ACCEL² simulation

Constraining growth rate from SNIa peculiar velocities and galaxy density field

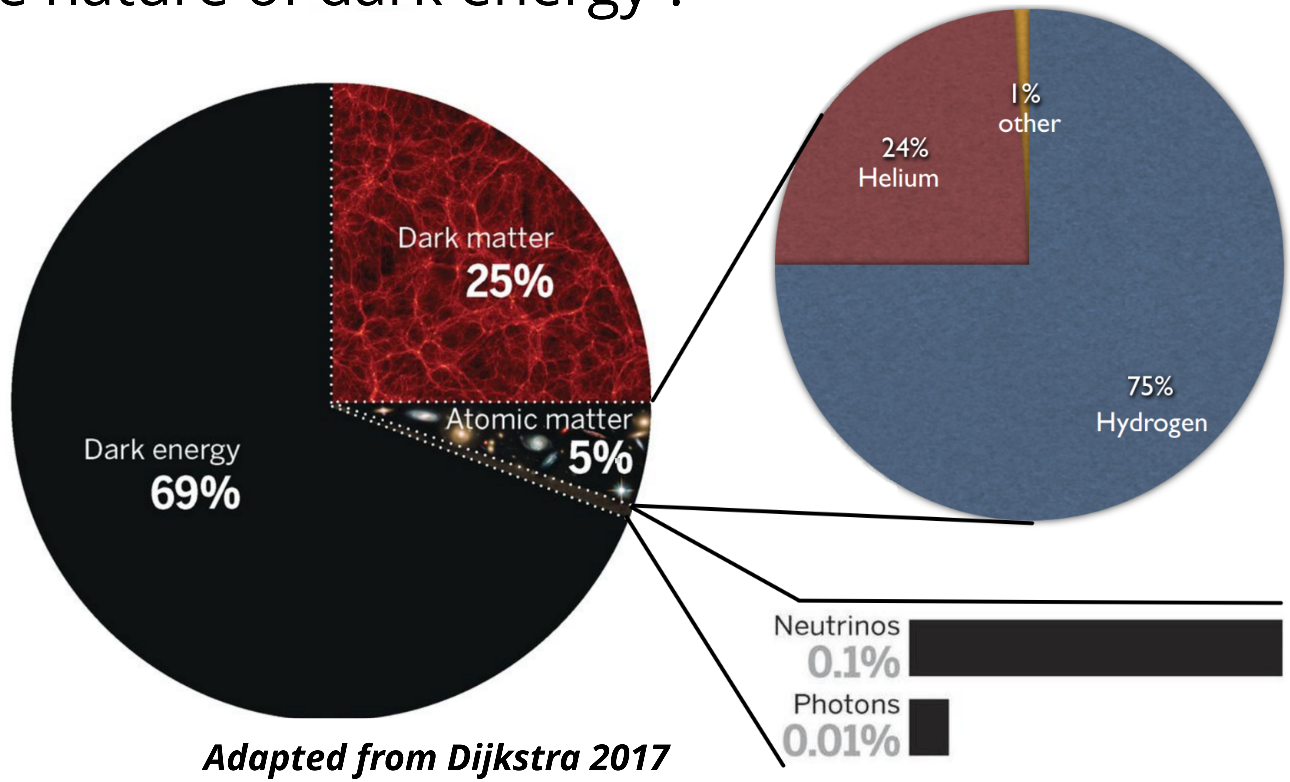
Corentin Ravoux - LPCA Clermont-Ferrand

with Bastien Carreres, Damiano Rosselli, Julian Bautista, Benjamin Racine, Dominique Fouchez, Fabrice Feinstein and many others

ADE & CoPhy webinar - 1 april 2025

Dark Energy

- What is the nature of dark energy ?

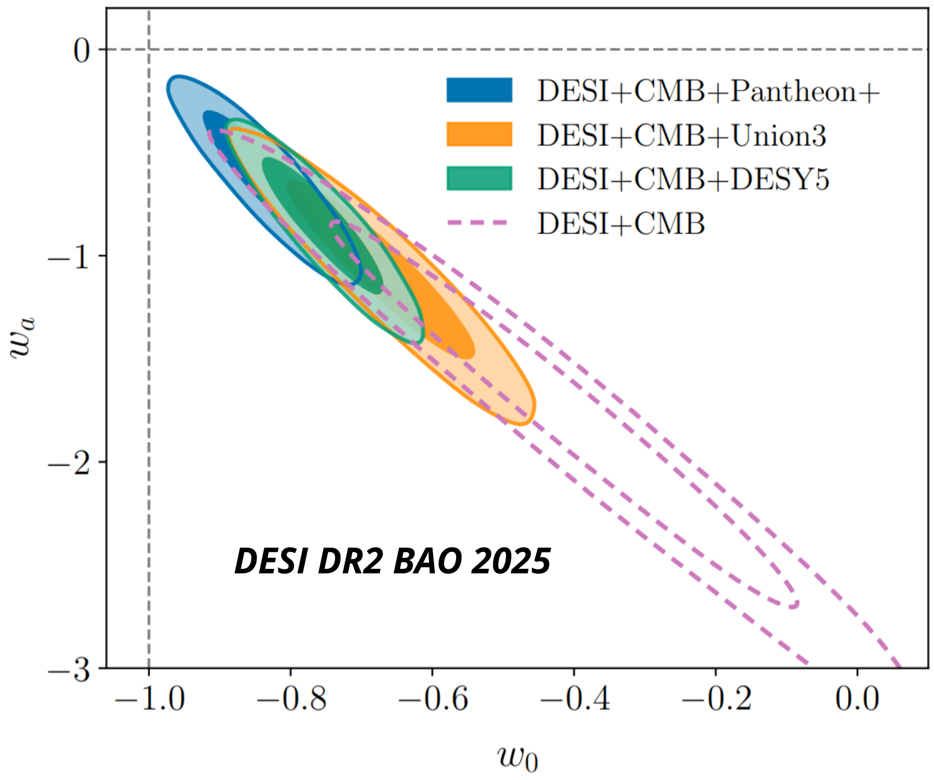


Adapted from Dijkstra 2017

Dark Energy

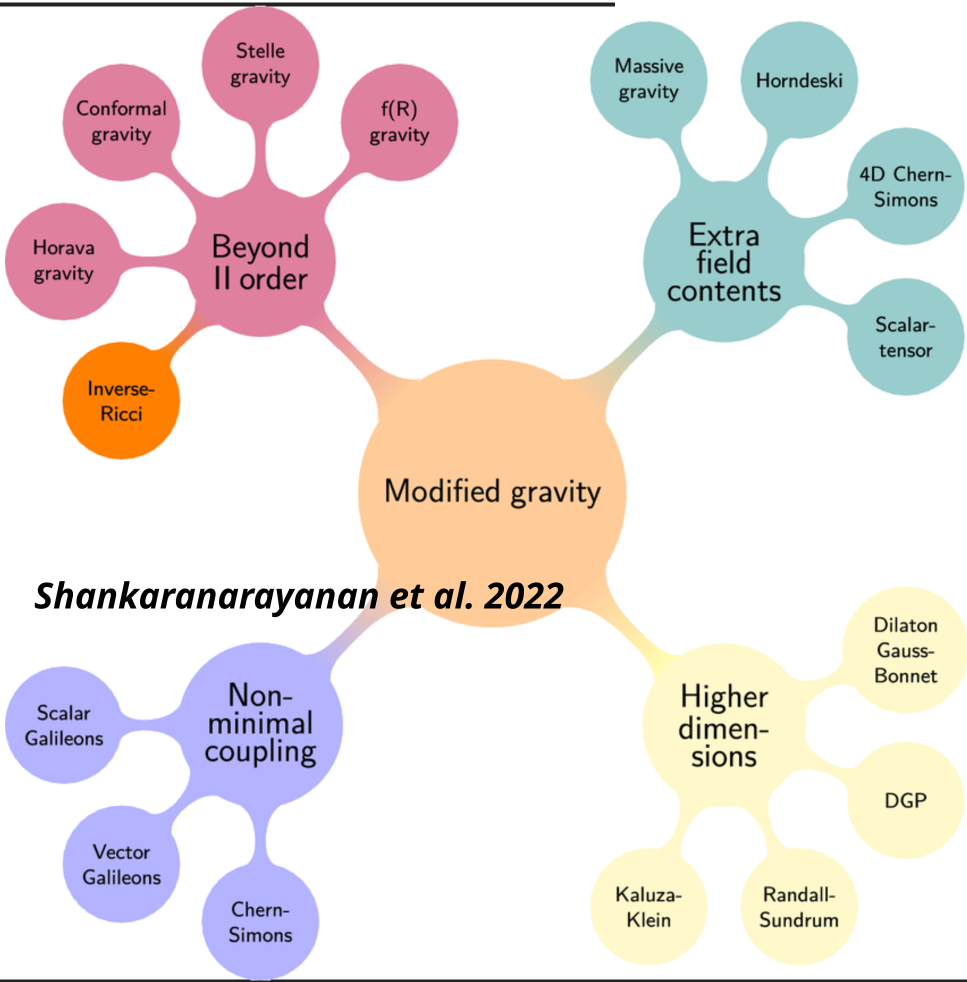
- What is the nature of dark energy ?
- Cosmological constant model (Λ) not very satisfying

Recent DESI DR2 BAO + CMB + SNIa results: stronger hints for varying dark energy



Dark Energy

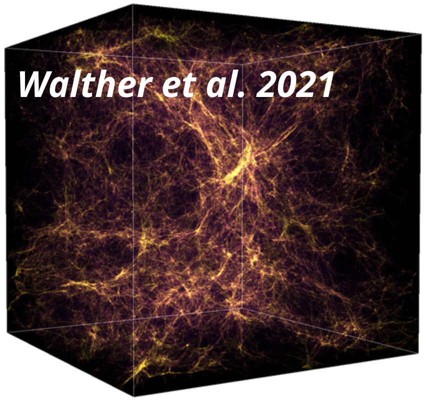
- What is the nature of dark energy ?
- Cosmological constant model (Λ) not very satisfying
- **Other models** :
  - Fifth force, additional field ?
  - Modified gravity theories ?
- **Current objective**: Probing large-scale structure formation to constrain dark energy/modified gravity.

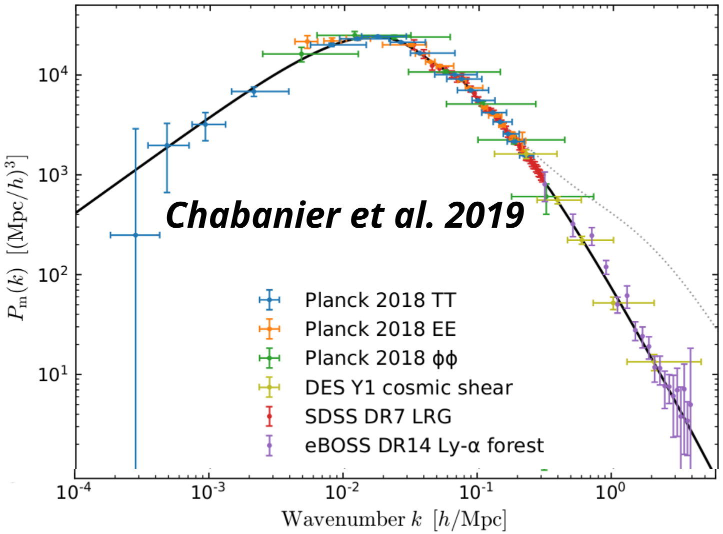


Shankaranarayanan et al. 2022

Cosmic web

- Matter perturbations in the primordial Universe grew to form the cosmic web
- **Static description**: matter density contrast (δ) follows a linear power spectrum at large-scales
- **Dynamic description**: Cosmic web peculiar velocities (v_p) caused by gravitation





$f\sigma_8$ parameter

- Linear perturbation theory:

Cosmic web velocities

Normalized matter overdensity

$$\nabla \cdot \mathbf{v}(\mathbf{r}) \propto -aH(f\sigma_8)\hat{\delta}(\mathbf{r})$$

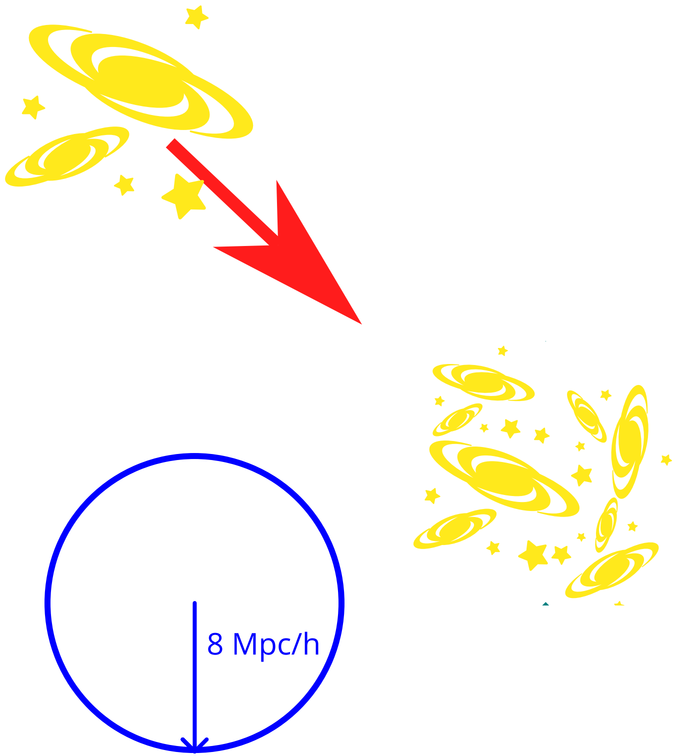
fσ8 parameter

- Linear perturbation theory:

Cosmic web velocities

Normalized matter overdensity

$$\nabla \cdot \mathbf{v}(\mathbf{r}) \propto -aH(f\sigma_8)\hat{\delta}(\mathbf{r})$$



Logarithmic growth rate of linear perturbations

$$f = \frac{d \log \delta_+}{d \log a} \simeq \Omega_{\rm m}^{0.55}$$ in GR

Amplitude of the matter perturbations in spheres of 8 Mpc/h comoving radius (matter power spectrum amplitude)

Measuring fσ8

Peculiar velocities (PV)	Redshift Space Distortions (RSD)

Peculiar velocities (PV)

- Direct estimation of velocities

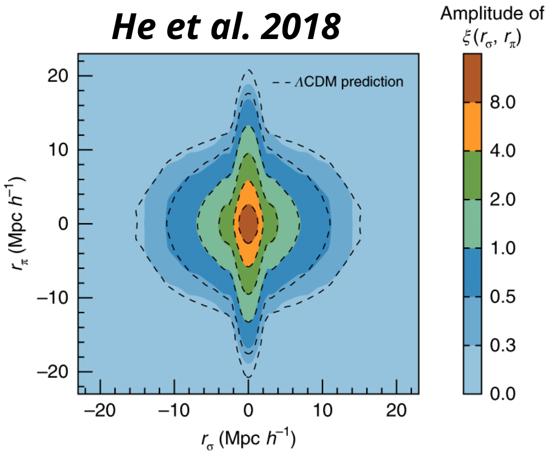
$$1 + z_{\rm obs} = (1 + z_{\rm cos})(1 + v_{\rm p}/c)$$

- Need an estimate of redshift ($z_{\rm obs}$) and distance (decoupling $z_{\rm cos}$ from $v_{\rm p}$)
- Compute statistics from PV

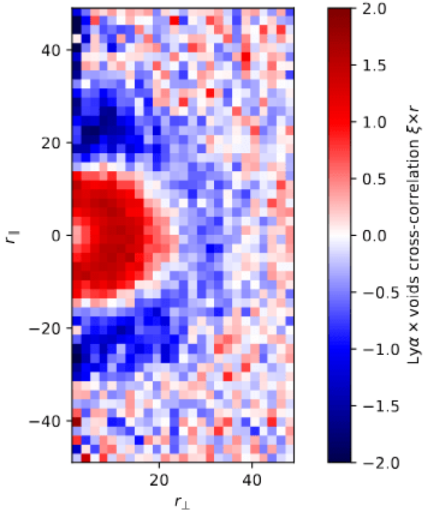
Redshift Space Distortions (RSD)

- Impact of PV on measurements
- **Example**: galaxy auto-correlation, void cross-correlation, ...

He et al. 2018

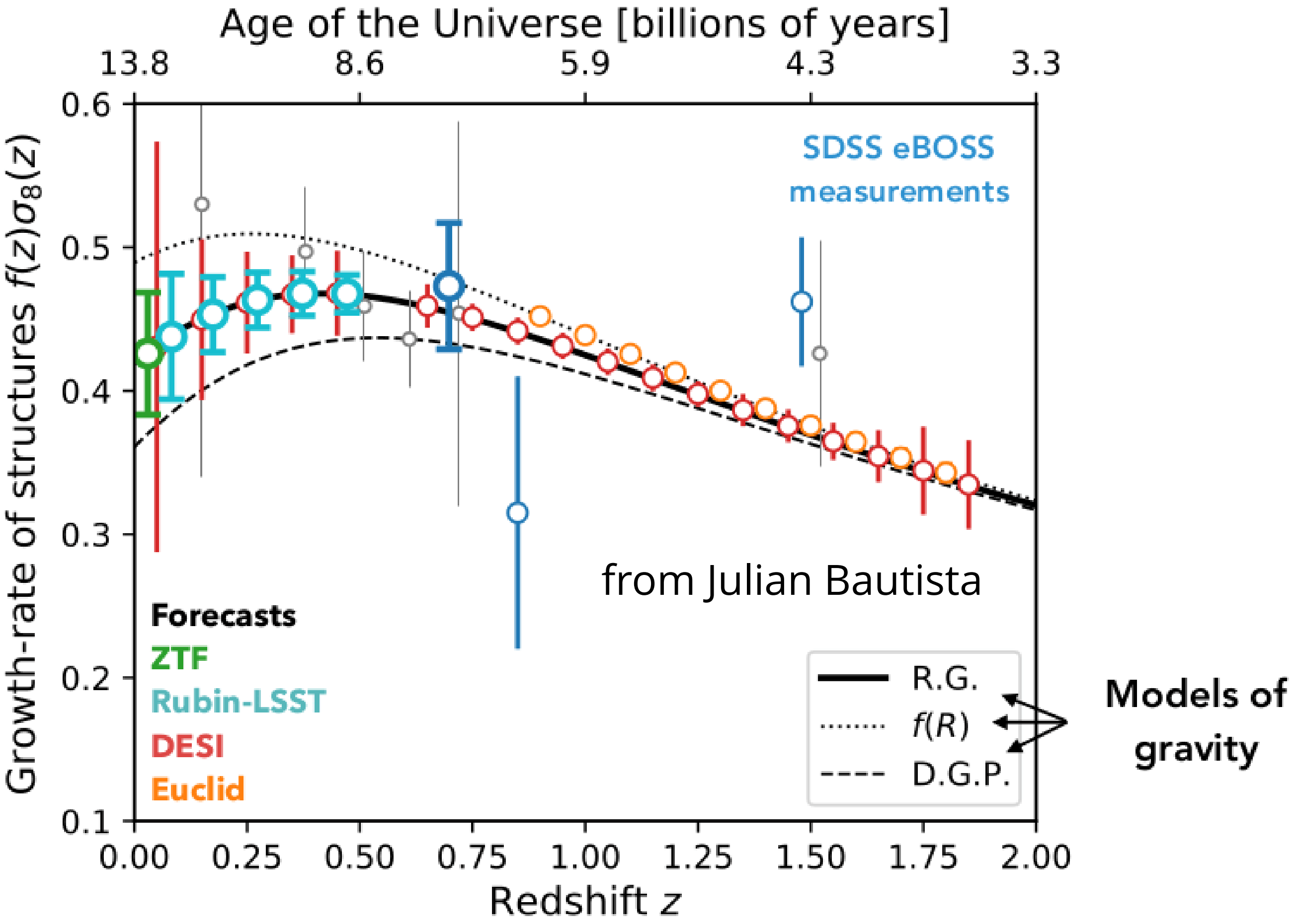


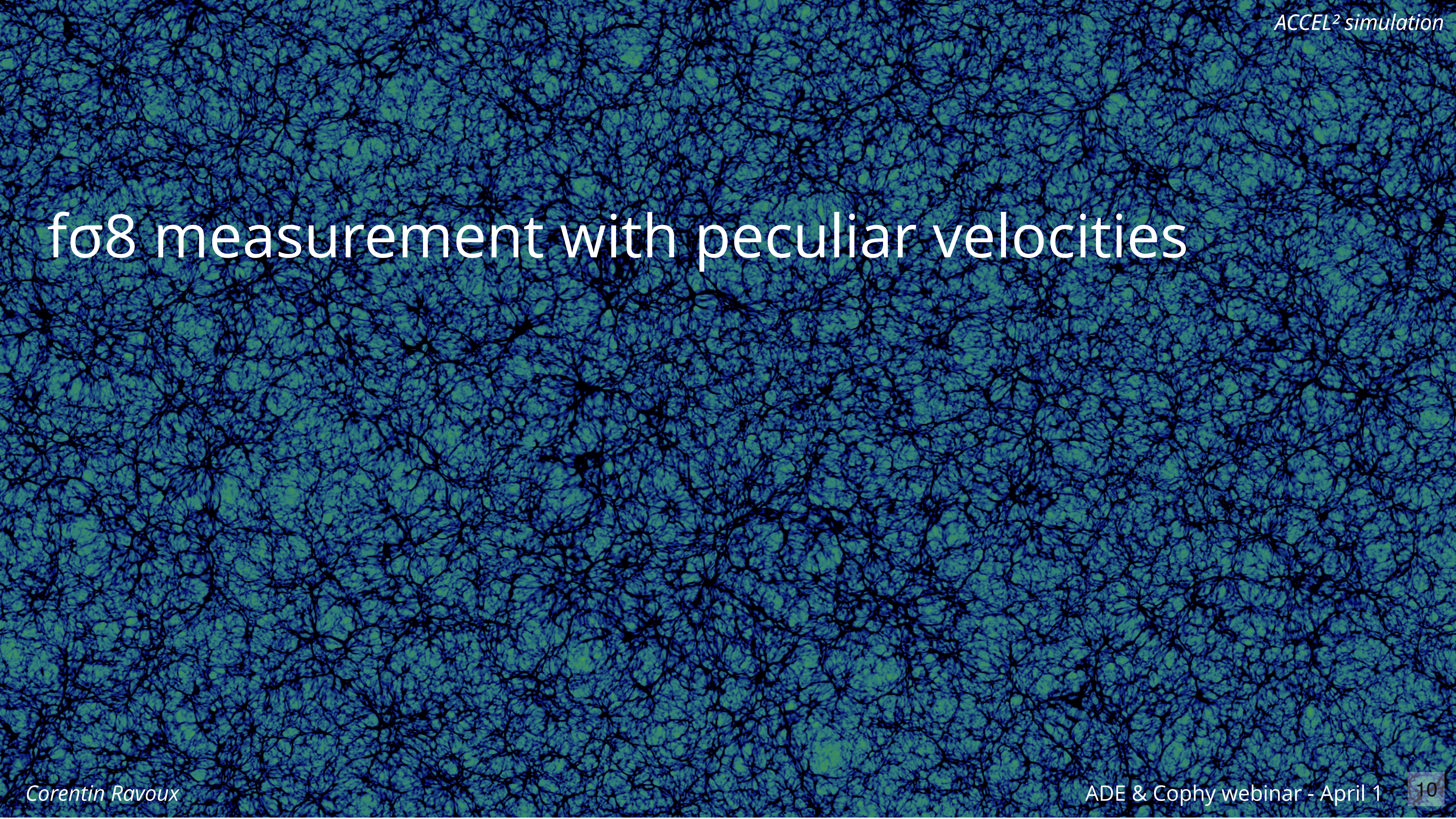
Ravoux et al. 2022



fσ8 constraints

- Constrain modified gravity models
- **RSD** very effective for high redshift
- **PV** for low redshift
- Improvement with method combination

