Laboratoire des 2 Infinis-Toulouse

Matrix Element Method at next-to-leading-order (NLO) for the measurement of the Higgs tri-linear coupling λ_{3H} in HH production in the at the LHC

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Introduction

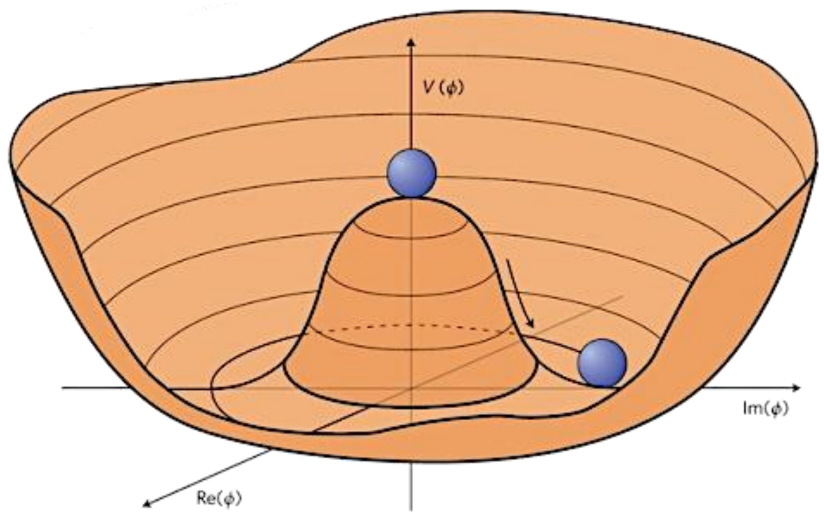
Higgs self-coupling λ_{3H}

Standard Model (assumption):

$$V(\phi^2) = \mu^2\phi^2 + \lambda\phi^4$$

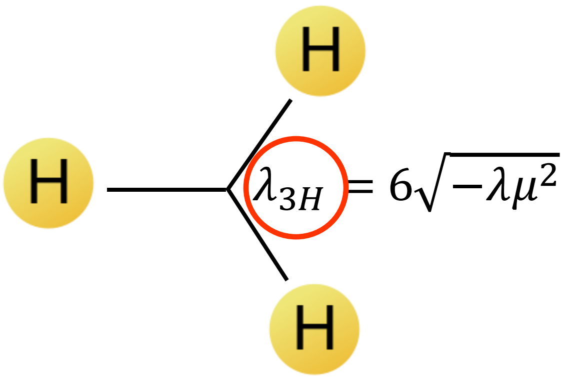
(for $\mu^2 < 0$ et $\lambda > 0$)

Mathematical expression of the **Higgs doublet ϕ** potential



$$V(h) = m_H{}^2\frac{h^2}{2} + \lambda_{3H}\frac{h^3}{3!} + \lambda_{4H}\frac{h^4}{4!} - \frac{v^4\lambda}{4}$$

Mathematical expression of the **Higgs boson h** potential after expansion around ground state.



Goal: Measure this λ_{3H} value experimentally, to confront with theoretical expectations.

$-1.2 < \lambda_{obs}/\lambda_{SM} < 7.2$	[ATLAS '24]
$-1.4 < \lambda_{obs}/\lambda_{SM} < 7.0$	[CMS '24]

Introduction

How to shorten this range to extract the value of λ_{3H} (or $\kappa_\lambda = \frac{\lambda_{3H}}{\lambda_{3H,SM}}$)?

And also how to take into account the large backgrounds in our samples?

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Introduction

Matrix Element Method [MEM]

<u>By definition:</u> [the (MEM)] is a statistically optimal multivariate method that maximizes the utilization of both the experimental and theoretical information available to an analysis.
It computes a probability (called "the Likelihood" $\mathcal{L}$(**h**,**x**)), to observe an event **x** under the hypothesis **h.**

<u>List of succeses:</u>
-[2004-2016] most precise measurement of the top-quark mass at Tevatron.
-[2015-2016] important role in the first evidence for single top production in the s-channel at the LHC.

<u>Likelihood mathematical expression (for particle physics*):</u>

Observed events (LO or NLO)

Total cross section

Transfer function

$$\mathcal{L}^{P}_{\text{process}}(\mathbf{h} \mid \mathbf{x}^{i})dx^{i} = \frac{(2\pi)^4}{\sigma_P^{obs}(pp \to F).s} \int_y \int_{q_1,q_2} \sum_{a_1,a_2} f_{a_1}(q_1) f_{a_2}(q_2) . \frac{|\mathcal{M}_{\mathcal{P}}(a_1 a_2 \to \mathbf{y}; \mathbf{h})|^2}{q_1 q_2} . W(\mathbf{x}^{i}, \mathbf{y}) \delta\Big(a_1 + a_2 - \sum_j^n y_j\Big) \mathrm{d}q_1 \mathrm{d}q_2 \mathrm{d}y^{4n}$$

Parton density functions

Matrix element (LO or NLO)

<u>Legend :</u>
: the experimental inputs
: the theoretical inputs

*in particle physics at colliders, we have the chance to have the ingredients that are necessary to compute the likelihood from first principles

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Introduction

Matrix Element Method [MEM]

- Given a fixed reconstructed event x^i, we integrate over all the possible configurations of events y that could be measured as x^i by the detector.
- The integrated value ($\mathcal{L}_{process}$) can be seen as the weight of the event x^i for the process p, under the hypothesis h.

Likelihood mathematical expression (for particle physics*):

Observed events (LO or NLO)

Total cross section

Transfer function

$$\mathcal{L}^{P}_{\text{process}}(\mathbf{h} \mid \mathbf{x}^i)d\mathbf{x}^i = \frac{(2\pi)^4}{\sigma_P^{obs}(pp \to F).s} \int_y \int_{q_1,q_2} \sum_{a_1,a_2} f_{a_1}(q_1) f_{a_2}(q_2). \frac{|\mathcal{M}_{\mathcal{P}}(a_1 a_2 \to \mathbf{y}; \mathbf{h})|^2}{q_1 q_2} \cdot W(\mathbf{x}^i, \mathbf{y}) \delta\Big(a_1 + a_2 - \sum_j^n y_j\Big) \mathrm{d}q_1 \mathrm{d}q_2 \mathrm{d}y^{4n}$$

Parton density functions

Matrix element (LO or NLO)

Legend :

: the experimental inputs

: the theoretical inputs

➔ Use: To search for $\boldsymbol{h}$ that maximizes $\mathcal{L}$

➔ hypothesis h can be:

- signal vs bkg
- λ_{3H} value

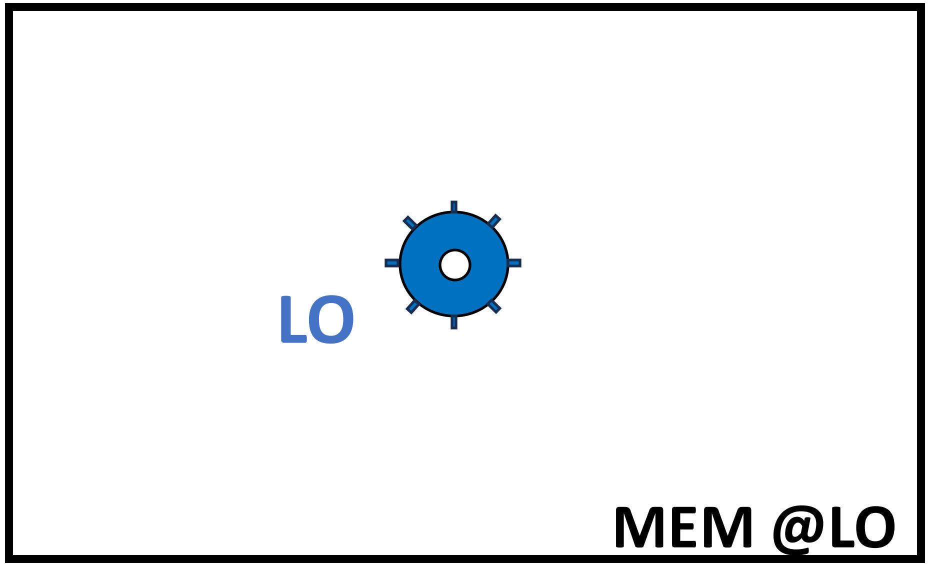
*in particle physics at colliders, we have the chance to have the ingredients that are necessary to compute the likelihood from first principles

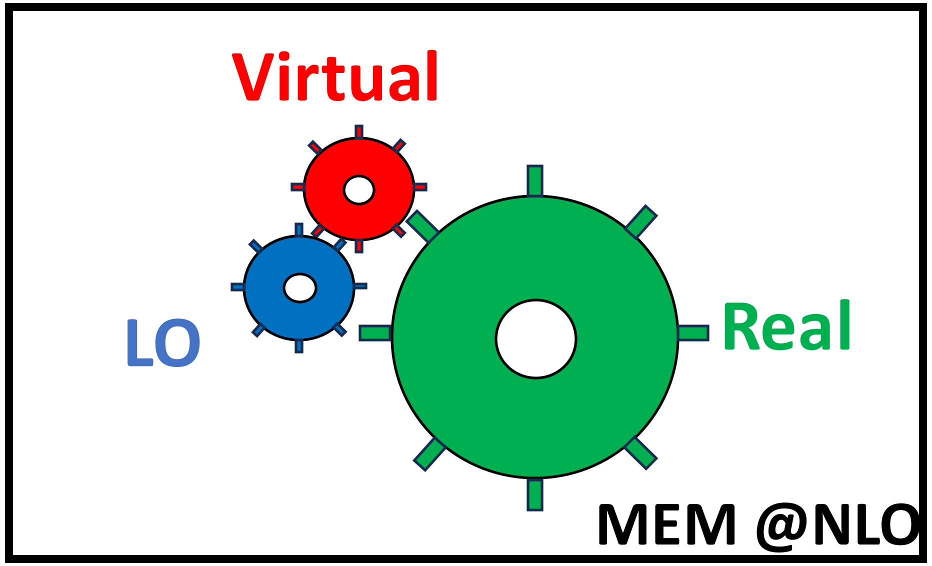
Introduction

Distinction between ME(M) analysis and Events order

- **<u>Matrix Element Method</u>** <u>[MEM]</u> **<u>analysis</u>** <u>:</u>

Can be **constructed** at different orders (the order of the Matrix Element using Feynman rules).



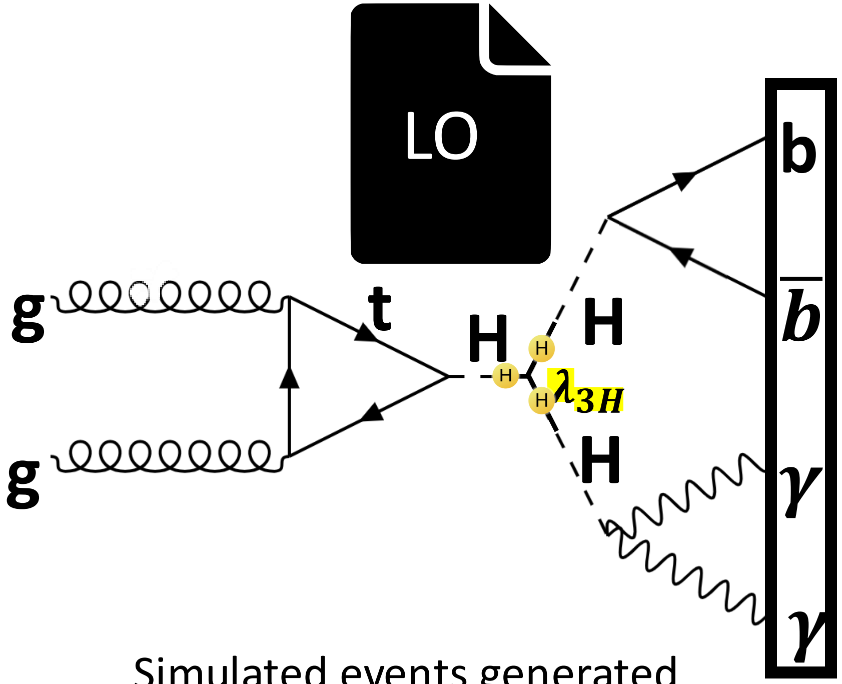


These symbols will be used throughout this presentation to indicate which order of the MEM is being presented.

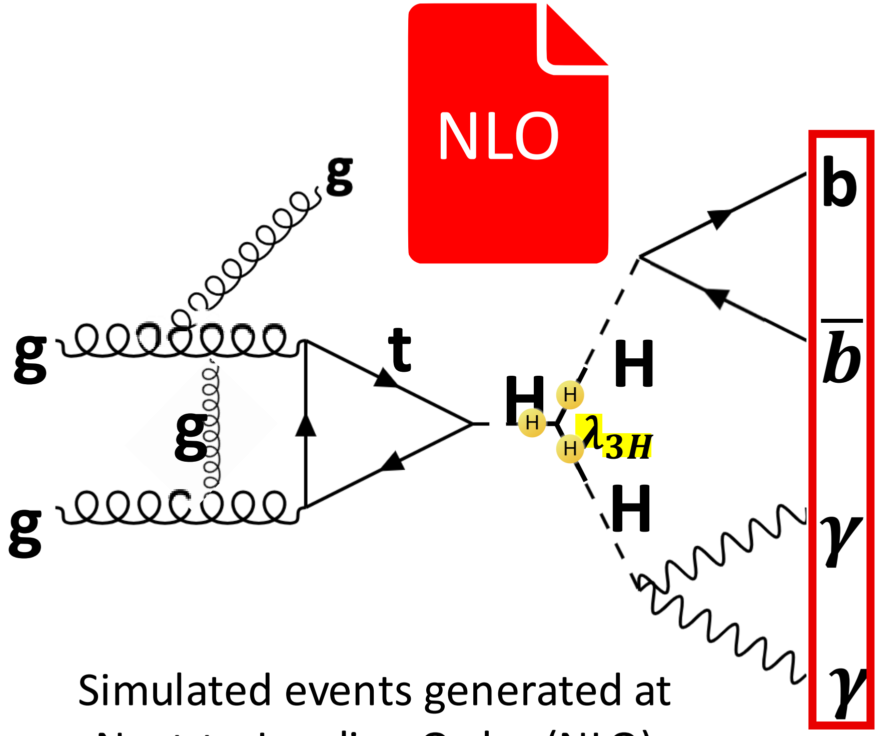
Introduction

Distinction between ME(M) analysis and Events order

- **<u>Events</u>**: Can be **generated** at different orders.
- <u>Example:</u> gluon fusion with $\mathrm{b}\bar{\mathrm{b}}\gamma\gamma$ final state. (dominant di-Higgs production mode)



Simulated events generated at Leading Order (LO)
(Triangle diagram only, here)



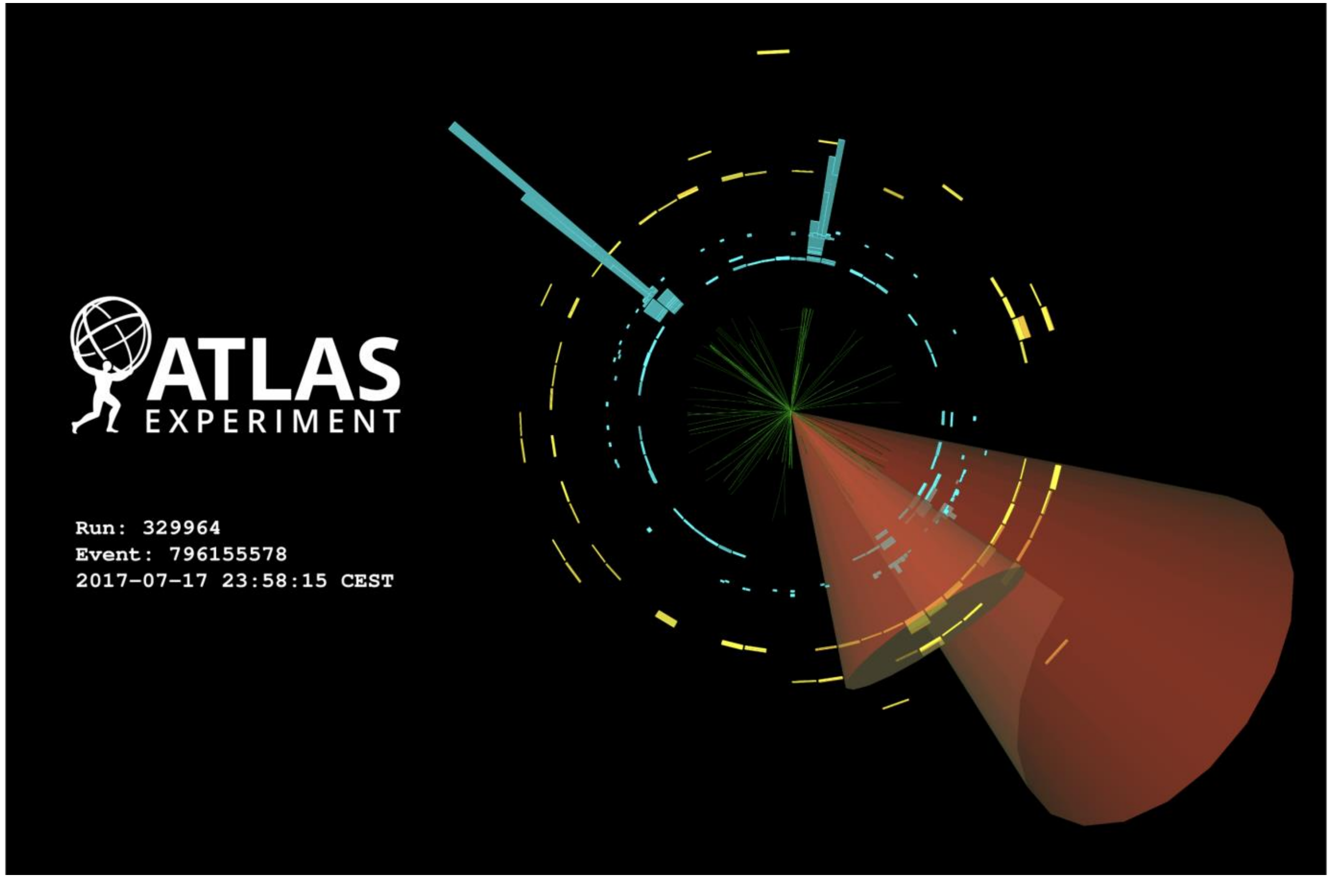
Simulated events generated at Next-to-Leading Order (NLO)
More « physically accurate »

Introduction

Event display (ATLAS): Final state

- Reco particles in x^i (**for both MEM@LO and MEM@NLO**): 4 Higgs decay daughters candidates **only**. Here: $\boxed{b\bar{b}\gamma\gamma}$

- Extra (real) radiation candidates will be **integrated over their 3 extra degrees of freedom**.

