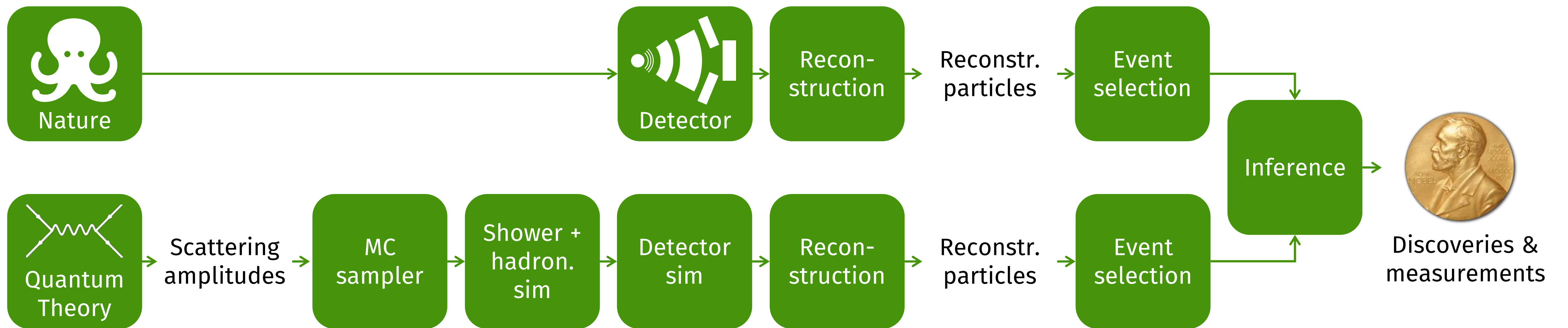
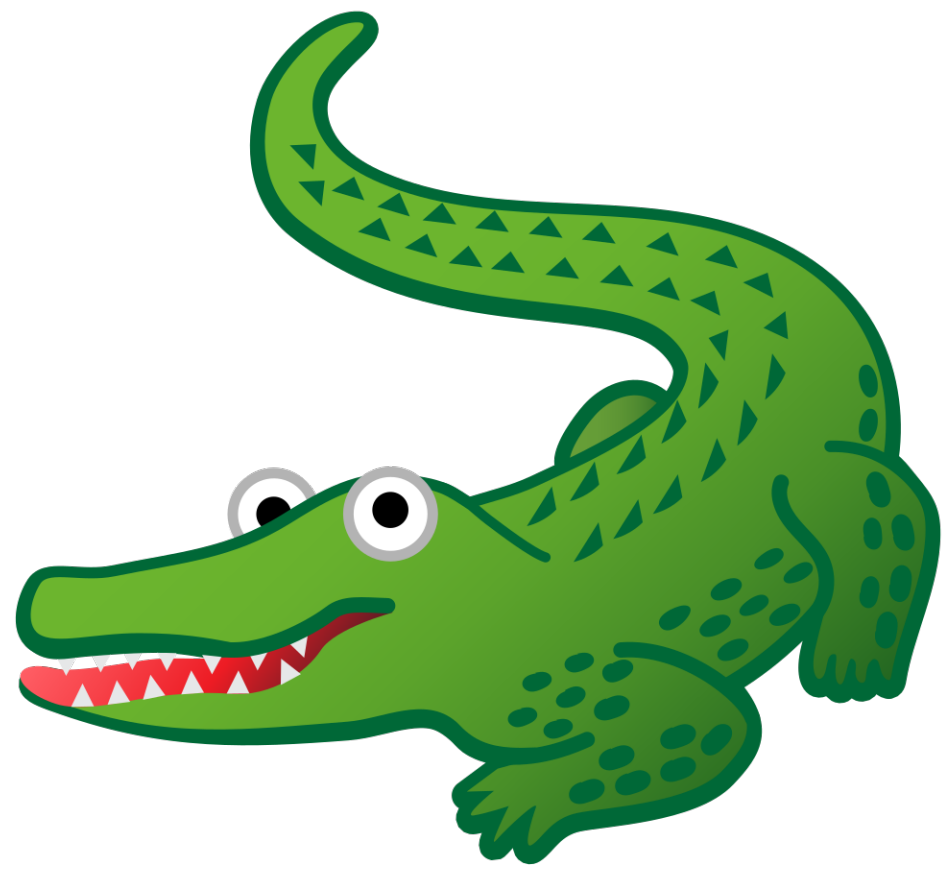