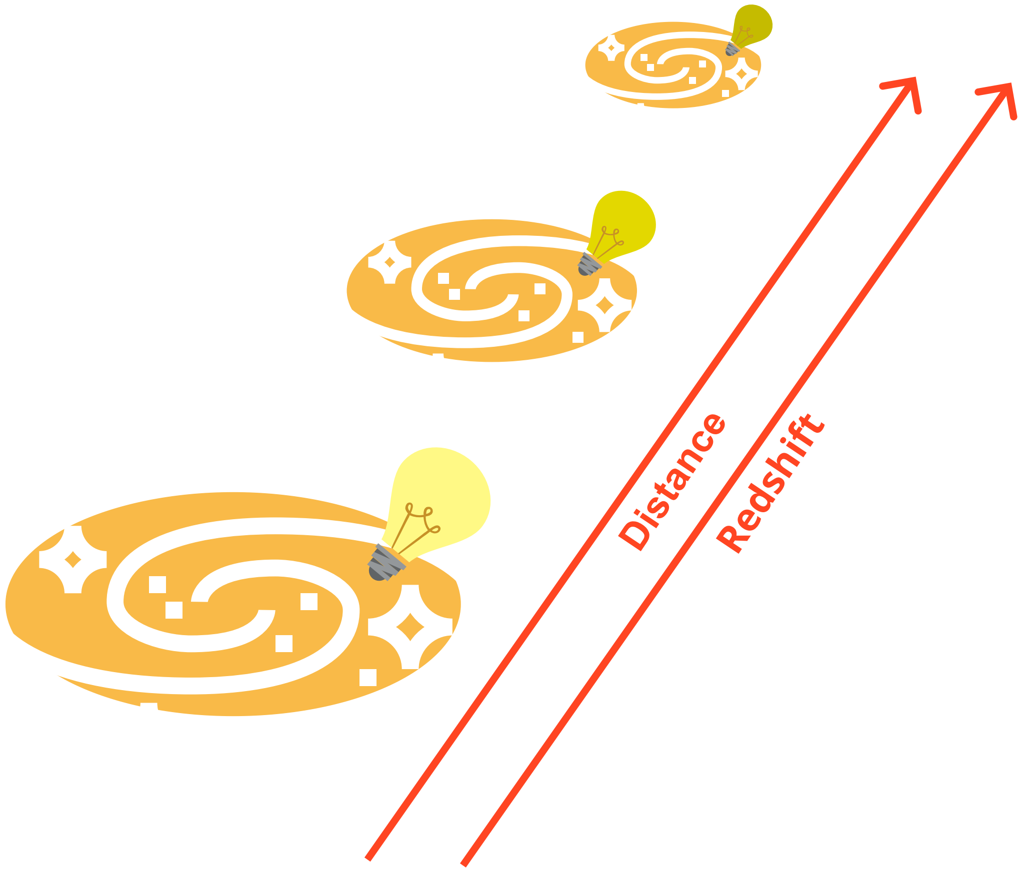



ACCEL² simulation

fσ8 measurement with peculiar velocities

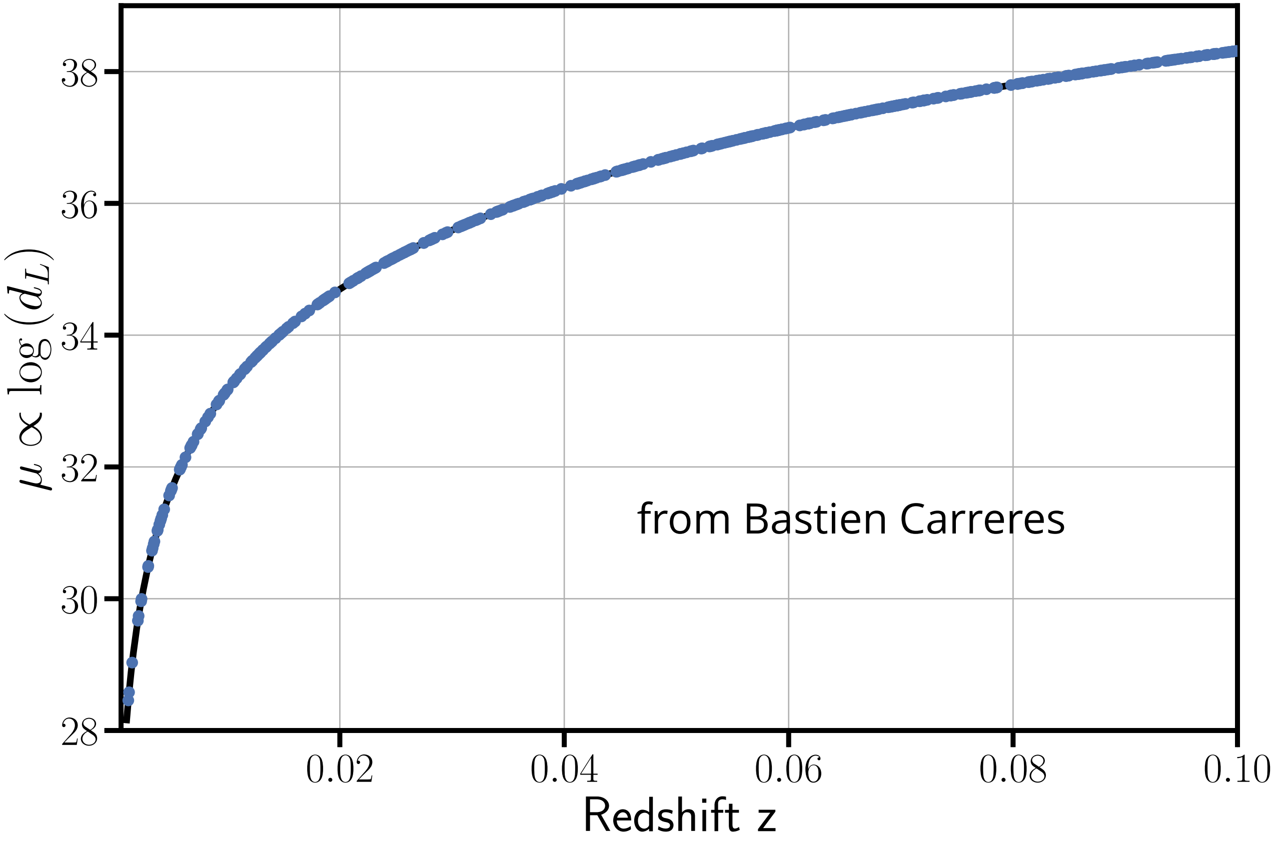
Hubble diagram

- **Standard candle** = object with fixed luminosity
- Luminosity + Redshift gives distance



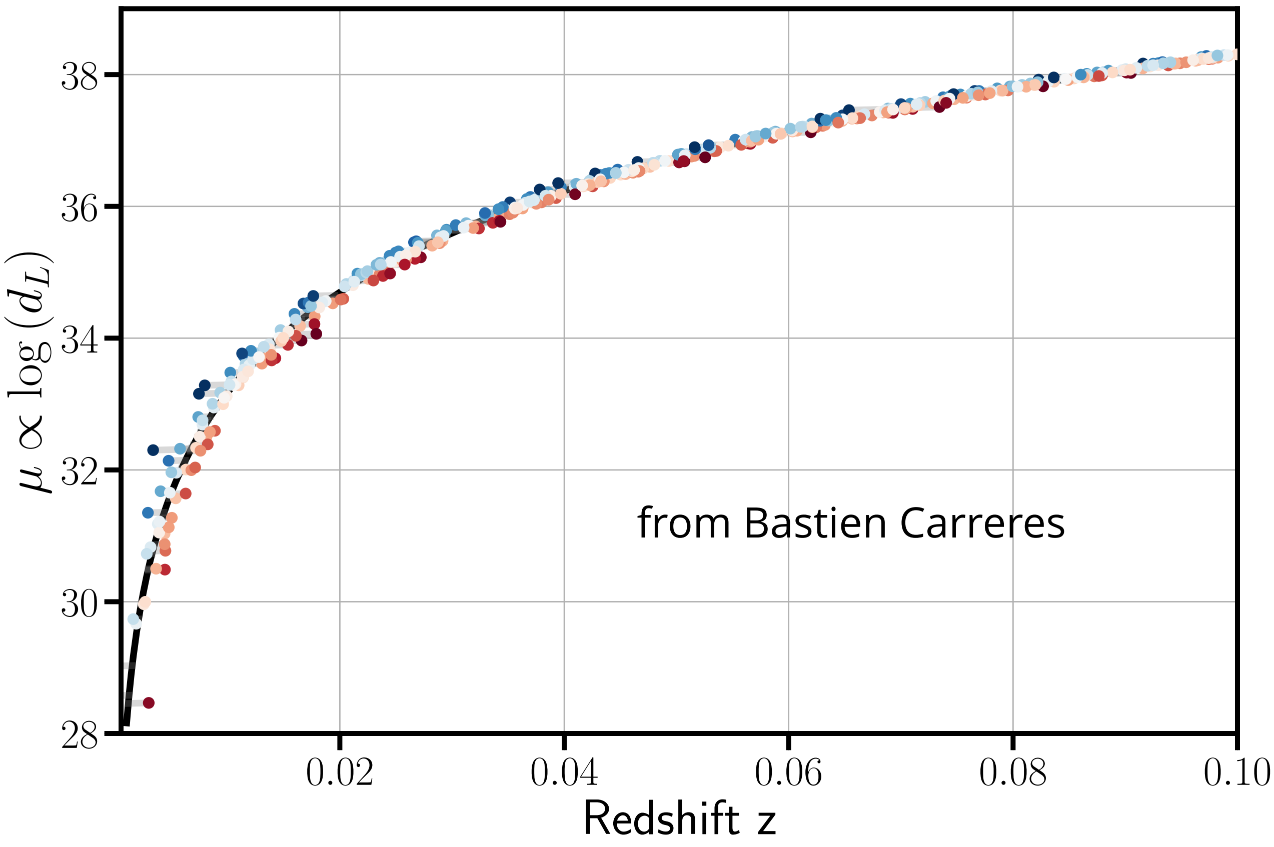
Hubble diagram

Perfect standard candle in an expanding Universe



Hubble diagram

Perfect standard candle in an expanding Universe



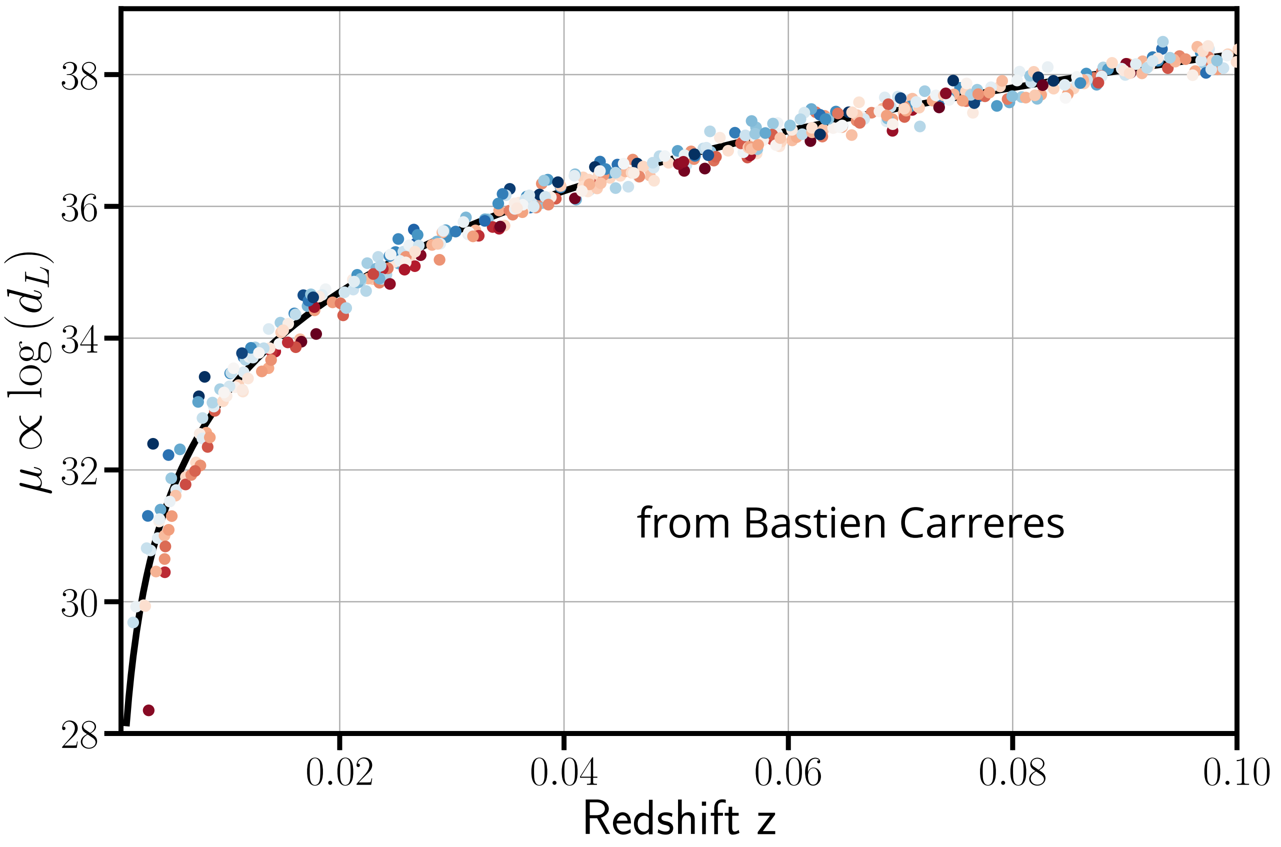
Peculiar velocities

Hubble diagram

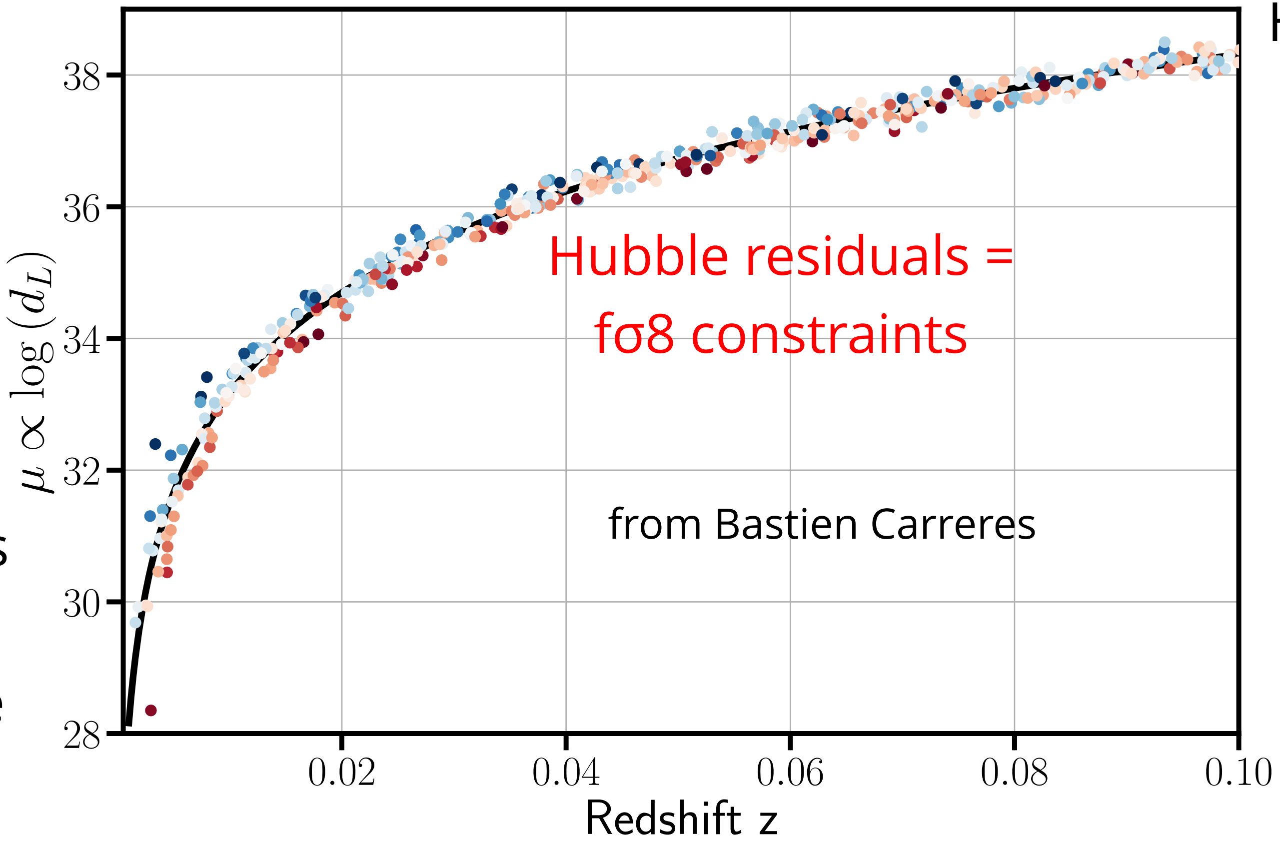
Standardizable candle in an expanding Universe

Noise sources (intrinsic scatter, local environment, instruments...)

Peculiar velocities



Hubble diagram



High redshifts = dark energy constraints (*w*, or *w0,wa*)

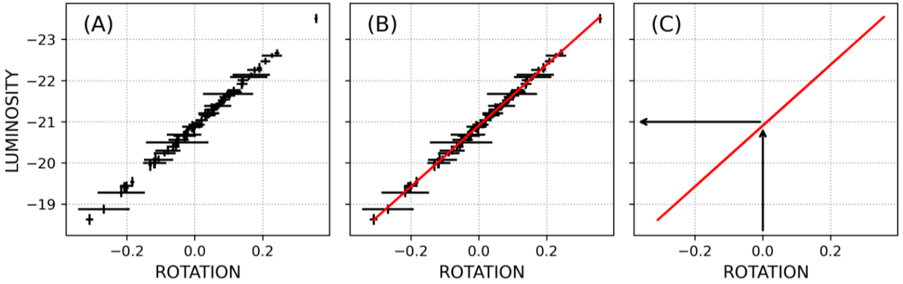
Low redshifts + calibration of the candle luminosity = H0 constraints