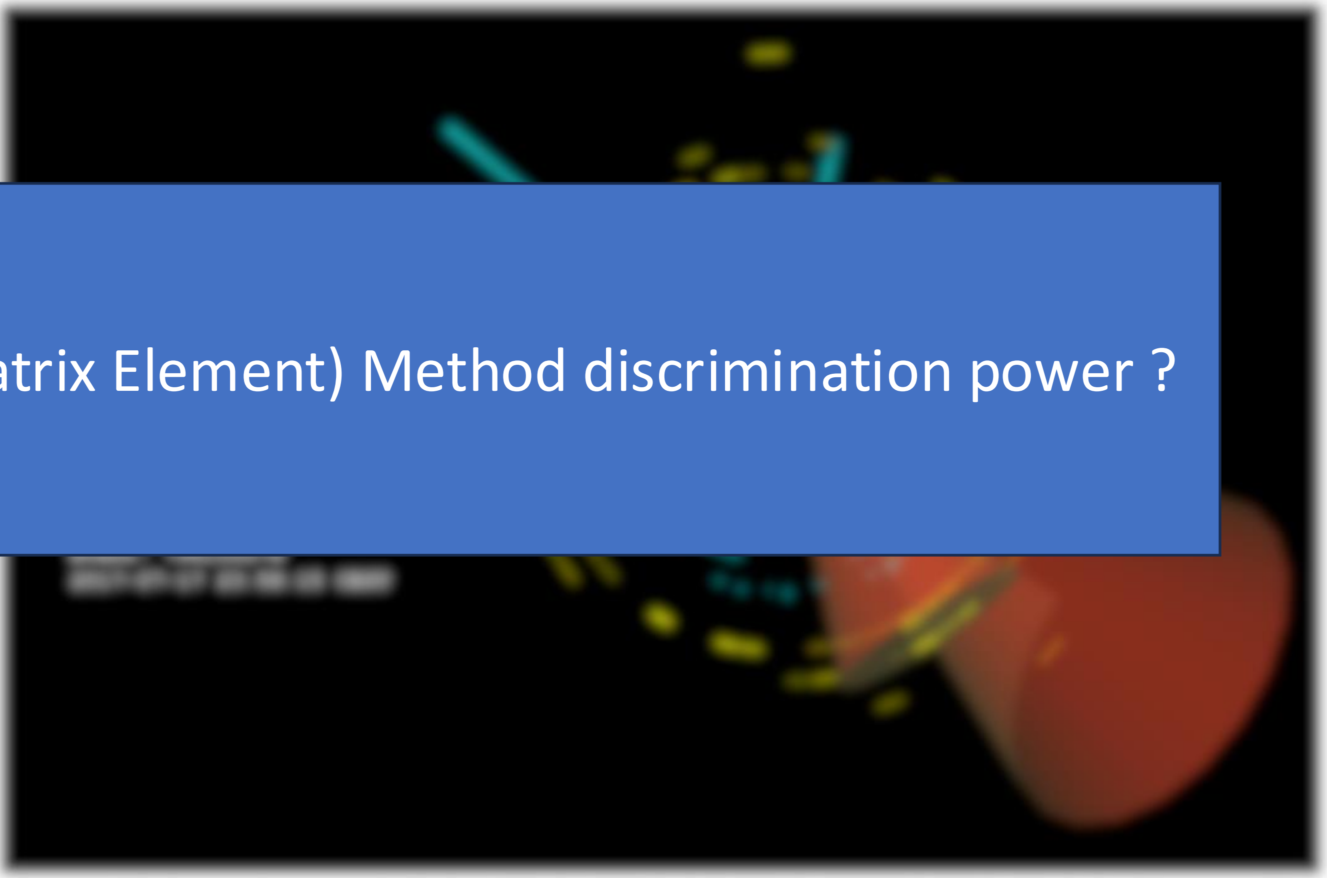


https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/PAPERS/HDBS-2018-34/figaux_02.png

Introduction

Event display (ATLAS): Final state

- Any extra (real) radiation candidates will be dealt by integrating over its 3 extra degrees of freedom



How to quantify the (Matrix Element) Method discrimination power ?

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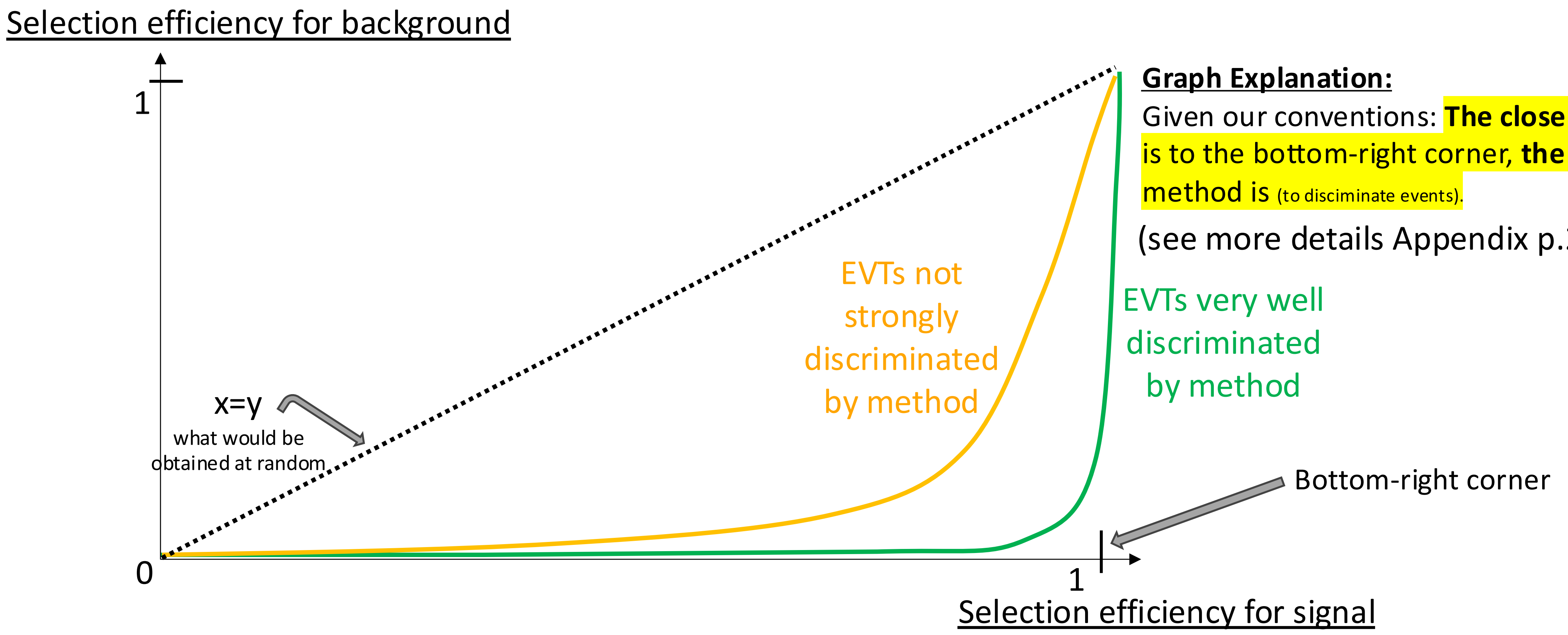
Introduction

Rejection/ROC curves

- ROC/Rejection curve : a graphical way to show a method's discrimination power. For our case: by using ratios of $\mathcal{L}_{process}$ (signal & background).



Graph Explanation:

Given our conventions: **The closer** the curve is to the bottom-right corner, **the better** the method is (to disciminate events).

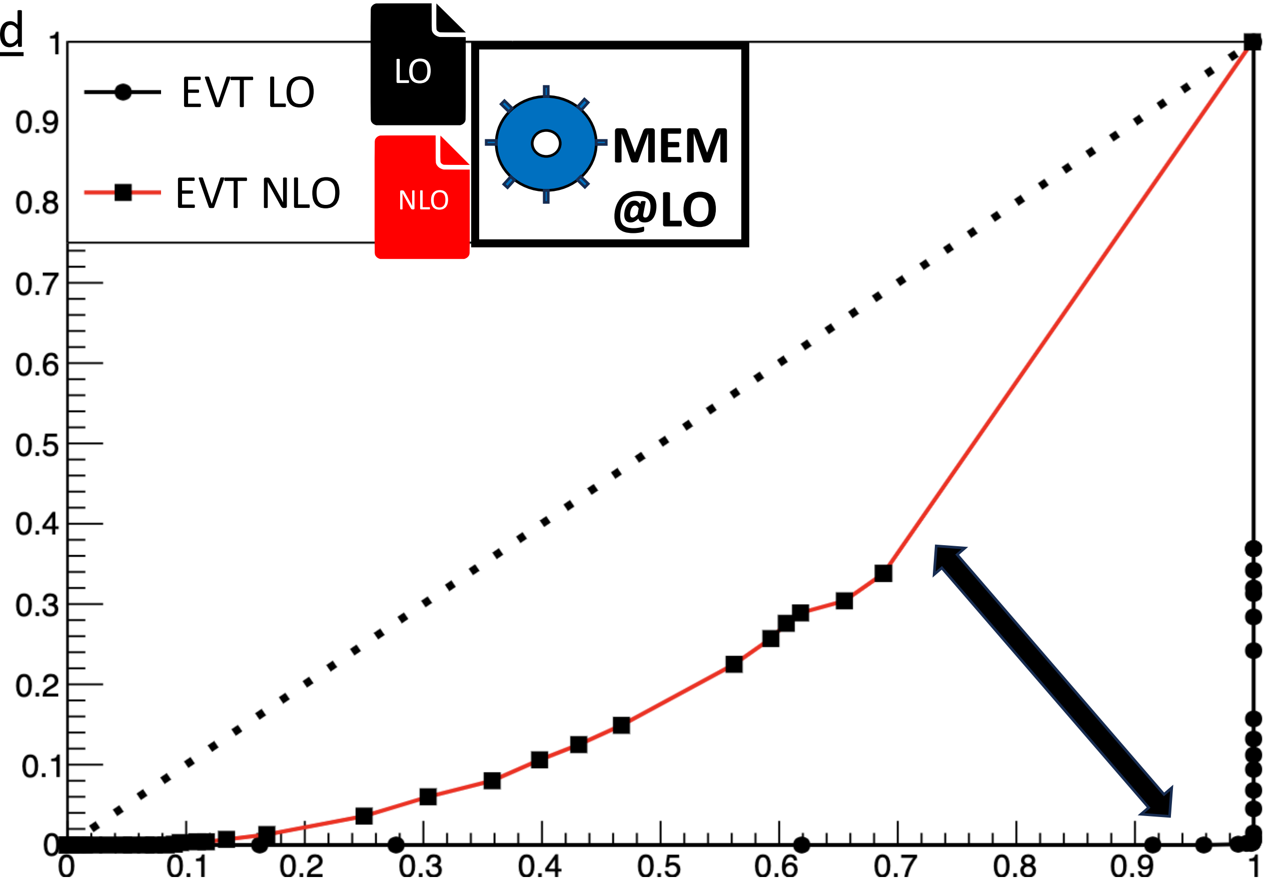
(see more details Appendix p.25)

Motivation

Using NLO events on MEM@LO

- Using MEM@LO to analyse events generated at LO and NLO:

background (ttH)



signal (gluon Fusion)

2 important results:

1) Very strong discrimination on LO events

2) Less in discrimination power on NLO events

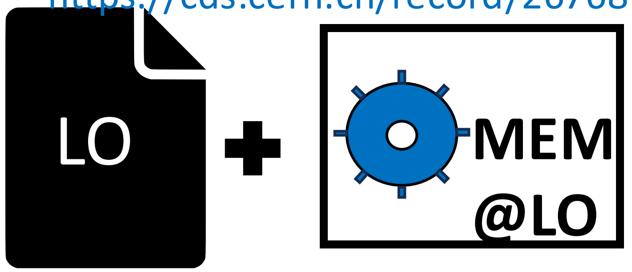
Goal of this presentation

Using NLO events on MEM@NLO

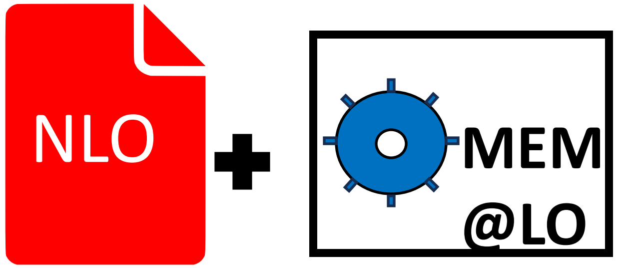
- <u>For this talk, we will present our work on constructing a new MEM@NLO formalism</u>

Work previously done [2019] (F.Eble, J.Stark)

https://cds.cern.ch/record/2676819



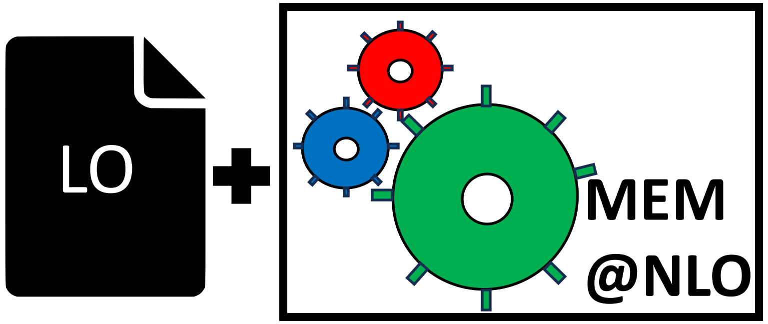
: Analysis of LO events with LO MEM



: Analysis of NLO events with LO MEM

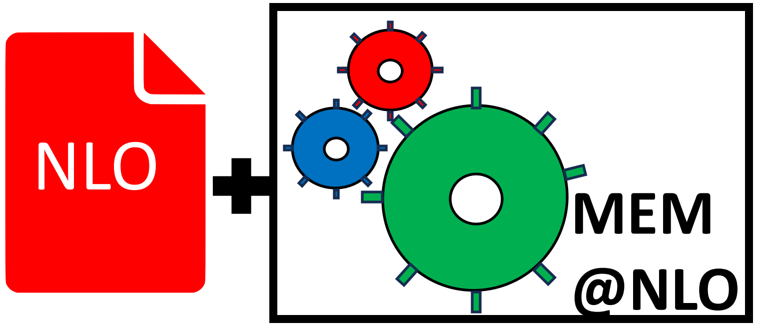
Our work [2022 -> 2025]

NEW!



: Analysis of LO events with NLO MEM

NEW!



: Analysis of NLO events with NLO MEM

To our knowledge, this work marks the first application of the MEM at NLO accuracy to the search for HH and the measurement of λ_{3H}

Results: MEM@NLO

Constructing the MEM at NLO

Matrix Element at NLO = LO + Virtual + Real (where LO == Born)

Main challenges:

1. Virtual and Real contributions (theoretically challenging).
   Already done by Heinrich, Jones, Kerner et al:1703.09252, implemented in the POWHEG-BOX-V2/ggHH for gluon fusion at NLO. **But:** No direct interface to access them for given PS points.

2. In the POWHEG-BOX-V2/ggHH implementation, the Higgs boson decays are generated using Pythia. But we need to add Higgs decay for the full matrix element. $\underbrace{gg \to HH}_{\text{ggF/ggHH}} \to b\bar{b}\gamma\gamma$

3. Different phase spaces: [LO and Virtual] share the same; But [Real] has its own.

This is very important for the dimension of the MEM integral.

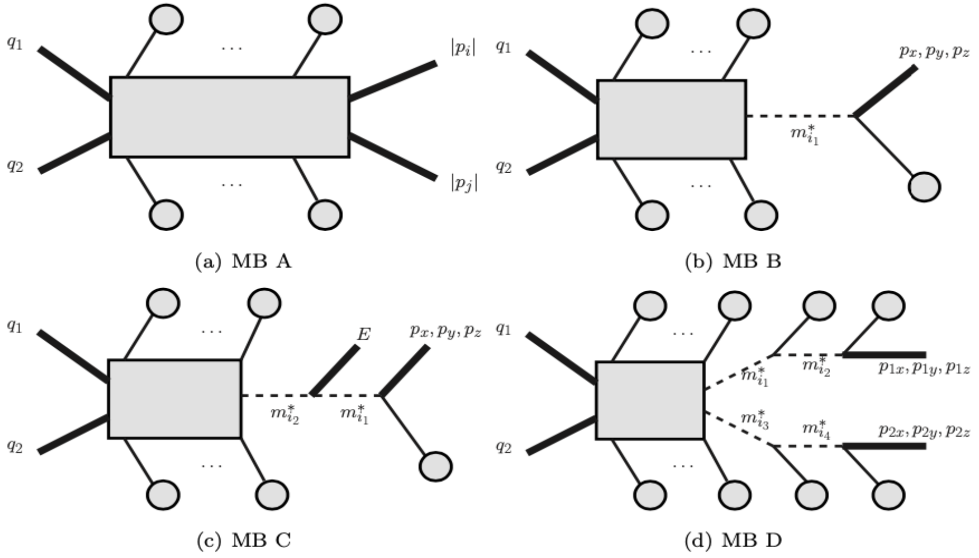
$\mathrm{d}q_1\mathrm{d}q_2\mathrm{d}y^{4n}$ $\mathrm{d}q_1\mathrm{d}q_2\mathrm{d}y^{4(n+1)}$ (due to extra real radiation)

Results: MEM@NLO

Need for a new main block

- In the literature, most MEM are at LO, or use a private algorithm to generate the extra radiation: **The Matrix Element Method tools** (easily available) **are not NLO friendly** !