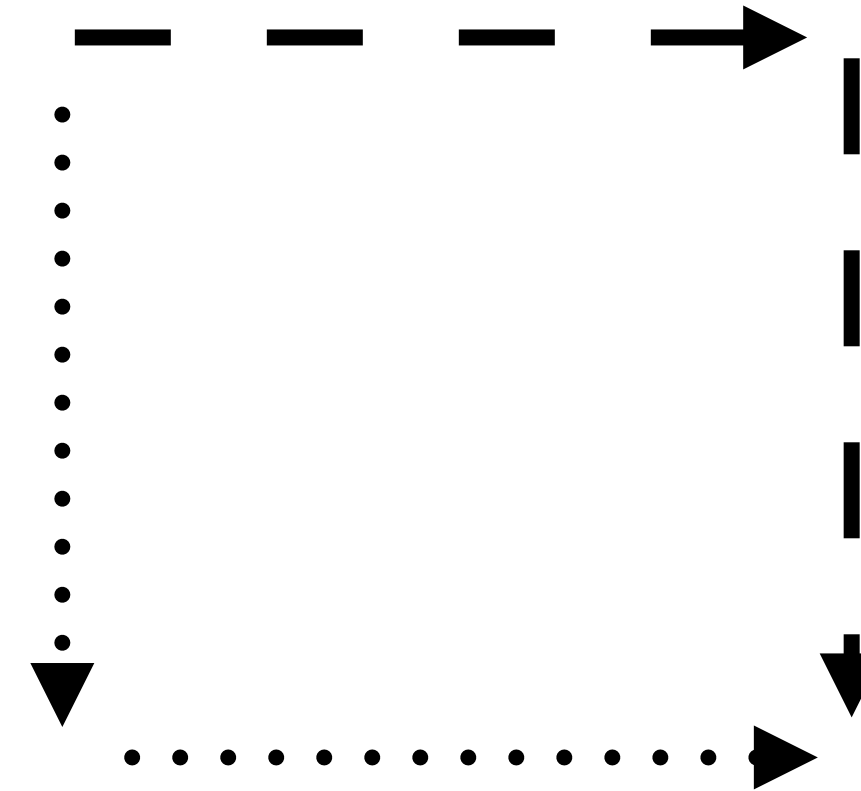
Lorentz-Equivariant
Geometric **A**lgebra
Transformer

=

$$\begin{pmatrix} 1 \\ \gamma^\mu \\ \sigma^{\mu\nu} \\ \gamma^\mu \gamma_5 \\ \gamma_5 \end{pmatrix}$$