"Not-so" standard candles

Tully-Fisher relation

- Spiral galaxies (Late-type)
- Absolute magnitude (M) / rotational velocity (W)
- Baryonic TF: Stellar and gas mass instead of magnitude (lower relation dispersion)

$$M = a + b \log W$$

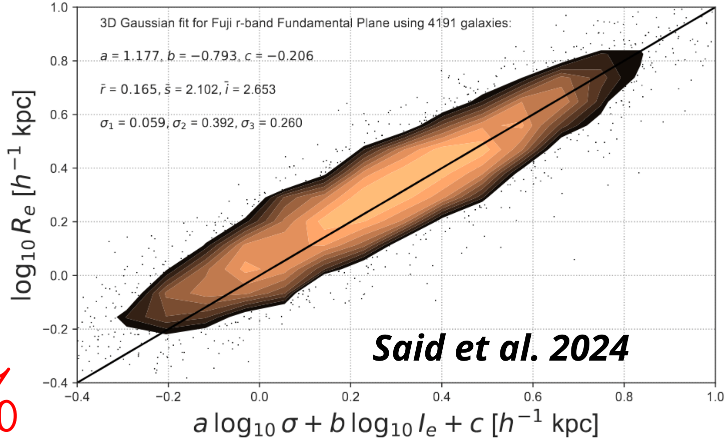


Said 2023

Fundamental plane relation

- Elliptical galaxies (Early-type)
- Effective radius (Re) / surface brightness (Ie) / stellar velocity dispersion (σ)

$$\log R_e = a \log \sigma + b \log I_e + c$$

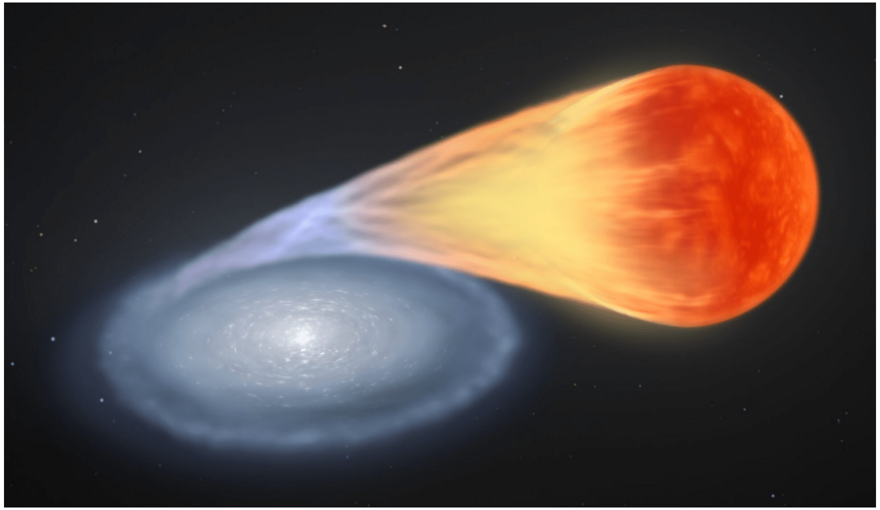


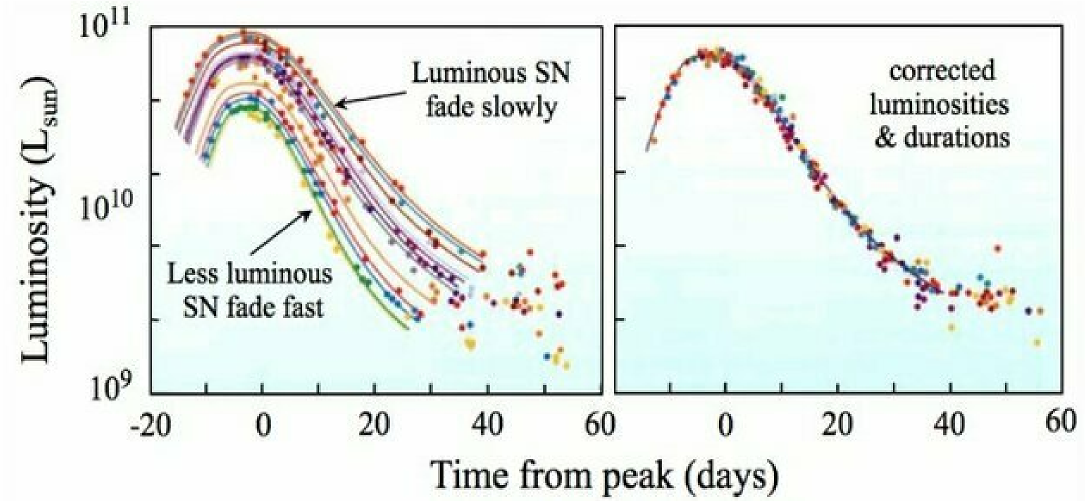
Said et al. 2024

$$\sigma_D / D \sim 20\%$$

Most standard candle: Type Ia supernovae

- **SNe Ia**: Thermonuclear explosion in a binary system with a white dwarf reaching Chandrasekhar mass





$$\sigma_D/D \sim 7\%$$

Most standard candle: Type Ia supernovae

- Distance modulus:

$$\mu_{\mathrm{obs},i} = m_{\mathrm{B},i} - M_{\mathrm{B},i}$$

- SNe Ia standardization = Tripp relation

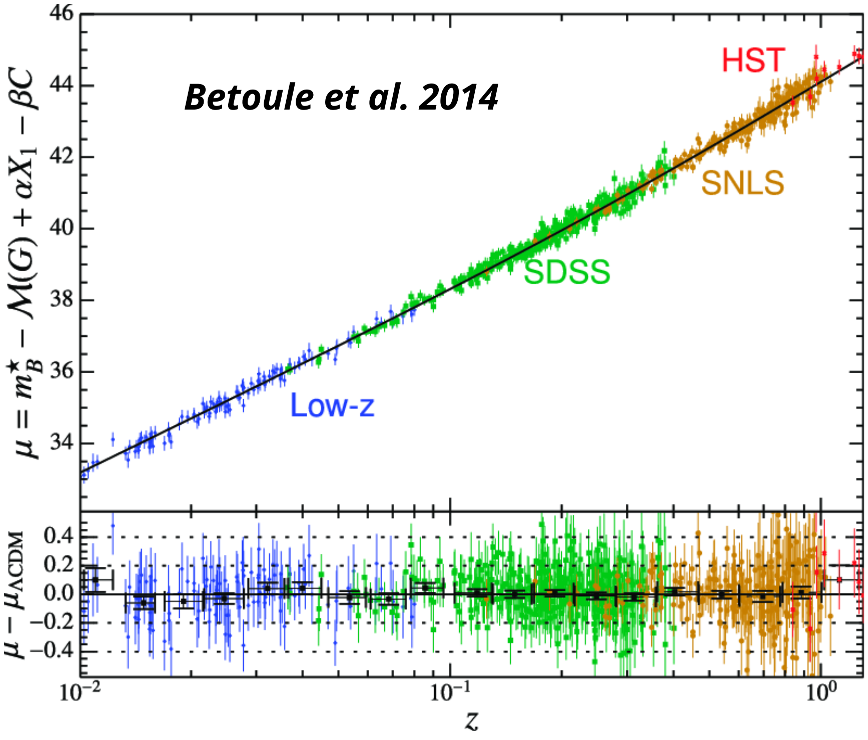
$$M_{\mathrm{B},i} = M_0 - \alpha x_{1,i} + \beta c_i + (\mathrm{opt})$$

Parameters common for all SN Ia

Light-curve stretch
"Luminous fade slowly"

SN color
"Redder is fainter"

Obtained from light-curve fitting (e.g., SALT2)



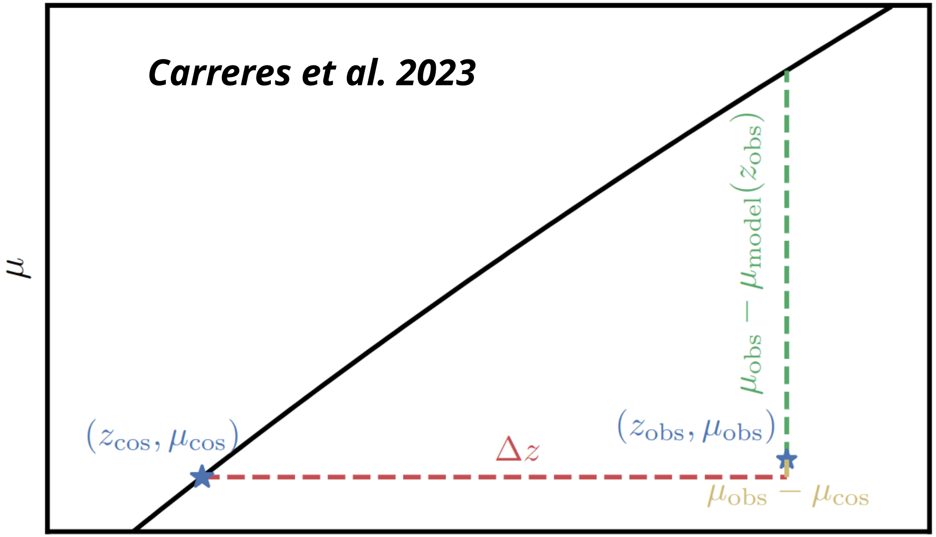
Most standard candle: Type Ia supernovae

- Hubble residuals:

$$\Delta\mu_i = \mu_{\mathrm{obs},i} - \mu_{\mathrm{model}}(z_{\mathrm{obs},i})$$
$$= \mu_{\mathrm{obs},i} - 5\log\left(\frac{d_{\mathrm{L}}(z_{\mathrm{obs},i})}{10\mathrm{pc}}\right)$$

- Velocity estimator:

$$\hat{v}_i = -\frac{\ln(10)c}{5}\left(\frac{(1+z_i)\,c}{H(z_i)\,r(z_i)} - 1\right)^{-1}\Delta\mu_i$$



PV and RSD constraint methods

- Growth rate measurement methods with peculiar velocities:

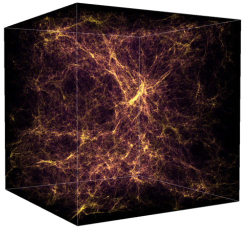
Density/velocity power spectra

$$\langle \delta_{\mathrm{g}}(k)\delta_{\mathrm{g}}(k)\rangle \; \langle \delta_{\mathrm{g}}(k)\mathbf{v}(k)\rangle \; \langle \mathbf{v}(k)\mathbf{v}(k)\rangle$$

Velocity reconstruction and comparison to density

$$\mathbf{v}_{\mathrm{measured}} \longleftrightarrow \nabla \cdot \mathbf{v}_{\mathrm{pred}} \propto -aHf\delta$$

Simulation based inference / Forward modeling



$$\longrightarrow (\delta_{\mathrm{g}}, \mathbf{v})$$

Likelihood-based field-level inference

Likelihood-based field-level inference

Carreres et al. 2023

Ravoux et al. 2025

Likelihood-based field-level estimator

- **Purpose**: Maximizing or sampling a likelihood computed from all coordinates of the data
- *p* **= fitted parameters**: **simultaneously** cosmology (fσ8) and field parameters (e.g., Hubble diagram parameters α, β, M0 for SN)

$$\mathcal{L}(p) \propto \exp\left[-\frac{1}{2} v^T(p) C_{vv}(p)^{-1} v(p)\right]$$

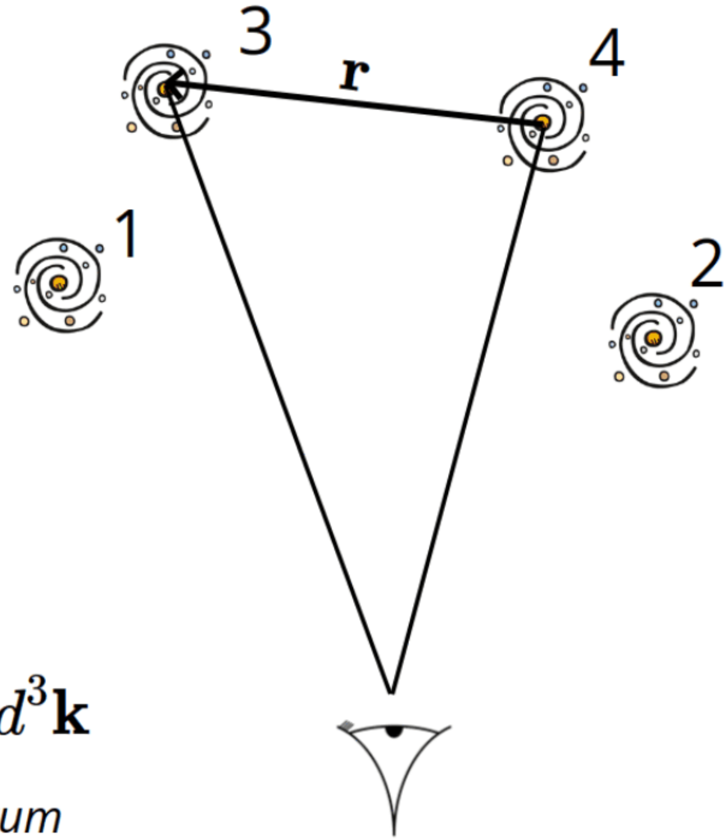
Parameters

Velocity from SNe Ia

Field covariance matrix computed from theory and SNe Ia coordinates

Computing the field covariance

$$\mathcal{L}(p) \propto \exp\left[-\frac{1}{2}\begin{bmatrix}1\\2\\3\\4\end{bmatrix}^T \begin{vmatrix} & C_{12} & C_{13} & C_{14} \\ C_{21} & & C_{23} & C_{24} \\ C_{31} & C_{32} & & C_{34} \\ C_{41} & C_{42} & C_{43} & \end{vmatrix}^{-1} \begin{bmatrix}1\\2\\3\\4\end{bmatrix}\right]$$



Field covariance

$$C_{\mathrm{nm}}(\mathbf{r_n}, \mathbf{r_m}) = \frac{1}{(2\pi)^3} \int_{\mathbf{k}} P_{\mathrm{nm}}(k, \mu_n, \mu_m) e^{i\mathbf{k}.\mathbf{r}} d^3\mathbf{k}$$

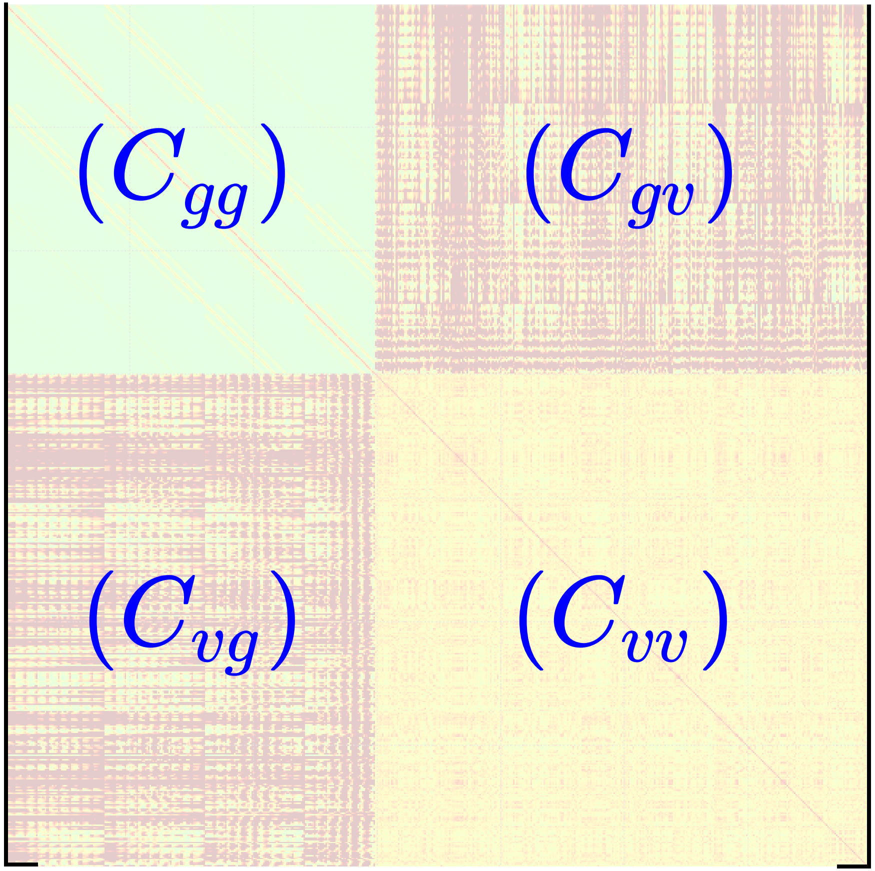
Theoretical power spectrum

Combining densities (RSD) and peculiar velocities

$$\mathcal{L}(p) \propto \exp\left[-\frac{1}{2} \begin{bmatrix} \delta \\ v \end{bmatrix}^T \begin{bmatrix} (C_{gg}) & (C_{gv}) \\ (C_{vg}) & (C_{vv}) \end{bmatrix}^{-1} \begin{bmatrix} \delta \\ v \end{bmatrix}\right]$$

Density field from galaxy surveys

SNe Ia velocities



Full field covariance matrix

General covariance calculation

- **Objective**: Improve/Extend the likelihood-based modeling

$$C_{\mathrm{ab}}(\mathbf{r_1},\mathbf{r_2}) = \frac{1}{(2\pi)^3}\int_{\mathbf{k}} P_{\mathrm{ab}}(k,\mu_1,\mu_2)e^{i\mathbf{k}.\mathbf{r}}d^3\mathbf{k}$$

gg, gv, vv

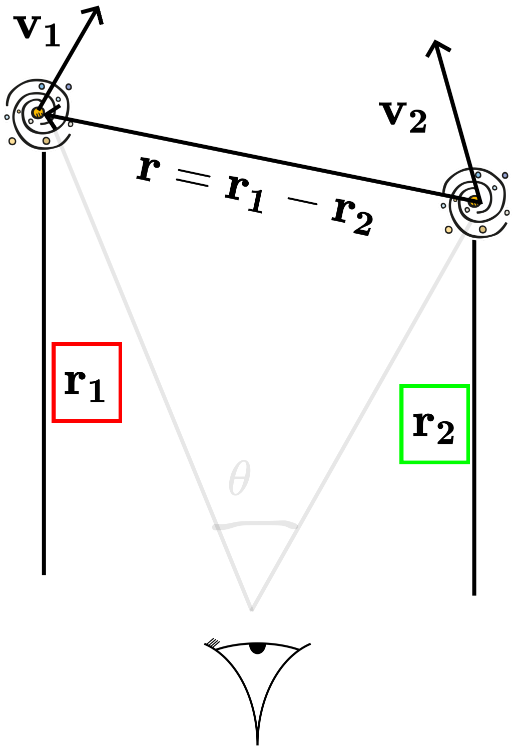
$$P_{\mathrm{ab}}(k,\mu) = \sum_n w_{\mathrm{ab},n}F_{\mathrm{ab},n}(k,\mu)\mathcal{P}_{\mathrm{ab},n}(k)$$

- General form of field power spectrum
- **Algorithmically-oriented calculations (Hankel transforms)**
- **Wide-angle** effect accounted in the integration

A word on wide-angle in clustering

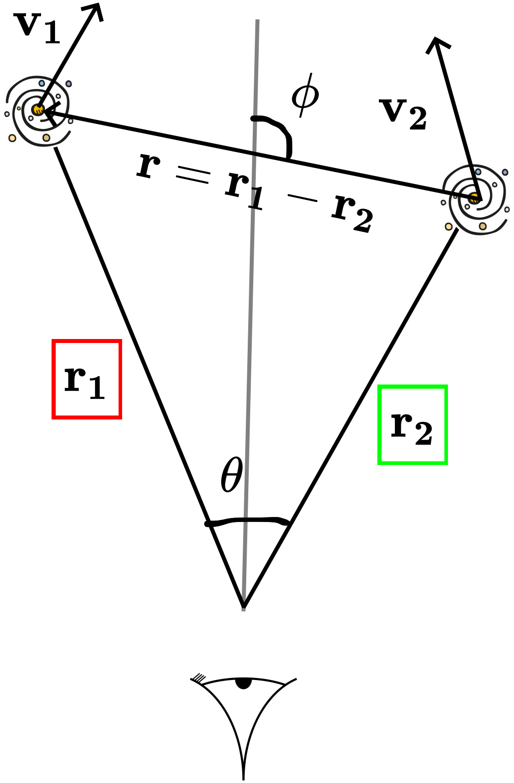
Plane parallel models

$$\mu = \mu_1 = \mu_2$$



Wide angle models

$$\mu_1 \neq \mu_2$$



$$\mu_i = \mathbf{k} \cdot \mathbf{r_i}$$

Plane parallel model

- Express power spectrum model in a general way:

$$P_{\mathrm{ab}}(k,\mu) = \sum_n w_{\mathrm{ab},n} F_{\mathrm{ab},n}(k,\mu) \mathcal{P}_{\mathrm{ab},n}(k)$$

Parameters to fit ($w_{\mathrm{ab},n}$)

Terms to integrate analytically ($F_{\mathrm{ab},n}$)

Power spectra terms to integrate numerically ($\mathcal{P}_{\mathrm{ab},n}$)

- vv "simple" example:

$$P_{\mathrm{vv}} = (f\sigma_8)^2 \times \left(a^2 H^2 \frac{\mu^2}{k^2}\right) \times \left(P_{\theta\theta}(k) D_u^2(k,\sigma_u)\right)$$

Plane parallel model

- Without loosing generality we can express the covariance:

$$C_{\rm ab}(\mathbf{r_1},\mathbf{r_2}) = \sum_n w_{{\rm ab},n} \sum_\ell N_{{\rm ab},\ell}(\phi)\mathcal{H}_\ell \left[\mathcal{P}_{{\rm ab},n}(k) M^\ell_{{\rm ab},n}(k)\right](r)$$

$$N_{{\rm ab},\ell}(\phi) = (2\ell+1)L_\ell(\cos{(\pi-\phi)})$$

$$M^\ell_{{\rm ab},n}(k) = \frac{1}{2}\int_{-1}^{1} d\mu' F_{{\rm ab},n}(k,\mu')L_\ell(\mu')$$

Plane parallel model

- Without loosing generality we can express the covariance:

$$C_{\mathrm{ab}}(\mathbf{r_1},\mathbf{r_2}) = \sum_n \boxed{w_{\mathrm{ab},n}} \sum_\ell N_{\mathrm{ab},\ell}(\phi)\mathcal{H}_\ell \left[\mathcal{P}_{\mathrm{ab},n}(k)M^\ell_{\mathrm{ab},n}(k)\right](r)$$

Linear sum of matrix with parameters to fit as coefficients

Plane parallel model

- Without loosing generality we can express the covariance:

$$C_{\mathrm{ab}}(\mathbf{r_1},\mathbf{r_2}) = \sum_n w_{\mathrm{ab},n} \sum_\ell N_{\mathrm{ab},\ell}(\phi) \mathcal{H}_\ell \left[\mathcal{P}_{\mathrm{ab},n}(k) M^\ell_{\mathrm{ab},n}(k) \right] (r)$$

Hankel transform

$$\mathcal{H}_\ell \left[f(k) \right] (r) = i^\ell \int_0^\infty \frac{k^2 dk}{2\pi^2} j_\ell(kr) f(k)$$

- **Most important part**: Algorithmically optimized way to compute power spectrum integral, with FFTLog algorithm

Plane parallel model

- Without loosing generality we can express the covariance:

$$C_{\rm ab}(\mathbf{r_1},\mathbf{r_2}) = \sum_n w_{{\rm ab},n} \sum_\ell N_{{\rm ab},\ell}(\phi)\mathcal{H}_\ell \left[\mathcal{P}_{{\rm ab},n}(k) M^\ell_{{\rm ab},n}(k)\right](r)$$

$$N_{{\rm ab},\ell}(\phi) = (2\ell+1)L_\ell(\cos(\pi-\phi))$$

$$M^\ell_{{\rm ab},n}(k) = \frac{1}{2}\int_{-1}^{1} d\mu' F_{{\rm ab},n}(k,\mu')L_\ell(\mu')$$

Terms pre-computed analytically with symbolic code generaltion

- **More complicated formalism derived and implemented for wide-angle**

flip Field Level Inference Package

- python package for likelihood-based field-level inference:

  https://github.com/corentinravoux/flip

  Flip article: **arxiv:2501.16852**

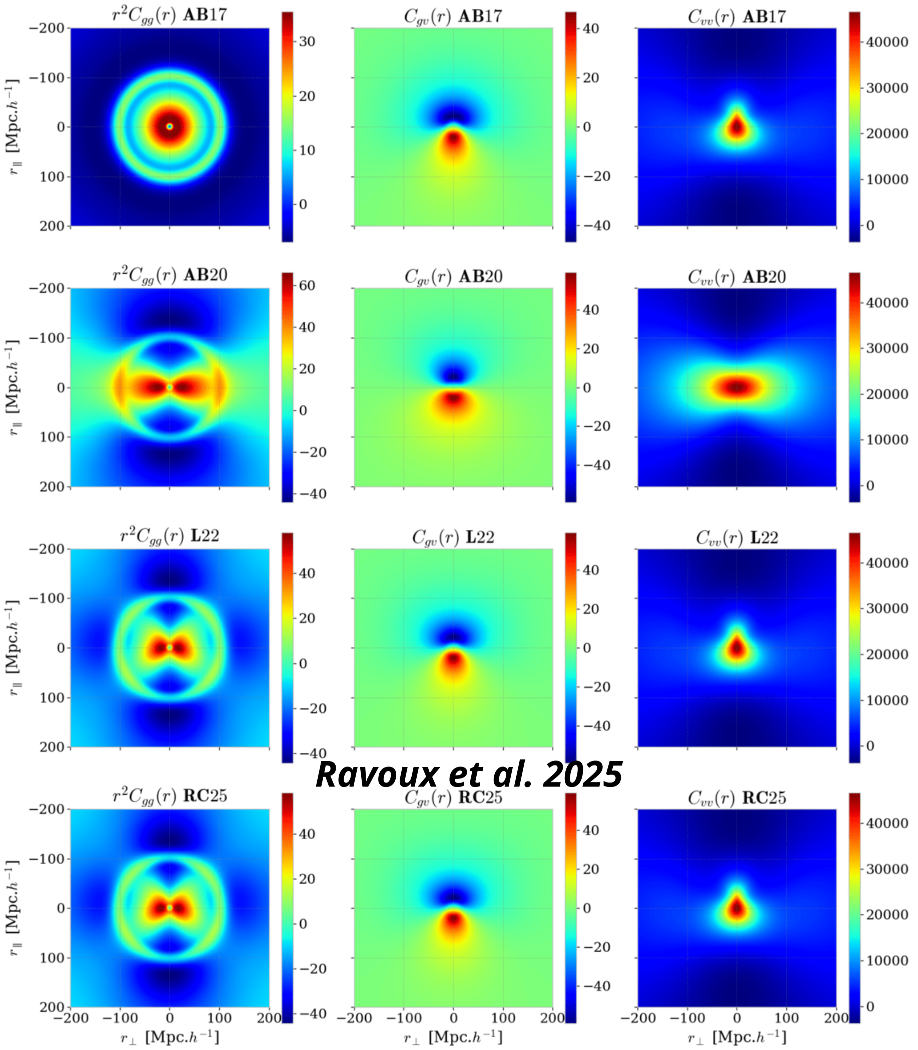
- **Fast generation of field covariance**
- MCMC and Minuit to fit fσ8
- General expression for vectors (SNIa, density, log distance, TF, FP...)

