Modelling DM with AGAMA

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Outline

- Why f(J) modelling?
- Application to Fornax & MW
- DFs for spheroidal components
- A new version of AGAMA
- Application to stellar streams
- Valuable contributions from 2nd → 3rd yr UG Tom Wright

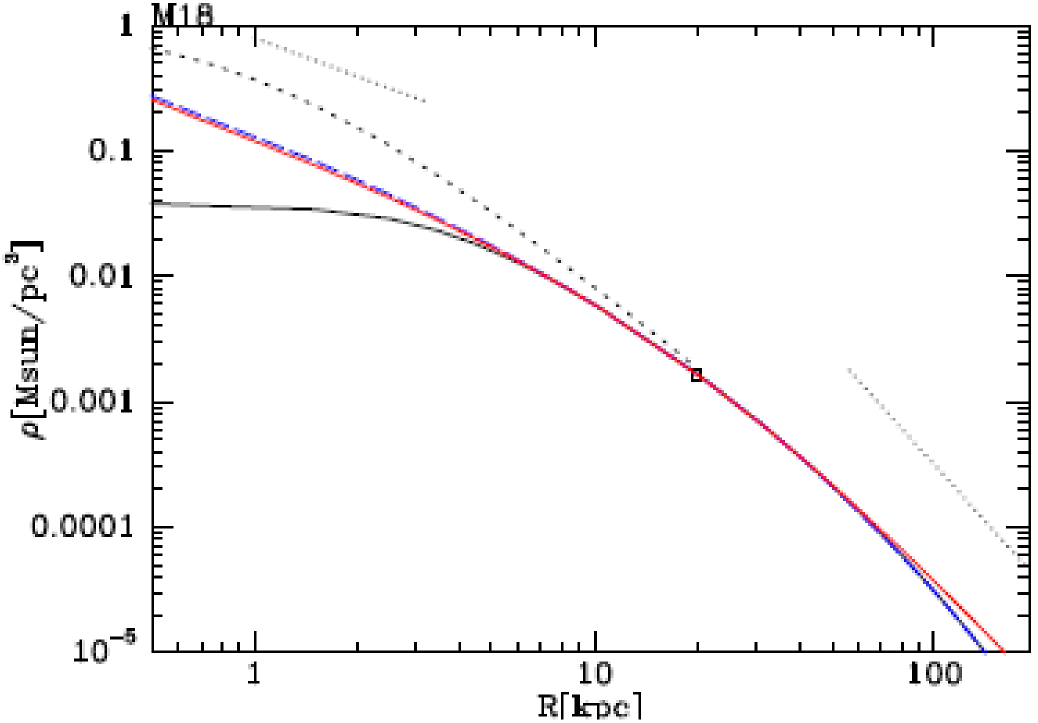
Why f(J) modelling?

- DM can only be mapped by its contribution to the grav field
- We map the grav field by its effect on stars
- For now this requires the assumption of a statistical steady state
- Traditionally use Jeans eqs but these suboptimal because
  - 1) $F(x)$ estimates dominated by $\rho(x)$, which is obscured by dust
  - 2) Jeans eqs don’t exploit shape of v-distributions, which are not affected by dust

f(J) modelling

- Adopt parametrised forms for f(J) for DM, & stars of various pops
  - (age, chemistry)
- Choose $\Phi_{gas}(x)$
- Make reasonable guess of $\Phi(x)$, solve for $\rho(x)$ by integrating over v
- Solve for new $\Phi(x)$ and iterate to convergence
- DM-only simulations provide better guidance re $f_{DM}(x,v)$ than $\rho_{DM}(x)$
- ρ_{DM} materially modified by gravity of baryons

