Summer School
on Particle and
Astroparticle Physics

Gravitational waves

Presented By
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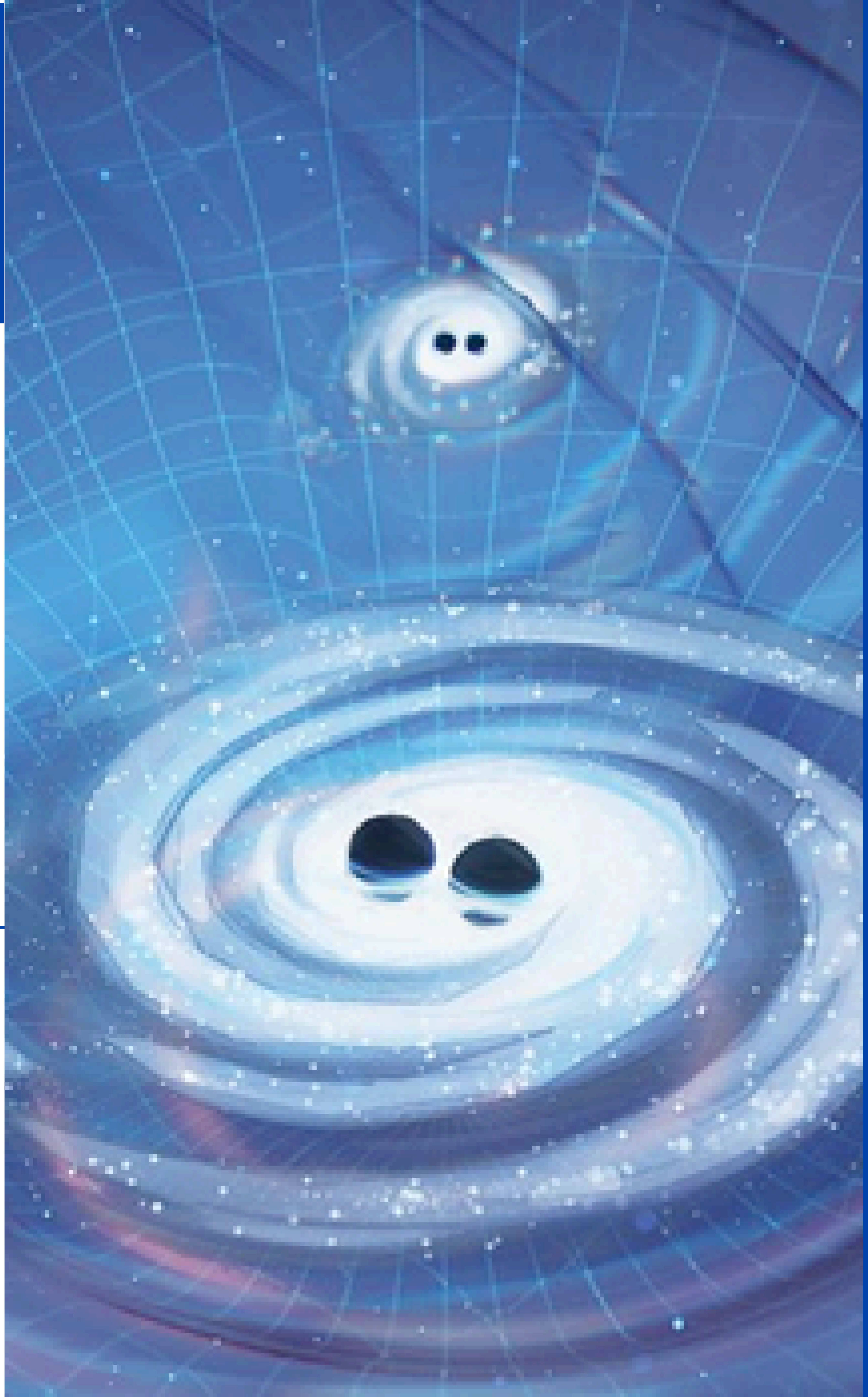
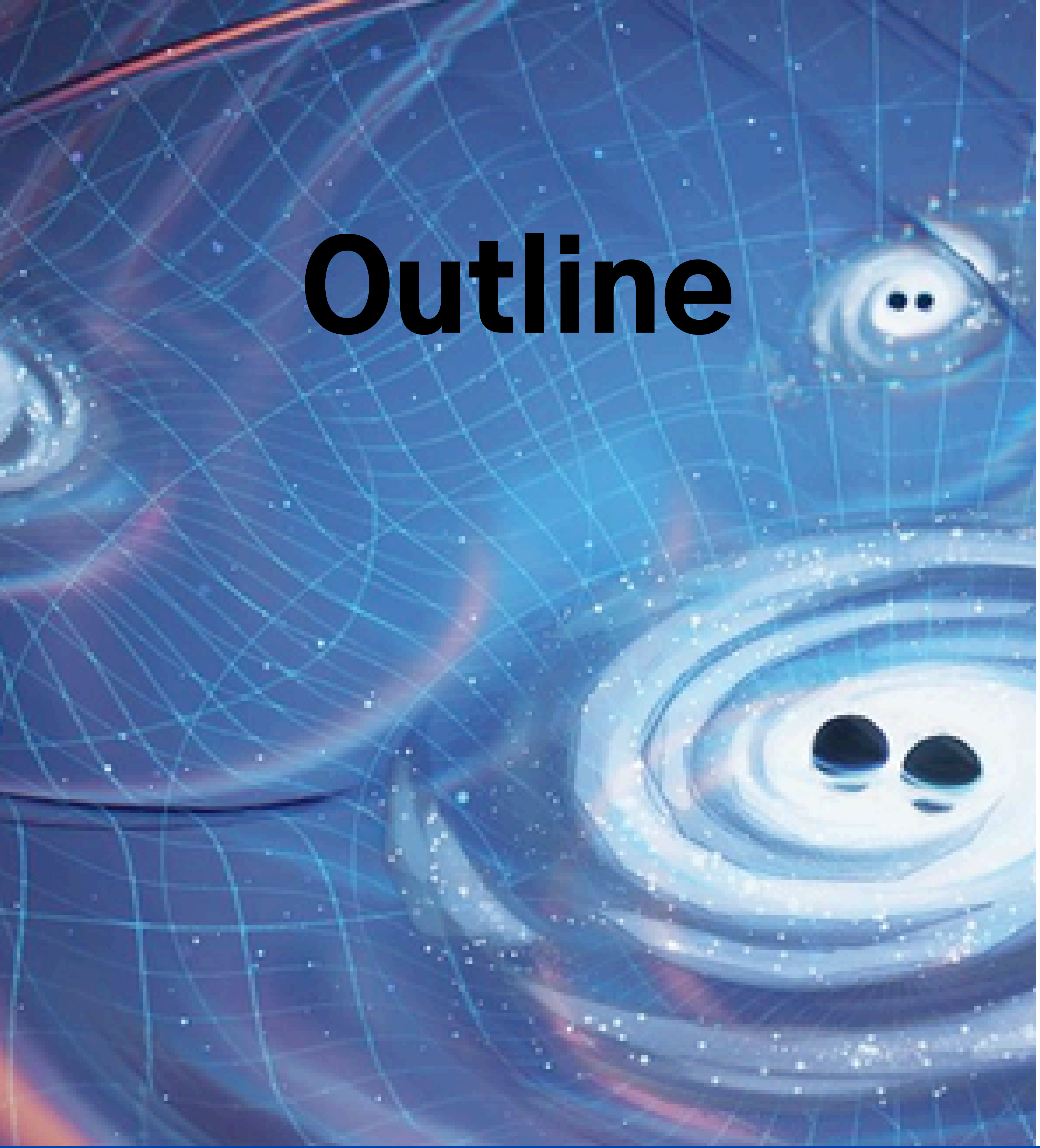
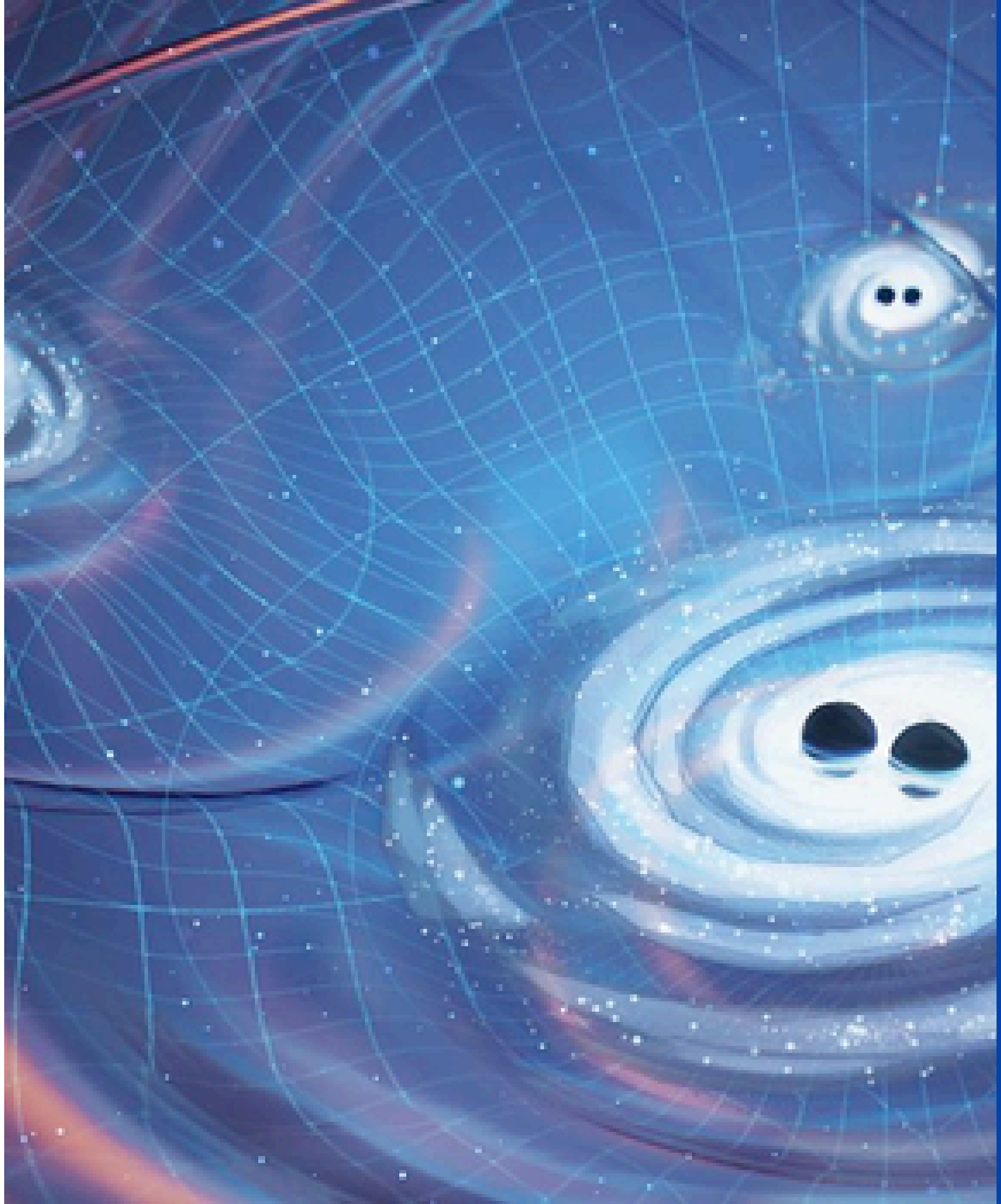
Laboratoire d'Annecy de Physique des Particules





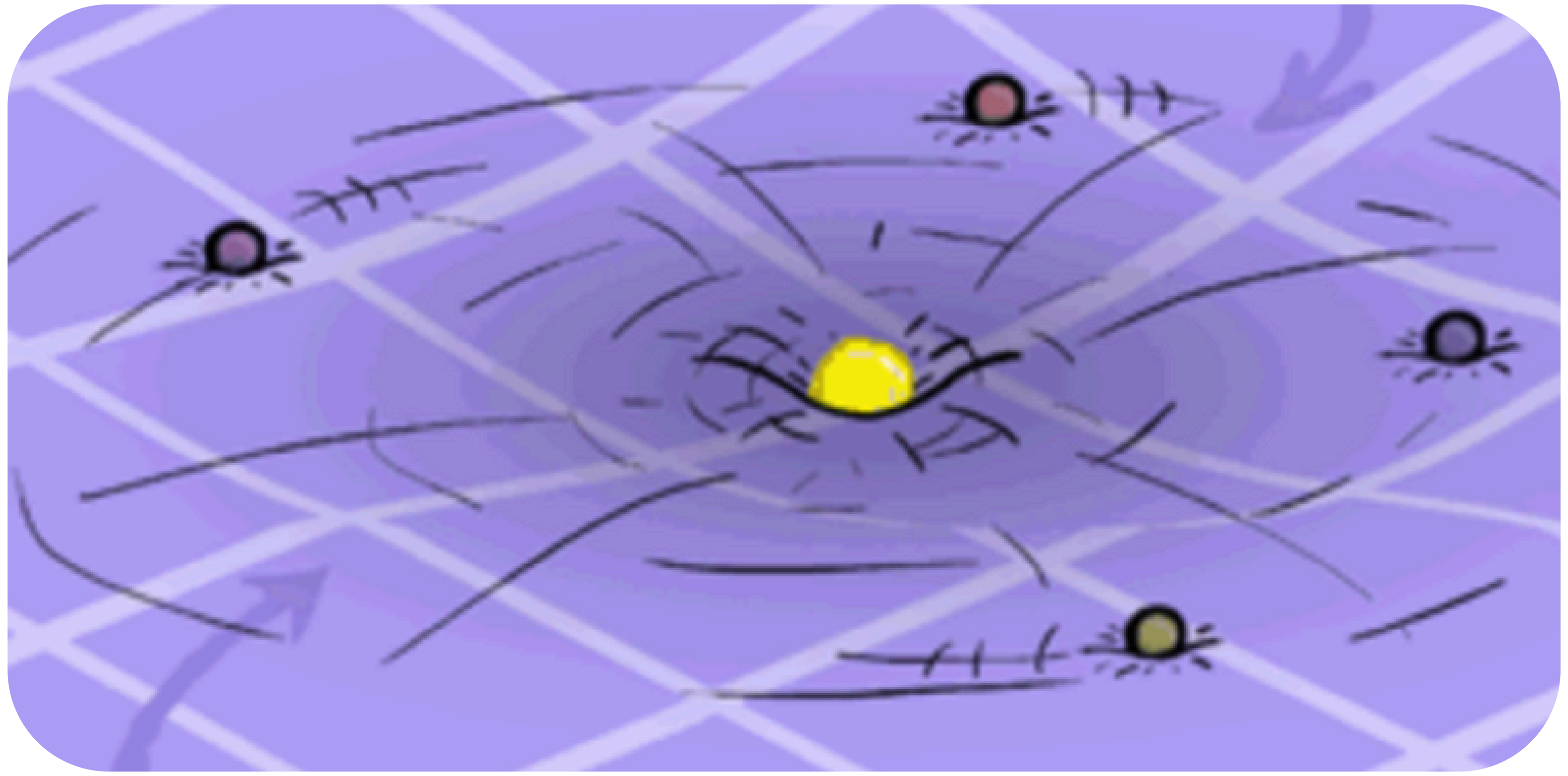
Outline

General Relativity



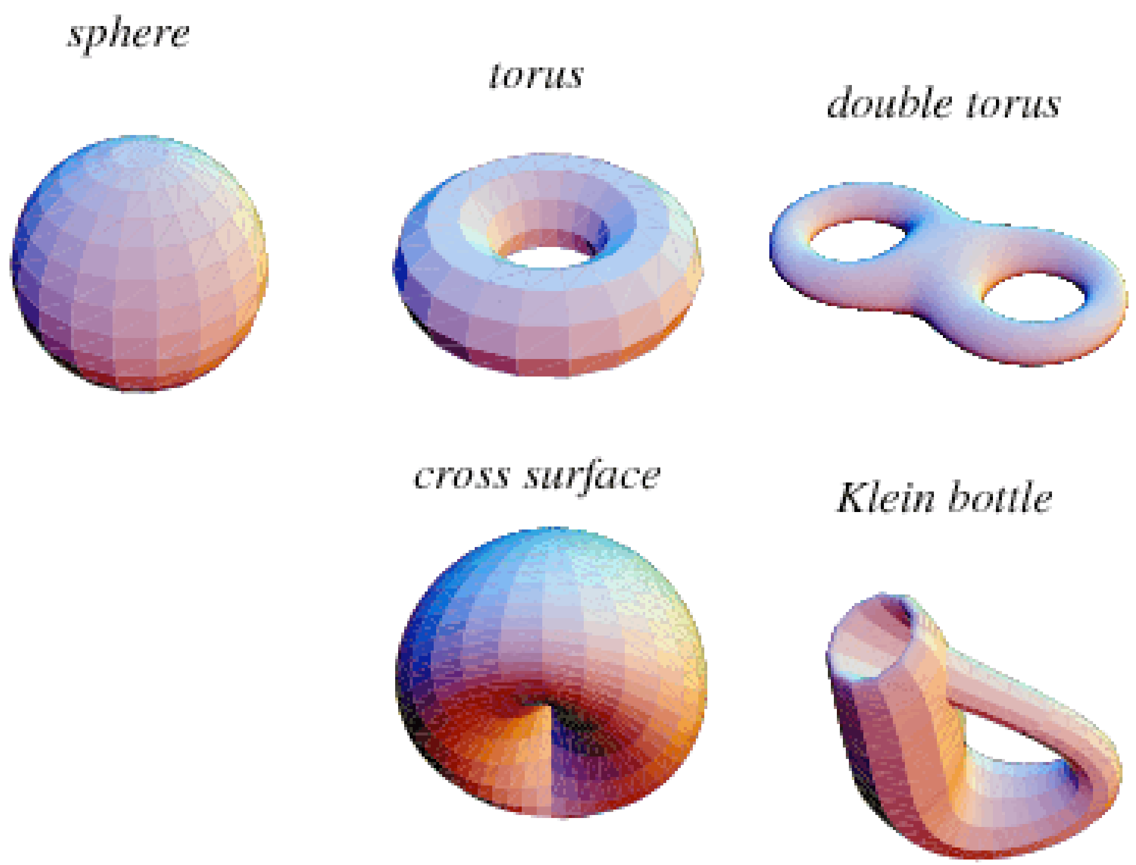
General Relativity

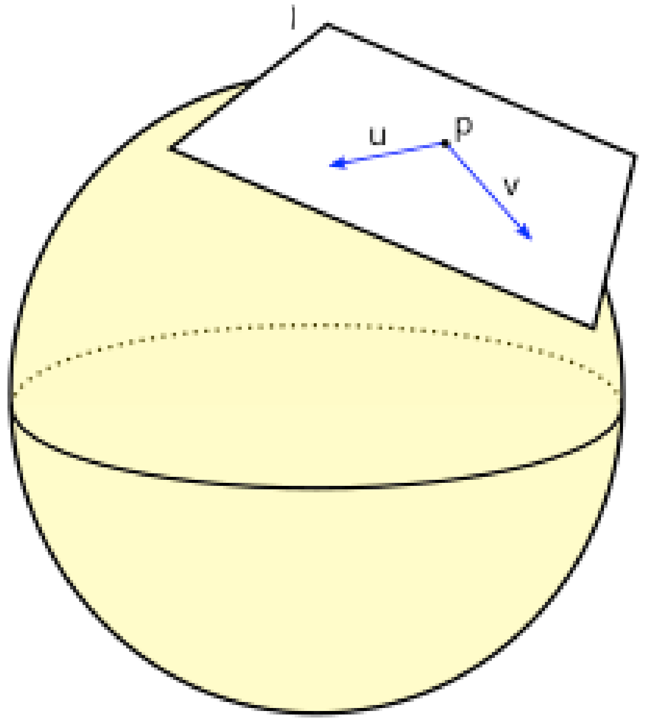
- General relativity (GR) is the best available theory of gravity
- It describes the interaction between massive bodies as an effect of the curvature of spacetime
- In Einstein's universe, objects moving freely under the effects of gravity simply follow ***geodesic*** paths dictated by the ***curvature of spacetime.***



Curved space time

- Einstein used Riemannian geometry to describe GR
- In the XIXth century Riemann studied curved spaces in any dimension
- ***Manifold***: generalizes the notions of curve (1D manifold) and surfaces (2D manifold) to spaces having more than 2D (n-D manifolds)
- A fundamental principle of this geometry: ***in a small neighbourhood of a point, non-Euclidean manifolds agree with Euclidean geometry***

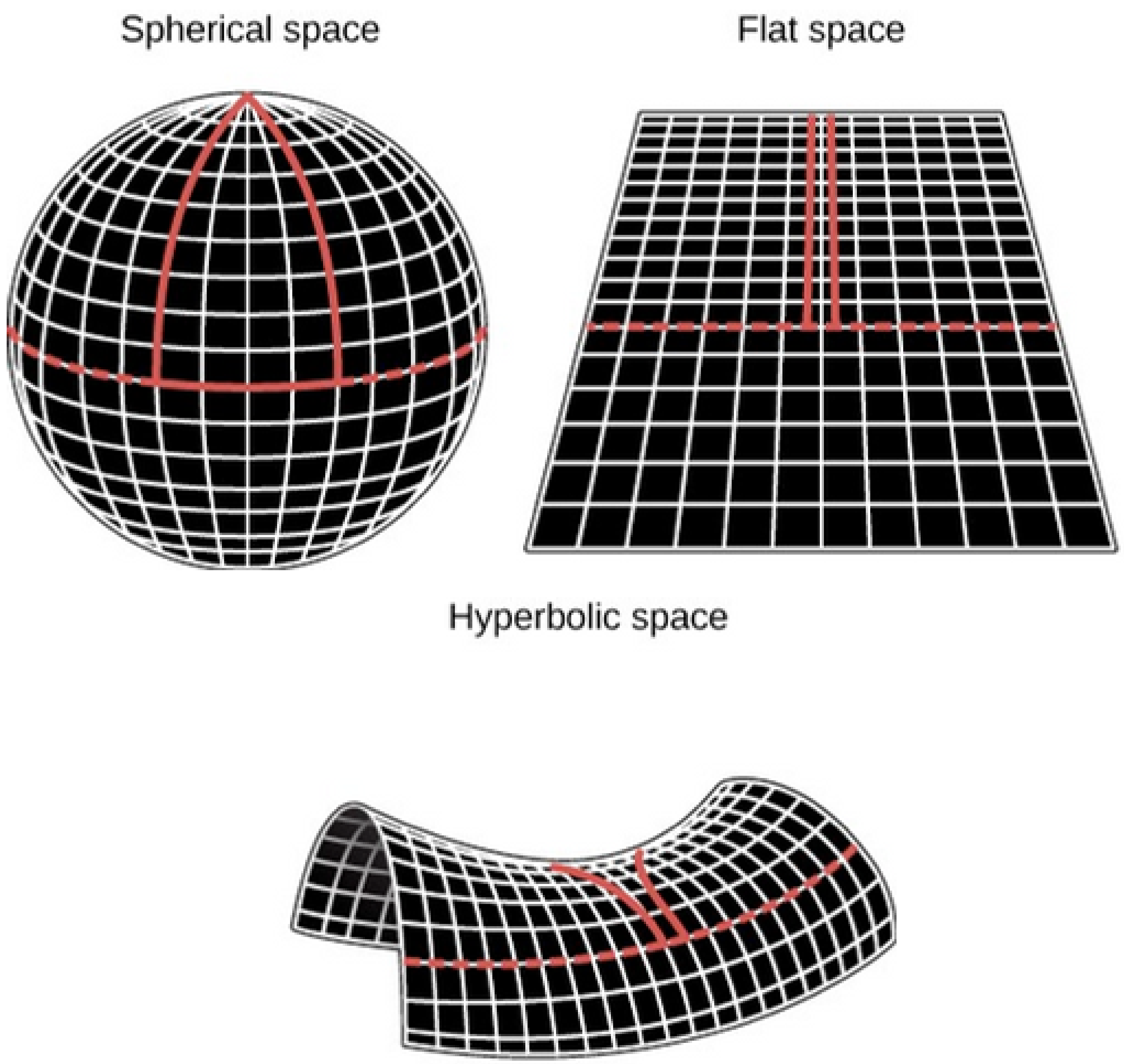




Curved space time

- The fundamental mathematical object of Riemannian geometry is the **metric, gij**
- The metric is represented by an n x n matrix that is symmetric

$$[g_{ij}] = \begin{pmatrix} g_{11} & g_{12} & \cdots & g_{1n} \\ g_{21} & g_{22} & \cdots & g_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ g_{n1} & g_{n2} & \cdots & g_{nn} \end{pmatrix}$$



Curved space time

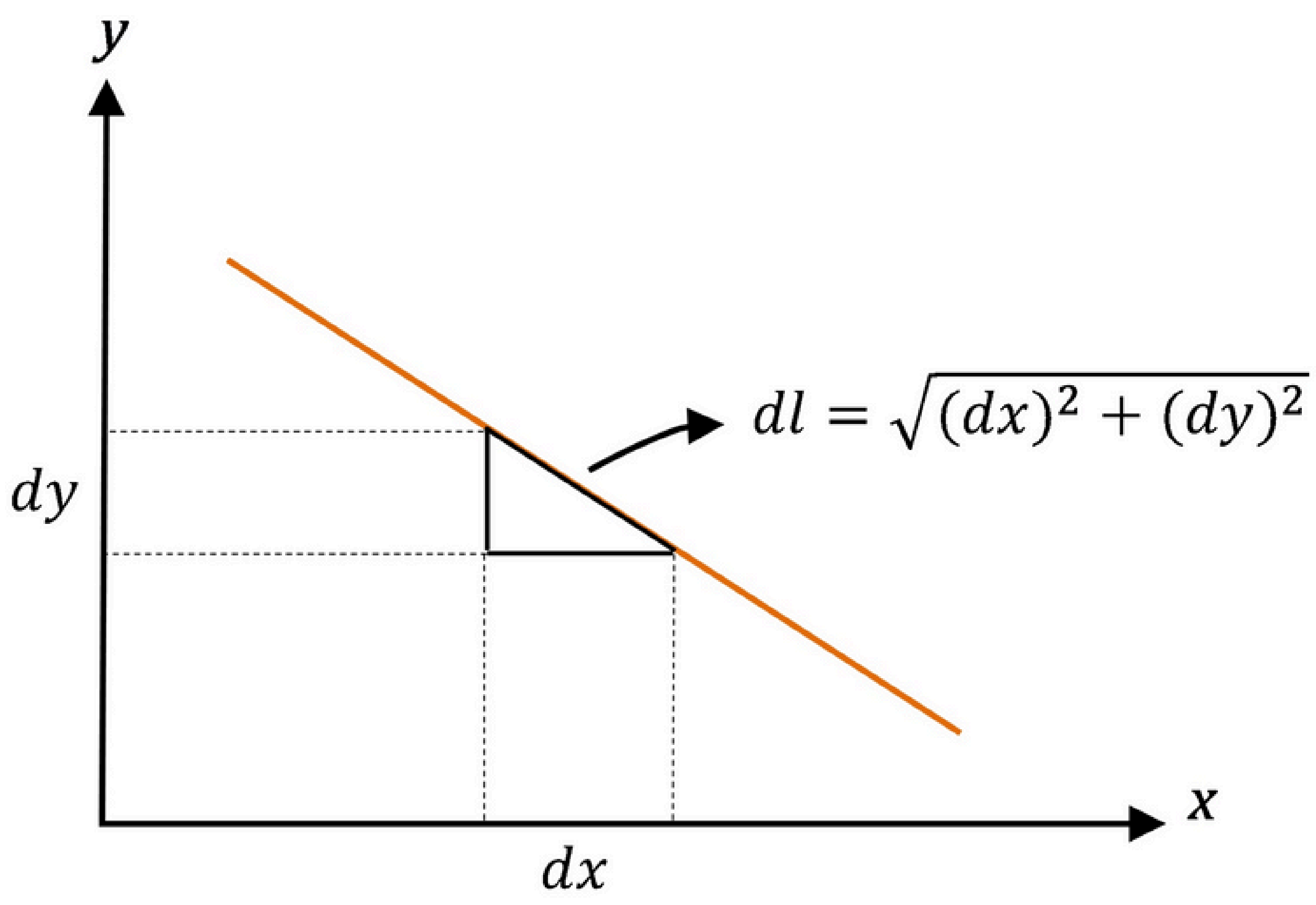
- The metric allows the calculation of the infinitesimal distance or **line element dl** between two points on any n-D manifold

$$(dl)^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} g_{ij} dx_i dx_j$$

- Note that dl is an invariant: it does not depend on the chosen coordinate system
- In a flat surface, the line element reduces to the usual:

$$[g_{ij}] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$(dl)^2 = (1)dxdx + (1)dydy + (0)dxdy + (0)dydx = (dx)^2 + (dy)^2$$

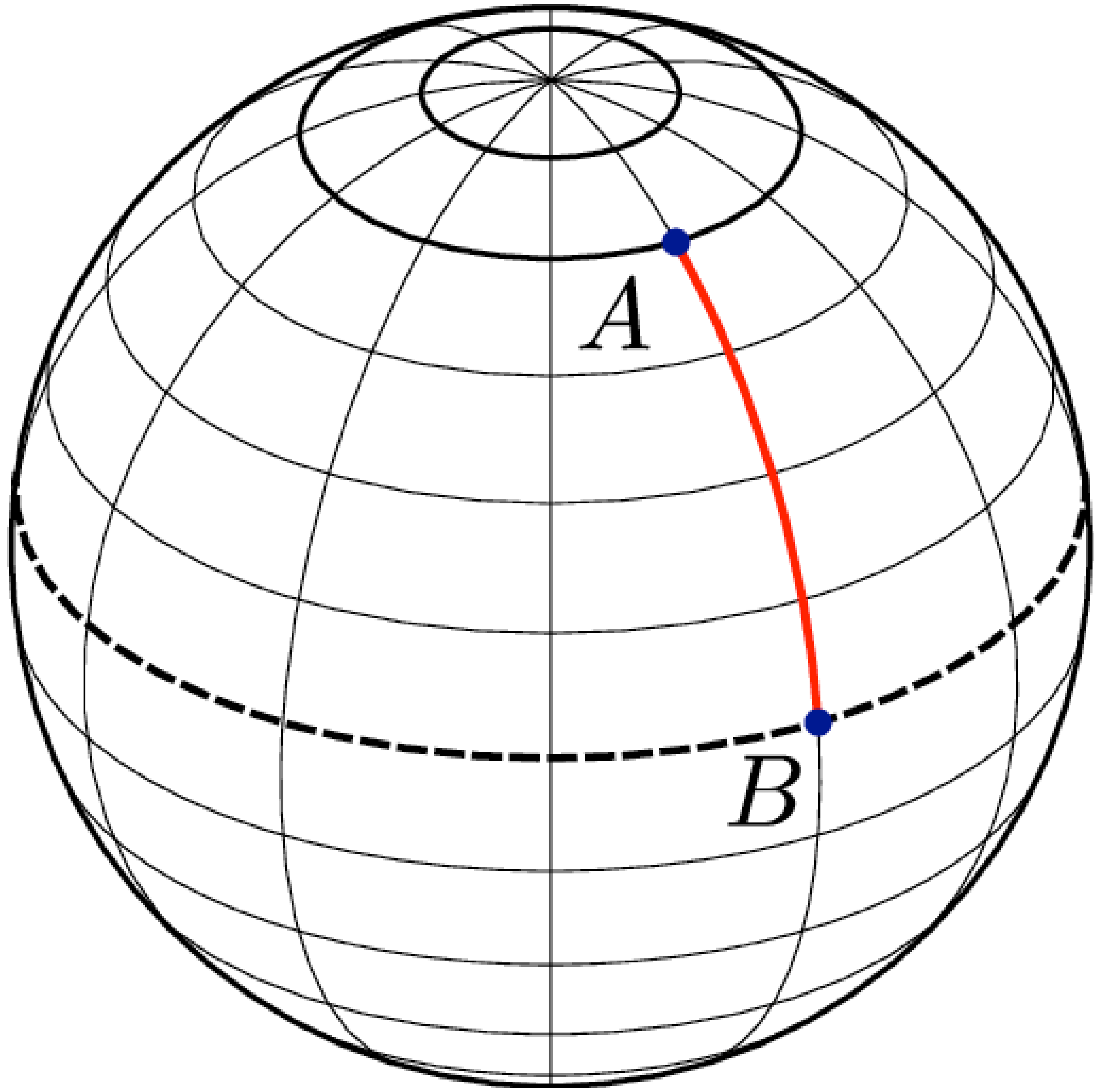


Curved space time

- ***Curvature*** and ***geodesics*** are two central properties of a manifold that are invariant
- Geodesics: locally shortest path between two points on a given manifold (it is a locally straight line). E.g.: shortest distance between points a and b:

$$\int_a^b dl = l(a,b) = \int_a^b \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} g_{ij} dx_i dx_j}$$

Geodesics in a sphere: great circles obtained as the intersection of the surface with a plane passing through its centre

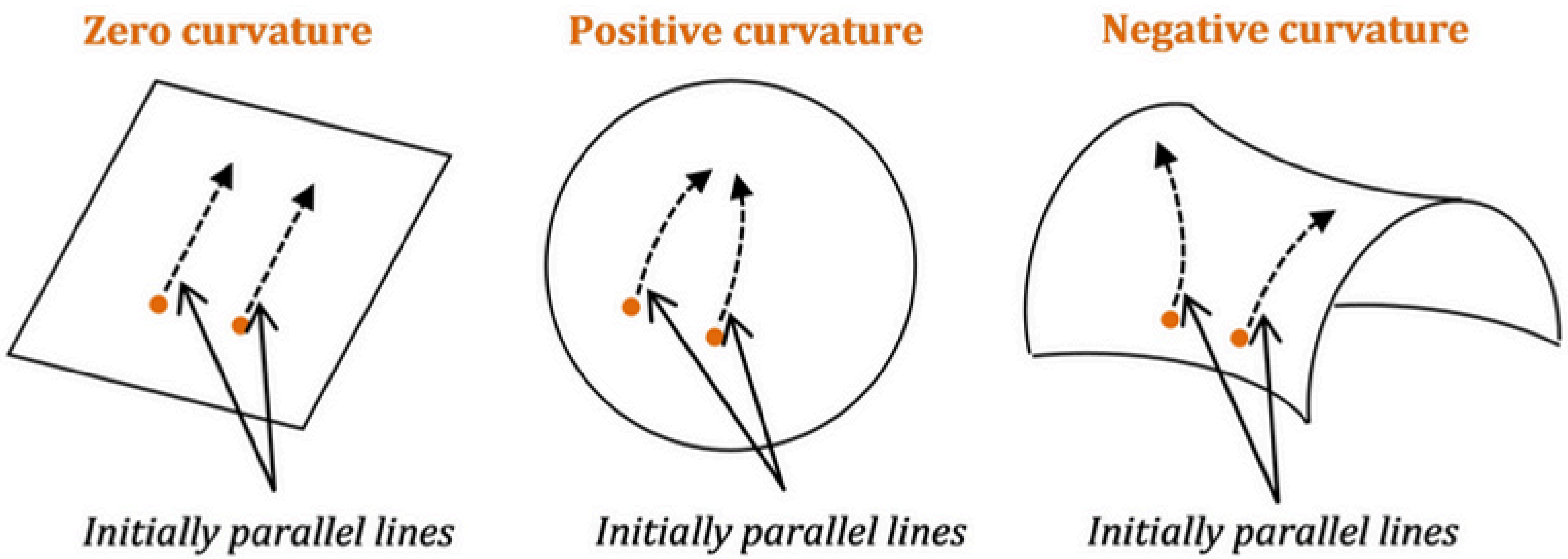


Curved space time

Curvature:

- A manifold can have only three curvature classes: null, positive, and negative
- In most cases, the curvature varies from point to point

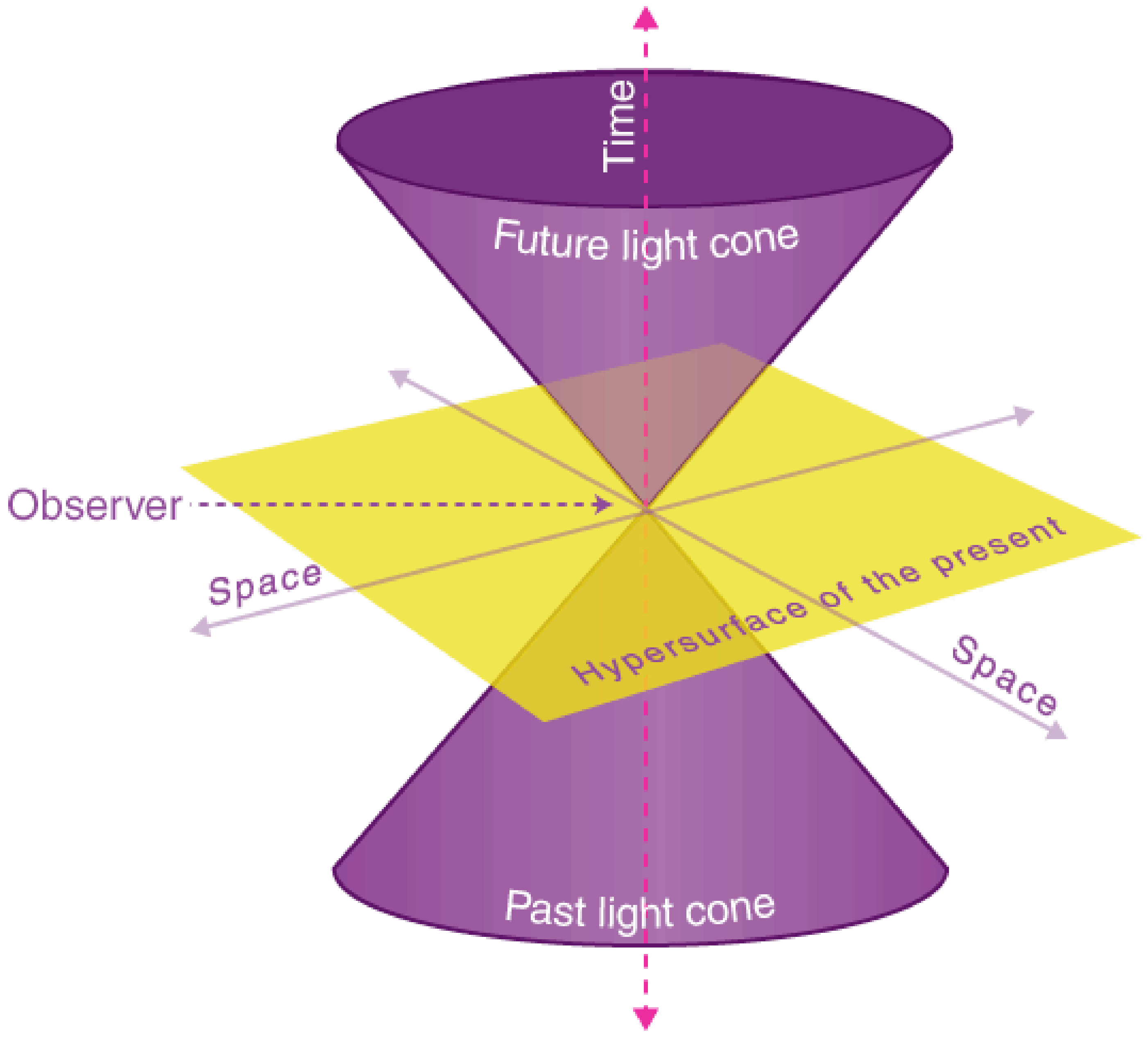
Geodesics are indicated by dashed black lines



General Relativity

- GR describes gravity as an effect of the curvature of spacetime, which means that without gravity the curvature is zero.
- This flat spacetime is the setting for the special theory of relativity (SR): special case of GR in which there is no gravity. 4D manifold with 3 spatial and 1 temporal coordinate (Minkowski metric)

$$(\eta)_{\alpha\beta} \equiv \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



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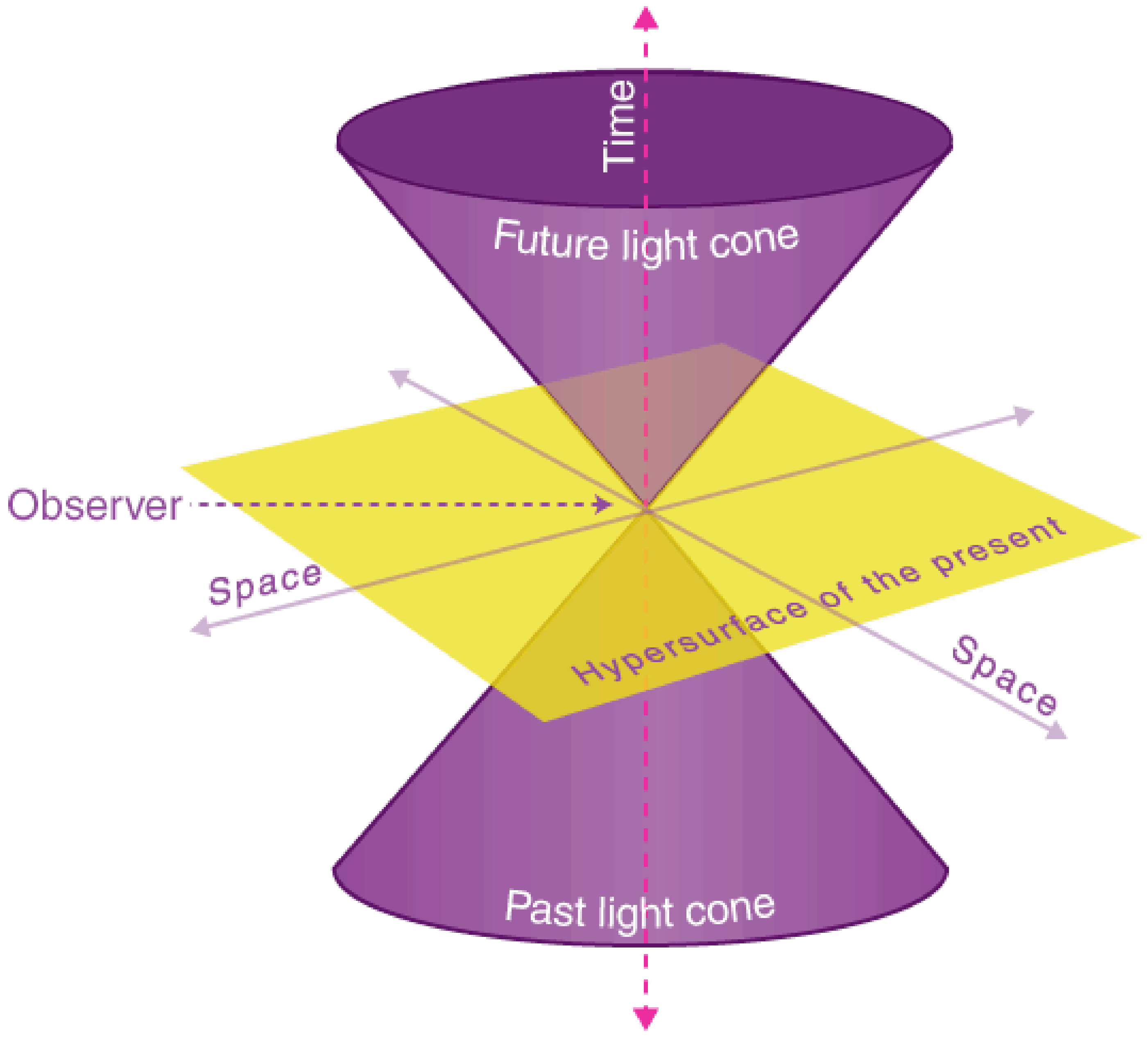
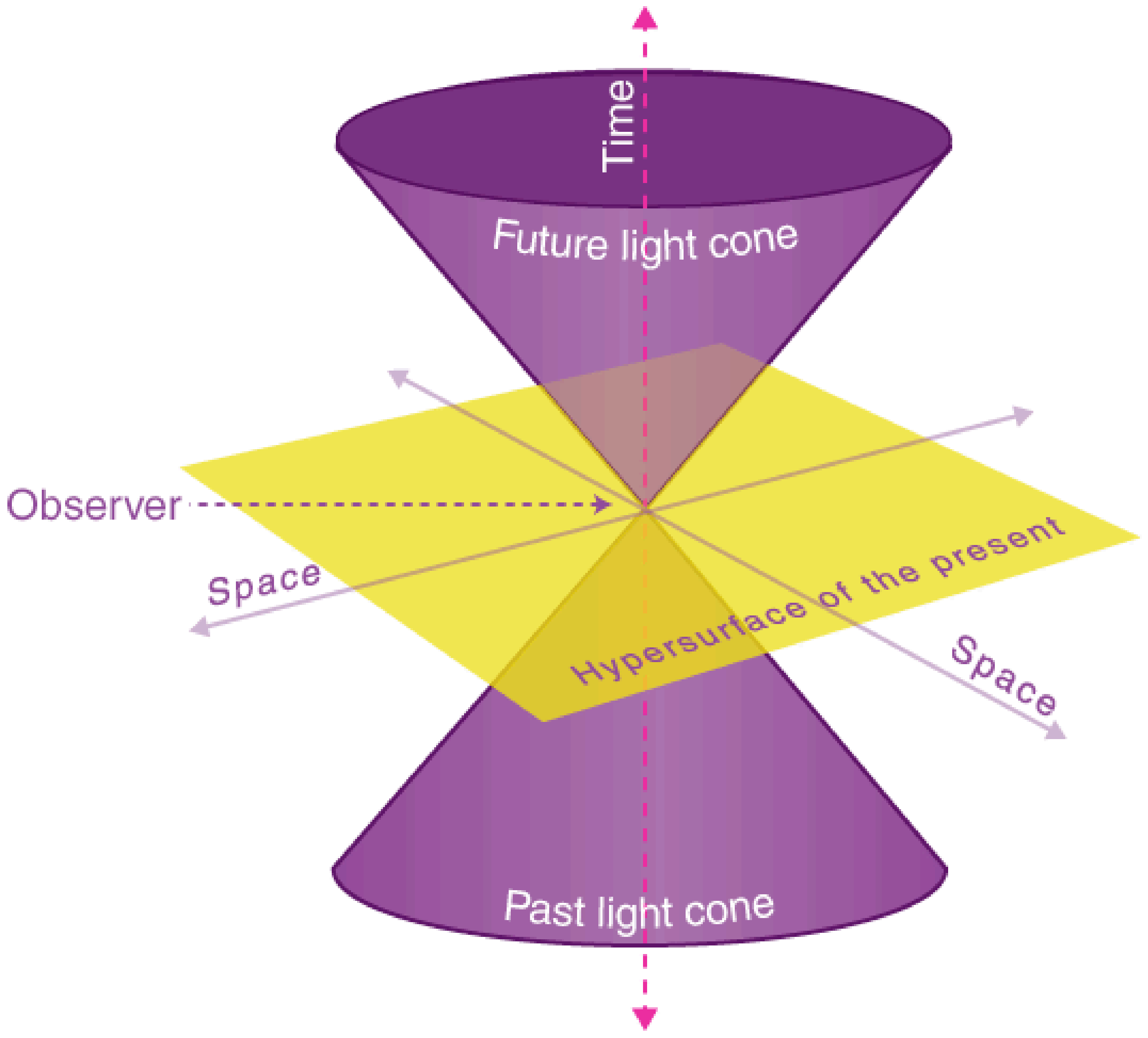
Time related elements



General Relativity

- The **infinitesimal distance between two points in flat spacetime** is denoted as **ds** and dl is used to define the infinitesimal distance between two points in flat space
- Spacetime metric can be defined as

$$ds^2 = -dx_0^2 + (dx_1^2 + dx_2^2 + dx_3^2) = -dx_0^2 + dl^2$$

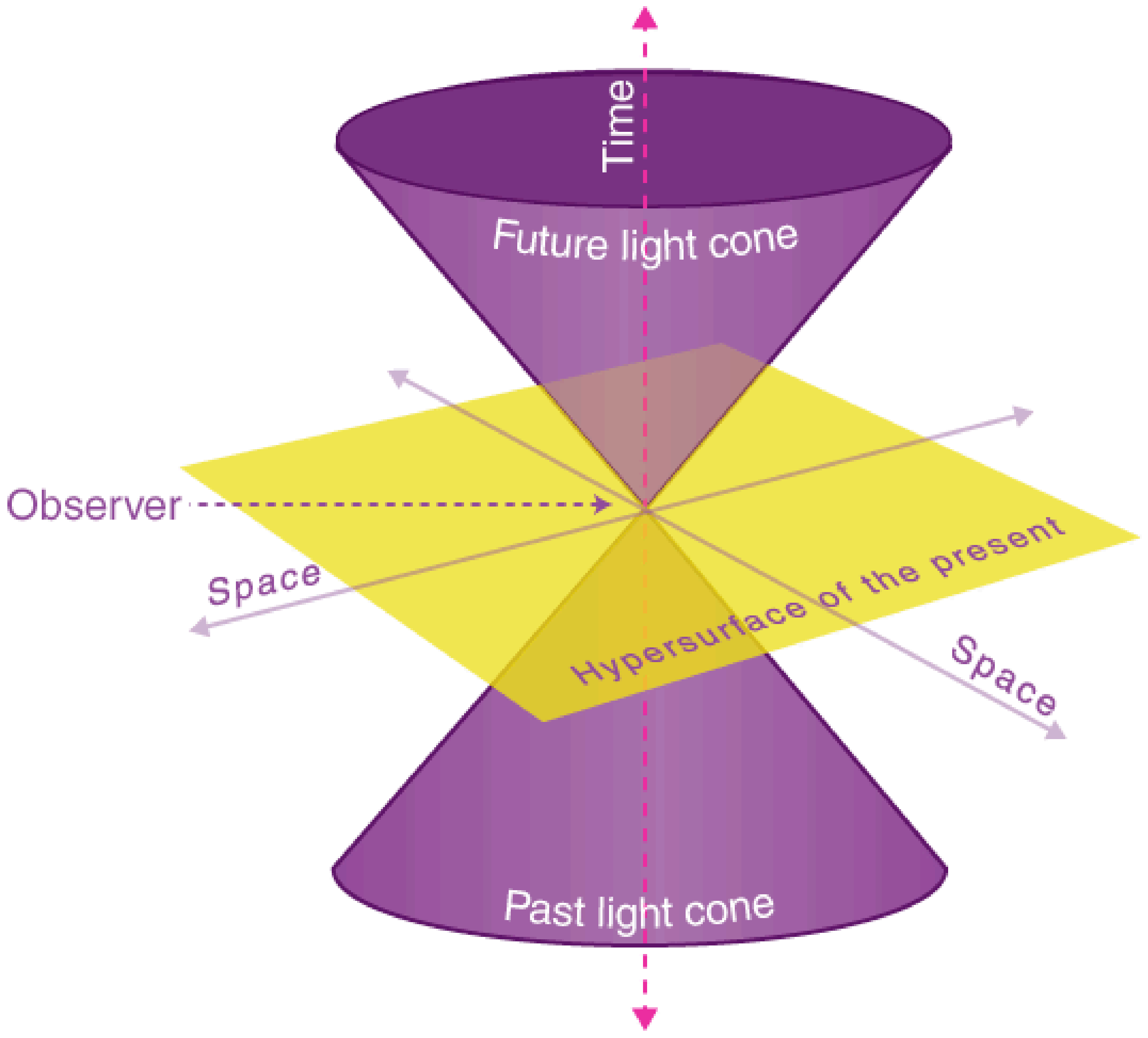


General Relativity

- Let us consider two events that occur at the same place in a reference frame S, but at different times separated by an infinitesimal interval dt.
- For an observer in S, if dx=dy=dz=0:

$$ds = -cdt \equiv -cd\tau$$

- τ: proper time - lapse measured by a "wristwatch" that travels with the observer in S; ds is the "displacement in time" made by the observer on his world line.