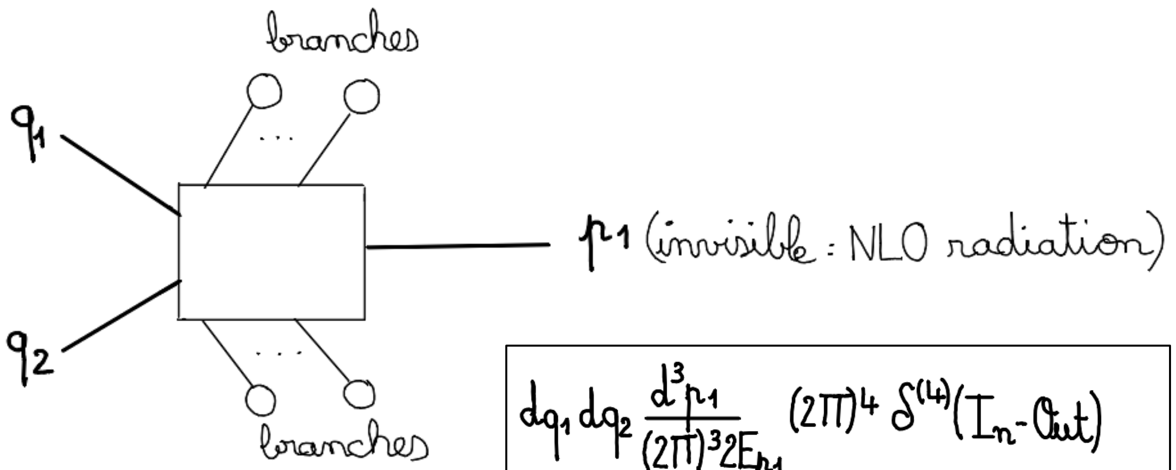
Definition « Main Blocks » : Choices of integration variable and substitutions made to break down a complex process into manageable configuration.



Issue: None of them can properly deal with an extra radiation

So we built our own in MoMEMta (MEM tool for integration).

Main Block « N » NEW!



Has extra radiation as integration variables

$$dq_1\, dq_2 \frac{d^3 p_1}{(2\pi)^3 2E_{p_1}} (2\pi)^4 \delta^{(4)}(I_n - Out)$$

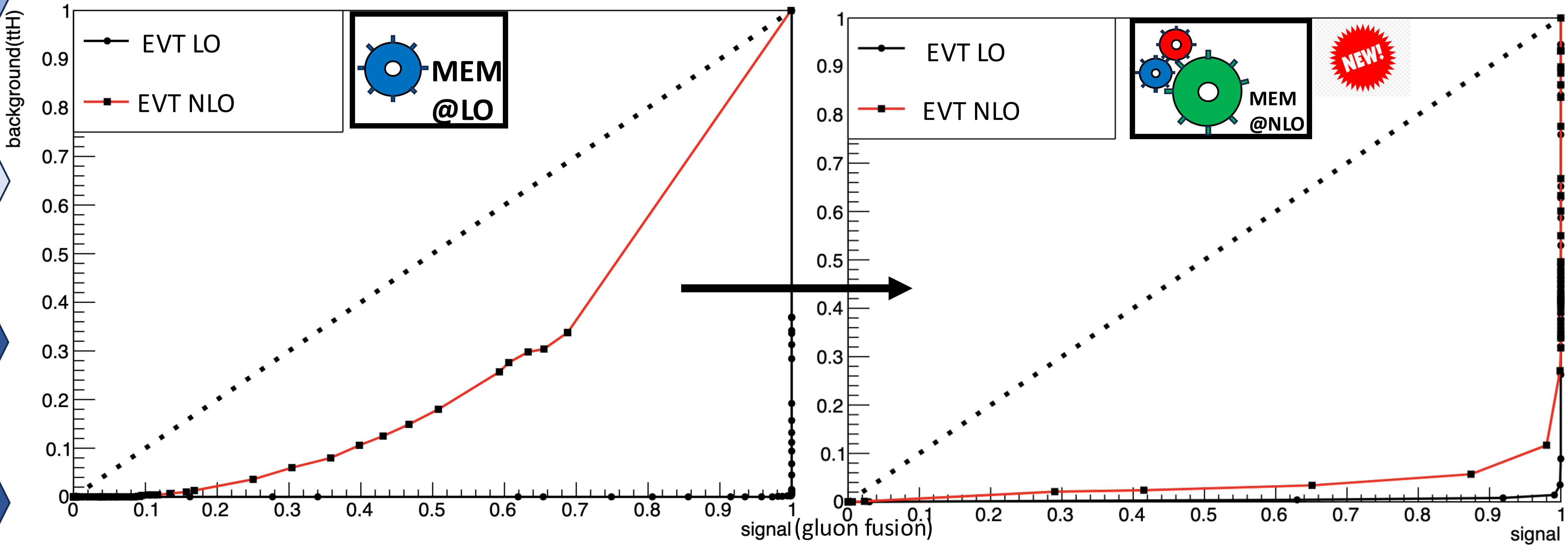
$$\downarrow$$

$$\prod \frac{1}{E_{p_1}} \cdot dp_{1z} \cdot |\det(J)|$$

Results: MEM@NLO

ROC curve [ttH background]

- Impact of Both (BV+Real): **background ttH**

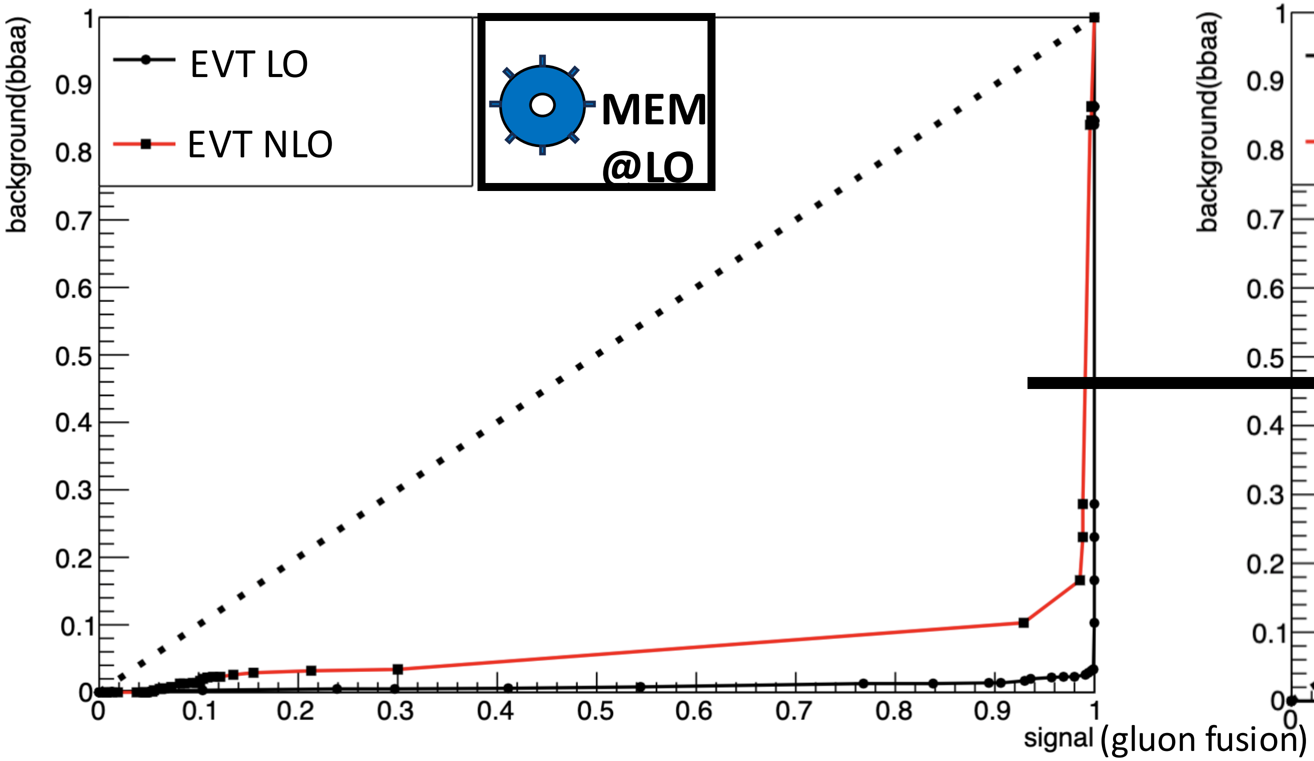


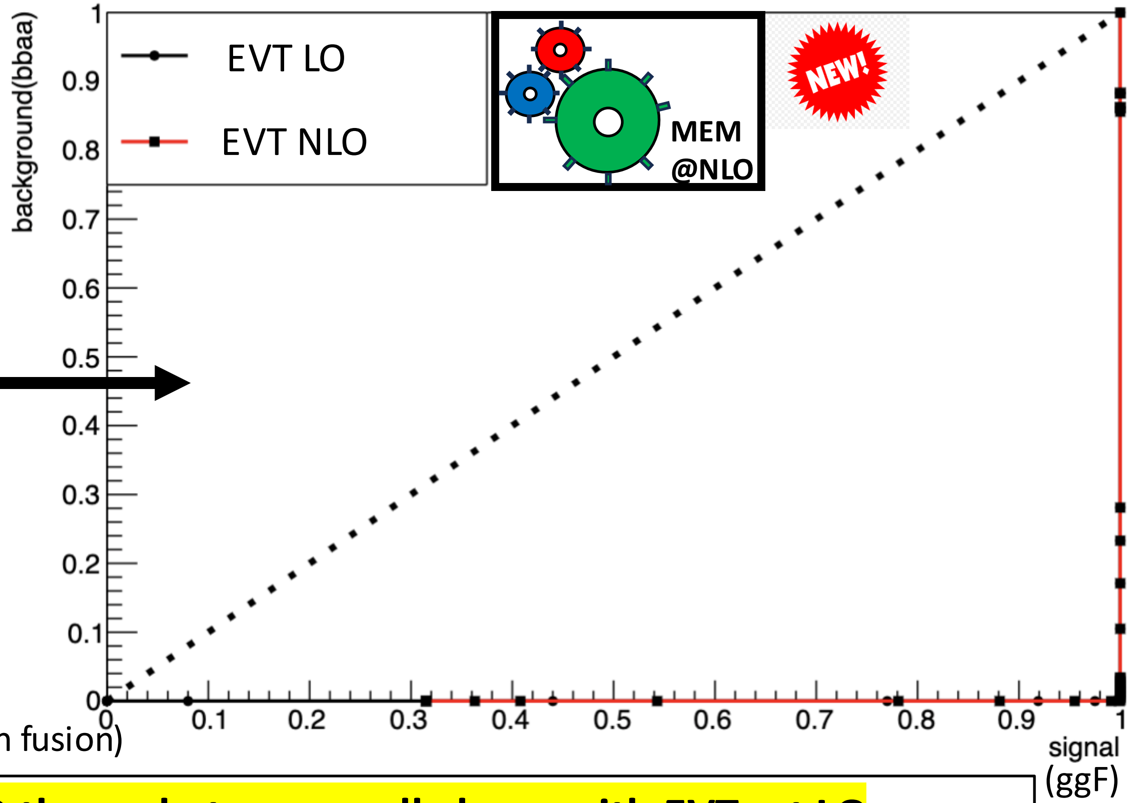
With NLO MEM: better discrimination than before on EVT NLO, huge improvement : Good!

Results: MEM@NLO

ROC curve [qcd background]

- Impact of Both (BV+Real): **background qcd**





We achieved the same efficiency with EVTs at NLO than what we usually have with EVTs at LO

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Extraction of κ_λ: Main idea

Likelihoods

- Now that we have seen the discrimination power of the MEM@NLO on NLO events using $\boldsymbol{\mathcal{L}_{process}}$ (i.e the integral seen earlier), we can continue our work by looking for $\boldsymbol{\lambda_{3H}}$ **value extraction** (or rather $\boldsymbol{\kappa_\lambda} = \frac{\lambda_{3H}}{\lambda_{SM}}$).

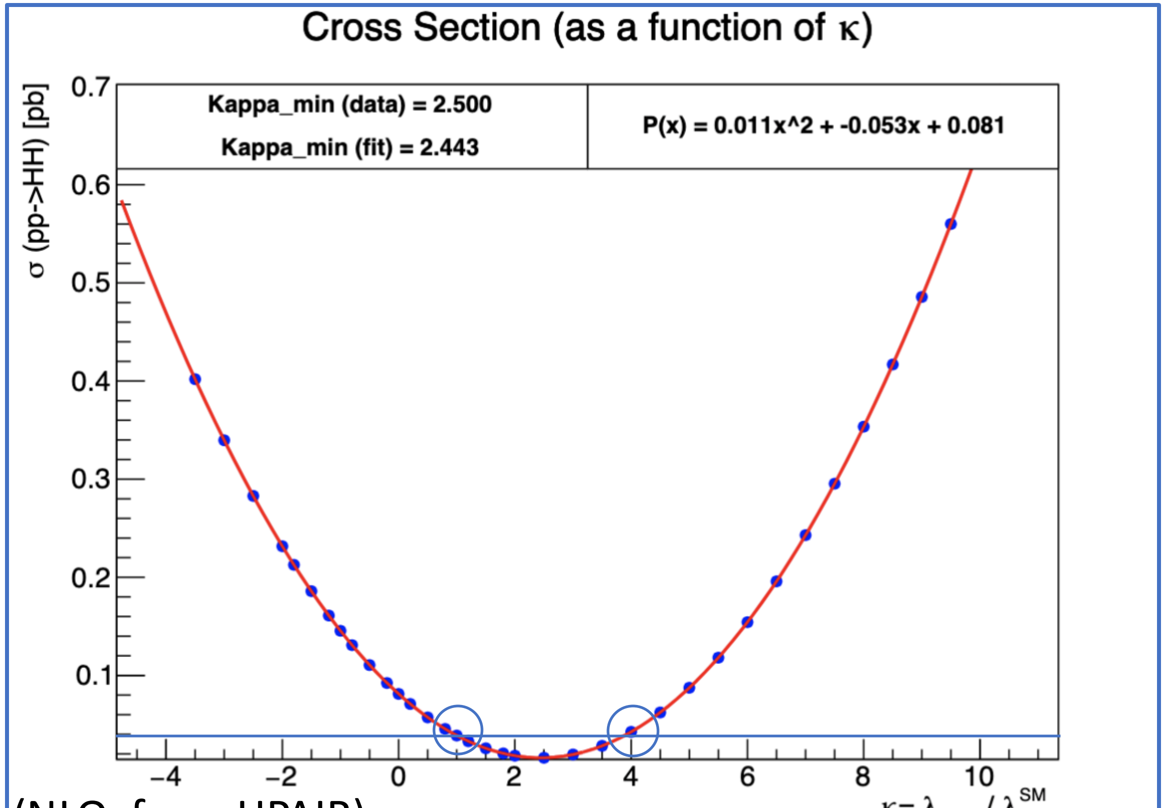
- Idea: We can construct three Likelihoods $\mathcal{L}$ (for the purpose of this analysis) for given $\boldsymbol{\kappa_\lambda}$ hypothesis values $\boldsymbol{\kappa_\lambda}$∈[-3.50 , 10]:
  - $\boldsymbol{\mathcal{L}_{kinematic}(\kappa_\lambda)}$, contructed directly from the MEM integral (i.e direcly constructed from $\boldsymbol{\mathcal{L}_{process}}$).
  - $\boldsymbol{\mathcal{L}_{yield}(\kappa_\lambda)}$, a theoretical prediction on the behavior of the number of events produced for given hypotheses (processes within the sample, integrated_luminosity, …)
  - $\boldsymbol{\mathcal{L}_{extended}(\kappa_\lambda)}$, the product of the two others.

(see Appendix p.27 & p.28 for more detail)

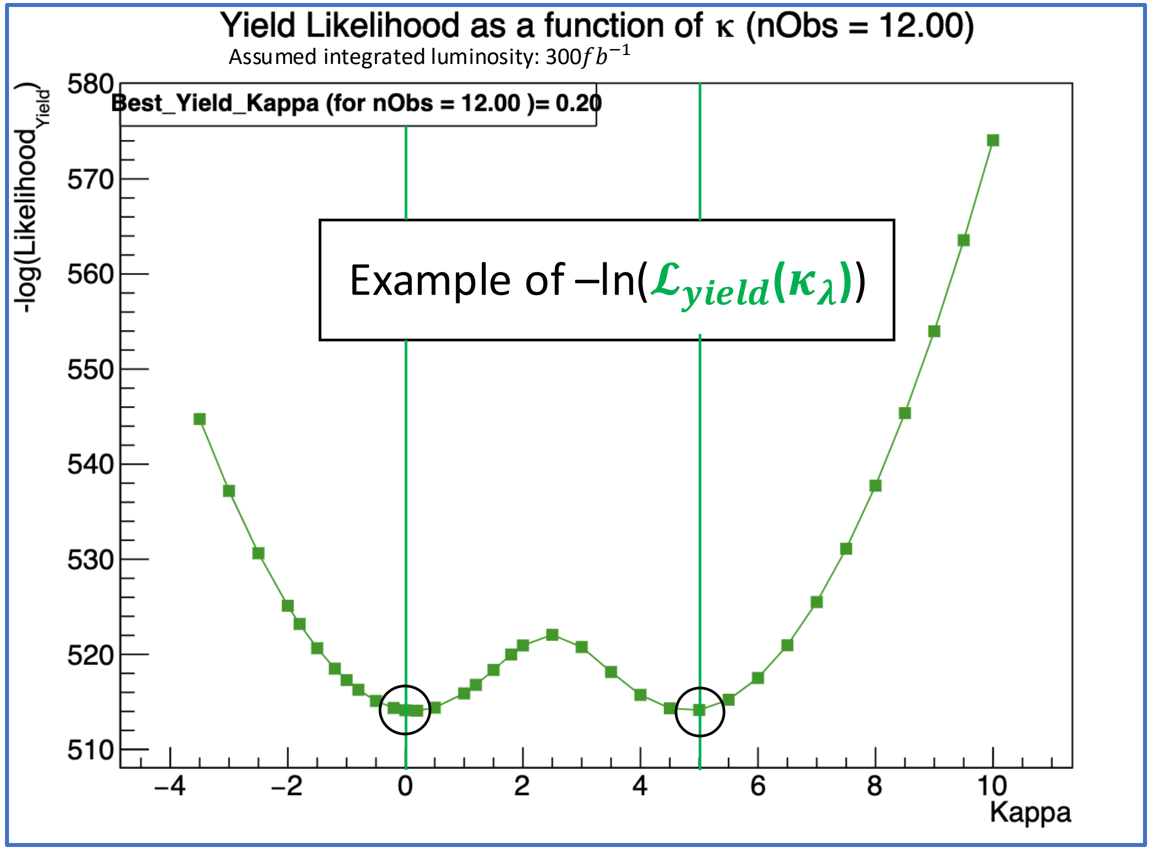
Extraction of κ_λ: Main idea

Likelihood Yield

The strong dependence of the production cross section on κ_λ is used in many analysis methods (like « Event counting methods »). In the case of our MEM, this information is inside $\mathcal{L}_{yield}(\kappa_\lambda)$.