Contributors:
Bastien Carreres,
Damiano Rosselli
and Alex G. Kim

Field covariance models

- Model comparison on a regular coordinate grid (*Adams & Blake 2017, Adams & Blake 2020, Lai et al. 2022)*

- New wide-angle model developed (**RC25**): improved stability in integration and for wider fitting parameter ranges

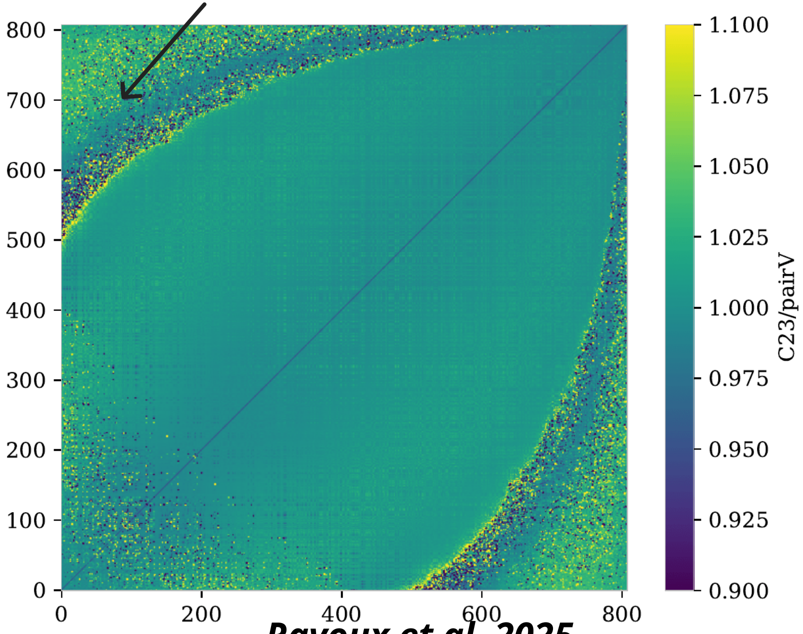


Ravoux et al. 2025

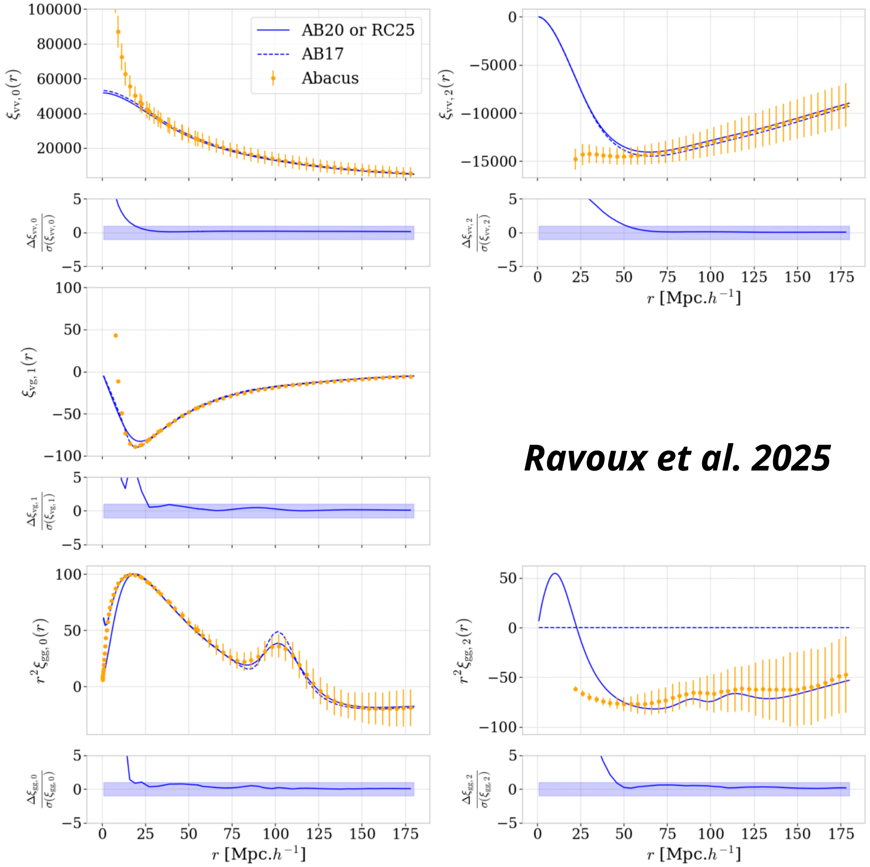
Validating *flip*

- Field covariance comparison with *pairV*
  Anthony Carr & David Parkinson

Disagreement due to different power spectrum model



Ravoux et al. 2025



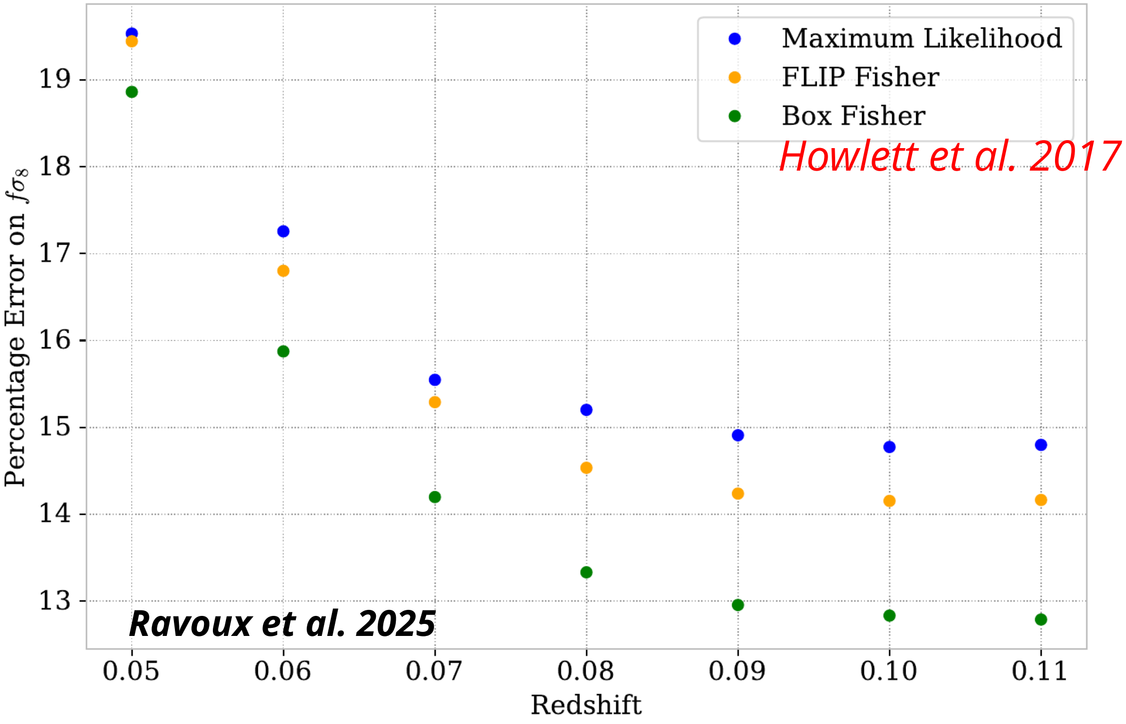
Ravoux et al. 2025

- Model validated on N-body simulations
  Tyann Dummerchat

Survey dependent Fisher forecast

Damiano Rosselli

- Field covariance matrix accounts for survey geometry
- Comparing ZTF simulation (*Carreres et al. 2023*) with Fisher forecasts
- **Error ~ 30% closer to likelihood-based than standard Fisher**



fσ8 constraints on ZTF & LSST simulations

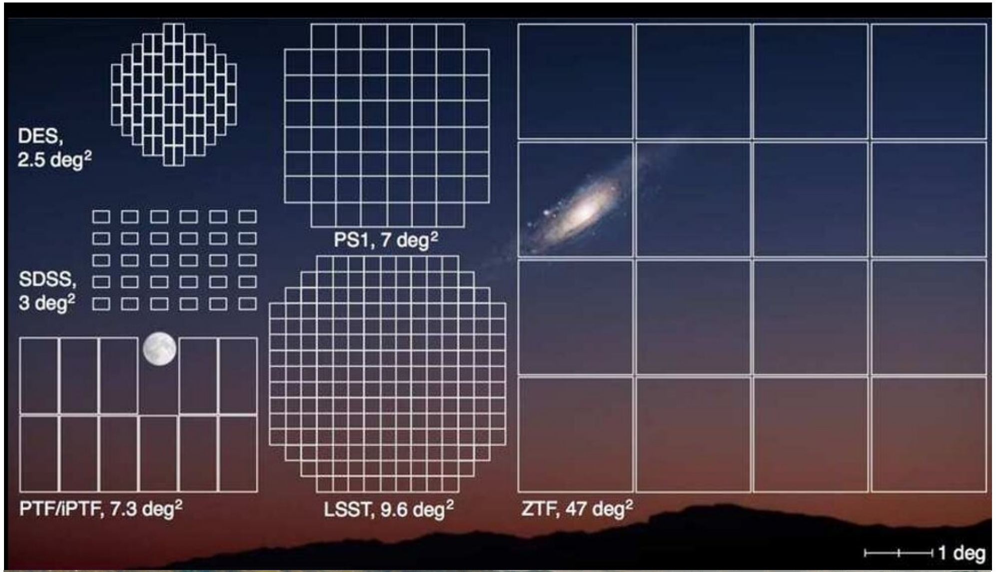
Carreres et al. 2023

Carreres et al. in prep.

Rosselli et al. in prep.

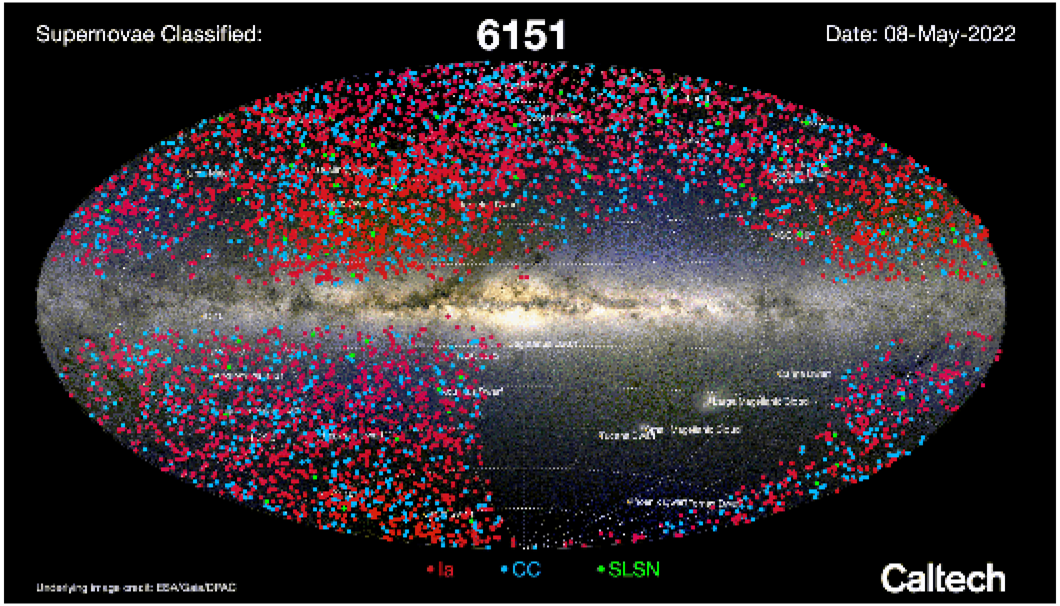
Zwicky Transient Facility (ZTF)

- ZTF = high-cadence photometric telescop in the Palomar observatory
- Very large field of view (47 deg²)
- Observing 3/4 of the sky every nights with 3 filters (g, r, i)

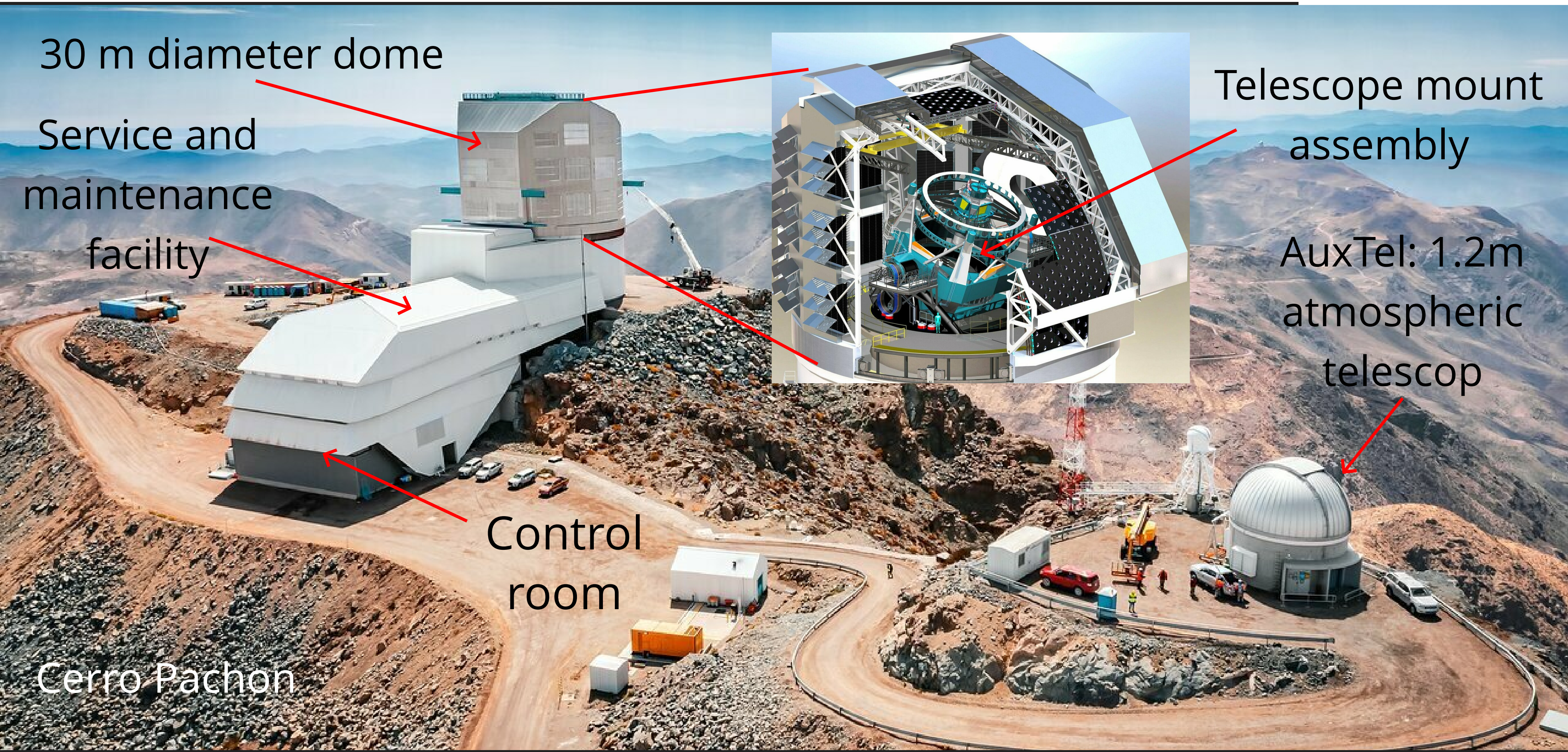


Zwicky Transient Facility (ZTF)

- **Transient sky**: Supernovae, gamma ray burst, tidal distruptive events, comets, asteroids
- Dedicated spectroscopic telescope measuring transient spectra (SEDmachine)
- **Latest release**: More than 3000 classified supernovae of type Ia



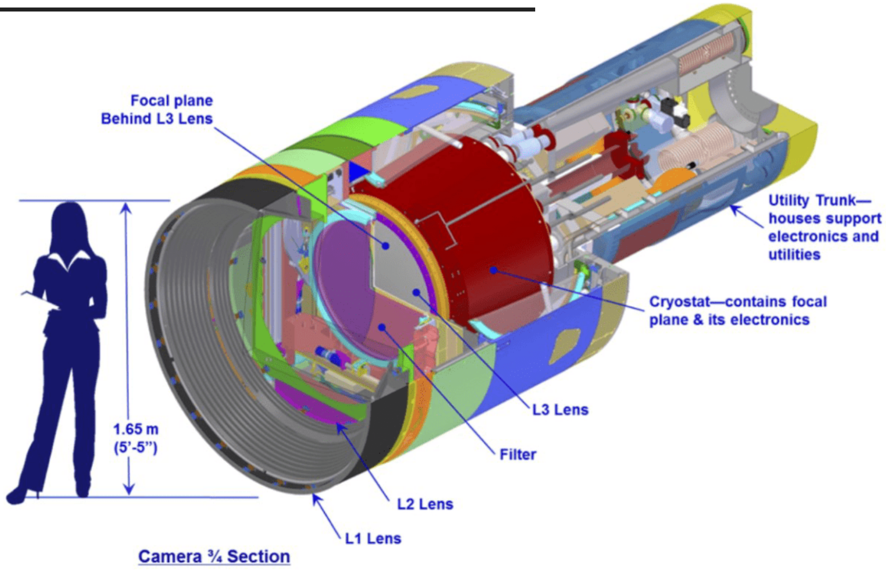
Rubin - LSST

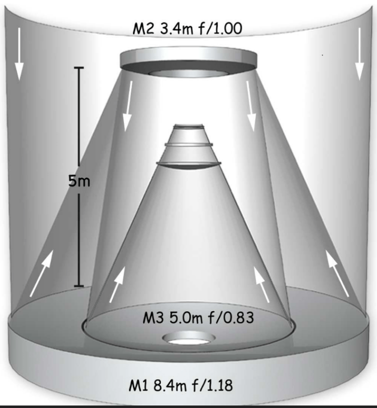


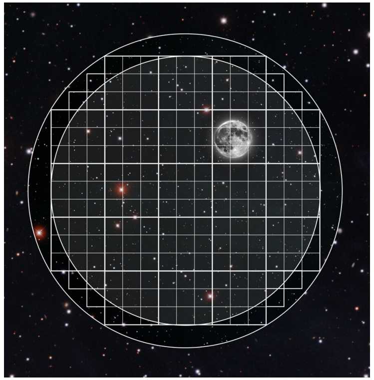
30 m diameter dome
Service and maintenance facility
Telescope mount assembly
AuxTel: 1.2m atmospheric telescop
Control room
Cerro Pachon

Rubin - LSST: instrument

- **Largest numerical camera in the world**: 3.2 Gpixels with 2s readout time
- 8.4 m primary mirror and 9.6 deg² field of view
- 6 filters: (u, g, r, i, z, y)
- 10-year photometric survey of half of the sky (~ 20 000 deg²)

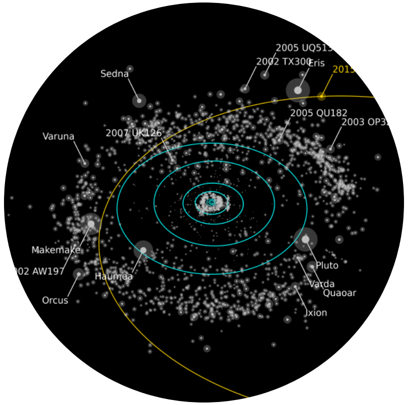






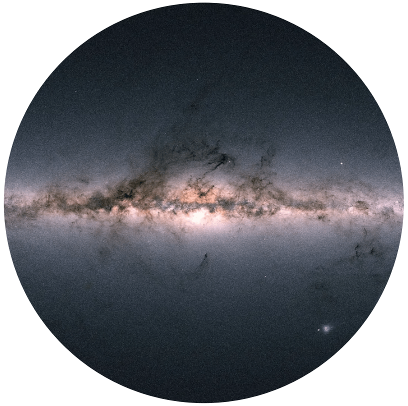
LSST survey: science

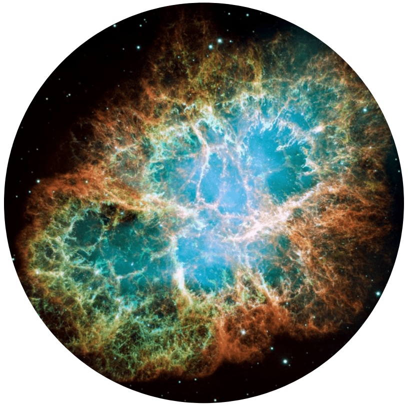
Dark matter & Dark energy
- Strong & Weak lensing
- BAO (angular and photo-z)
- Clusters, Supernovae cosmology



