Heating of disky systems with dark matter objects: Milky Way disk vs. exoplanetary systems

Théo Paré

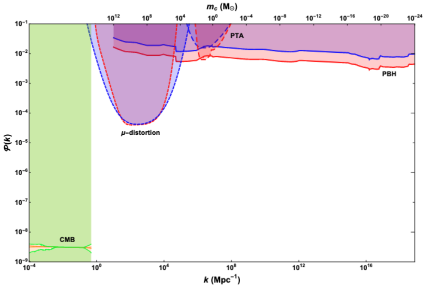
Laboratoire Univers et Particules de Montpellier

theo.pare@umontpellier.fr

September 11, 2025

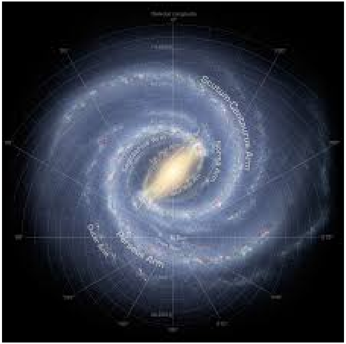
Context

- Not many probes on sub-galactic scales
- Structure formation theory predicts a large amount of small scale halos
- Subhalos' properties (min. mass, density profile, concentration) depend on dark matter nature and on the primordial power spectrum.



Gravitational heating

- First developed to study the dynamics of stellar clusters or any N-body system.
- Several existing works:
  - Dwarf galaxies [Penarrubia+ '25]
  - UFD [Graham & Ramani '24]
  - Wide binaries [Quinn+ '09]
- Independent of DM model (since DM is massive)
- MW disk / exoplanets : different scales to probe different DM halo scales



Velocity kick

- Velocity kick $\Delta\mathbf{v}$ induced by an encounter between a star and a DM halo

$$(\Delta\mathbf{v})^2 = \left(\frac{2GM_{DM}}{bv_{\mathrm{rel}}}\right)^2$$

- Impulsive approximation $T_{\mathrm{enc}} < P \Leftrightarrow b < b_{\mathrm{max}} = \frac{P}{2v_{\mathrm{rel}}}$
- Star -DM halo not gravitationally bound: $b > b_{\mathrm{min}} = \frac{G(M_\star + M_{DM})}{v_{\mathrm{rel}}^2}$
- Spatial extension of DM halo $\rightarrow$ penetrating encounters

$$(\Delta\mathbf{v})^2 = \left(\frac{2M_{DM}GI(b)}{bv_{\mathrm{rel}}}\right)^2$$

$$I(b) = 1 - \Theta(r_H - b)\frac{4\pi}{M_{DM}}\int_b^{r_H} \mathrm{d}x\rho(x)x\sqrt{x^2 - b^2}$$

$\rightarrow$ More concentrated halos produce more efficient heating

Heating rate

- Energy injected per unit time

$$\mathcal{H} = \int \mathrm{d}v_{\mathrm{rel}}\, f(v_{\mathrm{rel}}) \int \mathrm{d}M_{\mathrm{DM}} \frac{\mathrm{d}n}{\mathrm{d}M_{\mathrm{DM}}} \int 2\pi\, b\, \mathrm{d}b (\Delta v)^2$$

- For monochromatic mass function with point-like objects

$$\mathcal{H} = \frac{4\sqrt{2}\pi G^2 M_{\mathrm{DM}} \rho_{DM}}{\sigma_{\mathrm{DM}}} \ln \Lambda$$

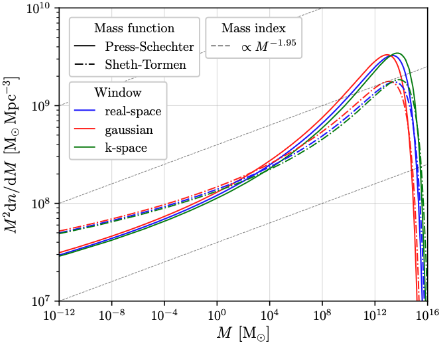
- Coulomb logarithm $\Lambda = b_{\mathrm{max}}/b_{\mathrm{min}}$

Cosmological mass function

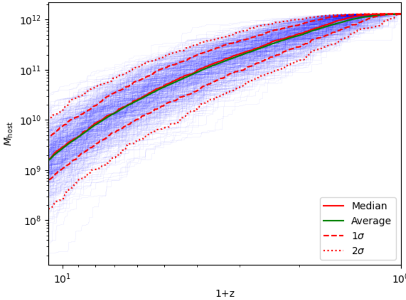
- Count the number of fluctuations growing larger than δ_c
- Extended Press Schechter formalism