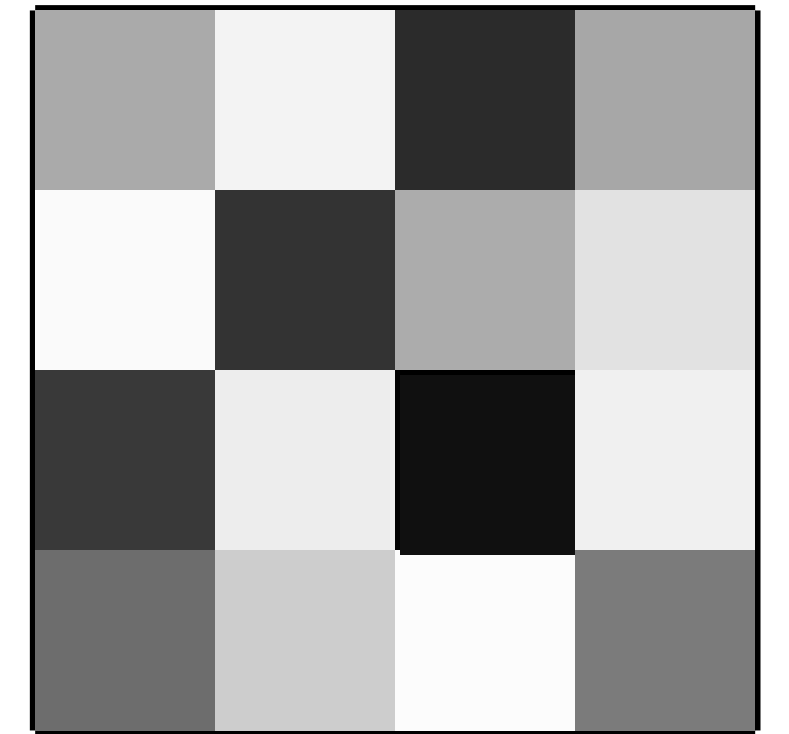
Geometric algebra
representations

+



Lorentz-Equivariant
layers

+



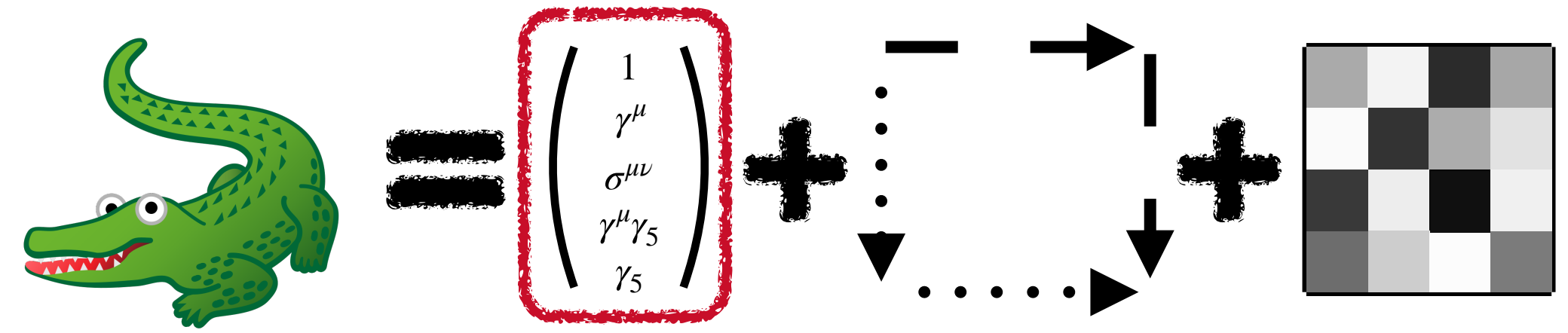
Transformer
architecture

GATr was originally
developed for E(3)
arXiv:2305.18415

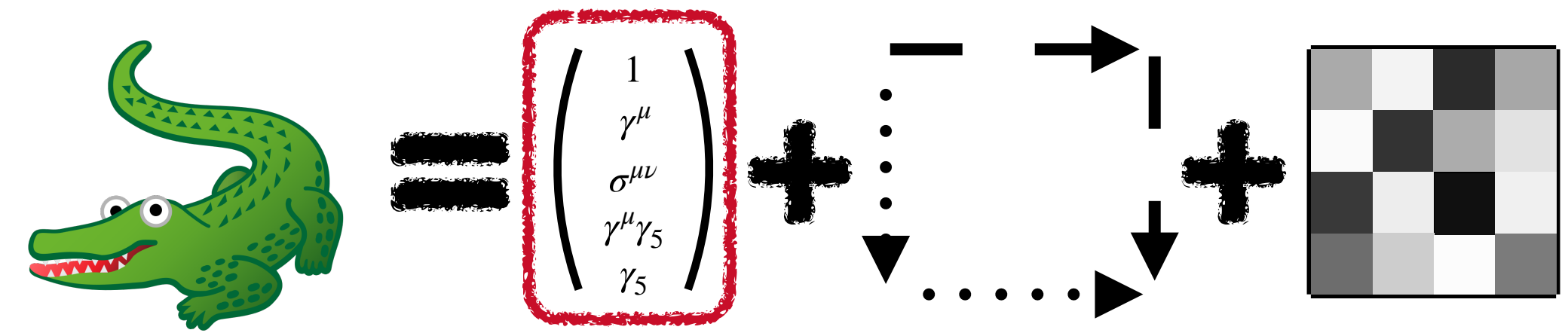
L-GATr

Geometric algebra = Clifford algebra

Geometric algebra = Vector space + geometric product $xy = \frac{\{x, y\}}{2} + \frac{[x, y]}{2}$



L-GATr



Geometric algebra = Clifford algebra

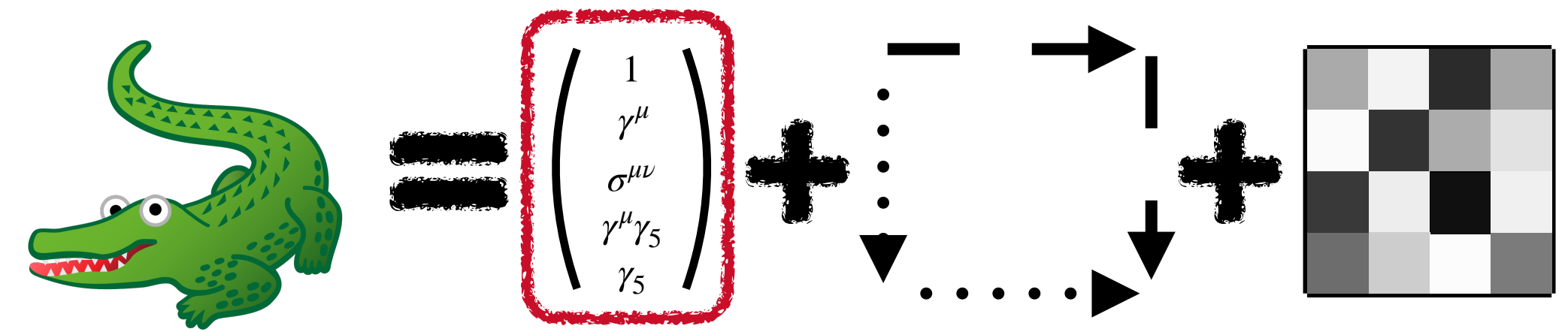
Geometric algebra = Vector space + geometric product $xy = \frac{\{x, y\}}{2} + \frac{[x, y]}{2}$