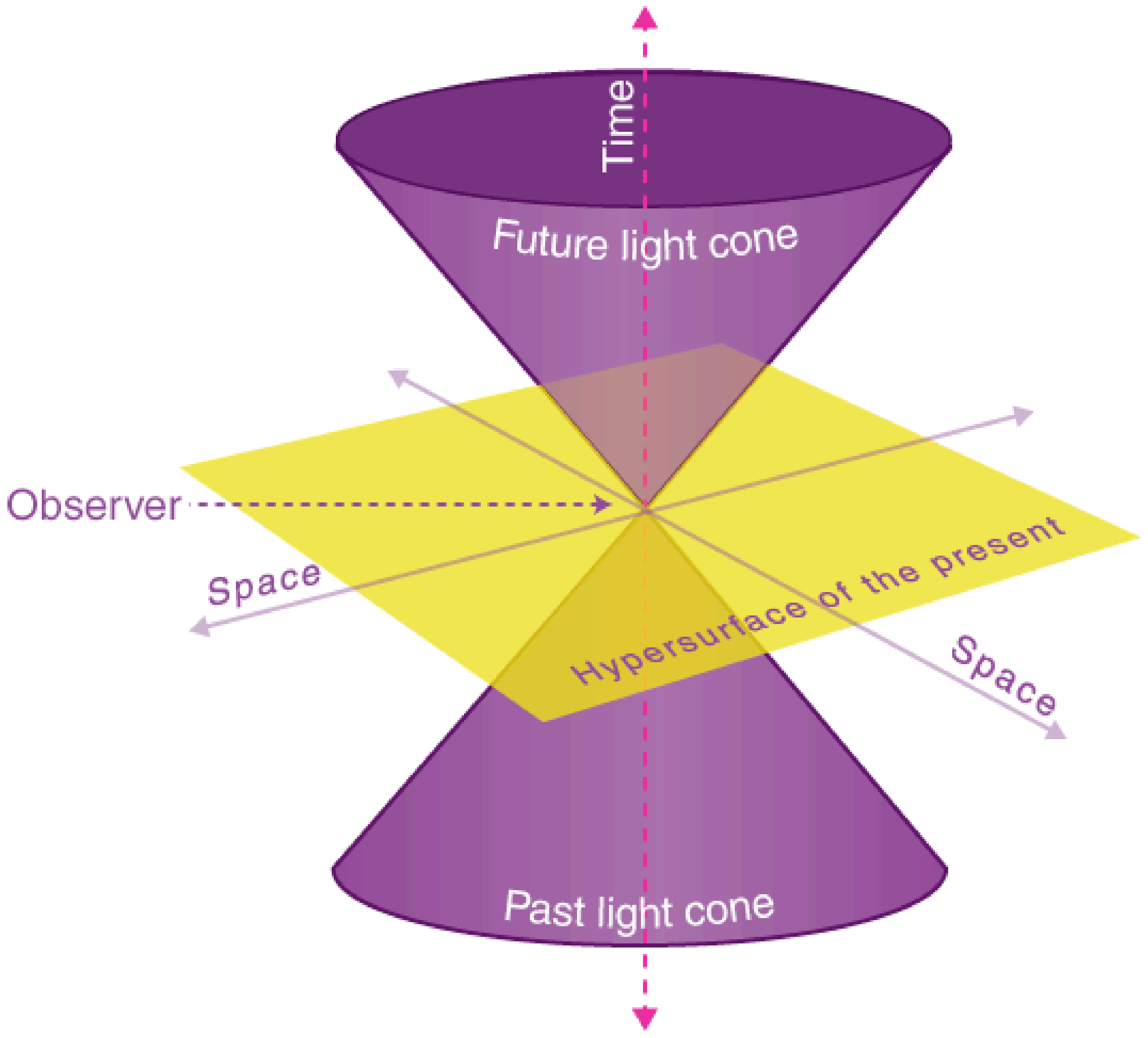


General Relativity

- Let us consider two events that occur at the same place in a reference frame S, but at different times separated by an infinitesimal interval dt.
- If we describe a light pulse, then dl=cdt, so:

$$ds = 0$$

- The light is said to describe a **null geodesic** in spacetime

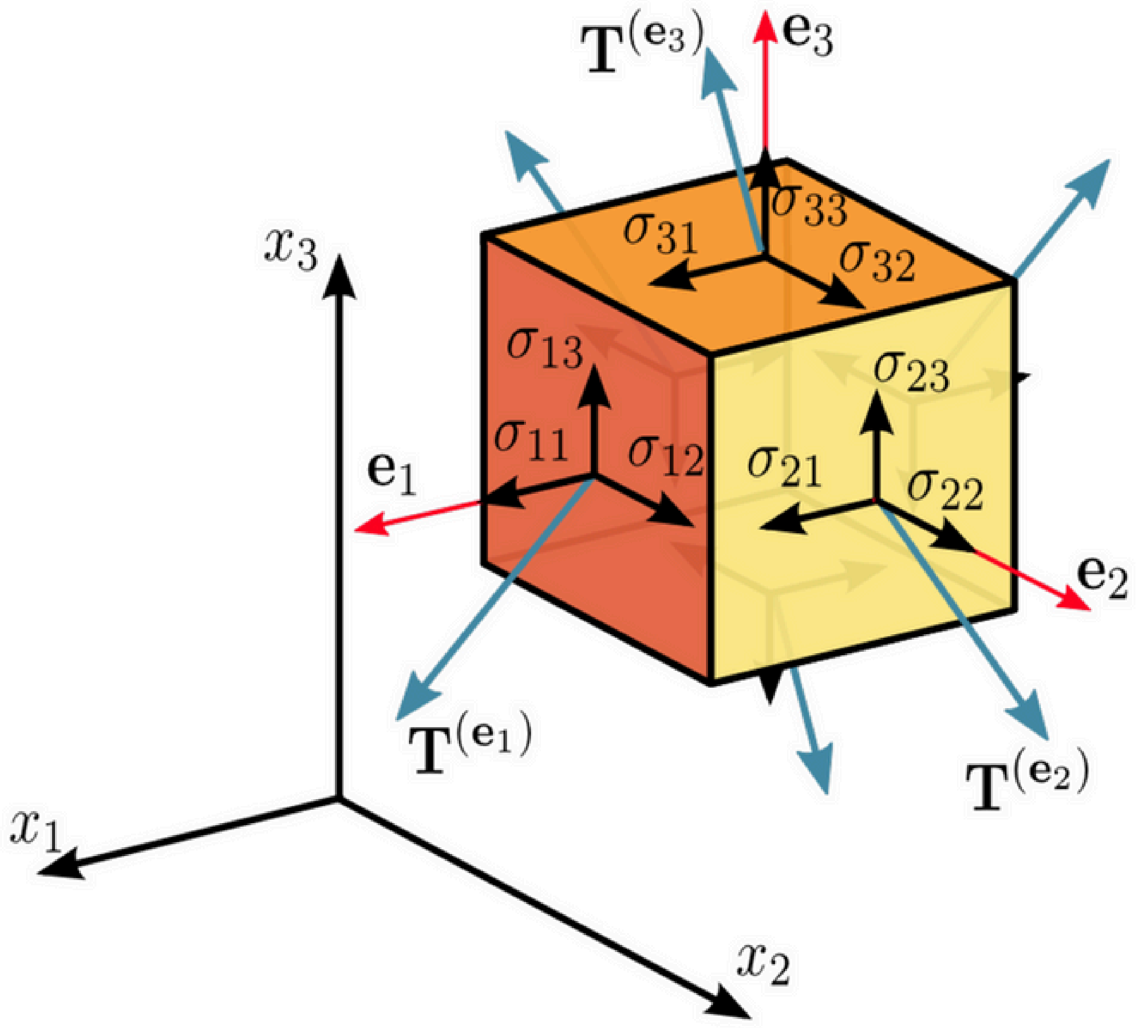


Small parenthesis: tensors

- Mathematical objects that generalize scalars, vectors, and matrices.
- A tensor has components that transform in a specific way under coordinate transformations → ideal for describing physical laws in any frame.
- Examples:
  - A scalar (like temperature) is a tensor of rank 0.
  - A vector (like velocity) is a tensor of rank 1.
  - The metric (which defines distances in spacetime) is a tensor of rank 2.

Electric conductivity of an anisotropic crystal

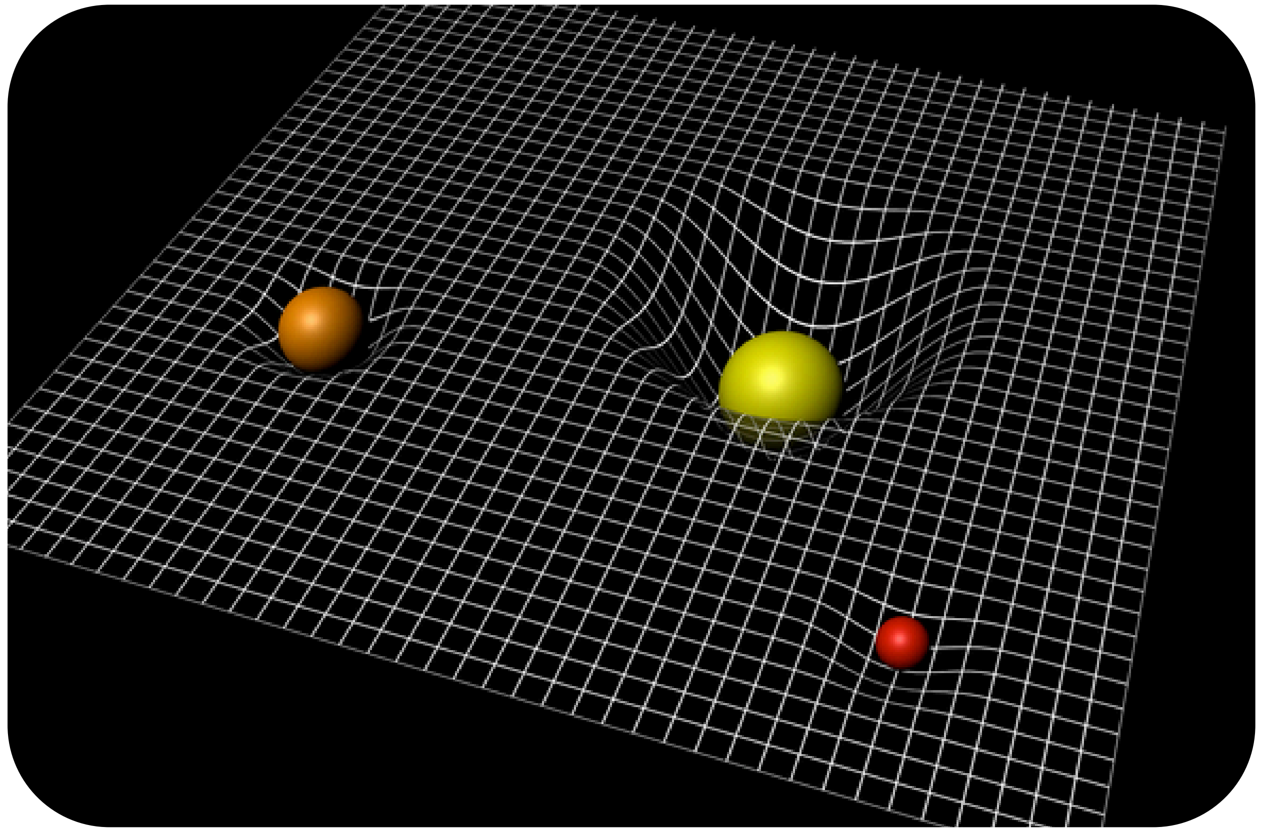
$$j^i = \sigma^i_j E^j \qquad \sigma^i_j E^j \equiv \sum_j \sigma^i_j E^j$$



General Relativity – Einstein equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\frac{8\pi G}{c^4}T_{\mu\nu}$$

General Relativity – Einstein equations



$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi G}{c^4} T_{\mu\nu}$$

Space-time curvature part

Mass/energy distribution part

General Relativity – Einstein equations

Ricci tensor: contraction of the Riemann tensor (obtained computing derivatives of the metric)

$$R_{\mu\nu} \equiv R^{\lambda}_{\mu\lambda\nu}$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi G}{c^4} T_{\mu\nu}$$

General Relativity – Einstein equations

Spacetime metric

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\frac{8\pi G}{c^4}T_{\mu\nu}$$

General Relativity – Einstein equations

Ricci scalar: contraction of the Ricci tensor with the metric

$$R = g^{\mu\nu} R_{\mu\nu} = R^{\mu}_{\mu}$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi G}{c^4} T_{\mu\nu}$$

General Relativity – Einstein equations

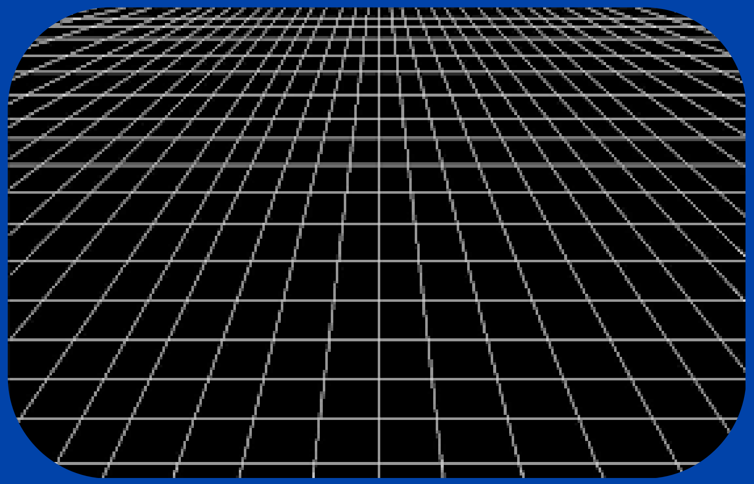
Energy momentum tensor: mass-energy content that produces the curvature

- Energy density: T_{00}
- Energy flux in ith direction: $T_{0i} = T_{i0}$
- Flux of i-momentum in j-direction T_{ij}

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi G}{c^4} T_{\mu\nu}$$

Derivation Gravitational waves

- Weak field limit: Minkowski + perturbation



$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad |h_{\mu\nu}| << 1$$

- We can simplify E. Eqs by assuming an appropriate gauge (Lorentz):

$$\Box \bar{h}_{\mu\nu} \equiv \partial^\sigma \partial_\sigma \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}$$

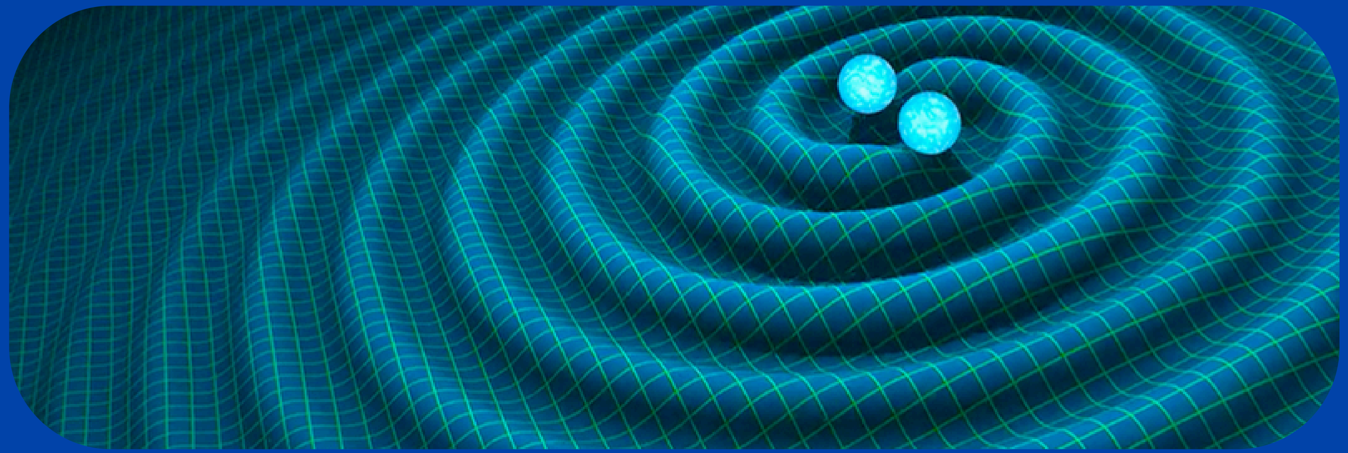
- Far away from any source of mass/energy ($T_{\mu\nu} = 0$), E. eqs. are a 4D wave equation with sols.: **plane waves**

$$\bar{h}^{\mu\nu} = A^{\mu\nu} e^{ik_\alpha x^\alpha}$$

Derivation Gravitational waves

- These are the **GRAVITATIONAL WAVES (GW)!!**

$$\bar{h}^{\mu\nu} = A^{\mu\nu} e^{ik_\alpha x^\alpha}$$



- Elements and properties:
  - $A^{\mu\nu}$ matrix with constant components
  - $k_\alpha = (\omega, k_i)$ wave vector, satisfying $k^\alpha k_\alpha = 0$
  - ω angular frequency of the wave
  -

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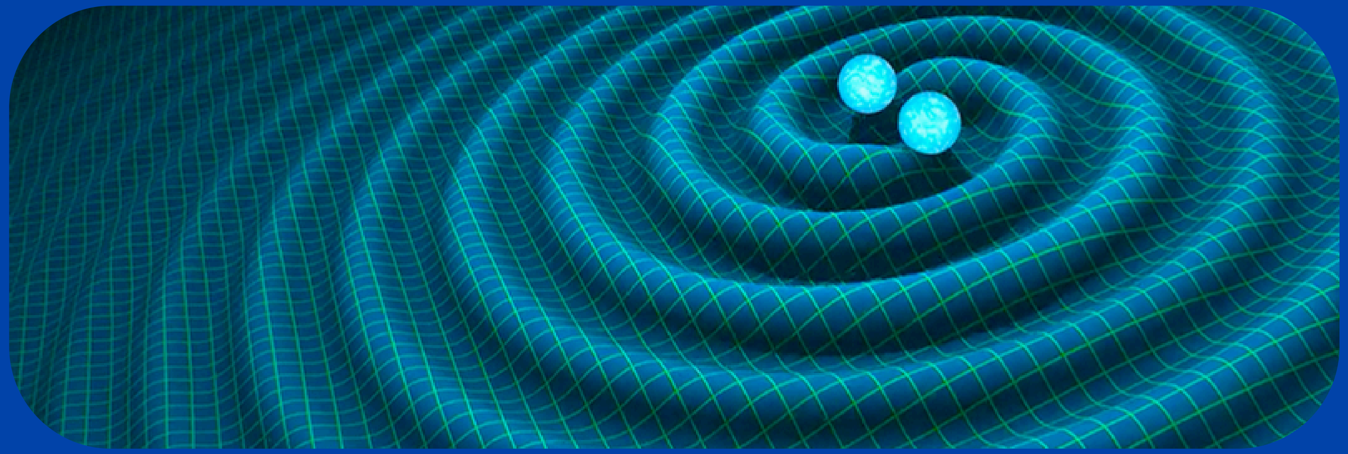

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  - $\omega^2 = |k_i|^2$, $|k_i| = \omega / v$

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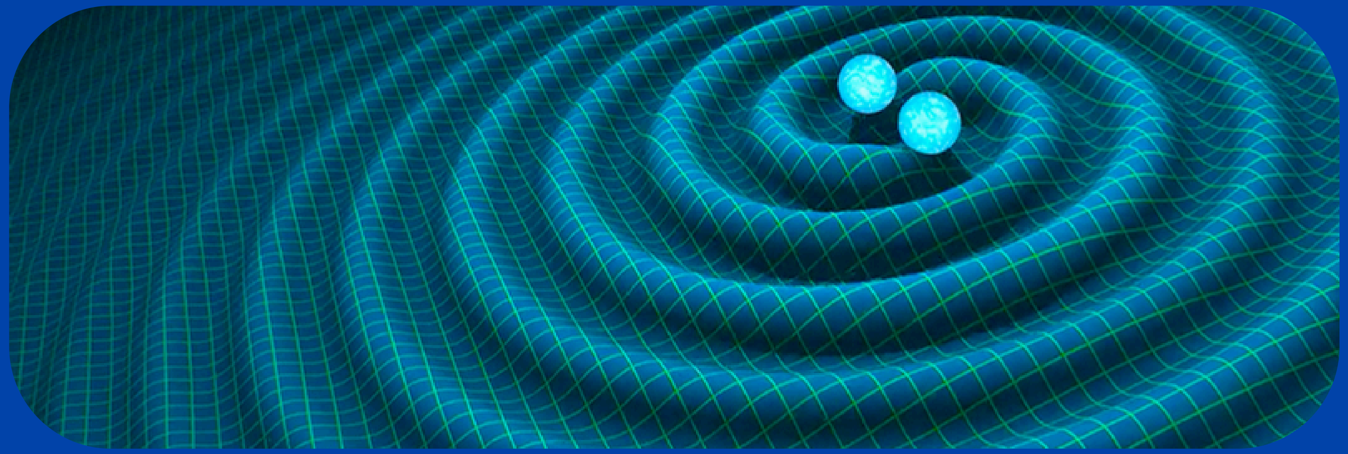

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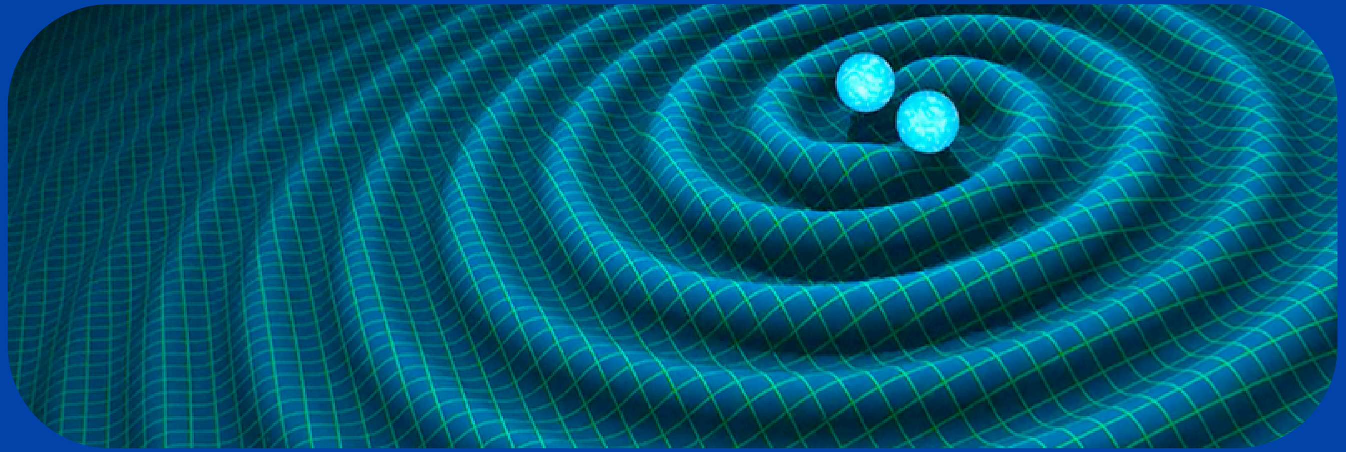
  - ω angular frequency of the wave
  - $\omega^2 = |k_i|^2$, $|k_i| = \omega / v$ $\longrightarrow$ $v = c = 1$

GWs travel at the speed of light

Derivation Gravitational waves

- These are the GRAVITATIONAL WAVES (GW)!!

$$\bar{h}^{\mu\nu} = A^{\mu\nu} e^{ik_\alpha x^\alpha}$$



- Elements and properties:
  - $A^{\mu\nu}$ matrix with constant components
  - $k_\alpha = (\omega, k_i)$ wave vector, satisfying $k^\alpha k_\alpha = 0$
  - ω angular frequency of the wave
  - Applying Lorentz gauge to E. eqs.: $k_\mu A^{\mu\nu} = 0$

GWs are transverse

GW polarizations

- Lorentz gauge + Transverse Traceless (TT) gauge: leave only 2 degrees of freedom in hμν

- For GWs propagating in the +z direction

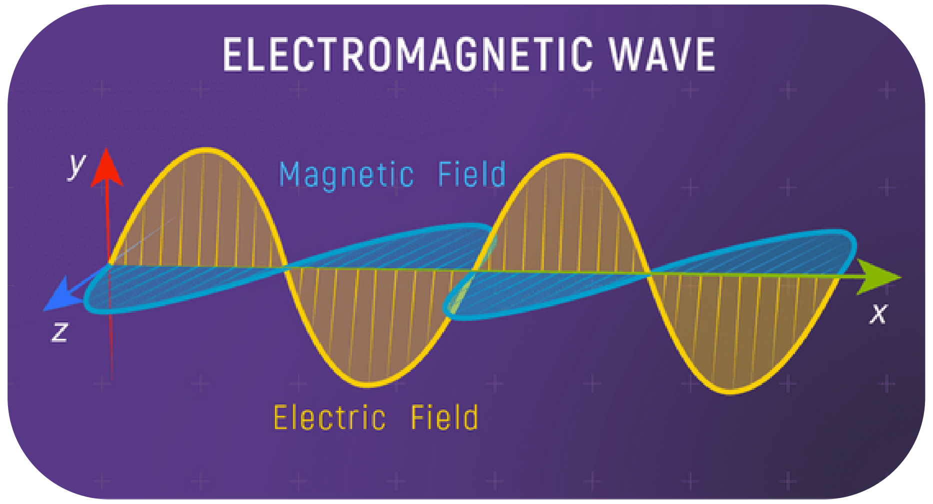
$$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_x & 0 \\ 0 & h_x & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

h+ is the plus polarization of the GW

$$h_+(t,z) \equiv A_+ \cos(\omega(t - z/c) + \phi_+)$$
$$h_x(t,z) \equiv A_x \cos(\omega(t - z/c) + \phi_x).$$

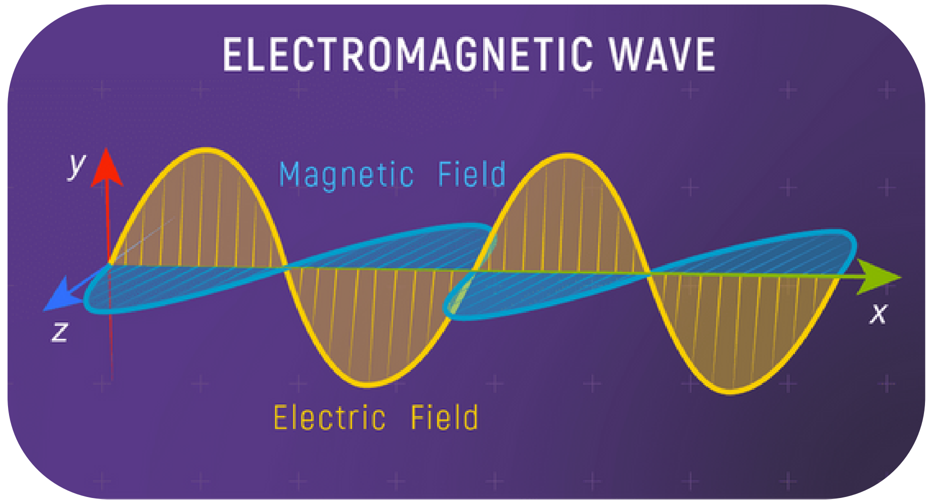
hx is the cross polarization of the GW

GW vs EM radiation



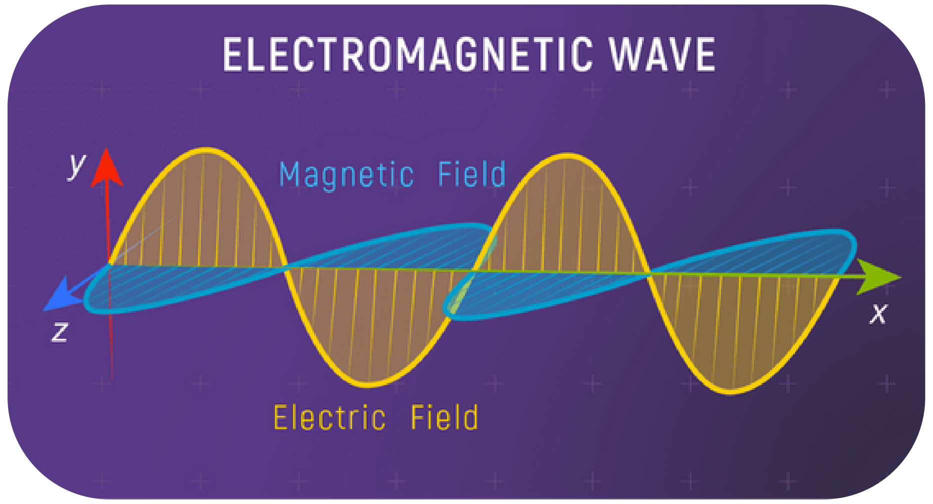
Electromagnetism	General Relativity
Accelerating charges emit EM radiation	Accelerating masses emit gravitational radiation

GW vs EM radiation



Electromagnetism	General Relativity
Accelerating charges emit EM radiation	Accelerating masses emit gravitational radiation
Travelling EM waves have their fields transverse to their direction of propagation	Travelling gravitational waves have their fields transverse to its direction of propagation

GW vs EM radiation



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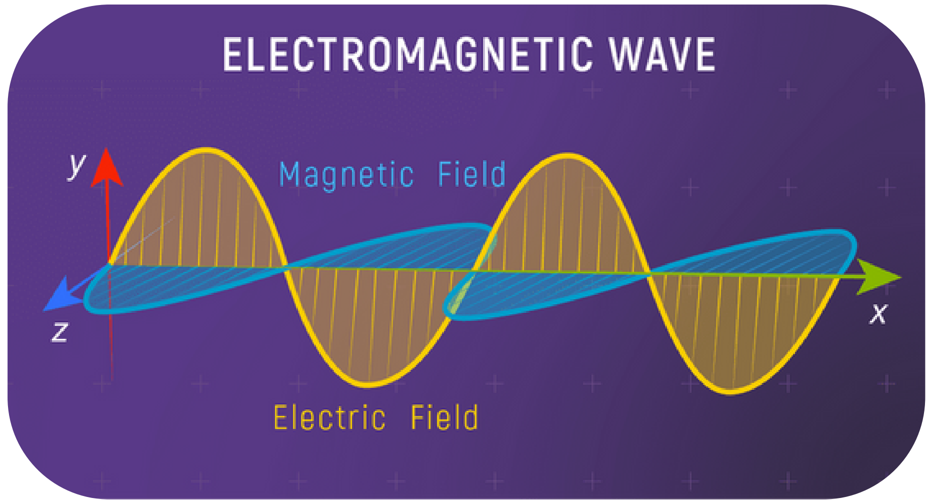
EM radiation has two polarizations	GWs have two polarizations

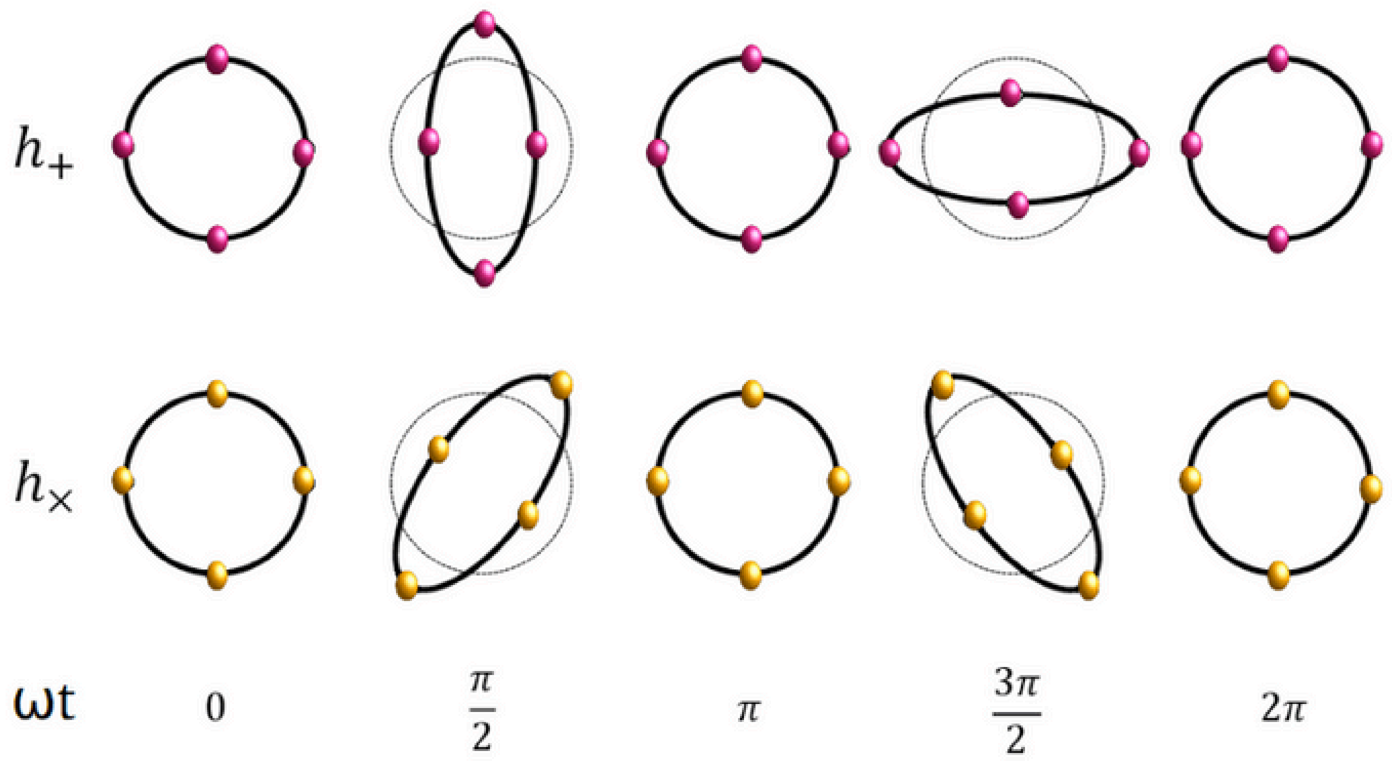
GW vs EM radiation



Electromagnetism	General Relativity
Accelerating charges emit EM radiation	Accelerating masses emit gravitational radiation
Travelling EM waves have their fields transverse to their direction of propagation	Travelling gravitational waves have their fields transverse to its direction of propagation
EM radiation has two polarizations	GWs have two polarizations
EM force is much stronger and easily measurable	GWs are weak and hard to measure

GW interactions with free-falling masses

- A GW passing through a particle at rest **in the TT gauge** leaves it at rest.
- However, the proper distance does change due to the passage of GWs.



- Keeping in mind that $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, the space-time interval ds^2 :

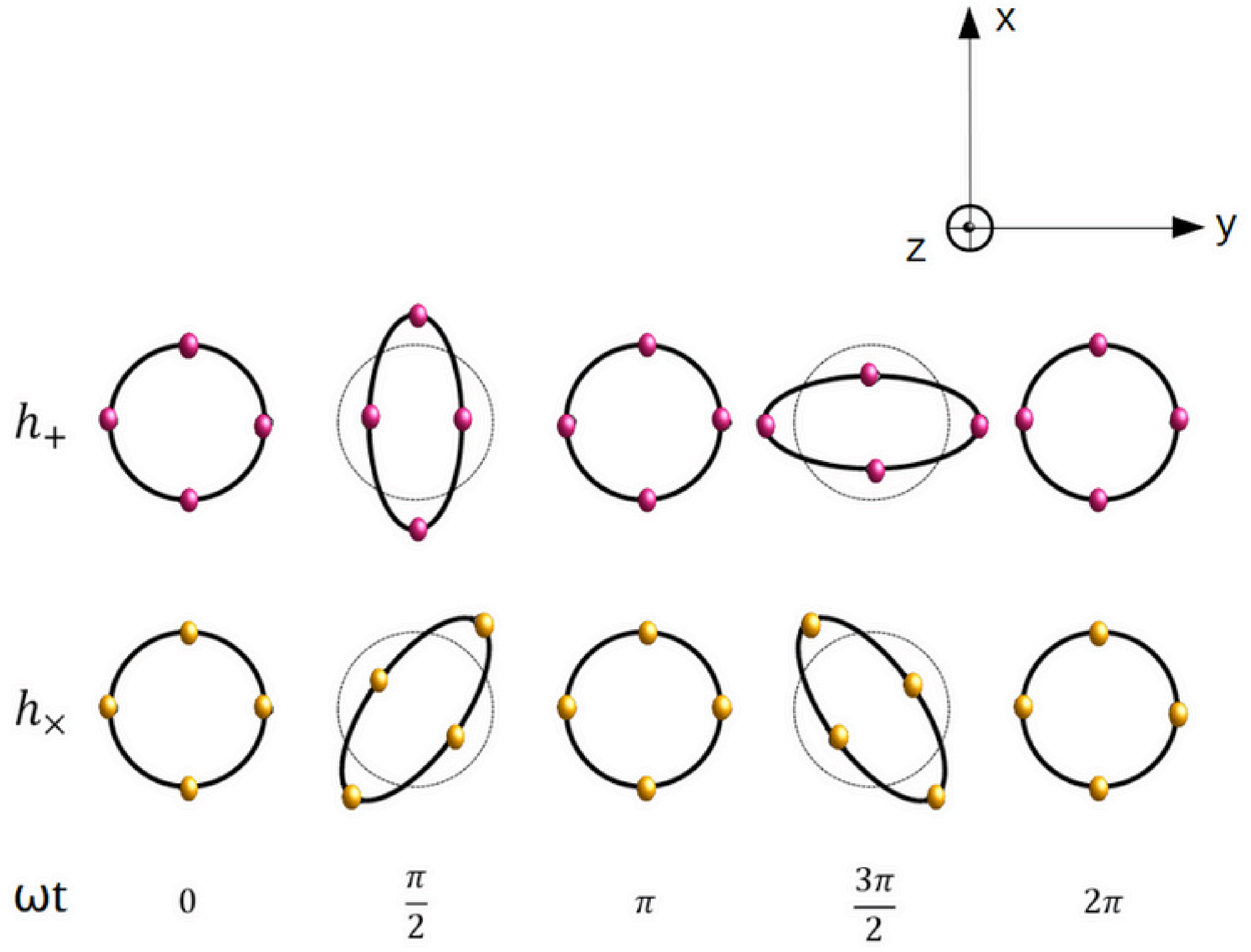
$$ds^2 = -dt^2 + dz^2 + (1 + h_+(t,z))dx^2 + (1 - h_x(t,z))dy^2 + 2h_x(t,z)dxdy$$

GW interactions with free–falling masses

- Proper distance at time t between two particles located at $(x_1, y_1, 0)$ and $(x_2, y_2, 0)$, and assuming $y_2 - y_1 = 0$:

$$ds = (1 + \frac{1}{2}h_+(t,0))(x_1 - x_2)$$

- Distance between two free masses changes as GWs pass by!! → Effect used to DETECT GWs with interferometers.