B & Vasliev 2023

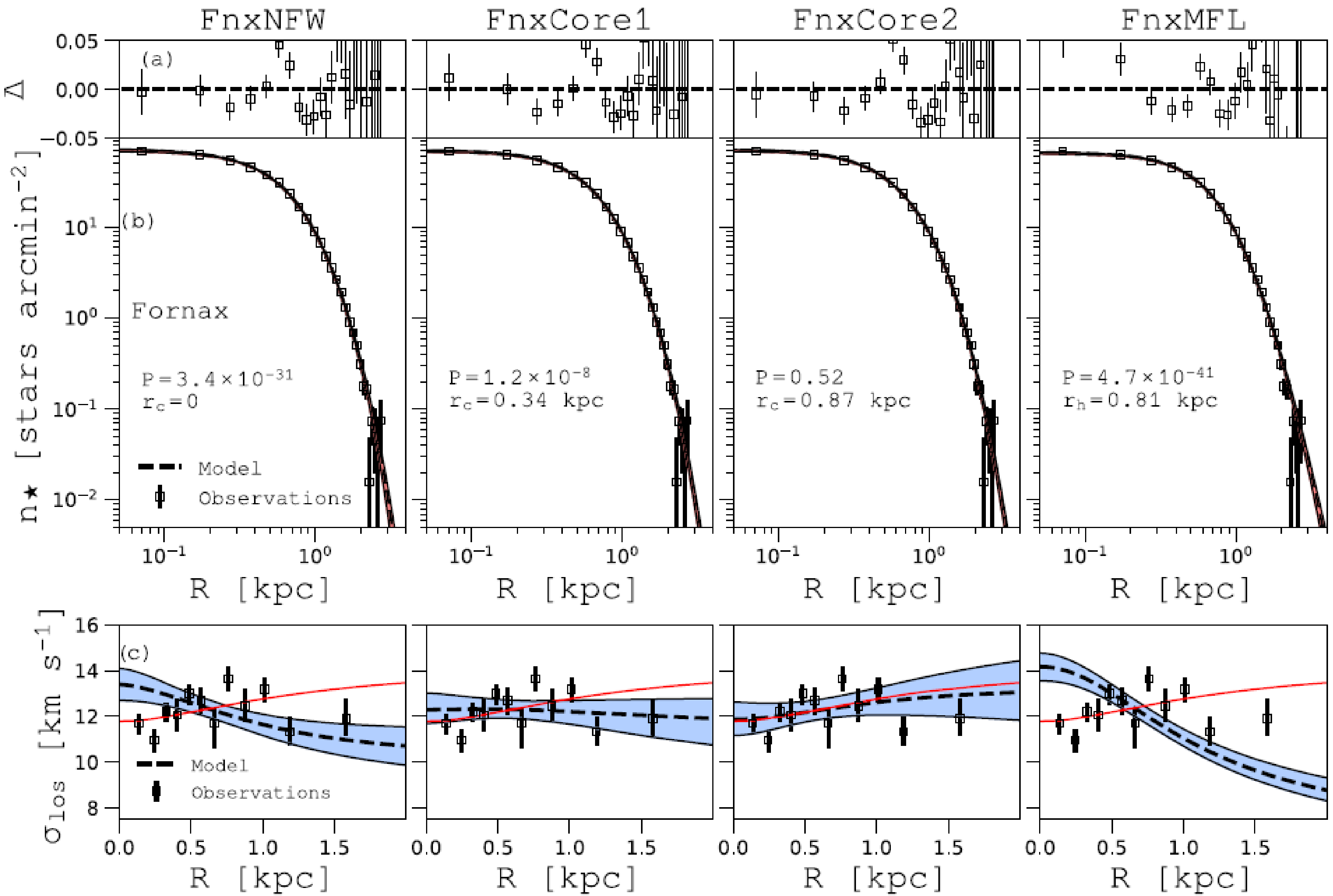


Connection to Schwarzschild modelling

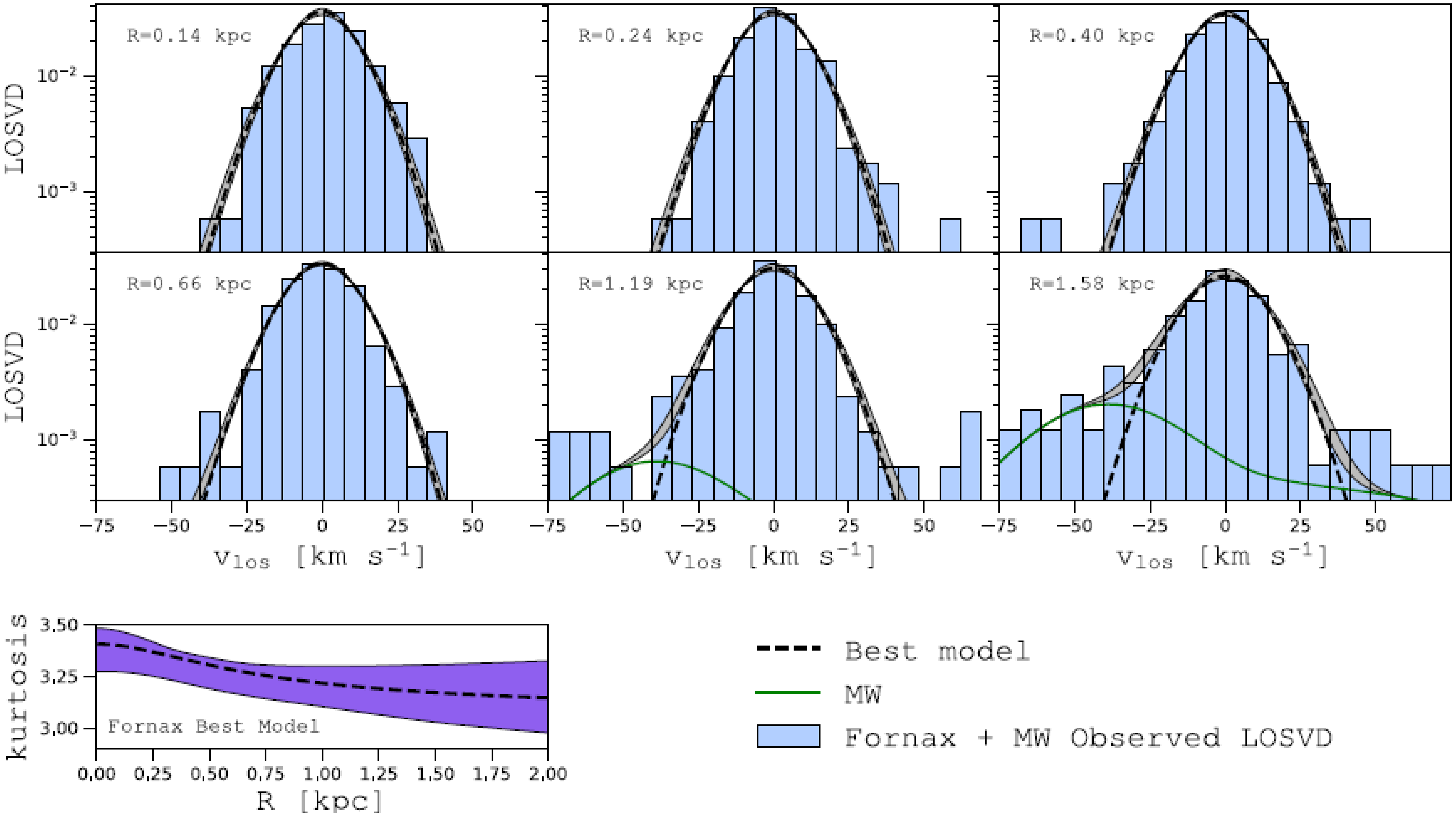
- f(J) models adopt analytic functions f(J|a,b,c..) of actions with parameters a,b,c..
- But one could assign orbits labelled by J weights w in the same way that Schwarzschild modellers weight orbit labelled by i.c.s
  - Then it wouldn't be necessary to fix Φ(x) up front
- Analytic f(J) assures DFs are smooth functions (is this an advantage?) and limit the # of parameters, facilitating parameters searches
  - But the real DF probably isn't reachable by the chosen forms….

Applications

- Pascale+ (2018,19) used f(J) to model dSph galaxies
  - Argued that cored f_{DM} required
- Piffl+ (2015), Cole & B (2016), B & Vasiliev (2023,24) used f(J) to model MW
  - Argued that circular speed is falling at R_0, estimated ρ_{DM} everywhere



Pascale+ 2018
on Fornax


R=0.14 kpc
R=0.24 kpc
R=0.40 kpc
R=0.66 kpc
R=1.19 kpc
R=1.58 kpc
LOSVD
10^{-2}
10^{-3}
−75 −50 −25 0 25 50
v_{los} [km s^{-1}]
kurtosis
3,50 3,25 3,00
Fornax Best Model
0,00 0,25 0,50 0,75 1,00 1,25 1,50 1,75 2,00
R [kpc]
Best model
MW
Fornax + MW Observed LOSVD