(NLO, from HPAIR)

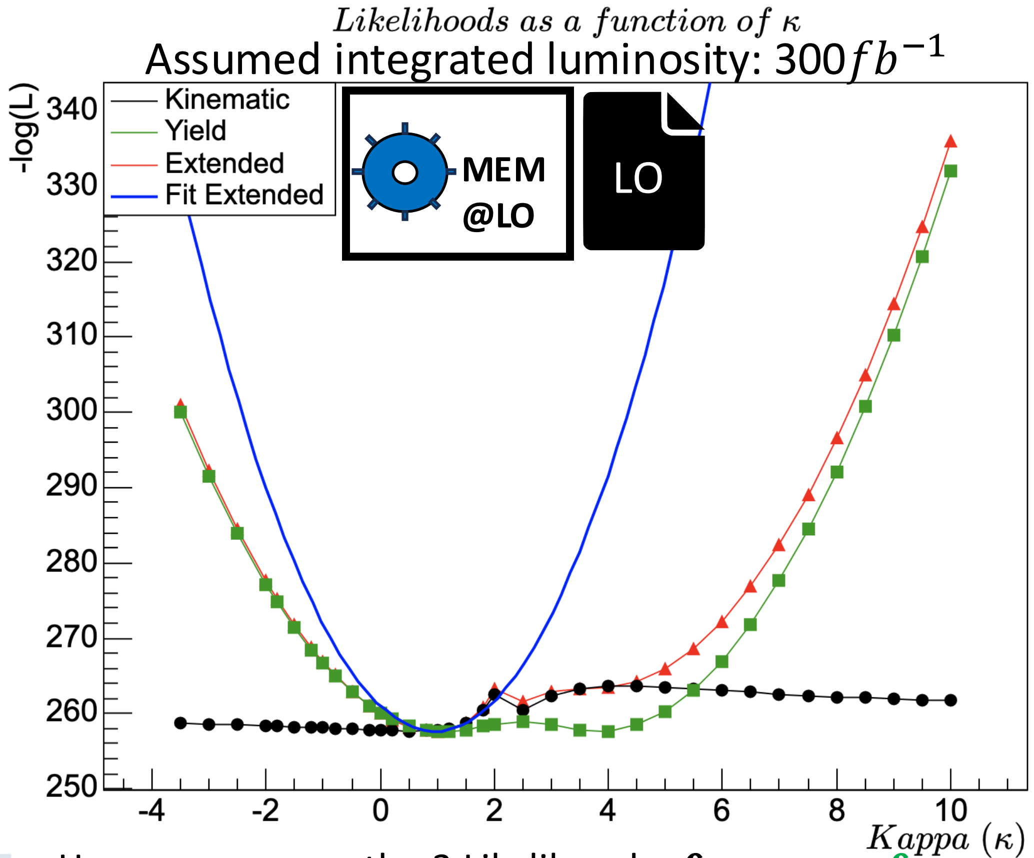


Example of $-\ln(\mathcal{L}_{yield}(\kappa_\lambda))$

Extraction of κ_λ: Results

Part 1: Signal only, MEM@LO

- (MEM@LO and events at LO)



⟵ For a given data subset

Here you can see the 3 Likelihoods: $\mathcal{L}_{kinematic}$, $\mathcal{L}_{yield}$, and $\mathcal{L}_{extended}$

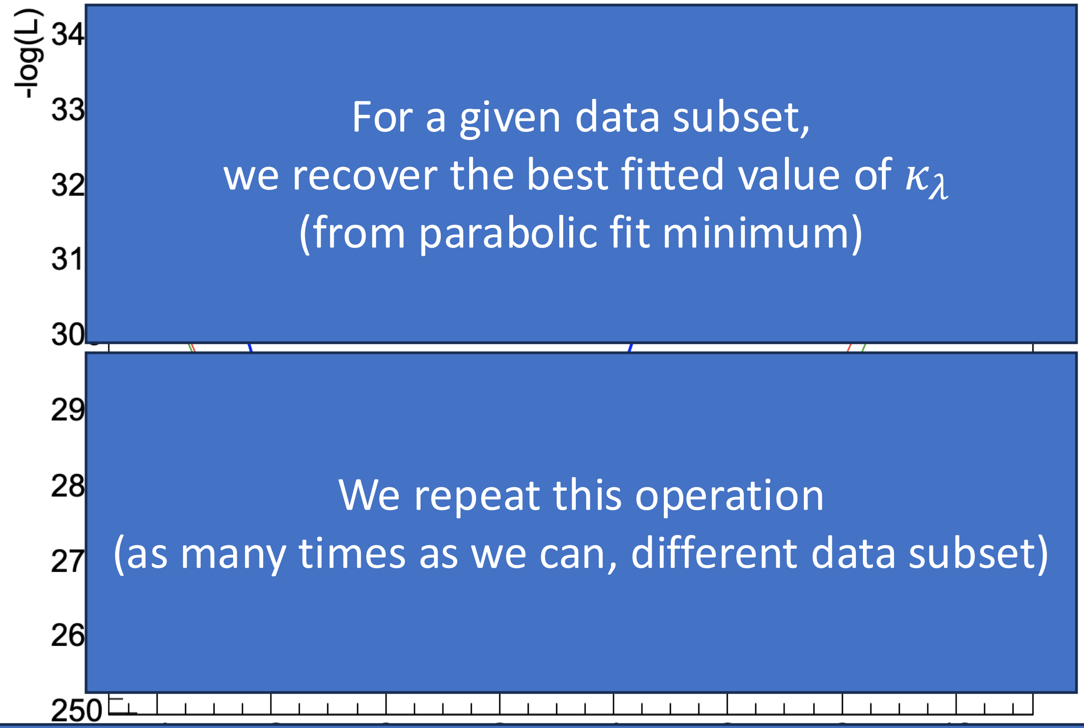
As well as a **Parabolic Fit** around $\mathcal{L}_{extended}$'s minimum.

Extraction of κ_λ: Results

Part 1: Signal only, MEM@LO

- (MEM@LO and events at LO)

Likelihoods as a function of κ



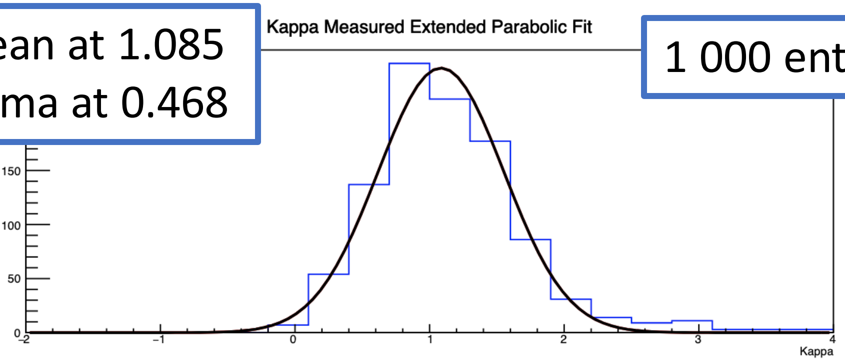
For a given data subset,
we recover the best fitted value of κ_λ
(from parabolic fit minimum)

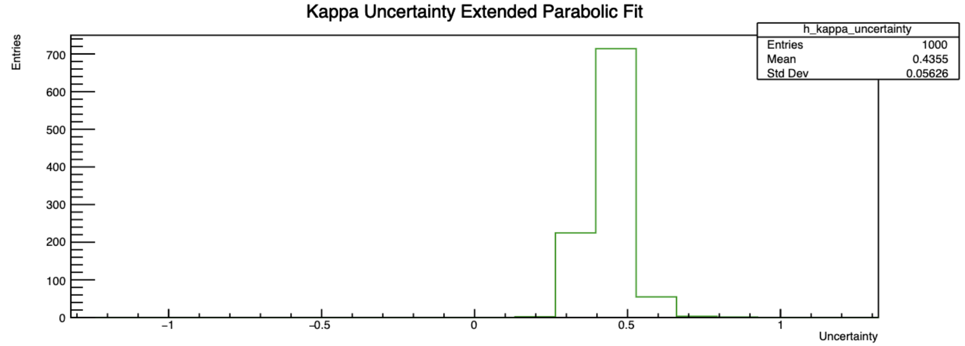
We repeat this operation
(as many times as we can, different data subset)

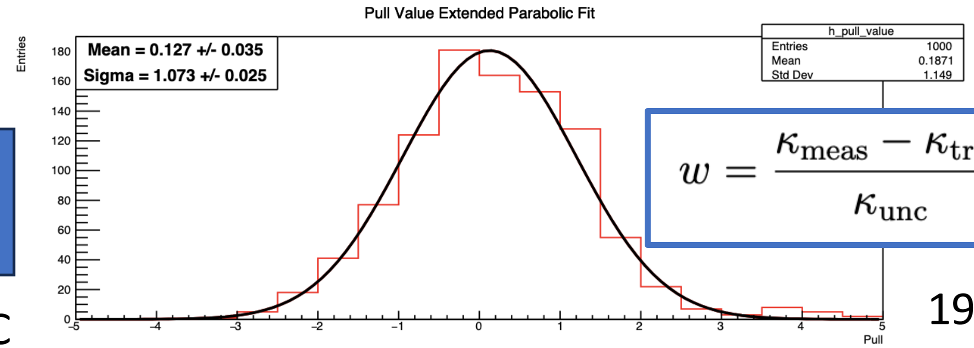
And from this we construct an histogram of all the best fitted κ_λ values

Mean at 1.085
Sigma at 0.468

1 000 entries







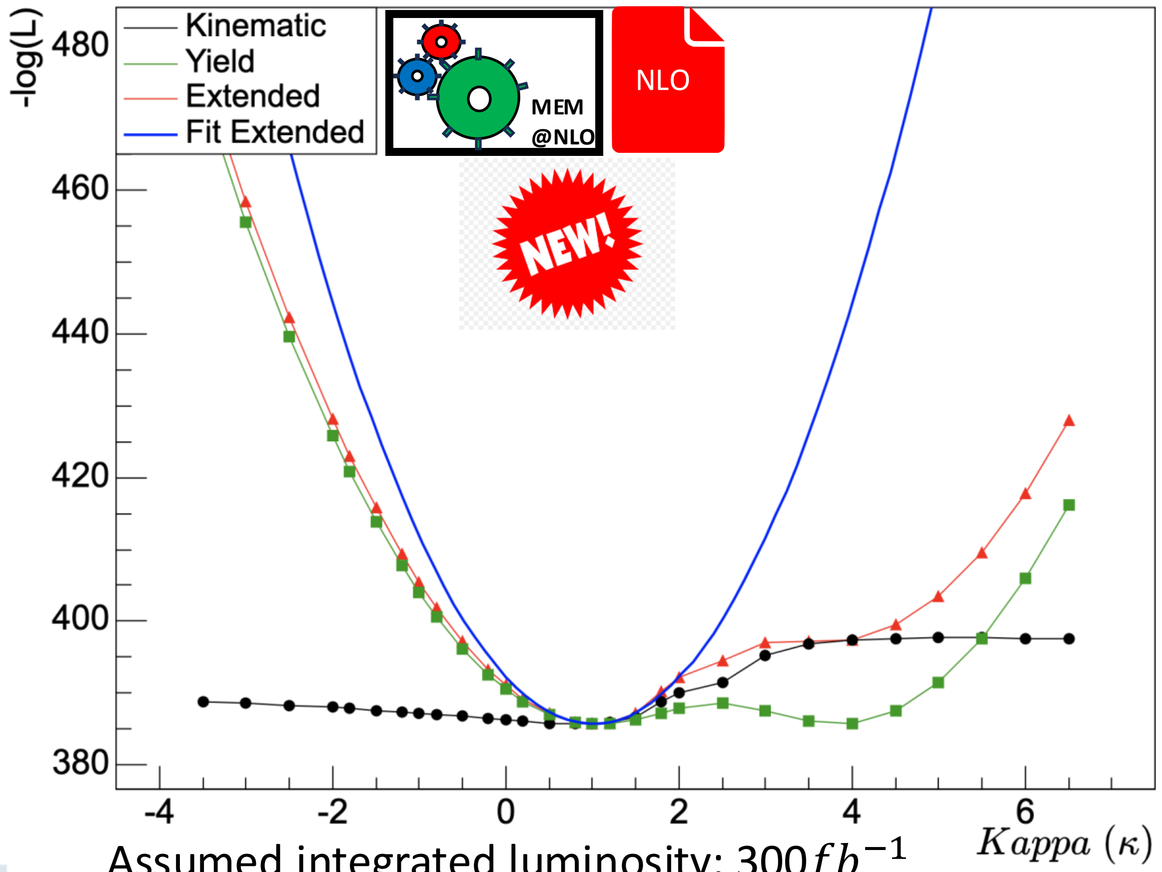
$$w = \frac{\kappa_{\text{meas}} - \kappa_{\text{true}}}{\kappa_{\text{unc}}}$$

Extraction of κ_λ: Results

Part 2: Signal only, MEM@NLO

- Result for MEM@NLO on NLO events

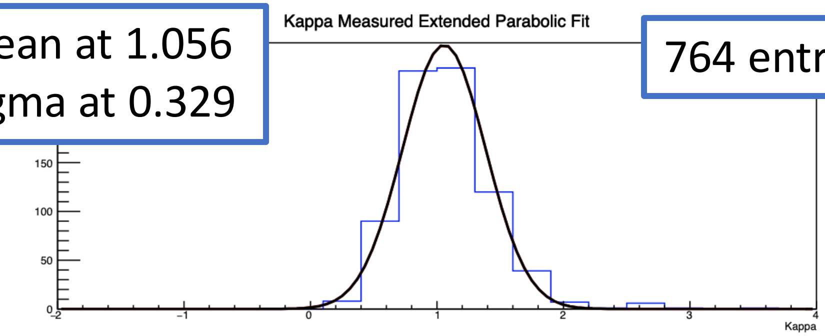
Likelihoods as a function of κ

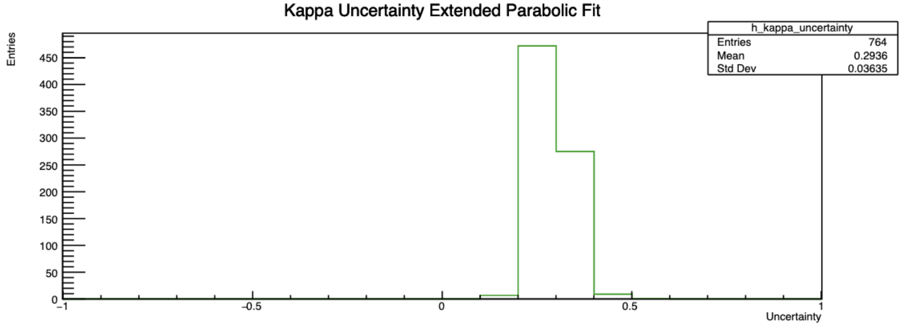


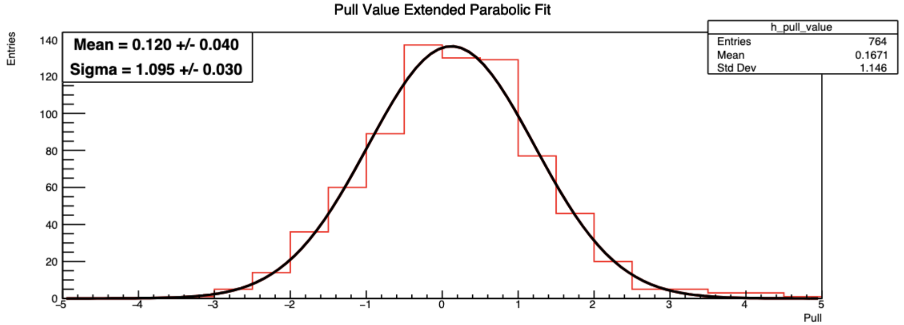
Assumed integrated luminosity: $300 fb^{-1}$

