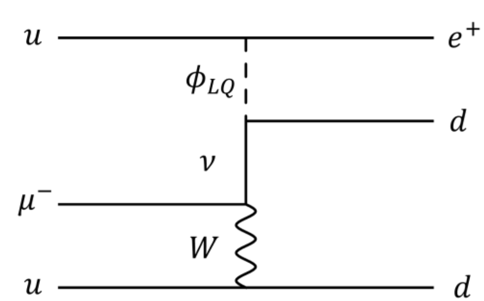
Probe to Lepton Number Violating (LNV) operators



Majorana ν, sterile ν



R-parity violating SUSY



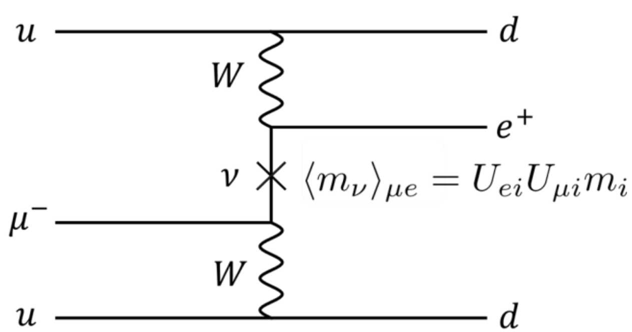
Composite, leptoquarks

$\mu^- \to e^+$ conversion $\mu^- + N(A,Z) \to e^+ + N'(A,Z-2)$

Minimal extension to allow $\mu^- \to e^+$ conversion

Must-have: mixing between flavor eigenstates
and source of LNV

→ **SM + Majorana neutrino**



$$\mathrm{BR}^{\mathrm{Majo.}}_{\mu^- \to e^+} = 1.6 \times 10^{-25} \frac{\left| \langle m_\nu \rangle_{\mu e} \right|^2}{m_e^2} \sim 10^{-33}\text{–}10^{-37}$$

P. Domin, S. Kovalenko, et al., PRC (2004)

Other extensions to allow $\mu^- \to e^+$ conversion

J. Berryman, A. Gouvea, K. Kelly, A. Kobach, PRD (2017)

Operator	Λ [TeV]	$T_{0\nu\beta\beta}$ $R_{\mu^- e^+}$
$(LL)(LH)e^c$	$4 \times 10^{6-7}$	$\ln(2)(\frac{\sqrt{2}}{G_F})^4 (q^2)^2 (\frac{16\pi^2}{y_\tau v^2})^2 \frac{\Lambda^2}{Q^{11}} \sim 10^{25}$–$10^{27}$ yr $(\frac{G_F}{\sqrt{2}})^2 (\frac{1}{q^2})^2 (\frac{y_\tau v^2}{16\pi^2})^2 \frac{Q^6}{\Lambda^2} \sim 10^{-38}$–$10^{-36}$
$(LL)(QH)d^c$	$2 \times 10^{4-5}$	$\ln(2)(\frac{\sqrt{2}}{G_F})^2 q^2 \frac{\Lambda^2}{Q^{11}} [(\frac{G_F}{\sqrt{2}})^2 \frac{1}{q^2} (\frac{y_b g^2 v^2}{(16\pi^2)^2})^2 + \frac{v^2}{\Lambda^4}]^{-1} \sim 10^{24}$–$10^{26}$ yr $\frac{1}{q^2} \frac{Q^6}{\Lambda^6} [(\frac{G_F}{\sqrt{2}})^2 \frac{1}{q^2} (\frac{y_b g^2 v^2}{(16\pi^2)^2})^2 + \frac{v^2}{\Lambda^4}] \sim 10^{-37}$–$10^{-36}$
$(LQ)(LH)d^c$	$1 \times 10^{7-8}$	$\ln(2)(\frac{\sqrt{2}}{G_F})^2 q^2 \frac{\Lambda^2}{Q^{11}} [(\frac{G_F}{\sqrt{2}})^2 \frac{1}{q^2} (\frac{y_b v^2}{16\pi^2})^2 + \frac{v^2}{\Lambda^4}]^{-1} \sim 10^{25}$–$10^{27}$ yr $\frac{1}{q^2} \frac{Q^6}{\Lambda^2} [(\frac{G_F}{\sqrt{2}})^2 \frac{1}{q^2} (\frac{y_b v^2}{16\pi^2})^2 + \frac{v^2}{\Lambda^4}] \sim 10^{-38}$–$10^{-36}$
$(L\bar{Q})(LH)\overline{u^c}$	$4 \times 10^{8-9}$	$\ln(2)(\frac{\sqrt{2}}{G_F})^2 q^2 \frac{\Lambda^2}{Q^{11}} [(\frac{G_F}{\sqrt{2}})^2 \frac{1}{q^2} (\frac{y_t v^2}{16\pi^2})^2 + \frac{v^2}{\Lambda^4}]^{-1} \sim 10^{25}$–$10^{27}$ yr $\frac{1}{q^2} \frac{Q^6}{\Lambda^2} [(\frac{G_F}{\sqrt{2}})^2 \frac{1}{q^2} (\frac{y_t v^2}{16\pi^2})^2 + \frac{v^2}{\Lambda^4}] \sim 10^{-38}$–$10^{-36}$
$(LL)(\bar{Q}H)\overline{u^c}$	2–7	This operator cannot contribute to $0\nu\beta\beta$. $\frac{v^2 Q^6}{q^2 \Lambda^2} [(\frac{G_F}{\sqrt{2}})^2 (\frac{y_d}{g^2})^2 (\frac{y_t}{16\pi^2})^2 + \frac{1}{\Lambda^4}] \sim 10^{-27}$–$10^{-24}$
$(LH)\overline{e^c}\,\overline{u^c}\,d^c$	$6 \times 10^{2-3}$	$\ln(2)(\frac{\sqrt{2}}{G_F})^2 q^2 \frac{\Lambda^2}{Q^{11}} [(\frac{G_F}{\sqrt{2}})^2 \frac{1}{q^2} (\frac{y_t y_b v}{(16\pi^2)^2})^2 + \frac{v^2}{\Lambda^4}]^{-1} \sim 10^{27}$–$10^{29}$ yr $\frac{1}{q^2} \frac{Q^6}{\Lambda^2} [(\frac{G_F}{\sqrt{2}})^2 \frac{1}{q^2} (\frac{y_t y_b y_\mu v^2}{(16\pi^2)^2})^2 + \frac{v^2}{\Lambda^4}] \sim 10^{-40}$–$10^{-38}$

Overall energy scale which generates m_ν = 0.5eV - 0.05eV

$0\nu 2\beta$ lifetime and BR($\mu^- \to e^+$) for the energy scale Λ