Generation of GWs

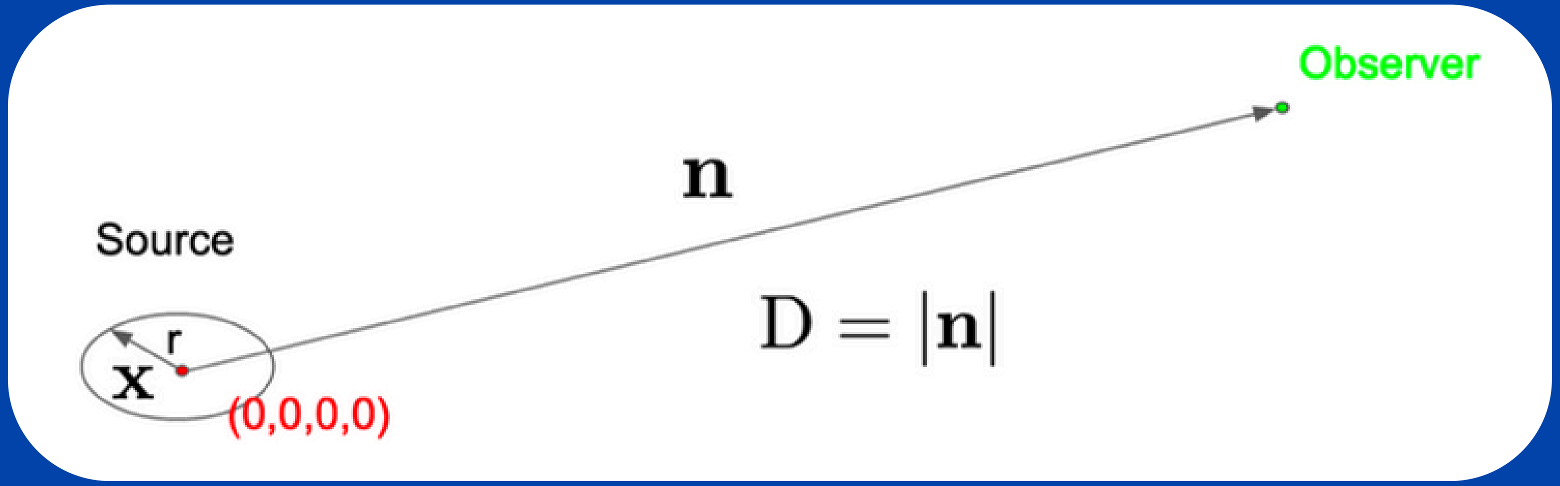
Any assymetrical body in rotation will generate GWs

Amplitude of GWs generated by Compact Binary Coalescences (CBCs) is measurable

Amplitude of GWs generated by light bodies is not measurable yet

GW emission, quadrupole formalism

- Given a source at



- A solution to E. eqs is given by

$$\bar{h}_{\mu\nu}(t, \mathbf{n}) = \frac{4}{D} \int d^3x T_{\mu\nu}(t - D, \mathbf{x})$$

GW emission, quadrupole formalism

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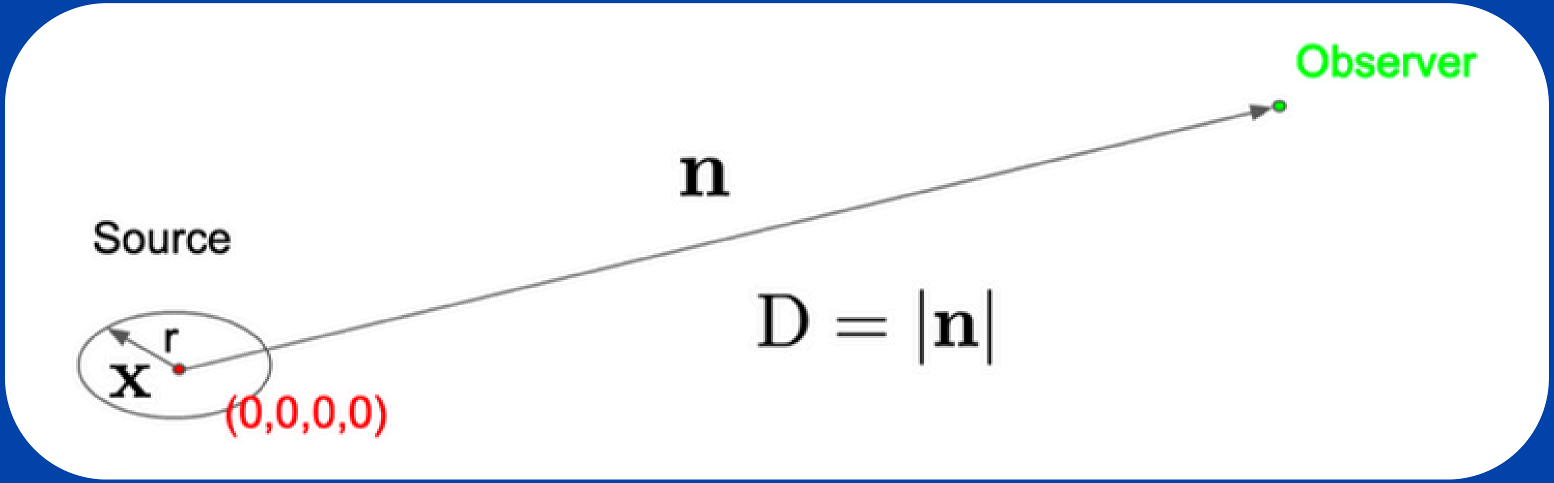
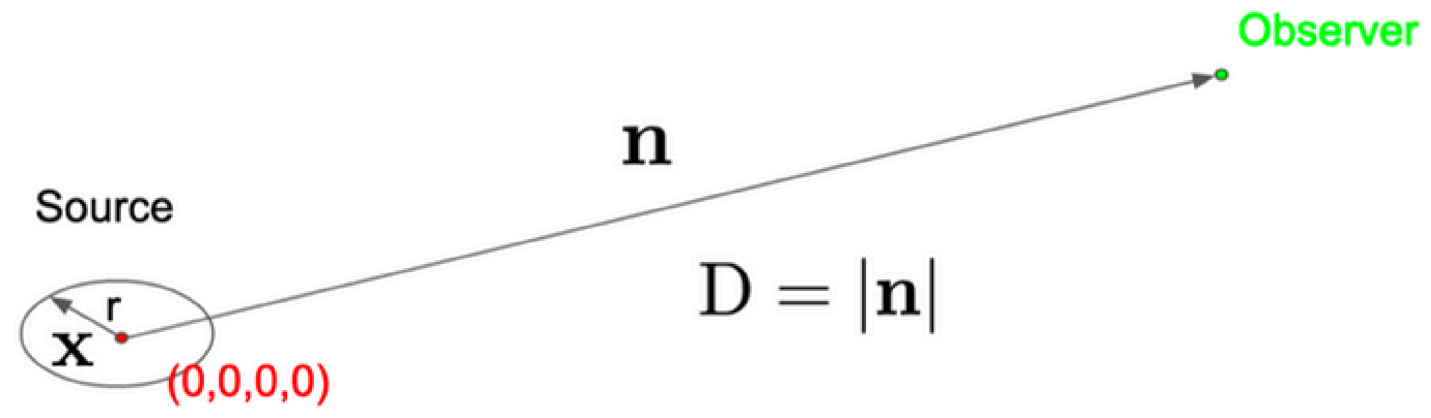


- A solution to E. eqs is given by

$$\bar{h}_{\mu\nu}(t, \mathbf{n}) = \frac{4}{D}\int d^3x T_{\mu\nu}(t - D, \mathbf{x})$$

amplitude of a GW decreases linearly with the distance

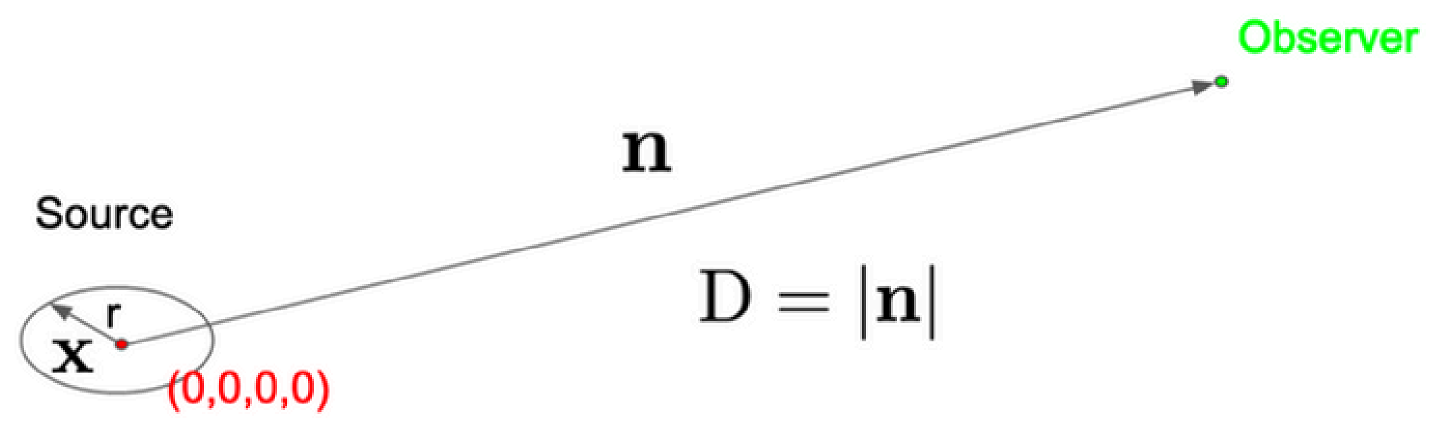
GW emission, quadrupole formalism



- Using the TT gauge the sol. can be further simplified:

$$h_{ij}^{TT}(t) = \frac{2}{D}\ddot{M}_{ij}(t-D)$$

GW emission, quadrupole formalism



- Using the TT gauge the sol. can be further simplified:

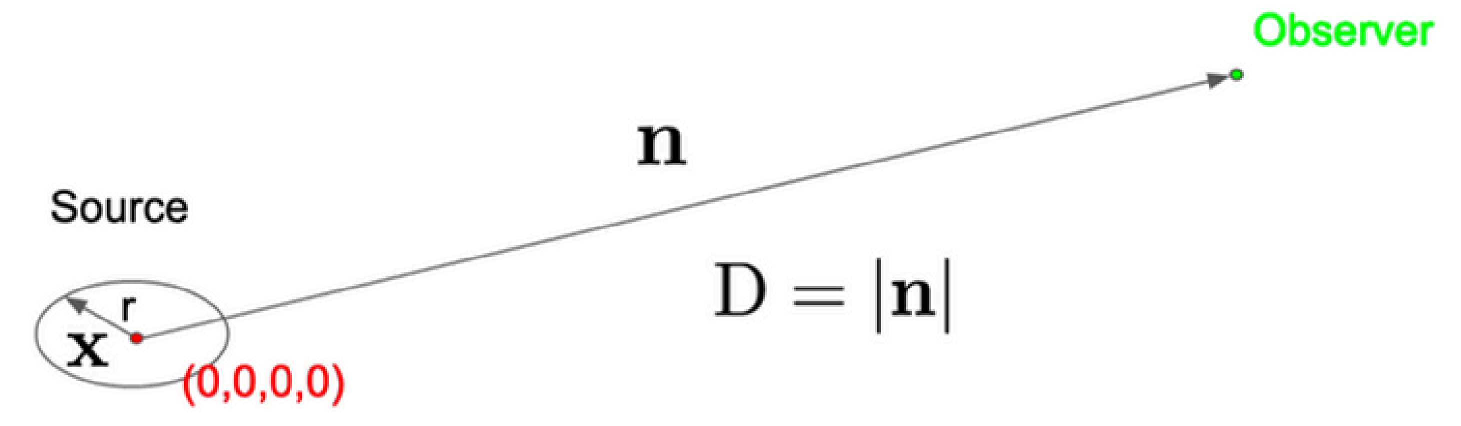
$$h_{ij}^{TT}(t) = \frac{2}{D}\ddot{M}_{ij}(t - D)$$

$$M_{ij}(t) \equiv \rho(t, \mathbf{x})(x_i x_j - \frac{1}{3}r^2\delta_{ij})d^3x$$

mass quadrupole momentum

ρ(t,x): mass distribution of the source

GW emission, quadrupole formalism



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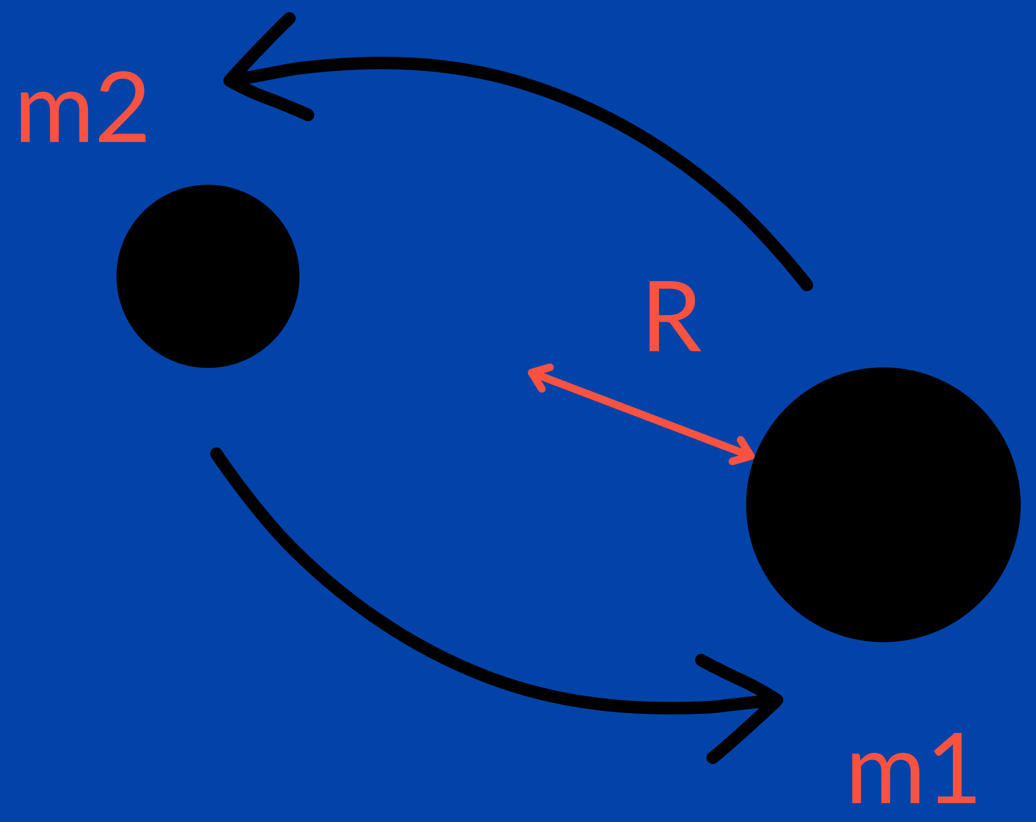
$$h_{ij}^{TT}(t) = \frac{2}{D}\ddot{M}_{ij}(t-D)$$

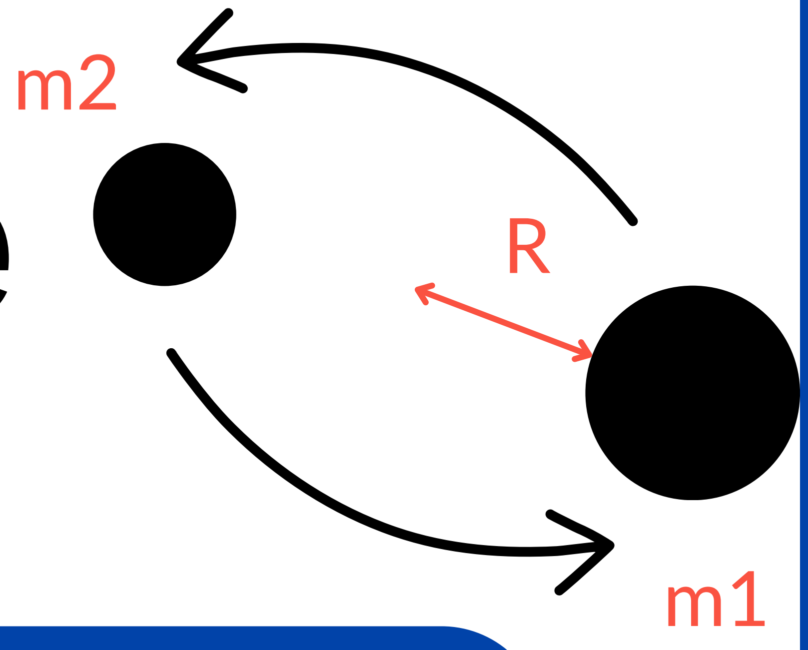
Sources whose mass have a varying quadrupolar moment will generate time and amplitude dependent GW

GW emission, quadrupole formalism – example

- Objective: determine magnitude of the strain produced by a binary system of objects with masses m1 and m2 in a circular orbit with radius R



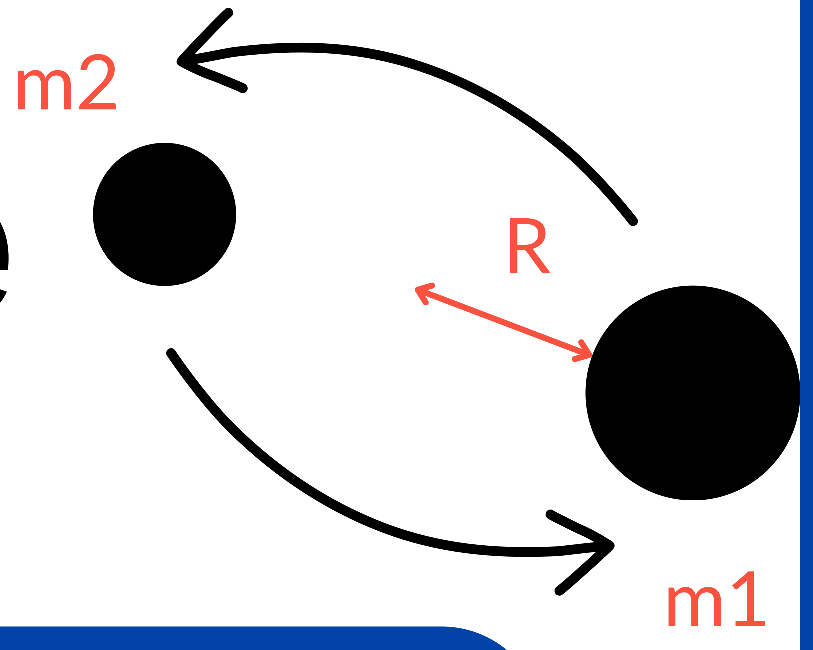
GW emission, quadrupole formalism – example



- h+ and hx can be expressed in terms of the mass momenta

$$h_+ = \frac{1}{D}(\ddot{M}_{11} - \ddot{M}_{22}), \quad h_x = \frac{2}{D}\ddot{M}_{12}$$

GW emission, quadrupole formalism – example



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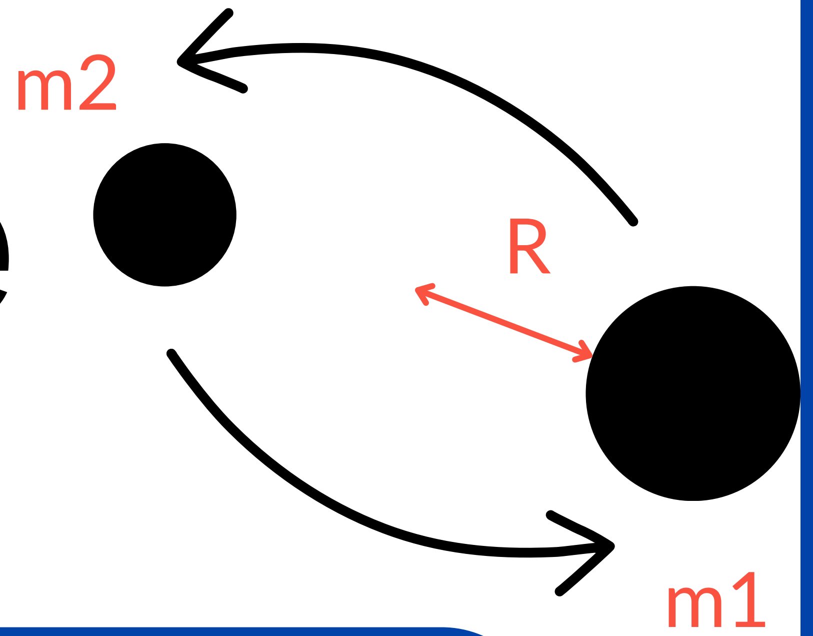
$$h_+ = \frac{1}{D}(\ddot{M}_{11} - \ddot{M}_{22}), \quad h_x = \frac{2}{D}\ddot{M}_{12}$$

- Non-null components of the quadrupole momentum of this system

$$M_{11} = \mu R^2 \frac{1 - \cos(2\omega_s t)}{2},$$
$$M_{22} = \mu R^2 \frac{1 + \cos(2\omega_s t)}{2},$$
$$M_{12} = -\frac{1}{2}\mu R^2 \sin(2\omega_s t),$$

GW emission, quadrupole formalism – example



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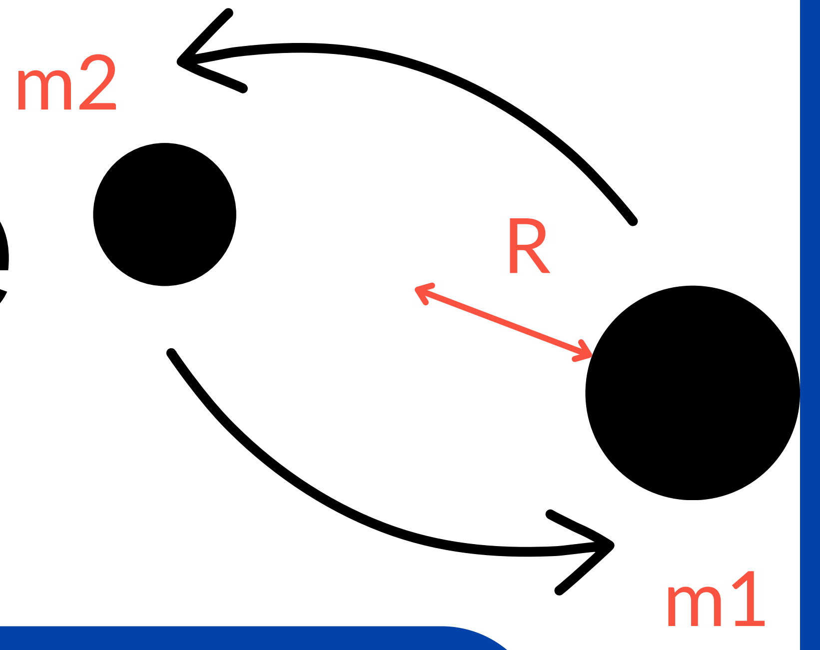
- Non-null components of the quadrupole momentum of this system

reduced mass of the system

$$\mu = \frac{m_1 m_2}{(m_1+m_2)}$$

$$M_{11} = \mu R^2 \frac{1 - \cos(2\omega_s t)}{2},$$
$$M_{22} = \mu R^2 \frac{1 + \cos(2\omega_s t)}{2},$$
$$M_{12} = -\frac{1}{2}\mu R^2 \sin(2\omega_s t),$$

GW emission, quadrupole formalism – example



- h+ and hx can be expressed in terms of the mass momenta

$$h_+ = \frac{1}{D}(\ddot{M}_{11} - \ddot{M}_{22}), \quad h_x = \frac{2}{D}\ddot{M}_{12}$$

- Non-null components of the quadrupole momentum of this system

reduced mass of the system

$$\mu = \frac{m_1 m_2}{(m_1+m_2)}$$

$$M_{11} = \mu R^2 \frac{1-\cos(2\omega_s t)}{2},$$

$$M_{22} = \mu R^2 \frac{1+\cos(2\omega_s t)}{2},$$

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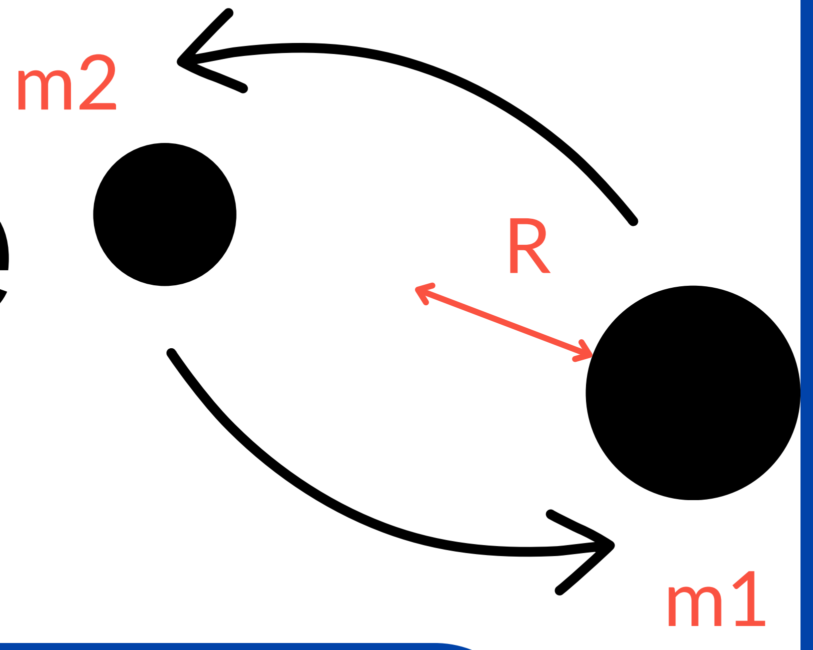
the orbital frequency

GW emission, quadrupole formalism – example

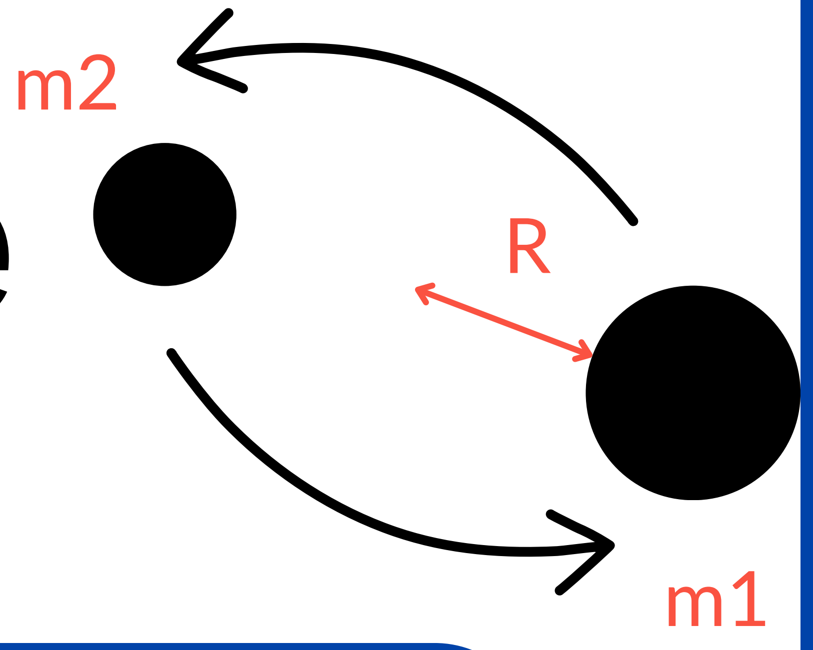


- Keeping in mind that $2\omega_s t = 2n\pi$ → $h_+ = \frac{1}{D} 2\ddot{M}_{11} = \frac{4}{D}\mu R^2 \omega_s, \quad h_x = 0$
- From Kepler's law ω is related to R as: $\omega_s^2 = G(m_1 + m_1)/R^3$ and hence

$$h_+ = \frac{4}{D}\frac{G}{c^4}\mu R^2 \omega_s^2 = \frac{4G^2}{Dc^4}\mu \frac{m_1 + m_2}{R}.$$

- Assuming masses are m1 = m2 = 1kg, D= 10^3 km and R = 1m → strain is

GW emission, quadrupole formalism – example



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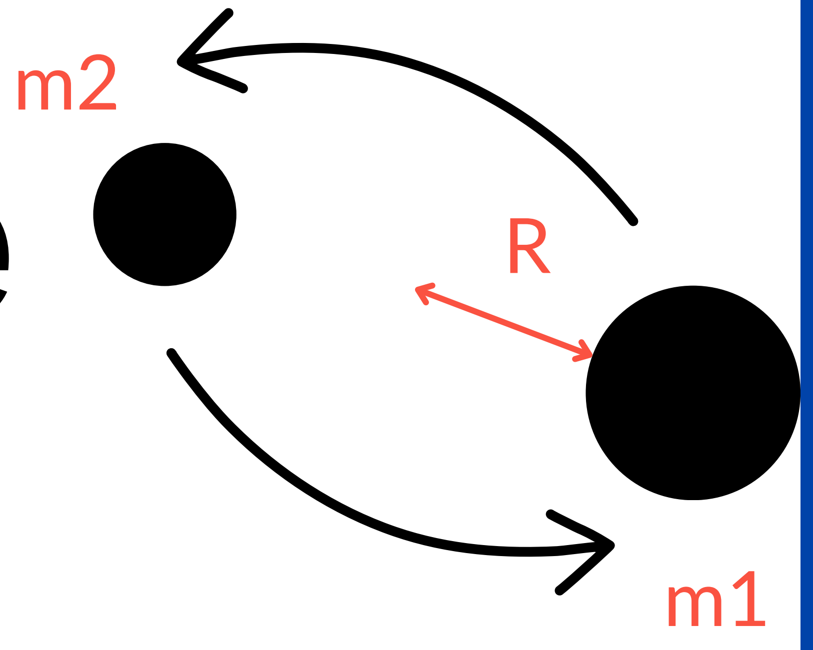
- Assuming masses are m1 = m2 = 1kg, D= 10^3 km and R = 1m → strain is **5.9×10^{-35}**

GW emission, quadrupole formalism – example



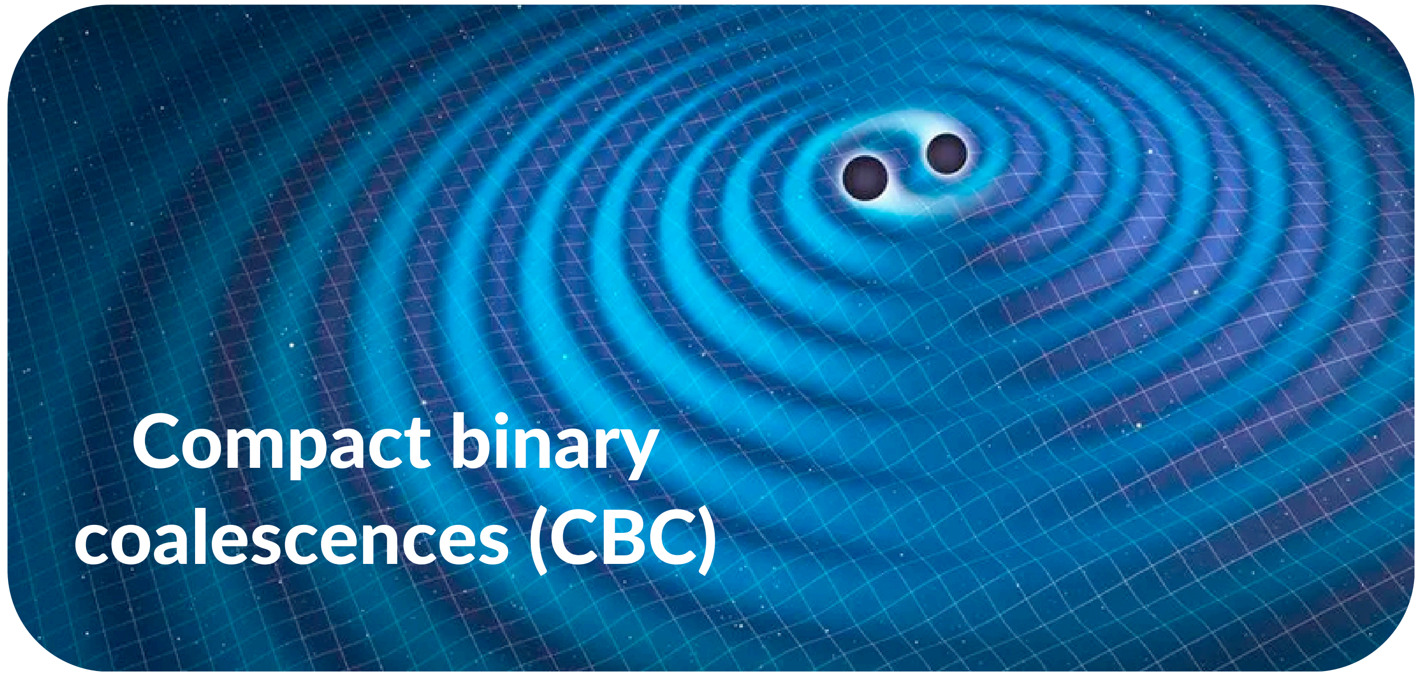
- In our detectors, strain is defined as: h = ΔL / L
  - ΔL: difference in length between the two arms
  - L: nominal length of an arm
- In Virgo (L = 3km), to detect h = 5.9×10^{-35} we would need to be sensitive to variations in length of: $\Delta L \sim 2 \times 10^{-31}$ m → unfeasible