Mean at 1.056
Sigma at 0.329

764 entries



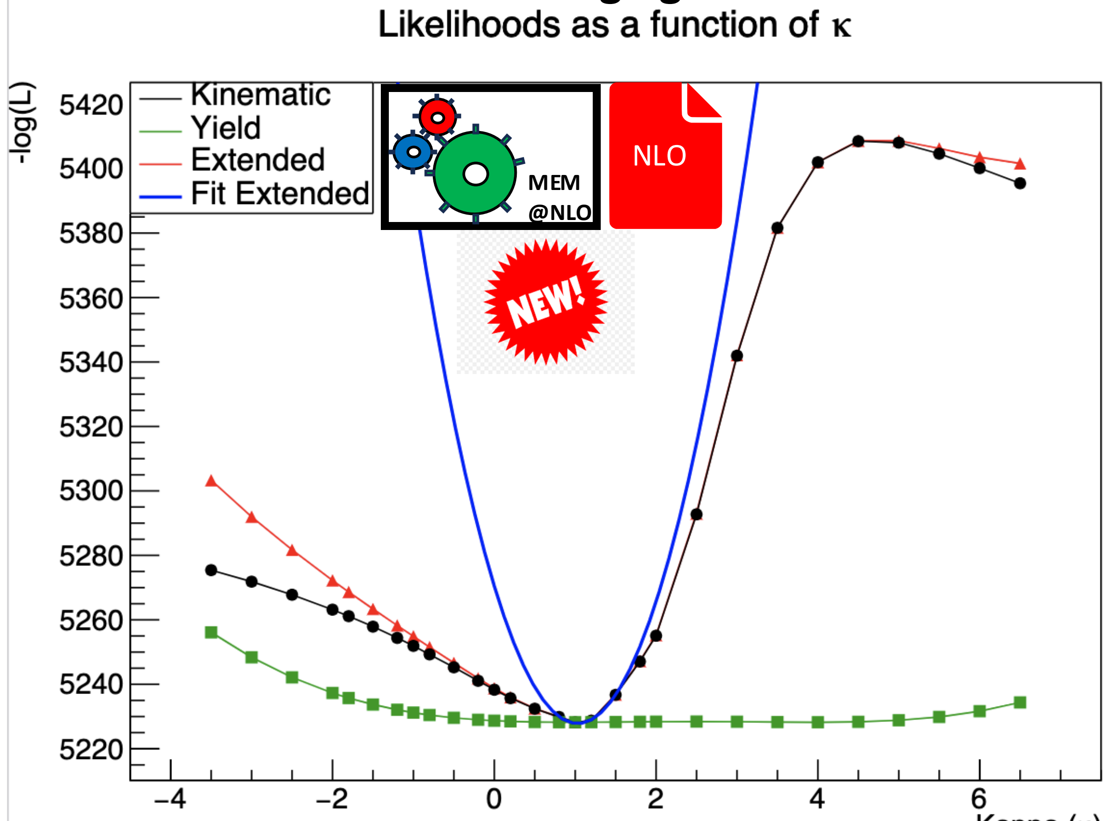




Extraction of κ_λ: Results

Part 3: with background [ttH], MEM@NLO

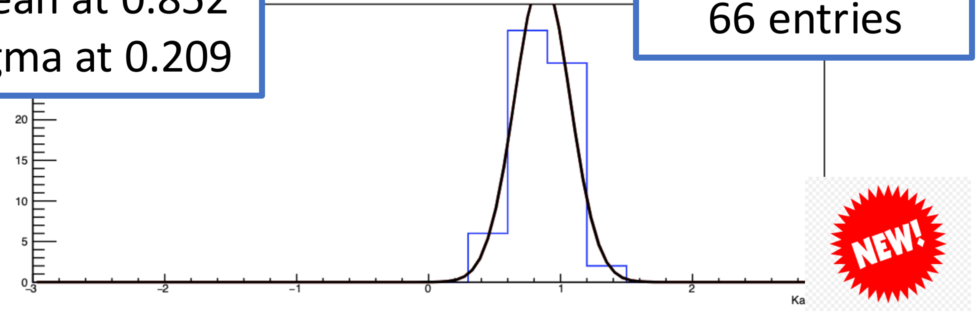
- To have more realistic samples (MEM@NLO on NLO events) : Add background events. As seen earlier, ttH is the process that will be **the most challenging for us**.

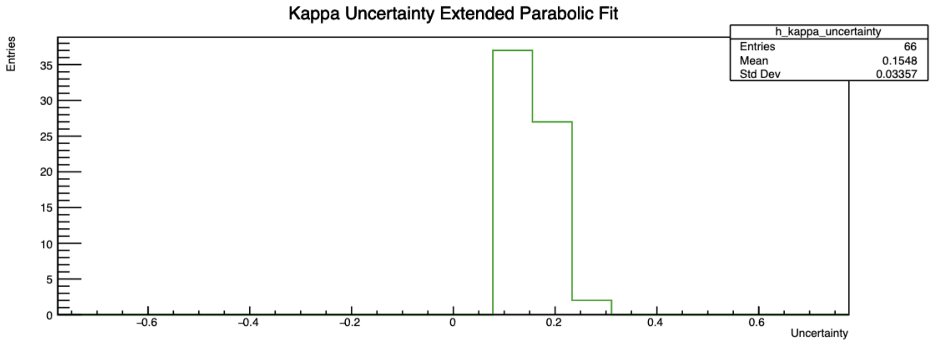


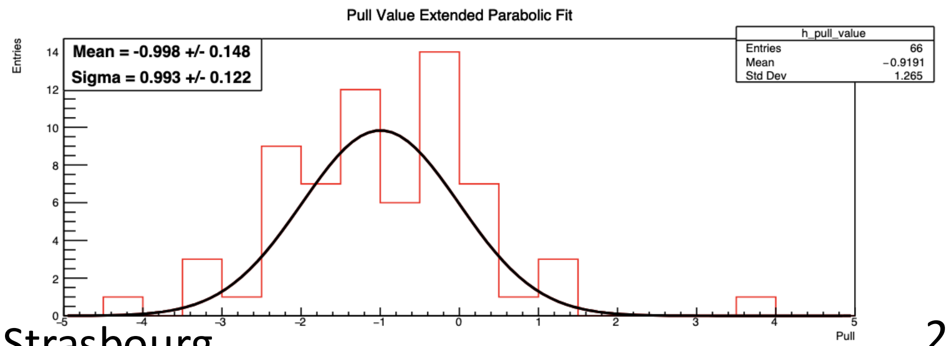
Assumed integrated luminosity: $300 fb^{-1}$

Mean at 0.852
Sigma at 0.209

66 entries







Extraction of κ_λ: Results

Part 3: with background [ttH], MEM@NLO

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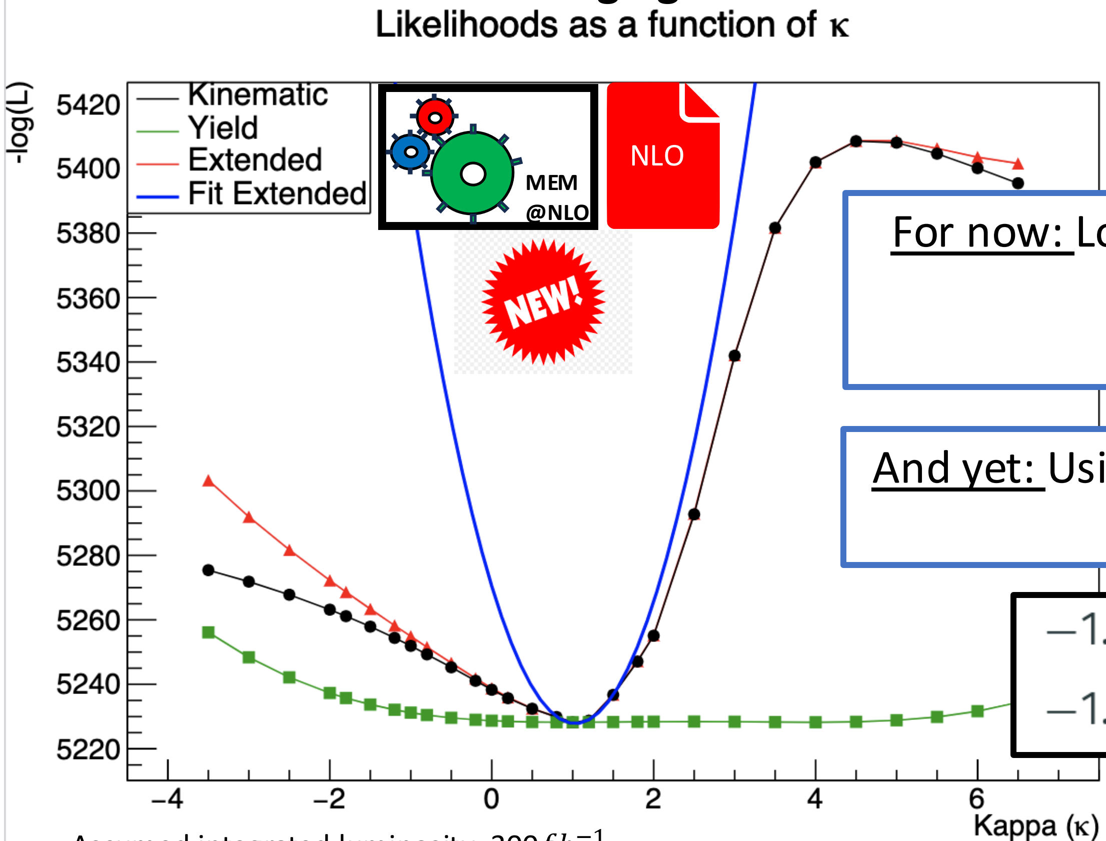
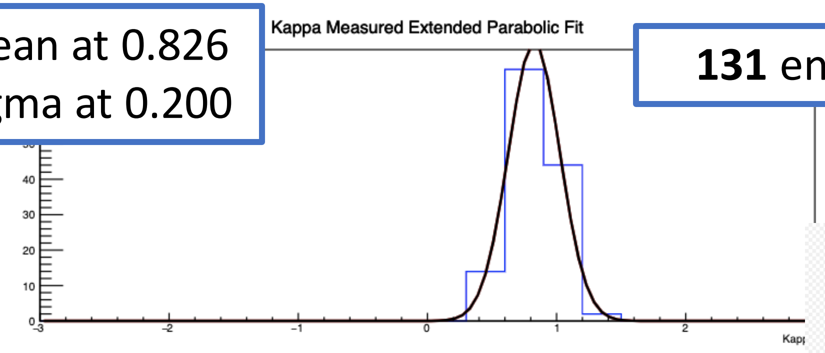


Assumed integrated luminosity: $300 fb^{-1}$

Mean at 0.826
Sigma at 0.200

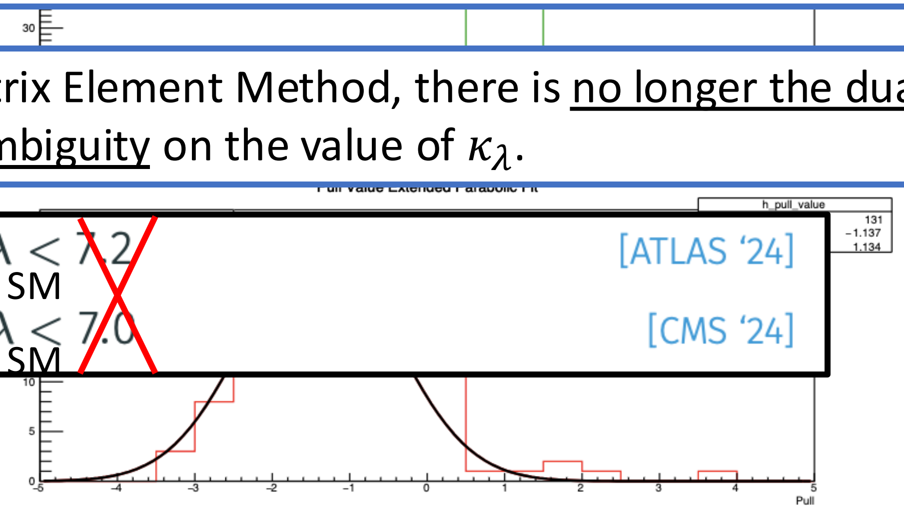


131 entries

NEW!

For now: Lower statistics (only **131** entries due to ttH high number of events per run)(**very fresh plot**). But more to come **soon.**

And yet: Using the Matrix Element Method, there is no longer the dual ambiguity on the value of κ_λ.



$-1.2 < \lambda_{obs}/\lambda_{SM} < 7.2$ [ATLAS '24]

$-1.4 < \lambda_{obs}/\lambda_{SM} < 7.0$ [CMS '24]

Conclusions and beyond

- The Matrix Element Method has had great successes in HEP, but is difficult to implement (in particular at NLO).
- We have successfully developped a MEM@NLO (new formalism that is meant to be as general as possible*).
- Current results show that our MEM@NLO has a great efficiency in recovering the κ_λ hypothesis for MC simulated samples (@NLO with background events), which is very promising.
- **Outlook:**
  - Increasing the statistics, adding other backgrounds (and together),
  - … and the end-goal would be to use this MEM@NLO on real data,
  - **Stay tuned!**

*as long as matrix element values can be provided. For our case: thanks to the work done by Heinrich et al. in POWHEG-BOX-V2/ggHH

Thank you
for your attention

APPENDIX

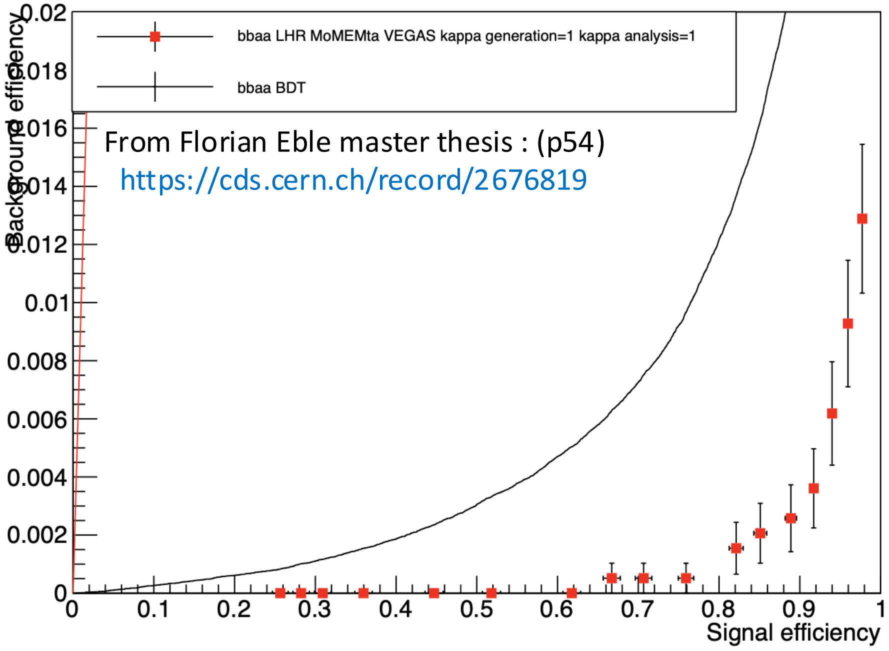
APPENDIX: Work done by F.Eble and J.Stark (and A.Lleres) [2019]

- Work done at MEM@LO (on LO generated events)

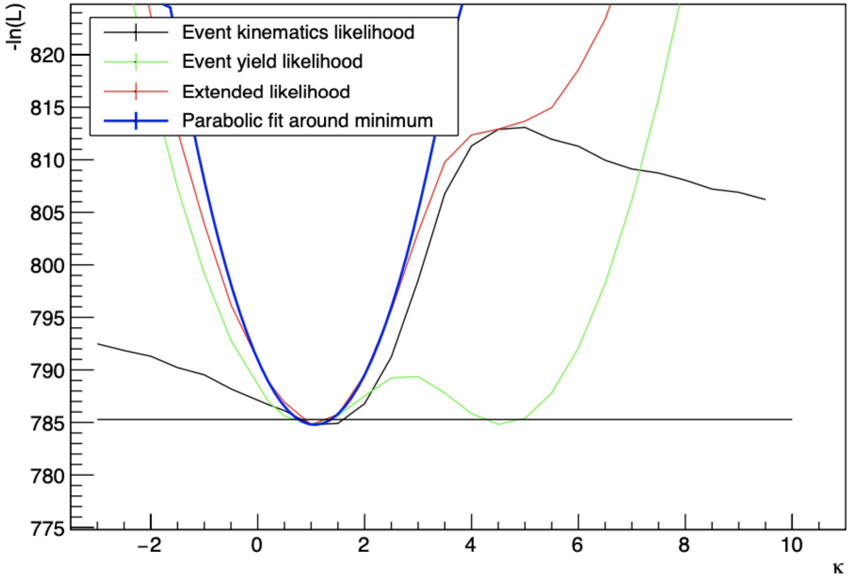
MEM @LO

LO

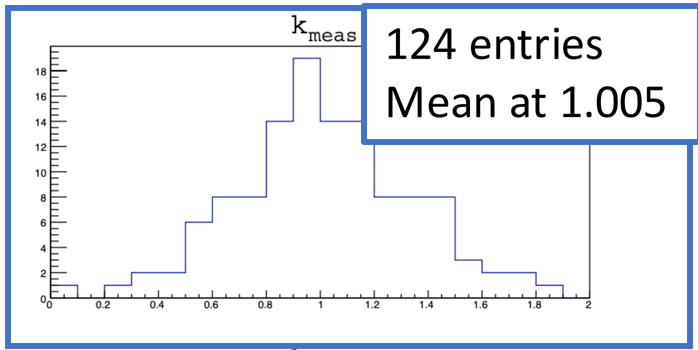
Rejection power



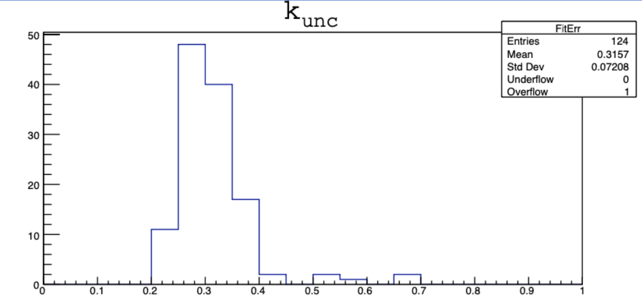
From Florian Eble master thesis : (p54)
https://cds.cern.ch/record/2676819

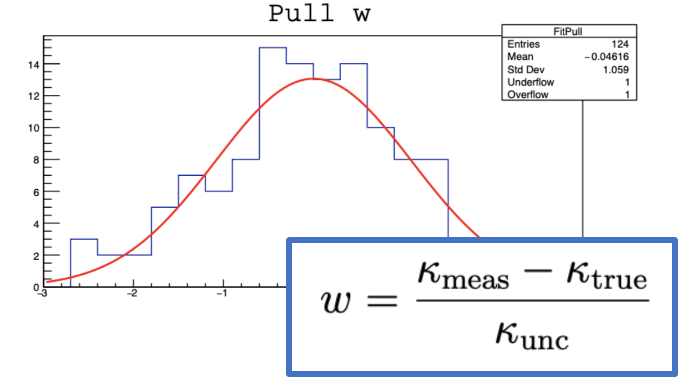


$\mathcal{L}_{kinematic}$, $\mathcal{L}_{yield}$, and $\mathcal{L}_{extended}$
As well as a **Parabolic Fit** around
$\mathcal{L}_{extended}$'s minimum



124 entries
Mean at 1.005





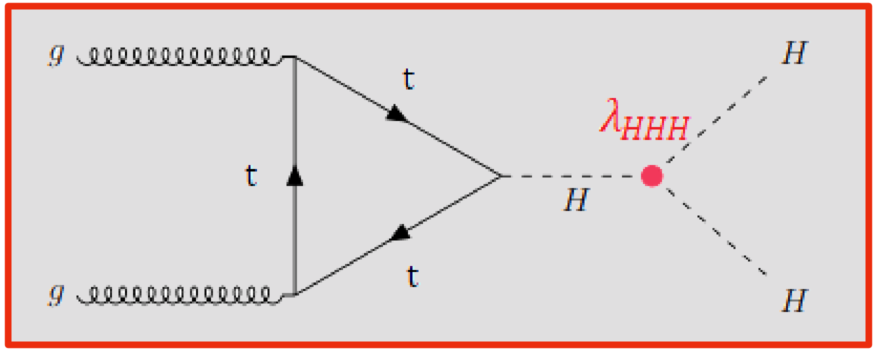
$$w = \frac{\kappa_{\text{meas}} - \kappa_{\text{true}}}{\kappa_{\text{unc}}}$$

APPENDIX: Why $b\bar{b}\gamma\gamma$ channel?