Solar System science
- Comets & asteroids
- Small body census

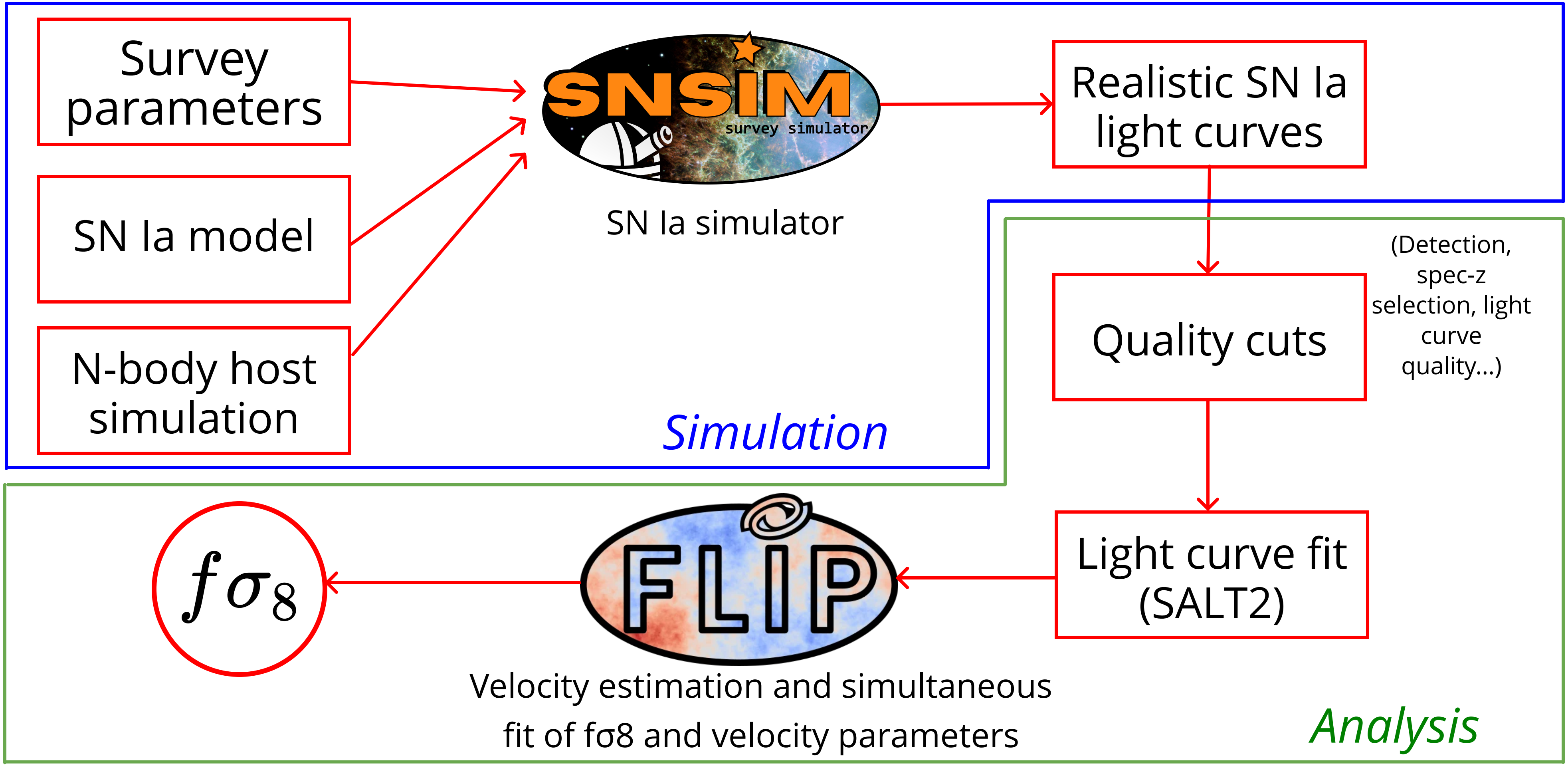
Mapping the Milky Way
- Structure and evolution
- Stellar properties





Transient sky
- Supernovae, variable stars
- Rare events (kilonovae, TDE)
- New classes of transients

SN Ia simulation and analysis

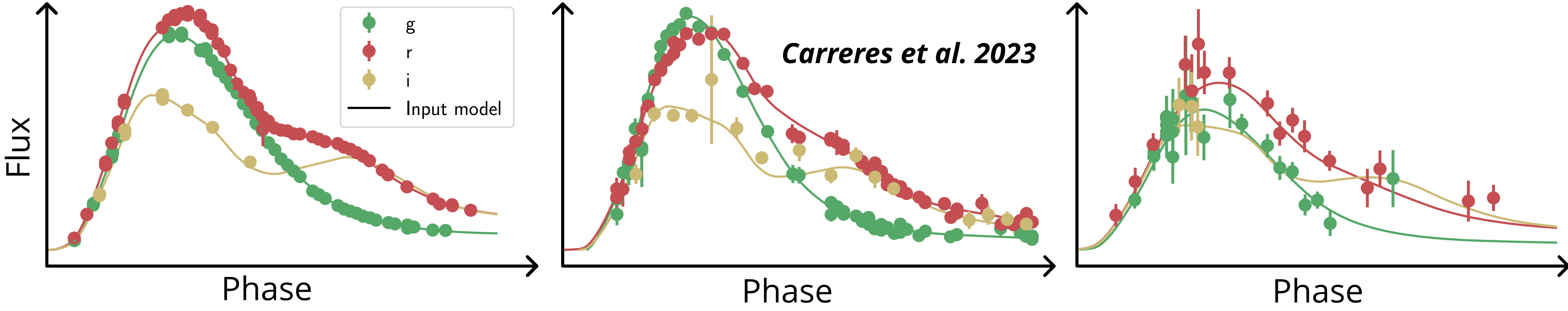


snsim

- **Survey parameters:** Cadence (ZTF, LSST), CCD gain, zero point, sky noise
- **SN Ia model:** Rate, SED, color, stretch, intrinsic scattering distributions, light-curve parameters
- **Host catalog**: Use velocities and associated clustering from N-body simulation

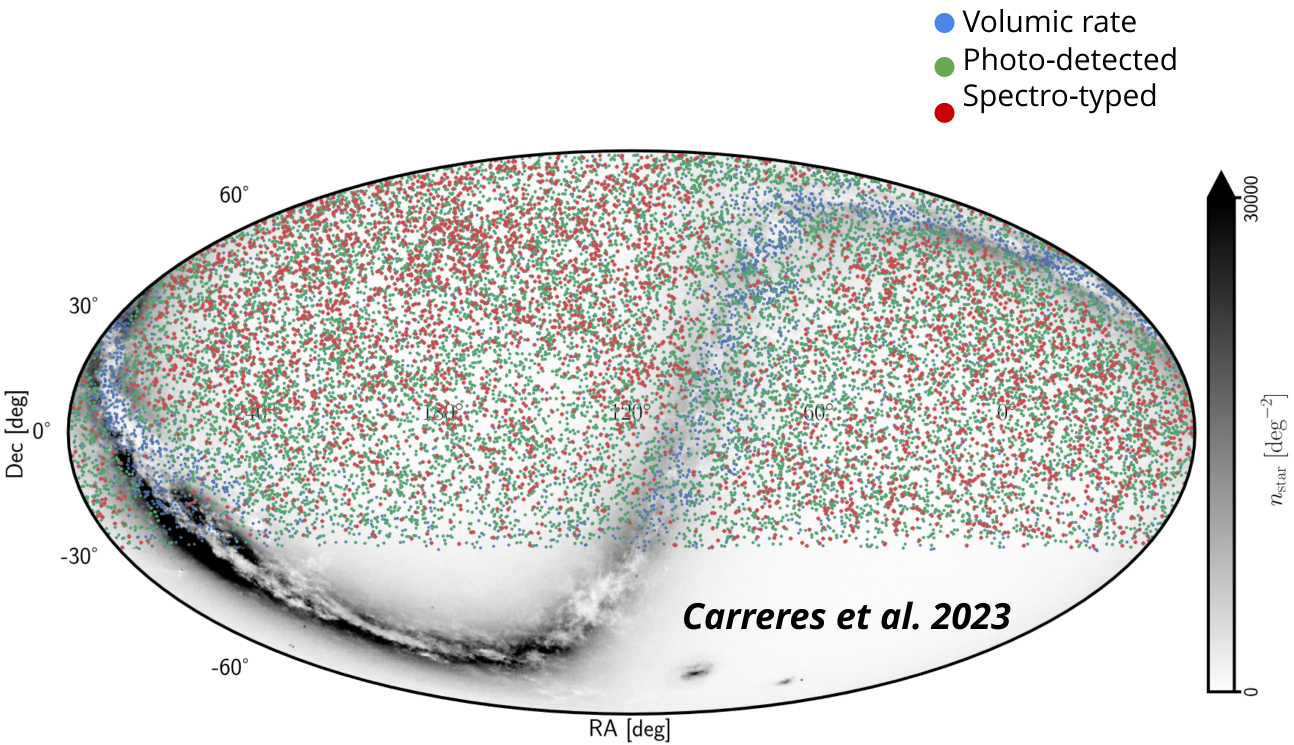
https://github.com/bastiencarreres/snsim

Main contributors:
Bastien Carreres,
Damiano Rosselli



ZTF simulations

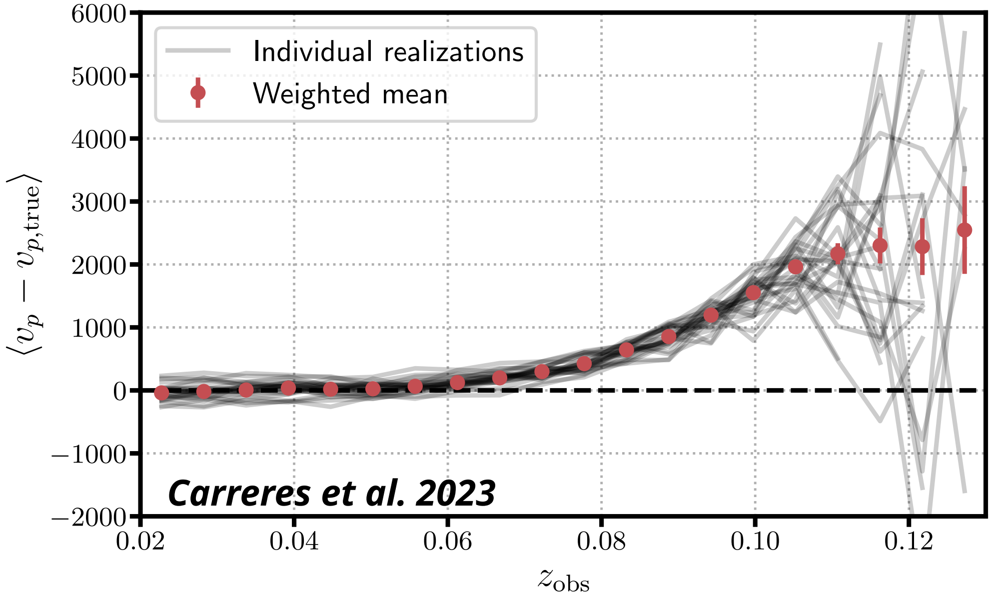
- 27 realizations from OuterRim (*Heitmann et al. 2019*) sub-volumes
- ZTF 6 years simulations
- **Cadence** from ZTF logs
- **Instrumental characterization**
- **Detection efficiency cuts**: ZTF and SEDmachine detection

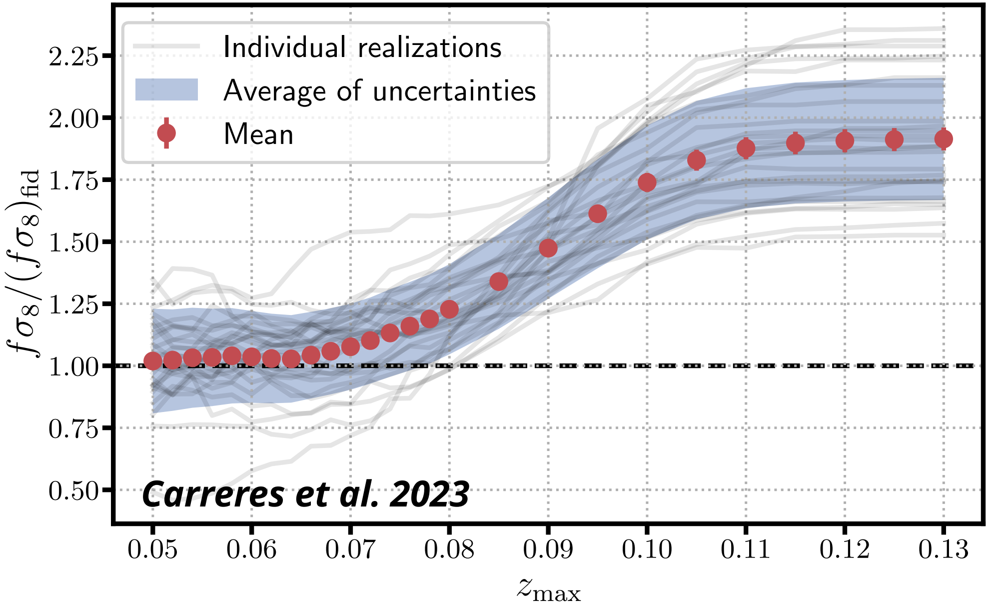


Carreres et al. 2023

ZTF results

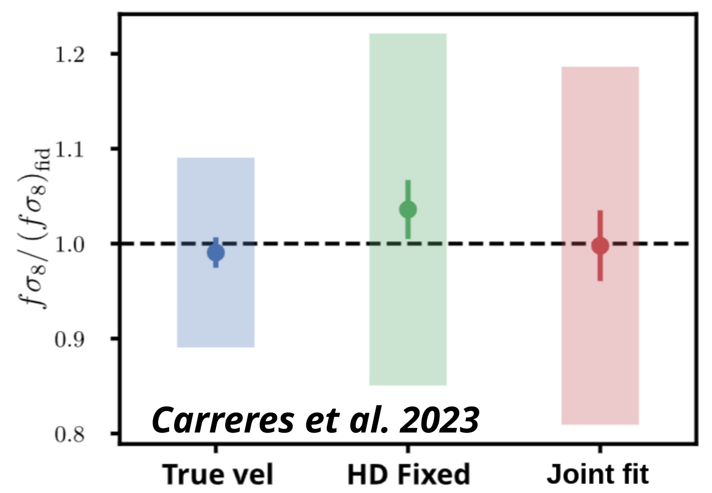
- SN Ia selection bias causes velocity & fσ8 bias at high redshift
- No fσ8 error bar improvement even with correction
- **Maximum redshift usable:** $z = 0.06$

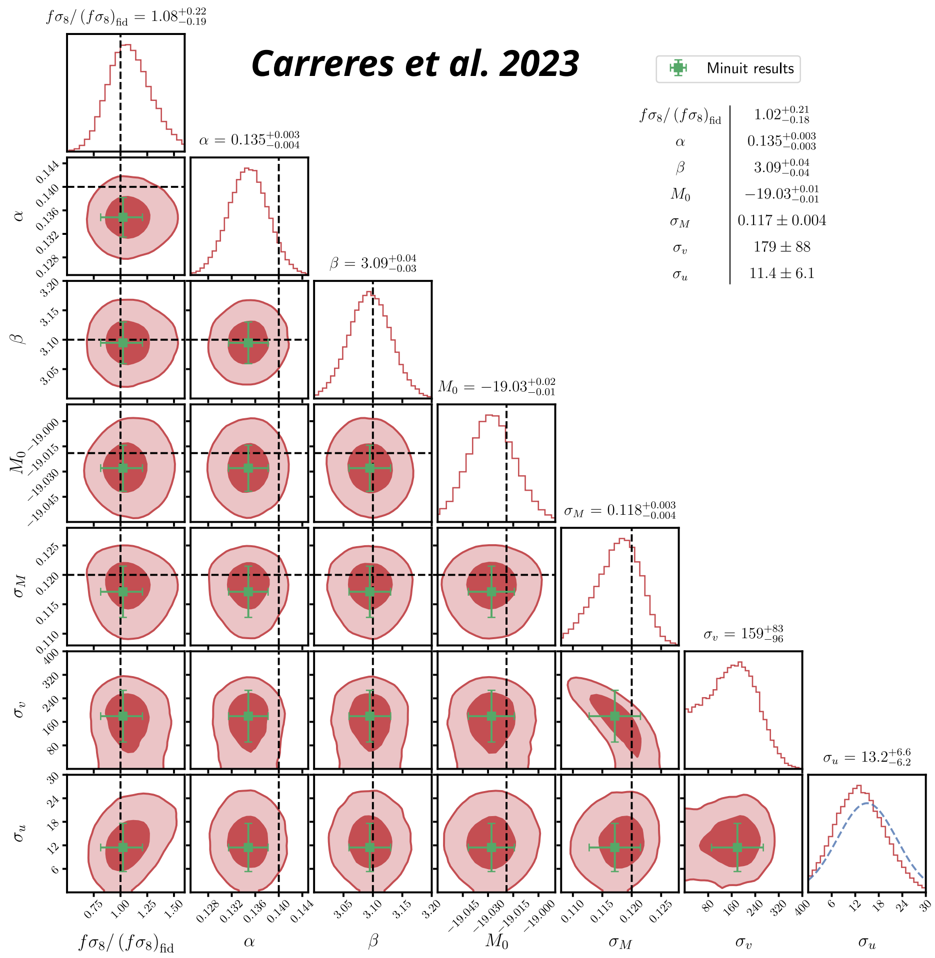




ZTF results

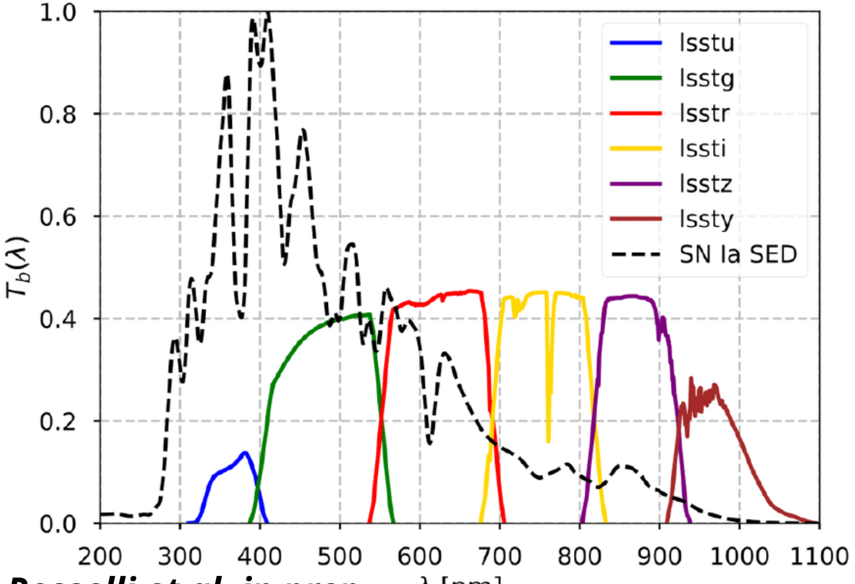
- fσ8 fit performed with Hubble diagram parameters
- no fσ8 bias on the 27 simulations
- **19 % precision measurement with only 1600 SNe Ia**



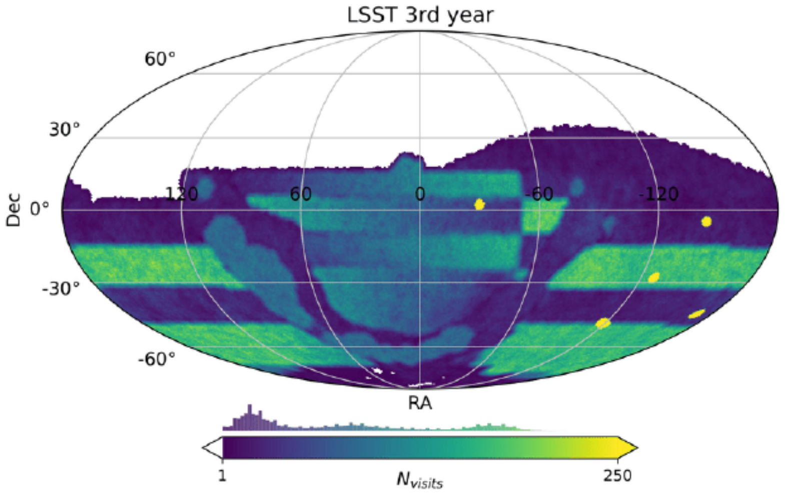


LSST simulations

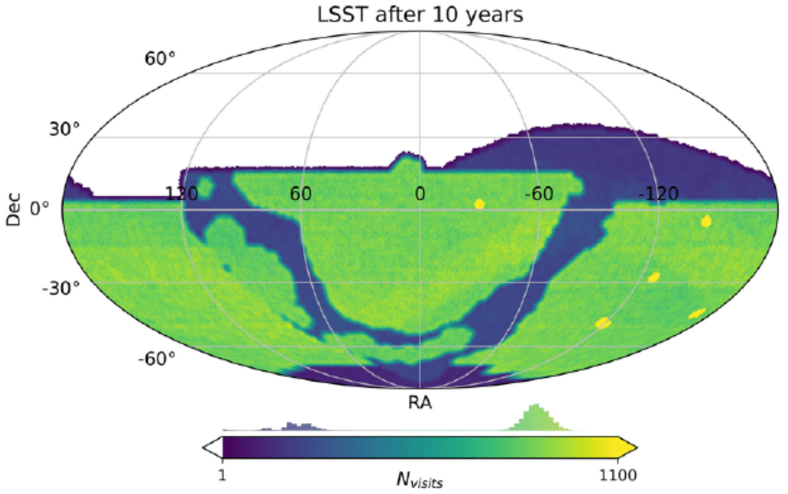
- 10 year simulation from Uchuu-UniverseMachine (*Behroozi et al. 2019*)
- Cadence and instrument properties from the observing strategy simulation v3.3



Rosselli et al. in prep.