Spacetime geometric algebra: Geometric algebra over vector space $\mathbb{R}^4$ with Minkowski metric $g = \text{diag}(1, -1, -1, -1)$

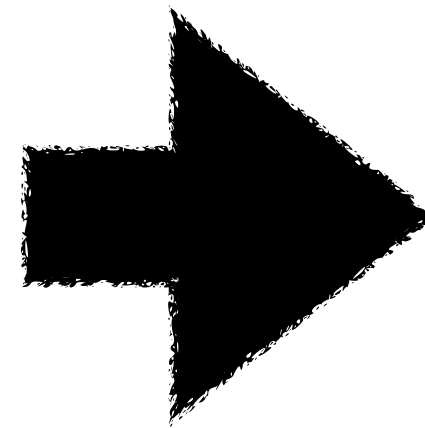
- Basis elements γ^μ are orthonormal: $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$
- Dirac algebra is the same up to $\mathbb{R} \rightarrow \mathbb{C}$

L-GATr

Geometric algebra representations



$$x^S \in \mathbb{R}$$



$$\begin{pmatrix} x^S \\ x_\mu^V \\ x_{\mu\nu}^B \\ x_\mu^A \\ x^P \end{pmatrix} \in \mathbb{R}^{16}$$

scalar channels

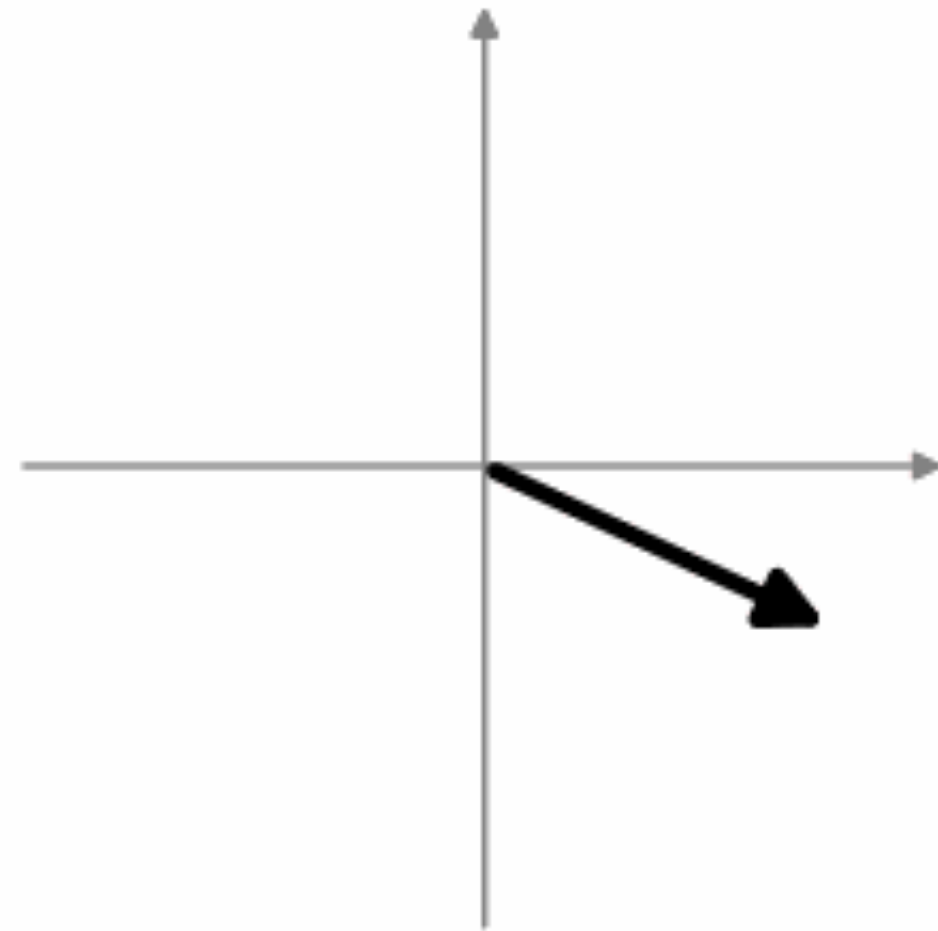
multivector channels

L-GATr

Invariance

$$\text{Crocodile} = \begin{pmatrix} 1 \\ \gamma^\mu \\ \sigma^{\mu\nu} \\ \gamma^\mu \gamma_5 \\ \gamma_5 \end{pmatrix} + \begin{matrix} \text{Diagram: A square with arrows forming a cycle (top: left to right, right: top to bottom, bottom: right to left, left: bottom to top) with a vertical ellipsis in the center. The square is outlined in red.