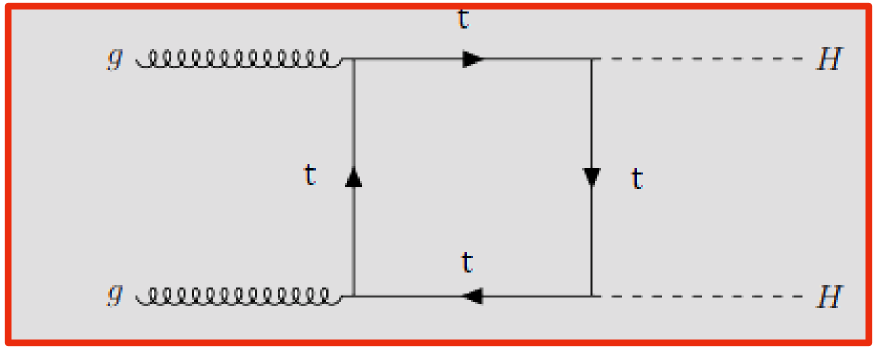
	bb	WW	ττ	ZZ	γγ
bb	34%				
WW	25%	4.6%			
ττ	7.3%	2.7%	0.39%		
ZZ	3.1%	1.1%	0.33%	0.069%	
γγ	0.26%	0.10%	0.028%	0.012%	0.0005%

Good compromise between excellent photon reconstruction, and b-jet very high Higgs-decay chanel

APPENDIX: full ggF process



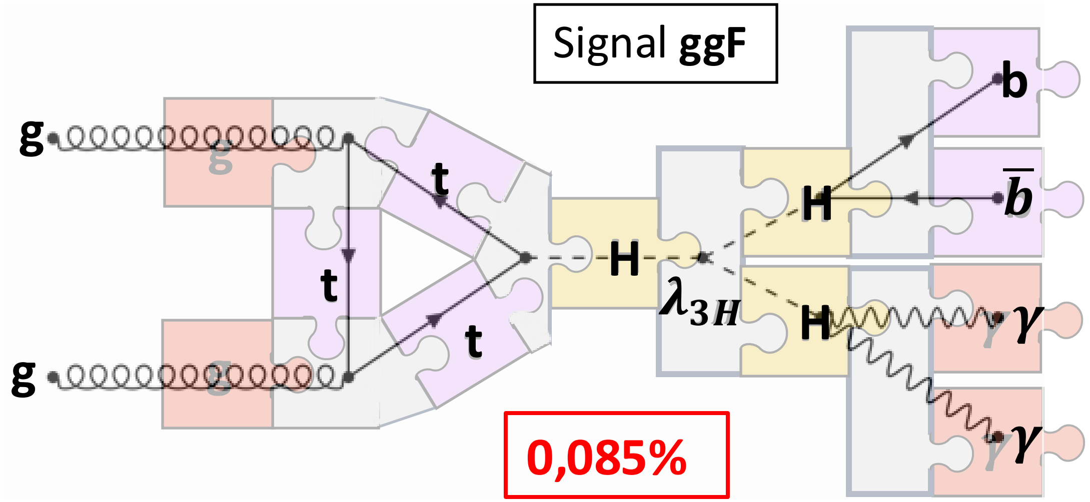
"Triangle" diagram

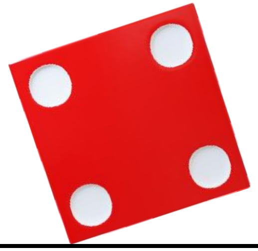


"Box" diagram

APPENDIX: How to know which process is at the origin of the $b\bar{b}\gamma\gamma$ measured in the data ? Indeed…

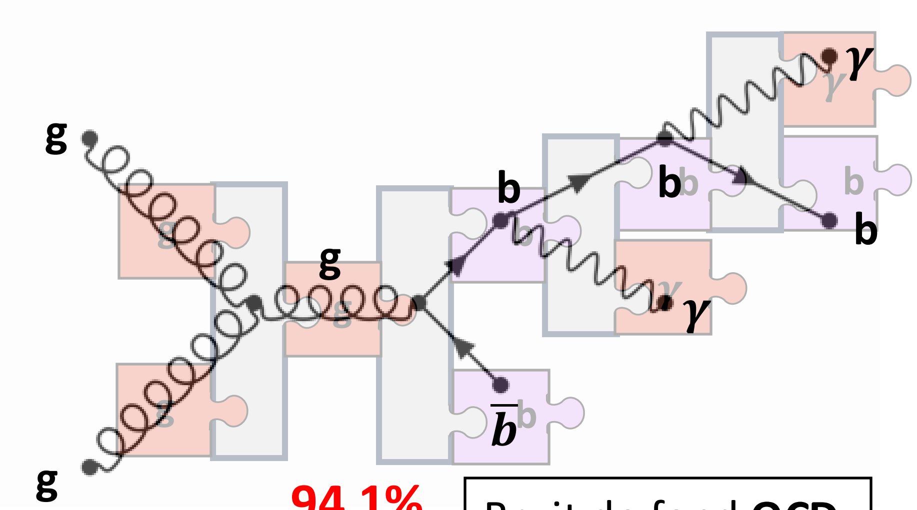
From one of my older presentation for "Doctoral school Day" (16/05/2024)





Comment savoir quel processus est à l'origine du $gg \rightarrow b\bar{b}\gamma\gamma$ mesuré?

(avec un boson de Higgs : <u>évènement rare</u>)

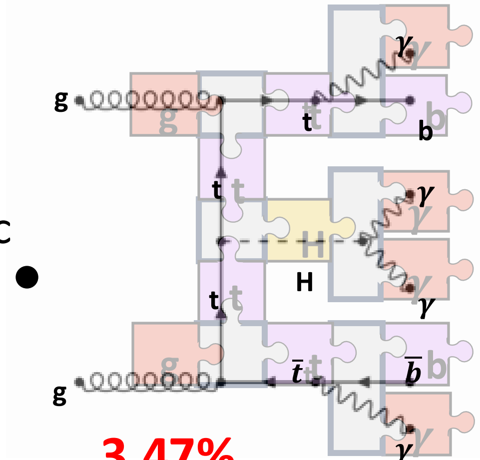


94,1%

Bruit de fond **QCD**

Etc

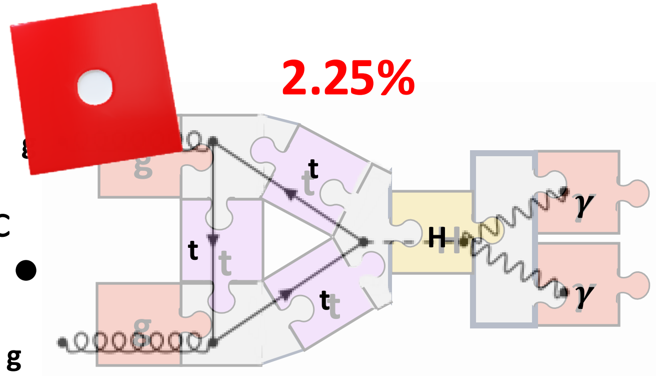
…



3.47%

Etc

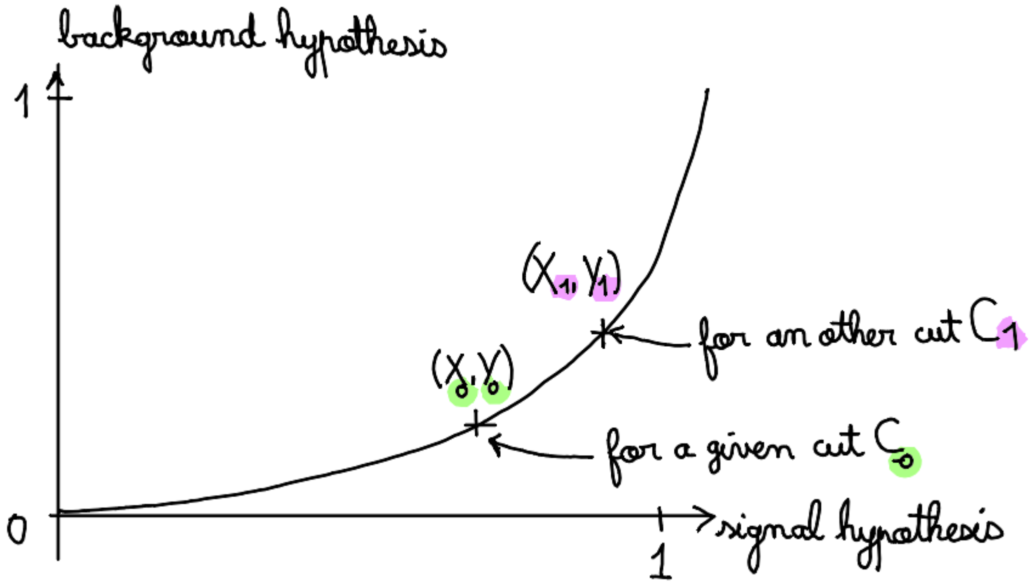
…



2.25%

D'autres bruit de fond?

APPENDIX: Rejection/ROC curves in more detail



Confusion matrix :

	actually positive	actually negative
predicted positive	TP	FP
predicted negative	FN	TN

- TP : Nombre d'EVT reconnus True signal
  (i.e EVTs pour lesquels : $\frac{\text{hyp-signal-(EVT\_signal)}}{\text{hyp-bckg-(EVT\_signal)}} > \text{Cut}$)
- FN : Nombre d'EVT reconnus False signal
  (i.e EVTs pour lesquels : $\frac{\text{hyp-signal-(EVT\_signal)}}{\text{hyp-bckg-(EVT\_signal)}} < \text{Cut}$)
- TN : Nombre d'EVT reconnus True background
  (i.e EVTs pour lesquels : $\frac{\text{hyp-signal-(EVT\_bckg)}}{\text{hyp-bckg-(EVT\_bckg)}} > \text{Cut}$)
- FP : Nombre d'EVT reconnus False background
  (i.e EVTs pour lesquels : $\frac{\text{hyp-signal-(EVT\_bckg)}}{\text{hyp-bckg-(EVT\_bckg)}} < \text{Cut}$)

$$X_{MEM} = \frac{TP}{(TP + FN)}$$

$$Y_{MEM} = \frac{FP}{(FP + TN)}$$

APPENDIX: Kinematic likelihood in more detail

Likelihood for a specific process:

$$\mathcal{L}^{p}_{process}(h|x^i) = \frac{d\sigma(pp \to x^i; h; W)}{d\sigma^{obs}_{p}(pp \to F)}$$

and $d\sigma(pp \to x^i; h; W)$ is called weight in the MEM vocabulary.

Likelihood of one event x^i:

$$\mathcal{L}^{i}_{event}(h|x^i) = \sum_{p=1}^{n_p} f_p \, \mathcal{L}^{p}_{process}(h|x^i)$$

Sample Kinematic likelihood :

$$\mathcal{L}_{sample}(h|x) = \prod_{i=1}^{N} \mathcal{L}^{i}_{event}(h|x^i)$$

Kinematic Likelihood: The likelihood one can construct from the MEM output.

APPENDIX: Yield likelihood in more detail

Event yield Likelihood: : The number of events observed (after a given set of event selection requirements) is a valuable piece of information for the extraction of κ. If we assume that we obtained N_{obs} from a poisson distribution from $N_{sig}(\kappa_{\text{test}})$, then:

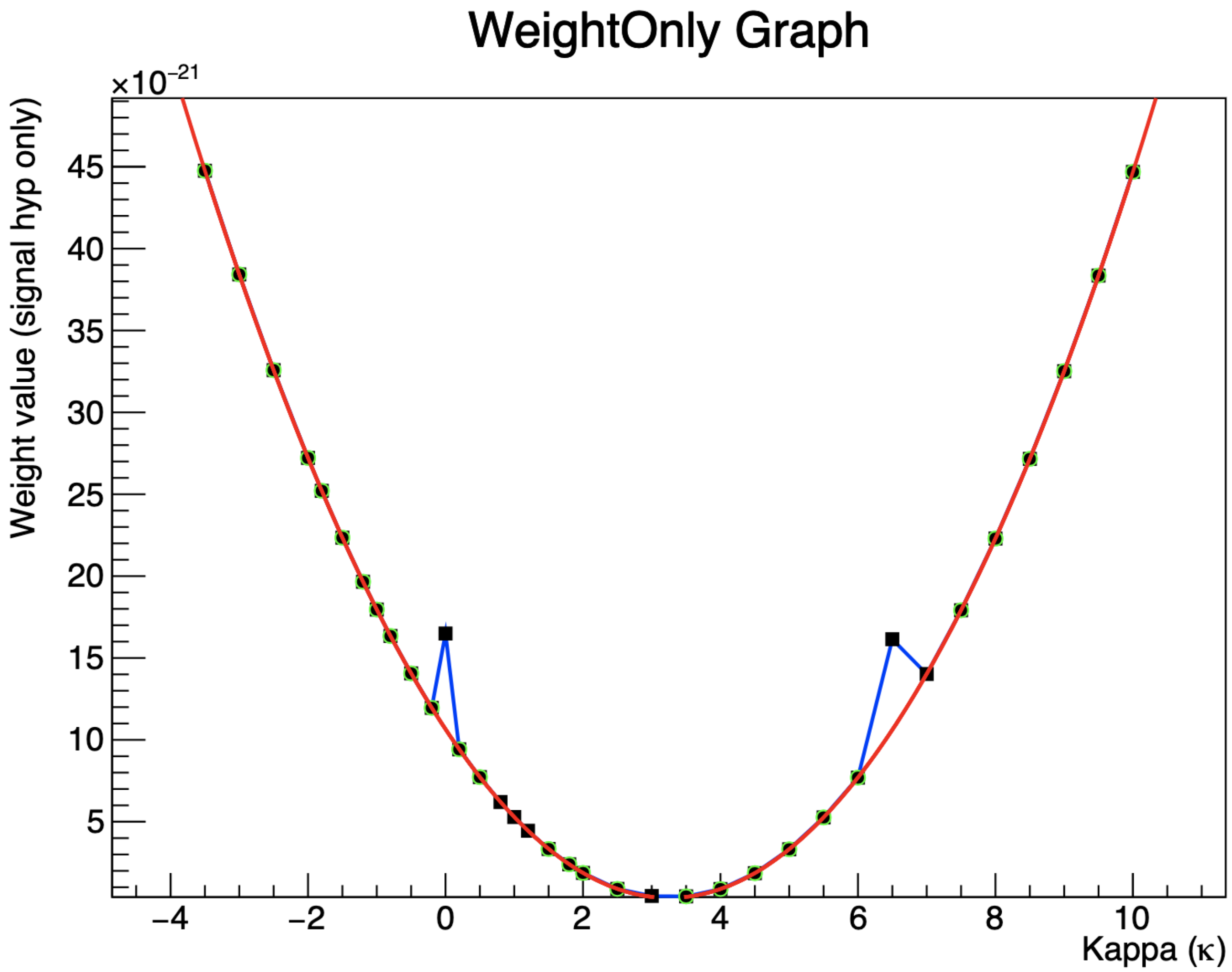
$$\mathcal{L}_{yield}(K) = e^{-(N_{bckg} + N_{sig}(K))} \cdot \frac{(N_{bckg} + N_{sig}(K))^{N_{obs}}}{N_{obs}!}$$

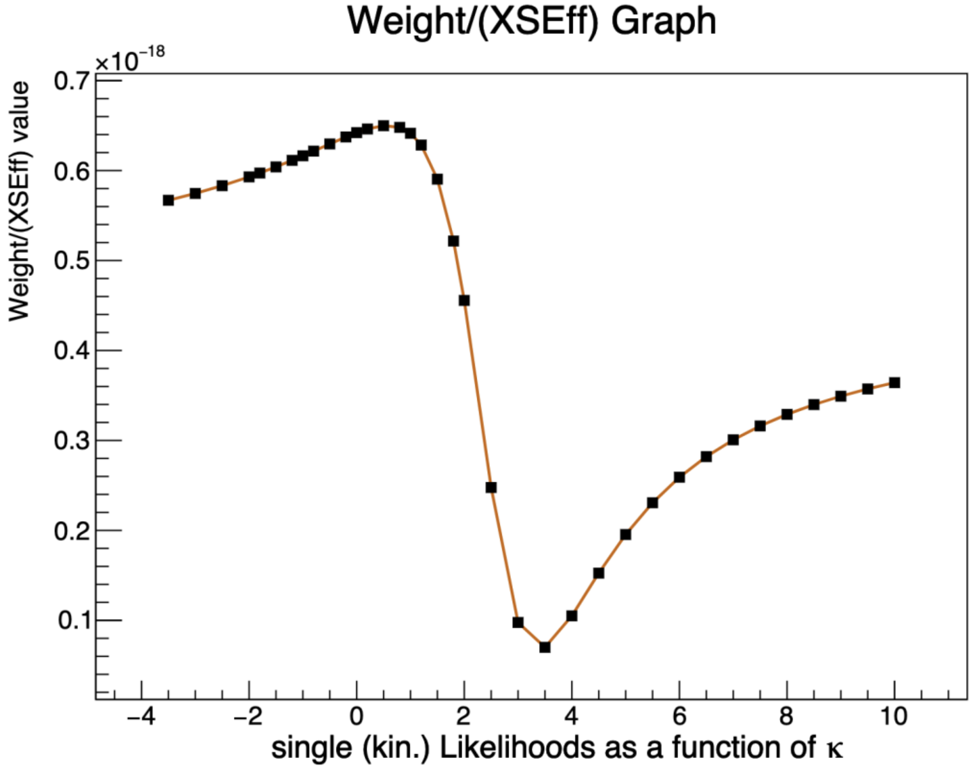
where: N_{bckg} is the expected number of event for all background, given the luminosity L.