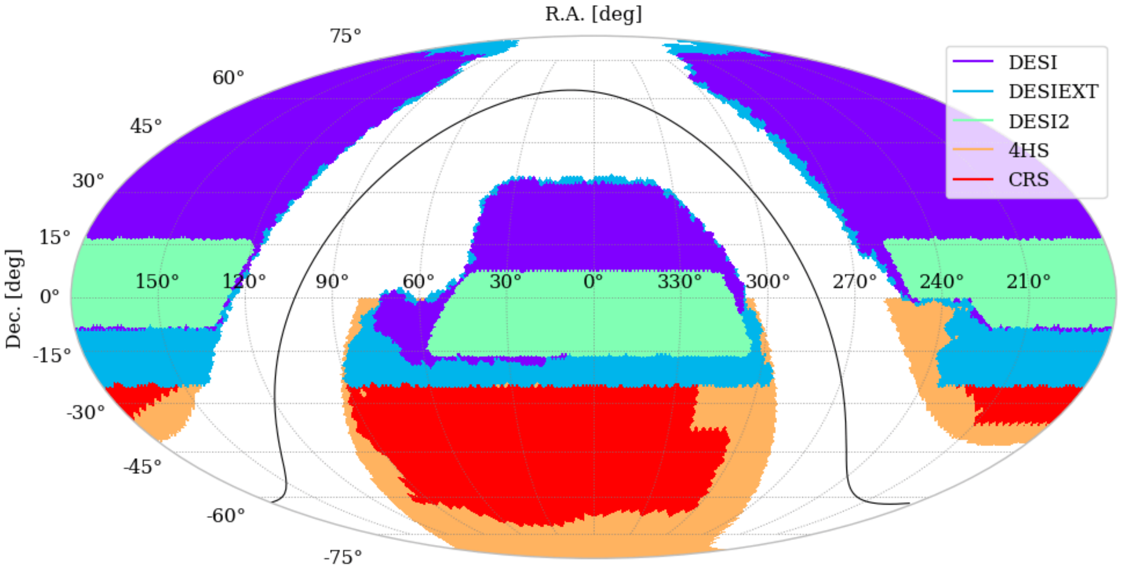


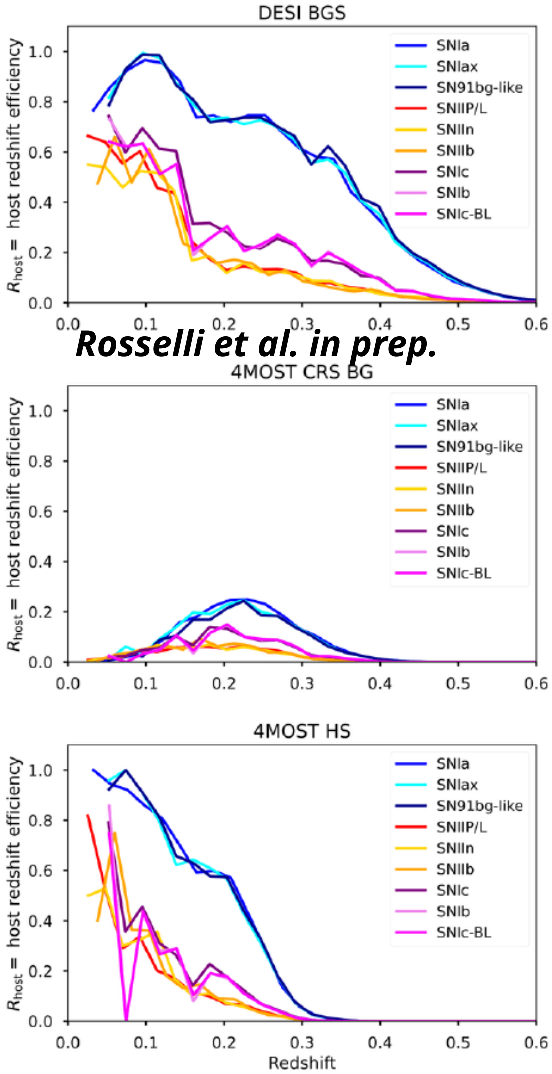
Rosselli et al. in prep.



LSST simulations

- **SN host redshift efficiency** (C. Ravoux, P. Gris, D. Rosselli, J. Bautista):
  - Use of DC2 LSST simulation with galaxy properties
  - Footprint and efficiency for different spectroscopic survey
  - Considering SNe variety contamination

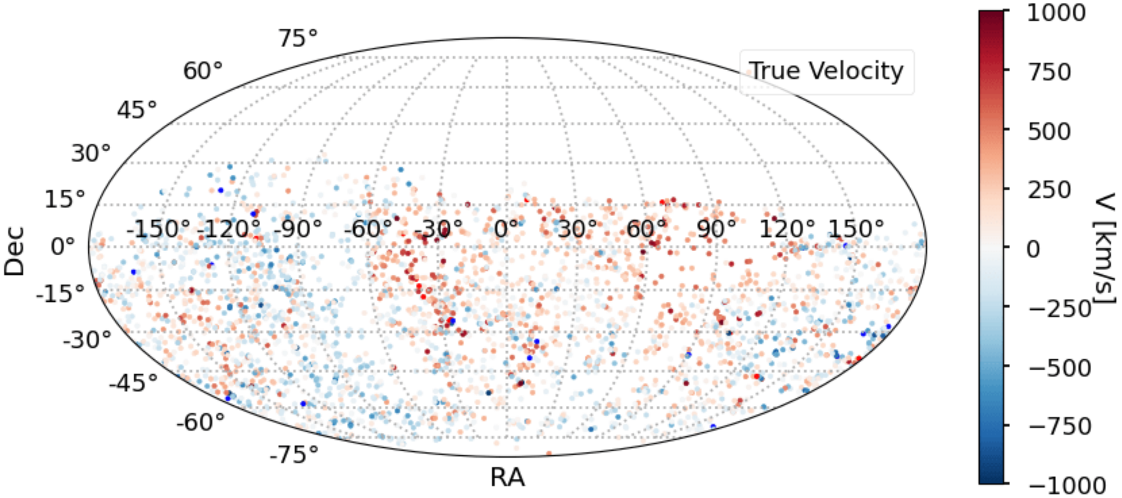




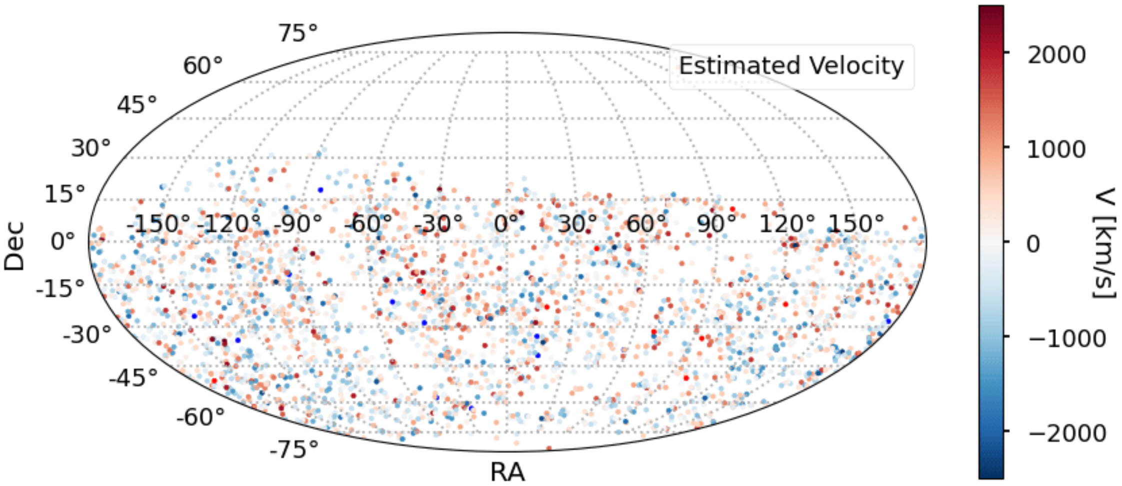
Rosselli et al. in prep.

LSST simulations

- **33,000 SNe Ia** with spectro-z at z < 0.16 (among 1M for the full sample)
- Photo-typing with *SuperNNovae* (*Moller et al. 2019*) gives **0.02% contamination from peculiar SN**

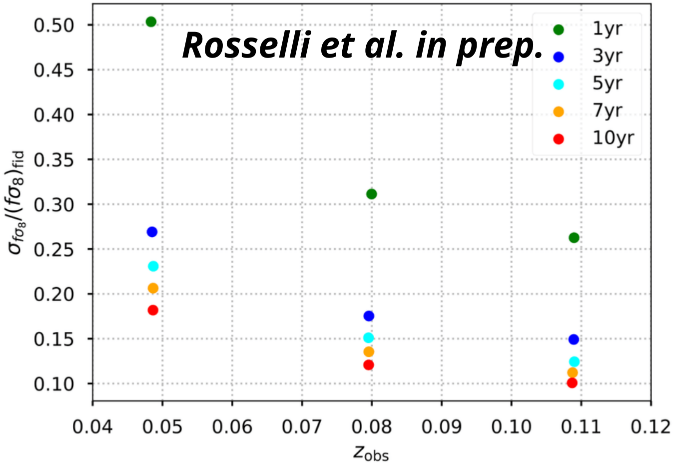


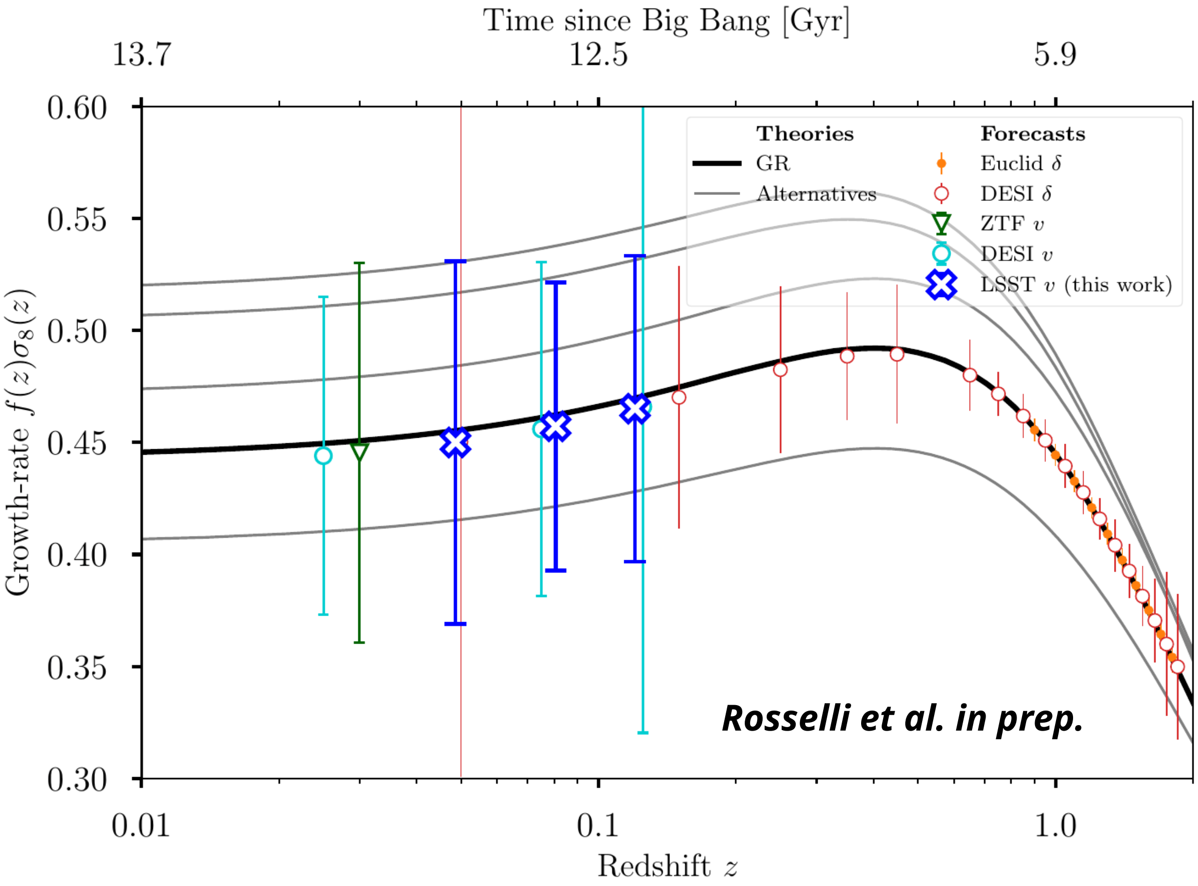
Rosselli et al. in prep.



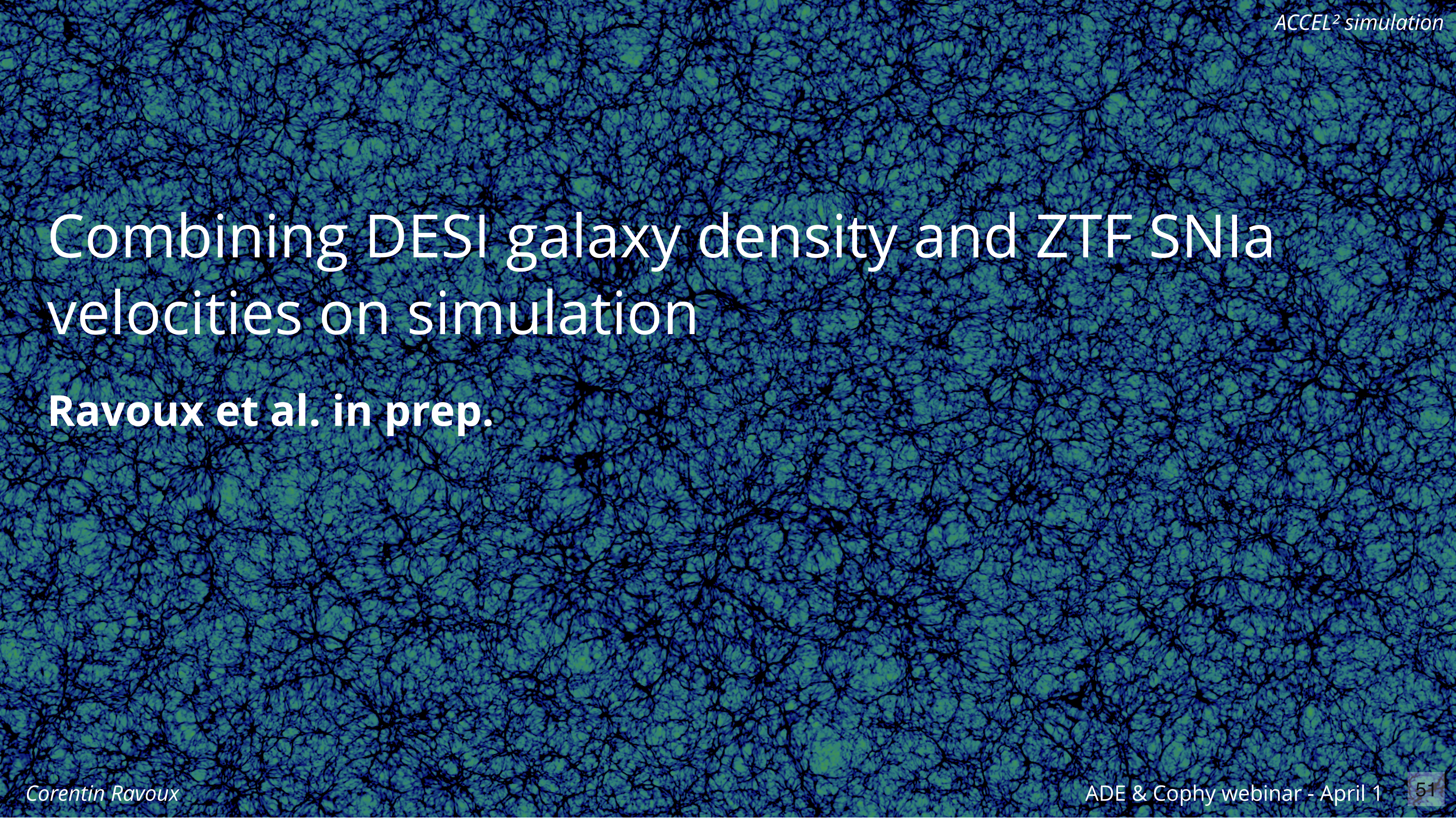
LSST results

- Unbiased fσ8 measurement with full Hubble diagram fit over 3 redshift points
- Full study on the impact of peculiar SN contamination





How we can do better ? ***Combination of PV with RSD***



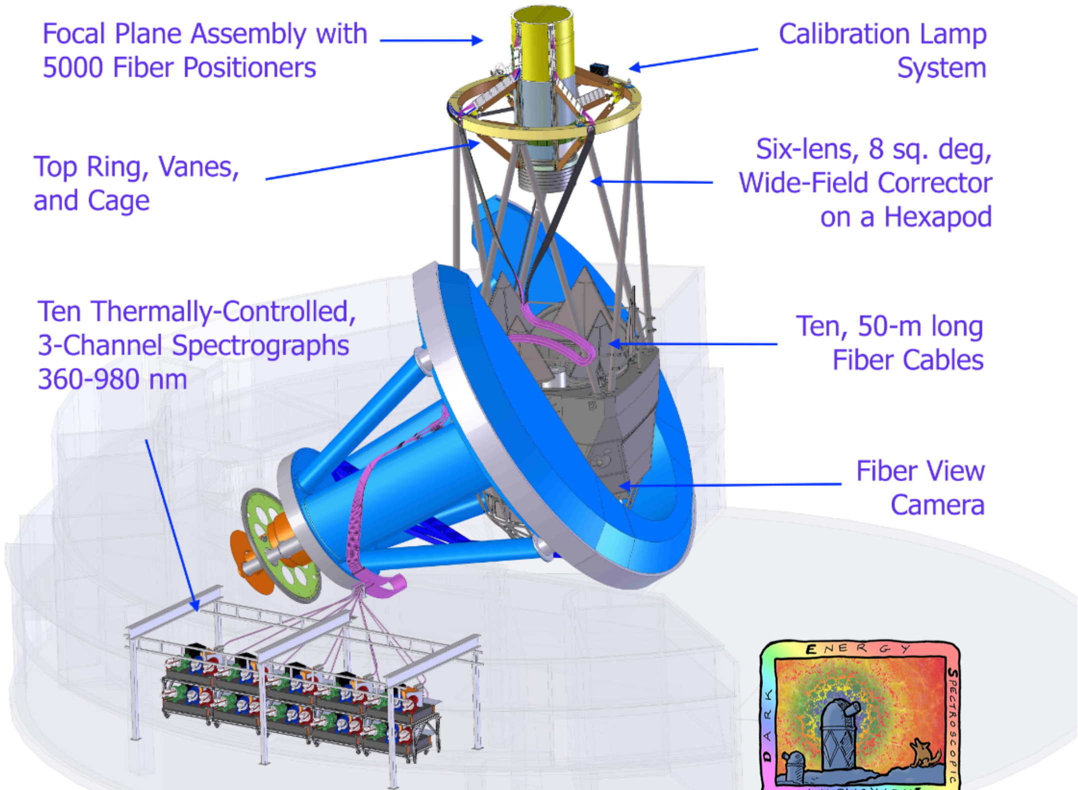
ACCEL² simulation

Combining DESI galaxy density and ZTF SNIa velocities on simulation

Ravoux et al. in prep.

DESI

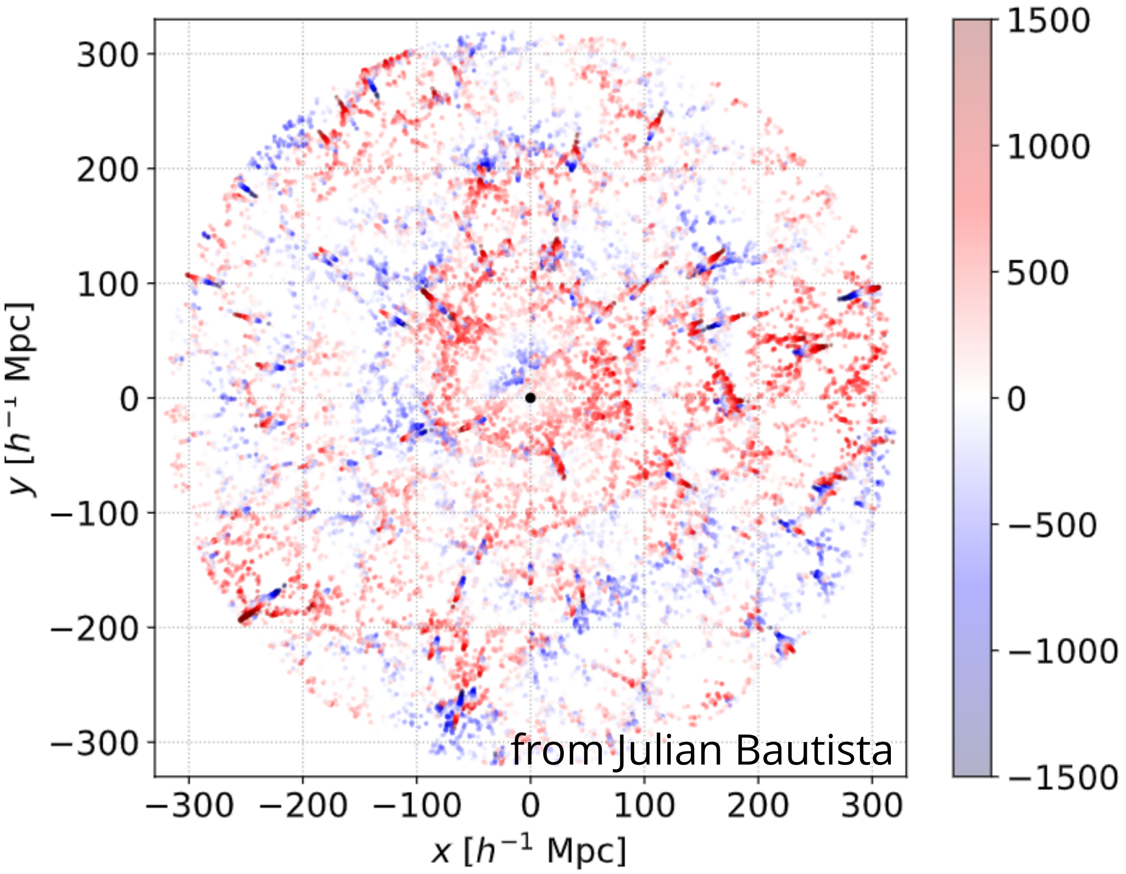
- 4m muti-object spectrograph at Kitt Peak Observatory
- 5000 robotic fiber positioners
- BAO and RSD measurements
- 40 M extra-galactic objects at Y5



DESI collaboration 2022

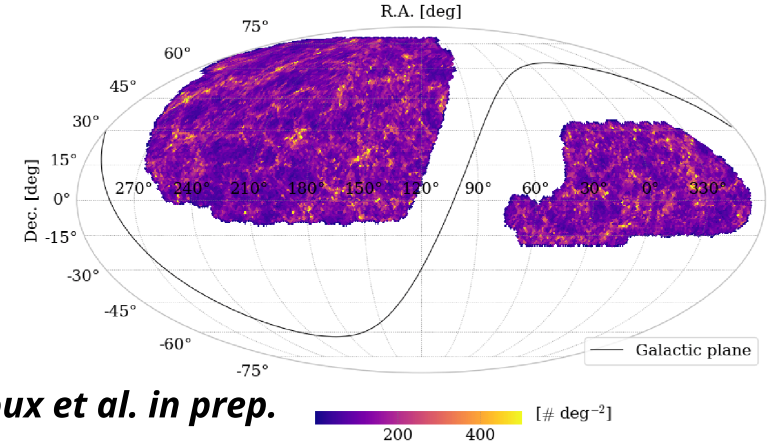
DESI x ZTF simulation

- 27 simulations from AbacusSummit (*Maksimova et al. 2019)*
- **Density**: DESI Bright Galaxy Survey (BGS) clustering matching for Y5 (J. Bautista, A. Smith)
- **Velocity:** ZTF SNIa Y6 simulation performed (following *Carreres et al. 2023*)

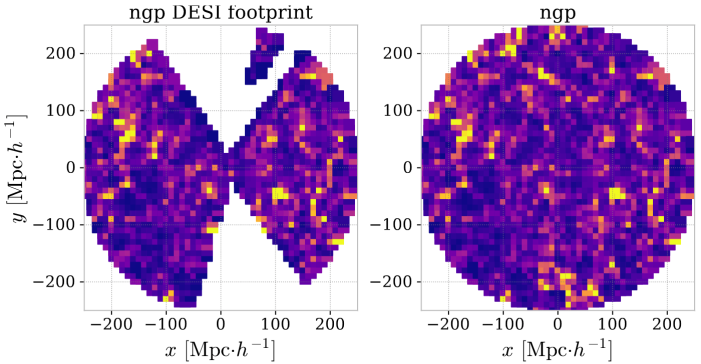


Galaxy field	Supernovae velocities

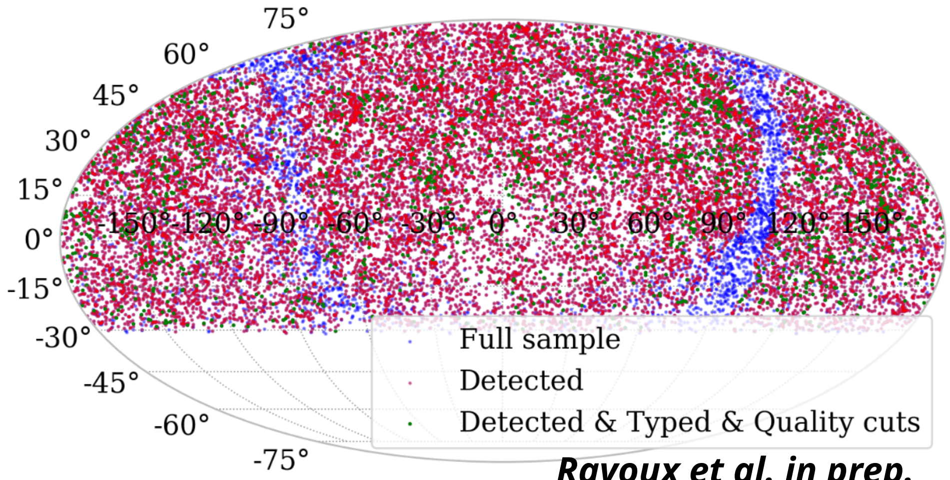
- DESI Y5 BGS footprint and density field gridding



Ravoux et al. in prep.