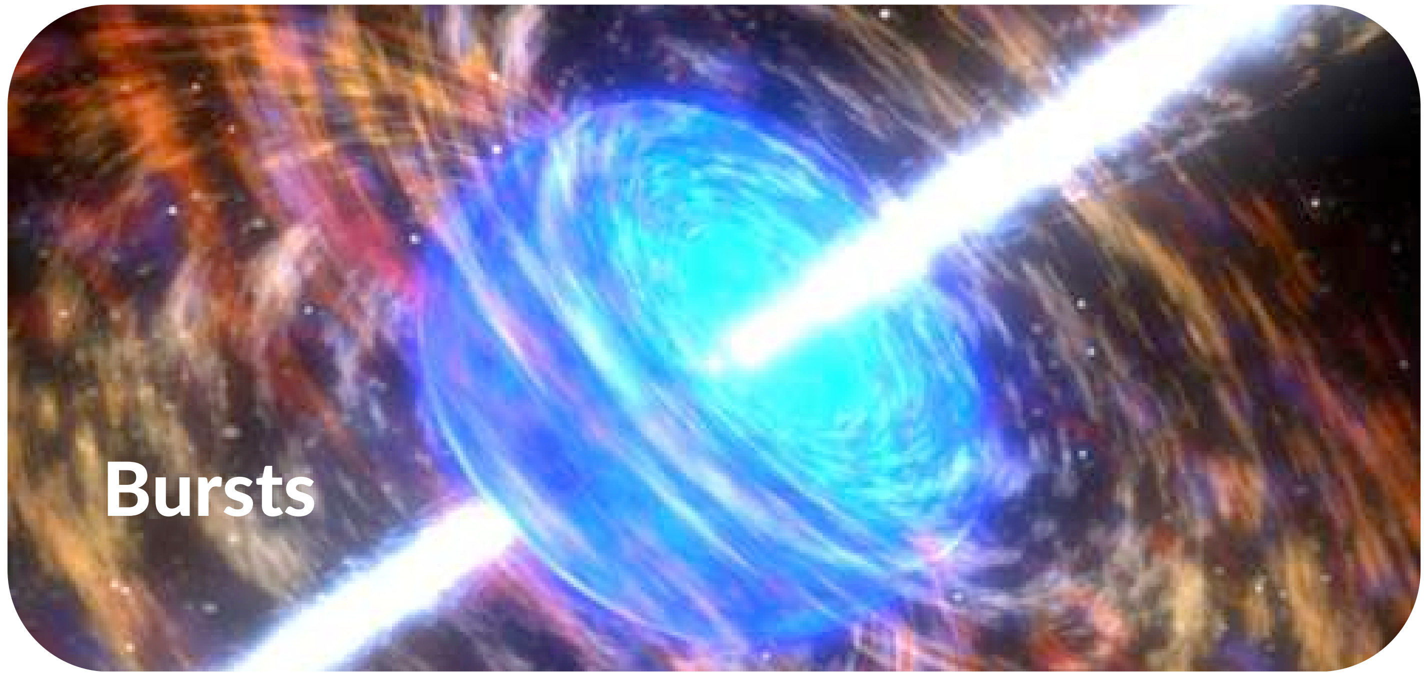
Sources of GWs

Modelled

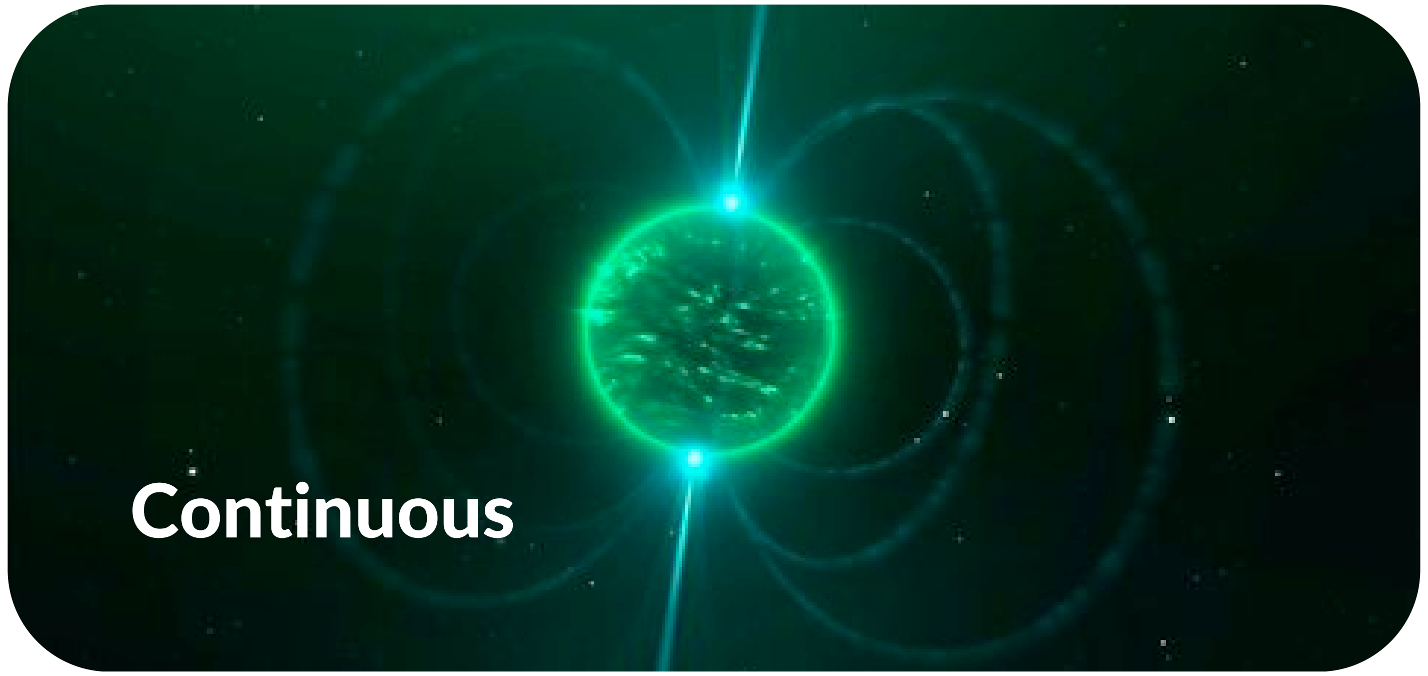
Unmodelled

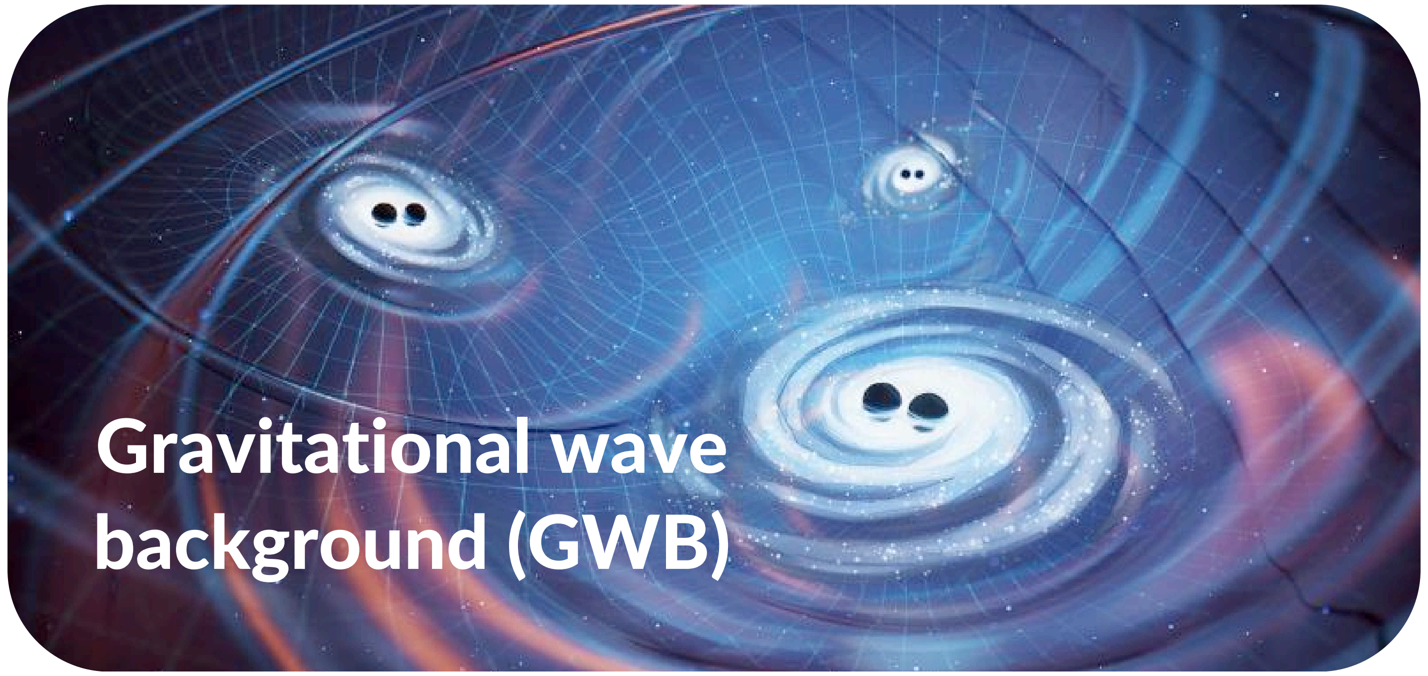
Short duration





Long duration



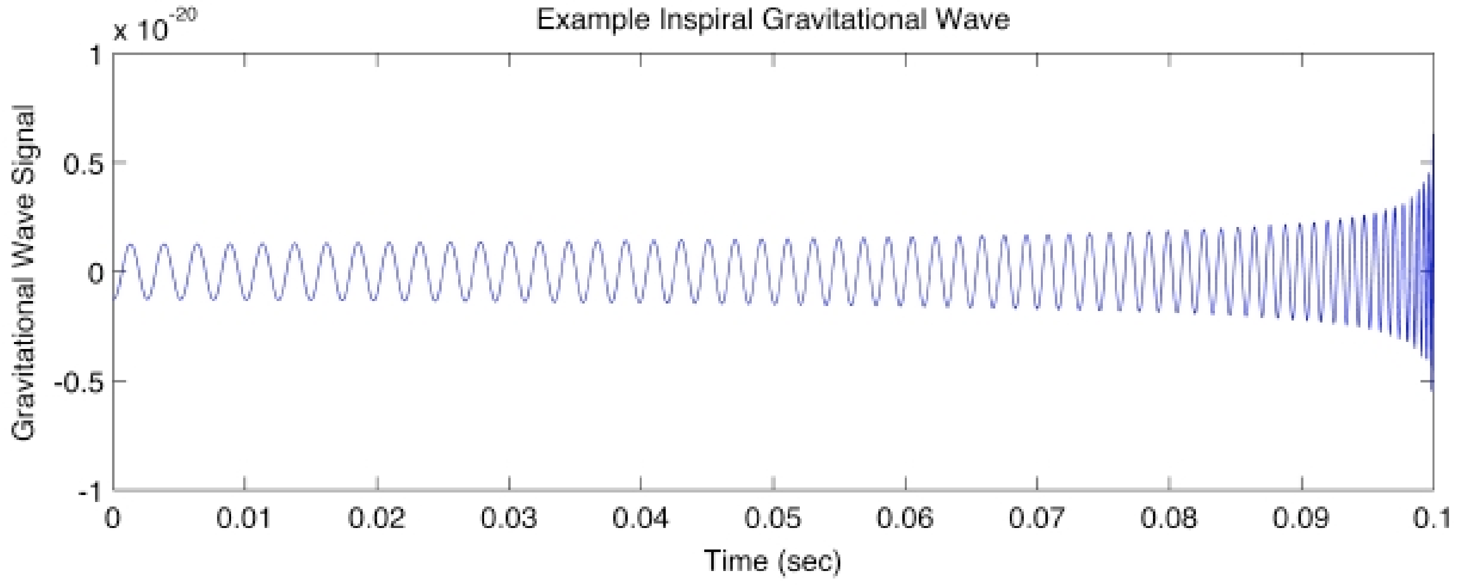


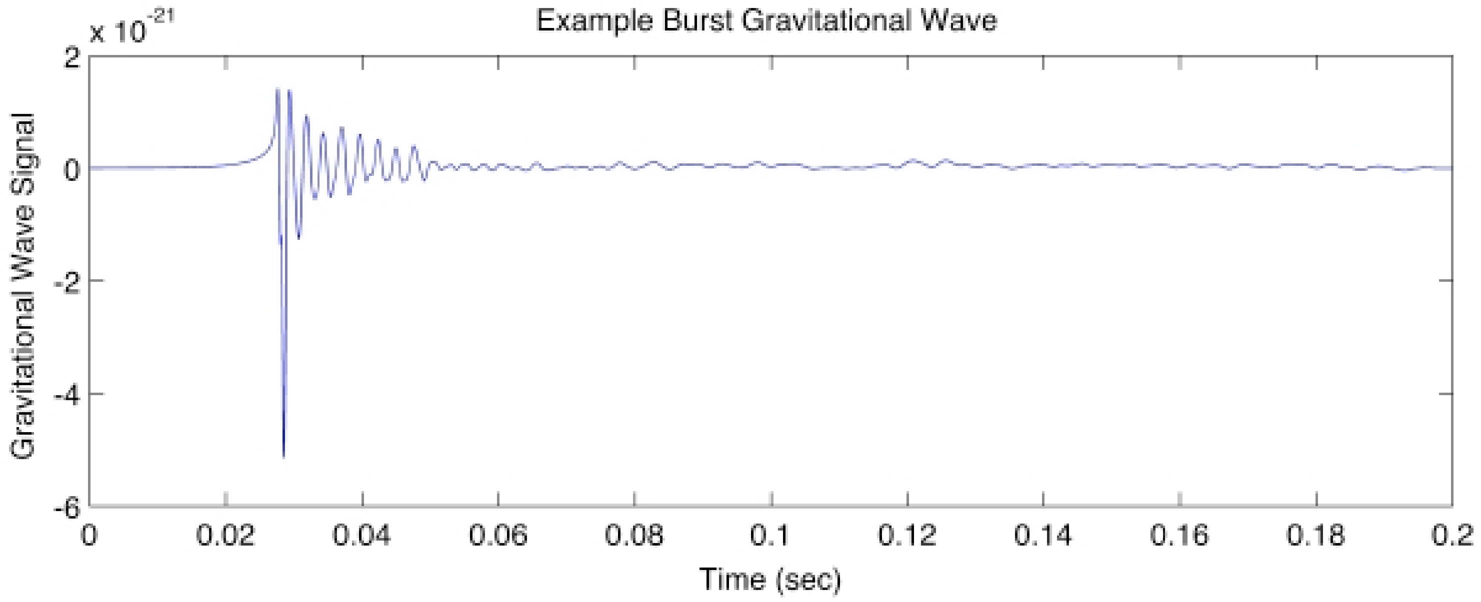
Sources of GWs

Modelled

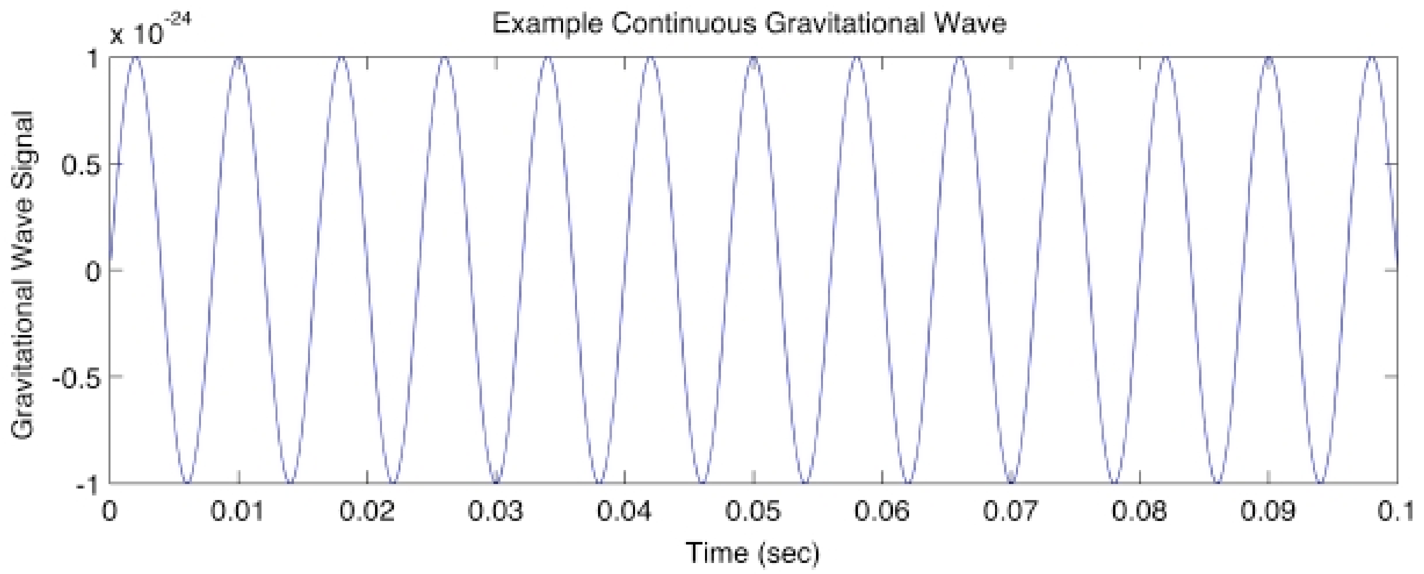
Unmodelled

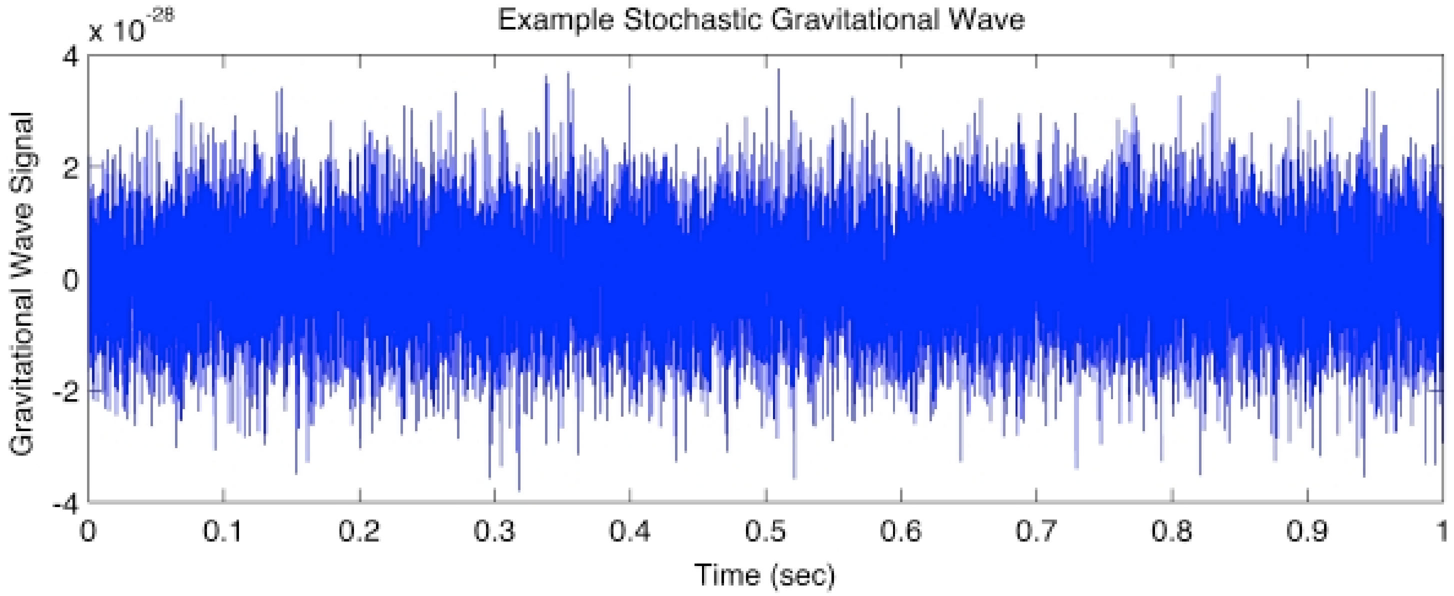
Short duration



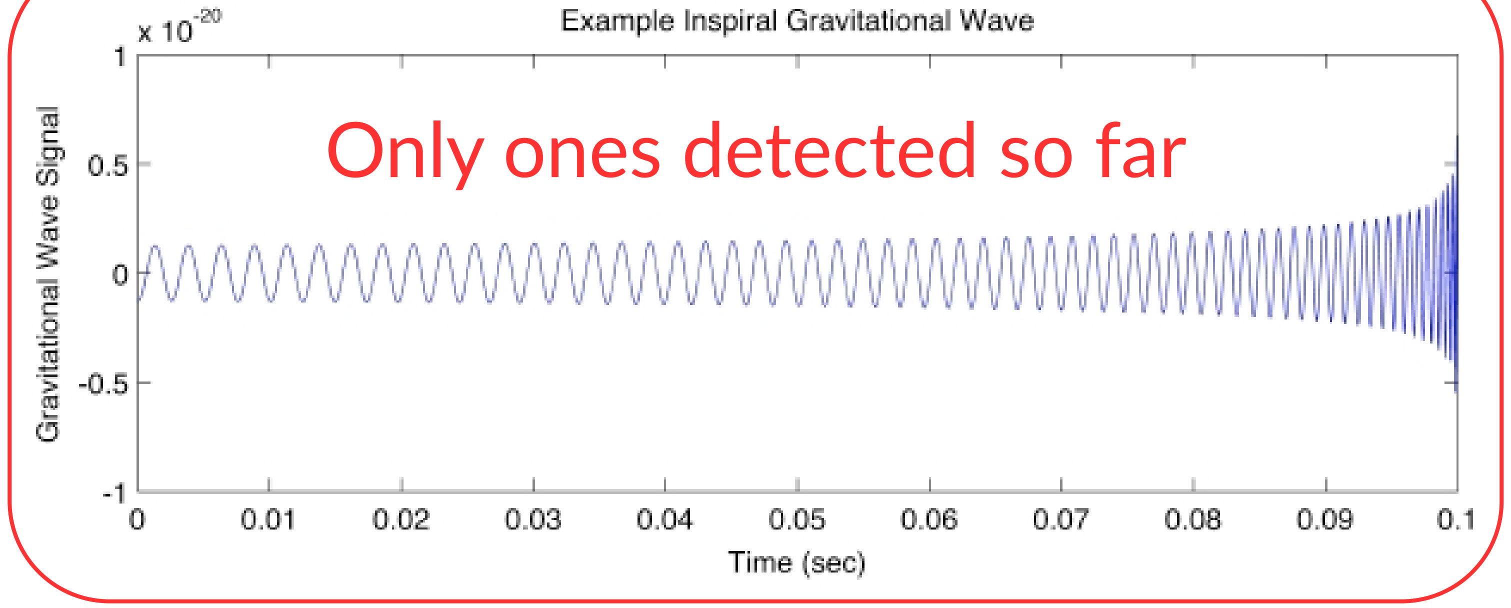
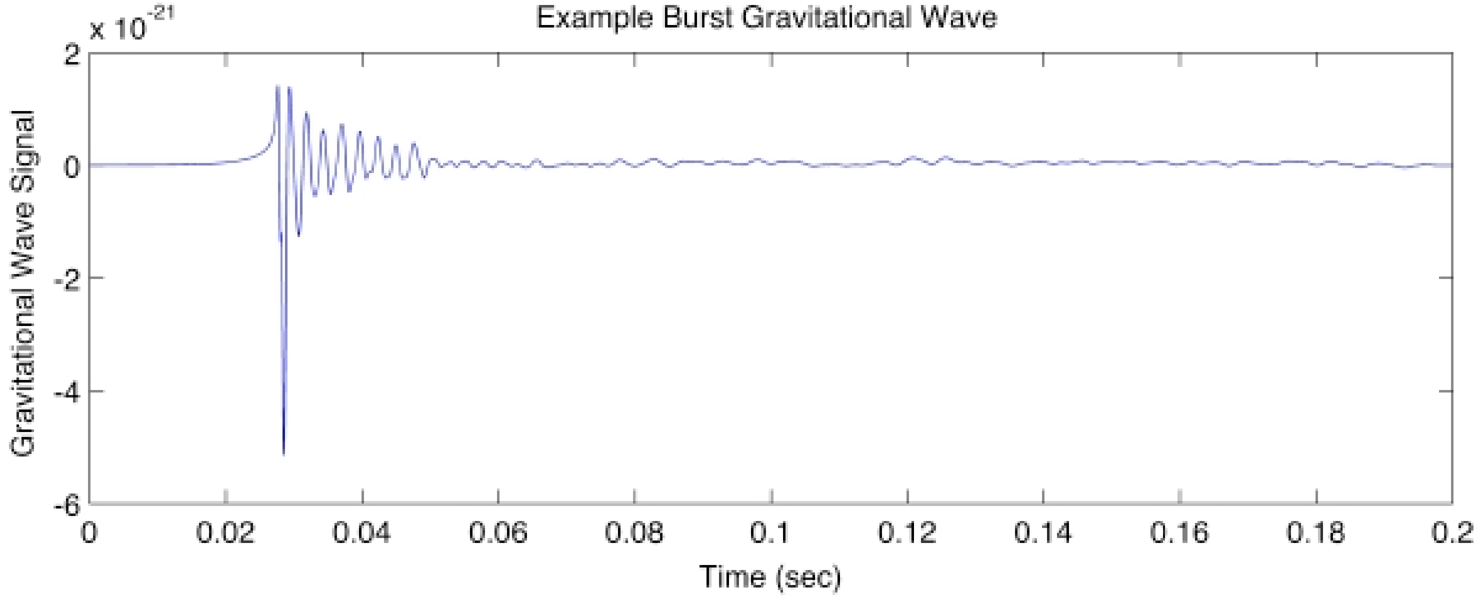
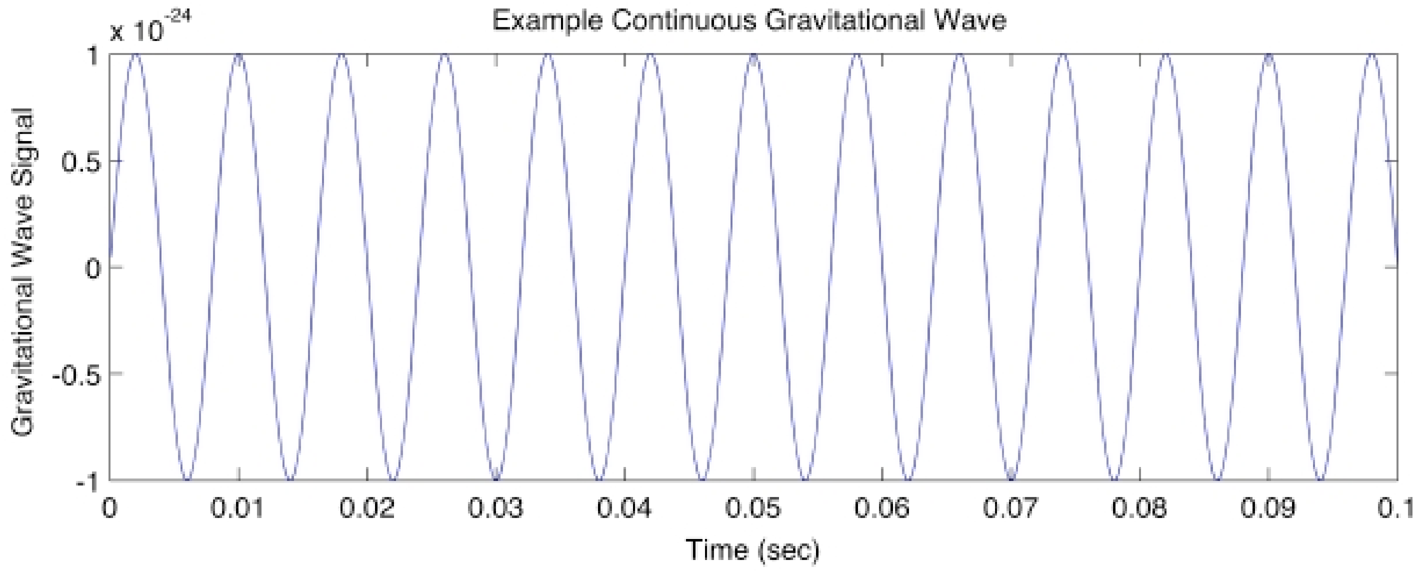
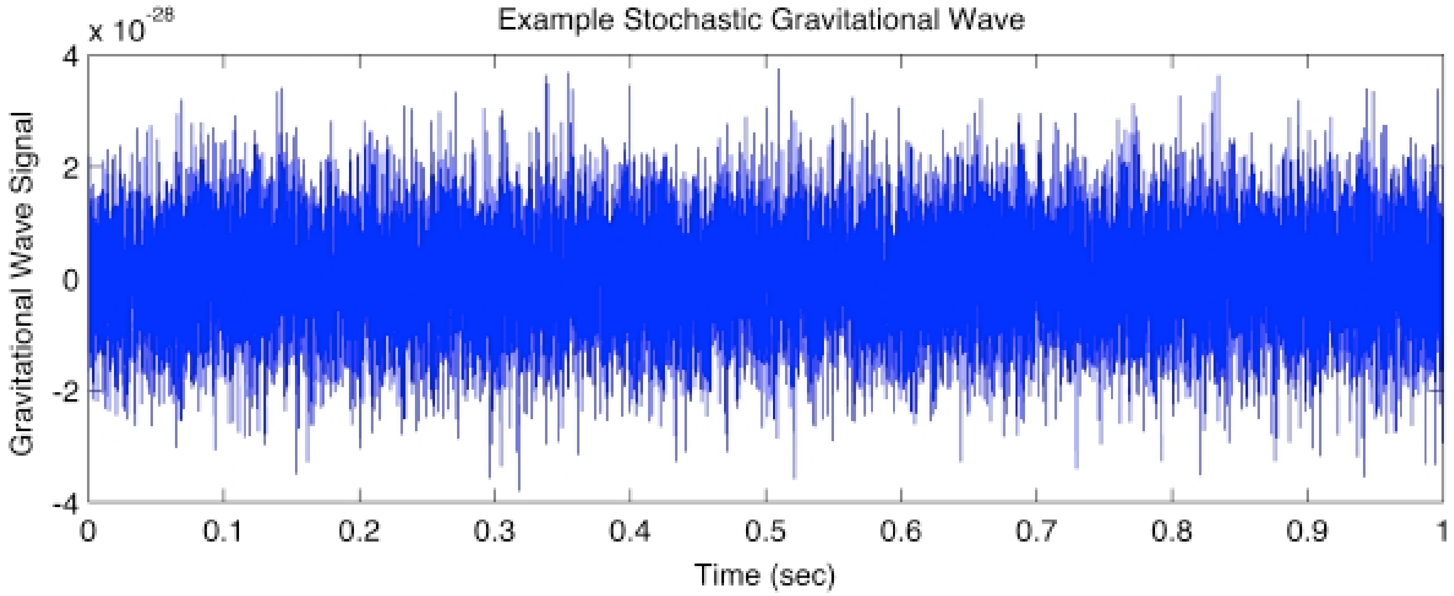


Long duration



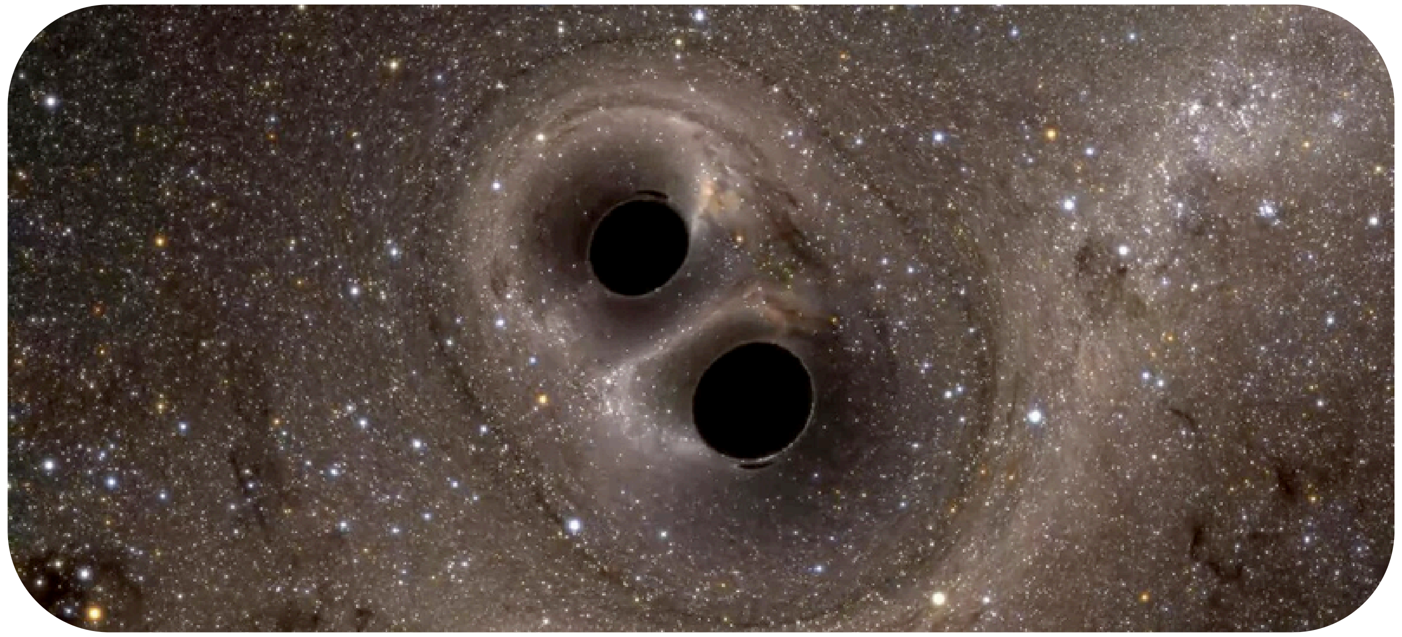


Sources of GWs

	Modelled	Unmodelled
Short duration	 Only ones detected so far	
Long duration		

Sources of GWs – CBCs

- Classification:
  - Binary of black holes (BBH)
  - Binary of Neutron Stars (BNS)
  - Binary of NS and BH (NSBH)



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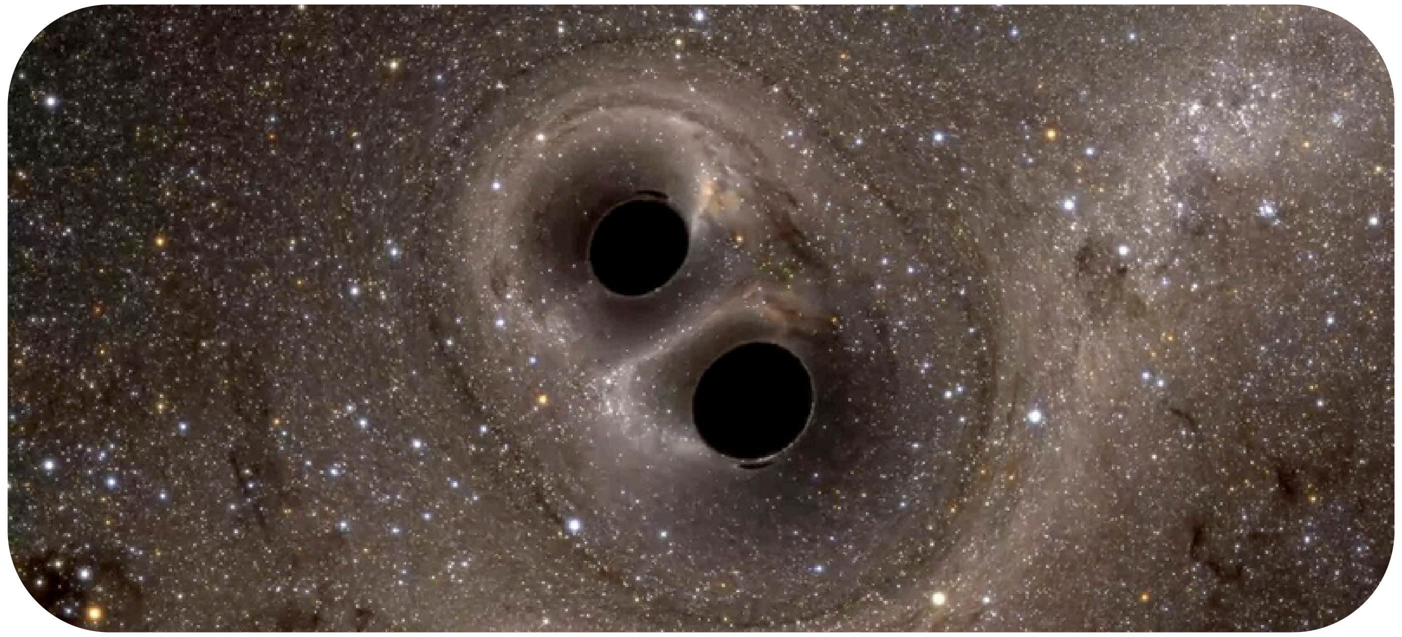
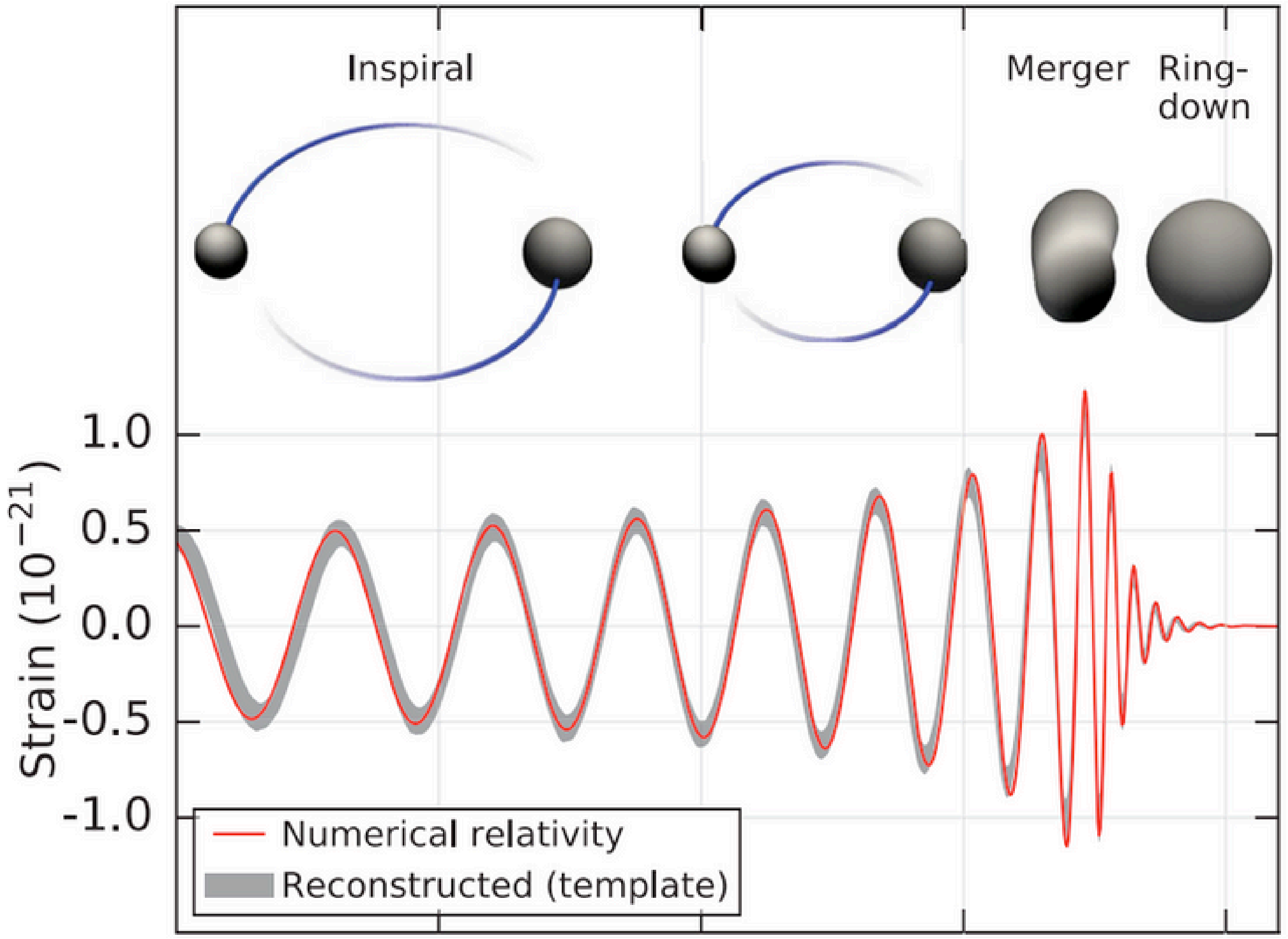
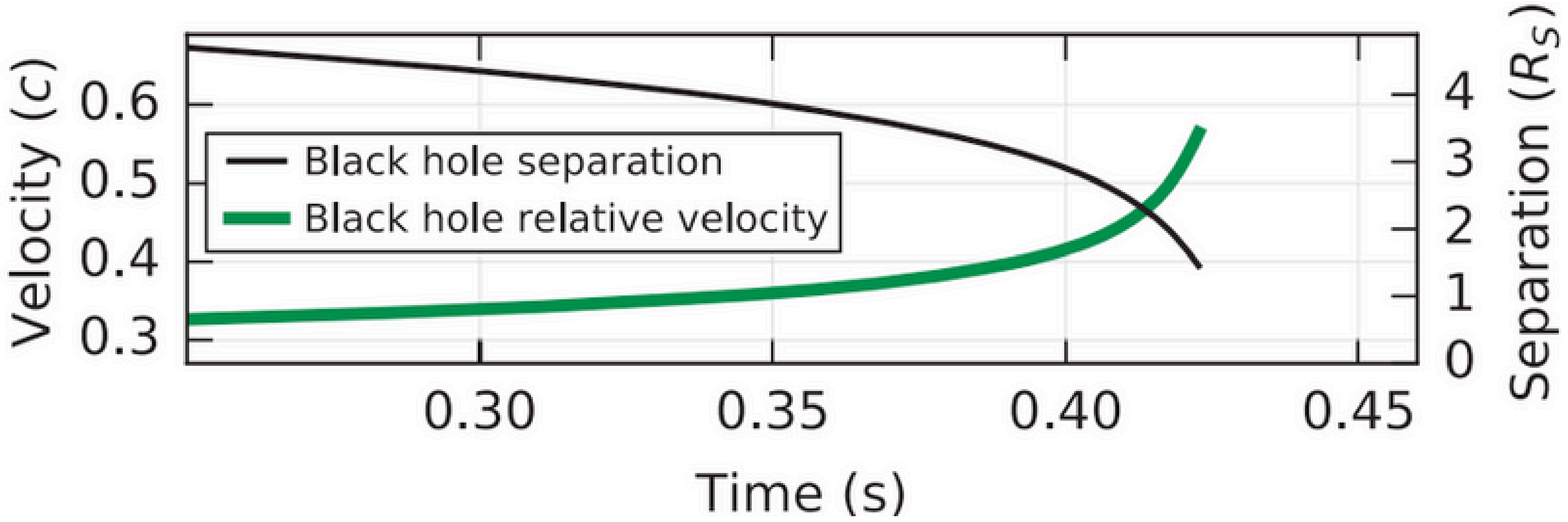




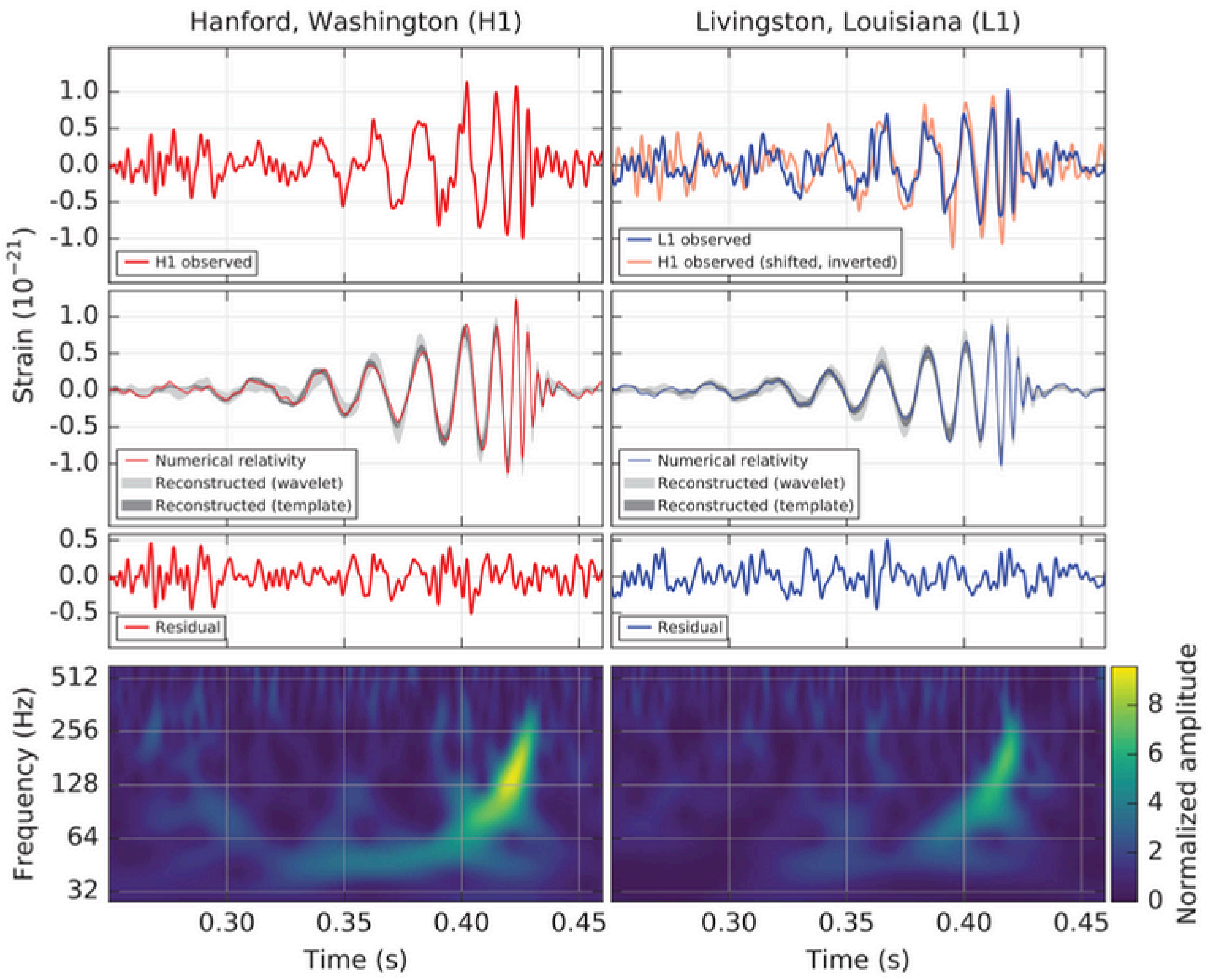


Vu, Nils. (2022). 10.25932/publishup-56226.

Sources of GWs – CBCs

First GW detection: **GW150914**

- 14^{th} of September 2015
- BBH: $36^{+5}_{-4}\,M_\odot$ and $29^{+4}_{-4}\,M_\odot$ at 90% CL
- Dimensionless spin magnitude of the more massive black hole < 0.7 at 90% CL
- Luminosity distance to the source 410^{+160}_{-180} Mpc
- Binary merges into a black hole of mass: $62^{+4}_{-4} M_\odot$ and spin $0.67^{+0.05}_{-0.07}$
- Combined matched filter SNR ratio of 24
- False alarm rate < 1 event per 203 000 years ~ significance > 5.1 σ



Abbott et al. (LIGO & Virgo), Phys. Rev. D 93, 122003 (2016). https://arxiv.org/abs/1602.03839

B. P. Abbott et al. (LIGO & Virgo), Phys. Rev. Lett. 116, 241102 (2016), arXiv:1602.03840

CBCs – search method

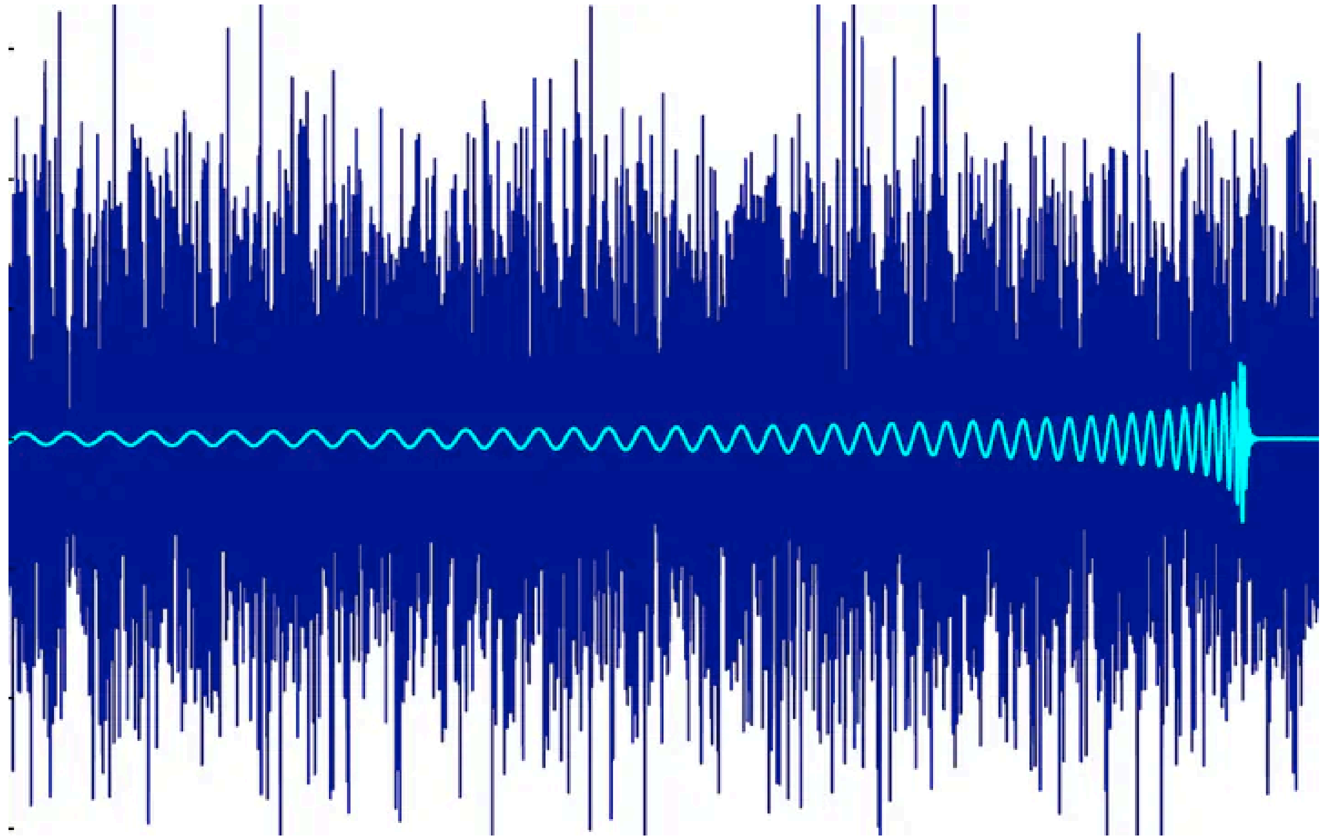
- The data, s, can be split in two parts: noise, n, and the gravitational wave, h

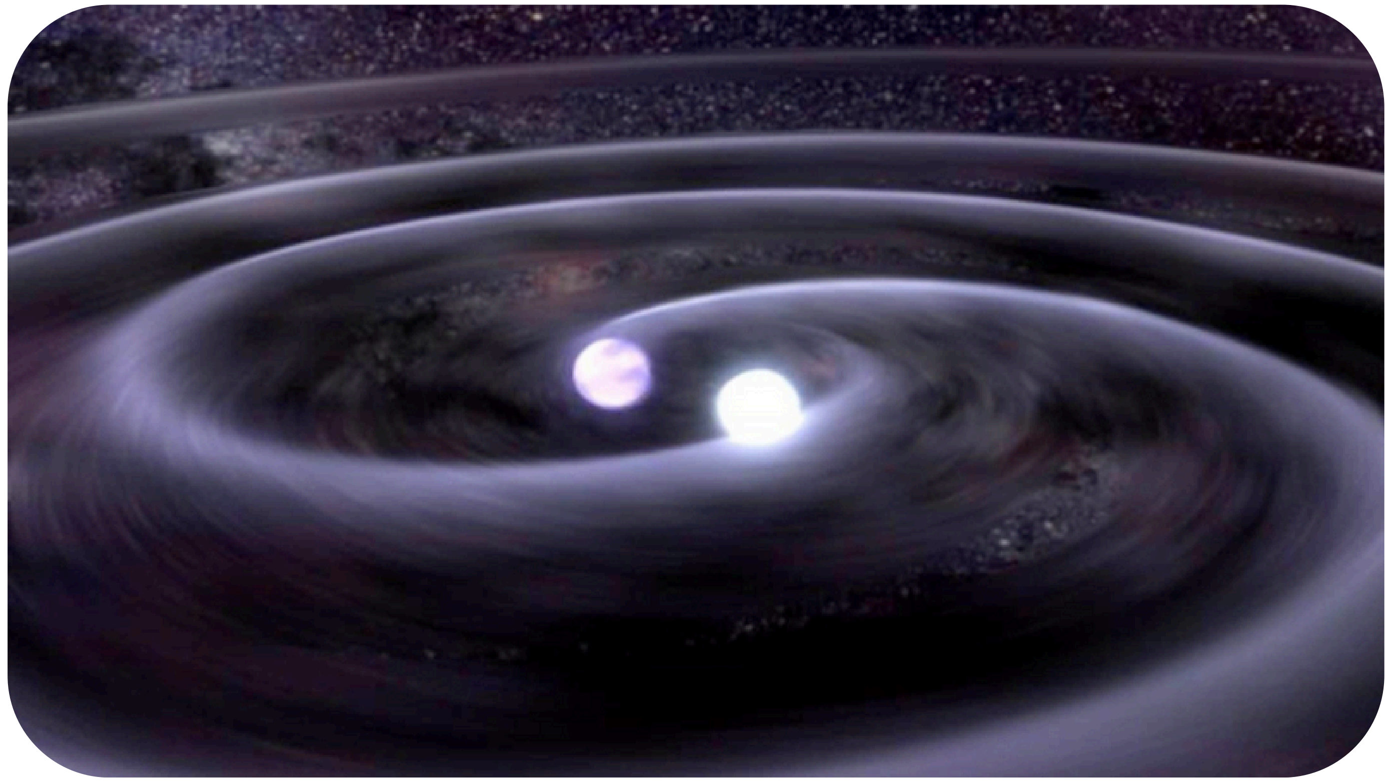
$$s(t) = n(t) + h(t) \xrightarrow{\text{FFT}} \tilde{s}(f) = \tilde{n}(f) + \tilde{h}(f)$$

- To analyse the data and find the GW, a matched filter is applied, using signal templates (inter-correlation between the data and template)

$$S = \int_{-\infty}^{+\infty} \tilde{s}(f)\tilde{Q}^*(f)\mathrm{d}f$$

Frequency -domain template





CBCs – search method

- We then need to figure out which is the filter that maximizes the SNR
- The significance of the filtered signal is given by the signal to noise ratio (SNR)

$$SNR = \frac{\langle S \rangle}{\sigma_N}$$

Expected value of S when the signal is present

- σN is the standard deviation of the filtered noise N:

$$\sigma_N^2 = \langle N^2 \rangle - \langle N \rangle^2 = \langle N^2 \rangle$$

- N can be expressed as:

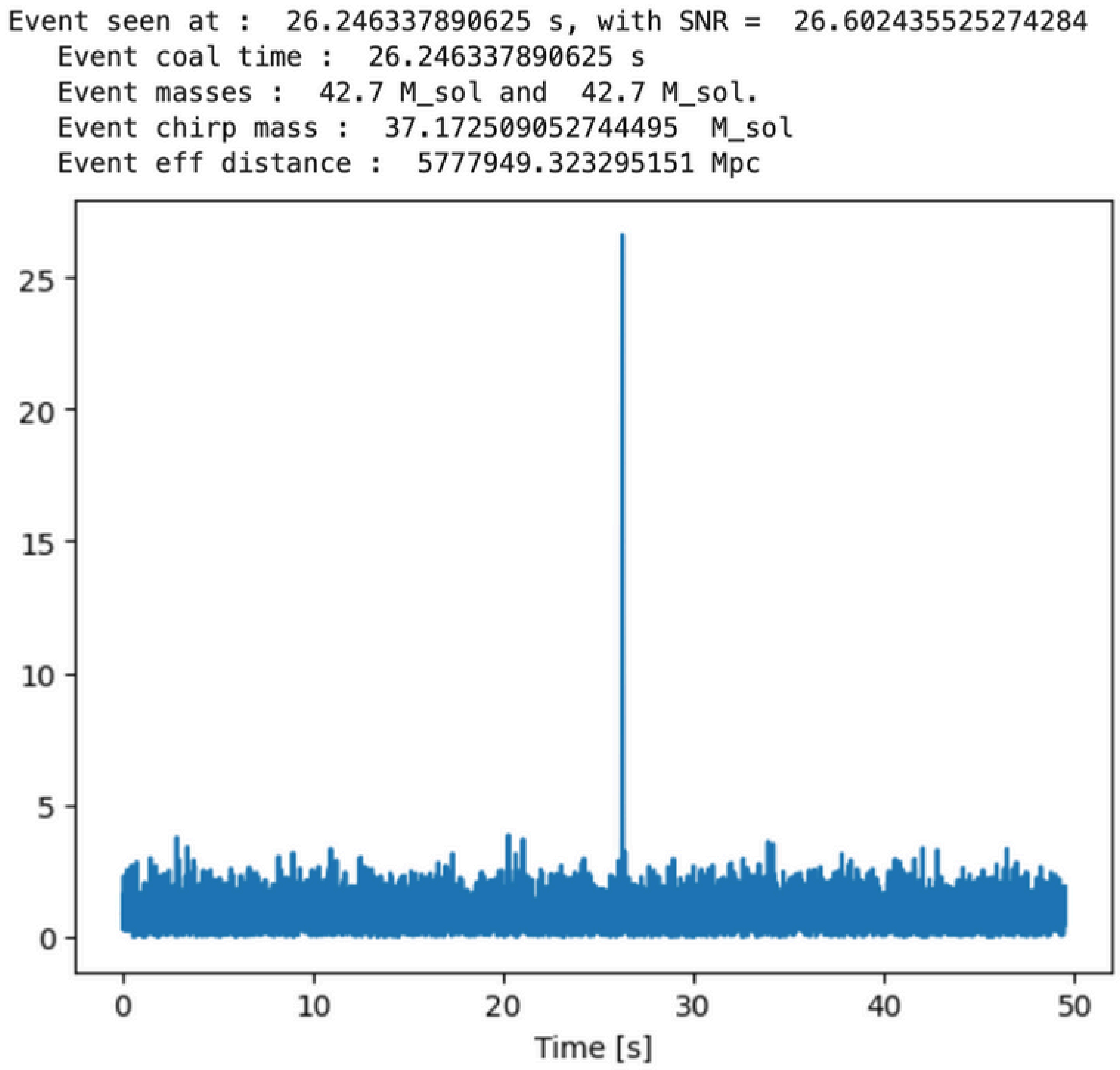
$$N = \int_{-\infty}^{+\infty} \tilde{n}(f)\tilde{Q}^*(f)\mathrm{d}f = S - \langle S \rangle$$

CBCs – search method

- The filtering is optimal if the template Q maximizes the SNR: it can be shown that the maximal SNR is obtained when the time of the wave tGW fits with the template and it has a value of α, which is a normalization factor in:

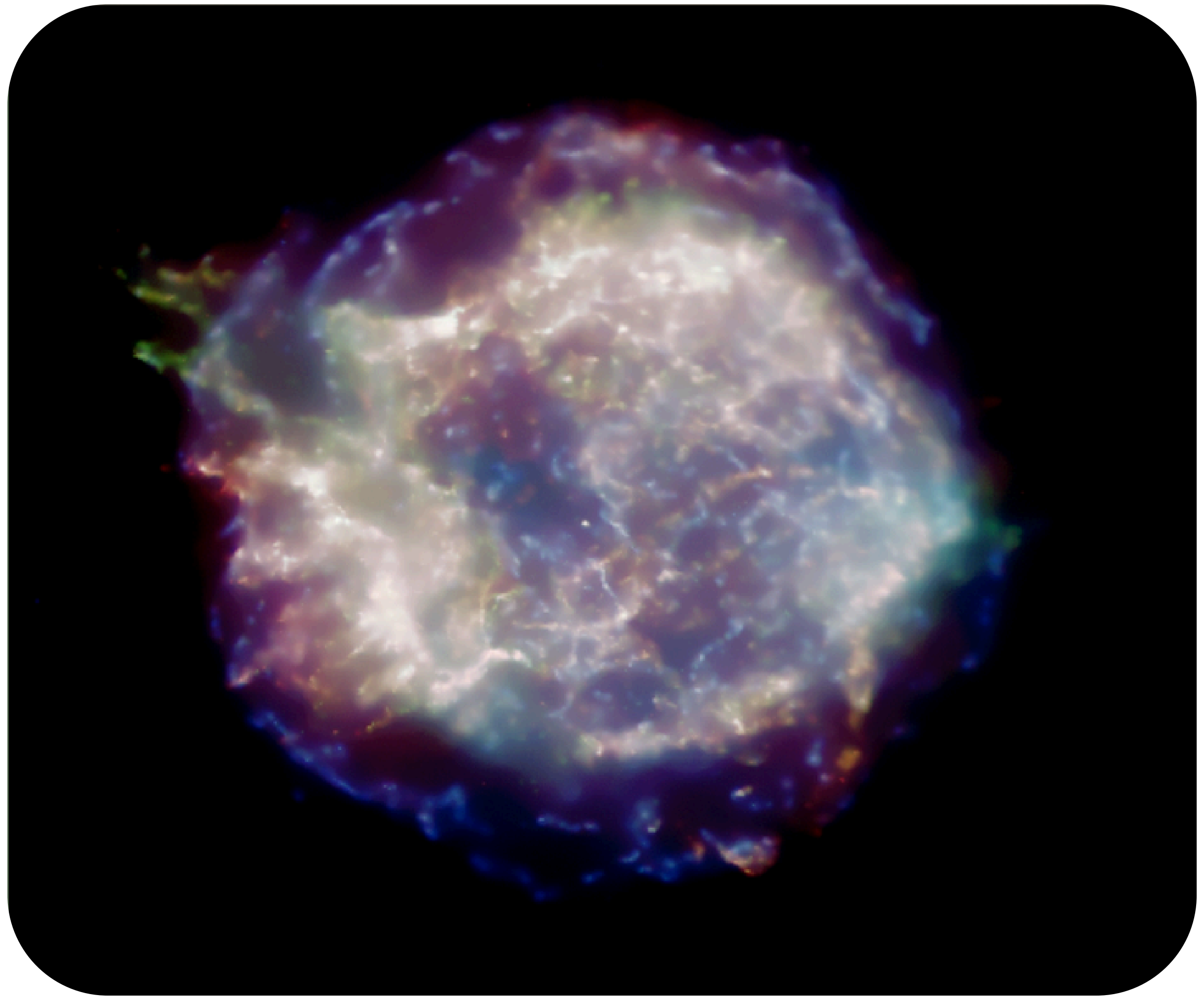
$$\tilde{h}(f) = 2\alpha\tilde{T}(f)e^{2i\pi f t_0}$$

- T (f) is the expected normalised waveform and t0 is a time shift



Sources of GWs – Bursts

- Short-duration (few ms up to a few minutes) unknown or unanticipated sources
- Expected from astrophysical sources:
  - stellar collapses, BNS post-merger signals
  - generators of gamma ray bursts, and other energetic phenomena
    - Energetic EM phenomena associated with isolated NS (magnetar flares, pulsar glitches, fast radio bursts, etc).
    - Instabilities in accretion disks around BHs
    - Cosmic strings



Sources of GWs – Bursts

- Search method:
  - Looking for zones of excess power in the data (in time-frequency domain). Either per detector, or with cross-correlated data.
  - There is elimination of event triggers that are coincident in time with anomalous events in PEM auxiliary channels

Sources of GWs – Bursts

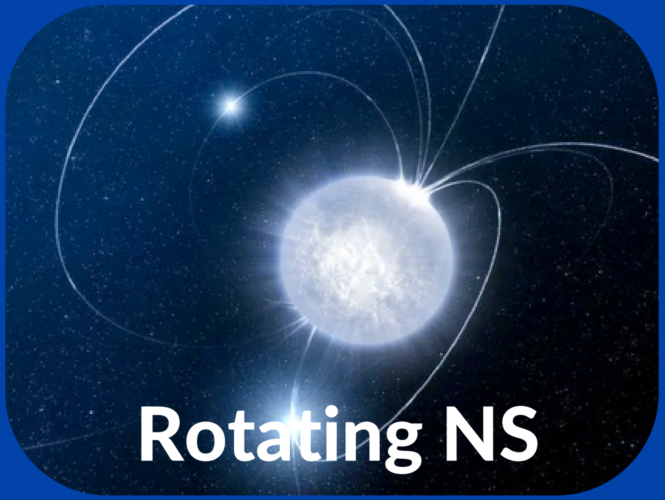
- Search method:
  - Real GW bursts will cause a simultaneous response in all 3 IFOs → triggers to be coherent in the cross-correlated data
  - Mean rate of background events: measuring mean rate of events that pass our coincident step after artificially shifting in time all the event triggers identified in one of the detectors.