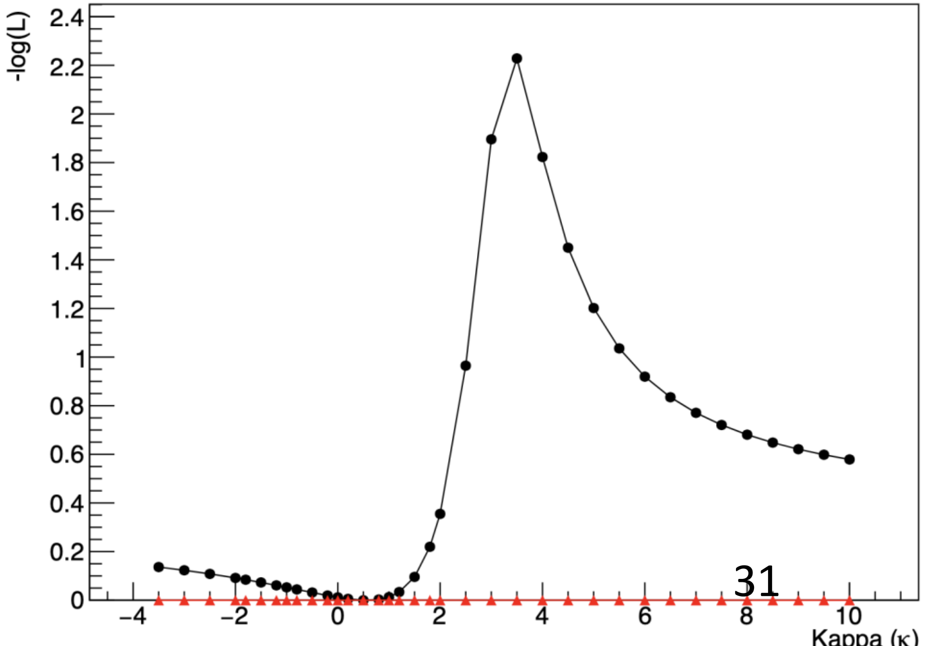
$N_{sig}(K)$ is the expected number of event for the signal given the value of K(-MEM) and luminosity L.

N_{obs} is the number of events to be observed.

APPENDIX: Evolution of weight wrt κ : Example (ggF $\kappa = 1.00$):



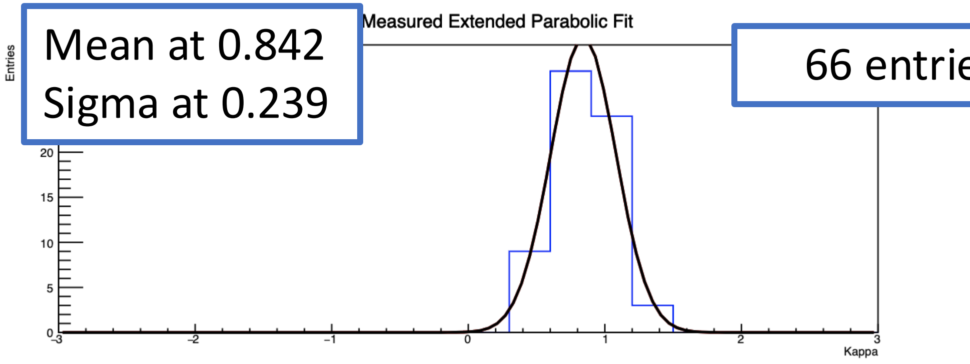


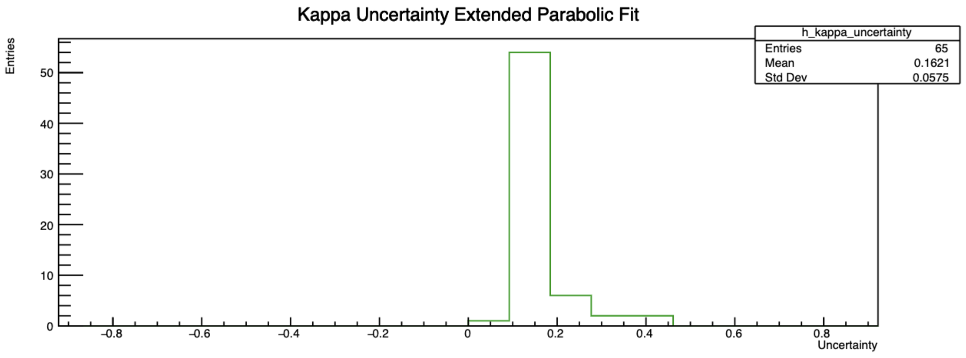


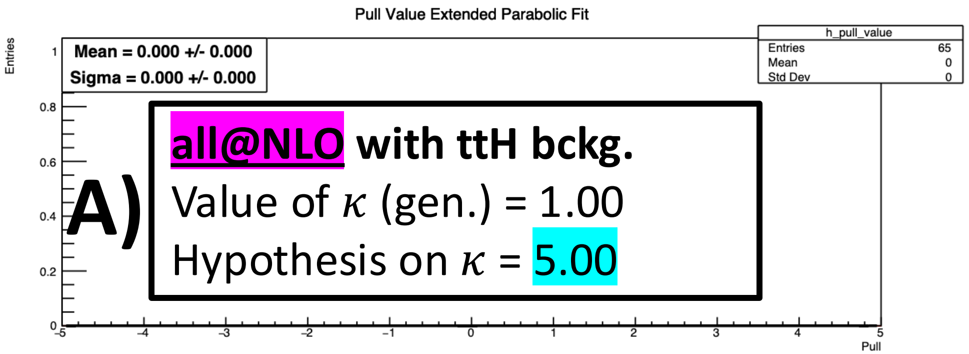
APPENDIX: Strenght of the MEM: Different tests (part 1/2)

Mean at 0.842
Sigma at 0.239

66 entries





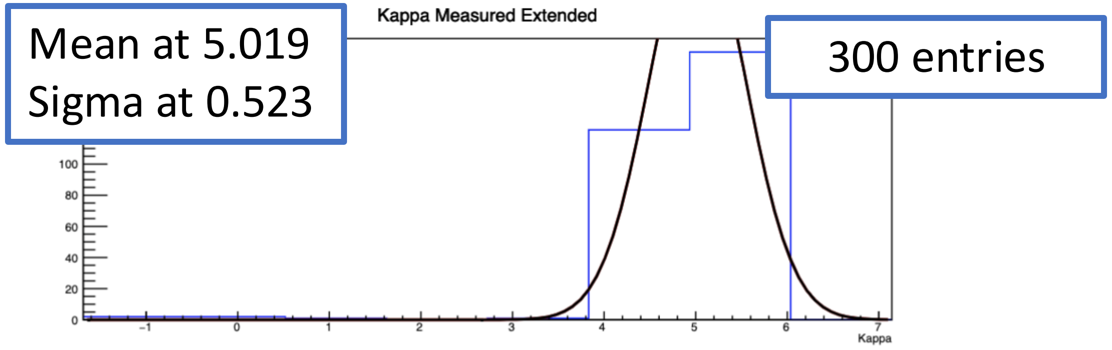


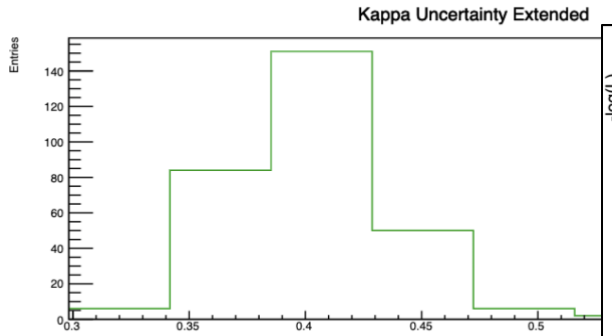
A)

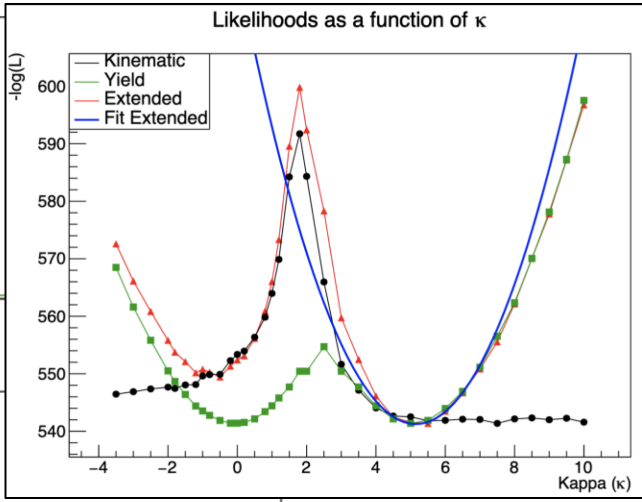
all@NLO with ttH bckg.
Value of κ (gen.) = 1.00
Hypothesis on κ = 5.00

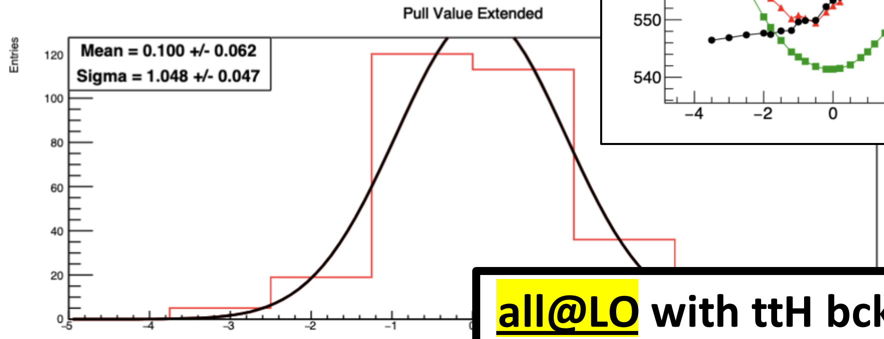
Mean at 5.019
Sigma at 0.523

300 entries









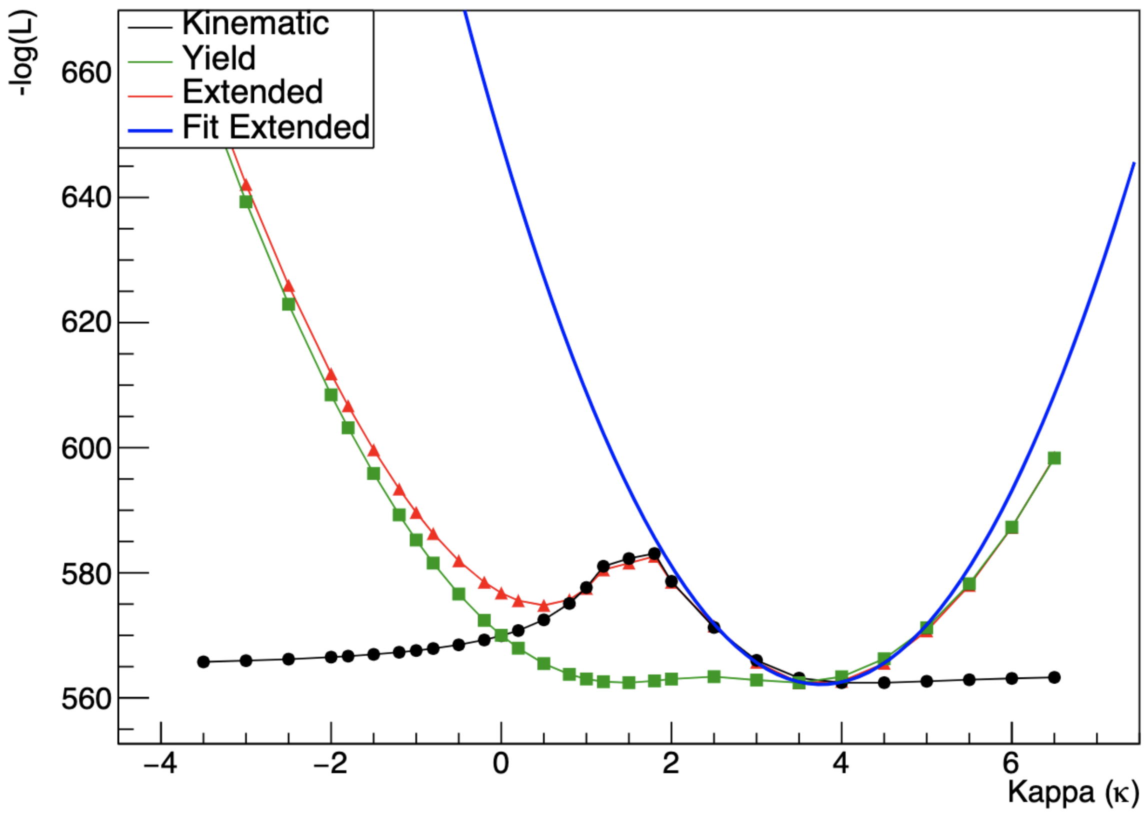
B)

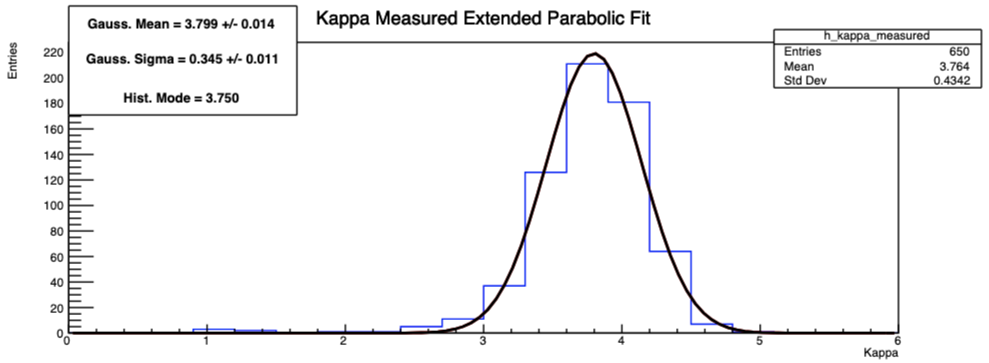
all@LO with ttH bckg.
Value of κ (gen.) = 5.00
Hypothesis on κ = 5.00

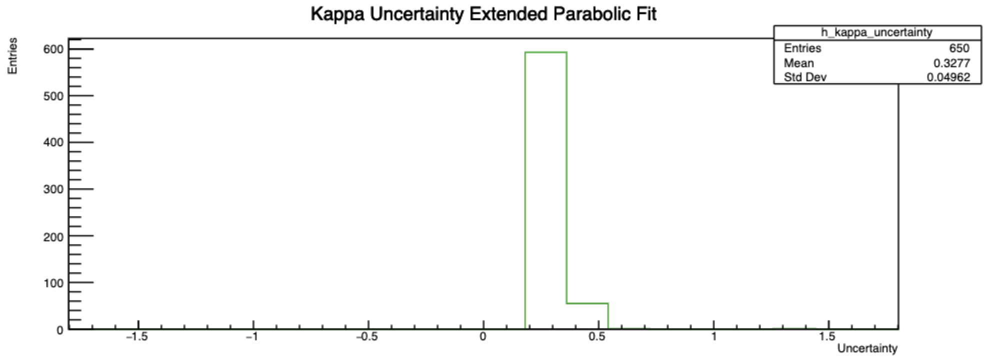
APPENDIX: Strenght of the MEM: Different tests (part 2/2)

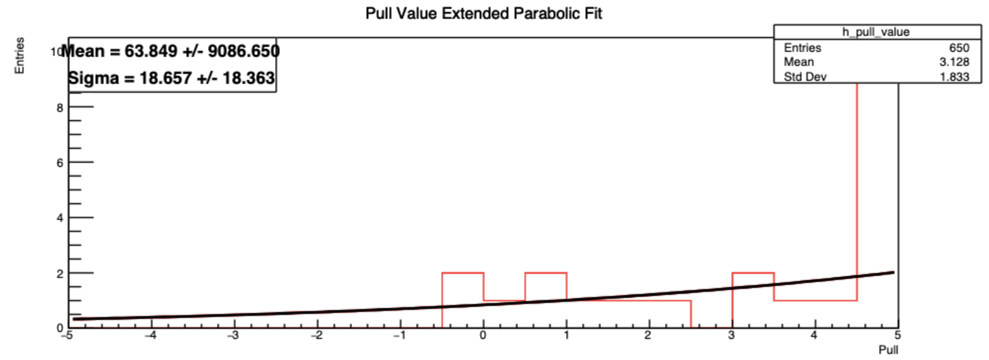
C) all@NLO « signal only test » but events are all background ttH only.
Value of κ (gen.) = none
Hypothesis on κ = 1.00