- *snsim* runs to mimic ZTF Y6 SN Ia field

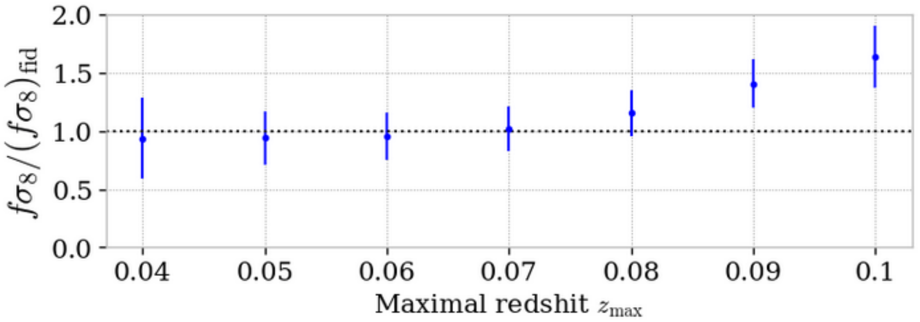


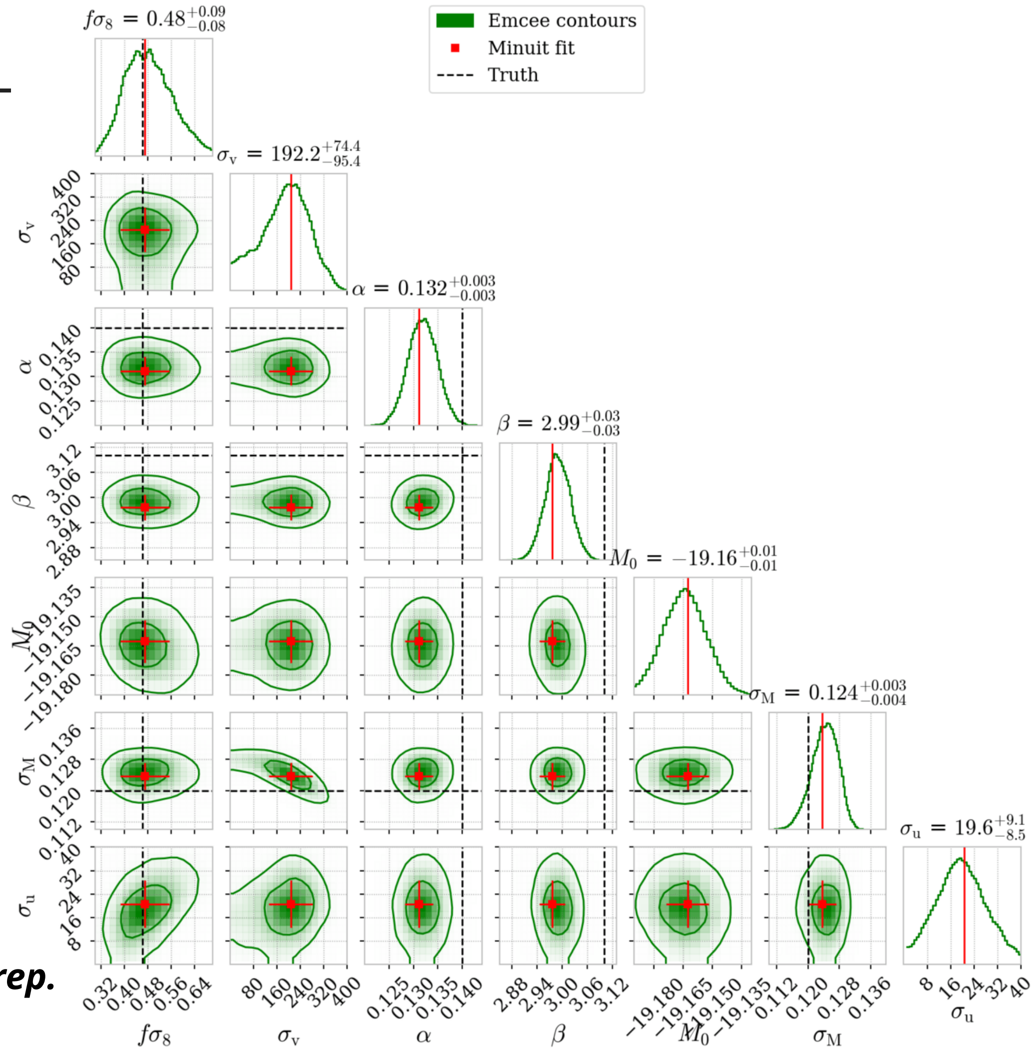
Ravoux et al. in prep.

SNSIM survey simulator

ZTF velocity fit

- Velocity fit including Hubble diagram parameters to reproduce *Carreres et al. 2023*
- Variation of analysis parameters (max redshift, SNIa density...)

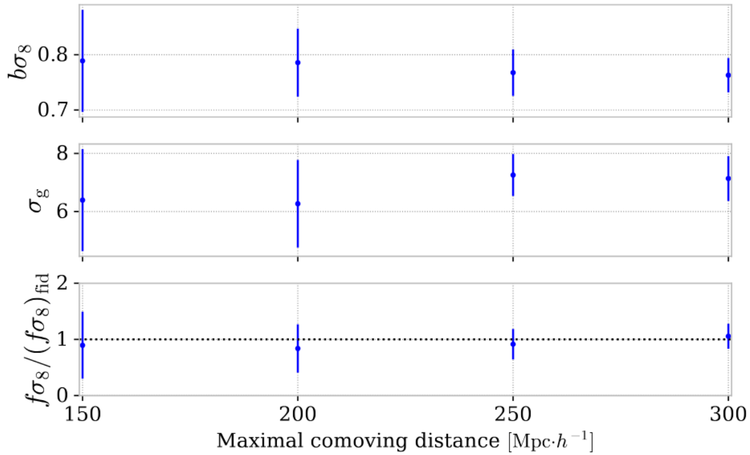




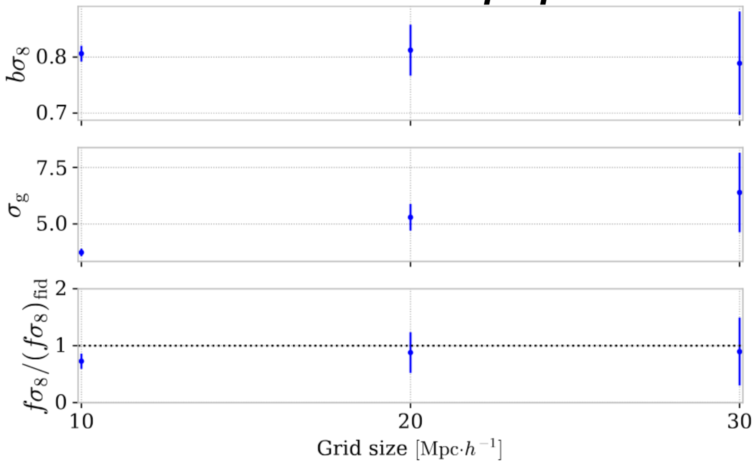
Ravoux et al. in prep.

DESI density fit

- Density grid with *pypower*
- Test varying **all density gridding parameters**: grid size, mesh cell size, interpolation scheme (ngc, cic, tsc, pcs), footprint (Y5 or full)
- Mean minuit fit on 27 mocks with average error bar



Ravoux et al. in prep.



DESI x ZTF joint fit

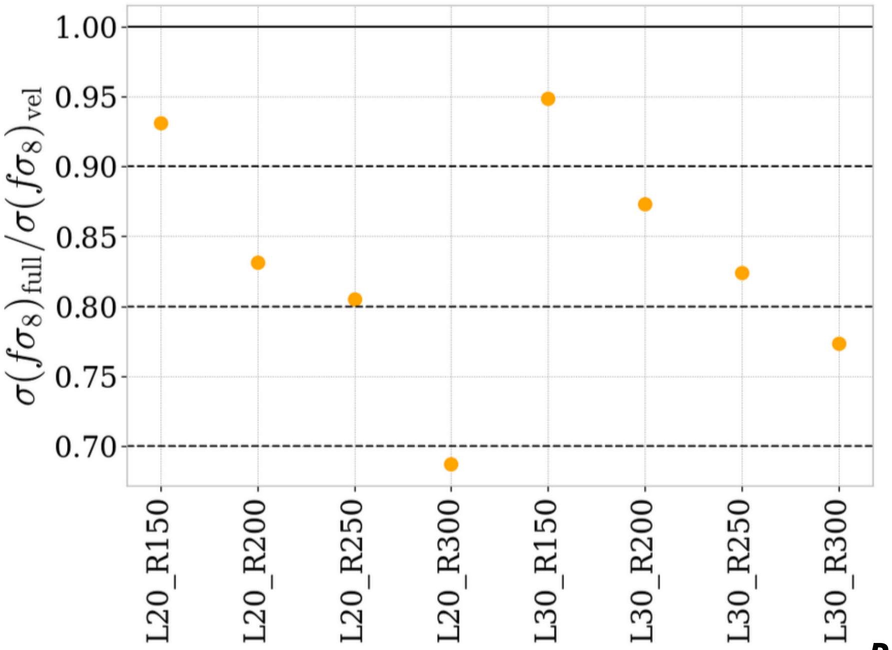
- For velocity x density, use of a new wide-angle model implemented in *flip*

$$
\begin{cases}
P_{\rm gg} = \left[b^2 P_{\rm mm}(k) + bf(\mu_1^2 + \mu_2^2)P_{\rm m\theta}(k) + f^2\mu_1^2\mu_2^2 P_{\theta\theta}(k)\right] \exp\left[\frac{-k^2(\mu_1^2+\mu_2^2)\sigma_{\rm g}^2}{2}\right] \\
P_{\rm gv} = iaH\frac{\mu_2}{k^2}\left(bfP_{\rm m\theta}(k) + f^2\mu_1^2 P_{\theta\theta}(k)\right) \exp\left[\frac{-k^2\mu_1^2\sigma_{\rm g}^2}{2}\right] D_u(k, \sigma_u) \\
P_{\rm vv} = (aHf)^2 \frac{\mu_1\mu_2}{k^2} P_{\theta\theta}(k) D_u^2(k, \sigma_u)
\end{cases}
$$

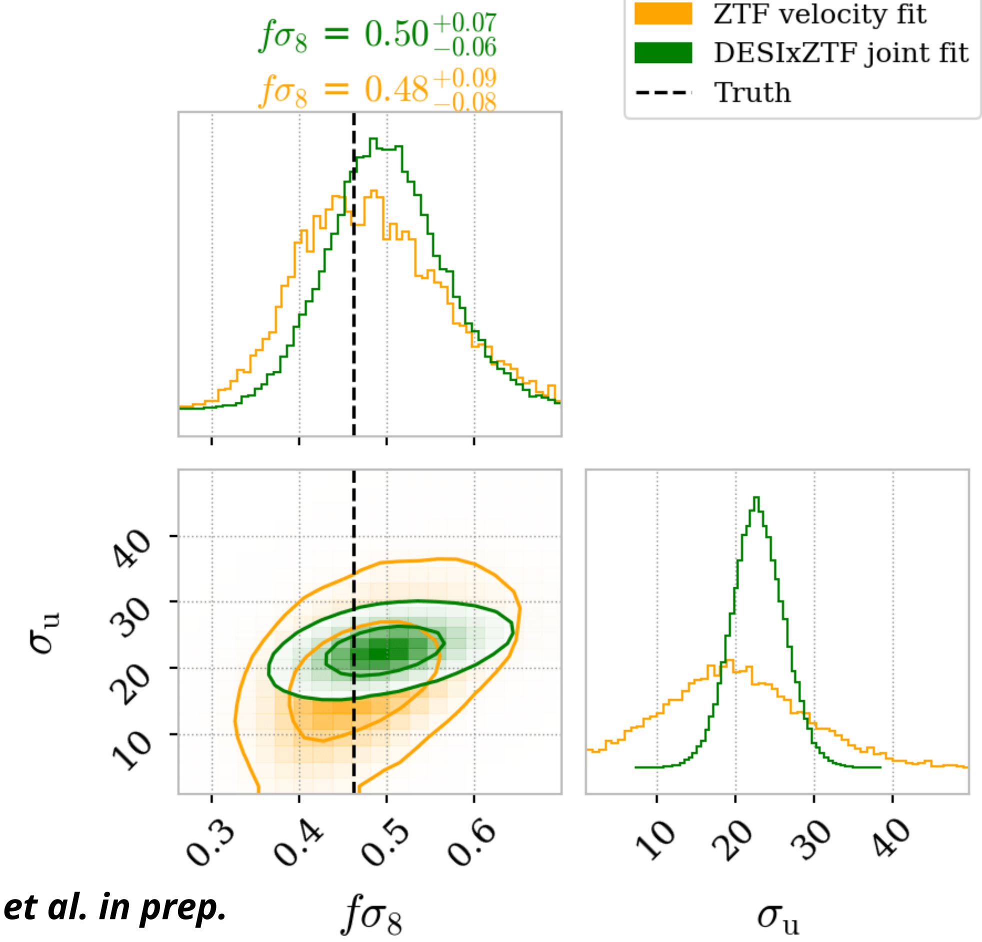
$$
\longrightarrow C_{\rm ab}(\mathbf{r_1}, \mathbf{r_2}) = \frac{1}{(2\pi)^3}\int_{\mathbf{k}} P_{\rm ab}(k, \mu_1, \mu_2) e^{i\mathbf{k}.\mathbf{r}} d^3\mathbf{k}
$$

DESI x ZTF joint fit

- SNIa velocity fit with Hubble diagram parameters and density field



Different density configurations

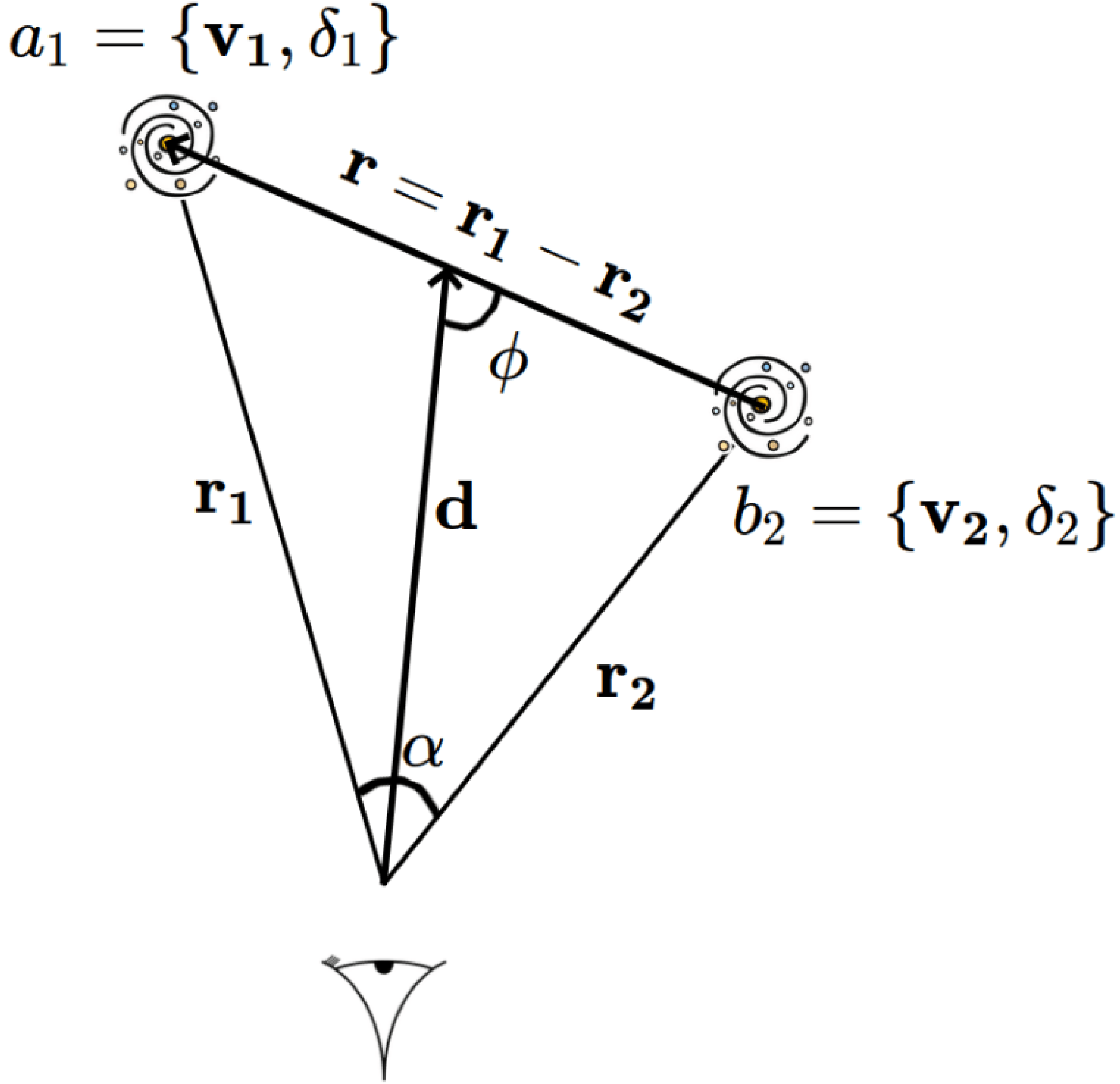


Ravoux et al. in prep.