Applications

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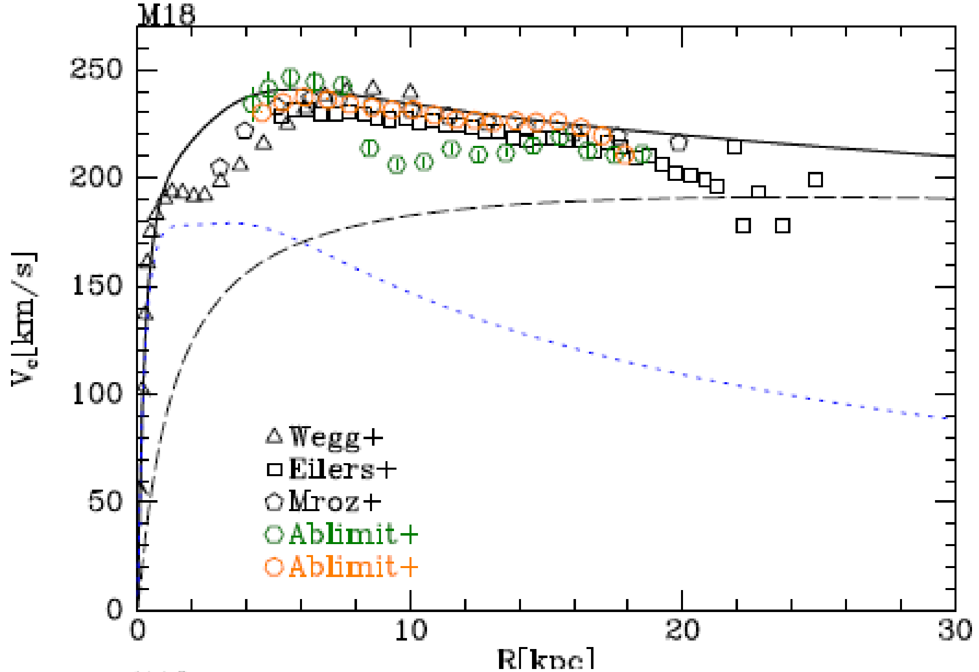
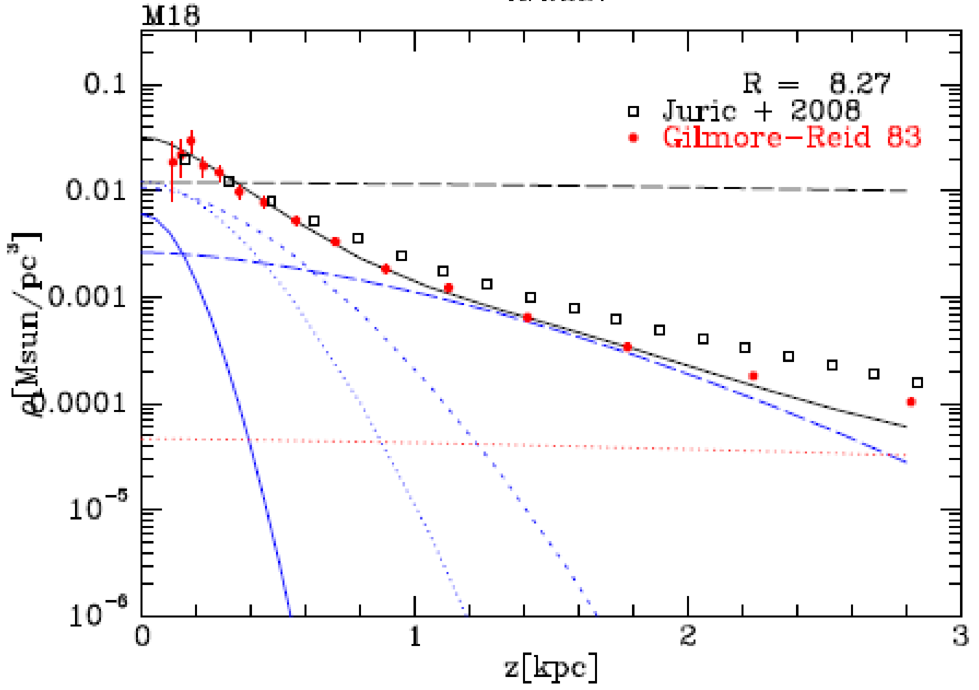
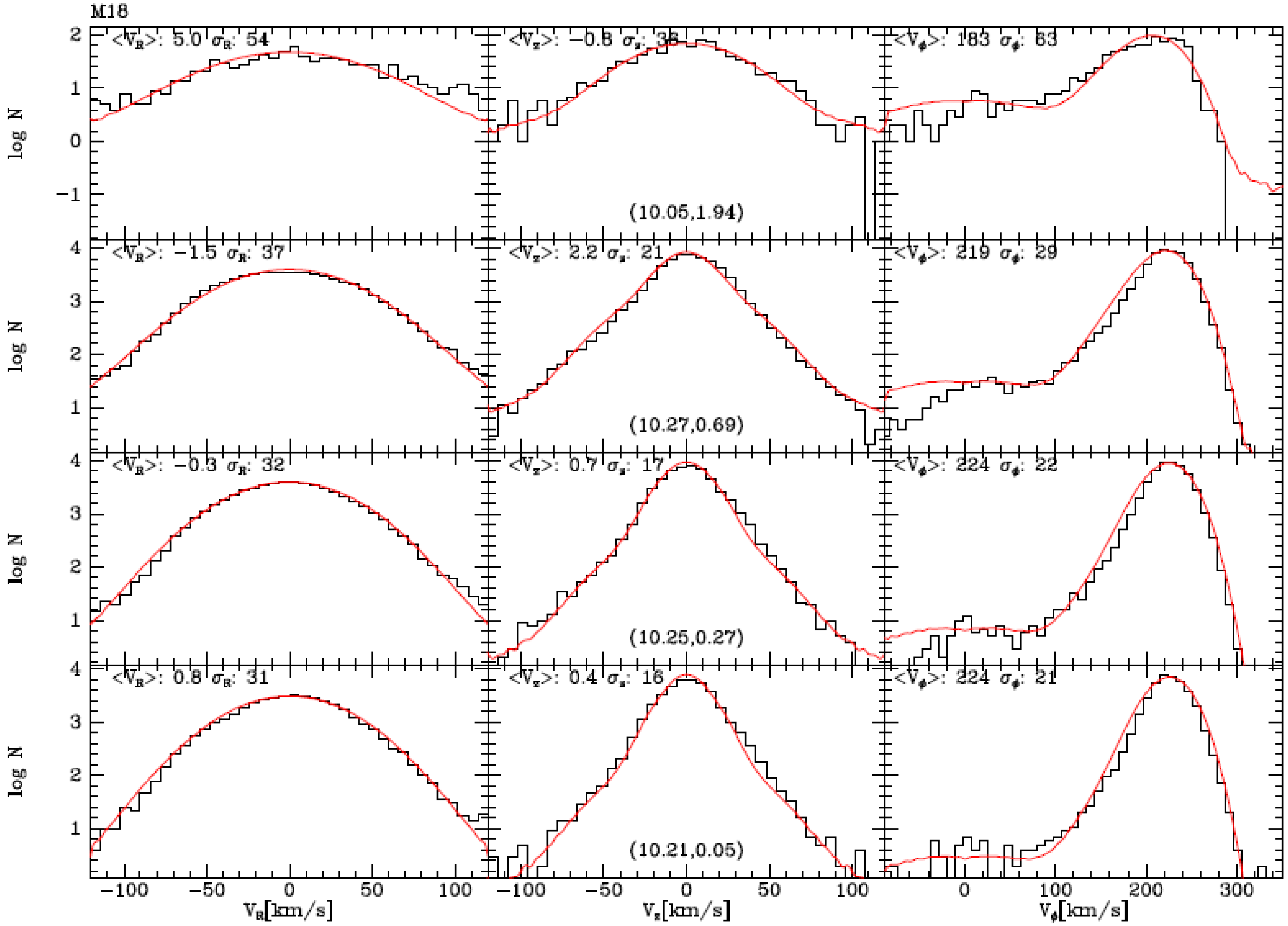




B & Vasiliev 2023



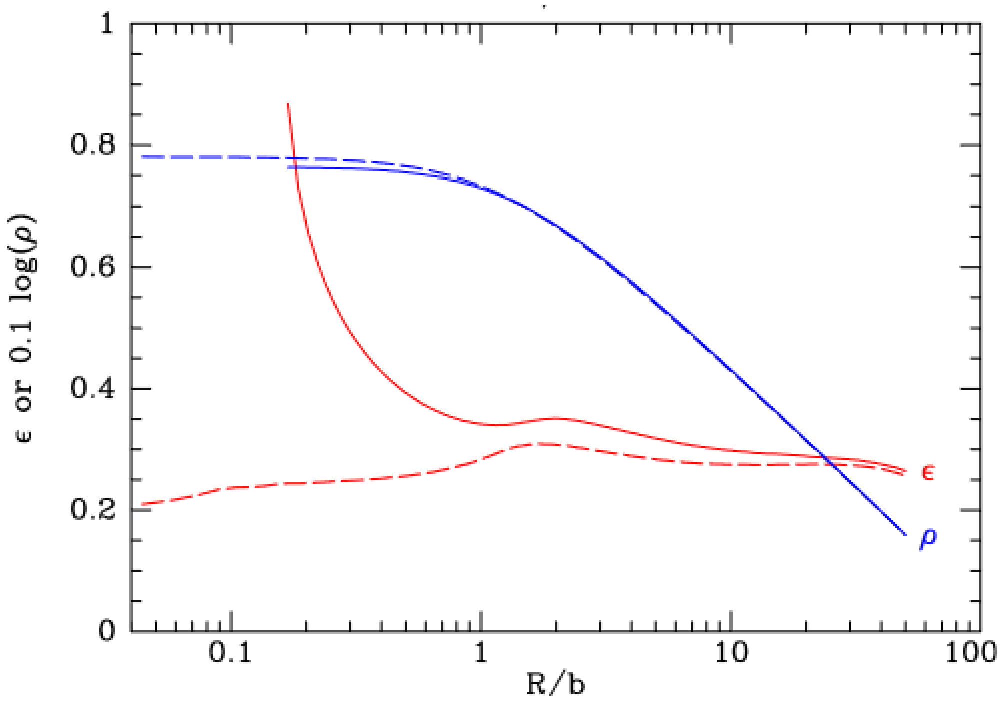
A serious limitation

- Pascale+ produced only spherical models
- Work on MW avoided DFs for DM and stellar halo with the required degree of velocity anisotropy