Likelihoods as a function of κ









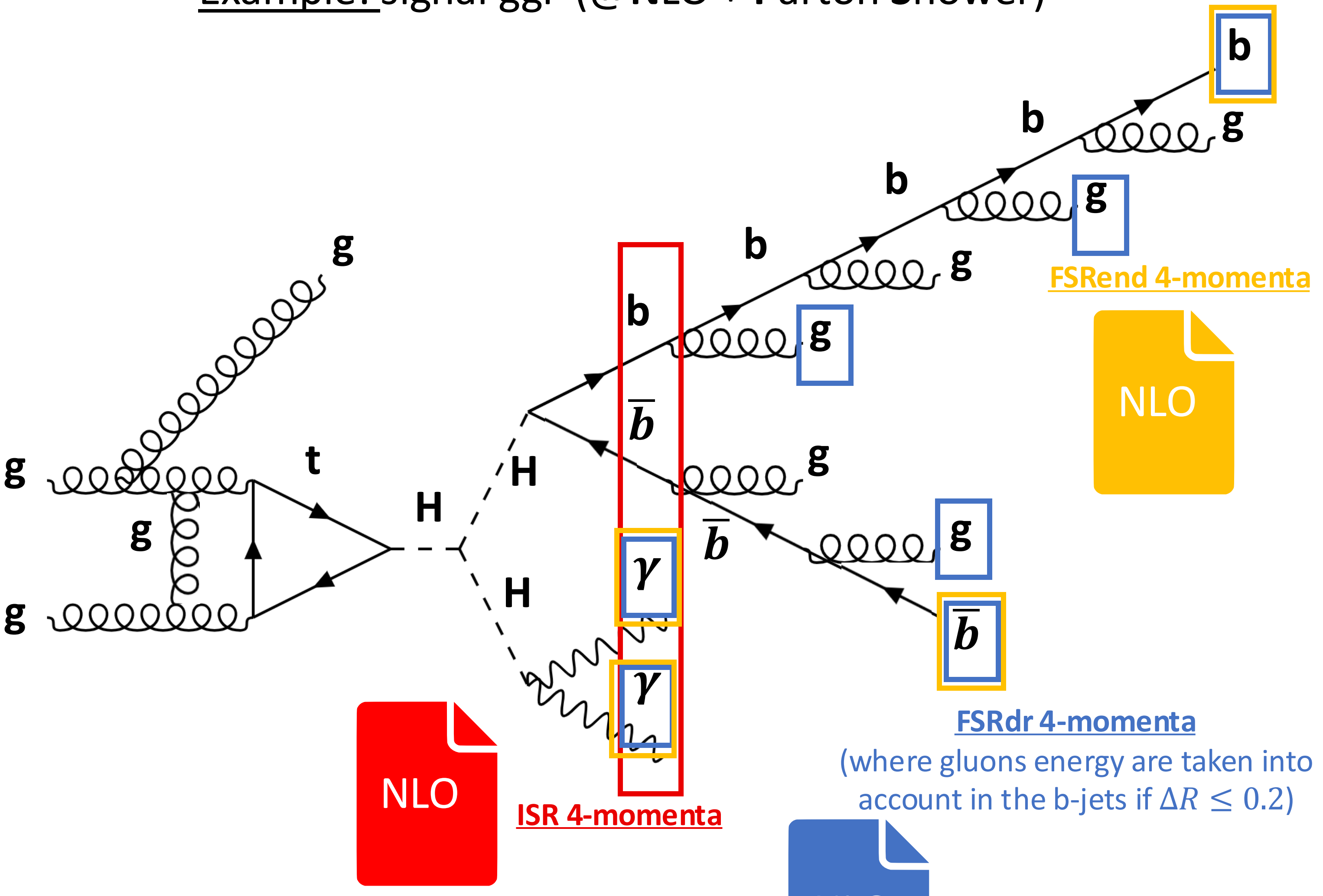
APPENDIX: MEM Degrees Of Freedom and choices for our analysis

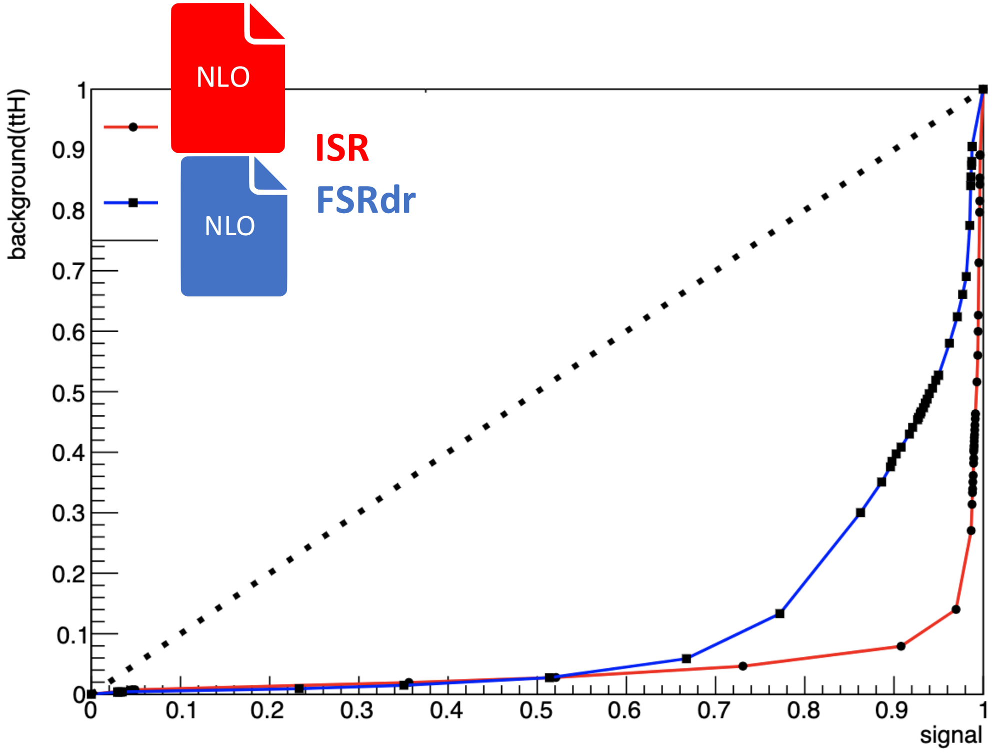
Process	Dimension of integration	Variables of integration (not mandatory choice)
ggF Di-Higgs @LO+Virtual	**2** (-> 0)	$(H_{2,Width})\ ;\ \gamma_{1,E}$
ttH @LO	**9** (-> 6)	$(H_{,Width},\ top_{1,Width},\ top_{2,Width});\ \text{perm}(b_3,b_4);\ \gamma_{1,E}\ ; b_{3,E}\ ;\ b_{4,E};\ q_1\ ;\ q_2$
qcd @LO	**2**	$\gamma_{1,E}\ ;\ \gamma_{2,E}$
ggF Di-Higgs@NLO_Real	5 (-> **3**)	$(H_{1,Width}\ ;\ H_{2,Width})\ ;\ \gamma_{1,E}\ ;\ b_{3,E}\ ;\ g_{pz}$
ttH @NLO_Real	12 (-> **9**)	[LO] + $g_{px}\ ;\ g_{py}\ ;\ g_{pz}$
qcd @NLO_Real	**5**	$\gamma_{1,E}\ ;\ \gamma_{2,E}\ ;\ b_{3,E}\ ;\ b_{4,E}\ ;\ g_{pz}$

As far as we know: Never been done

APPENDIX: NLO eventS, different meanings (impact of Parton shower)

- Example: signal ggF (@**N**LO + **P**arton **S**hower)





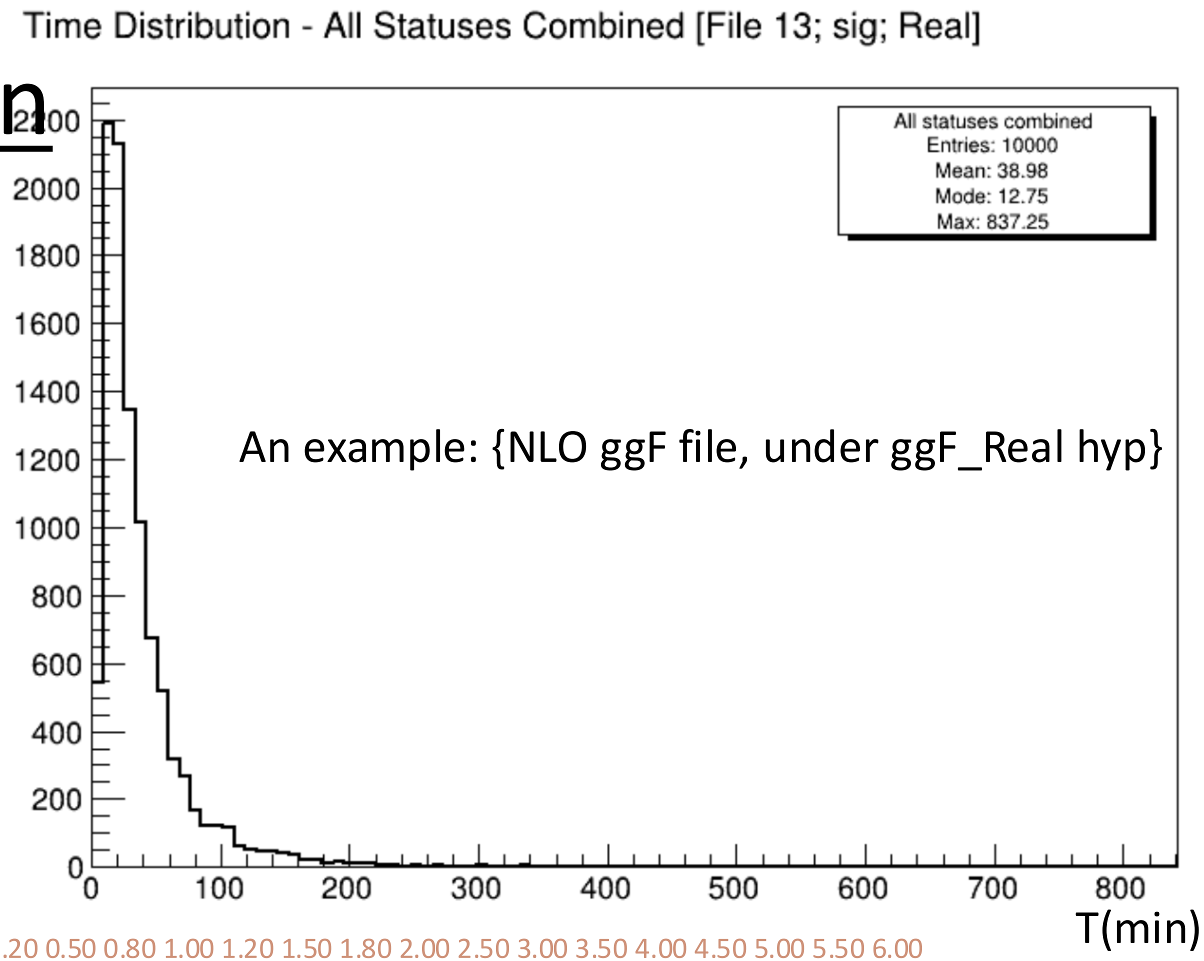
APPENDIX: Time of computation

10 000evts each

ggF | ttH | qcd | single Higgs

10 000evts each

ggF | ttH | qcd | single Higgs



An example: {NLO ggF file, under ggF_Real hyp}

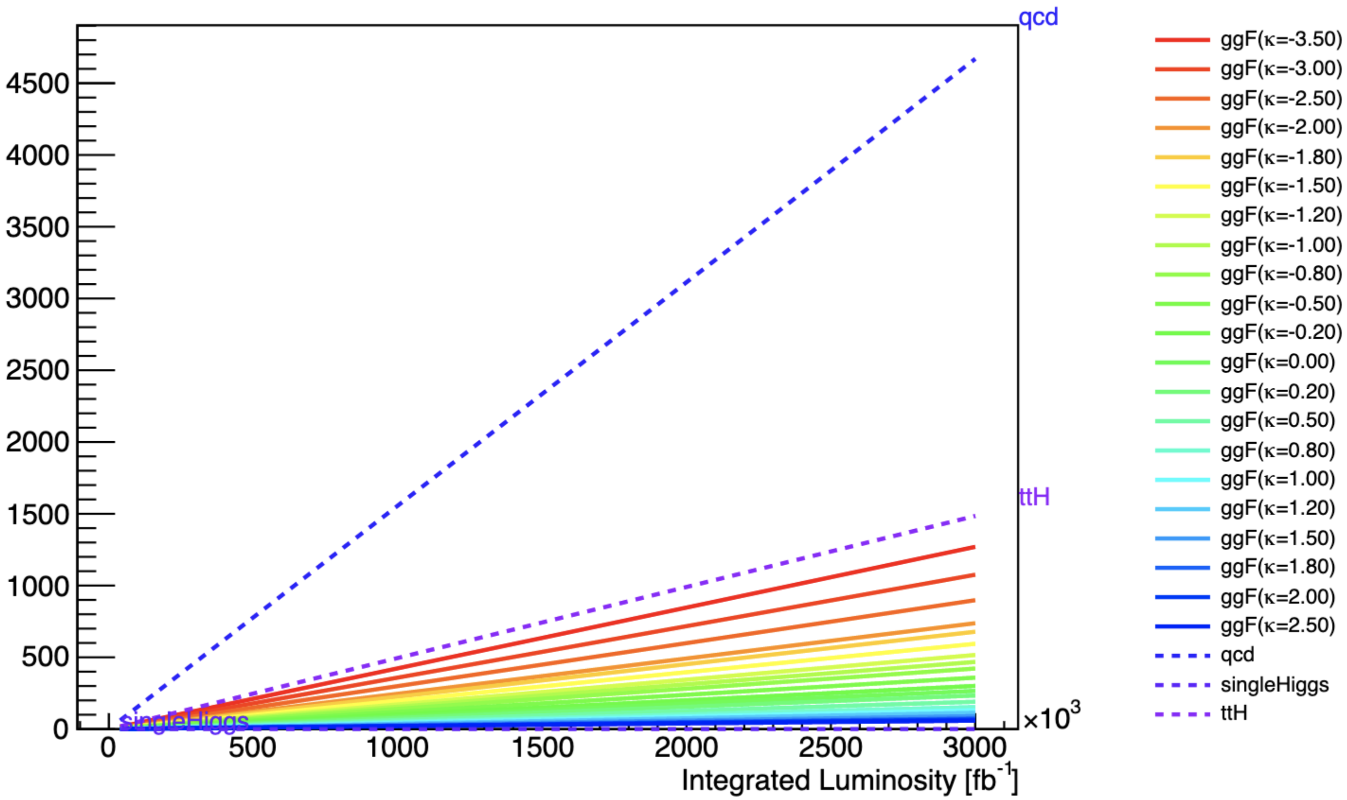
Kappa hypothesis = "-3.50 -3.00 -2.50 -2.00 -1.80 -1.50 -1.20 -1.00 -0.80 -0.50 -0.20 0.00 0.20 0.50 0.80 1.00 1.20 1.50 1.80 2.00 2.50 3.00 3.50 4.00 4.50 5.00 5.50 6.00 6.50 7.00 7.50 8.00 8.50 9.00 9.50 10.00 » x2 (BV and Real)

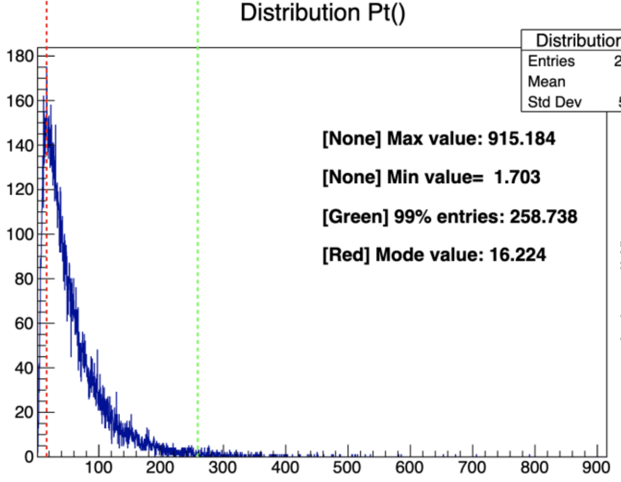
ttH hypothesis ;
qcd hypothesis;
singleHiggs hypothesis;
Total: 36*2 + 3 hypothesis (for NLO) = 75.

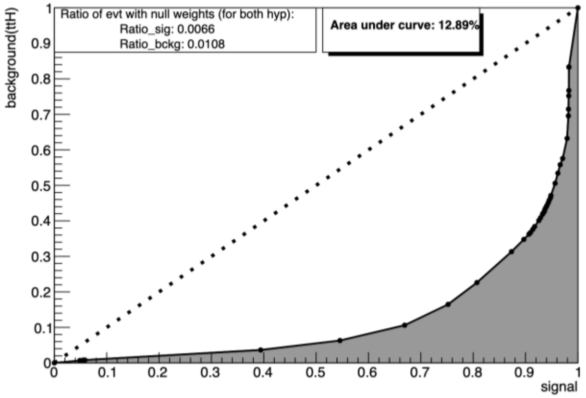
MEM Computation time: From minutes to days for a single given event (it varies depending on the process and hypothesis chosen of course).

APPENDIX: Impact of cut choices

Expected number of Events vs Integrated Luminosity (for our choices of cuts: ATLAS pre-selection classics +/-):







Impact of choice of cut on pt (min) for the NLO radiation: