Decoherence and measurement process in Quantum Mechanics

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The measurement is not a unitary process

- $\hat{\rho} = |\psi\rangle\langle\psi| = \sum_{i,j} c_i c_j^* |o_i\rangle\langle o_j| = \sum_i |c_i|^2 |o_i\rangle\langle o_i| + \underbrace{\sum_{i\neq j} c_i c_j^* |o_i\rangle\langle o_j|}_{\textit{coherence}}$

- $\hat{\rho}' = \sum_i |c_i|^2 |o_i\rangle\langle o_i| = \sum_i \hat{P}_i\,\hat{\rho}\,\hat{P}_i, \quad \text{where} \quad \hat{P}_i \equiv |o_i\rangle\langle o_i|$ (experimental evidence)

Pre-measure phase : entanglement between subsystems

$$\mathcal{S} \quad \begin{array}{ll} \nearrow & \mathcal{H}_{\mathcal{S}} \\ \rightarrow & \hat{O}_{\mathcal{S}} |o_i\rangle = o_i |o_i\rangle \\ \searrow & |\psi_{\mathcal{S}}\rangle = \sum_i c_i |o_i\rangle \end{array}$$

$$\mathcal{A} \quad \begin{array}{ll} \nearrow & \mathcal{H}_{\mathcal{A}} \\ \rightarrow & \hat{O}_{\mathcal{A}} |a_i\rangle = a_i |a_i\rangle \\ \searrow & |\mathcal{A}\rangle = \sum_i a_i |a_i\rangle \end{array}$$

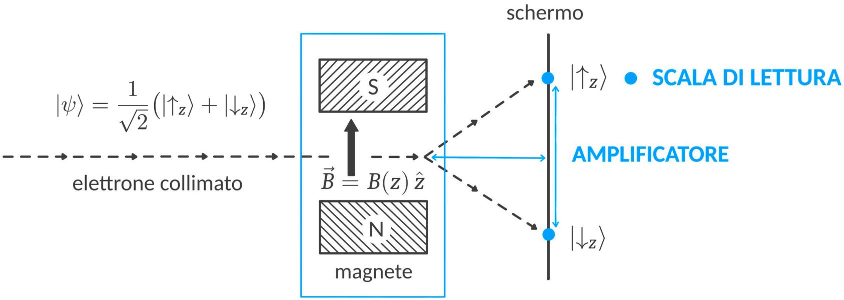
$$\boxed{|\Psi_{\mathcal{S}\mathcal{A}}(0)\rangle \equiv |\psi_{\mathcal{S}}(0)\rangle \otimes |\mathcal{A}(0)\rangle \qquad \mathcal{S},\ \mathcal{A}\ \text{completely decoupled systems}}$$

$$\hat{U}_{\mathcal{S}\mathcal{A}}(\tau, 0) = exp\left\{-\frac{i}{\hbar}\int_0^\tau dt\ \hat{H}_{\mathcal{S}\mathcal{A}}(t)\right\}, \quad \text{where} \quad \hat{H}_{\mathcal{S}\mathcal{A}}(t) \equiv \varepsilon_{\mathcal{S}\mathcal{A}}(t)\, \hat{O}_{\mathcal{S}} \otimes \hat{O}_{\mathcal{A}}$$

$$\boxed{|\Psi_{\mathcal{S}\mathcal{A}}(\tau)\rangle = \sum_i c_i |o_i\rangle |a_i\rangle \qquad \mathcal{S},\ \mathcal{A}\ \ \text{entangled systems}}$$

Ex. Stern-Gerlach's Apparatus

Stern-Gerlach's apparatus it is an example of a quantum measuring apparatus that converts a microscopic quantum property (the spin of an electron) into a macroscopically observable experimental result (the position of the particle on the screen).



MISURATORE : $\hat{H}_{int} = -\vec{\mu} \circ \vec{B}$, *dove* $\vec{\mu} = g_s(\frac{e\hbar}{2m})\vec{S}$

Problem of bi-orthogonal basis degeneration

It is demonstrated, through the **Uniqueness Theorem of the tri-orthogonal decomposition**, that by introducing a third system, the **environment** $\mathcal{E}$, the ambiguity regarding the measured quantity is resolved.