Adding density improves fs8 constraints up to 30%

Conclusion

- Large modeling and simulation preparatory work for ZTF, LSST and combination with density fields
- Dedicated sofware for simulation *(snsim)* and analysis *(flip)*
- **Next steps**:
  - Tests on Pantheon+ data (Anthony Carr, KASI), DEBASS data (Bastien Carreres, Duke) and ZTF data (Raphael Kebadian, CPPM)
  - Implementation of non-linear models (EFTofLSS)
  - Combination with other velocity tracers (TF, FP)


$a_1 = \{\mathbf{v_1}, \delta_1\}$
$\mathbf{r} = \mathbf{r_1} - \mathbf{r_2}$
ϕ
$\mathbf{r_1}$
$\mathbf{d}$
$b_2 = \{\mathbf{v_2}, \delta_2\}$
$\mathbf{r_2}$
α

AB17 (Adams & Blake 2017) - Wide-angle model without RSD on density

Fields ab	Term number n	$w_{\mathrm{ab},n}$	$F_{\mathrm{ab},n}$	$\mathcal{P}_{\mathrm{ab},n}$
gg	0	$(b\sigma_8)^2$	1	$P_{\mathrm{mm}}(k)$
gv	0	$b\sigma_8 f\sigma_8$	$(iaH)\frac{\mu_2}{k}$	$P_{\mathrm{m}\theta}(k)D_{\mathrm{u}}(k,\sigma_{\mathrm{u}})$
vv	0	$(f\sigma_8)^2$	$(aH)^2\frac{\mu_1\mu_2}{k^2}$	$P_{\theta\theta}(k)D_{\mathrm{u}}^2(k,\sigma_{\mathrm{u}})$

AB20 (Adams & Blake 2020) - Plane-parallel model with RSD on density

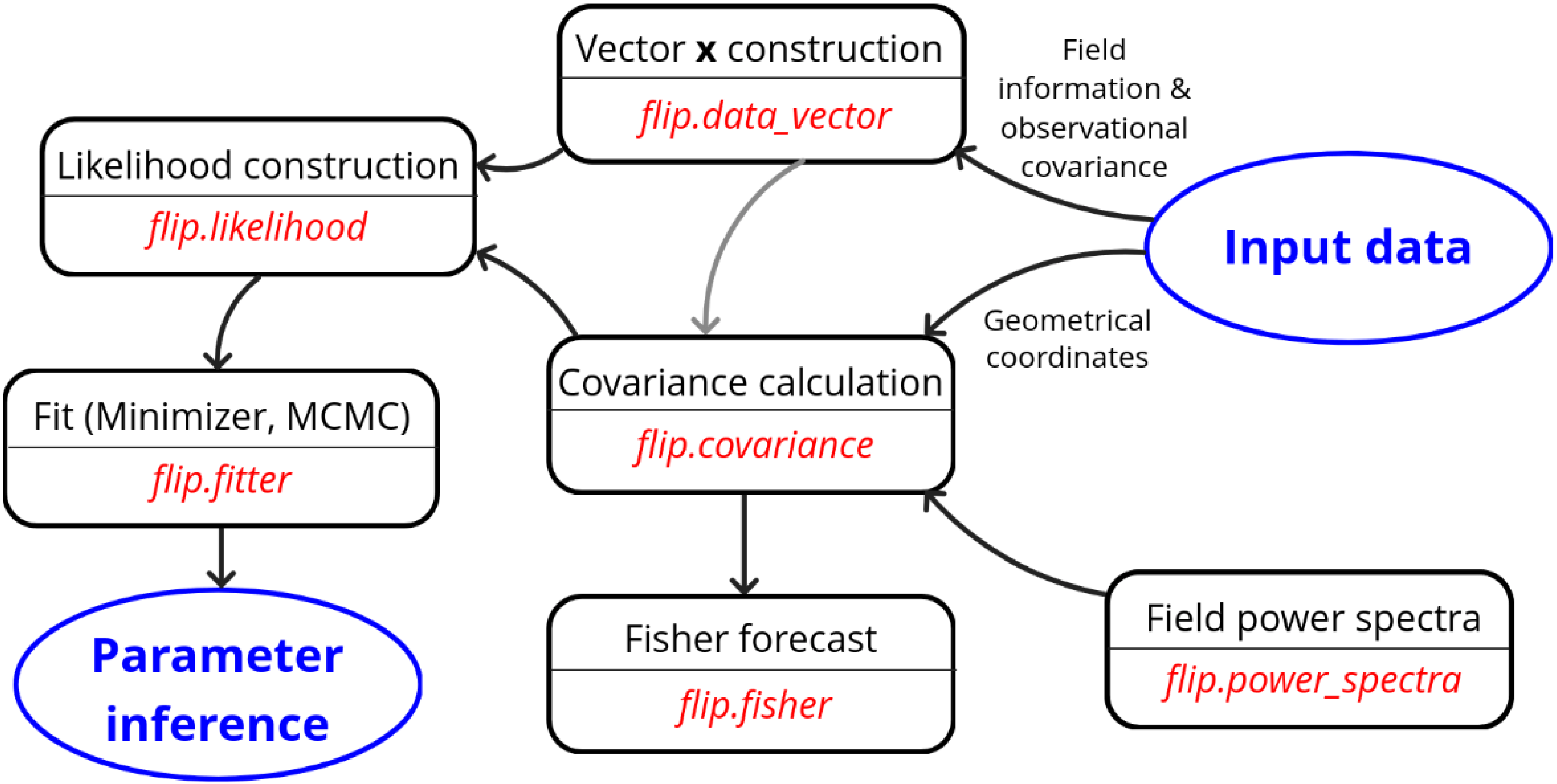
Field ab	Term n	$w_{\mathrm{ab},n}$	$F_{\mathrm{ab},n}$	$\mathcal{P}_{\mathrm{ab},n}$
gg	0	$(b\sigma_8)^2$	$\exp\left[-(k\sigma_g\mu)^2\right]$	$P_{\mathrm{mm}}(k)$
	1	$(b\sigma_8)^2\beta_f$	$2\mu^2\exp\left[-(k\sigma_g\mu)^2\right]$	$P_{\mathrm{m}\theta}(k)$
	2	$(b\sigma_8)^2\beta_f^2$	$\mu^4\exp\left[-(k\sigma_g\mu)^2\right]$	$P_{\theta\theta}(k)$
gv	0	$b\sigma_8 f\sigma_8$	$(iaH)\frac{\mu}{k}\exp\left[-\frac{(k\sigma_g\mu)^2}{2}\right]$	$P_{\mathrm{m}\theta}(k)D_{\mathrm{u}}(k,\sigma_{\mathrm{u}})$
	1	$(f\sigma_8)^2$	$(iaH)\frac{\mu^3}{k}\exp\left[-\frac{(k\sigma_g\mu)^2}{2}\right]$	$P_{\theta\theta}(k)D_{\mathrm{u}}(k,\sigma_{\mathrm{u}})$
vv	0	$(f\sigma_8)^2$	$(aH)^2\frac{\mu^2}{k^2}$	$P_{\theta\theta}(k)D_{\mathrm{u}}^2(k,\sigma_{\mathrm{u}})$

L22 (Lai et al. 2022) - Wide-angle model with RSD and Taylor expansion of FoG

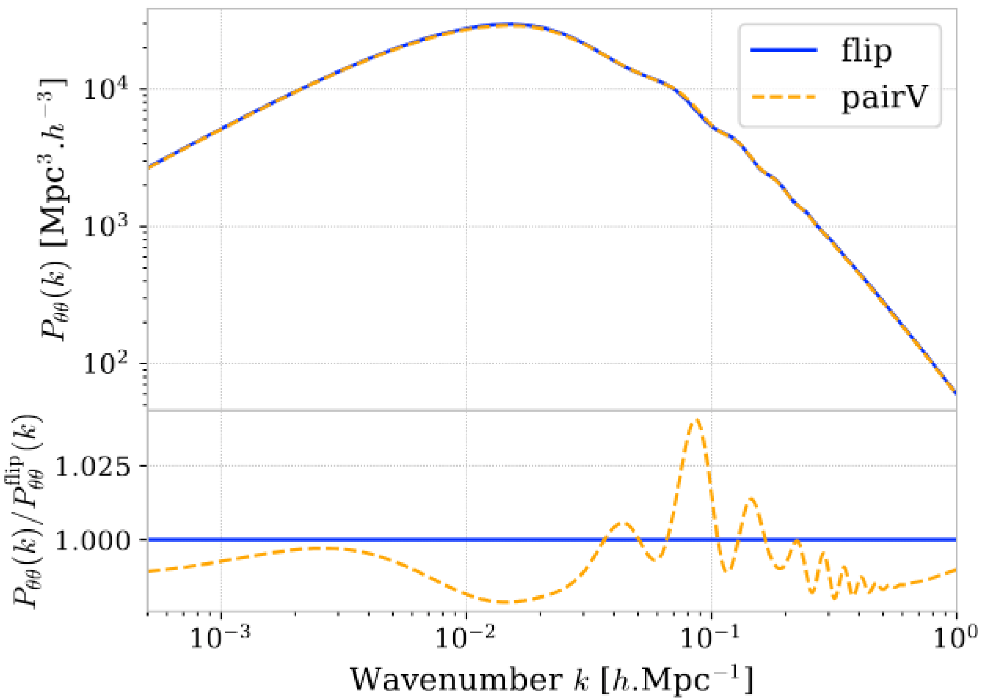
Field ab	Term n	$w_{\mathrm{ab},n}$	$F_{\mathrm{ab},n}$	$\mathcal{P}_{\mathrm{ab},n}$
gg	$0, m$	$(b\sigma_8)^2\sigma_g^{2m}$	$\sum_{p,q,p+q=m}\left(\frac{(-1)^{p+q}}{2^{p+q}p!q!}\right)k^{2(p+q)}\mu_1^{2p}\mu_2^{2q}$	$P_{\mathrm{mm}}(k)$
	$1, m$	$(b\sigma_8)^2\beta_f\sigma_g^{2m}$	$\sum_{p,q,p+q=m}\left(\frac{(-1)^{p+q}}{2^{p+q}p!q!}\right)k^{2(p+q)}\mu_1^{2p}\mu_2^{2q}(\mu_1^2+\mu_2^2)$	$P_{\mathrm{m}\theta}(k)$
	$2, m$	$(b\sigma_8)^2\beta_f^2\sigma_g^{2m}$	$\sum_{p,q,p+q=m}\left(\frac{(-1)^{p+q}}{2^{p+q}p!q!}\right)k^{2(p+q)}\mu_1^{2p+2}\mu_2^{2q+2}$	$P_{\theta\theta}(k)$
gv	$0, m$	$(b\sigma_8)^2\beta_f\sigma_g^{2m}$	$(iaH)\left(\frac{(-1)^m}{2^m m!}\right)k^{2m-1}\mu_2\mu_1^{2m}$	$P_{\mathrm{m}\theta}(k)D_{\mathrm{u}}(k,\sigma_{\mathrm{u}})$
	$1, m$	$(f\sigma_8)^2\sigma_g^{2m}$	$(iaH)\left(\frac{(-1)^m}{2^m m!}\right)k^{2m-1}\mu_2\mu_1^{2m+2}$	$P_{\theta\theta}(k)D_{\mathrm{u}}(k,\sigma_{\mathrm{u}})$
vv	0	$(f\sigma_8)^2$	$(aH)^2\frac{\mu_1\mu_2}{k^2}$	$P_{\theta\theta}(k)D_{\mathrm{u}}^2(k,\sigma_{\mathrm{u}})$

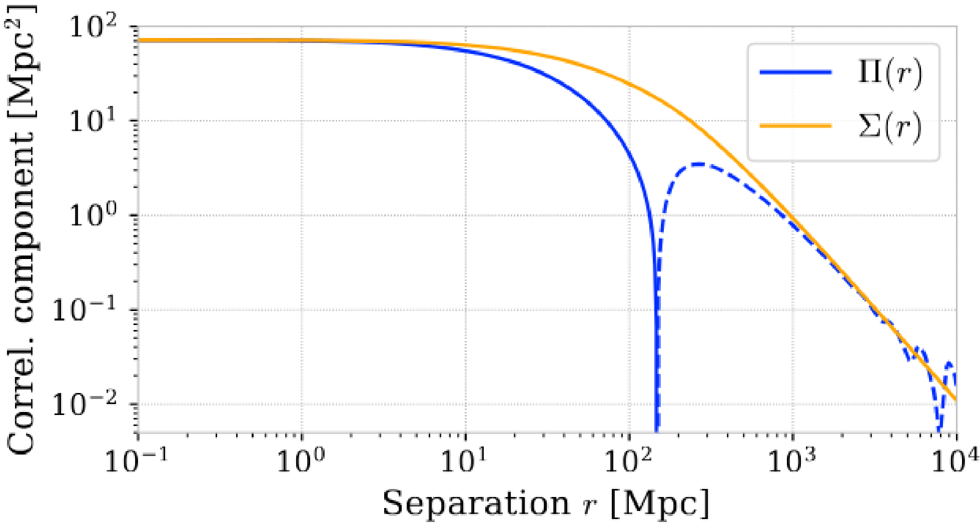
RC25 This study - Wide-angle model with RSD

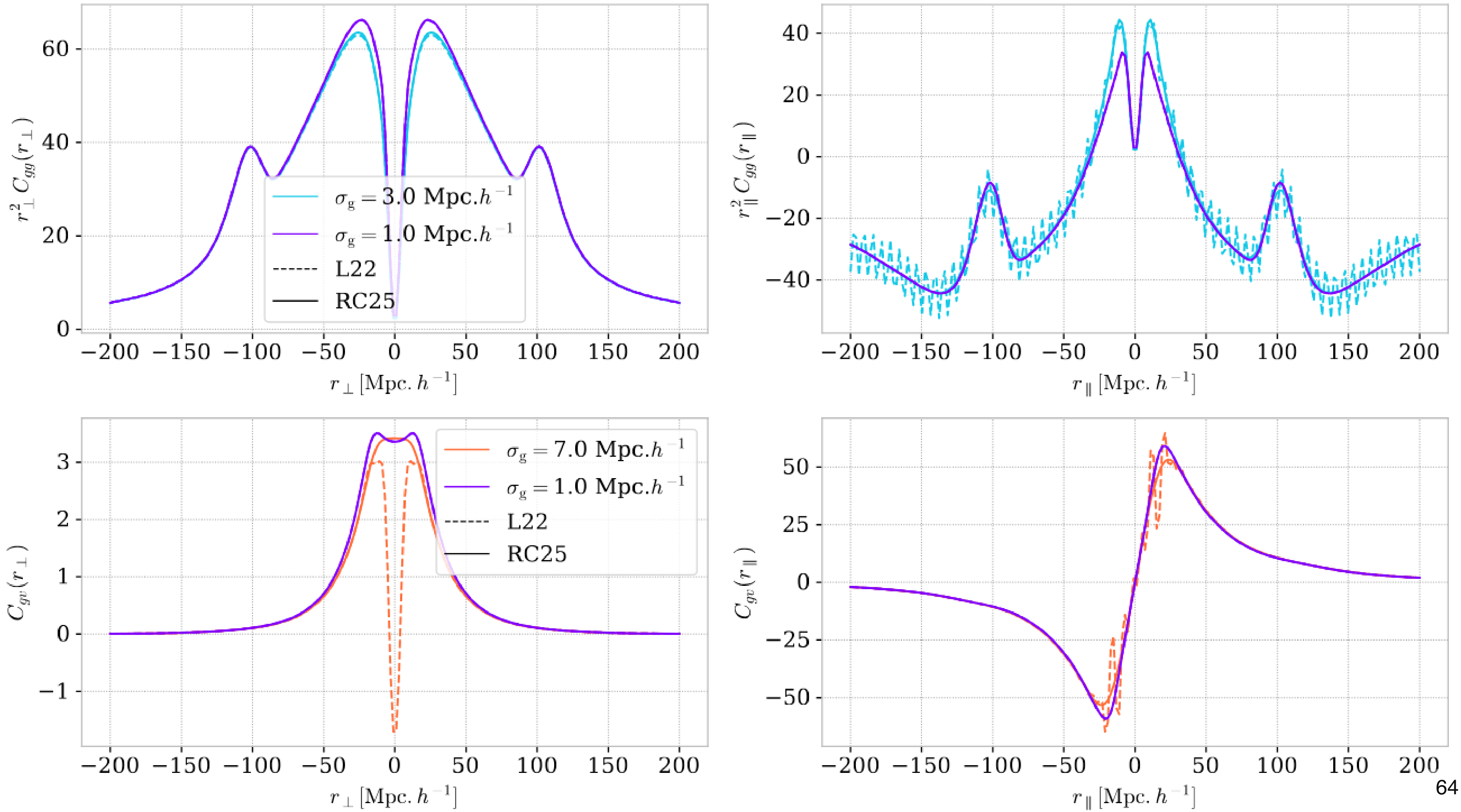
Field ab	Term n	$w_{\mathrm{ab},n}$	$F_{\mathrm{ab},n}$	$\mathcal{P}_{\mathrm{ab},n}$
gg	0	$(b\sigma_8)^2$	$\exp\left[-\frac{k^2\sigma_g^2(\mu_1^2+\mu_2^2)}{2}\right]$	$P_{\mathrm{mm}}(k)$
	1	$(b\sigma_8)^2\beta_f$	$(\mu_1^2+\mu_2^2)\exp\left[-\frac{k^2\sigma_g^2(\mu_1^2+\mu_2^2)}{2}\right]$	$P_{\mathrm{m}\theta}(k)$
	2	$(b\sigma_8)^2\beta_f^2$	$\mu_1^2\mu_2^2\exp\left[-\frac{k^2\sigma_g^2(\mu_1^2+\mu_2^2)}{2}\right]$	$P_{\theta\theta}(k)$
gv	0	$(b\sigma_8)^2\beta_f$	$(iaH)\frac{\mu_2}{k}\exp\left[-\frac{(k\sigma_g\mu_1)^2}{2}\right]$	$P_{\mathrm{m}\theta}(k)D_{\mathrm{u}}(k,\sigma_{\mathrm{u}})$
	1	$(f\sigma_8)^2$	$(iaH)\frac{\mu_2\mu_1^2}{k}\exp\left[-\frac{(k\sigma_g\mu_1)^2}{2}\right]$	$P_{\theta\theta}(k)D_{\mathrm{u}}(k,\sigma_{\mathrm{u}})$
vv	0	$(f\sigma_8)^2$	$(aH)^2\frac{\mu_1\mu_2}{k^2}$	$P_{\theta\theta}(k)D_{\mathrm{u}}^2(k,\sigma_{\mathrm{u}})$



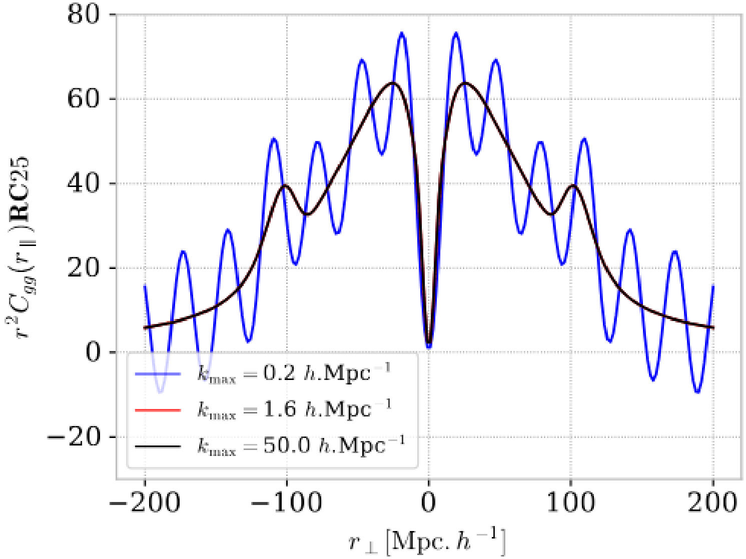
Vector x construction
flip.data_vector
Field information & observational covariance
Likelihood construction
flip.likelihood
Input data
Geometrical coordinates
Fit (Minimizer, MCMC)
flip.fitter
Covariance calculation
flip.covariance
Fisher forecast
flip.fisher
Field power spectra
flip.power_spectra
Parameter inference

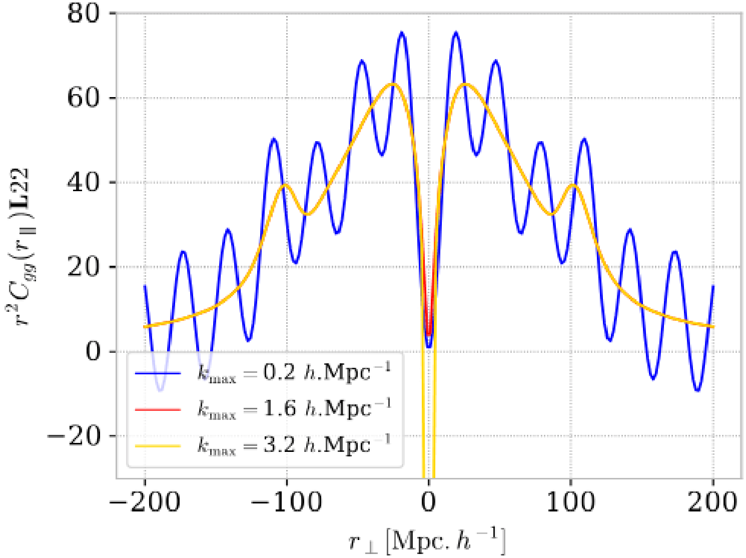

flip
pairV
$P_{\theta\theta}(k)$ [$\mathrm{Mpc}^3.h^{-3}$]
$P_{\theta\theta}(k)/P_{\theta\theta}^{\mathrm{flip}}(k)$
Wavenumber k [$h.\mathrm{Mpc}^{-1}$]

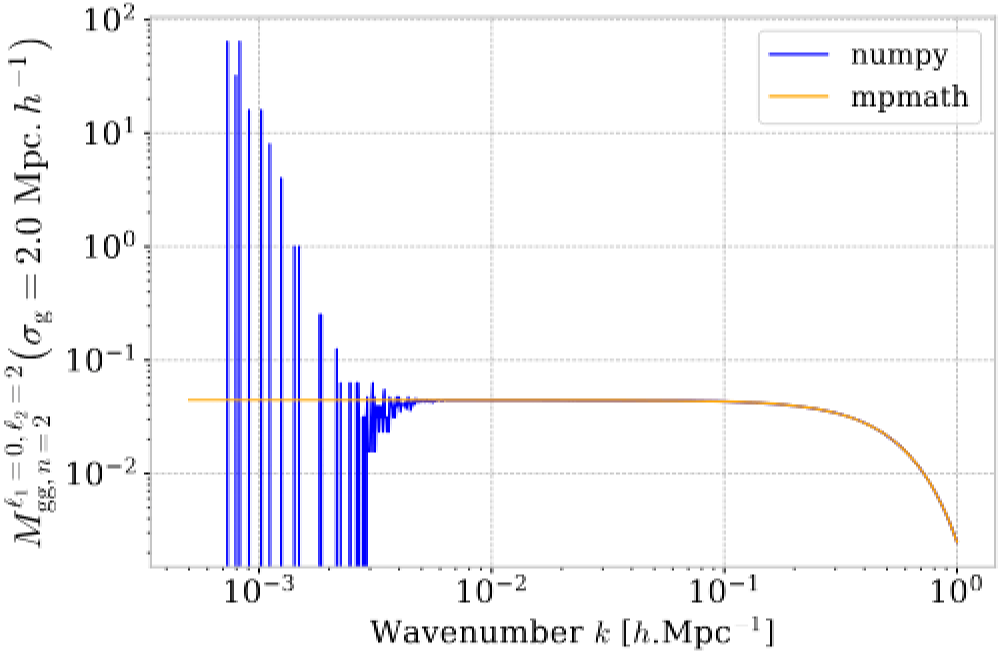

$\Pi(r)$
$\Sigma(r)$
Correl. component [Mpc^2]
Separation r [Mpc]



$r_\perp^2 C_{gg}(r_\perp)$
$\sigma_g = 3.0$ Mpc.h^{-1}
$\sigma_g = 1.0$ Mpc.h^{-1}
L22
RC25
$r_\perp$ [Mpc. h^{-1}]
$r_\parallel^2 C_{gg}(r_\parallel)$
$r_\parallel$ [Mpc. h^{-1}]
$C_{gv}(r_\perp)$
$\sigma_g = 7.0$ Mpc.h^{-1}
$\sigma_g = 1.0$ Mpc.h^{-1}
L22
RC25
$r_\perp$ [Mpc. h^{-1}]
$C_{gv}(r_\parallel)$
$r_\parallel$ [Mpc. h^{-1}]


$r^2 C_{gg}(r_\parallel)$ **RC25**
$r_\perp$ [Mpc. h^{-1}]
$k_{\rm max} = 0.2\ h.{\rm Mpc}^{-1}$
$k_{\rm max} = 1.6\ h.{\rm Mpc}^{-1}$
$k_{\rm max} = 50.0\ h.{\rm Mpc}^{-1}$


$r^2 C_{gg}(r_\parallel)$ **L22**
$r_\perp$ [Mpc. h^{-1}]
$k_{\rm max} = 0.2\ h.{\rm Mpc}^{-1}$
$k_{\rm max} = 1.6\ h.{\rm Mpc}^{-1}$
$k_{\rm max} = 3.2\ h.{\rm Mpc}^{-1}$


$M^{\ell_1=0,\,\ell_2=2}_{\rm gg,\,n=2}(\sigma_g = 2.0\ {\rm Mpc}.\ h^{-1})$
Wavenumber k [h.Mpc^{-1}]
numpy
mpmath