

# Initiation to astroparticle theory & cosmology



*Camille Flammarion (Paris, 1888) "L'Atmosphère: Météorologie Populaire"*

Pasquale D. Serpico

LAFPT<sub>h</sub>

# Initiation to astroparticle theory & cosmology

## dark matter



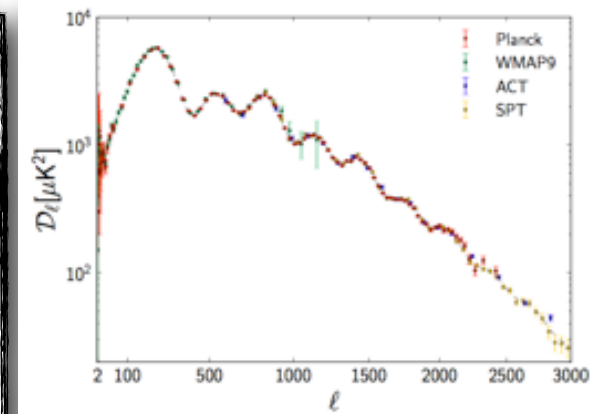
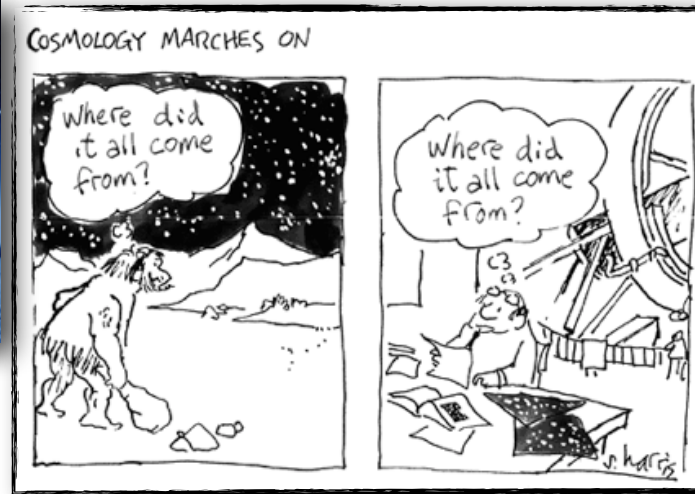
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# Goals



In a broad sense, physics (science?) cares of understanding:

What everything is made of, and how it works

Astroparticle physics takes this statement literally!

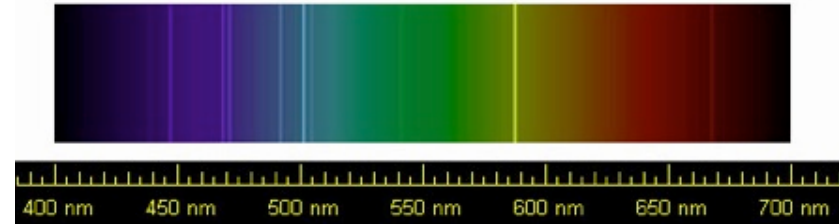
Relies on an assumption of *universality*, fruitful since Newton, at least

# New physics from the sky?

History invites us to be optimists!

587.49 nm

**1868:** soon after new tool (spectroscopy) introduced in astro, new “particle” (atom) identified first via astrophysics:  
**He in solar spectrum** (Janssen & Lockyer), only discovered on Earth in 1885 (uranium ore's)

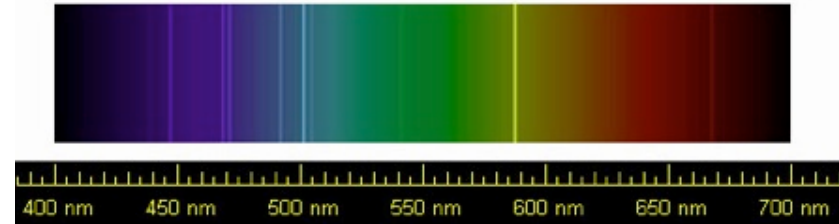




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The Nobel Prize in Physics 1936

Victor F. Hess, Carl D. Anderson

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Victor Franz Hess



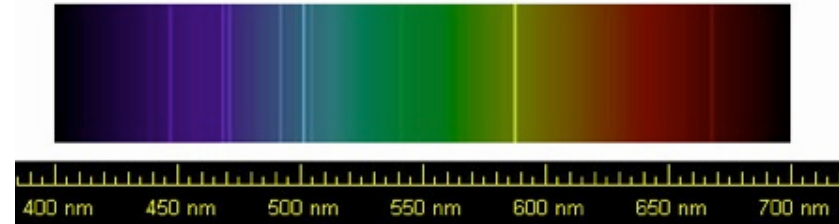
Carl David Anderson

The Nobel Prize in Physics 1936 was divided equally between Victor Franz Hess “for his discovery of cosmic radiation” and Carl David Anderson “for his discovery of the positron”.

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**Into XX1st century:** from pioneering experiences by Davis to final confirmation by SNO, systematically detected less  $\nu$ 's than predicted from the sun: first evidence for  **$\nu$  oscillations** (hence  **$m \neq 0$** )!

→ Related to Andrés López Moreno's lectures

**Why? The cosmos offers an extreme range of parameters, often inaccessible in the Lab!**



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# Disclaimer

**Astroparticle physics is a huge field!**

Some sub-branches:

## **I. Particle cosmology**

*(processes and constituent of cosmologically interesting phenomena)*

## **II. High-energy astroparticle physics**

*(processes involving high-energy, non-thermal phenomena, cosmic rays...)*

—————→ Related to S. Caroff's lectures

## **III. Stellar astroparticle physics**

*(processes involving thermal phenomena as in stellar burning, exotic particle emission and anomalous cooling, altered supernova dynamics...)*

—————→ Partially related to M. Giannotti's lectures

Decided to touch almost exclusively I, with related cosmo notions

Also: meant to be an *inspirational lecture*, but don't get fooled:

Lots of hard calculations and difficult measurements underlie current cosmo research!

# Outline

- ▶ Intro to dark matter (DM): original evidence and interpretation
- ▶ Modern evidence: heavily relying on cosmology. Cosmology detour!
- ▶ Pillars of the hot big bang model.
- ▶ “Newtonian cosmology”: basic solutions & link with ingredients of the standard cosmological model
- ▶ Cosmology beyond smoothness and importance for the DM
- ▶ Towards DM identification: The WIMP benchmark for DM models



# “Classical” notion of dark matter

In a number of **astrophysical bound systems**,  
“**Gravitational mass**”  $\gg$  “**Luminous mass**”  
DM = excess of the former wrt the latter

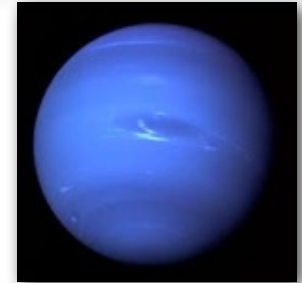
- I. **No** implication, yet, that DM is “**exotic**”. Might still be ordinary matter which does not shine (e.g. dim stars, planets, cold and/or rarefied gas, etc.)
- II. Implicitly assumes that the **theory of gravity** used (Einstein GR, most often in its Newtonian limit) is **correct**.
- III. **matter** (as opposed e.g. to radiation): its effects are inferred in bound systems, so that **DM clusters and forms structures** (very different, for instance, from the cosmological constant)

# Dark matter(s) common in astrophysics

**Not shocking to infer presence of “extra stuff” via gravity**

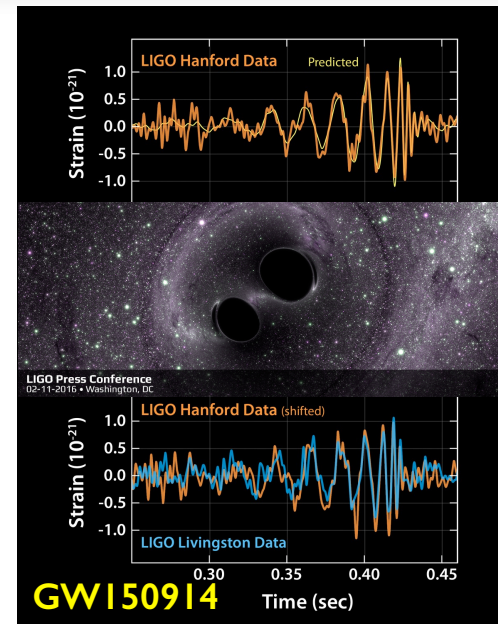
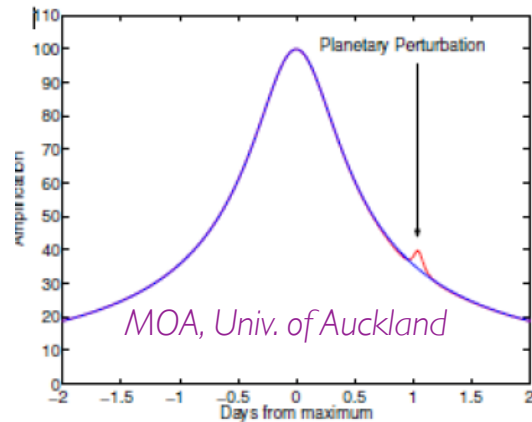
Le Verrier and independently Adams interpreted irregularities in Uranus orbit as due to perturbation by a yet unknown planet, calculating its orbital elements “by inversion”

On September 24, 1846 Galle found that “the planet whose place you [Le Verrier] have [computed] really exists” (“indirect DM detection”)



*Indirect detection of former Solar System DM by Voyager 2*

Microlensing used to discover e.g. brown dwarfs or exoplanets!



Only way BH mergers via GW are detected: virtually forever!

**Inferring the existence of objects from their gravitational effect is familiar in astrophysics!**

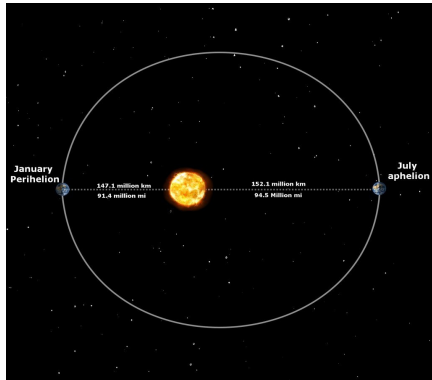
*DM has however some peculiarity that we'll mention later and that makes it particularly interesting*



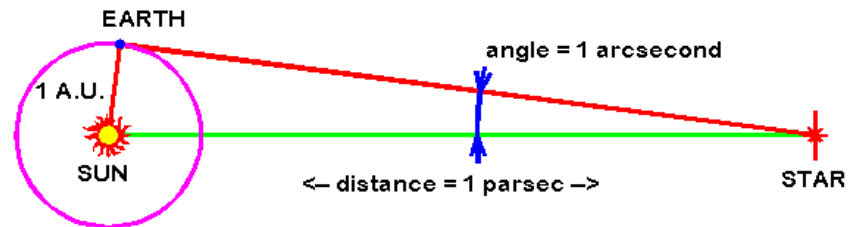
# A reminder of scales

Largely covered by Y. Génolini, M. Cagliari, Z. Gao...

astronomical unit ( $\sim 1.5 \cdot 10^{11}$  m)

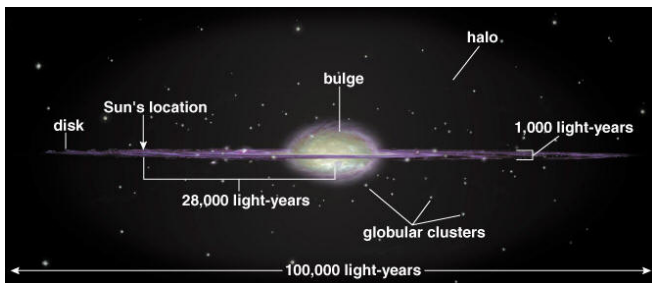


parsec ( $\sim 3 \cdot 10^{16}$  m  $\sim 3.26$  ly)

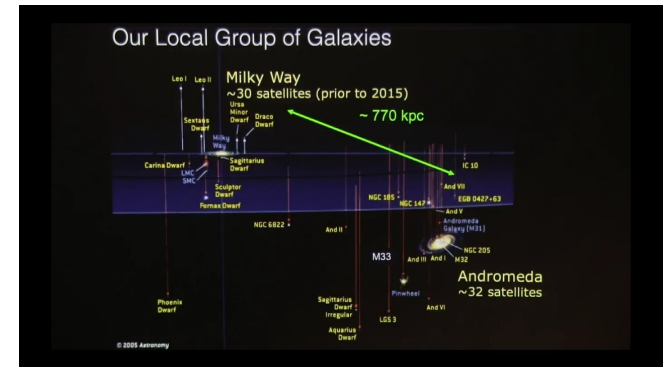


Linked to parallax measurement of distance

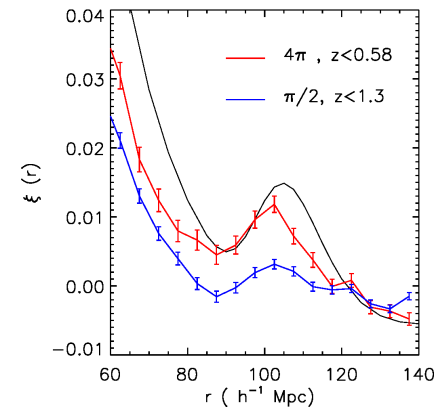
$\sim 10$  kpc



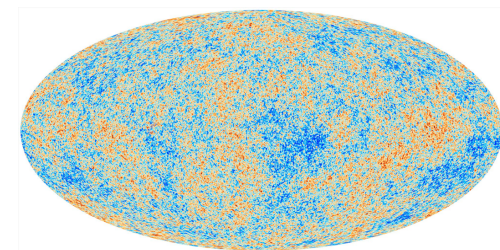
$\sim 1$  Mpc



$\sim 100$  Mpc

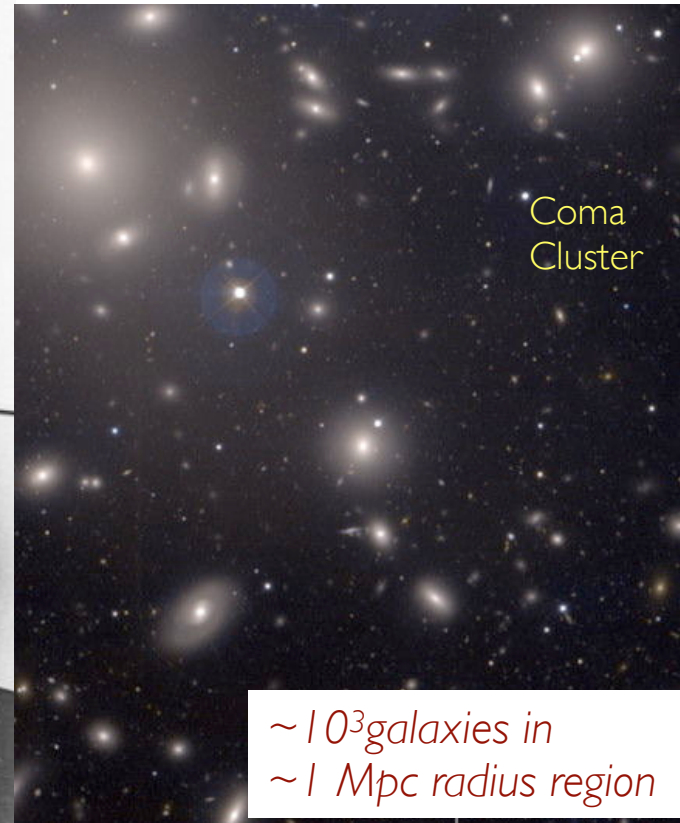
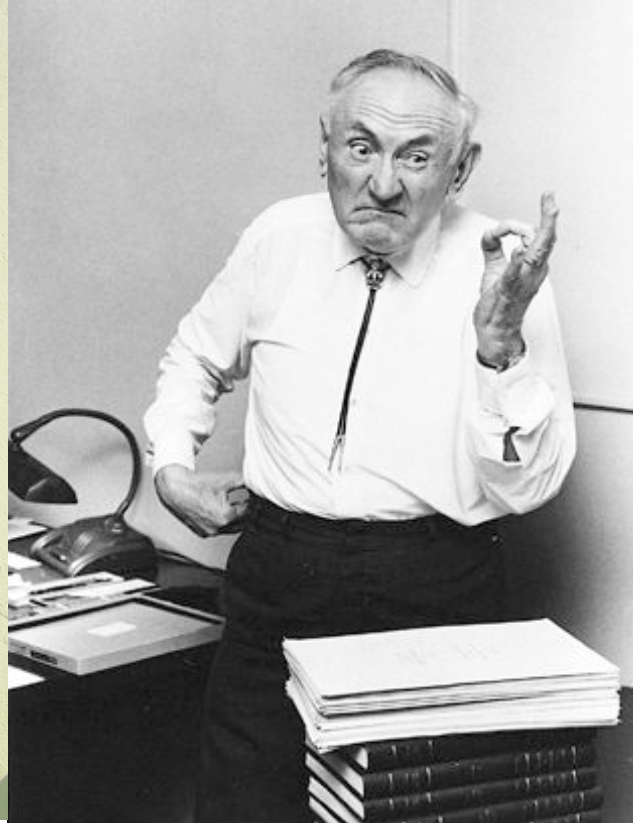


Up to  $\sim 30$  Gpc



# Galaxy motion in Clusters - Zwicky's finding

Varna, Bulgaria



$\sim 10^3$  galaxies in  
 $\sim 1$  Mpc radius region

Puzzle follows from application of the **Virial Theorem**

$$2\langle T \rangle + \langle U_{tot} \rangle = 0$$

*Die Rotverschiebung von extragalaktischen Nebeln\**, *Helvetica Physica Acta* (1933) 6, 110–127.

*"On the Masses of Nebulae and of Clusters of Nebulae"*, *ApJ* (1937) 86, 217

Scale: 0.1–1 Mpc

\*Nebula=Early XXth century name for what we call now galaxy

# Sketch of Zwicky's method

*Remember the gravitational potential energy of a self-gravitating homogeneous sphere of radius  $R$*

$$\langle U_{tot} \rangle \simeq -\frac{3}{5} \frac{G_N M^2}{R}$$

$$2\langle T \rangle + \langle U_{tot} \rangle = 0 \quad \longrightarrow \quad M \langle v^2 \rangle \simeq \frac{G_N M^2}{R}$$

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3 x dispersion  
from doppler shifts  
in spectra

$$M_{tot} \simeq \mathcal{O}(1) \frac{\langle v^2 \rangle R}{G_N}$$

inferred from  
distance  
& angular size

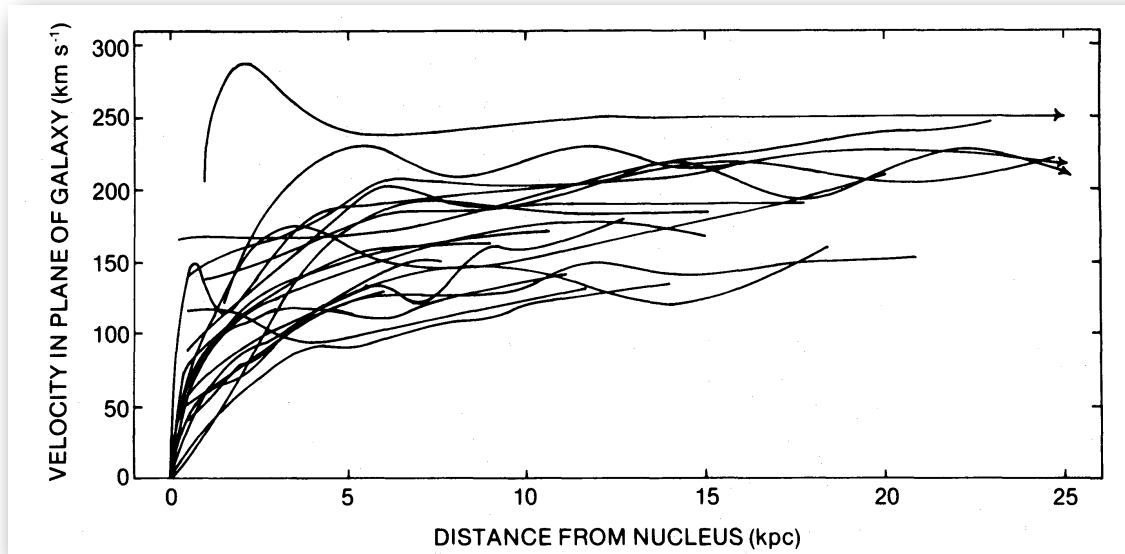
weakly depends on geometry/distribution of Galaxies in the cluster

Found  $M \sim 100-1000 \times M_{lum}$  (from converting luminosity into mass)



# Anomalies in spiral galaxy rotation curves

From ~1970, astronomers like V. Rubin and W. K. Ford Jr. exploited major advances (21 cm surveys, improved spectroscopy...) to obtain many rotational curves to their faint outer limits



Vera Rubin

*V. C. Rubin and W. K. Ford, Jr.,*

*"Rotation of the Andromeda Nebula from a Spectroscopic Survey of Emission Regions,"*

*ApJ 159, 379 (1970) [... ] V. C. Rubin, N. Thonnard and W. K. Ford, Jr.,*

*"Rotational properties of 21 SC galaxies with a large range of luminosities and radii, from NGC 4605  $R = 4\text{ kpc}$  to UGC 2885  $R = 122\text{ kpc}$ ," ApJ 238, 471 (1980).*

Scale: ~10 kpc

By the '80, many people started to take the DM problem seriously  
(partly due to technical refinements, part sociology?)

Opening a Window of Discovery  
on the Dynamic Universe

The telescope will produce  
the deepest, widest, image of the Universe:

27-ft (8.4-m) mirror, the width of a singles tennis court

3200 megapixel camera

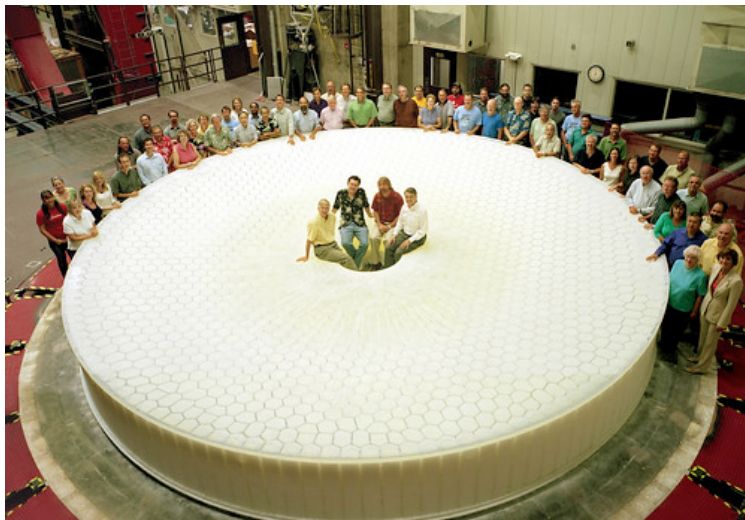
Each image the size of 40 full moons

37 billion stars and galaxies

10 year survey of the sky

Up to 10 million alerts, 1000 pairs of exposures,

20 Terabytes of data .. every night!



Formerly known as LSST,  
In Chile

Strong French implication  
(just after US)

*LAPP hosts a vibrant group!*

# Where is the problem? $M v_{\text{rot}}^2 \simeq \frac{G_N M^2}{R}$ (V.T. rotationally supported Galaxy)

- observed

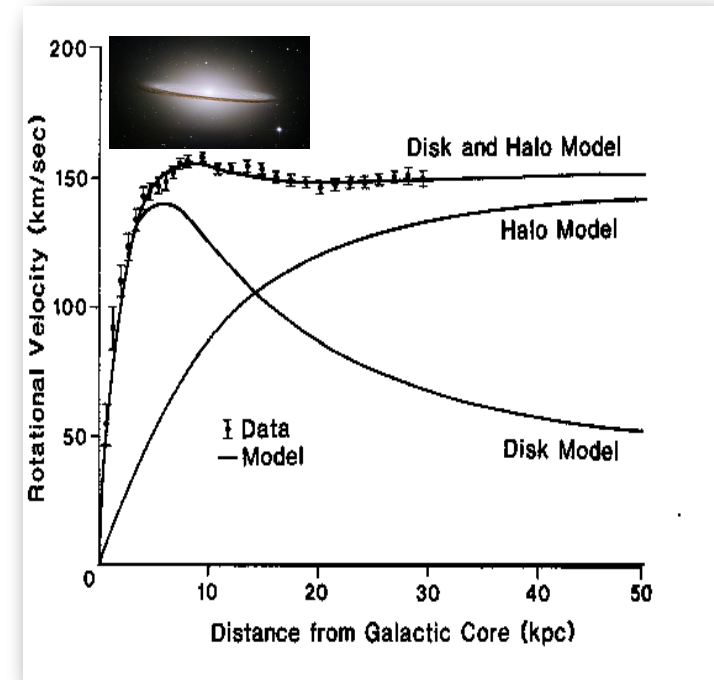
$$v_{\text{rot}}^2 = \frac{G M(R)}{R} \simeq \text{const.} \quad M(R) = \int_0^R 4\pi r^2 \rho(r) dr$$

- predicted based on visible light

$$v_{\text{rot}}^2 \propto \frac{1}{R}$$

Data well described by an additional component extending to distance  $\gg$  visible mass scale, with a profile

$$\rho(r) \propto r^{-2} \quad (\text{clearly not valid at asymptotically large } r!)$$



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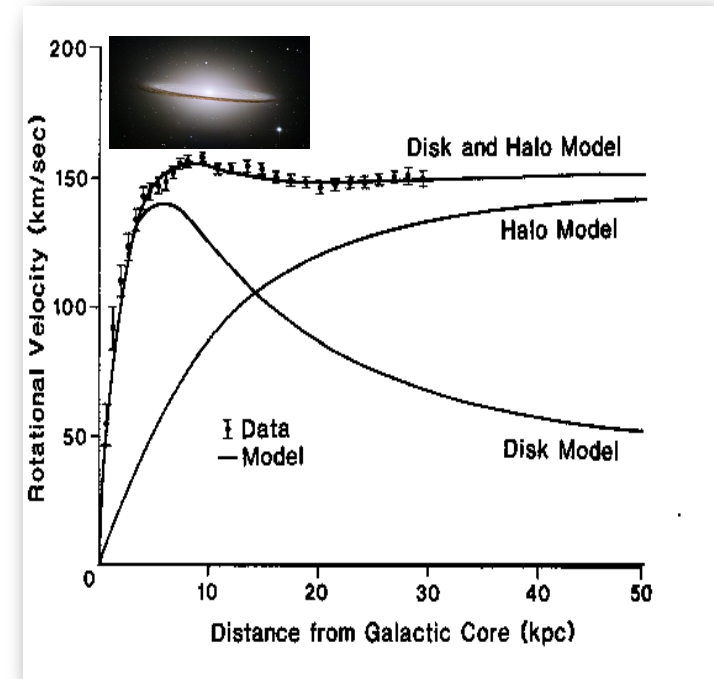
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## Caveats on this evidence

Determination of “local” Galactic DM requires a multi-parameter fit (including stellar disk, gas, bulge...)

Such techniques, as well as analogous ones used to infer DM in other systems (like dwarf Galaxies) are extremely important for direct and indirect searches of DM,

*but people often forget about it...*

Yet not most crucial or unambiguous ones to infer DM existence and properties

*...yet often presented as “smoking gun”!*

# MOND (only for Galaxy rotations!)

Alternative, phenomenological way to reproduce Galaxy rotation curves

*Milgrom +80*

Acceleration-dependent  
Modification of Newton's law

$$\mathbf{F} = m \mu \left( \frac{a}{a_0} \right) \mathbf{a}$$

$$\mu(x) \rightarrow 1$$

$$x \gg 1$$

Newtonian limit

$$\mu(x) \sim x$$

$$x \ll 1$$

MOND regime

**Exercise:** Prove that, using MOND force for a circular orbit, one gets a constant velocity at low accelerations. Compute this velocity in terms of  $a_0$

Attempts to similarly explain DM pheno in other systems (clusters, cosmology...) failed

This empirical law (besides having no theory behind) fails if applied to clusters, to the kSZ... and even detailed Galactic data

**In a DM context, interpreted as effective/emergent, not fundamental**



# Modern evidence

# From classical to modern DM evidence

In a number of **astrophysical bound systems**,  
“**Gravitational mass**”  $\gg$  “**Luminous mass**”  
DM = excess of the former wrt the latter

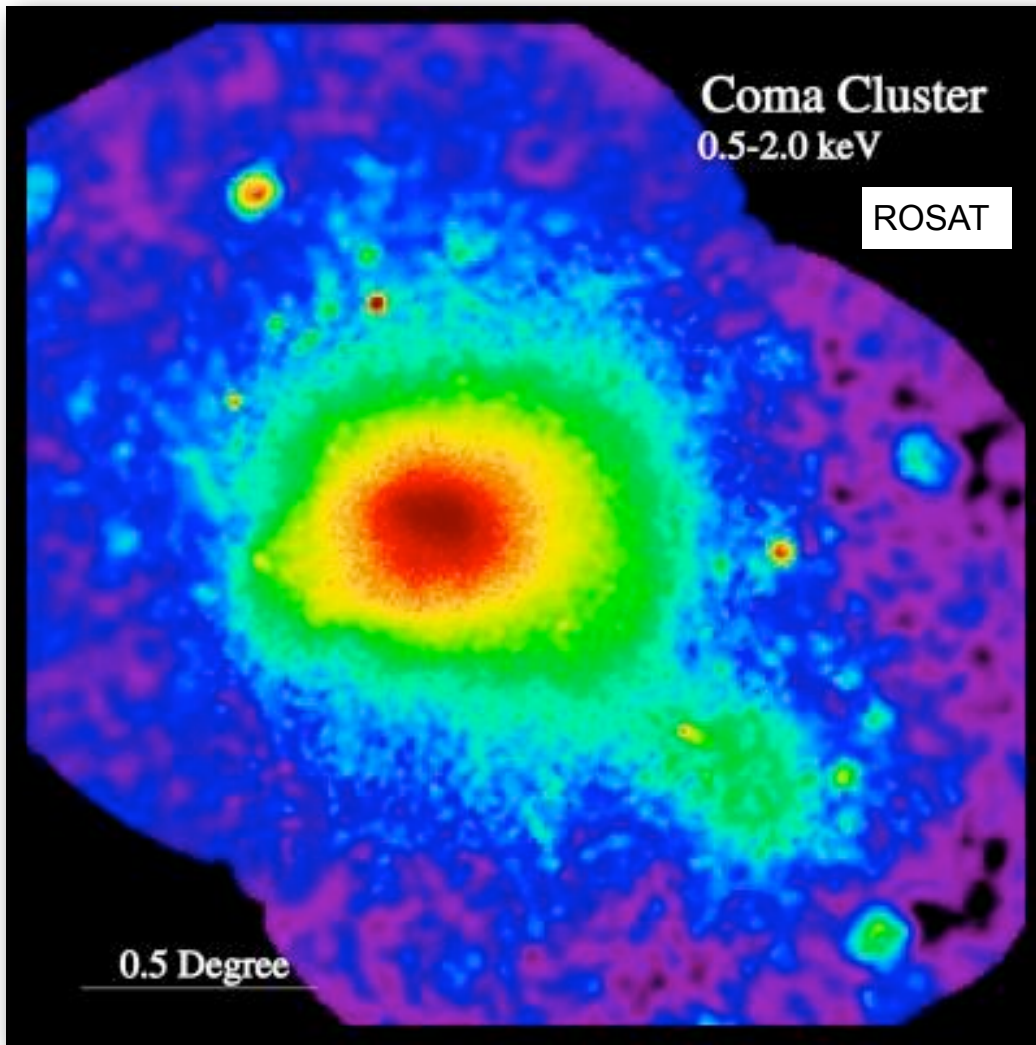
**Modern cosmological evidence greatly clarifies the situation!**

- I. Indication that **DM is “exotic”**: Cannot be made of ordinary matter which does not shine (e.g. dim stars, planets, cold and/or rarefied gas, etc.)
- II. Numerous tests that the **theory of gravity** used **leads to consistent results**
- III. Evidence extends beyond gravitationally bound objects, also to perturbative and **theoretically well-controlled regime (CMB & Large scales)**

# Clusters: seeing the invisible

**Most of the “ordinary” mass** in clusters (not true for galaxies!) in the form of **hot intergalactic gas**, which can be traced via X rays: **X-luminosity and spectrum provide mass profile!**

*Why? Estimate the gas temperature via the virial theorem, linking kinetic energy to temperature...*



Again, a factor  $\sim 7$  more mass than those in gas form is inferred (also its profile can be traced...)

Gets rid of unknown velocity distribution of DM, relies on spherical symmetry of the observable baryonic gas in hydrostatic equilibrium

See for example  
Lewis, Buote, and Stocke, *ApJ* (2003), 586, 135

# Sketch of the method

Spherical symmetric, hydrostatic equilibrium for the gas:

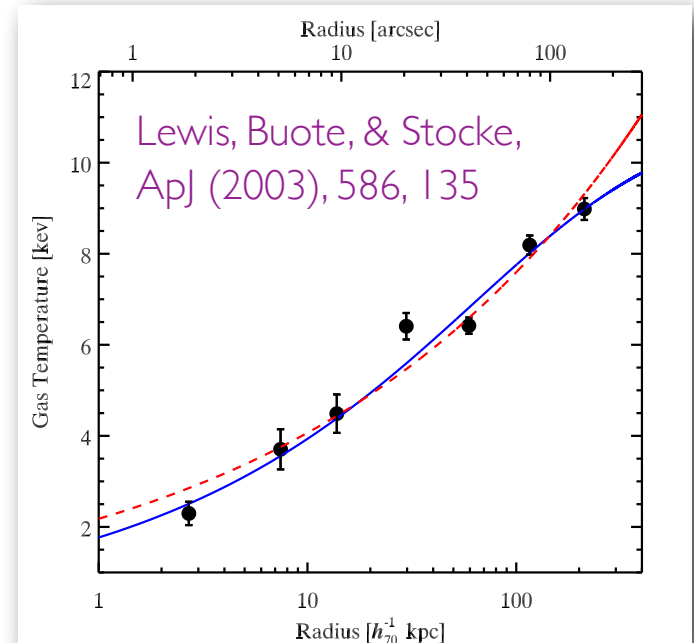
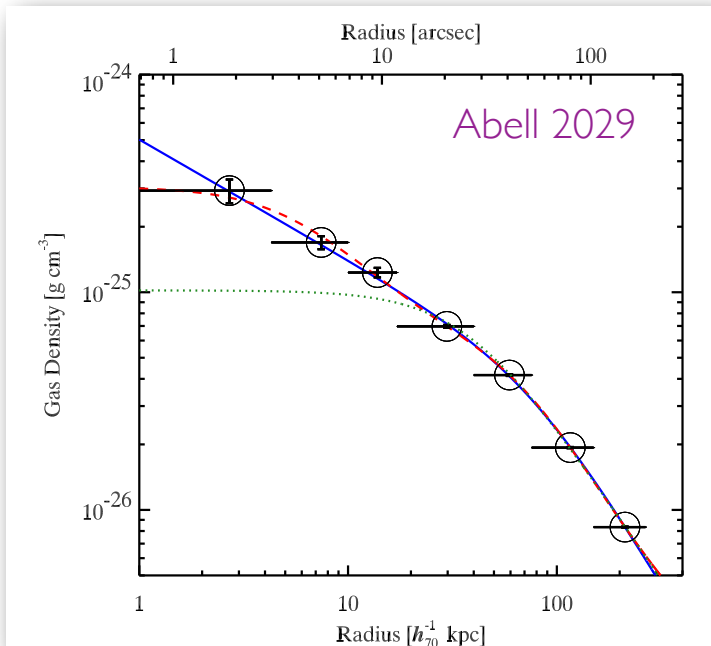
Newton's law in the fluid limit (shell)

$$dF = - \frac{G_N M(r) \rho_g(r) S}{r^2} dr \quad \longrightarrow \quad \frac{dP_g}{dr} = - \frac{G_N M(r) \rho_g(r)}{r^2}$$

Use perfect gas EOS (for gas)

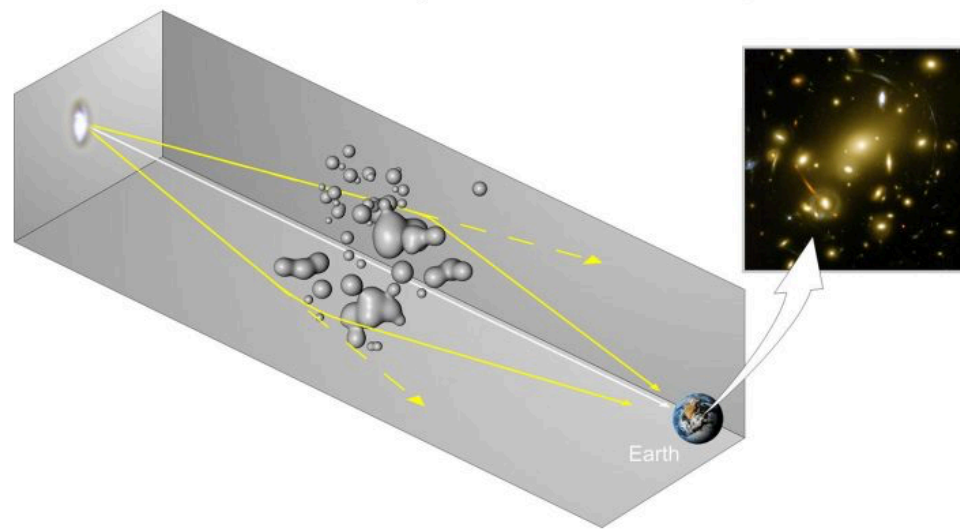
$$P_g = \frac{\rho_g}{\mu m_p} k_B T_g \quad \longrightarrow \quad M(r) = - \frac{r T_{\text{gas}}}{\mu m_u G_N} \left[ \frac{d \log \rho_{\text{gas}}}{d \log r} + \frac{d \log T_{\text{gas}}}{d \log r} \right]$$

The method **does not depend on gas density normalisation** (which controls the baryonic mass)!