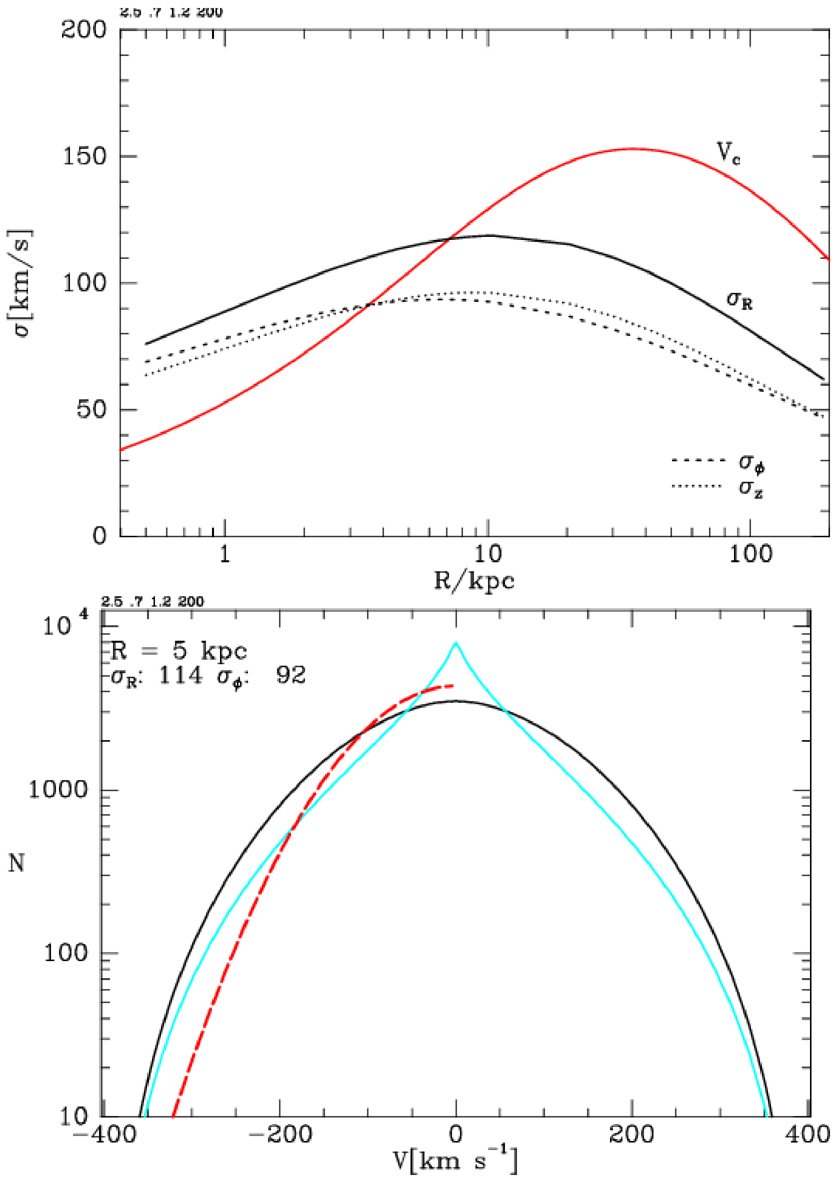
B 2014

DFs for spheroids

- Henon's isochrone has known f(H) and H(J) so we can write down f(J) for the ergodic isochrone
- Binney (2014) produced flattened isochrones by changing the weights on J_r and J_z in f(J)
    - Radial anisotropy produced by lowering weight on J_r
    - Flattening produced by increasing weight on J_z
- But
    - Resulting models had unphysical shapes near centre
    - Piffl & B (2015) encountered unphysical $N(v_\phi)$ when using this technique to make stellar halo radially biased



- NFW-type dark halo
- (Posti+ 2015)

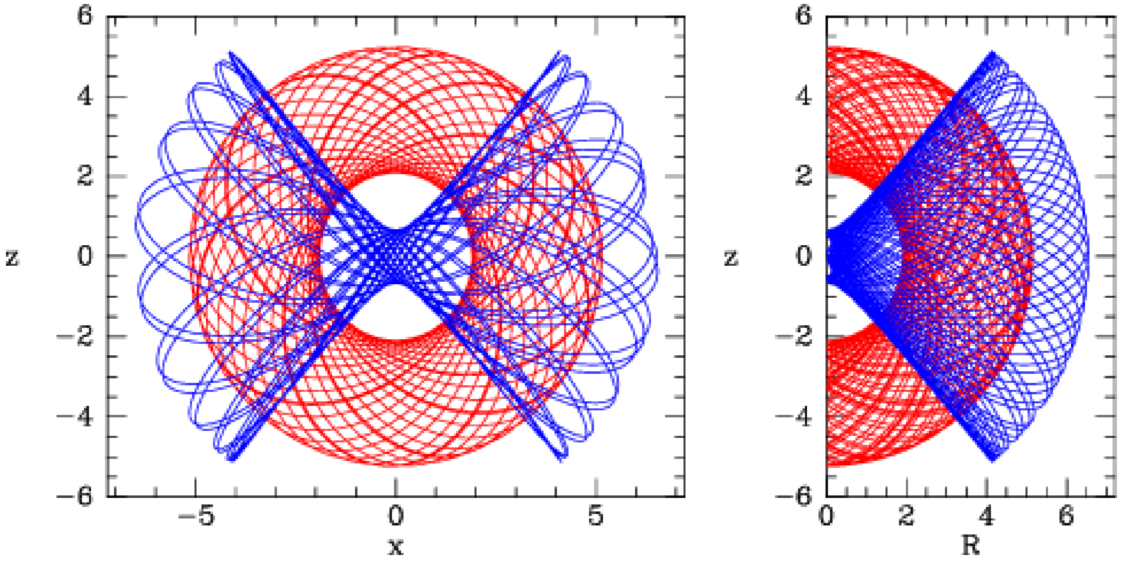


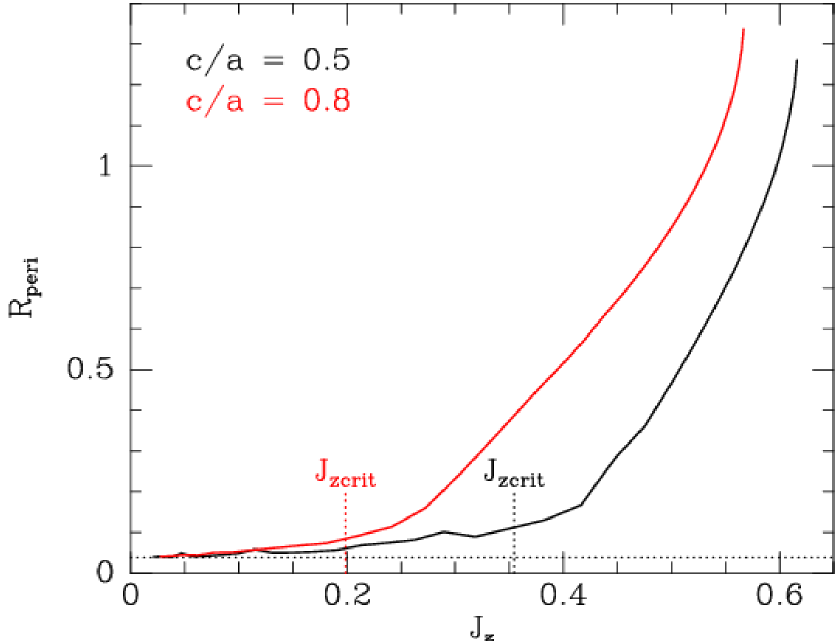
The problem

- At r = 0 of spherical system, all directions equivalent so N(v) must be isotropic
  - They are all determined by $f(J_r, L=0)$ so isotropy guaranteed
  - At $x=\epsilon$, $N(v_x)$ determined by $f(J_r, L=0)$ but $N(v_y)$ depends on L-dependence
  - So L-dependence has to align with J_r dependence as $L \rightarrow 0$
- At R=0 of flattened system $N(v_x) = N(v_y)$ on z axis
  - Both determined by J_r or J_z dependence of f(J) when on axis
  - But J_ϕ dependence important for $N(v_y)$ when $x = \epsilon$
  - So J_ϕ dependence has to align with J_r or J_z dependence as $J_\phi \rightarrow 0$

Orbits at low J_ϕ

- Orbits with $J_\phi = 0$ move in (x,z) plane in a barred Φ
- Orbits divide into boxes ($J_z < J_{zcrit}$) & loops ($J_z > J_{zcrit}$)
- This hard distinction vanishes at $J_\phi = \epsilon$ but a distinction remains
- At $J < J_{zcrit}$, J_r dependence has to align with J_ϕ dependence as $J_\phi \to 0$
- At $J > J_{zcrit}$, J_z dependence has to align with J_ϕ dependence as $J_\phi \to 0$





A way to arrange this

- As $J_\phi \to 0$ we require 0 = $\frac{\partial f}{\partial v_\phi} = \frac{\mathrm{d}f}{\mathrm{d}H}\left(\Omega_r \frac{\partial J_r}{\partial v_\phi} + \Omega_z \frac{\partial J_z}{\partial v_\phi} + \Omega_\phi R\right)$
- So cancellations are required
- Consideration of ergodic DF shows cancellations iff

$$\frac{\partial f/\partial J_z}{\partial f/\partial J_\phi} \to 1 \text{ as } J_\phi \to 0 \text{ with } J_z > J_{z\mathrm{crit}}$$

$$\frac{\partial f/\partial J_r}{\partial f/\partial J_\phi} \to 2 \text{ as } J_\phi \to 0 \text{ with } J_z < J_{z\mathrm{crit}}.$$