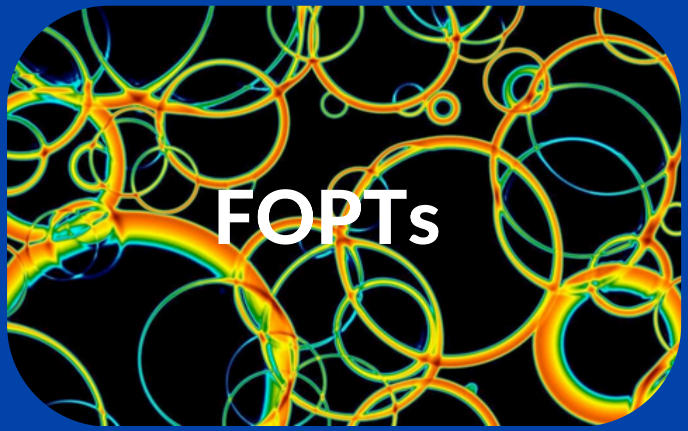
Sources of GWs – Continuous

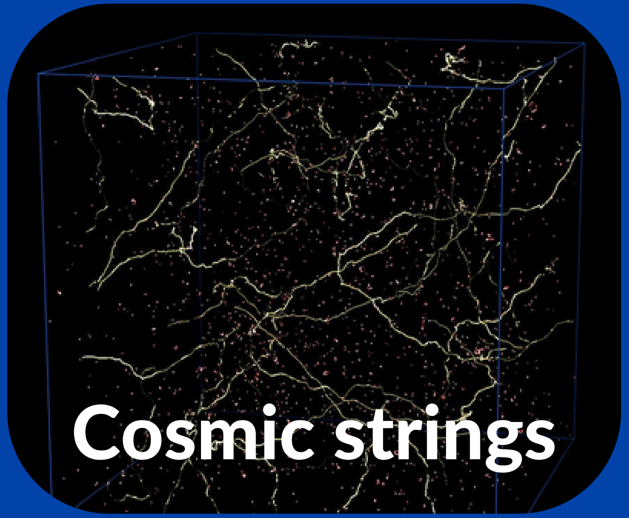
- Expected to be produced by a single spinning massive object like a NS.
- Any bumps or imperfections in the spherical shape of this star will generate GW as it spins.
- If the spin-rate of the star stays constant, so too will the GW it emits: the GW has continuously the same frequency and amplitude

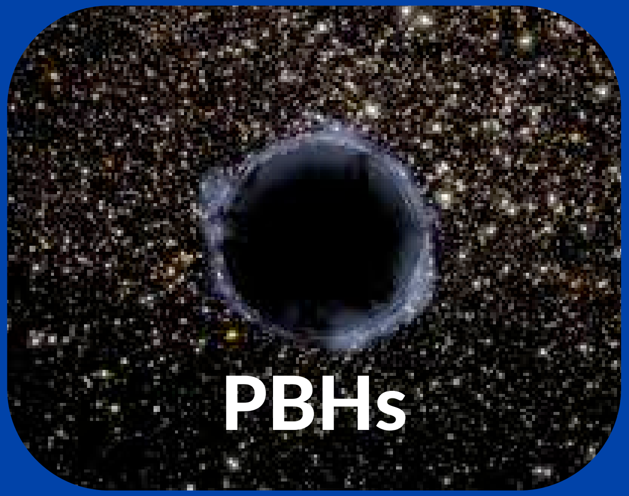
Sources of GWs – GWB

Superposition of random GW signals produced by a large number of weak, independent and unresolved sources

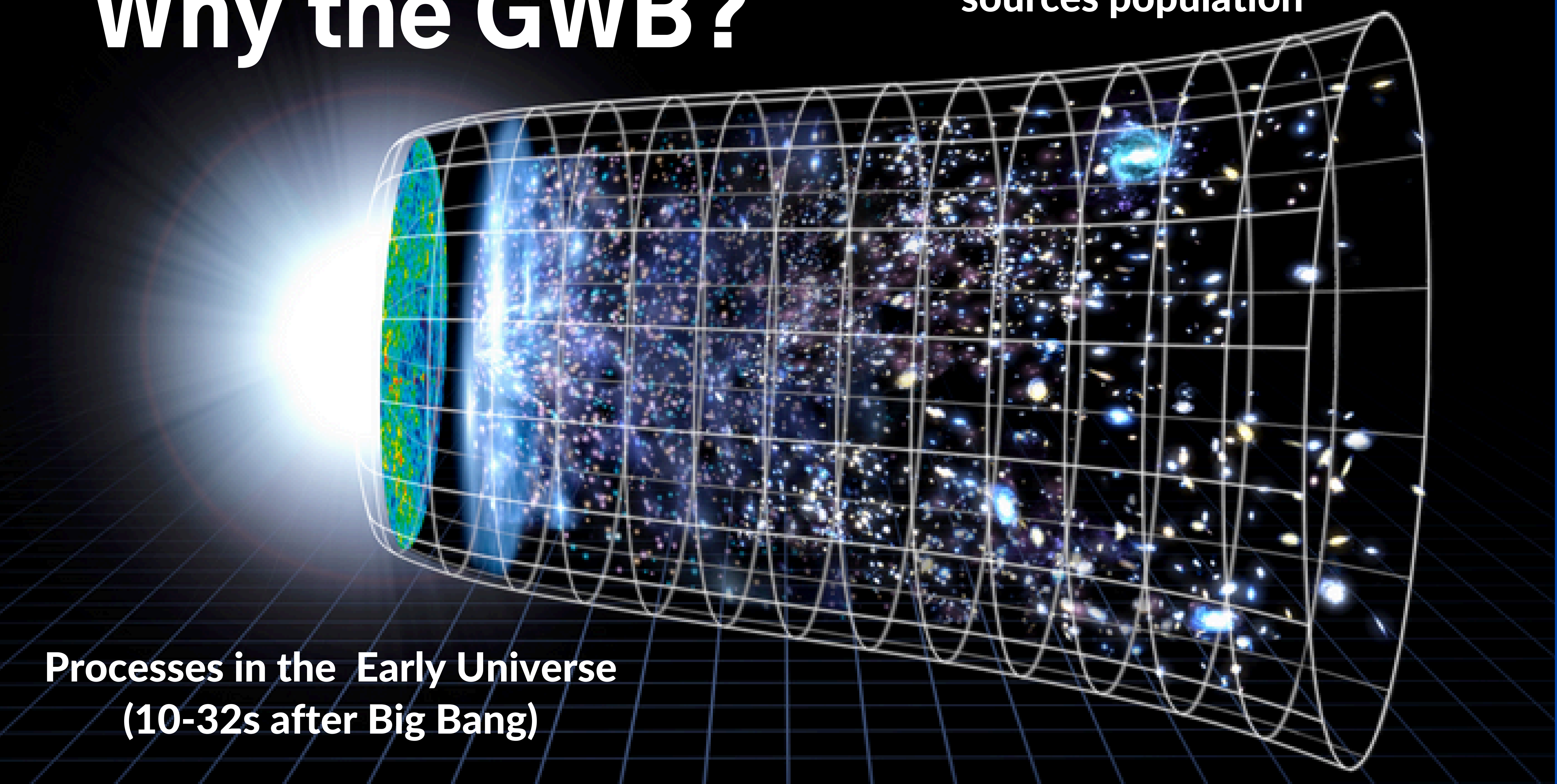








Why the GWB?


Astrophysical
sources population
Processes in the Early Universe
(10-32s after Big Bang)

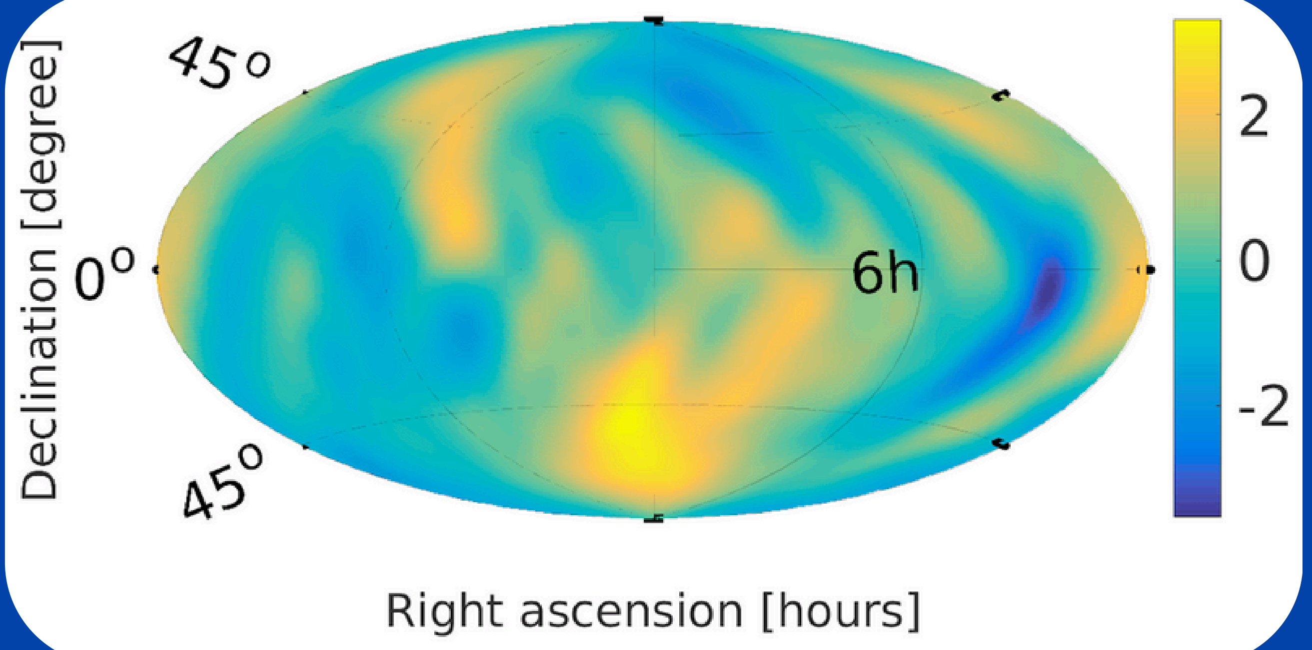
Sources of GWs – GWB

- Statistically: probability distribution or moments
- Large number of independent sources: GWB is Gaussian

$$\langle h_{ab}(t,\vec{x})\rangle, \quad \langle h_{ab}(t,\vec{x})h_{cd}(t',\vec{x}')\rangle$$

Assumptions

- Isotropic
- Stationary
- Unpolarized
- Gaussian



$$\langle h_A(f,\hat{n})h^*_{A'}(f',\hat{n}')\rangle = \frac{1}{16\pi}S_h(f)\delta(f-f')\delta_{AA'}\delta^2(\hat{n},\hat{n}')$$

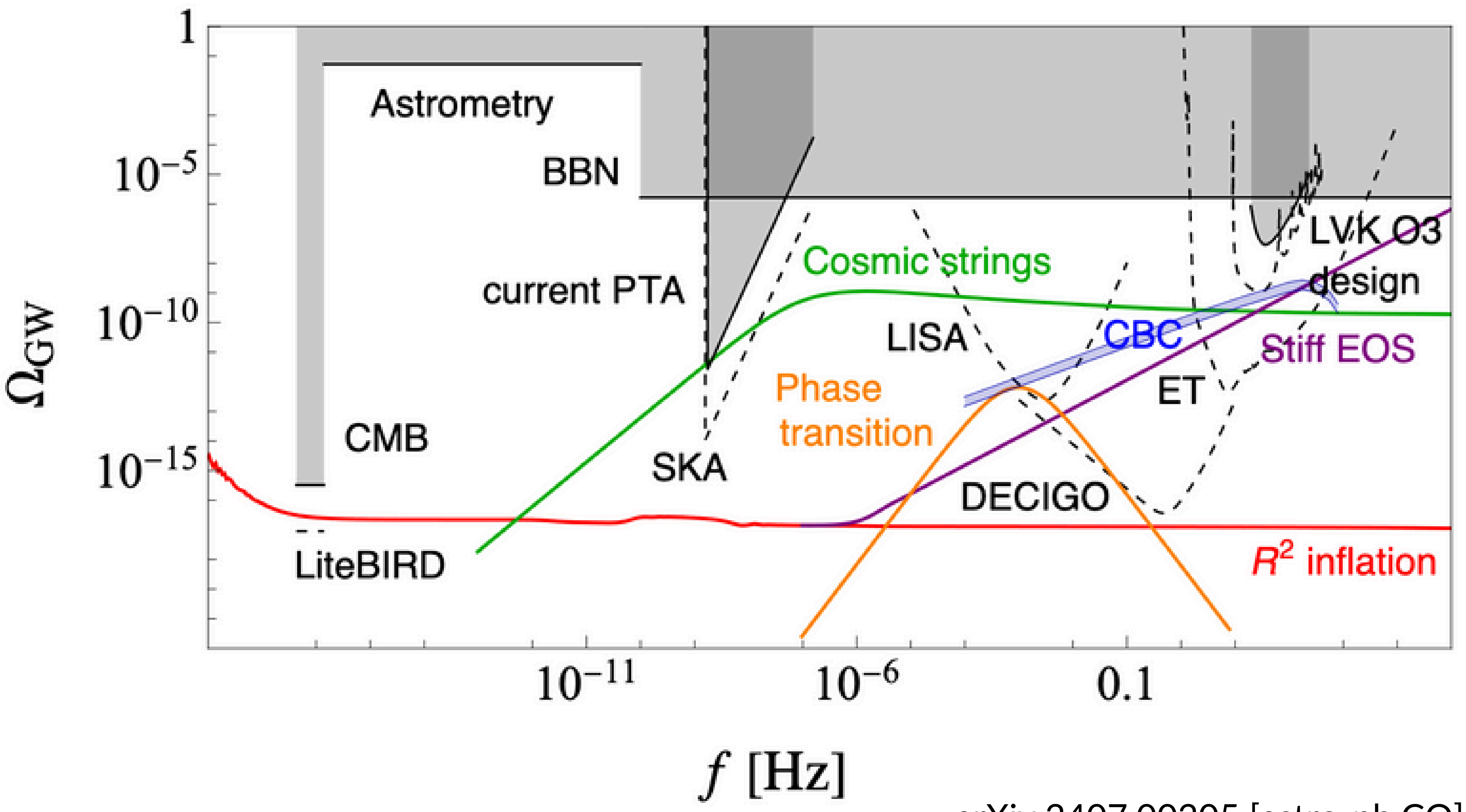
Sources of GWs – GWB

Fractional energy density spectrum in GWs

$$\Omega_{gw}(f) = \frac{1}{\rho_c}\frac{d\rho_{gw}}{d\ln f}$$

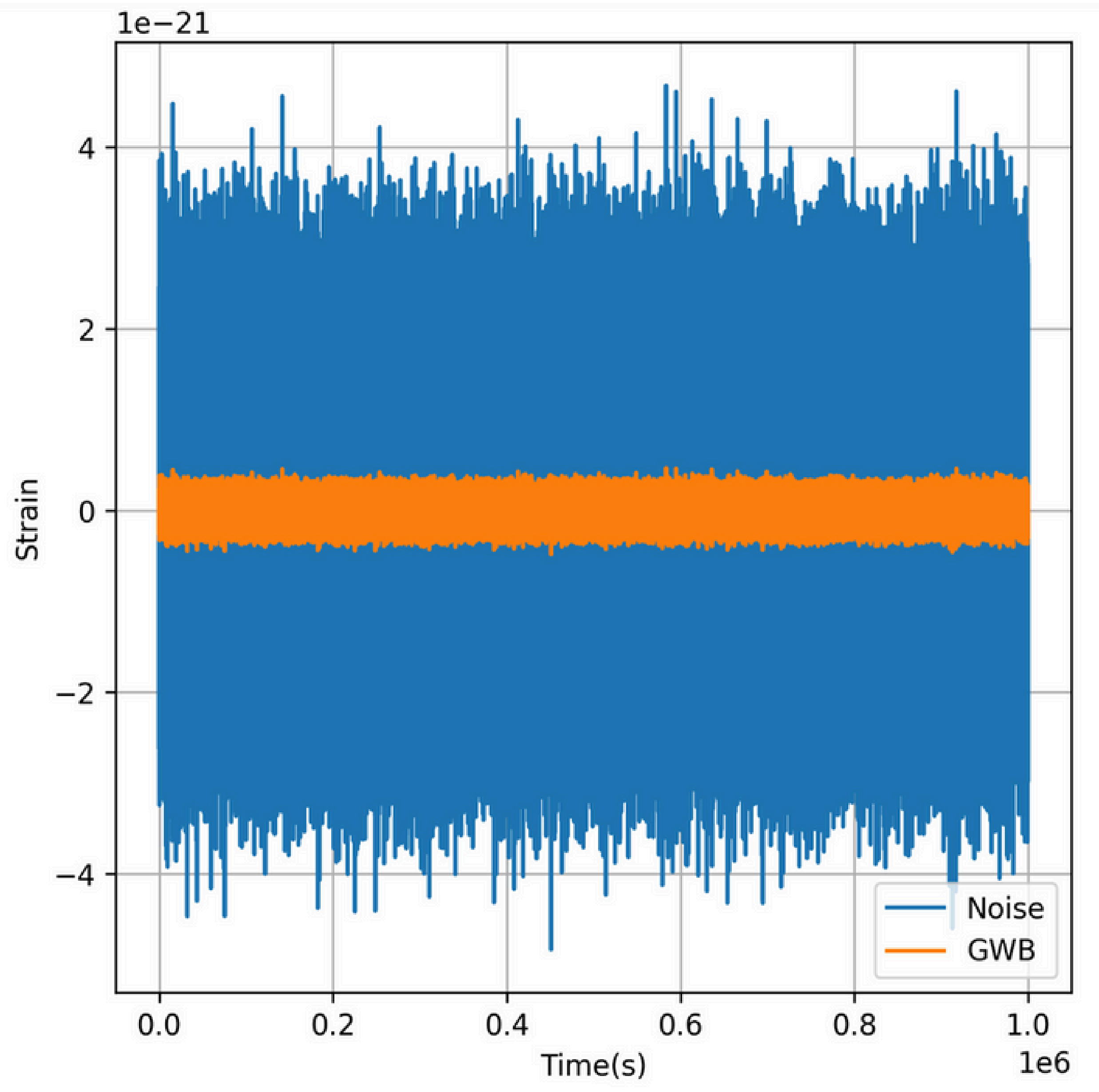
$$\rho_{GW} = \frac{c^2}{32\pi G}\langle \dot{h}_{ab}(t,\mathbf{x})\dot{h}^{ab}(t,\mathbf{x})\rangle$$

$$\Omega_{GW}(f) = \frac{2\pi^2}{3H_0^2} f^3 S_h(f)$$



arXiv:2407.00205 [astro-ph.CO]

Sources of GWs – GWB

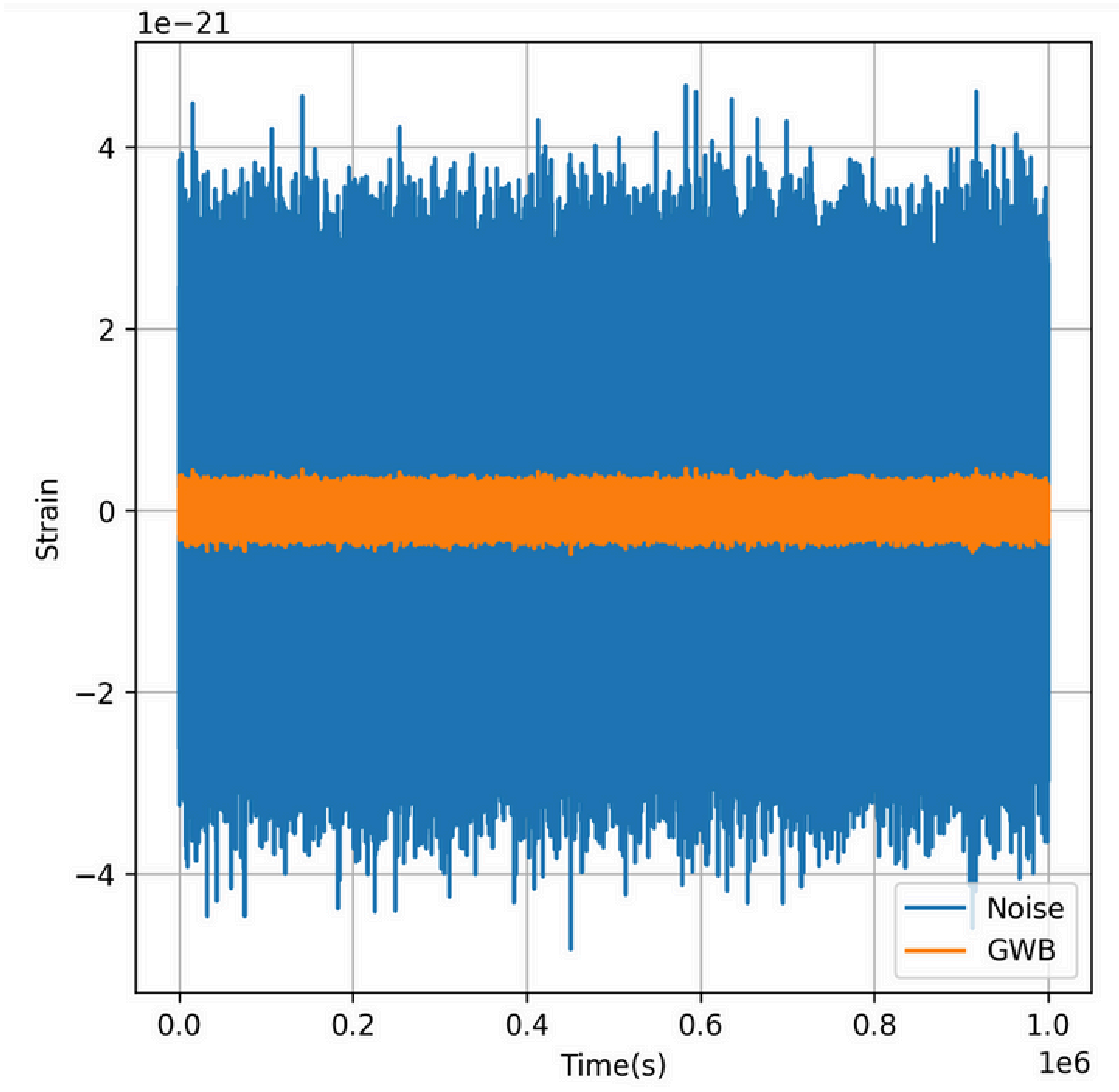


Cross correlation search

$$
\begin{aligned}
s_1(t) &= n_1(t) + h_1(t), \\
s_2(t) &= n_2(t) + h_2(t) .
\end{aligned}
$$

$$\mathrm{SNR} = \frac{3H_0^2\sqrt{T}}{10\pi^2}\left(\int_{-\infty}^{\infty} \mathrm{d}f \, \frac{\Omega_{\mathrm{GW}}^2(|f|)\gamma_{12}^2(|f|)}{|f|^6 P_1(|f|)P_2(|f|)}\right)^{1/2}$$

Sources of GWs – GWB



Cross correlation search

$$s_1(t) = n_1(t) + h_1(t),$$
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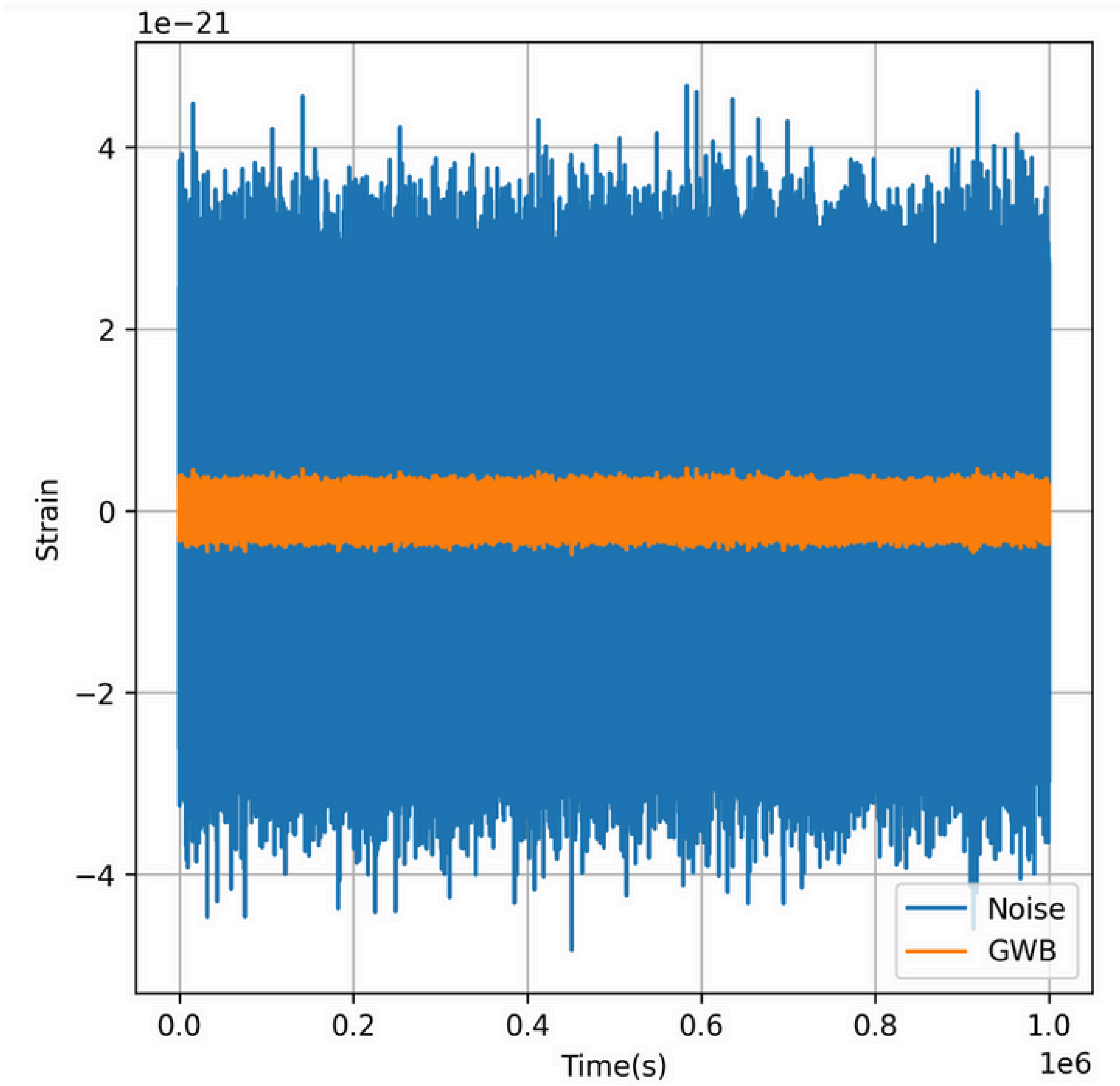
$$\mathrm{SNR} = \frac{3H_0^2\sqrt{T}}{10\pi^2}\left(\int_{-\infty}^{\infty} \mathrm{d}f\, \frac{\Omega_{\mathrm{GW}}^2(|f|)\gamma_{12}^2(|f|)}{|f|^6 P_1(|f|)P_2(|f|)}\right)^{1/2}$$

Sources of GWs – GWB



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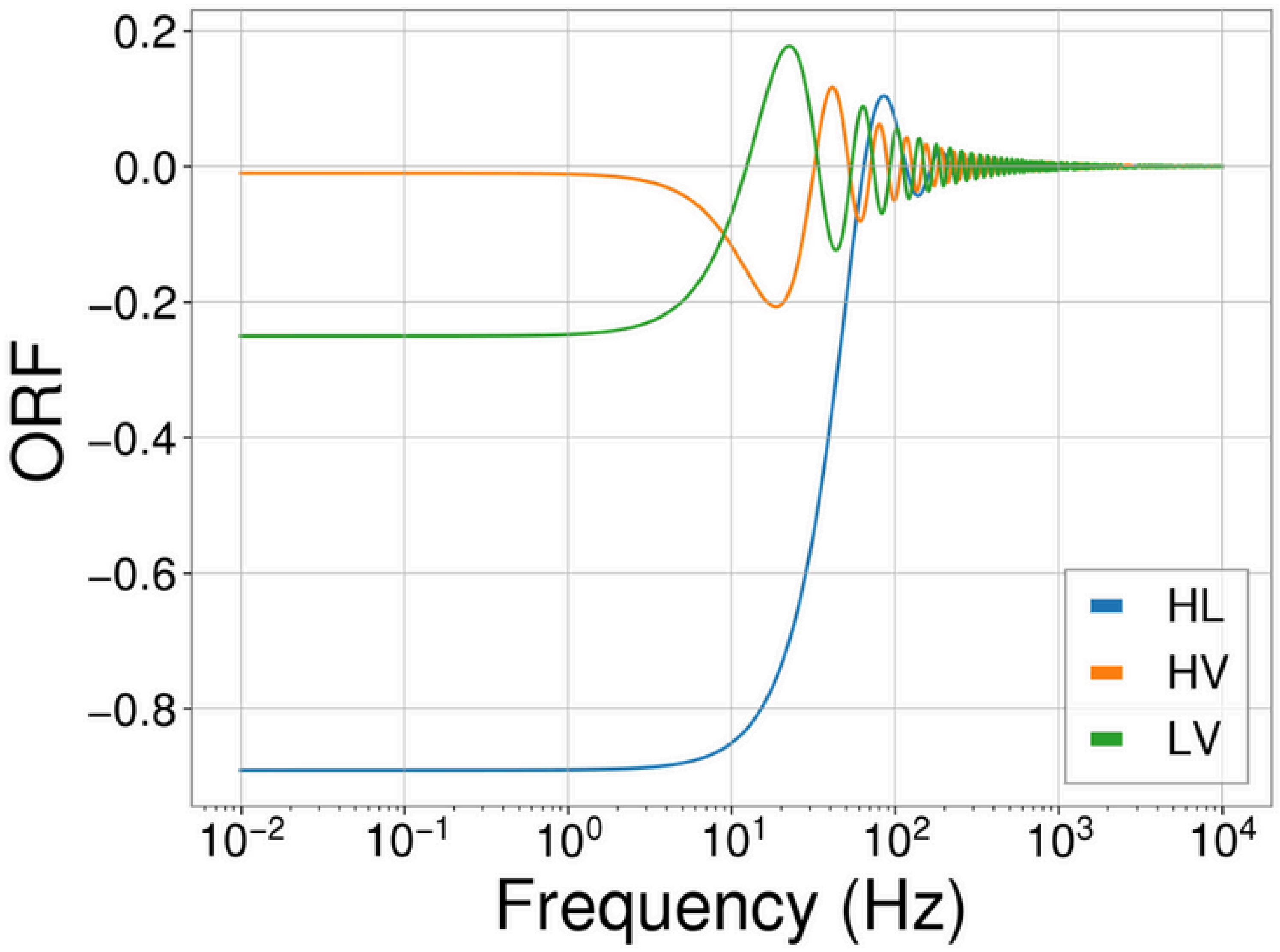
Sources of GWs – GWB



Cross correlation search

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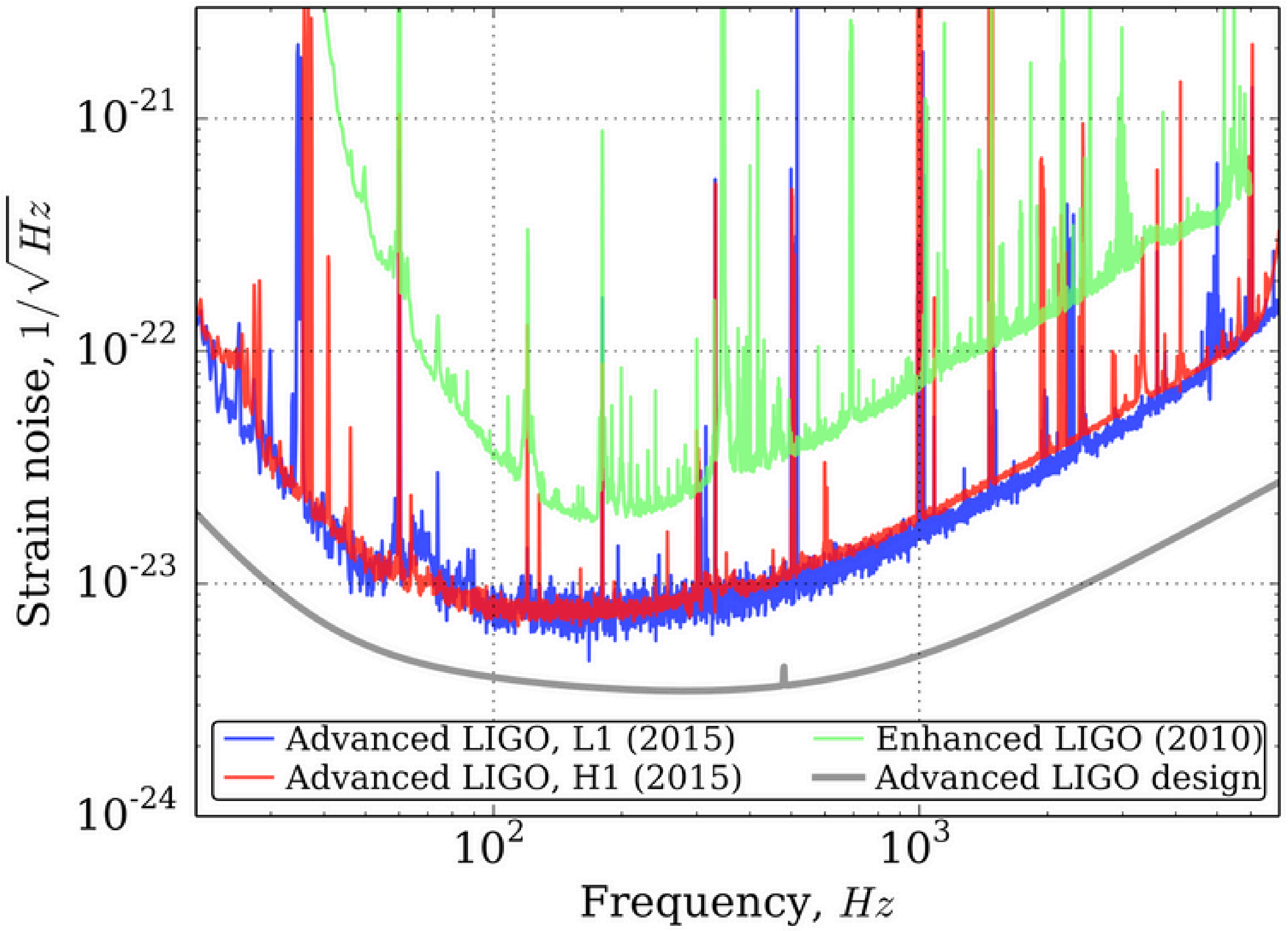
Sources of GWs – GWB

Noise power spectra



Cross correlation search

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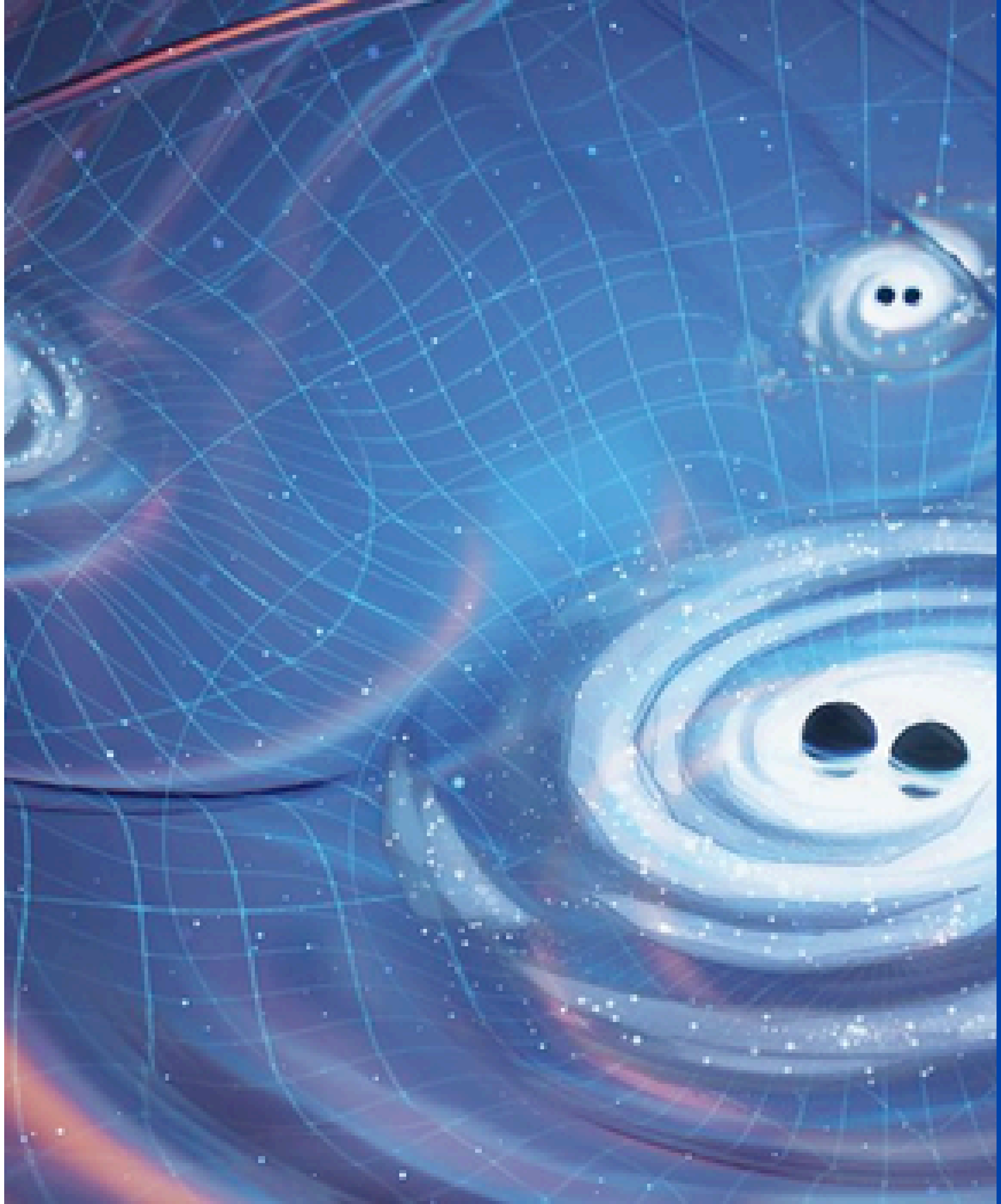
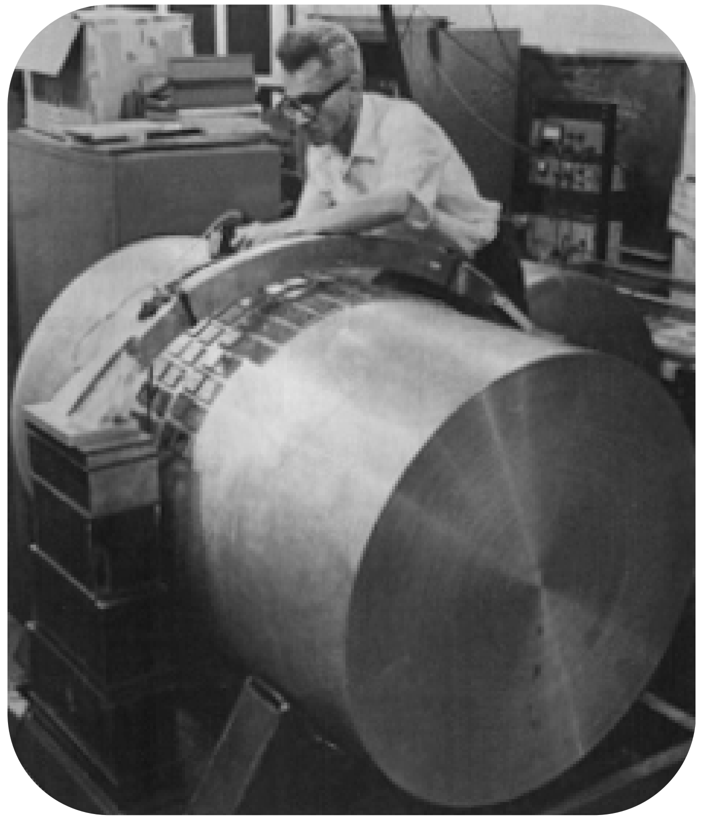
Ground based GW detectors

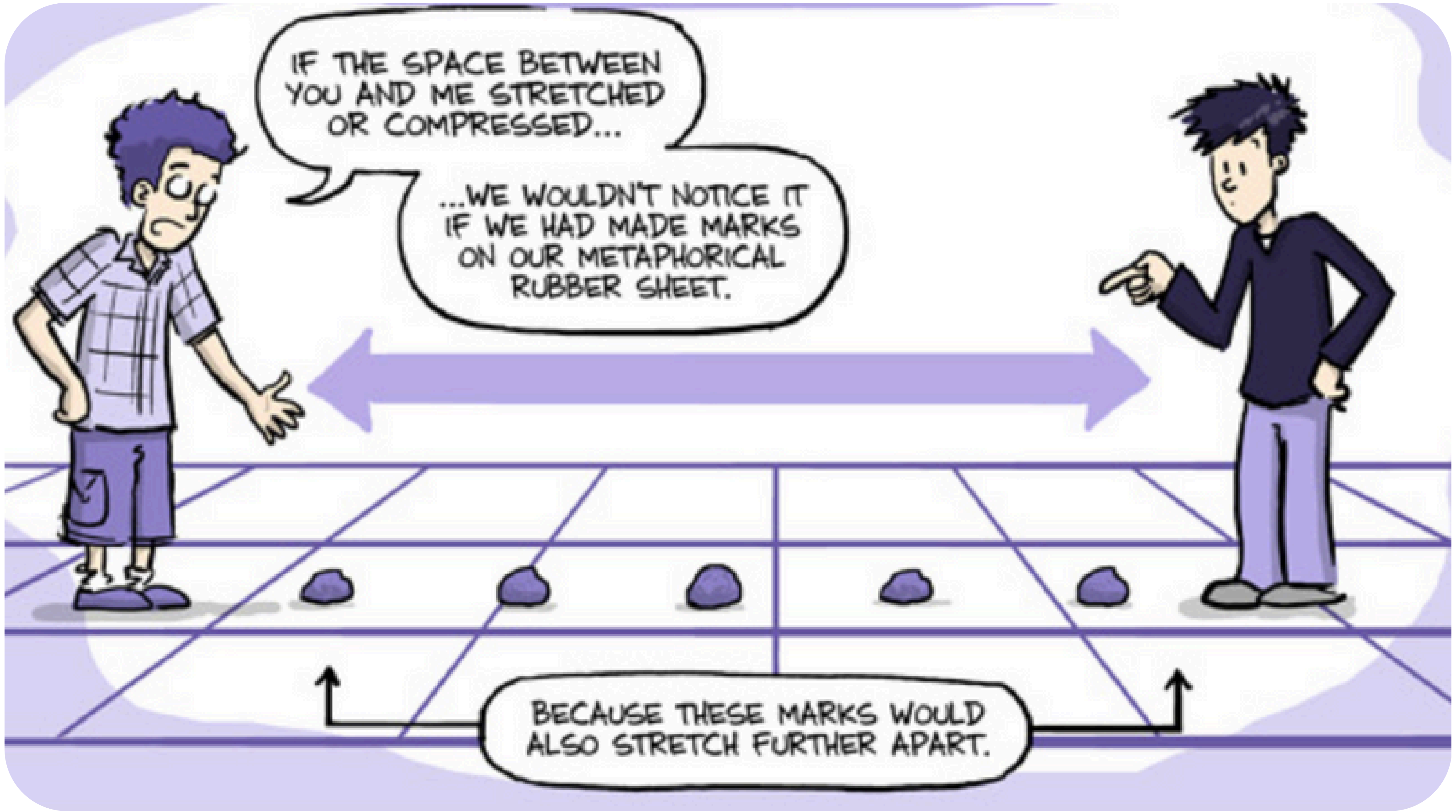


History of GW detectors

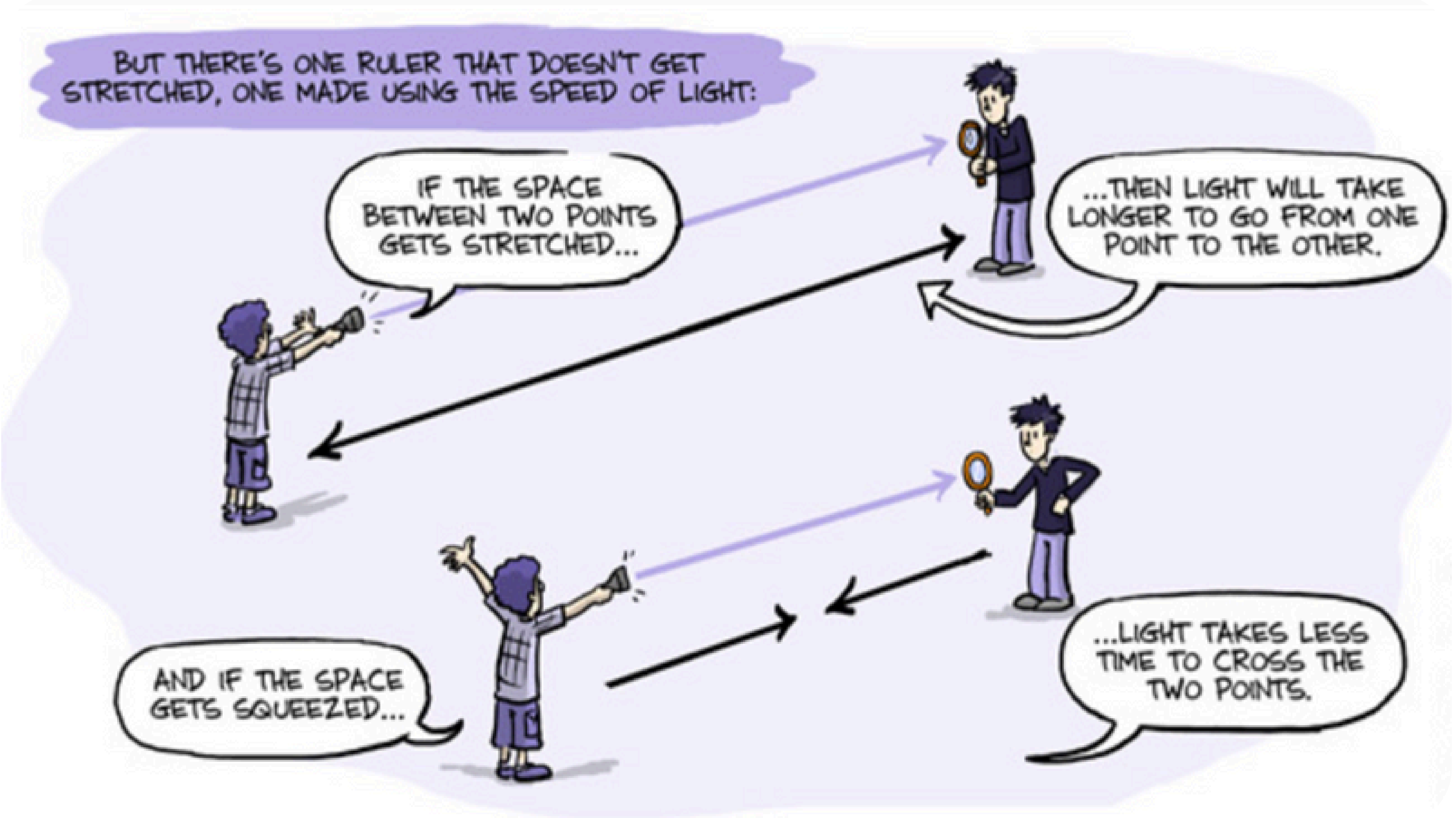
- Resonant bar antennas (Weber, 1960s): fundamental mode excited by passage of GW
- Laser interferometers (Moss, Miller, Forward, Weiss, 1970s):
  - 1st one: Hughes Research Laboratory (Forward, 1978)
  - kms length detectors in the 1990s