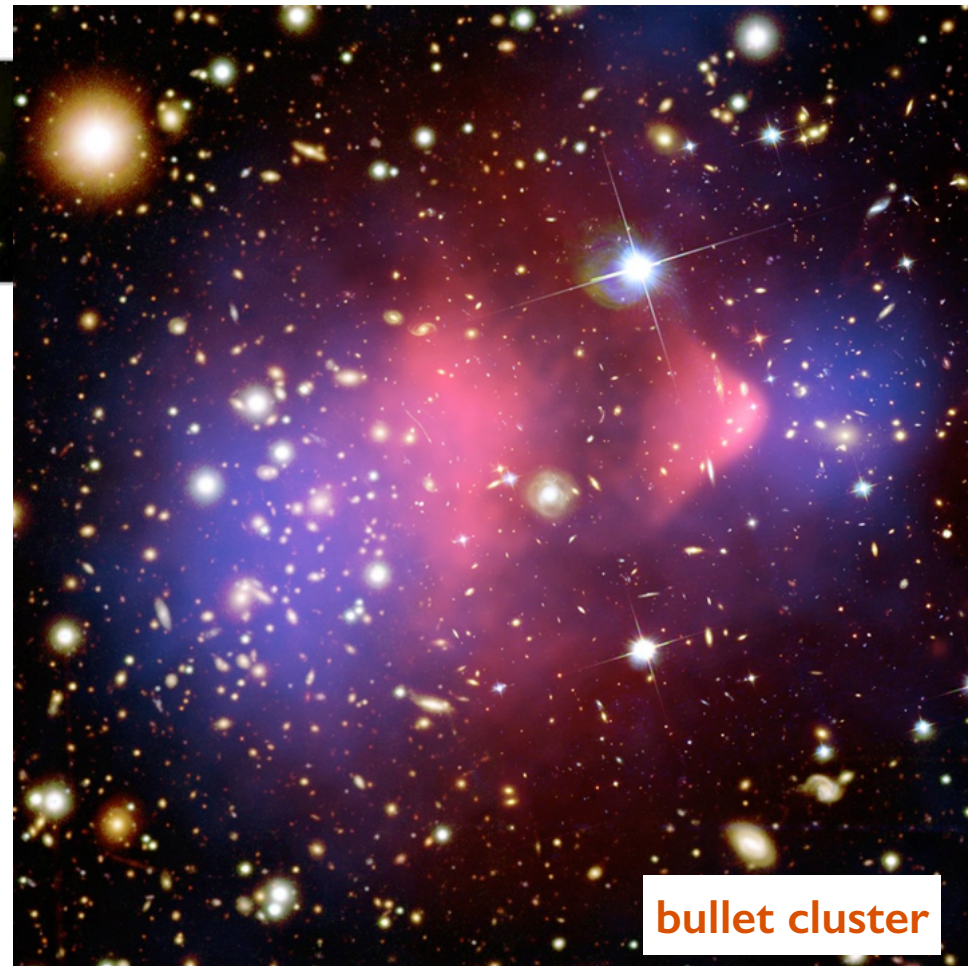


# mass-gas segregation in colliding clusters

Baryonic gas (imaged via X-rays) shocked in the collision and stays behind.  
Overall mass (causing lensing), like the subdominant galaxies pass through each other



most of the mass  $\neq$  collisional gas  
( $\sim$ equality expected if law of gravity altered)

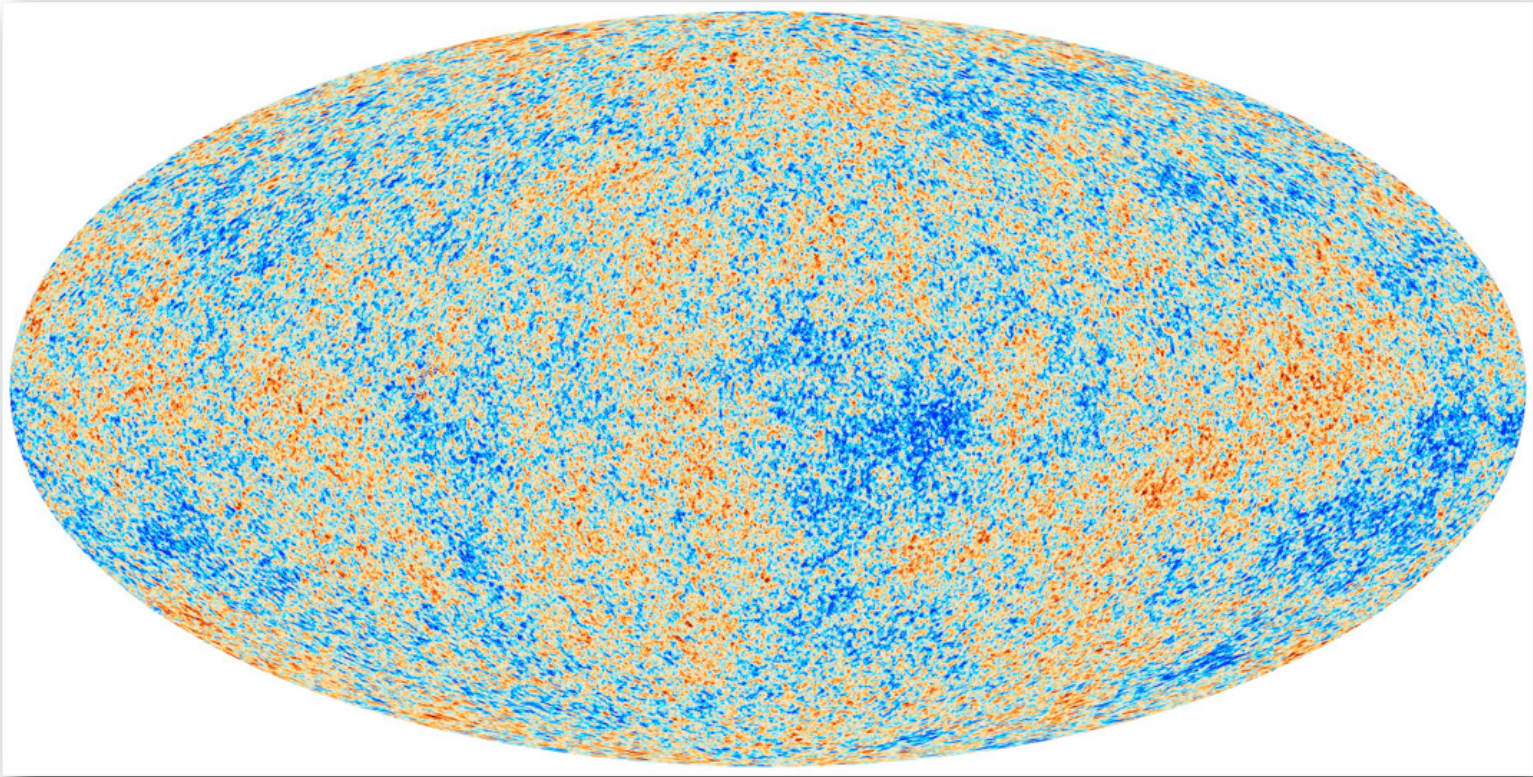


bullet cluster



# CMB, growth and pattern of large scale structures

This picture, plus linear theory is a robust proof of the existence of DM!