Between the '70s and '80s, the idea spread within the scientific community that the "collapse" of the wave function could be explained in terms of **decoherence processes resulting from the interaction of the system with the environment**.

Environment-induced superselection rules - Zurek's model

FUNDAMENTAL ASSUMPTIONS OF ZUREK'S MODEL

I. All physical systems are *open quantum systems*
II. The environment can be modeled as a *heat bath*

Example: 2-level system

- Particle $\mathcal{S}$: $|\psi\rangle = c_1 |\uparrow\rangle + c_2 |\downarrow\rangle\,,$ where $\{|\uparrow\rangle\,, |\downarrow\rangle\}$ basis ON of $\mathcal{H}_{\mathcal{S}}$
- Apparatus $\mathcal{A}$: $|\mathcal{A}_0\rangle = d_1 |a_\uparrow\rangle + d_2 |a_\downarrow\rangle\,,$ where $\{|a_\uparrow\rangle\,, |a_\downarrow\rangle\}$ basis ON of $\mathcal{H}_{\mathcal{A}}$
- Reservoir $\mathcal{E}$: $|\mathcal{E}\rangle = \sum_{i=1}^{N} h_i |e_i\rangle\,,$ where $\{|e_i\rangle\}_{i=1,\dots,N},$ basis ON of $\mathcal{H}_{\mathcal{E}}$

1. $\mathcal{S}$, $\mathcal{A}$, $\mathcal{E}$ completely decoupled systems

$$|\Psi_{\mathcal{SAE}}(0)\rangle \equiv |\psi\rangle\,|\mathcal{A}_0\rangle\,|\mathcal{E}(0)\rangle$$

2. $(\mathcal{S}+\mathcal{A})$, $\mathcal{E}$ decoupled systems

$$|\Psi_{\mathcal{SAE}}(t_1)\rangle = \Big(g_1\,|\uparrow, a_\uparrow\rangle + g_2\,|\downarrow, a_\downarrow\rangle\Big)\,|\mathcal{E}(t_1)\rangle$$

$$\Rightarrow\quad \hat{\tilde{\rho}}_{\mathcal{SA}}(t_1) = Tr_{\mathcal{E}}\left[\hat{\rho}_{\mathcal{SAE}}(t_1)\right] = \begin{pmatrix} |g_1|^2 & g_1^*\,g_2 \\ g_1\,g_2^* & |g_2|^2 \end{pmatrix}$$

Without coupling to the environment, the apparatus entangled with the system would yield a superposition of macroscopic outcomes (Schrödinger's cat paradox).

3. $\mathcal{S}, \mathcal{A}, \mathcal{E}$ (partially) entangled systems

$$|\Psi_{\mathcal{SAE}}(t_2)\rangle = g_1\,|\uparrow, a_\uparrow, \epsilon_1\rangle + g_2\,|\downarrow, a_\downarrow, \epsilon_2\rangle\,, \quad \text{where} \quad \langle\epsilon_1|\epsilon_2\rangle \neq 0$$

$$\Rightarrow \quad \hat{\hat{\rho}}_{\mathcal{SA}}(t_2) = Tr_{\mathcal{E}}\left[\hat{\rho}_{\mathcal{SAE}}(t_2)\right] = \begin{pmatrix} |g_1|^2 & g_1^*\,g_2\,\langle\epsilon_1|\epsilon_2\rangle \\ g_1\,g_2^*\,\langle\epsilon_2|\epsilon_1\rangle & |g_2|^2 \end{pmatrix}$$

Only the coherences of the reduced density matrix of $\mathcal{S} + \mathcal{A}$ are reduced by a factor $|\langle\epsilon_2|\epsilon_1\rangle| \leq 1$ due to entanglement with the surrounding environment. Successive interactions with the reservoir cause the coherences to asymptotically approach zero (the faster the environment is *hotter*).

Thanks for your attention!