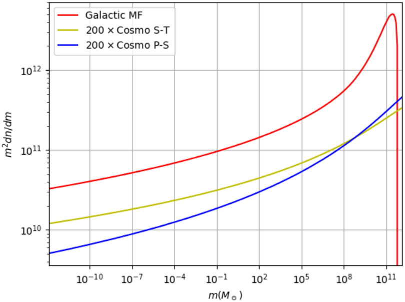
$$\frac{\mathrm{d}n}{\mathrm{d}M} = \frac{\bar{\rho}}{M^2} \frac{\delta_c}{\sqrt{2\pi S}} \left| \frac{\mathrm{d}\ln S}{\mathrm{d}\ln M} \right| \exp\left(-\frac{\delta_c^2}{2S} \right)$$

- NFW halos with concentration-mass relation from [Diemer+ '19]
- Semi-analytical approach from [Hiroshima+ '22] to reproduce merger tree simulations to compute the galactic mass function



Galactic mass function





$$N(m) = f(S(m), \delta_c(z+\Delta z)|S(M_{\mathrm{host}}), \delta(z)) \frac{M_{\mathrm{host}}}{m} \left|\frac{\mathrm{d}S}{\mathrm{d}m}\right| G\left(\sqrt{\frac{S(M_{\mathrm{host}})}{S(m)}}, \frac{\delta_c(z+\Delta z)}{\sqrt{S(m)}}\right)$$

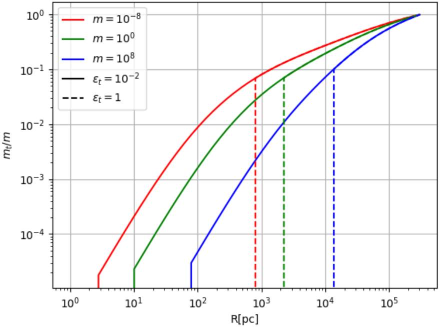
$$f(S(m), \delta_c(z+\Delta z)|S(M_{\mathrm{host}}), \delta(z)) = \frac{\delta_c(z+\Delta z) - \delta_c(z)}{\sqrt{2\pi}\,[S(m) - S(M_{\mathrm{host}})]^{3/2}} \exp\left[-\frac{(\delta_c(z+\Delta z) - \delta_c(z))^2}{2(S(m) - S(M_{\mathrm{host}}))}\right]$$

Tidal effects

- Tidal stripping from the galactic gravitational field

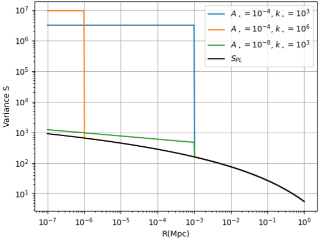
$$r_t = R\left[\frac{m(r_t)}{3M(R)f(R)}\right]^{1/3}$$

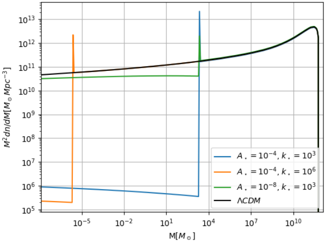
- Tidal destruction criterion $\epsilon_t = r_t/r_s$
- Other tidal effects from the disk and the stars.



Ultra compact mini-halos

- Extra power at a given scale $\mathcal{P}_{\mathcal{R}} = \mathcal{P}_{\rm PL} + \mathcal{A}_\star k_\star \delta(k - k_\star)$
- UCMH can make for a significant fraction of DM
- Very concentrated $\rightarrow$ very efficient heating
- Other modifications to the primordial power spectrum: PBH halos with Poisson fluctuations



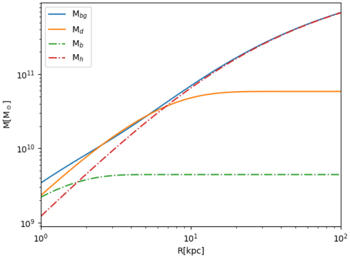


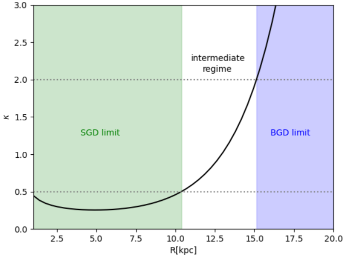
MW modeling

- Multiple contribution (DM halo, disk, bulge ...)
- Assuming inside-out growth and uncorrelated vertical and radial motion of stars
- Hydrostatic equilibrium + Poisson eq.

$$\frac{\partial^2(\phi_d + \phi_{bg})}{\partial z^2} = 4\pi G(\rho_d + \rho_{bg}^{\text{eff}})$$

$$-\frac{\sigma_z^2}{\rho_d}\left(\frac{\partial \rho_d}{\partial z}\right) = \frac{\partial(\phi_d + \phi_{bg})}{\partial z}$$