Detection of GWs

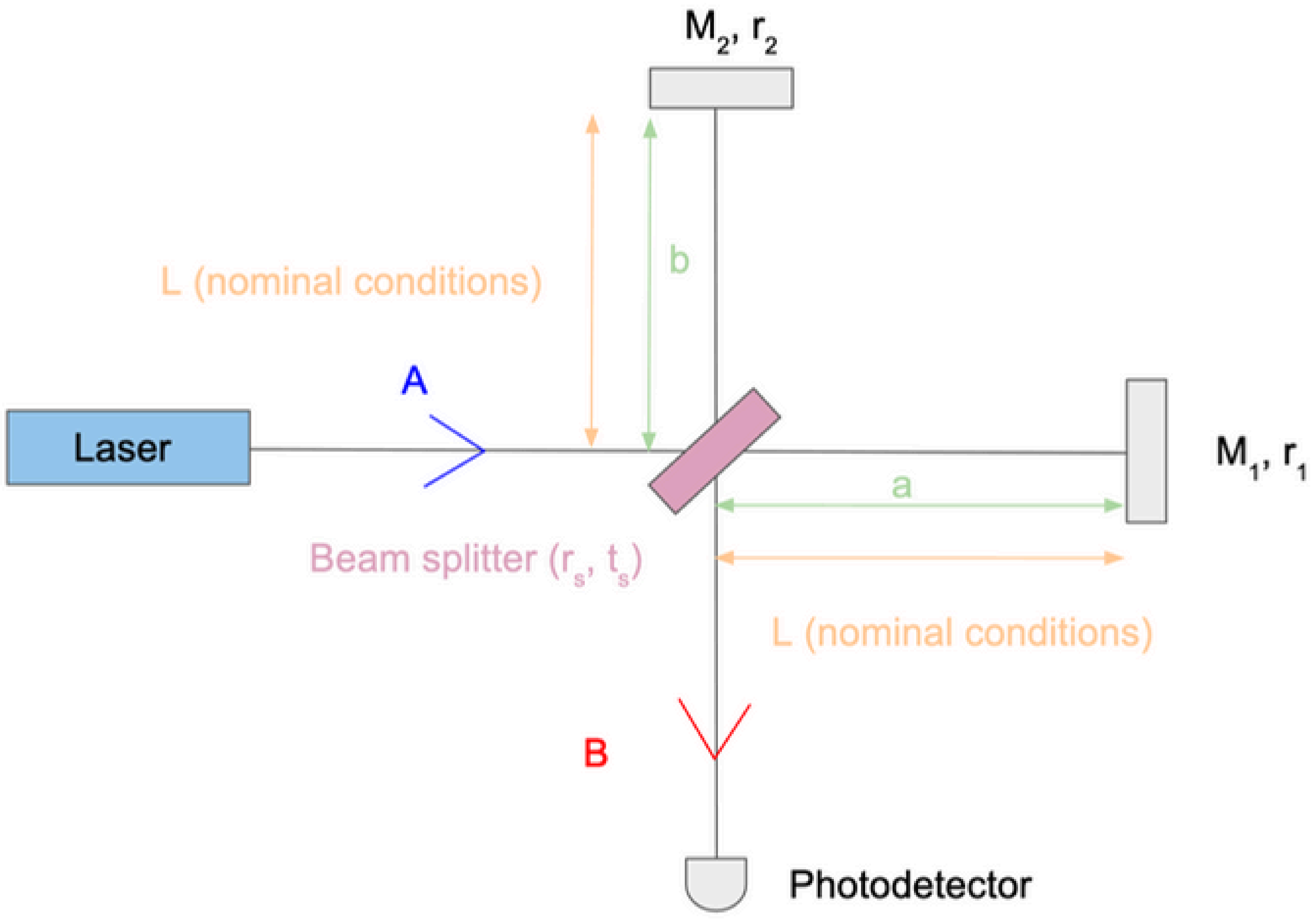


Detection of GWs



Second generation GW detectors

Simple Michelson interferometer



Effect of a GW propagating within the IFO

- In vacuum, light follows null geodesics, so a light ray propagating in the x,y directions of a Michelson IFO follows

$$0 = c^2dt^2 - dx^2 - dy^2 - dz^2 + 2h_x dxdy + h_+(dx^2 - dy^2)$$

- The interaction with the GW does not modify the direction of propagation of the light ray. Only effect: phase change in the light that will be derived in what follows.
- For simplicity, we assume propagation in x

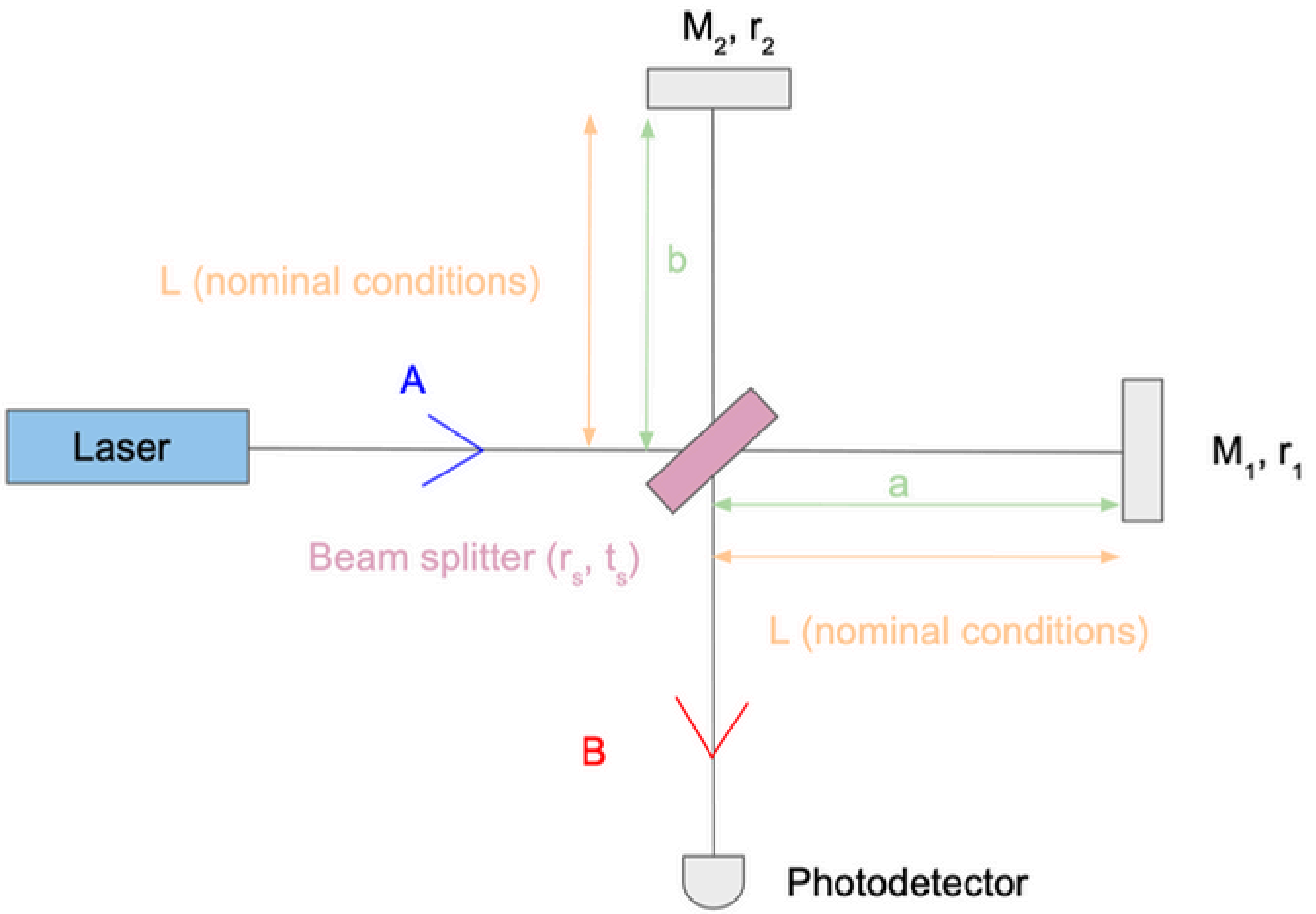
$$0 = c^2dt^2 - dx^2 + h_+dx^2 \Rightarrow dx = \pm cdt\left[1 + \frac{1}{2}h_+(t)\right]$$

$\frac{1}{\sqrt{1-h_+}}$ on h_+ since $h_+ << 1$

Effect of a GW propagating within the IFO

Solution of $dx = \pm c dt\left[1 + \frac{1}{2}h_+(t)\right]$ for a round trip experiment in which light is emitted from the origin of coordinates at to and received at x = L at t1:

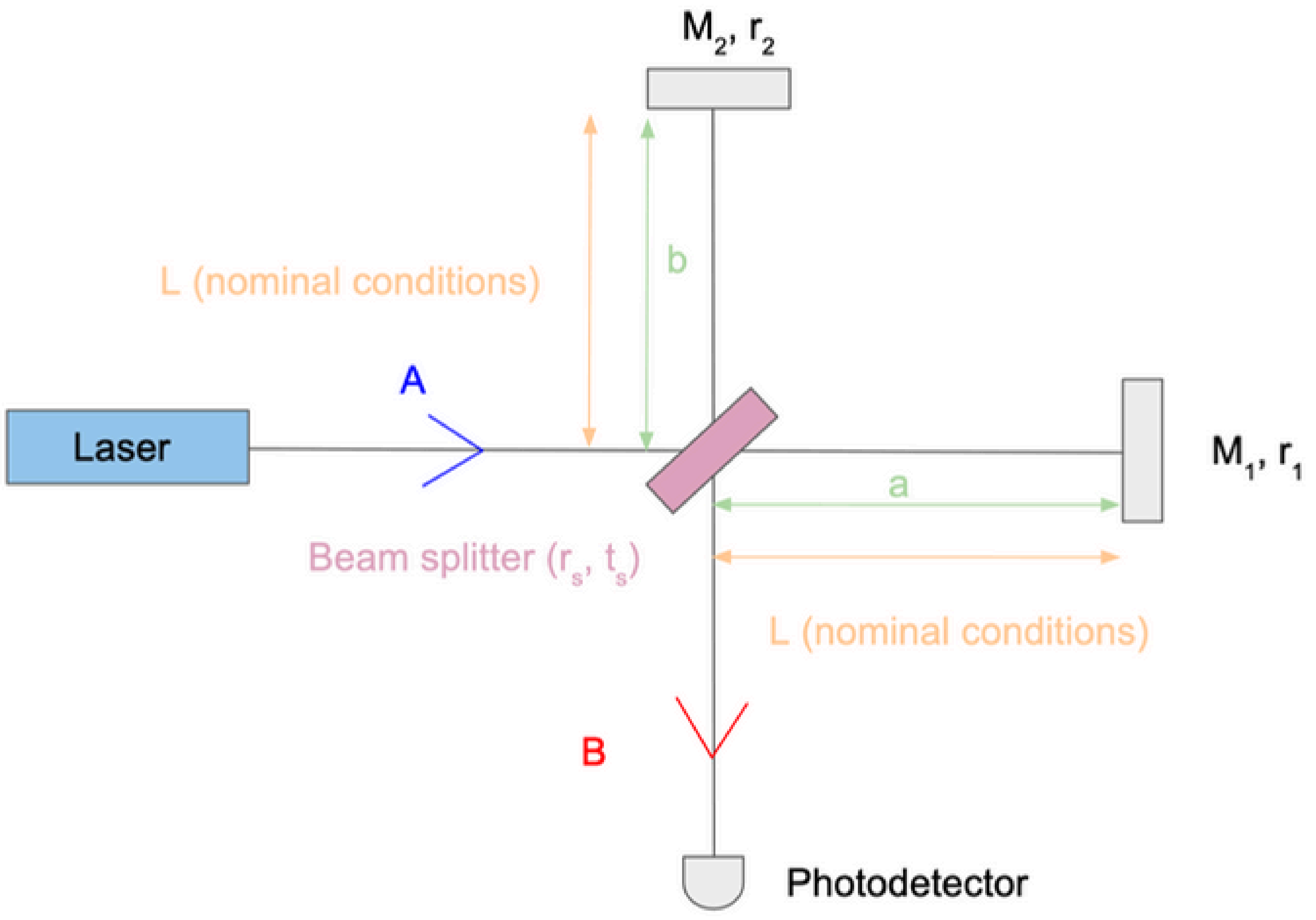
$$L = c(t_1 - t_o) + \frac{1}{2}c\int_{t_o}^{t_1} h_+(u)du$$



Effect of a GW propagating within the IFO

Light at x = L is then reflected back and reaches the origin of coordinates at t2, since it is backward propagation the solution leads to

$$-L = c(-t_2 + t_1) - \frac{1}{2}c\int_{t_1}^{t_2} h_+(u)du$$



Effect of a GW propagating within the IFO

- Deducting the two solutions we get

$$2L = t_2 - t_o + \frac{1}{2} c \int_{t_o}^{t_2} h_+(u) du$$

- We rewrite t2 as t (detection time) and to ≡ tr (retarded time). tr is the time at which light was initially emitted, given by the detection time t minus the time it takes the light to do a round trips, i.e.: 2L/c → the lower limit of the integral can be substituted by t − 2L/c. For a monochromatic GW of frequency νg = Ω/(2π), i.e.: h+(t) = h cos(Ωt), the solution:

$$t_r = t - \frac{2L}{c} + h \frac{L}{c} \mathrm{sinc}\left(\frac{\Omega \mathrm{L}}{\mathrm{c}}\right) \cos\left(\Omega(\mathrm{t} - \mathrm{L/c})\right)$$

Effect of a GW propagating within the IFO

- If we denote t_r by $t_r^{(x)}$, the field that reaches the beam splitter at t from arm x:

$$E^{(x)}(t) = -\frac{1}{2}E_o e^{-i\omega_L t_r^{(x)}} = -\frac{1}{2}E_o e^{-i\omega_L (t-2L/c)+i\Delta\phi_x}$$

- Where Δϕx is the phase shift the light has acquired due to the trip along the x arm:

$$\Delta\phi_x(t) = -h\frac{\omega_L L}{c}\mathrm{sinc}\left(\frac{\Omega \mathrm{L}}{\mathrm{c}}\right)\cos\left(\Omega(\mathrm{t}-\mathrm{L/c})\right)$$

Effect of a GW propagating within the IFO

- Doing the same for the y propagation, we get: $\Delta\phi_y = -\Delta\phi_x$
- Total phase difference acquired by the light in the detector due to a GW passing through is:

$$\Delta\phi_{\text{Mich}} \equiv \Delta\phi_x - \Delta\phi_y = 2\Delta\phi_x$$

- Total field reaching the output of the interferometer

$$E_{\text{tot}}(t) = E^{(x)}(t) + E^{(y)}(t) = -iE_o e^{-i\omega_L(t-2L/c)} \sin[\Delta\phi_x(t)]$$

Effect of a GW propagating within the IFO

- Power detected at the output photodetector

$$P = |E_{\rm tot}|^2 = P_o \sin^2[\Delta\phi_x(t)]$$

- We need to have Δ ϕx(t) as large as possible to recover the value of h
- Δ ϕx(t) is maximized for ΩL/c = π/2 → Loptimal = πc/(2Ω).
- Given than Ω = 2πfGW (fGW is the frequency of the GW) --> optimal length:

$$L_{\rm optimal} = \frac{c}{4f_{GW}} \sim 74948km\frac{1Hz}{f_{GW}} \sim 750km\frac{100Hz}{f_{GW}}$$

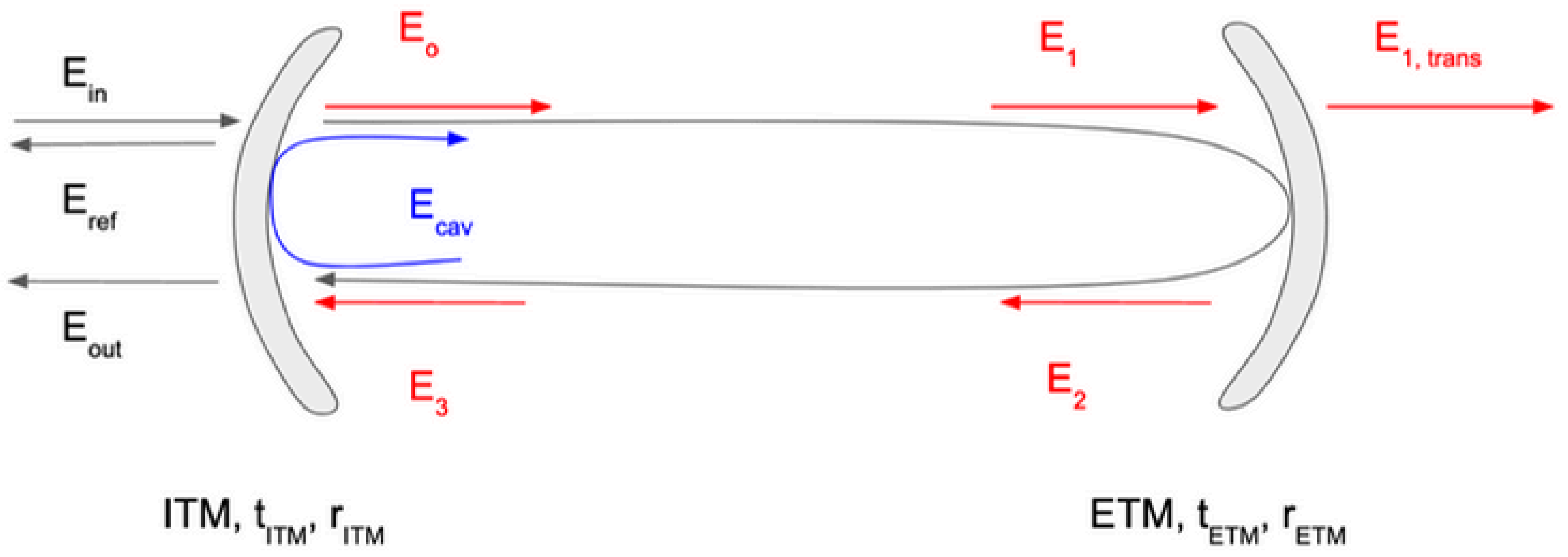
Effect of a GW propagating within the IFO

$$L_{\text{optimal}} = \frac{c}{4f_{GW}} \sim 74948km\frac{1Hz}{f_{GW}} \sim 750km\frac{100Hz}{f_{GW}}$$

- For a GW with a frequency of 100Hz → length of the arm required would be technologically and financially impossible to build.
- We need to find an alternative way of making the optical path length -> **Fabry–Pérot resonant cavity**

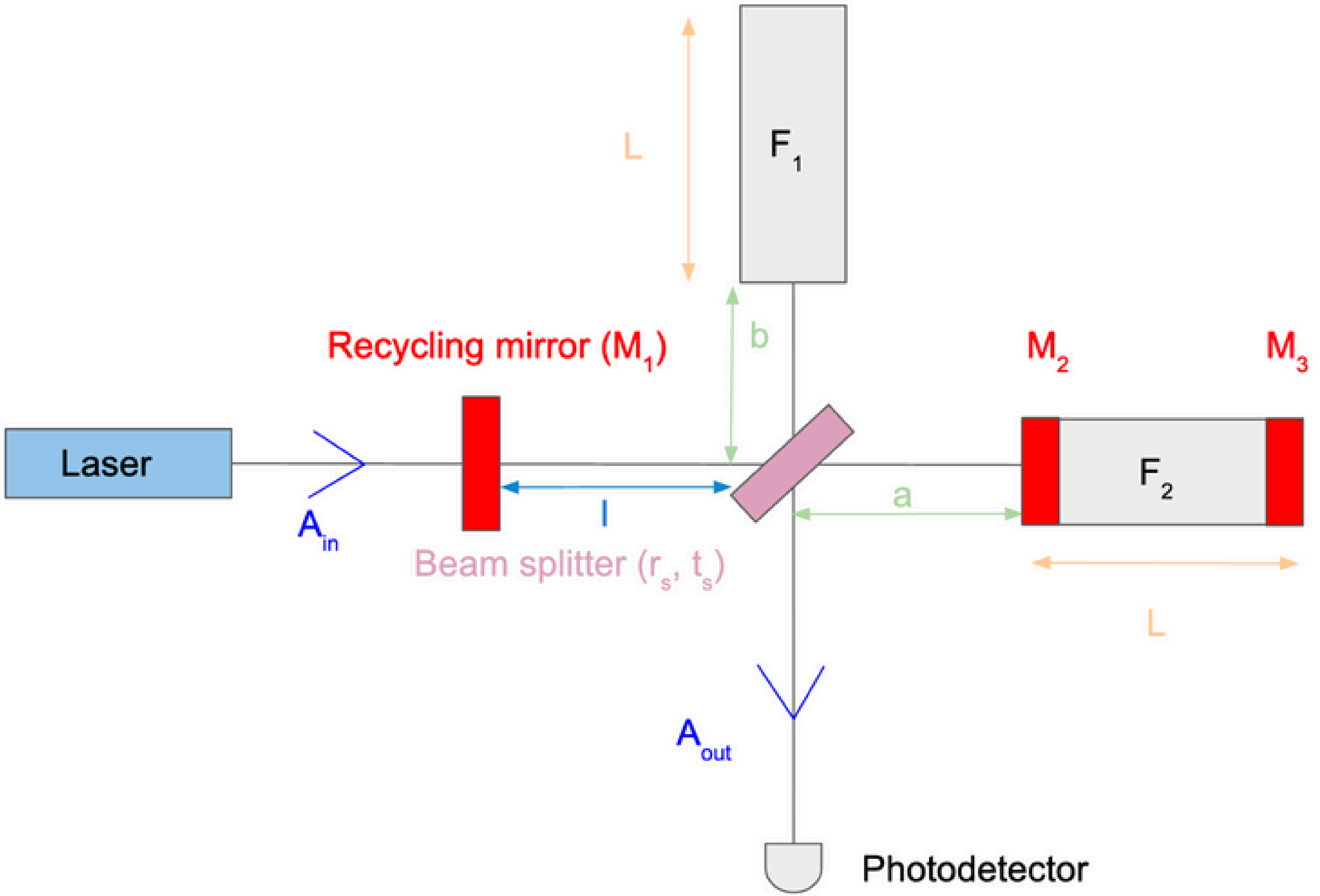
Second generation GW detectors

Fabry-Perot resonant cavity



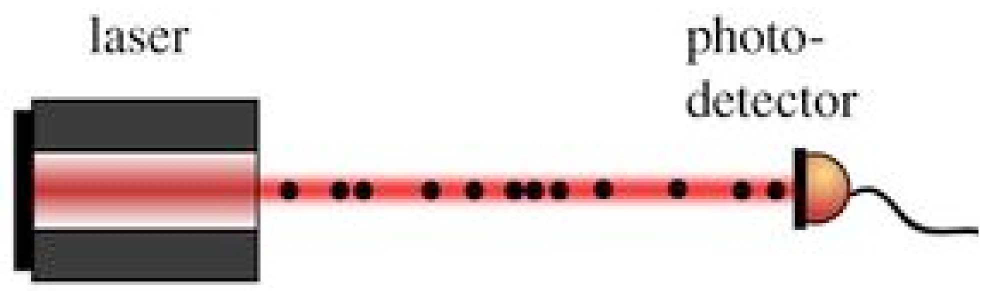
Second generation GW detectors

Michelson interferometer + Fabry-Perot resonant cavities



Noise sources in GW detectors

- Stationarity: noise's statistical properties do not vary much over time
- Origin of the noise:
  - Fundamental noises: intrinsic to the detector
  - Control noise: introduced in the system or amplified by control loops used to maintain the IFO locked
  - Technical noises: coming from the implementation of the IFO, such as power noises.
  - Environmental noises: magnetic fields that couple to the detector, or scattered light



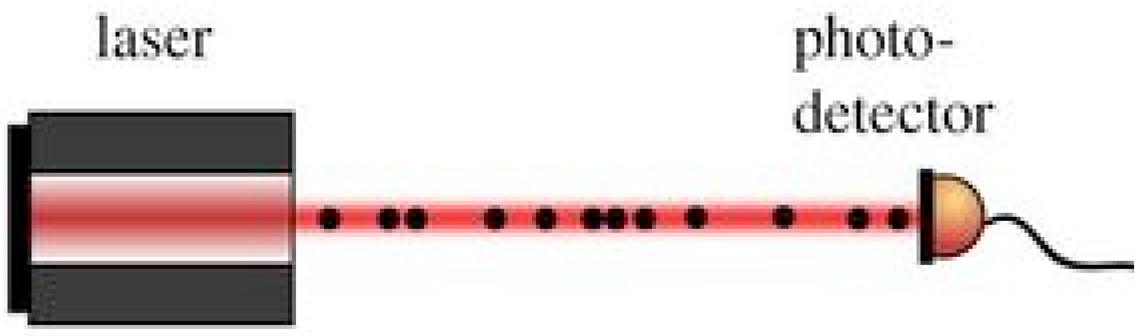
visualization of shot noise

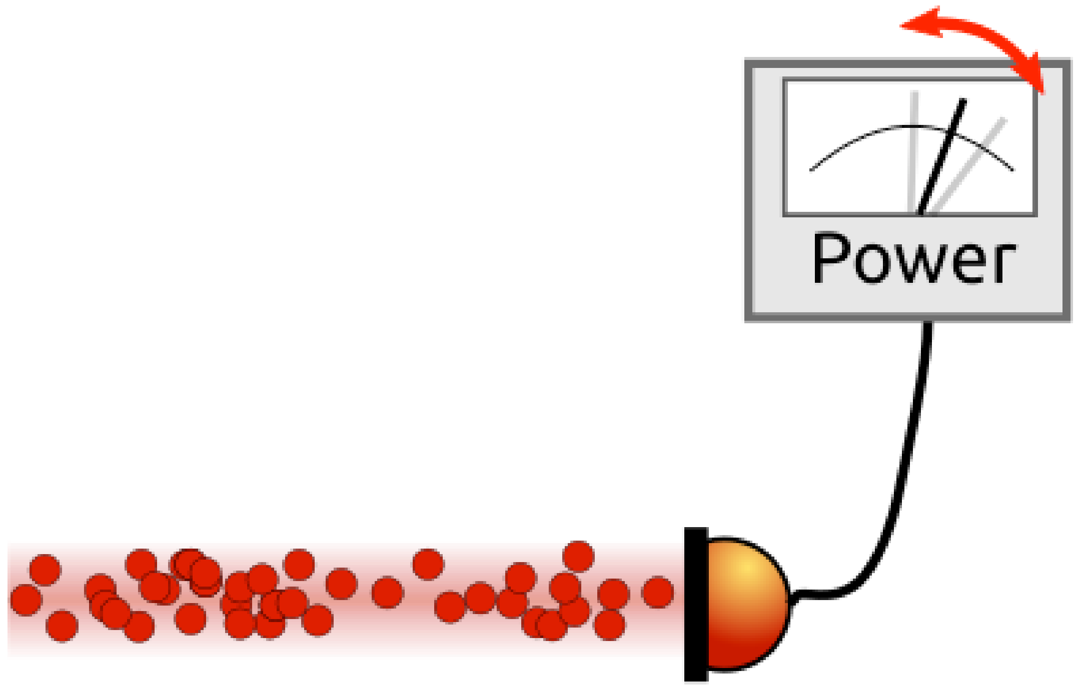
Noise sources in GW detectors

- Shot noise (SN):
  - Due to discrete nature of laser light (photons). After each observation time, the number of photons reaching the PD will be different → fluctuation in the power observed
  - Strain sensitivity due to SN

$$S_n^{1/2}(f)|_{\rm shot} = \frac{1}{8\mathcal{F}L}\sqrt{\frac{4\pi\hbar\lambda_L c}{\eta P_{bs}}}\sqrt{1+\left(\frac{f}{f_p}\right)^2}$$

  - A way of reducing SN: increasing the power reaching the beam splitter (Pbs)
    - increase of the laser power
    - and/or a power recycling cavity





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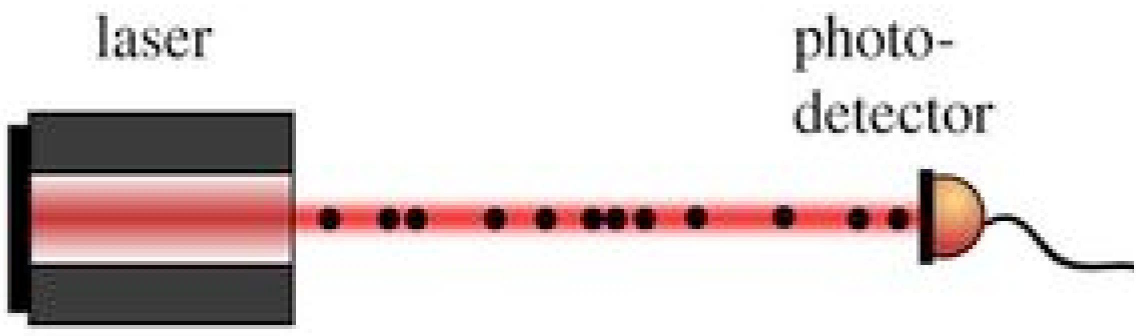
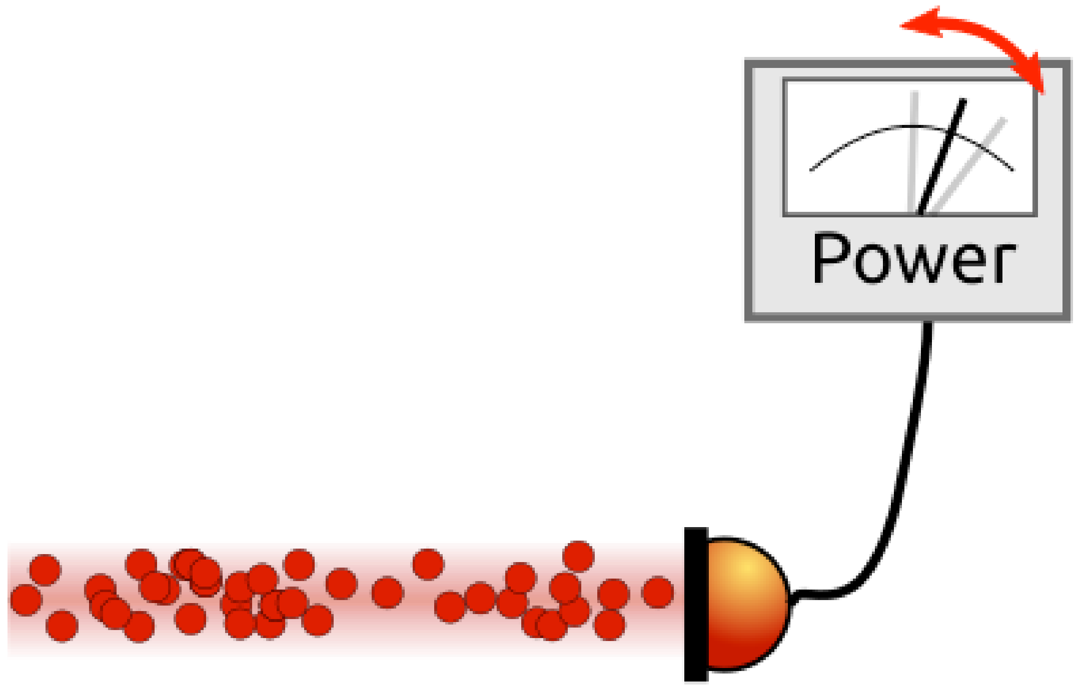


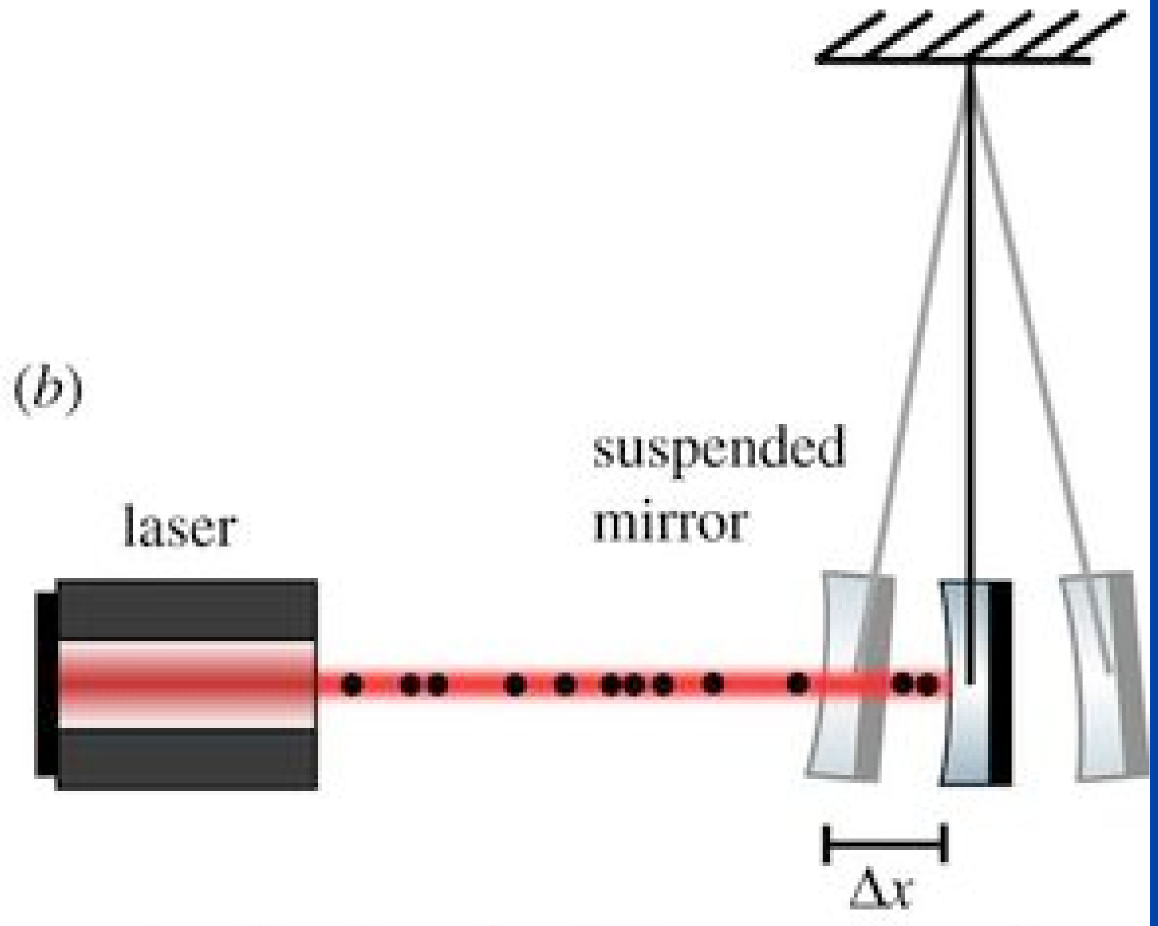


Noise sources in GW detectors

- Radiation pressure (RP):
  - Exerted by the light impinging in the mirrors and then reflecting back
  - Since the number of photons fluctuates → the radiation pressure will also fluctuate → force that shakes the mirrors and grows as √Pbs
  - Strain sensitivity due to RP

$$S_n^{1/2}(f)|_{\text{rad pressure}} = \frac{16\sqrt{2}\mathcal{F}}{ML(2\pi f)^2}\sqrt{\frac{\hbar P_{bs}}{2\pi\lambda_L c}}\frac{1}{\sqrt{1+(f/f_p)^2}}$$

  - A way of reducing the RP: reducing Pbs



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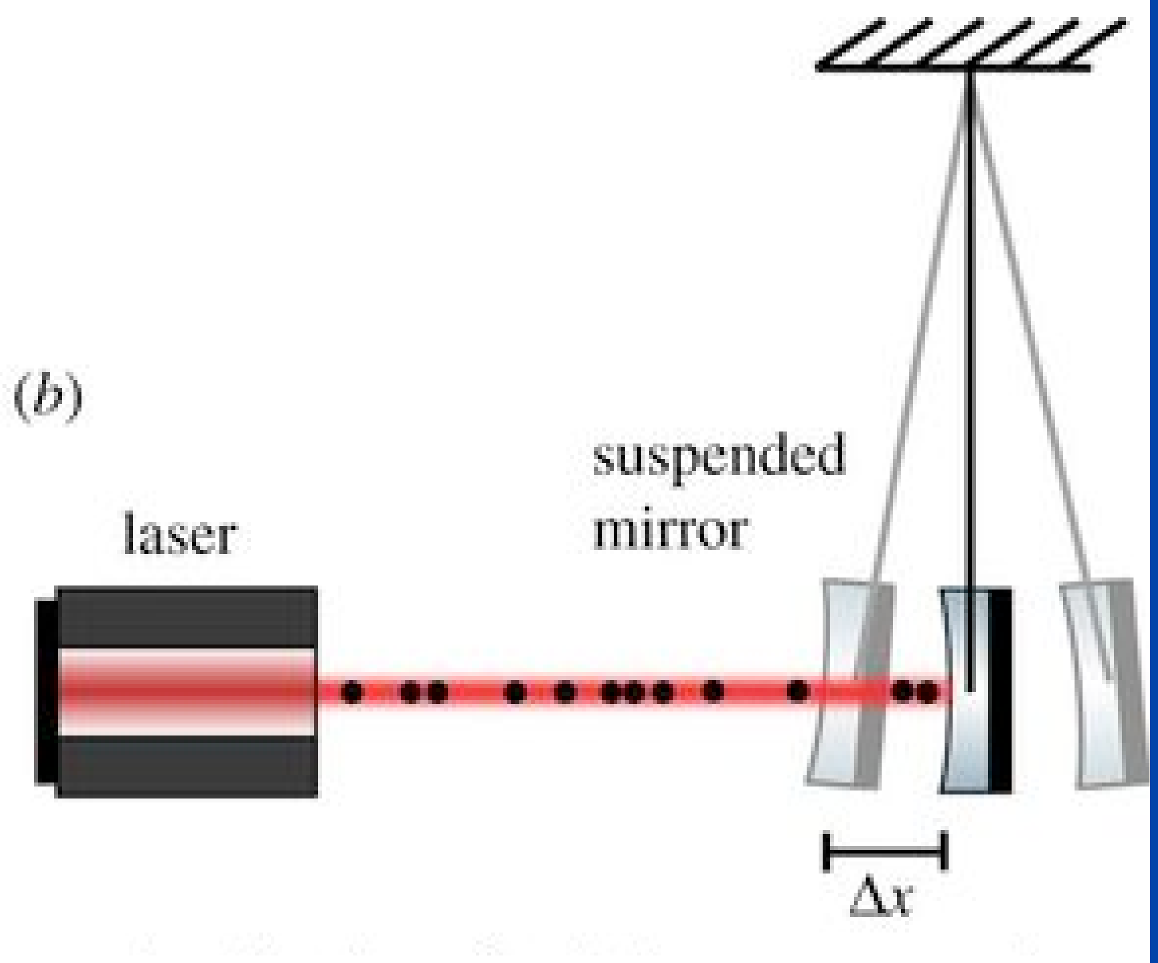


Noise sources in GW detectors

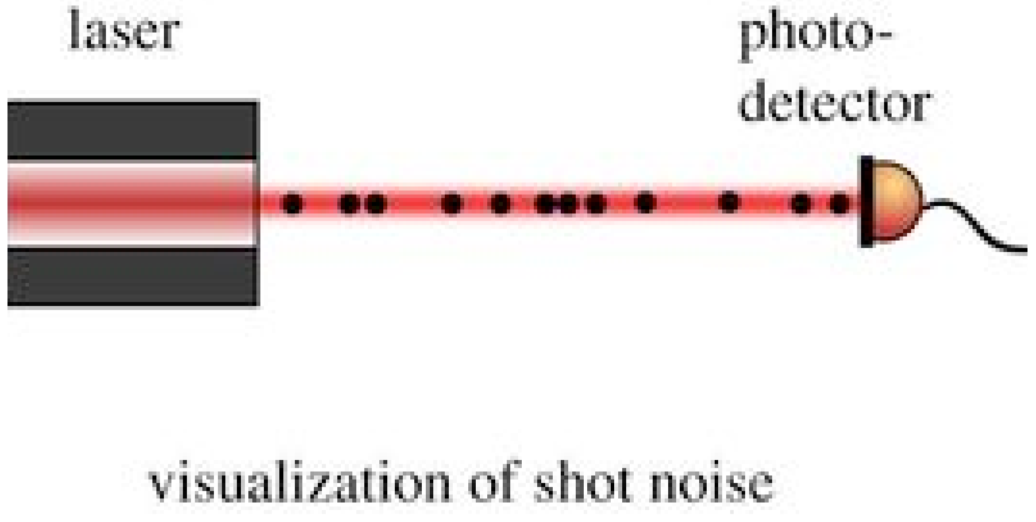
- Combined effect of SN and RP: **optical read-out noise.**
- Total strain sensitivity:

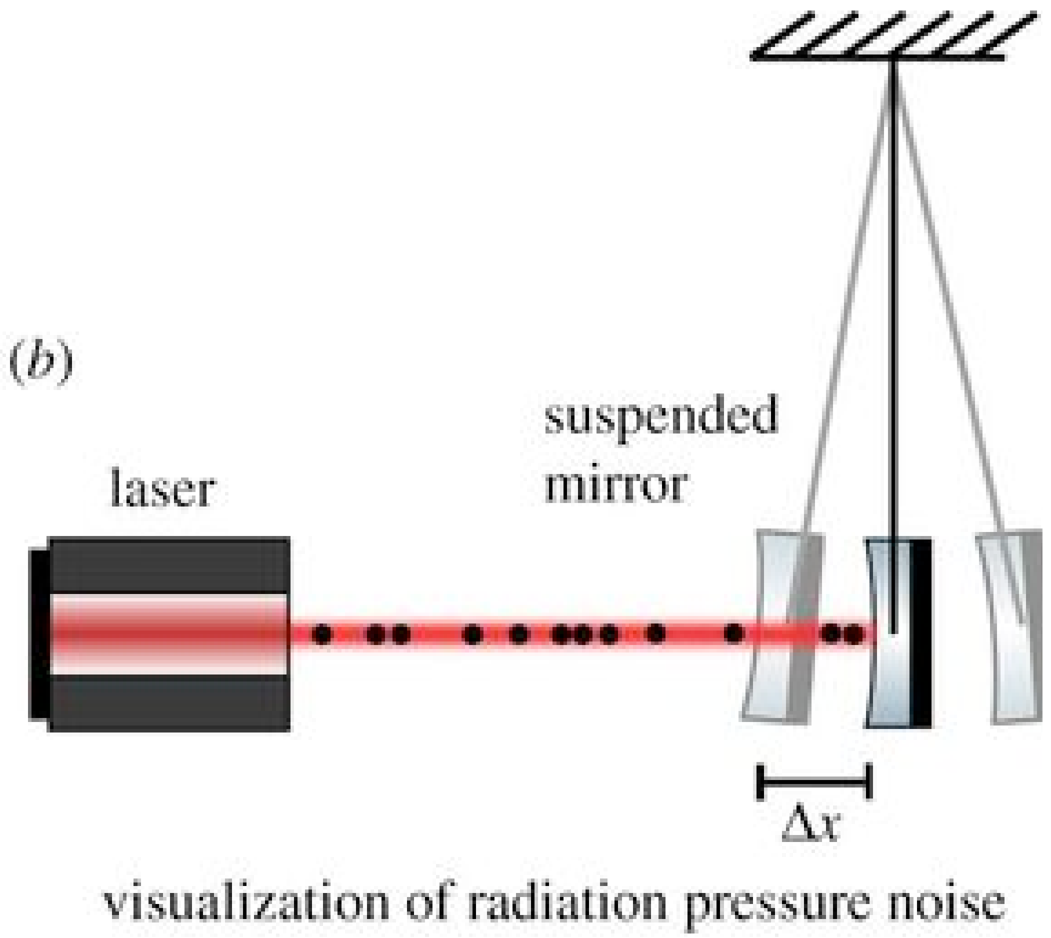
$$S_n^{1/2}(f)|_{\rm opt} = \frac{1}{L\pi f_o}\sqrt{\frac{\hbar}{M}}\left[1+(f/f_p)^2+(f/f_p)^4\frac{1}{1+(f/f_p)^2}\right]^{1/2}$$

$$f_o = \frac{8\mathcal{F}}{2\pi}\sqrt{\frac{P_{bs}}{\pi\lambda_L c M}}$$

- Optimal value of fo is that for which the SN and RP contributions are equal → corresponding optimal value of strain

$$S_{\rm SQL}^{1/2}(f) = \frac{1}{2\pi f L}\sqrt{\frac{8\pi}{M}}$$





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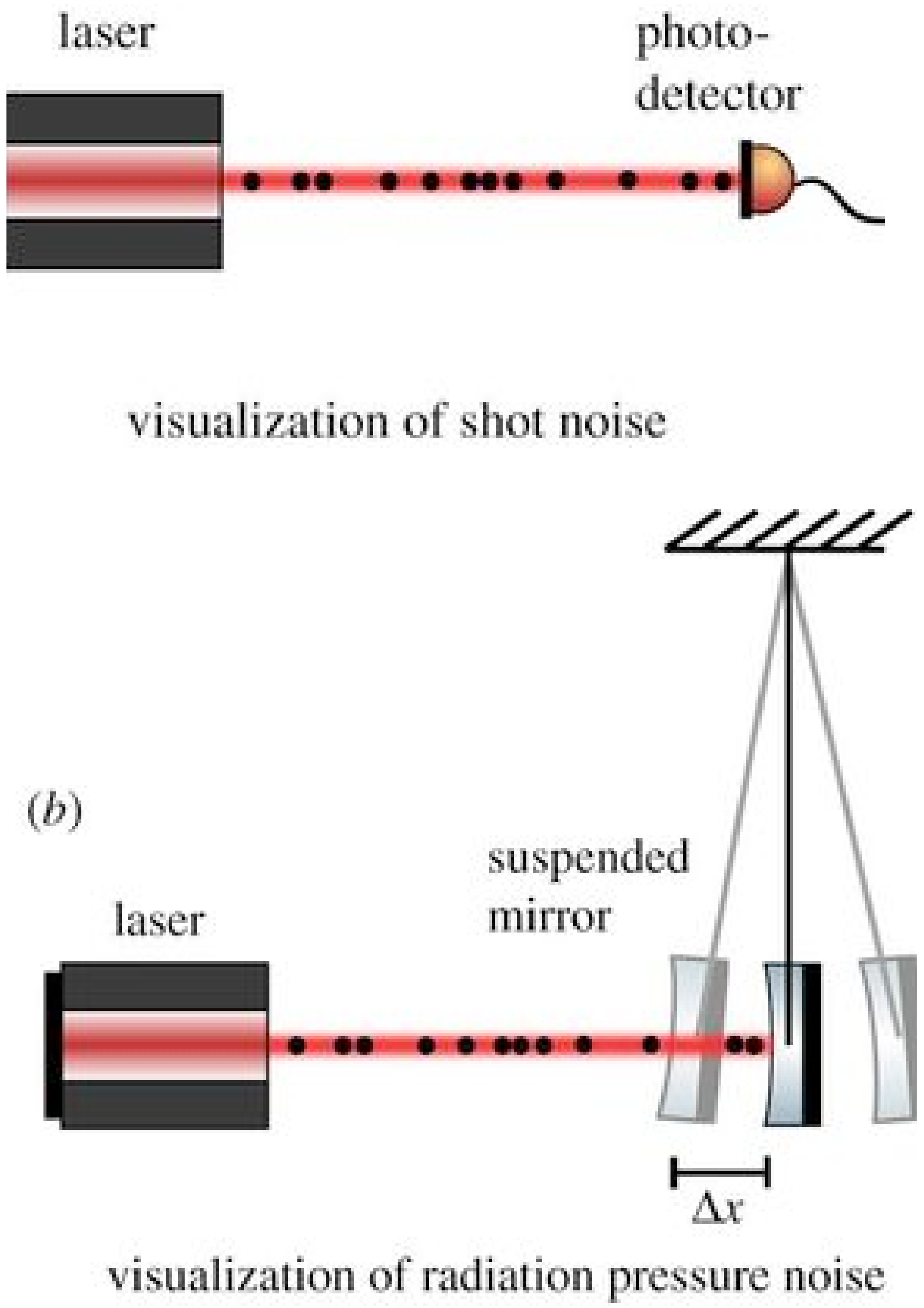
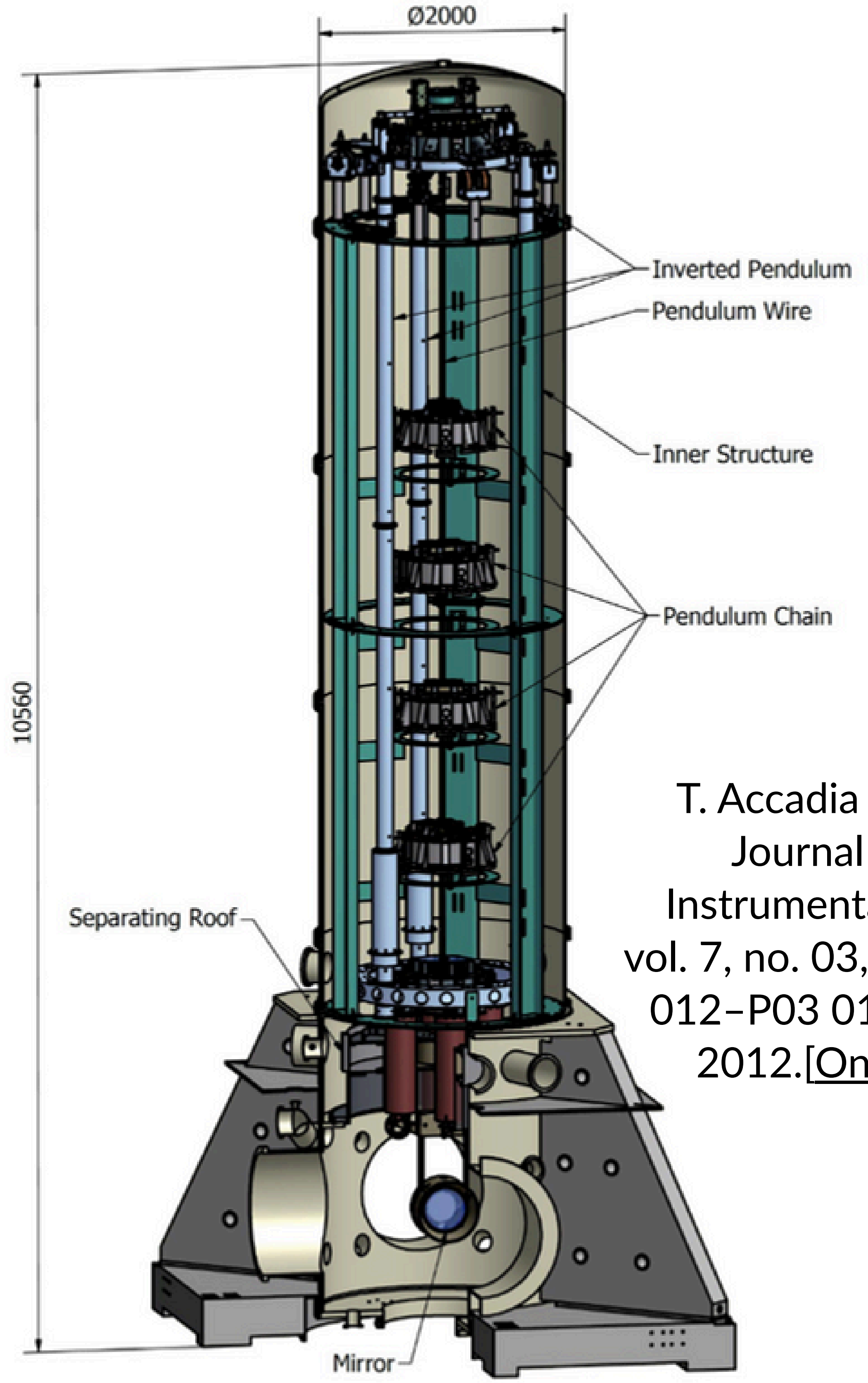
Standard quantum limit (SQL)



Noise sources in GW detectors

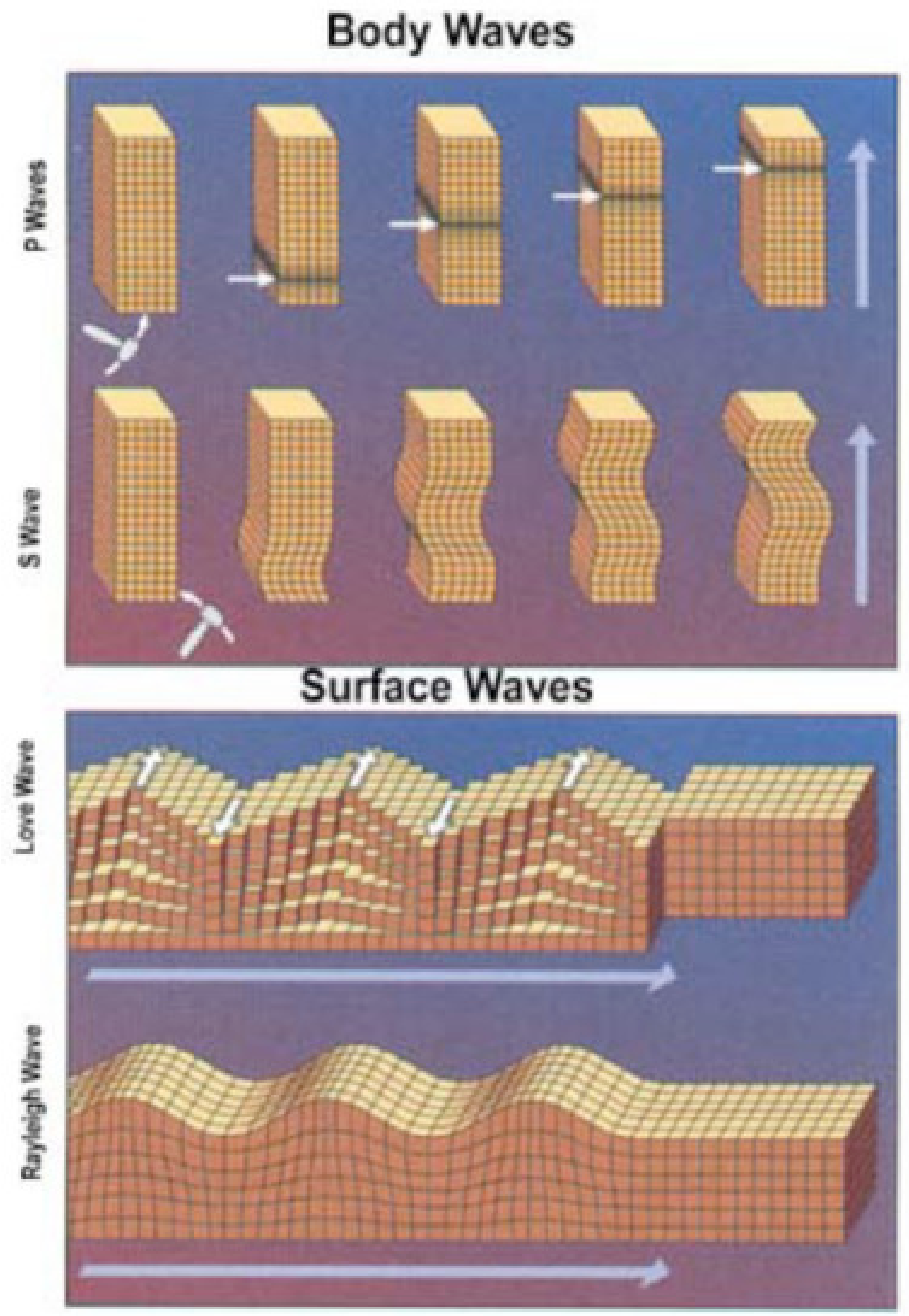
- Seismic motion:
  - Types:
    - Earth's ground is in constant motion,
    - Human activity (means of transport, walking, daily activities,...),
    - Weather conditions such as winds,
    - Micro-seismic background which shakes the suspension mechanisms and thus the mirrors.
  - The seismic noise must be attenuated → Achieved with a set of pendulums in cascade: **superattenuator** in Virgo



T. Accadia et al., Journal of Instrumentation, vol. 7, no. 03, pp. P03 012–P03 012, mar 2012.[Online]

Noise sources in GW detectors

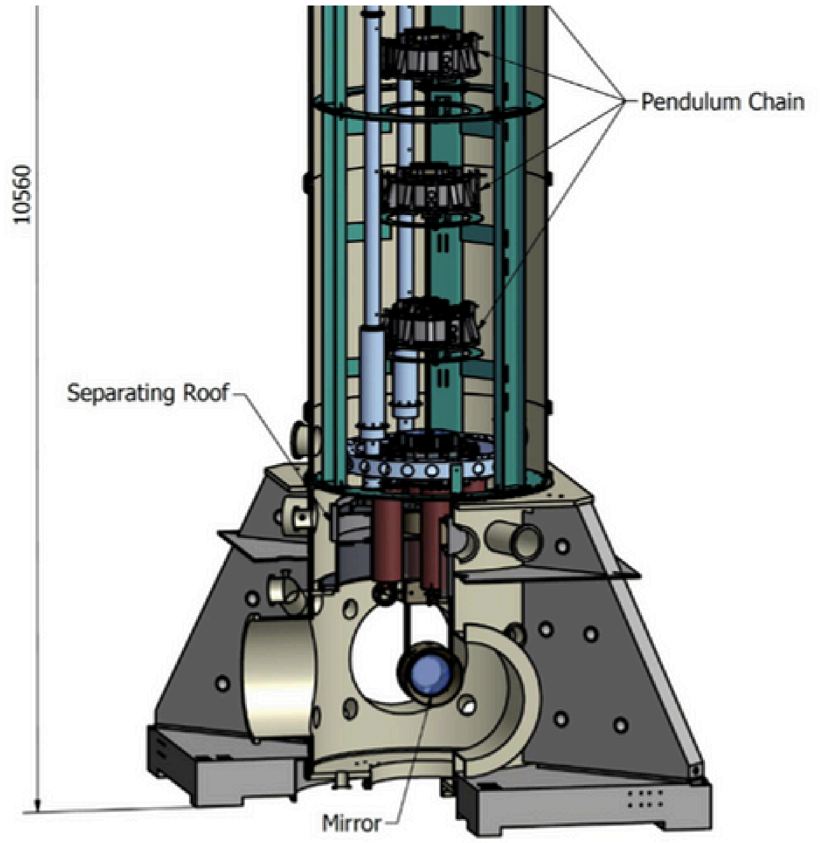
- Newtonian noise (NN, ***gravity gradient noise***): due to the Newtonian gravitational forces of objects that are moving.
- E.g.: test masses in the IFO are subject to gravity perturbations due to the propagation of seismic waves, atmospheric changes, …
- To model the NN, the local seismic field nearby the test masses is monitored. This movement results in a time-varying gravitational force that cannot be screened out from the detectors, though the noise can be reduced.

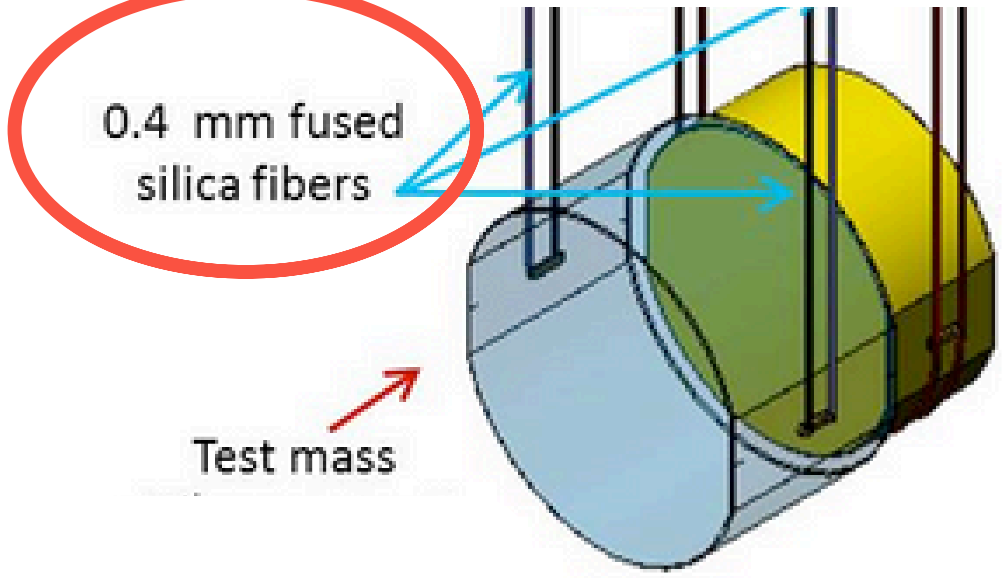


L. Trozzo et al. Galaxies 2022, 10(1), 20; https://doi.org/10.3390/galaxies10010020

Noise sources in GW detectors

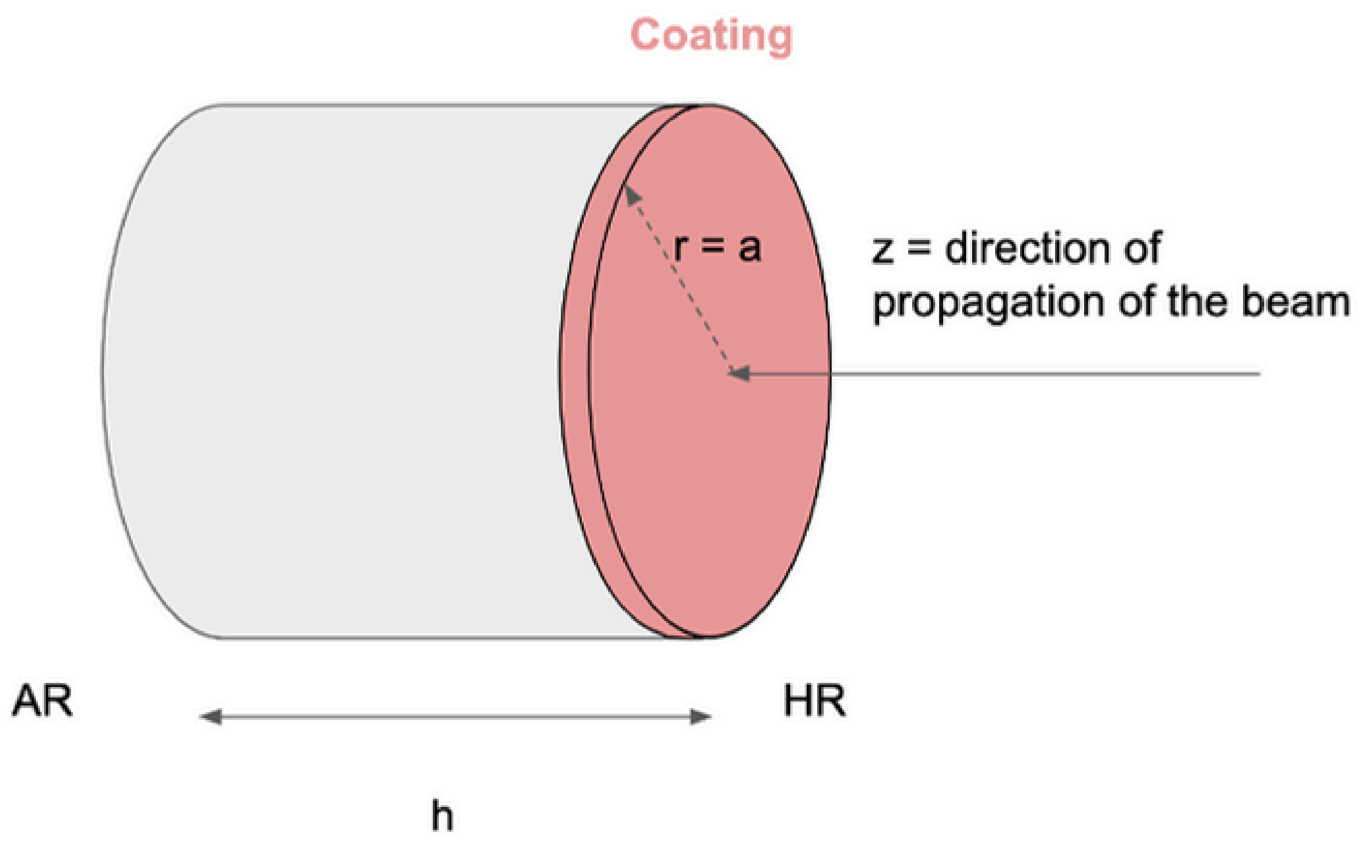
- Thermal noise (TN)
  - Induces vibrations in the mirrors and the suspensions. Most important ones:
    - Suspension TN. Any vibration induced in the suspension of the test masses results in a displacement noise.
      - Pendulum thermal fluctuations(swinging motion) → horizontal displacement of the mirrors.
      - Vertical thermal fluctuations.
      - Violin modes: vibrations that can be described in terms of fluctuations of the normal modes of he wires holding the mirrors.





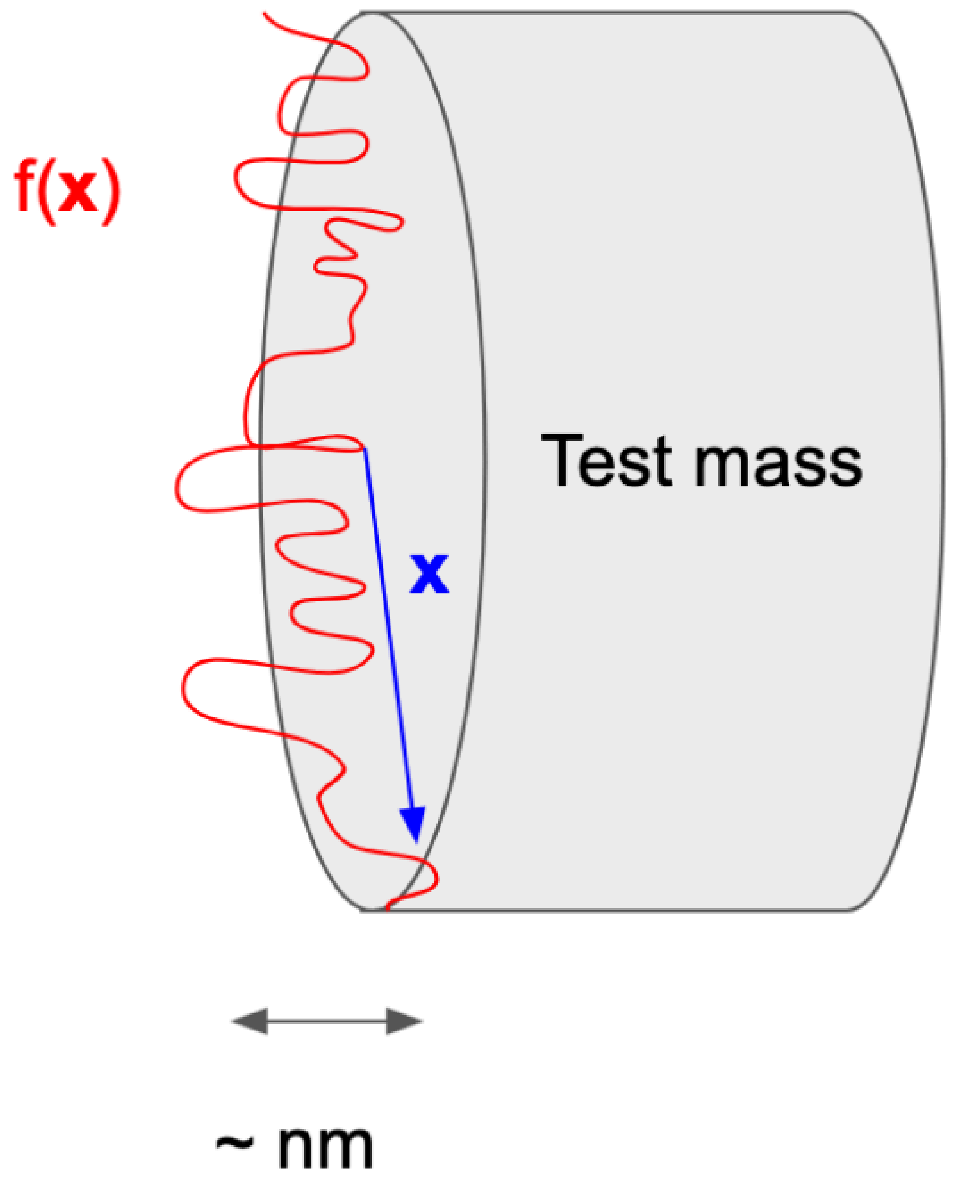
Noise sources in GW detectors

- Thermal noise (TN)
  - Test mass TN: thermal fluctuations within the test masses themselves.
    - Brownian motion: atoms of a mirror at temperature T have Brownian motion due to their kinetic energy
    - Thermo-elastic fluctuations: In a finite volume, the temperature fluctuates → displacement noise through the expansion of the material (bulk and coating).
    - Thermo-refractive fluctuations. The refractive index of the coatings is a function of the temperature.



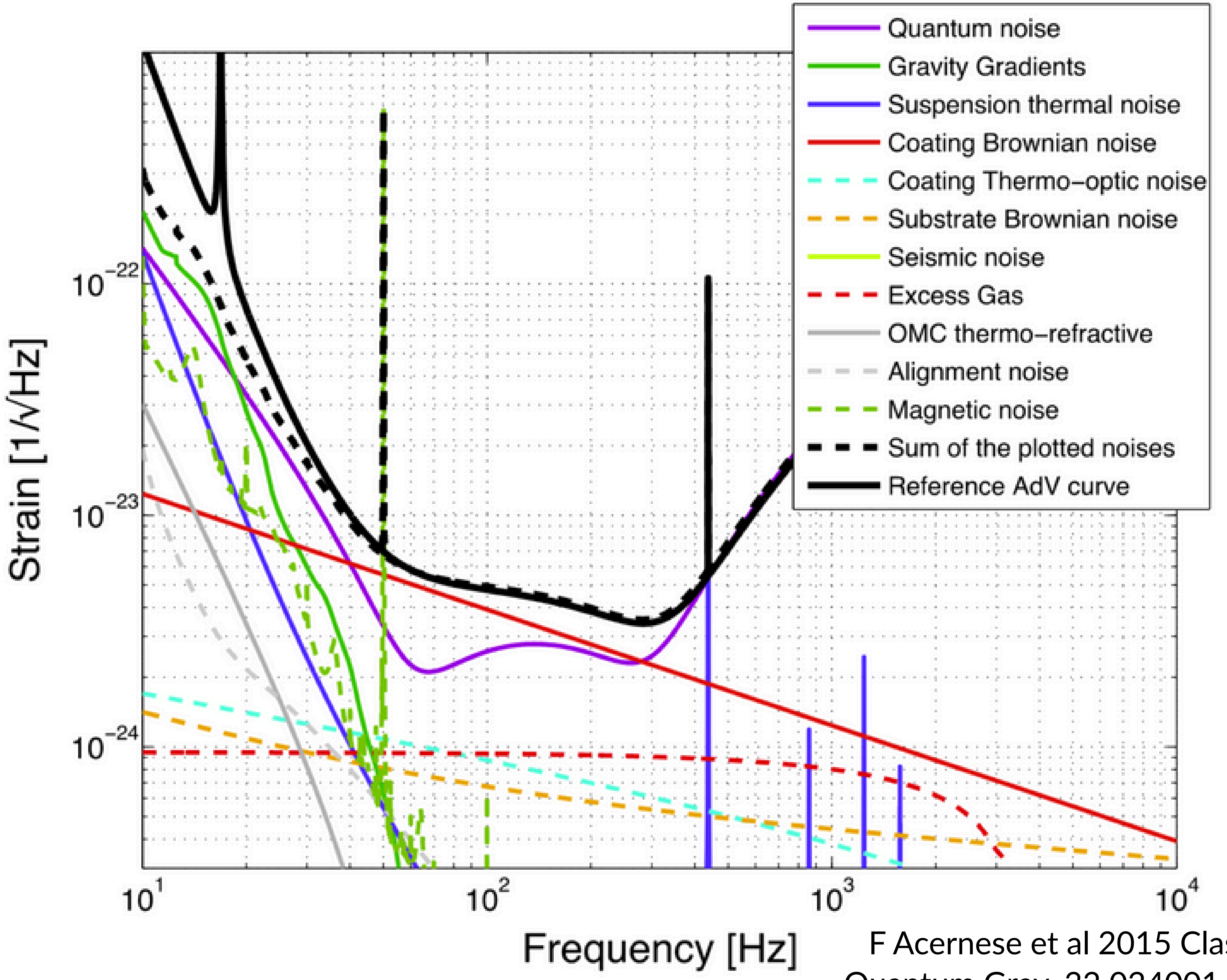
Noise sources in GW detectors

- Scattered light (SL):
  - Light coming from the laser that does not follow the designed path in the optical system.
  - Some sources of SL:
    - Imperfections in the surface of the coating over mirrors, e.g.: point absorbers.
    - Spurious reflections due to a non-ideal anti-reflective coating.
    - Optical components with a limited aperture → diffraction
  - Total losses in the mirrors in the current IFOs are very low: amplitude of the SL is just a few parts per million.
  - SL may backscatter and recouple to the main cavity mode, introducing a shift in the phase of the main mode



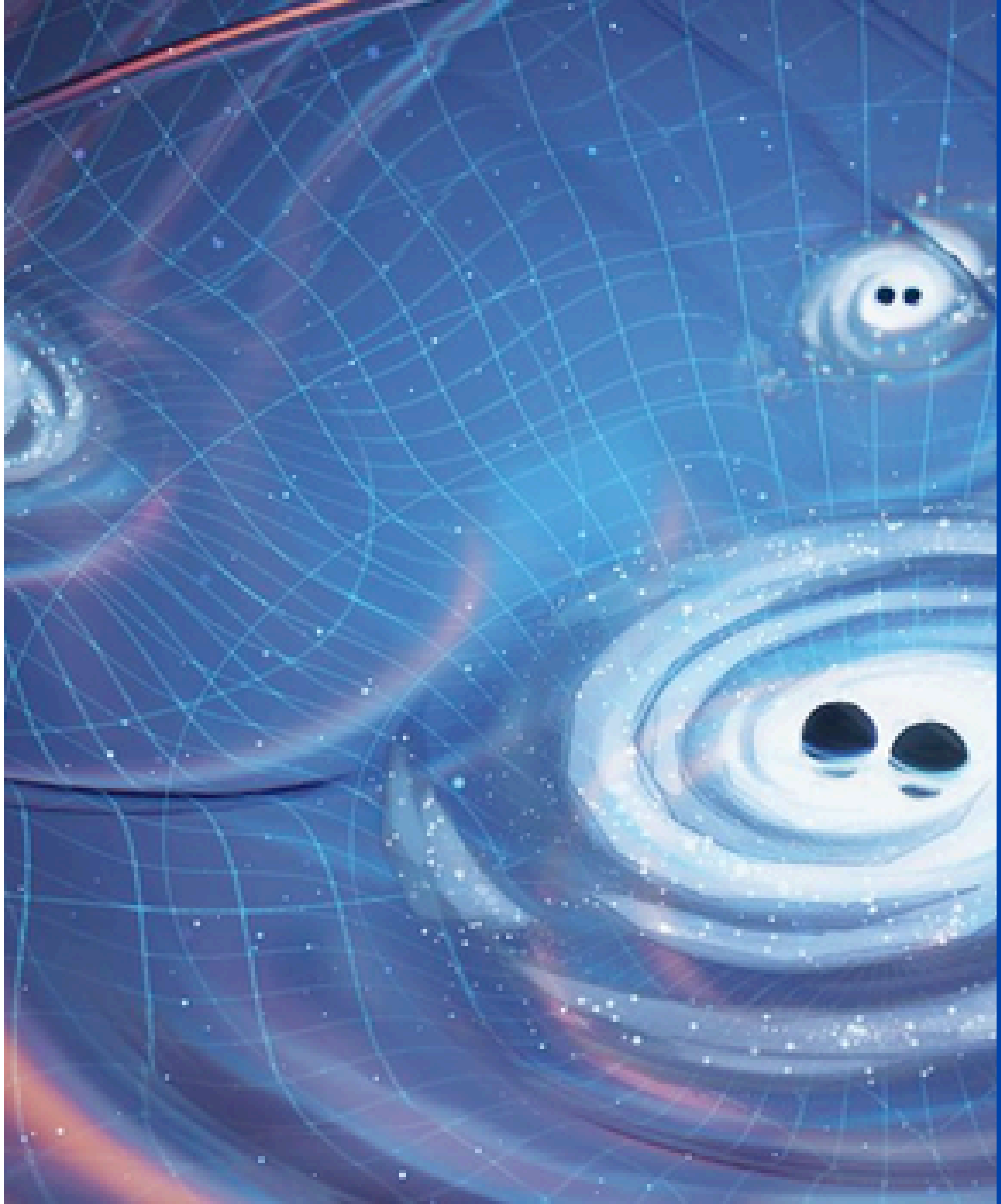
Noise sources in GW detectors

The ensemble of noises results in the detector's **sensitivity curve** (black curve)

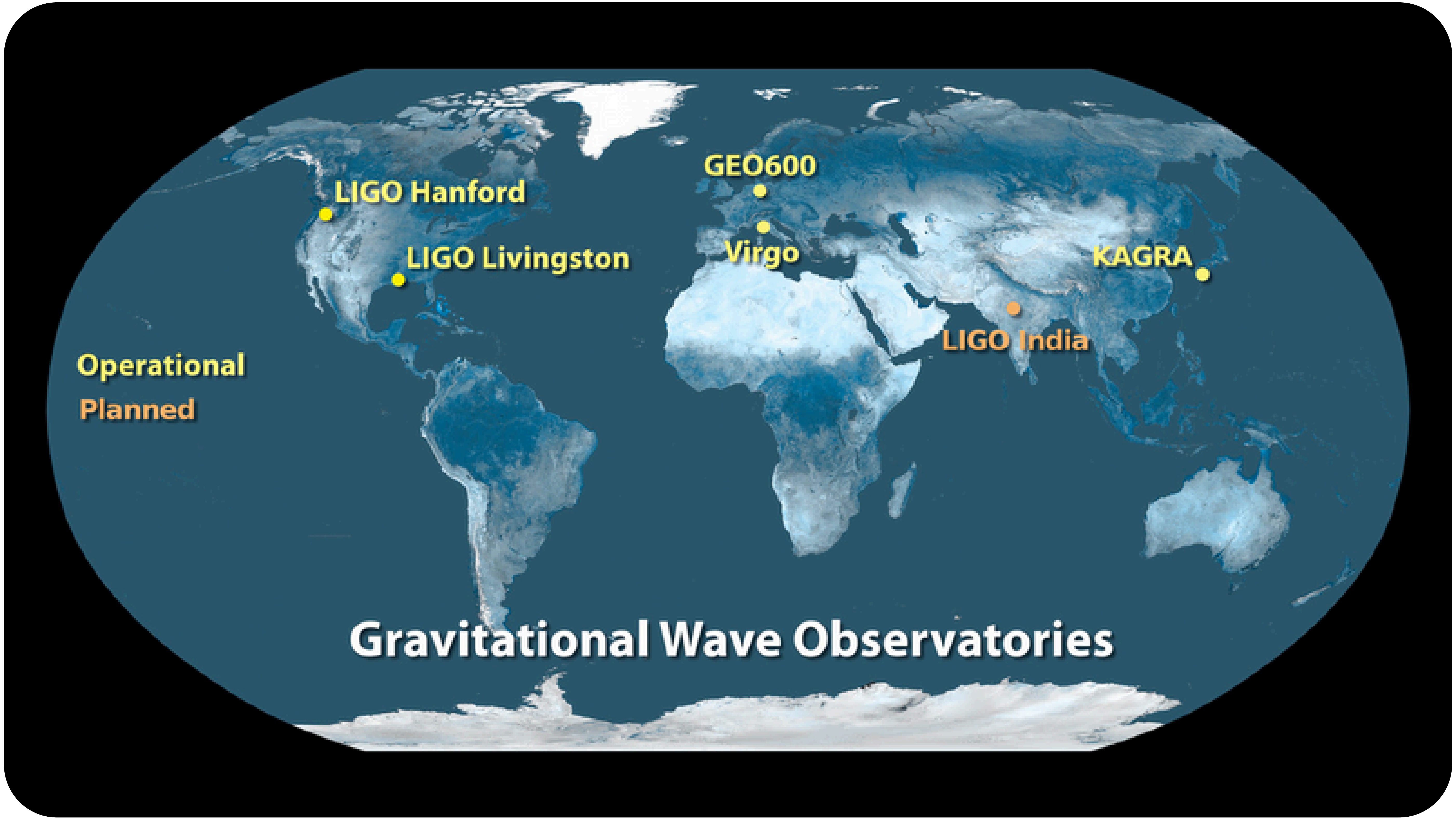


F Acernese et al 2015 Class. Quantum Grav. 32 024001, link

Where do we stand?



Current network of detectors



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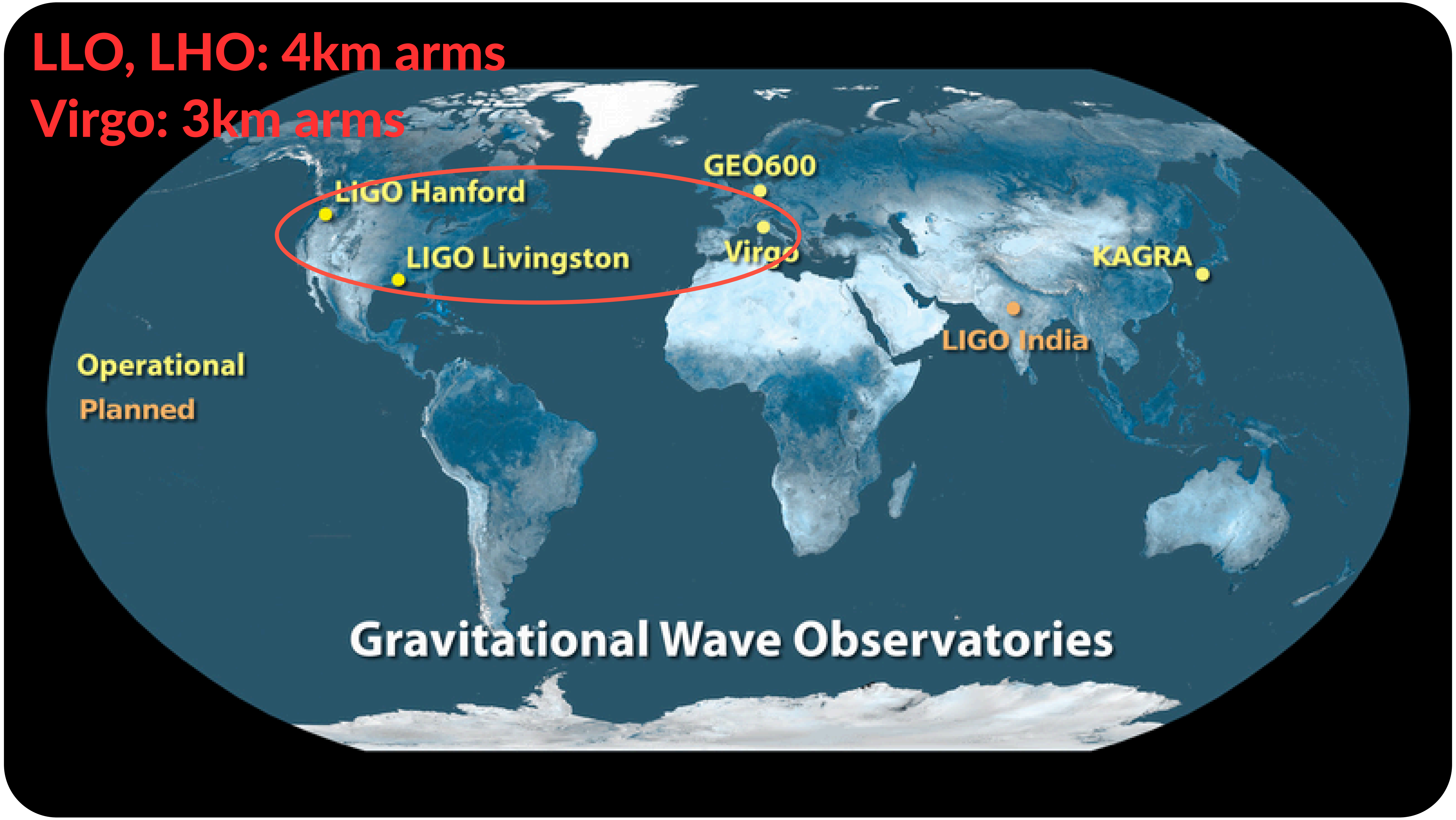

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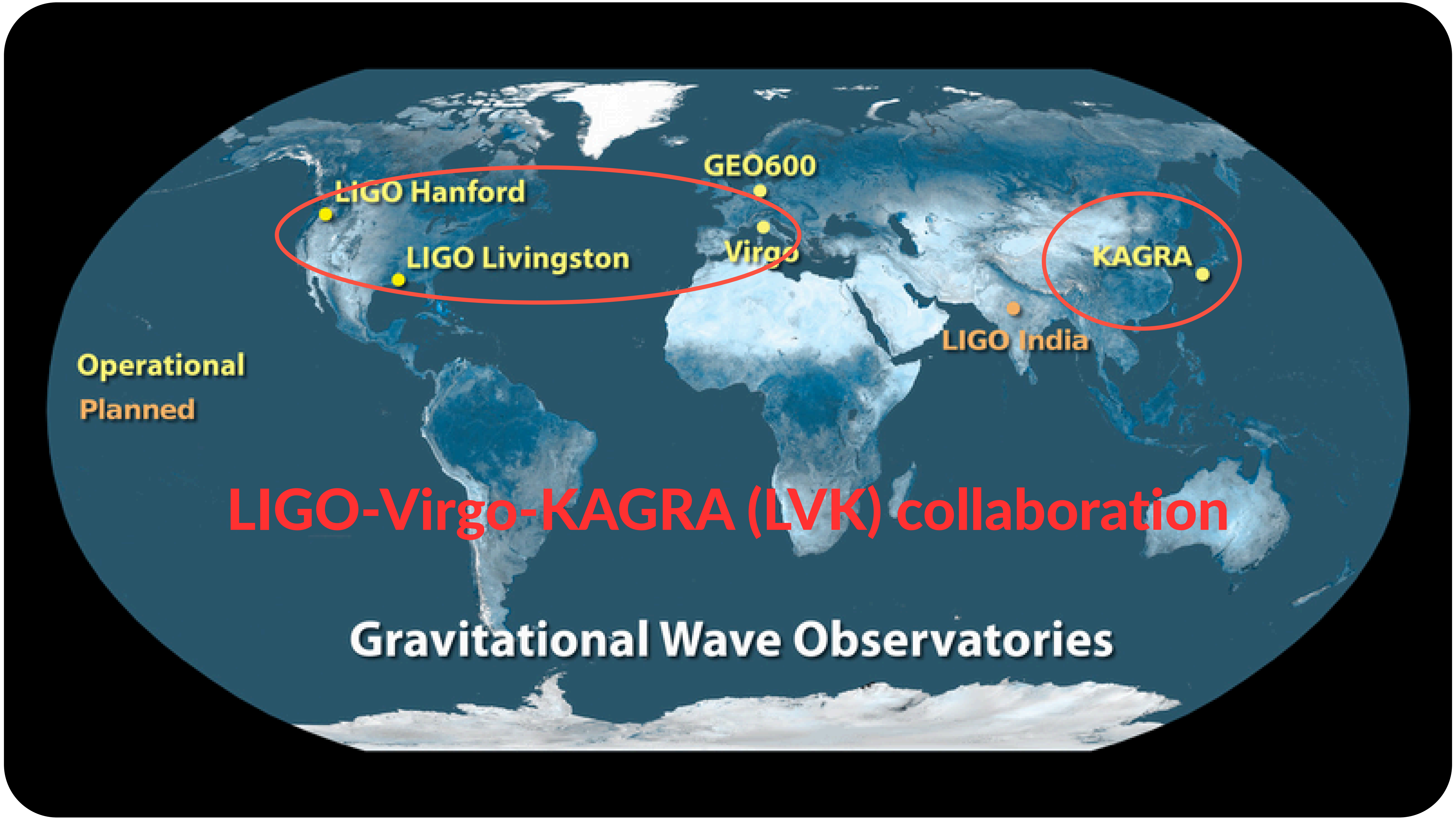
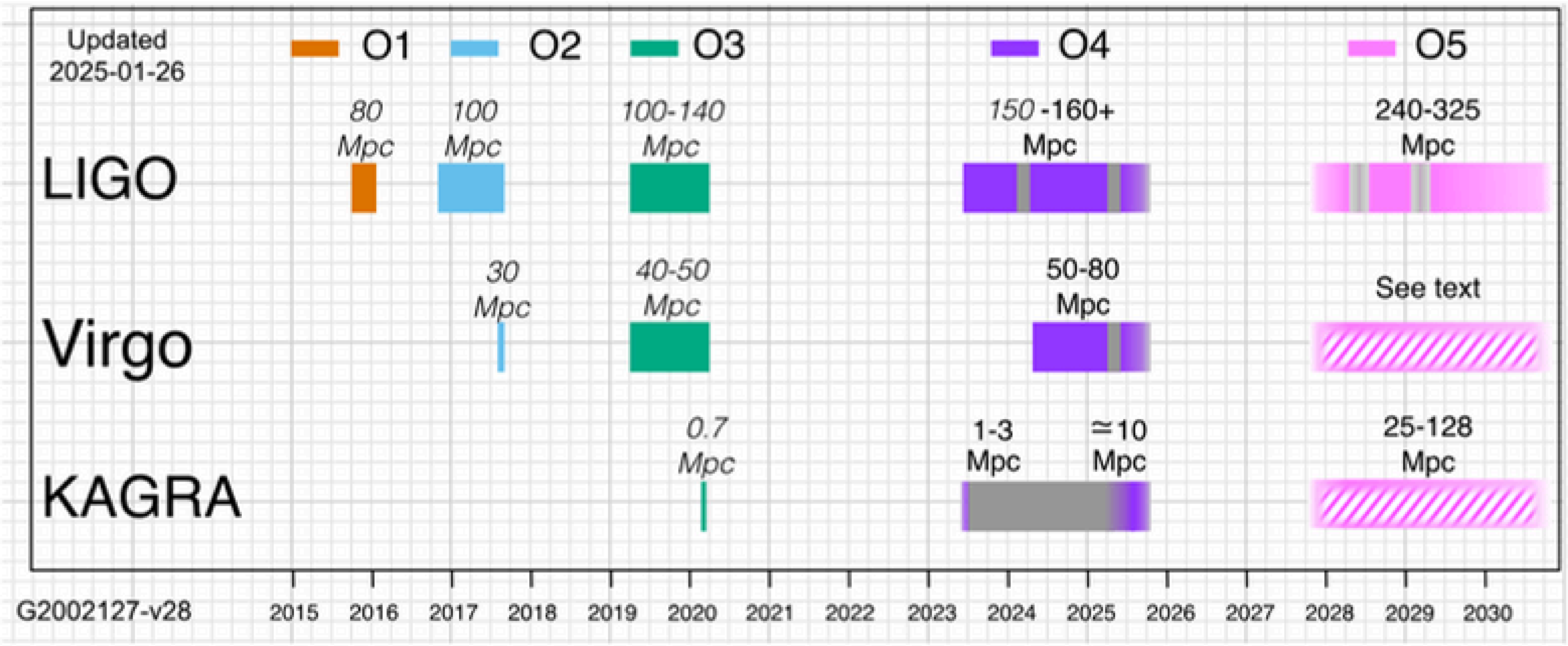
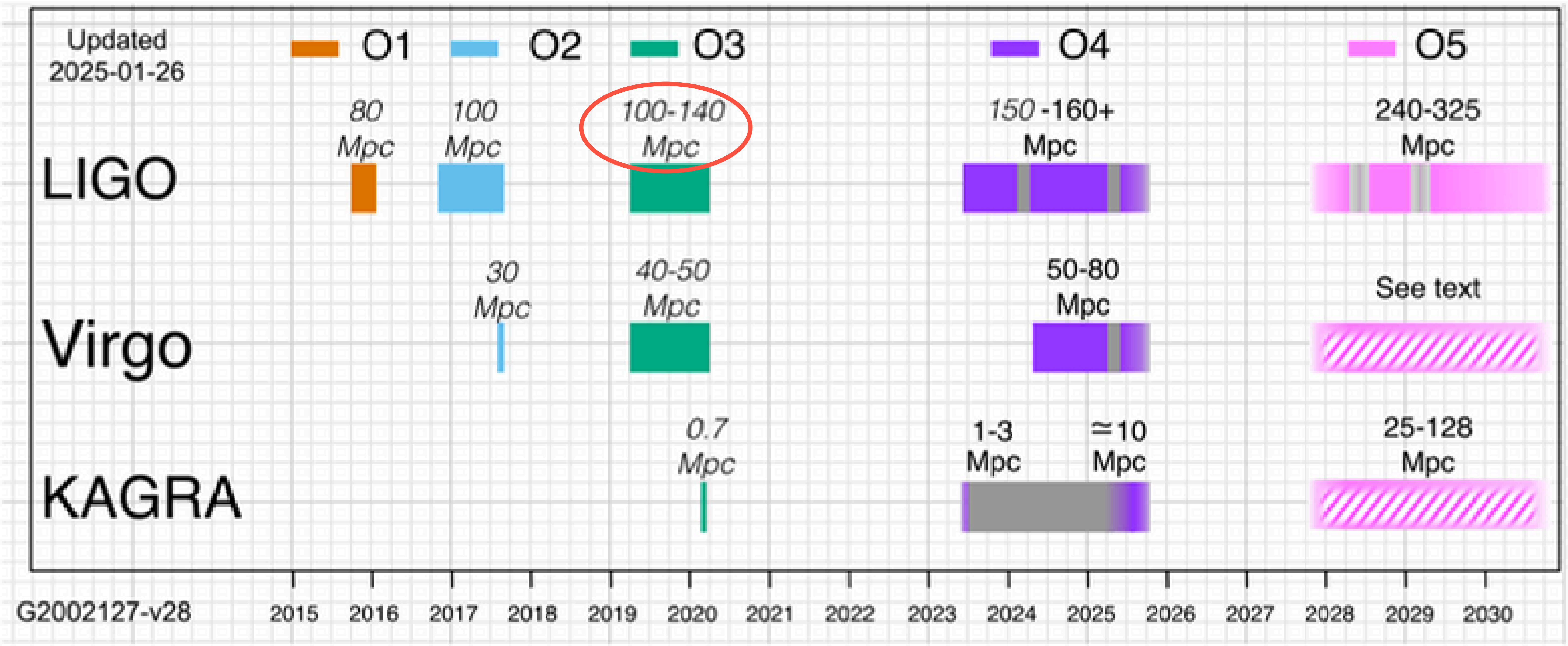


