Higgs mechanism

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Global symmetries

- $U(1) : \phi \to \phi' = \exp\{ieQ\}\phi$

$$\mathcal{L}_0 = i\overline{\psi}(x)\gamma^\mu\partial_\mu\psi(x) - m\overline{\psi}\psi, \tag{1}$$

- $SU(2)_L \otimes U(1)_Y : \psi(x) \xrightarrow{G} \psi'(x) \equiv \exp\{iy_1\beta\}U_L\phi,$

$$U_L \equiv \exp\{i\frac{\sigma_i}{2}\alpha^i\}, \quad Y = Q - \frac{\sigma_3}{2}.$$

$$\psi_1(x) = \begin{pmatrix} u \\ d \end{pmatrix}_L, \qquad \psi_2(x) = u_R, \qquad \psi_3(x) = d_R. \tag{2}$$

$$\mathcal{L}_0 = \sum_{j=1}^{3} i\overline{\psi}_j(x)\gamma^\mu\partial_\mu\psi_j(x). \tag{3}$$

The Goldstone's theorem

$$\mathcal{L} = \partial_\mu \phi^\dagger \partial^\mu \phi - V(\phi), \qquad V(\phi) = \mu^2 \phi^\dagger \phi + h(\phi^\dagger \phi)^2, \tag{4}$$

Invariant under global phase transformations.
For $\mu^2 < 0$, the minimum of the potential is given for

$$|\phi_0| = \sqrt{\frac{-\mu^2}{2h}} \equiv \frac{v}{\sqrt{2}}. \tag{5}$$

Hence, there is an infinite number of minima, $\phi_0 = \frac{v}{\sqrt{2}} e^{i\theta}$.
Choosing $\theta = 0$ as the ground state, the symmetry gets spontaneously broken.

The Goldstone theorem:
When a global symmetry gets spontaneously broken, a massless boson then appears for every broken generator.

Spontaneous Symmetry Breaking (SSB)

$$\partial_\mu \phi(x) \to D_\mu \phi(x) = [\partial_\mu + ieQA_\mu(x)]\,\phi(x) \tag{6}$$

The new Lagrangian takes the form:

$$\mathcal{L}(x) = [D^\mu \phi]^* \, [D_\mu \phi] - \mu^2 \phi^\dagger \phi - h(\phi^\dagger \phi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}. \tag{7}$$

We take $\phi(x) = \frac{1}{\sqrt{2}}[v + \varphi_1(x) + i\varphi_2(x)]$. Then,

$$\begin{aligned}\mathcal{L}(x) = &\frac{1}{2}\partial^\mu \varphi_1(x) \partial_\mu \varphi_1(x) - \frac{1}{2}(2hv^2)\varphi_1^2(x) \\ &- \frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) + \frac{1}{2}(eQv)^2 A_\mu(x) A^\mu(x) \\ &+ \frac{1}{2}\partial^\mu \varphi_2(x) \partial_\mu \varphi_2(x) + eQvA^\mu(x)\partial_\mu \varphi_2(x) + \text{int. terms}\end{aligned} \tag{8}$$

$$\phi(x) = \begin{pmatrix} \phi^{(+)}(x) \\ \phi^{(0)}(x) \end{pmatrix}, \qquad \mathcal{L} = \partial_\mu \phi^\dagger \partial^\mu \phi - \mu^2 \phi^\dagger \phi - h(\phi^\dagger \phi)^2. \tag{9}$$

Replacing the ordinary derivative with a covariant one:

$$D^\mu \phi(x) = \left[\partial^\mu + ig\widetilde{W}^\mu(x) + ig' y_\phi B^\mu(x)\right] \phi(x). \tag{10}$$

We can rewrite the field as

$$\phi(x) = e^{i\frac{\sigma_i}{2}\theta^i(x)} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}. \tag{11}$$

Now we can use the local gauge invariance to get rotate away any dependence on $\theta^i(x)$.

Higgs mechanism

$U(1)_{QED}$ still remains a symmetry:

$$\phi'(x) = \frac{e^{i(Y+T_3)\theta}}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} = \phi(x). \tag{12}$$

The kinetic part of the Lagrangian gives rise to the terms:

$$(D_\mu \phi)^\dagger D^\mu \phi \to \frac{1}{2}\partial_\mu H \partial^\mu H + (v+H)^2 \left\{ \frac{g^2}{4} W_\mu^\dagger W^\mu + \frac{g^2}{8\cos^2\theta_W} Z_\mu Z^\mu \right\}. \tag{13}$$

Consequently,

$$M_Z = \frac{1}{2\cos\theta_W} vg, \qquad M_W = \frac{1}{2} vg, \qquad M_H = \sqrt{2hv^2}. \tag{14}$$

Higgs mechanism

$$\mathcal{L}_Y = -c_1(\bar{u}, \bar{d})_L \phi d_R - c_2(\bar{u}, \bar{d})_L \widetilde{\phi} u_R - c_3(\bar{\nu}_e, \bar{e})_L \phi e_R + \mathrm{h.c.}, \qquad (15)$$

where

$$\widetilde{\phi} = i\sigma_2 \phi^* = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix}^* = \begin{pmatrix} \phi^{(0)*} \\ -\phi^{(-)} \end{pmatrix} \qquad (16)$$

is the C-conjugate of ϕ.

In the unitary gauge, it takes the form

$$\mathcal{L}_Y = -\frac{1}{\sqrt{2}}(v + H)\left\{c_1 \bar{d} d + c_2 \bar{u} u + c_3 \bar{e} e\right\}. \qquad (17)$$

Then,

$$m_d = c_1 \frac{v}{\sqrt{2}}, \qquad m_u = c_2 \frac{v}{\sqrt{2}}, \qquad m_e = c_3 \frac{v}{\sqrt{2}}. \qquad (18)$$

Conclusions

When the ordinary derivative is replaced with a covariant derivative, the Goldstone bosons get instead 'reabsorbed' by the bosons, acquiring mass in the process.

Therefore, we have learnt a mechanism that

- Provides every particle their corresponding mass.
- Leaves the photon massless.
- Introduces a new scalar boson, H.

THANK YOU