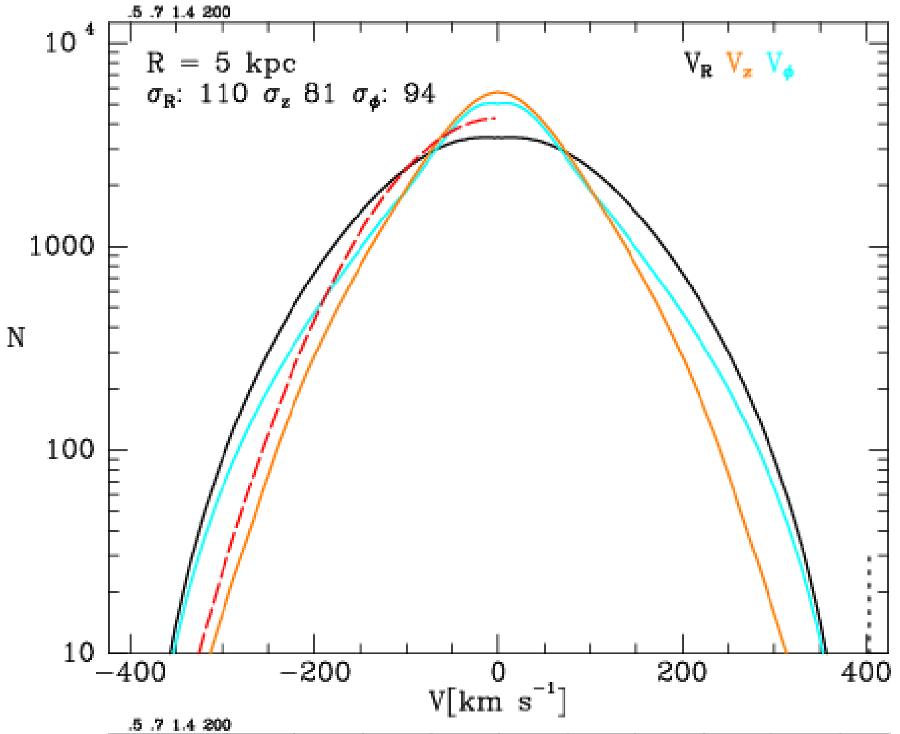
- Define auxiliary DF

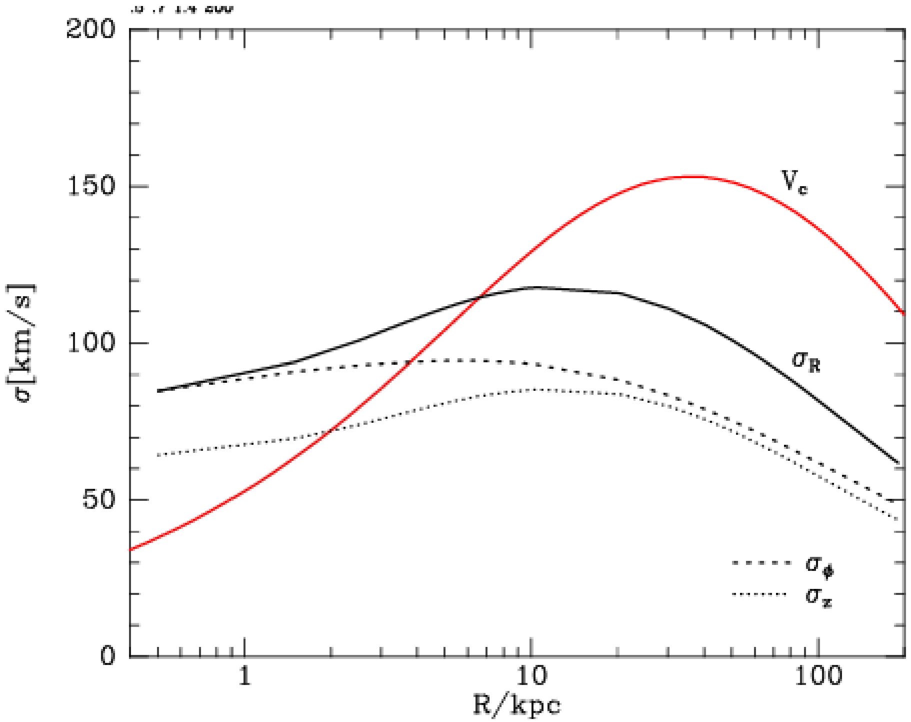
$$f'(\mathbf{J}) \equiv \begin{cases} f(J_r + \frac{1}{2}(|J_\phi| - \epsilon), J_z, \epsilon) & J_z < J_{z\mathrm{crit}} \\ f(J_r, J_z + (|J_\phi| - \epsilon, \epsilon)) & J_z > J_{z\mathrm{crit}} \end{cases}$$

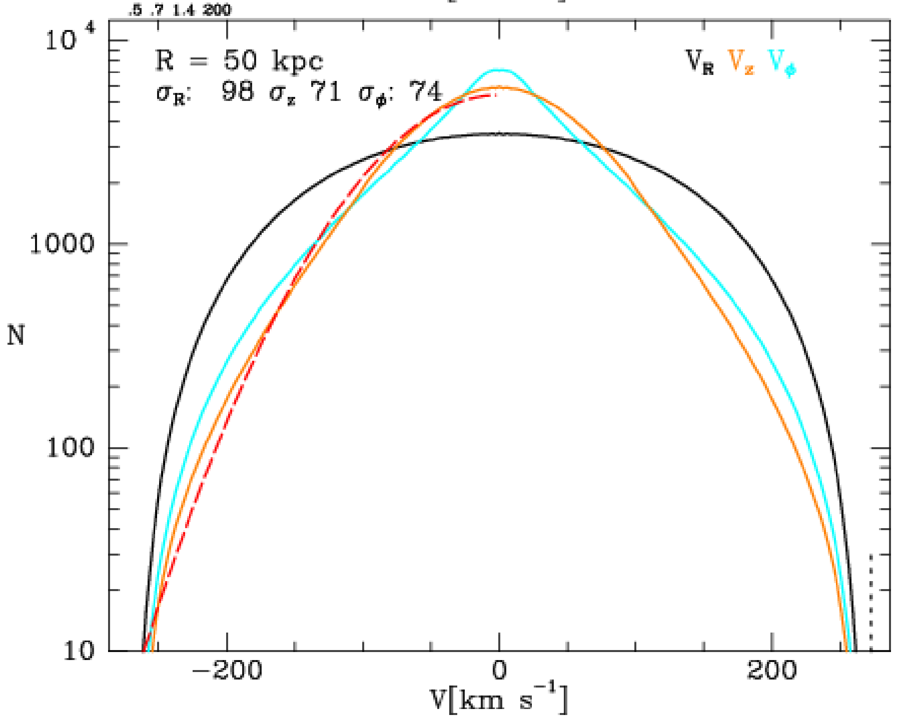
- Use DF $f''(\mathbf{J}) = w f'(\mathbf{J}) + (1-w) f(\mathbf{J})$. where $\lim_{J_\phi \to 0} w(\mathbf{J}) = 1$ and $\lim_{J_\phi \to 0} \nabla_{\mathbf{J}} w = 0$,

An example

- NFW type Φ(R,z) from ρ(R,z) flattened to c/a = 05
- DF Posti type f($0.7J_r + 1.4J_z + |J_\phi|.$)