Link between $\mathcal{H}$ and the velocity dispersion

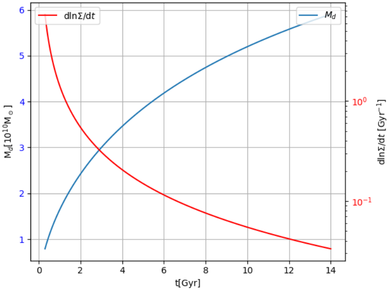
- Vertical action is an adiabatic invariant

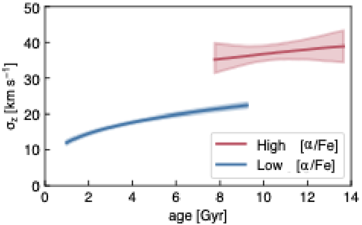
$$\frac{\mathrm{d}J_z}{\mathrm{d}t} = \frac{P}{4\pi}\mathcal{H}$$

- Going to velocity dispersion:

$$\frac{\mathrm{d}(\sigma_z^2)}{\mathrm{d}t} = \frac{2}{3}\sigma_z^2\frac{\mathrm{d}\ln\Sigma}{\mathrm{d}t} + \kappa\langle\mathcal{H}\rangle$$

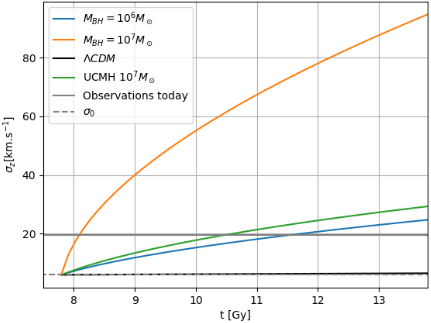
- σ_z is sourced by collision and adiabatic evolution of the disk
- Thin vs thick disk





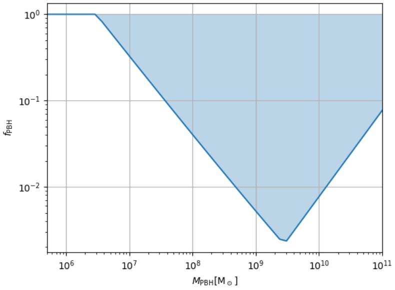
σ_z evolution from heating

- Star age and migration computed from [Frankel+ '20]
- ΛCDM halos are destroyed and produce negligible heating

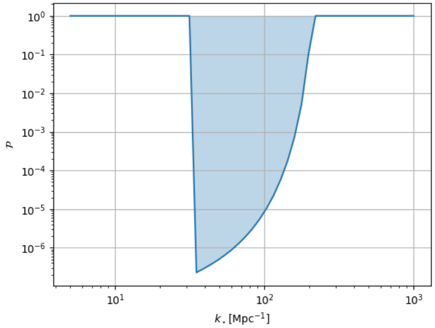


Preliminary Results

PBH



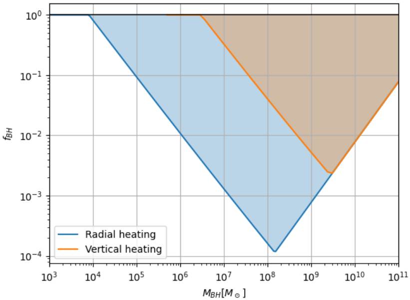
UCMH



Conservatives constraints since we ignore other heating mechanisms (central bar, gas clouds, spiral arms)

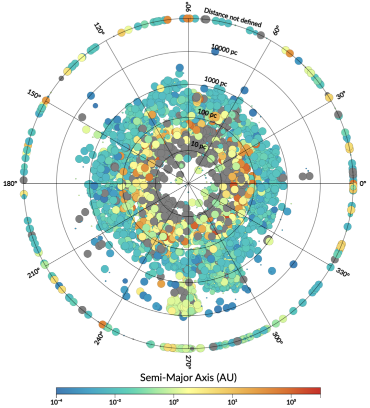
Radial heating

- Observations show way larger migration / radial heating (see Neige Frankel's talk) than what can be explained with DM collisions
- Need another mechanism to explain the migration → only a fraction of observed radial heating can come from collisions (see Chris Hamilton's talk)

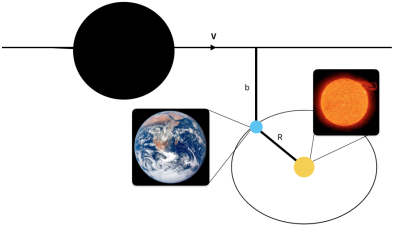


Exoplanets

- Same heating mechanism with Keplerian potential
- 7000 + planets discovered today, and more observation missions planned