LVK observation plan



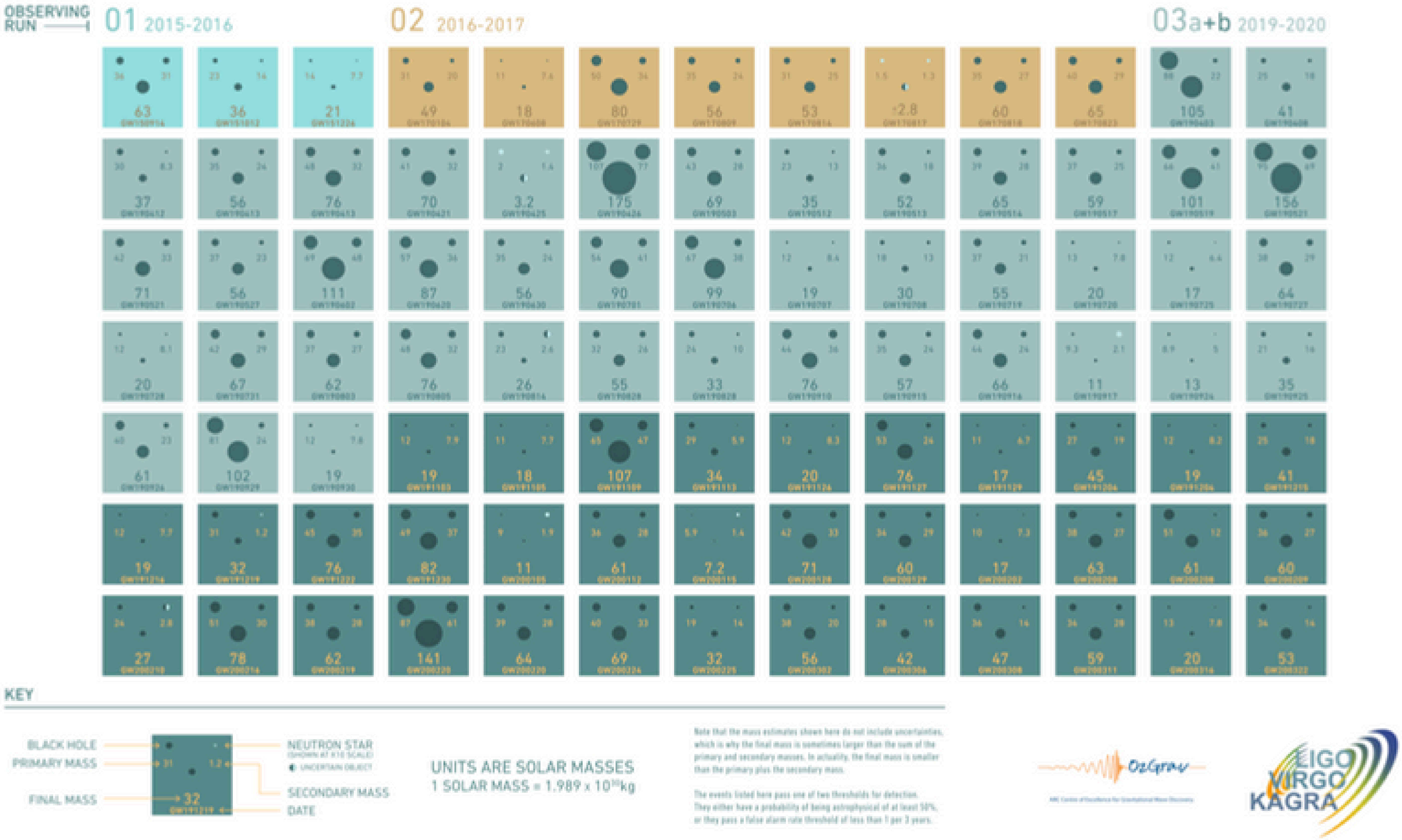
LVK observation plan

BNS range: distance at which the detector can observe a binary neutron star coales-
cence with masses 1.4M − 1.4M at an SNR of 8

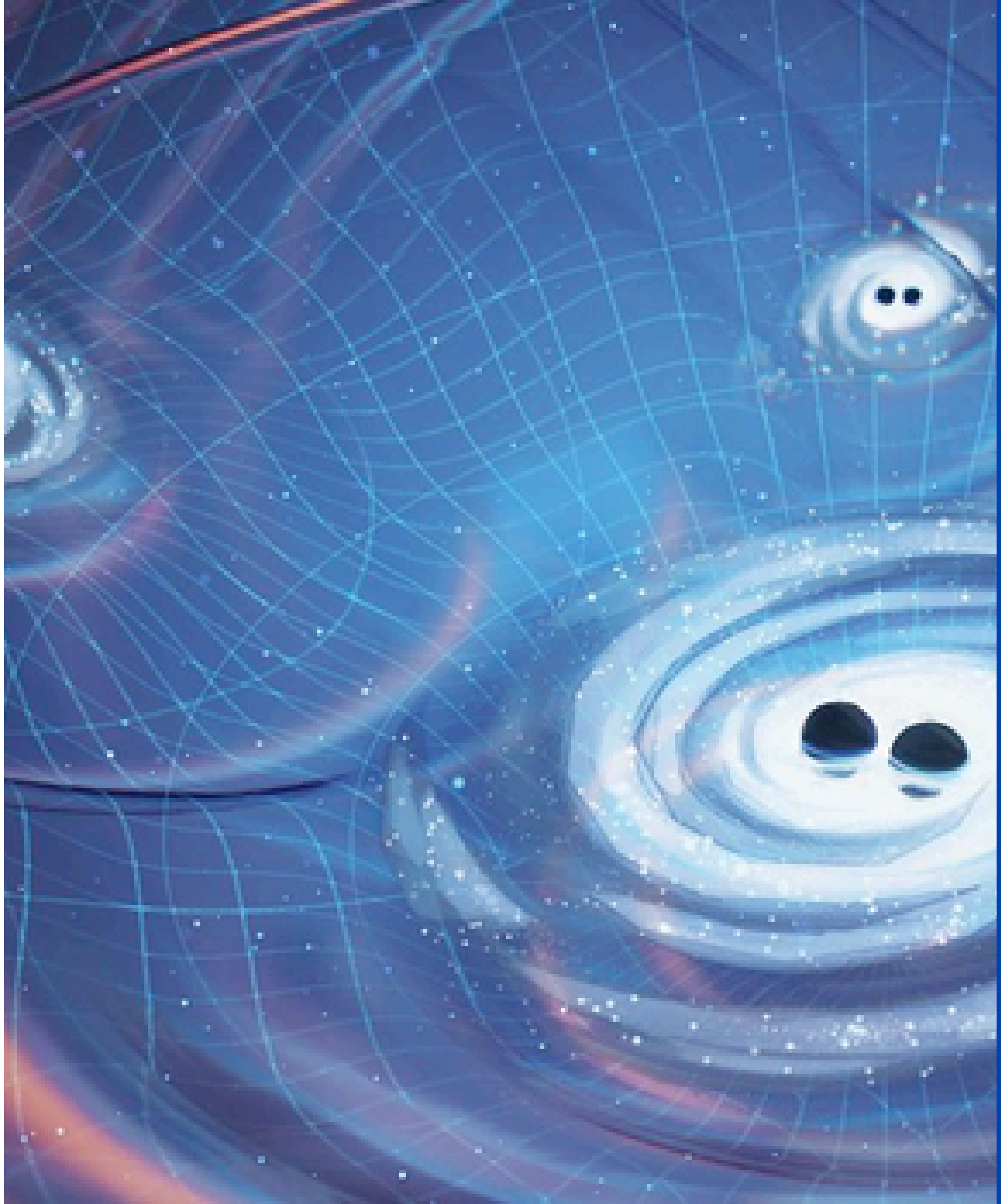


Catalog of GW Events

BH: black dots
NS: blue dots

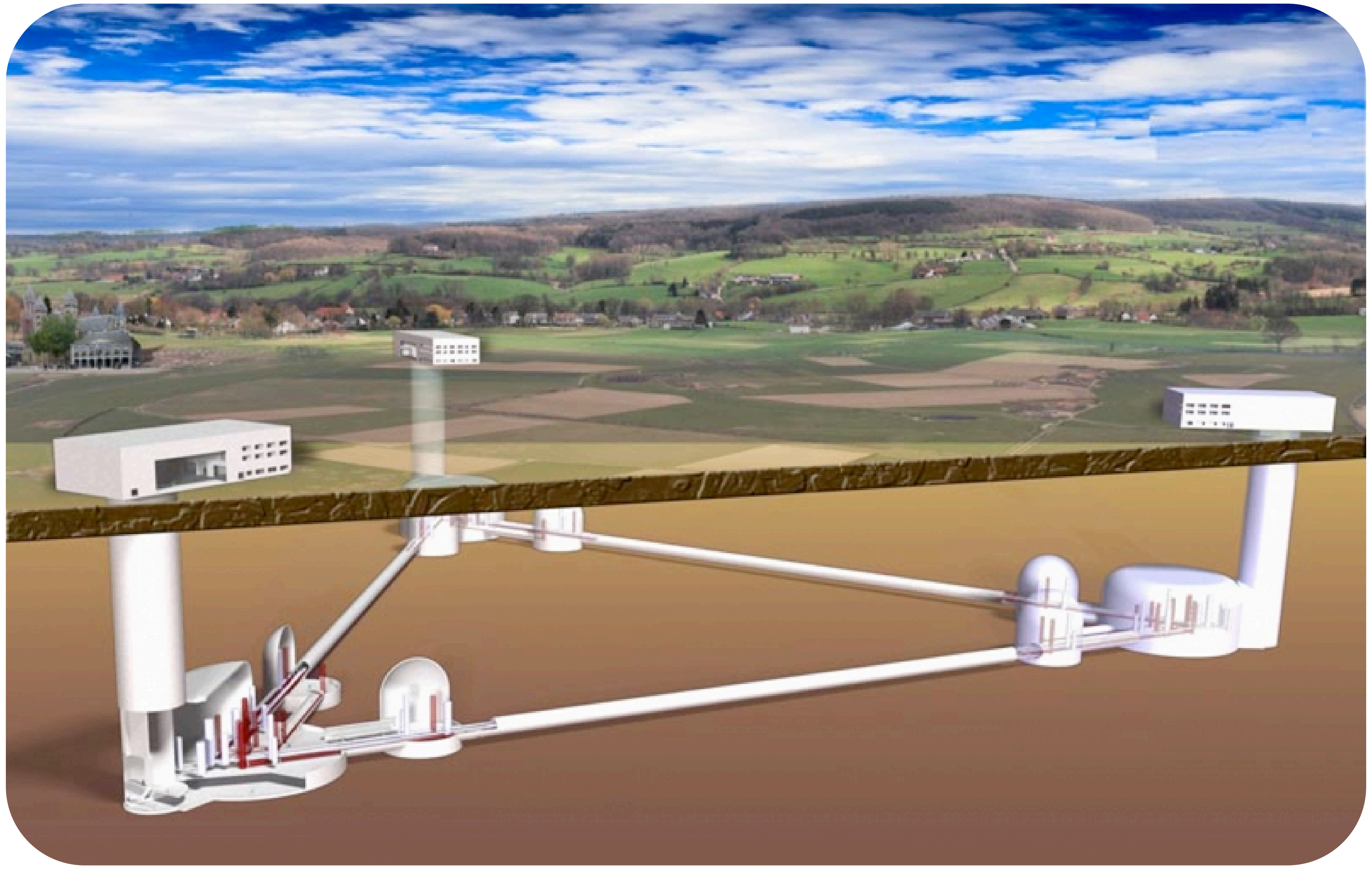


Future detectors



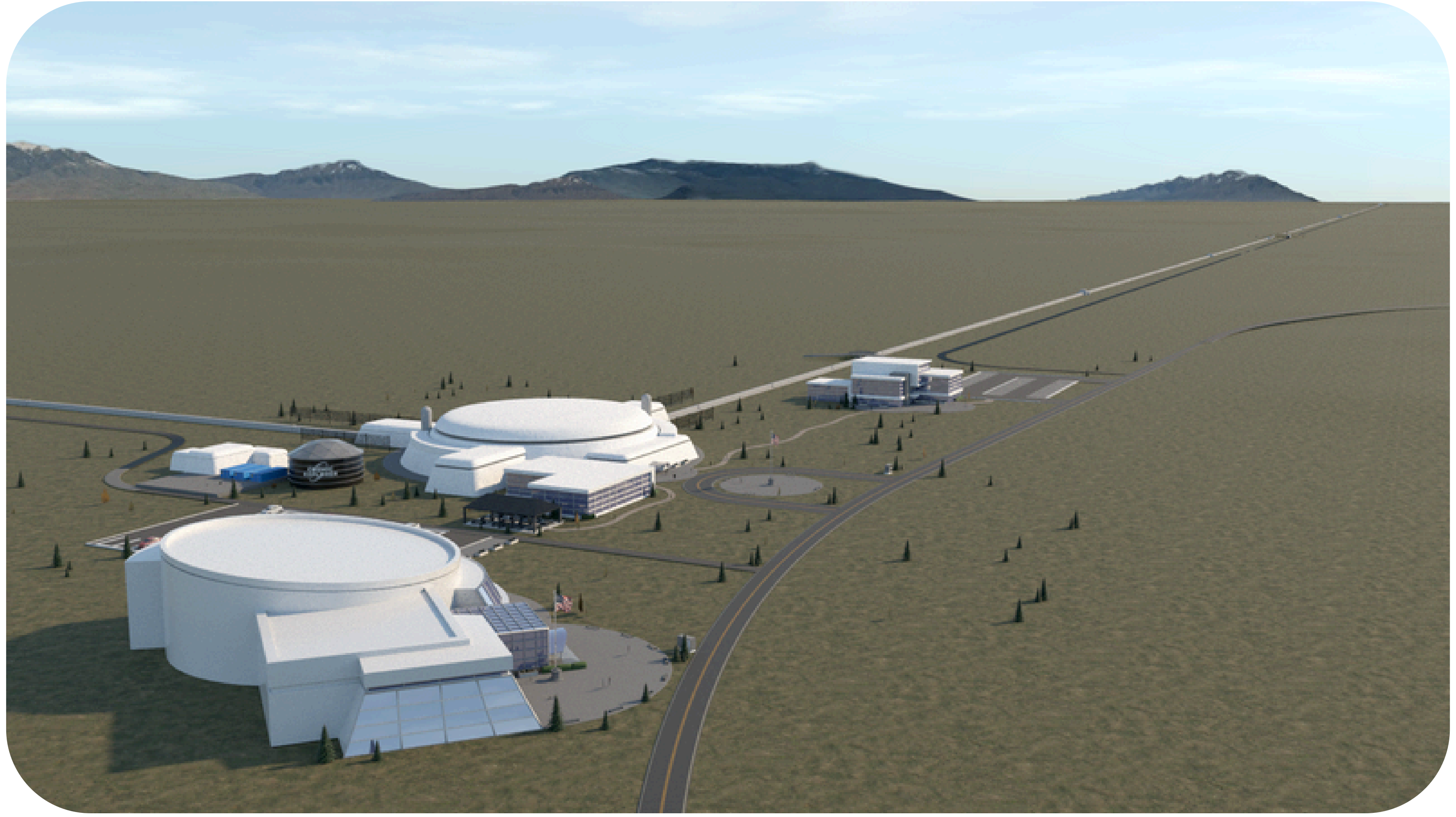
Einstein Telescope (ET)

- Proposed underground infrastructure to host a 3rd gen GW detector in Europe.
- Improved sensitivity by increasing the size of the interferometer arms to 10km, and by:
  - Cryogenic system to cool some of the main optics to 10 – 20K.
  - New quantum technologies to reduce the fluctuations of the light.
  - Infrastructural and active noise-mitigation measures to reduce environmental perturbations.



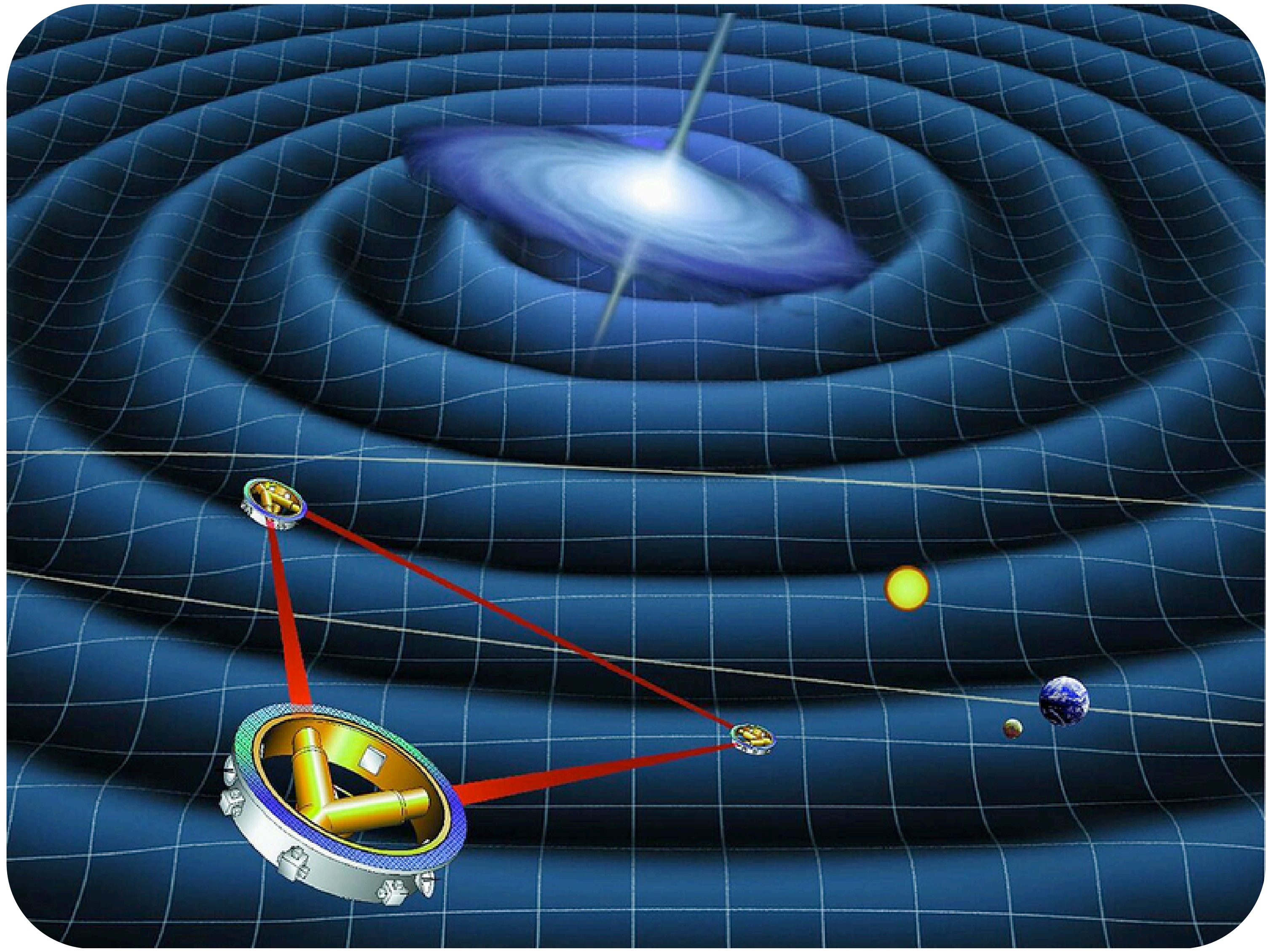
Cosmic Explorer (CE)

- 3rd gen GW detector concept in USA: two facilities, one with 40 km arms and another with 20 km arms, each housing a single L-shaped detector.
- The IFOs will have ultrahigh-vacuum beam tubes, roughly 1 m in diameter, built in an L-shape on the surface of flat and seismically quiet land.

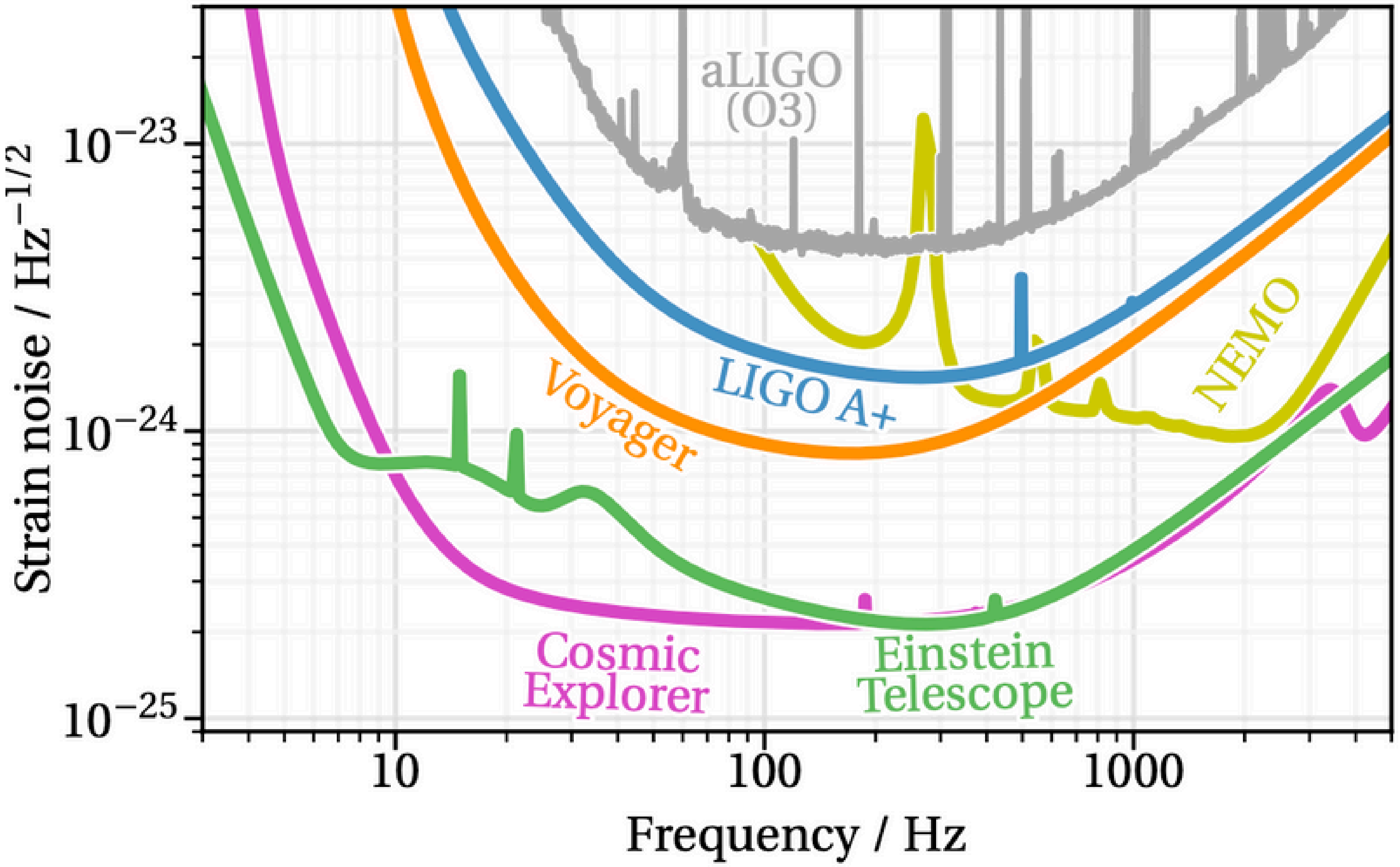


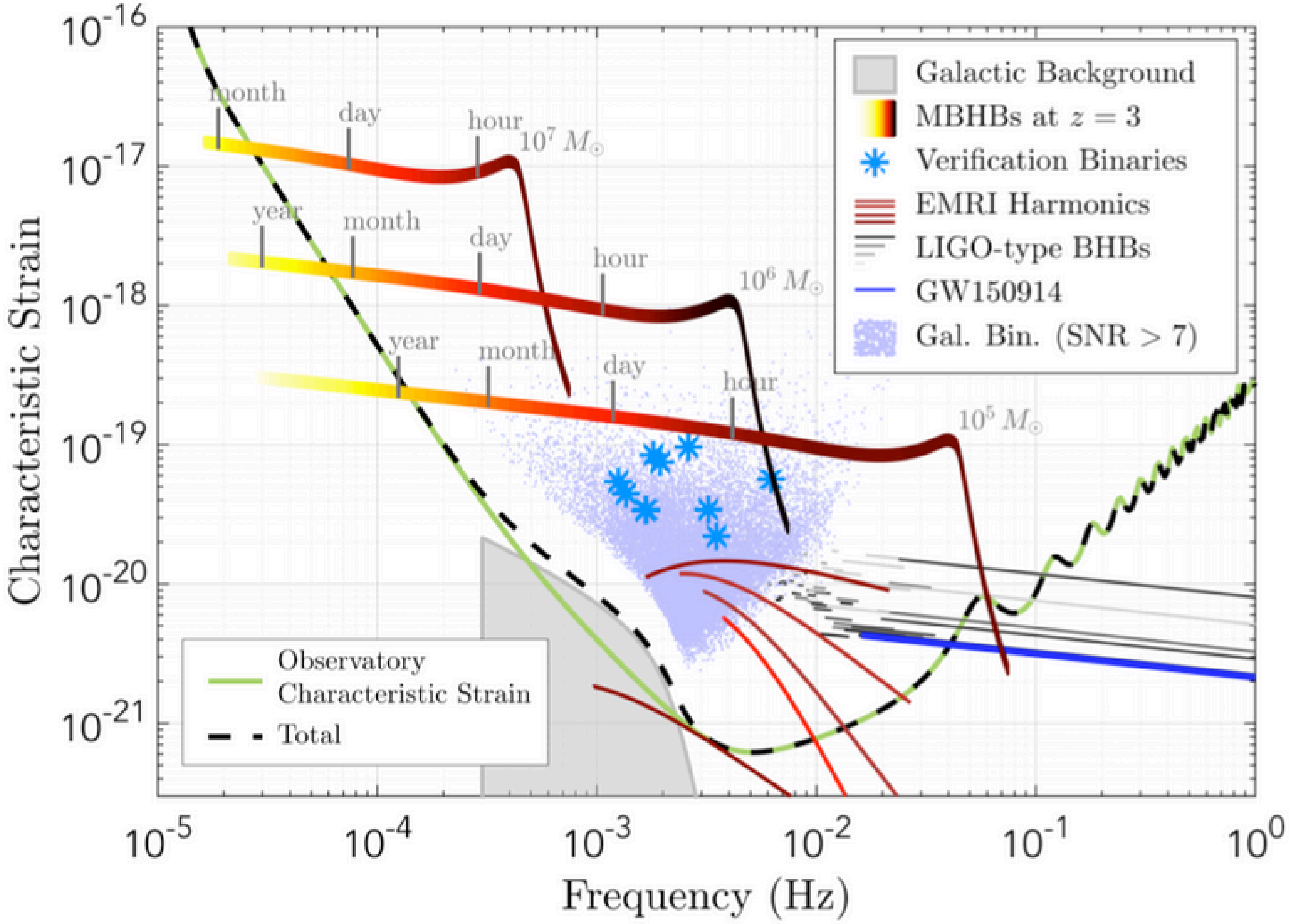
LISA

- Space-based GW detector constructed of 3 spacecraft separated by millions of miles, trailing tens of millions of miles.
- These 3 spacecraft relay laser beams back and forth between the different spacecraft and the signals are combined to search for GWs.
- NASA is a partner in the ESA-led mission, which is scheduled to launch in the mid-2030s.



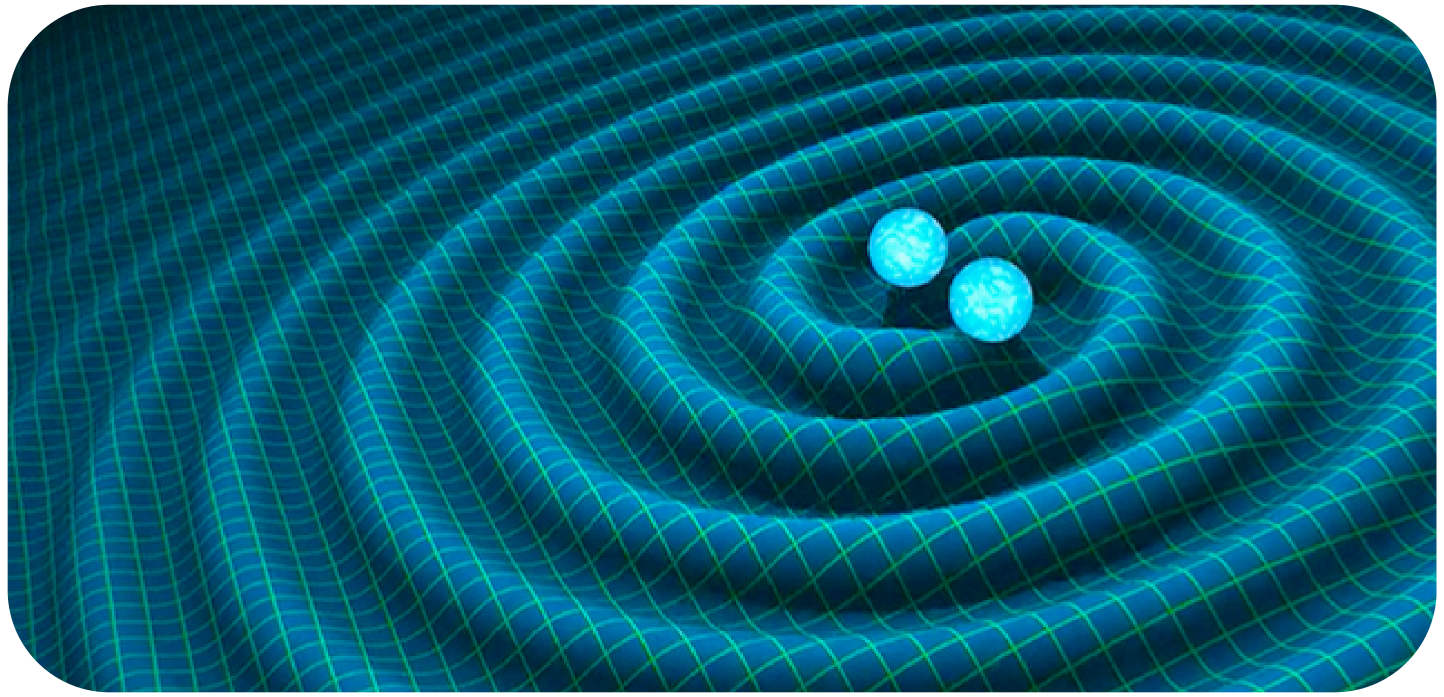
Comparison of sensitivity curves

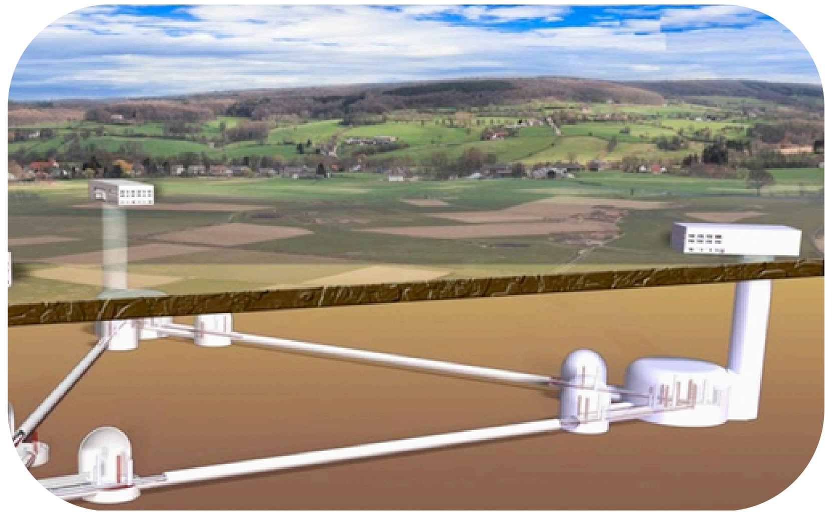


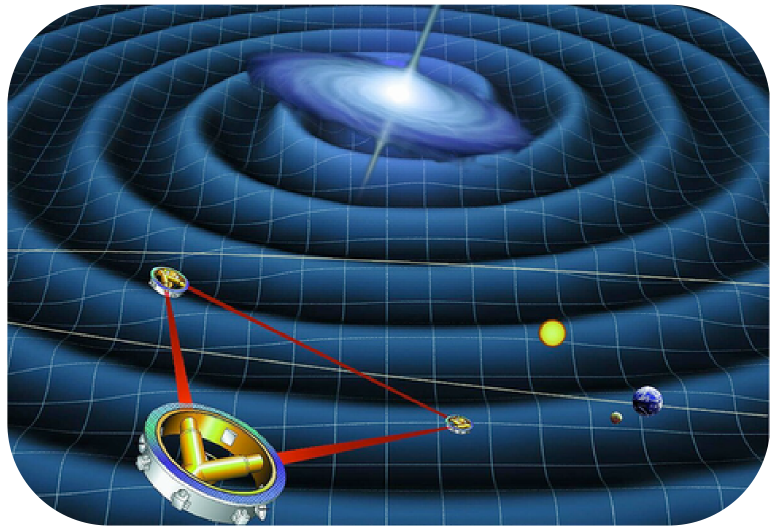


Conclusions

- GWs are a solution of Einstein's equations
- Complementary view of the Universe.
- GWs carry valuable information on their sources (either of astrophysical or cosmological origin)
- GWs can be deteted with upgraded versions of Michelson interferometers with FP cavities
- So far, only CBCs have been detected
- A new generation of detectors is planned, which will widen our understanding of the Universe







Further reading

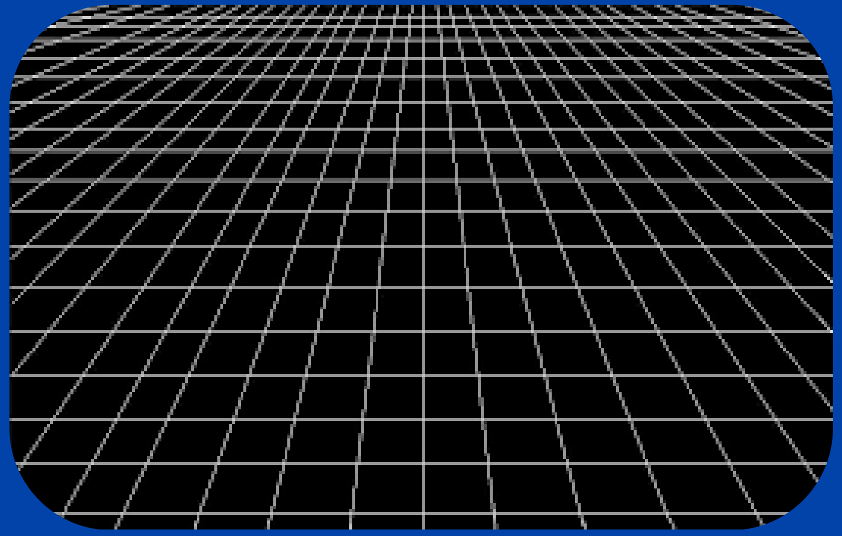
- Maggiore, M. (2007). Gravitational waves. *Vol. 1: Theory and experiments.* Oxford University Press. https://doi.org/10.1093/acprof:oso/9780198570745.001.0001
- Maggiore, M. (2018). *Gravitational Waves. Vol. 2: Astrophysics and Cosmology.* Oxford University Press.
- Peter R. Saulson, *Fundamentals of Interferometric Gravitational Wave Detectors*, 2nd Edition, World Scientific, April 2017. DOI: https://doi.org/10.1142/10116
- Peter Saulson (Author), David Reitze & Hartmut Grote (Eds.), *Advanced Interferometric Gravitational-wave Detectors*, World Scientific Publishing, 2019. DOI: 10.1142/10181

BACKUP

Derivation Gravitational waves

- Weak field limit: Minkowski + perturbation

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$ $$|h_{\mu\nu}| << 1$$



- We can simplify E. Eqs by assuming:

$$\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h$$

$$x^{\mu} \to x'^{\mu} = x^{\mu} + \xi^{\mu}(x)$$

trace reverse of $h_{\mu\nu}$

$$h \equiv h^{\mu}_{\mu}$$

ξ^{μ} are four arbitrary functions

Derivation Gravitational waves

- LHS of E. eqs reduces to:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{1}{2}(\partial^\sigma \partial_\mu \bar{h}_{\sigma\nu} - \partial^\sigma \partial_\sigma \bar{h}_{\mu\nu} + \partial_\nu \partial_\alpha \bar{h}_{\mu\alpha} - \eta_{\mu\nu}\partial^\alpha \partial_\beta \bar{h}_{\alpha\beta})$$

- Lorentz gauge further simplifies E. eqs:

$$\partial^\nu \bar{h}_{\mu\nu} = 0 \longrightarrow \Box \bar{h}_{\mu\nu} \equiv \partial^\sigma \partial_\sigma \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4}T_{\mu\nu}$$

- Far away from any source of mass/energy ($T_{\mu\nu} = 0$), E. eqs. are a 4D wave equation with sols.: **plane waves**

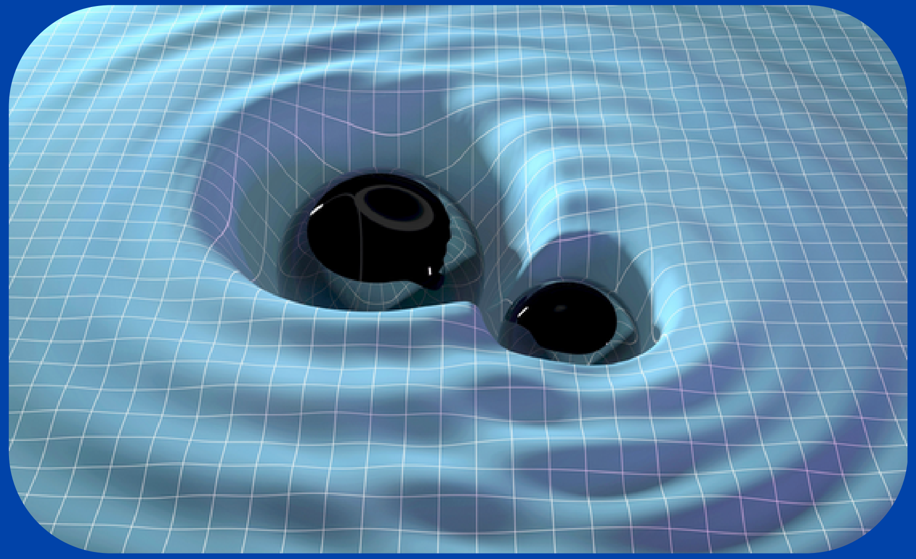
$$\bar{h}^{\mu\nu} = A^{\mu\nu} e^{ik_\alpha x^\alpha}$$

General Relativity

- On top of the Lorentz gauge, we can use the Transverse Traceless (TT) gauge: particles at rest remain at rest during and after GW passage

- The TT gauge is defined by:

$$h^{0\mu} = 0 \quad , \quad h_i^i = 0 \quad , \quad \partial^j h_{ij} = 0$$



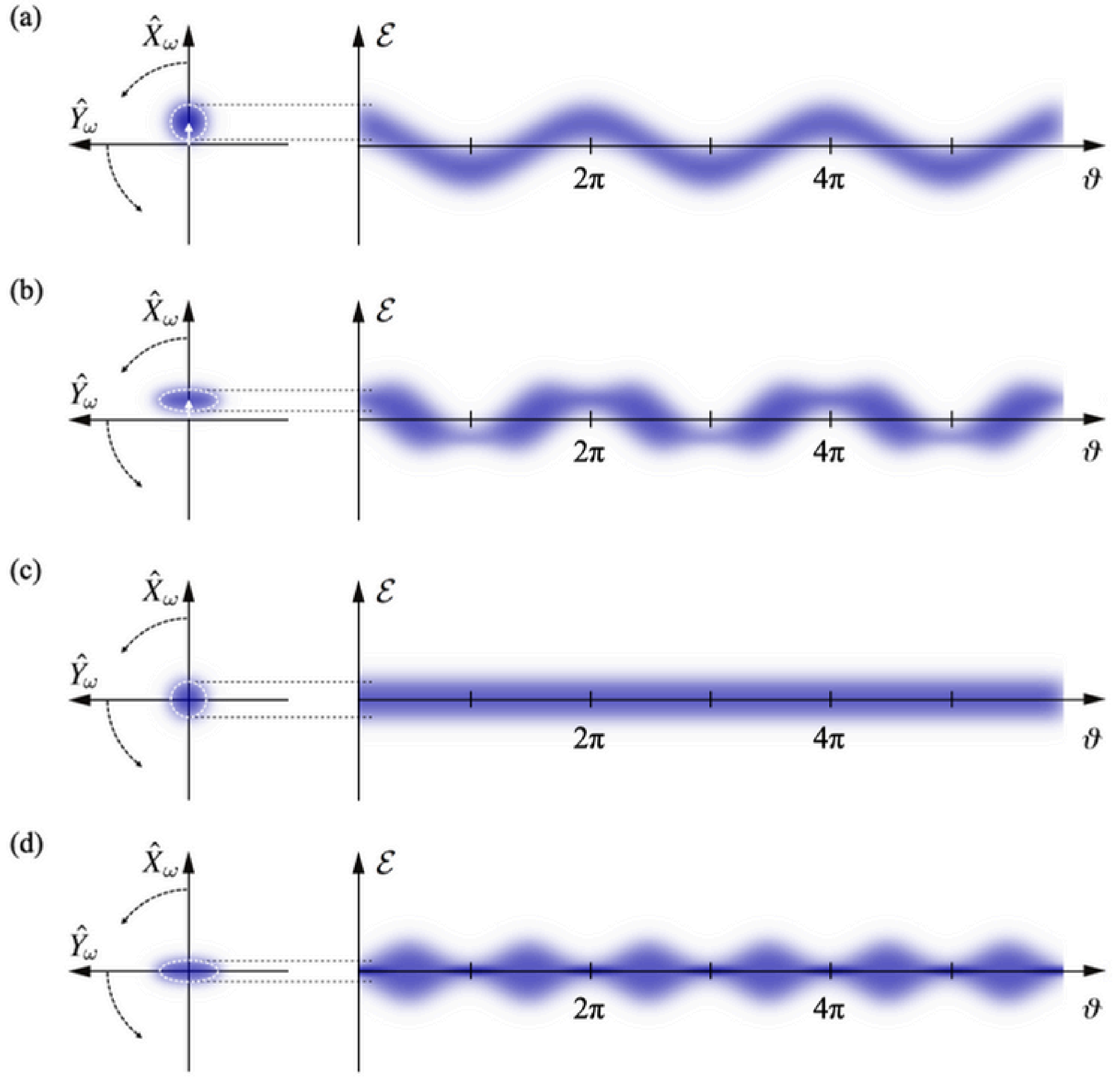
  - metric is purely spatial hμ0 = 0
  - wave is excited transversely to its direction of propagation ∂jhij = 0
  - wave is traceless hii = 0.

Noise sources in GW detectors

- Limiting value of SSQL(f) is a manifestation of the Heisenberg uncertainty principle (HUP).
- Two observables can be measured on a quantum system:
  - Electric field strength
  - System's photon number
- Corresponding dimensionless operators
  - phase quadrature: ˆX
  - amplitude quadrature: ˆY

$$\Delta\hat{X}\Delta\hat{Y} \geq S_{\mathrm{SQL}}^{1/2}(f)$$

- These satisfy the HUP → it allows reduction of the uncertainty in one quadrature when increasing the uncertainty in the other: ***squeezed states of light (quantum squeezing)***



R. Schnabel, Physics Reports, vol. 684, pp. 1–51, 2017,[Online]