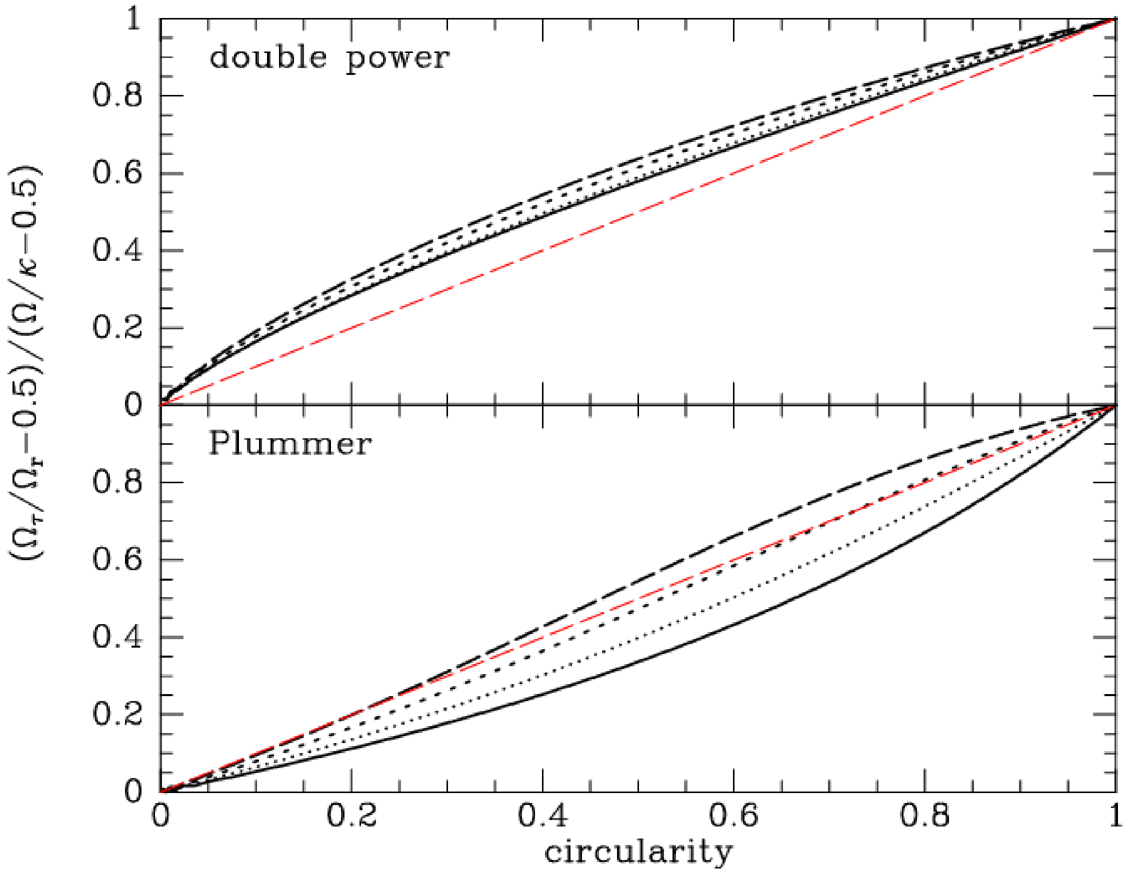


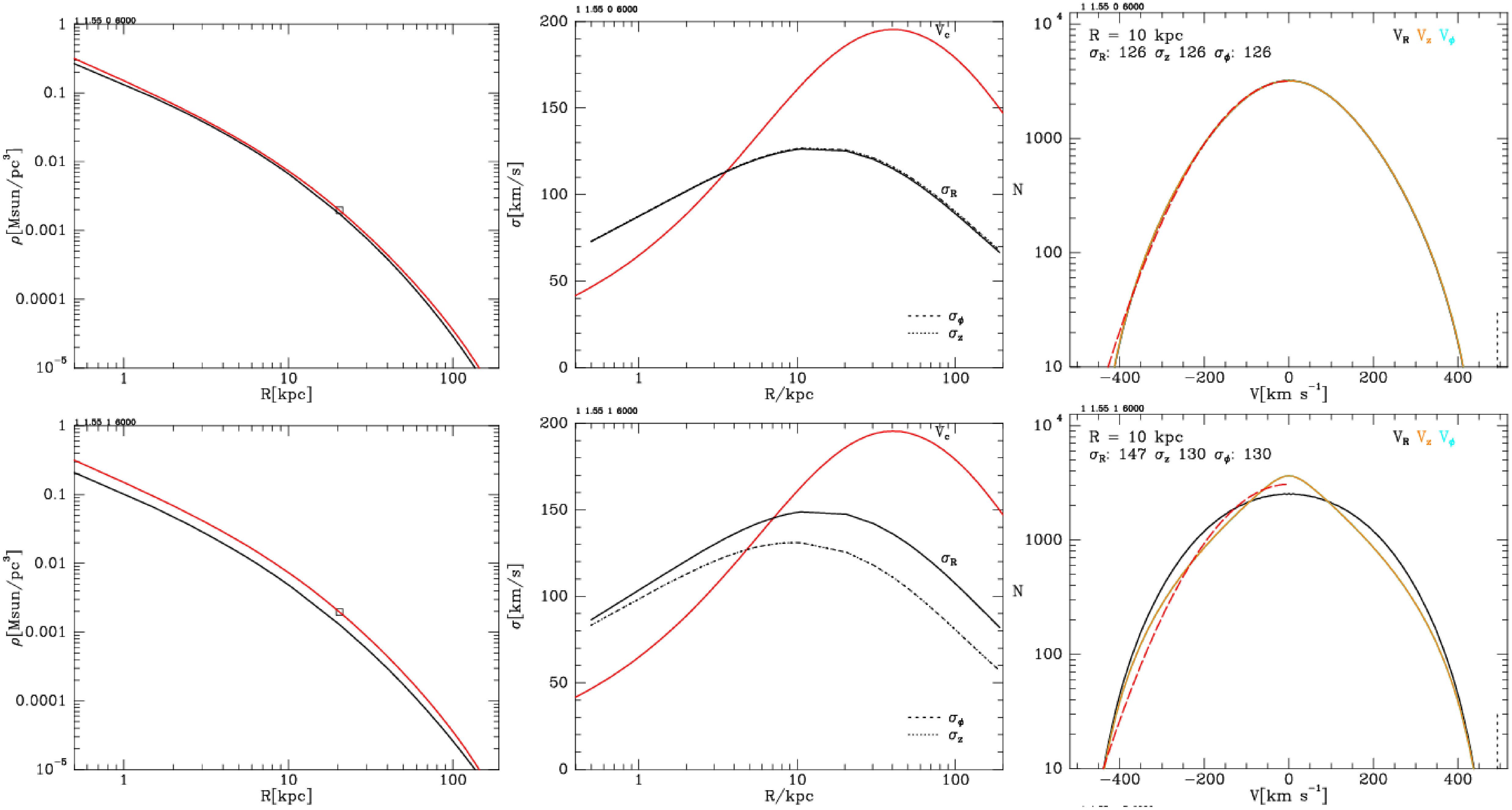




Spherical models

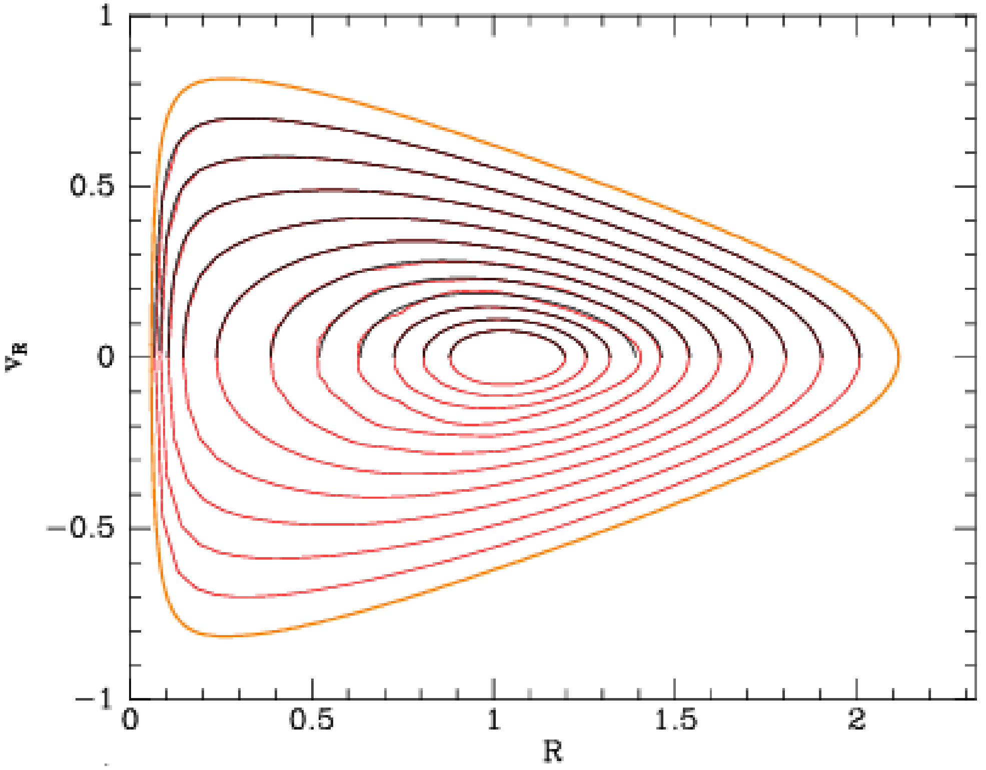
- Ergodic DF f(E) always available by Eddington
- But H(J) rarely available
- Can compute good approx. to H(J) by using approx. to Ω_t/Ω_r
- Integrate $0 = dH = \Omega_r dJ_r + \Omega_t dL$ from (J_r, L) to $(0, L_c)$ and use $E(L_c)$
- Modify procedure to generate anisotropic models



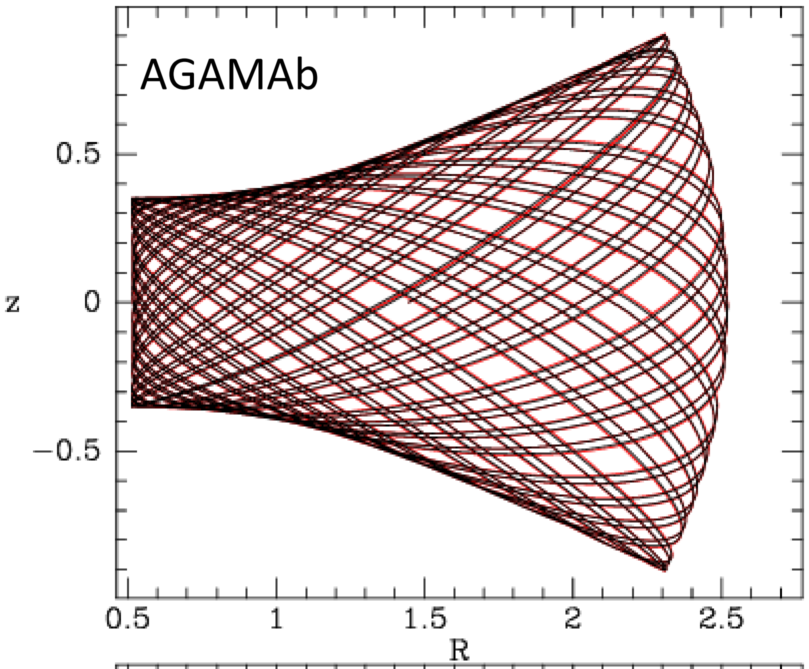

1 1.55 0 6000
ρ[Msun/pc^3]
R[kpc]
σ[km/s]
R/kpc
V_c
σ_R
σ_φ
σ_z
R = 10 kpc
σ_R: 126 σ_z 126 σ_φ: 126
V_R V_z V_φ
N
V[km s^-1]
1 1.55 1 6000
σ_R: 147 σ_z 130 σ_φ: 130

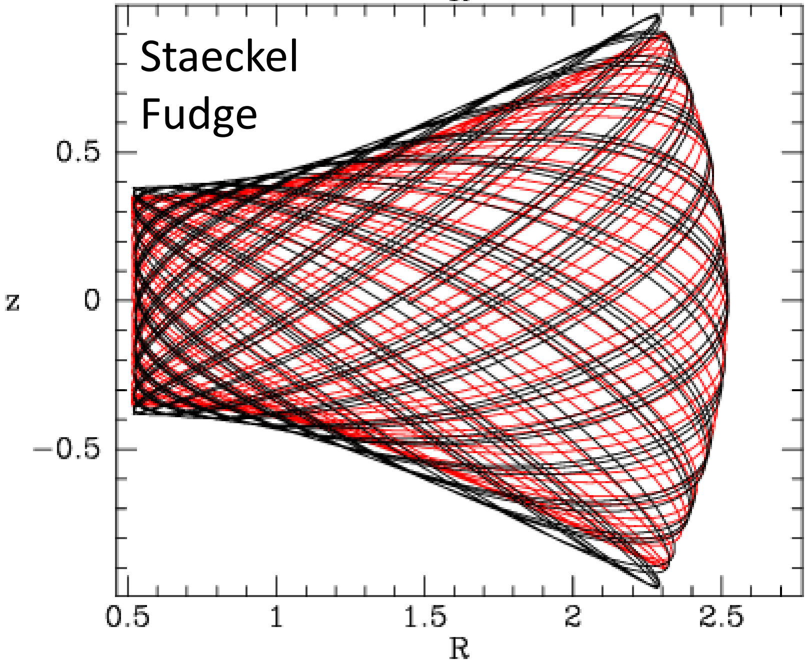
A new AGAMA

- Facilities for handling obs data
  - Sky coordinates (RA, dec) or (l, b) proper motions, Vlos, etc
  - Obscuration by dust
  - Luminosity functions of pops with given DF
  - Line of sight sampling
- Function to compute J_{zcrit} as func of E or J_r
  - New DFs
- Native torus mapper that can
  - handle highly eccentric orbits
  - Interpolate seamlessly
  - Includes an action finder (x,v) → (θ,J)
  - Windows & Linux versions
  - New Python interface