} \\ \end{matrix} + \begin{matrix} \text{Diagram: A 4x4 checkerboard pattern of black and white squares.} \\ \end{matrix}$$

Input



Output
(not invariant)

0.8



Output
(invariant)

0.8

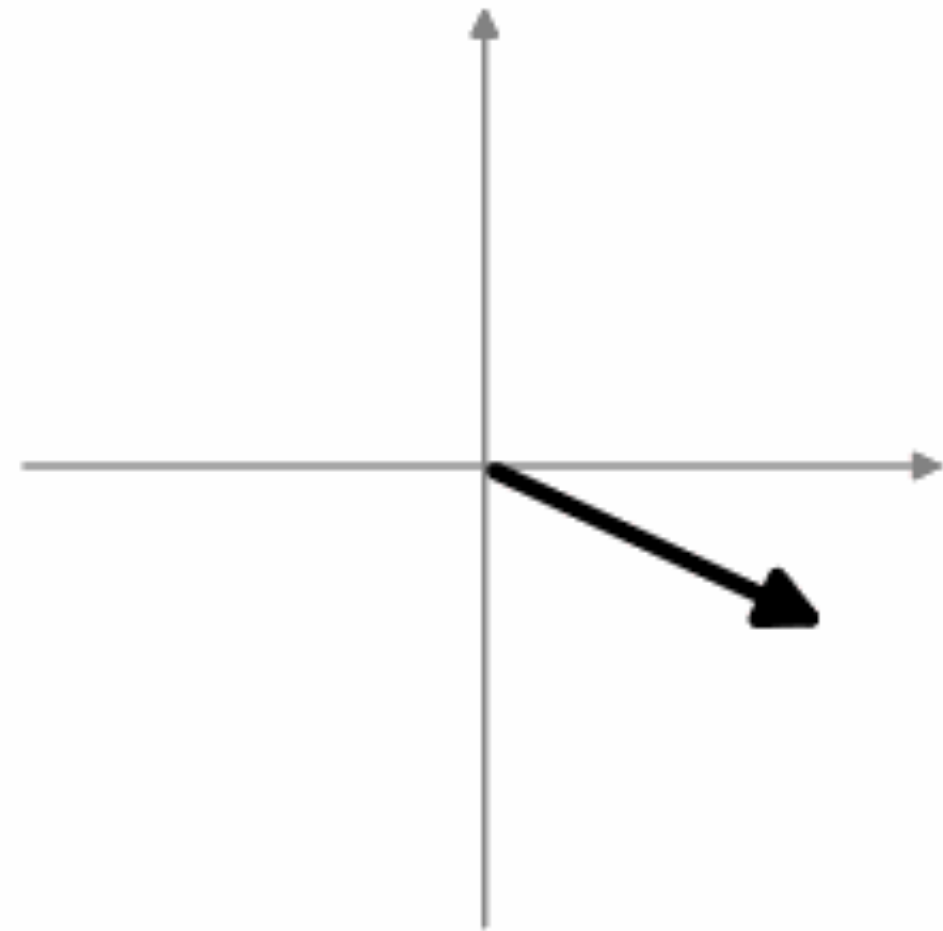


L-GATr

Invariance

$$\text{Crocodile} = \begin{pmatrix} 1 \\ \gamma^\mu \\ \sigma^{\mu\nu} \\ \gamma^\mu \gamma_5 \\ \gamma_5 \end{pmatrix} + \begin{matrix} \text{Diagram: A square with arrows forming a cycle (top: left to right, right: top to bottom, bottom: right to left, left: bottom to top) with a vertical ellipsis in the center. The square is outlined in red.} \\ \end{matrix} + \begin{matrix} \text{Diagram: A 4x4 checkerboard pattern of black and white squares.} \\ \end{matrix}$$

Input



Output
(not invariant)

0.8



Output
(invariant)

0.8