Velocity kick



Difference between the velocity kick on the planet and the one on the star

$$(\Delta v)^2 = \left(\frac{2GM_{\rm DM}}{v_{\rm rel}\, b}\right)^2 \frac{2R_p^2}{2R_p^2 + 3b^2}$$

$\rightarrow$ Extra suppression of very distant encounters

Gets messier when considering halo size:

$$\langle (\delta \mathbf{V})^2 \rangle = \left(\frac{2GM_{\rm DM}}{v_{\rm rel}\, b}\right)^2 \frac{3b^2(I_p - I_\star)^2 + 2R_p^2 I_p^2}{3b^2 + 2R_p^2}$$

Heating rate

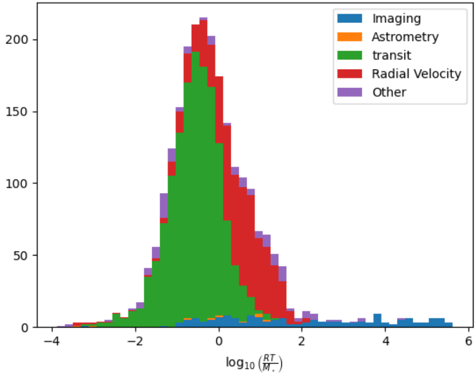
- For point-like objects with monochromatic mass function

$$\mathcal{H} = \frac{2\sqrt{2}G^2 M_{\rm DM} \rho_{DM}}{\sigma_{\rm DM}} \ln \frac{3 + 2(R_p/b_{\rm min})^2}{3 + 2(R_p/b_{\rm max})^2}$$

- $b_{\rm min} \ll R_p \Rightarrow ln\left(\frac{3+2(R_p/b_{\rm min})^2}{3+2(R_p/b_{\rm max})^2}\right) \sim \ln\left(\frac{R_p}{b_{\rm min}}\right)$:
  Coulomb logarithm with $b_{\rm max} \approx R_p$.
- $b_{\rm min} \gg R_p \Rightarrow \ln\left(\frac{3+2(R_p/b_{\rm min})^2}{3+2(R_p/b_{\rm max})^2}\right) \sim \left(\frac{R_p}{b_{\rm min}}\right)^2$
  Heating suppressed for large masses $\mathcal{H} \propto M_{\rm DM}^{-1}$

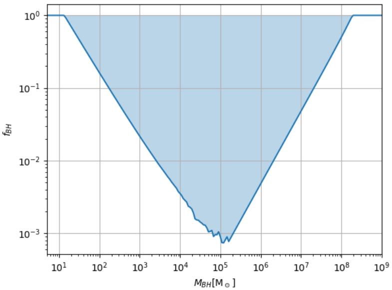
Observable

- Relevant quantity $\frac{E_{\rm inj}}{|E|} = \frac{\mathcal{H}\, TR}{G\, M_\star}$
- Orbital radius can vary from 10^{-3} a.u. to 10^{4} a.u.
- vertical heating $\rightarrow$ orbital plane inclination shift
- radial heating $\rightarrow$ orbit eccentricity
- If $E_{\rm inj}/|E| > 0$: ejection

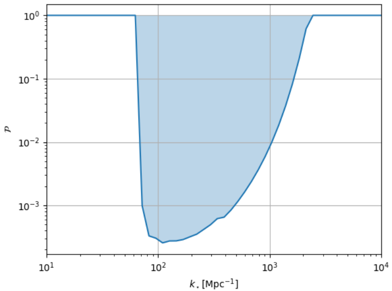


Results

PBH



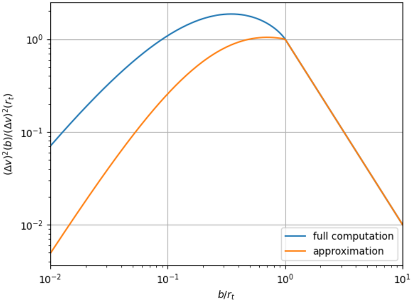
UCMH



Thank you

Penetrating encounters

- Two ways to deal with penetrating encounters
  1. Only take the mass inside the impact parameter $m(b)$ ignoring the effect of outer shell
  2. Fully compute the heating by integrating the e.o.m. over time



Frame Title

$$\frac{P}{2} = \int_{-z_m}^{z_m} \frac{\mathrm{d}z}{\sqrt{2(\phi(z_m) - \phi(z))}}$$

$$J_z \equiv \frac{1}{2\pi} \oint p \mathrm{d}q = \frac{1}{2\pi} \int_{-z_m}^{z_m} \mathrm{d}z \sqrt{2(\phi(z_m) - \phi(z))}$$