Action finder

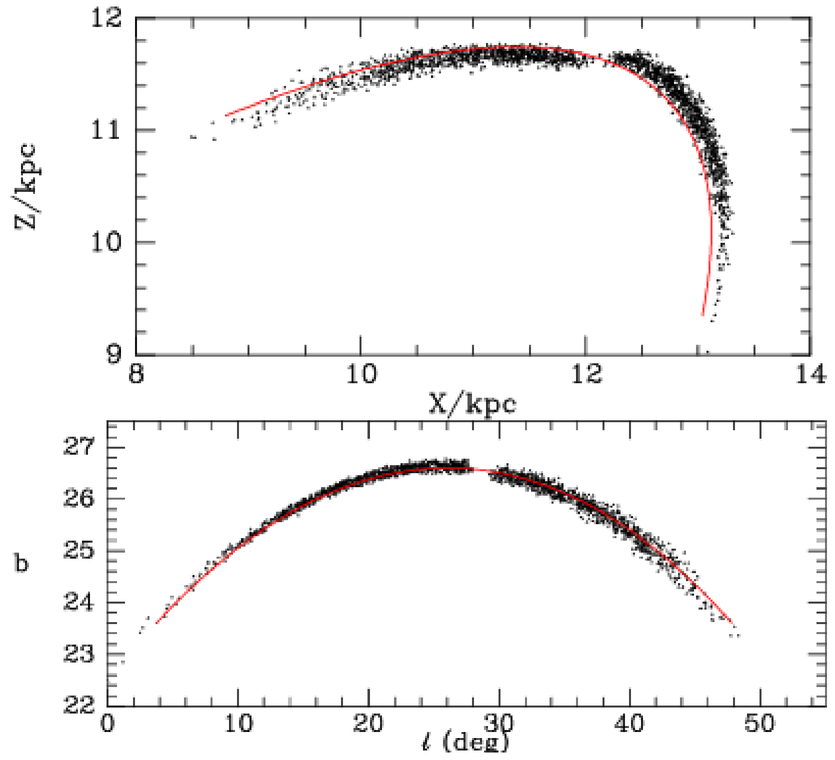


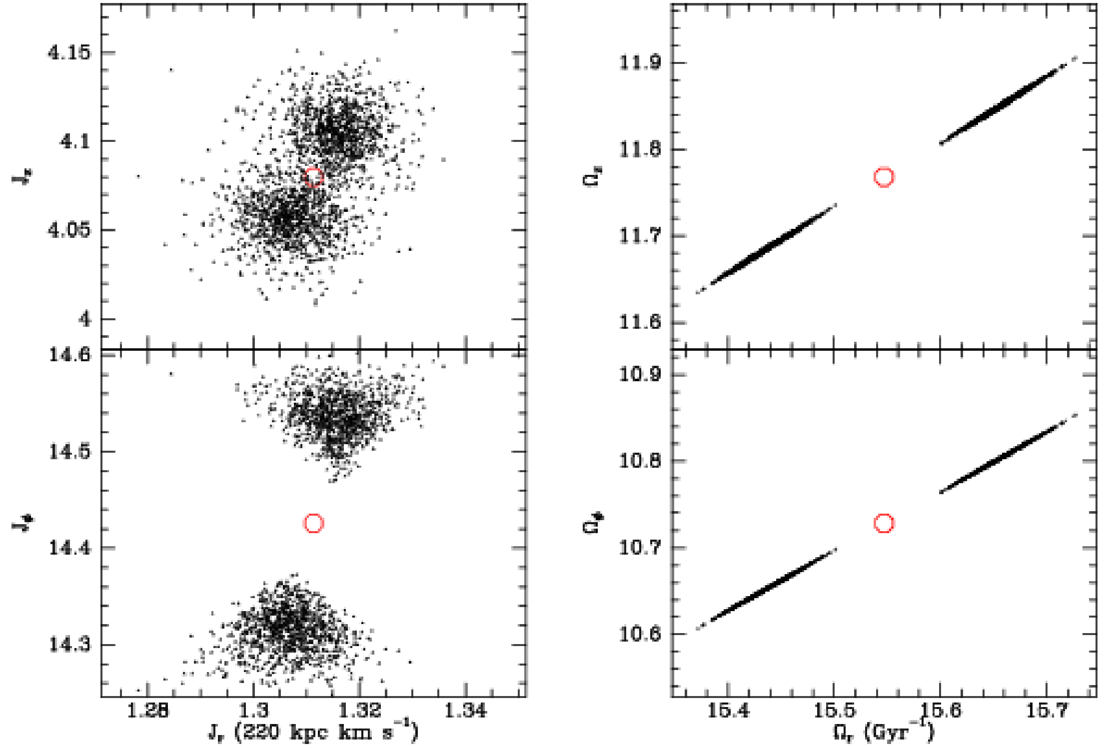


A stellar stream created in 11 seconds

- Create 5^3 grids of tori ahead/behind progenitor
- Place 2,000 stars by Gaussianly sampling action and using

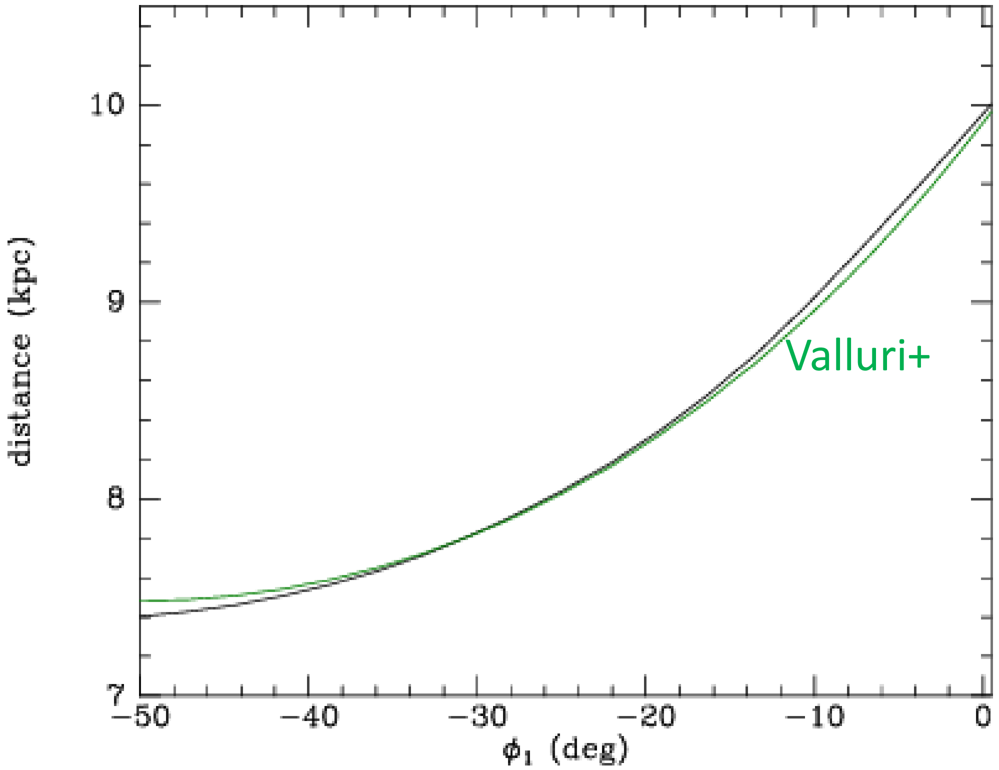
$$\boldsymbol{\theta} = \boldsymbol{\theta}_0 + \Delta\boldsymbol{\theta} + \boldsymbol{\Omega}_{\rm prog}(t - t_{\rm drft}) + \boldsymbol{\Omega} t_{\rm drft},$$

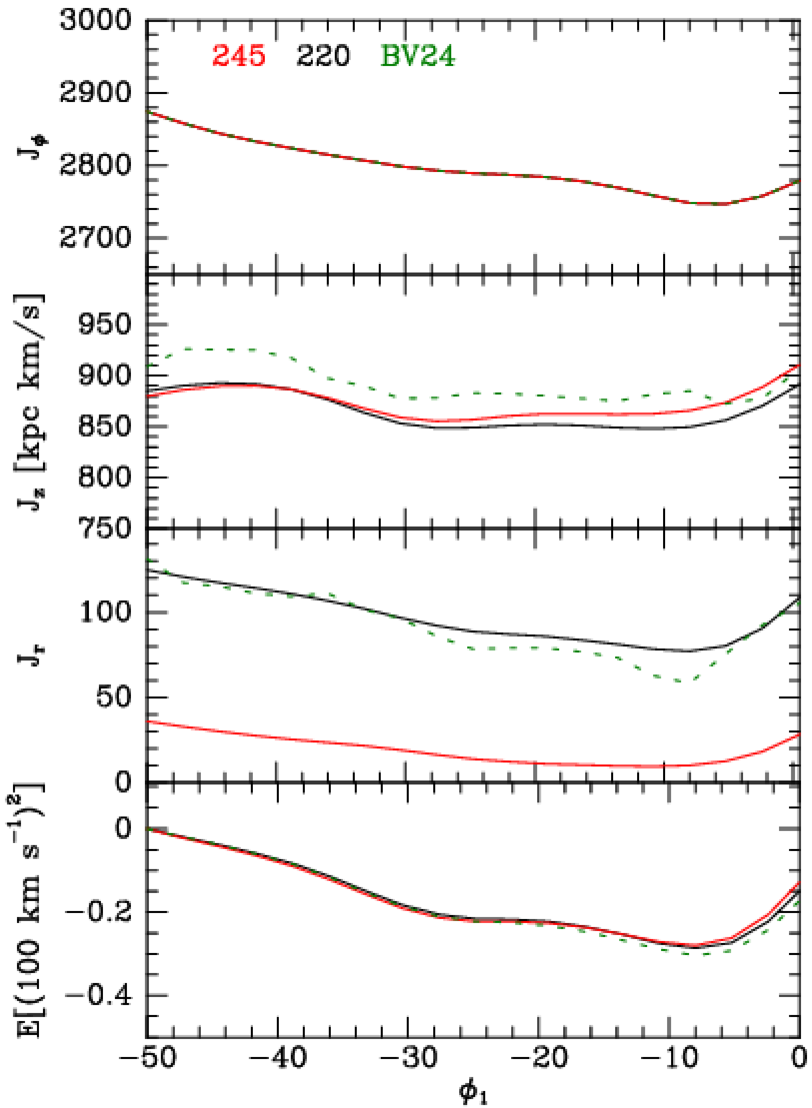




Application to GD1

- Actions should vary little (& systematically) along stream
- Tiny correction to distances of Valluri+ 2025 hold J_ϕ constant





Conclusions

- Forward modelling of stellar systems mandatory
- N-body models are very hard to taylor to specific galaxies
- Can map DM only by assuming steady state
  - → exploitation of Jeans thm
- Actions are by far the best constants of motion
- Natural to start with f(J) – analytic or free-form
- Fornax & MW modelled using analytic f(J)
- DFs for spheroidal cpts lack required velocity anisotropy
- This problem now understood & resolved
- AGAMA provides powerful tools for f(J) modelling
- A more powerful release will appear soon
- We'll use it to explore shape of MW's dark halo