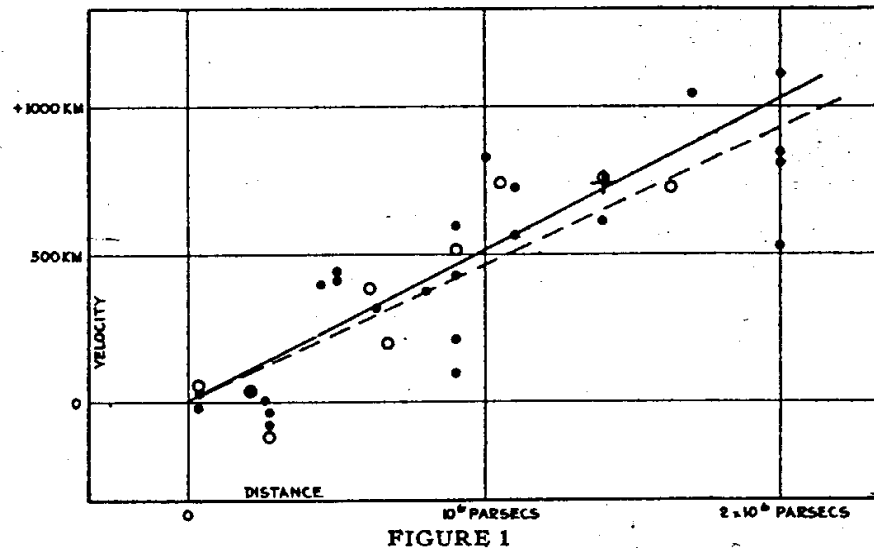


Why? We need a detour in cosmology, to understand (some of) it!

Scale: > Gpc

# Smooth cosmology

# First Pillar: Hubble-Lemaître law



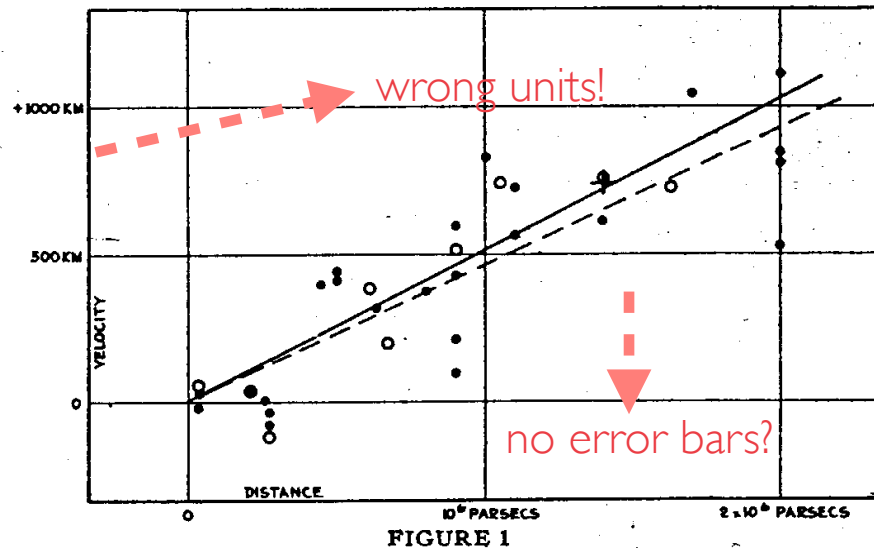
In 1927-29, they found out that all **galaxies sufficiently far away present a reddening** of their light (interpreted as 'relative velocity') **proportional to their distance**

$$cz \approx v = H_0 d$$

(Actually, it's only valid to first order...)



# First Pillar: Hubble-Lemaître law



Hopefully, you did a better job with Marina Cagliari & Zucheng Gao!

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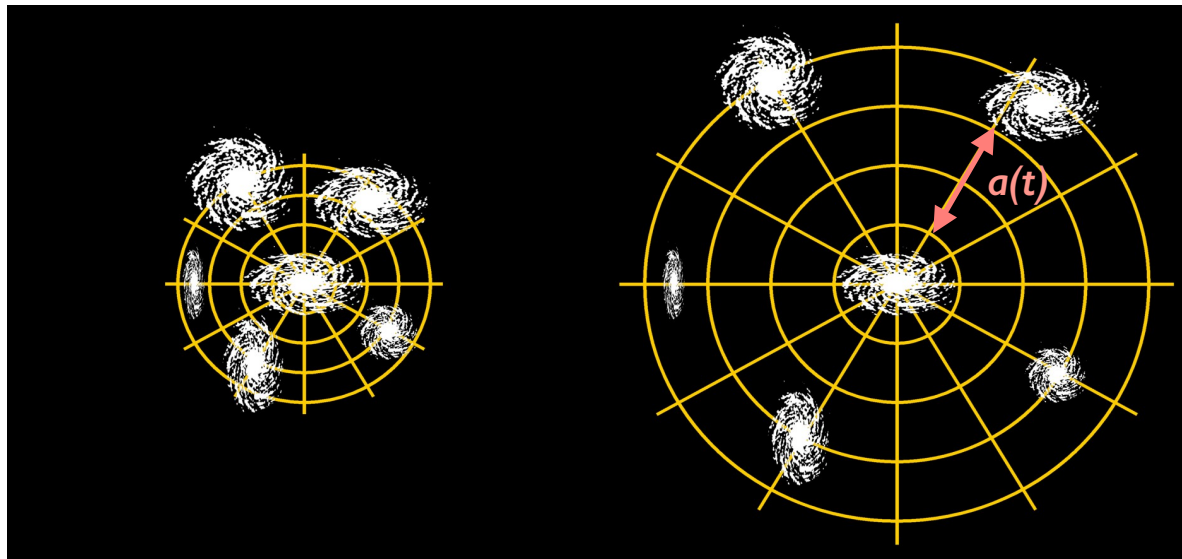


# Cosmological interpretation

Just like the pitch of a siren getting **farther** (**closer**) **lowers** (**increases**) its frequency (**Doppler effect**) the light of Galaxies moving **away from** (**towards**) us becomes **redder** (**bluer**)

$$\frac{\nu - \nu_0}{\nu_0} = \frac{\Delta \nu}{c}$$

Can be explained (consistently with relativity) if the space between galaxies gets stretched, rather than being an expansion 'within' space.



**Redshift  $z$**   $(1 + z) \equiv \frac{\lambda}{\lambda_0} = \frac{\nu_0}{\nu} = \frac{E_0}{E} = \frac{a_0}{a(t)}$

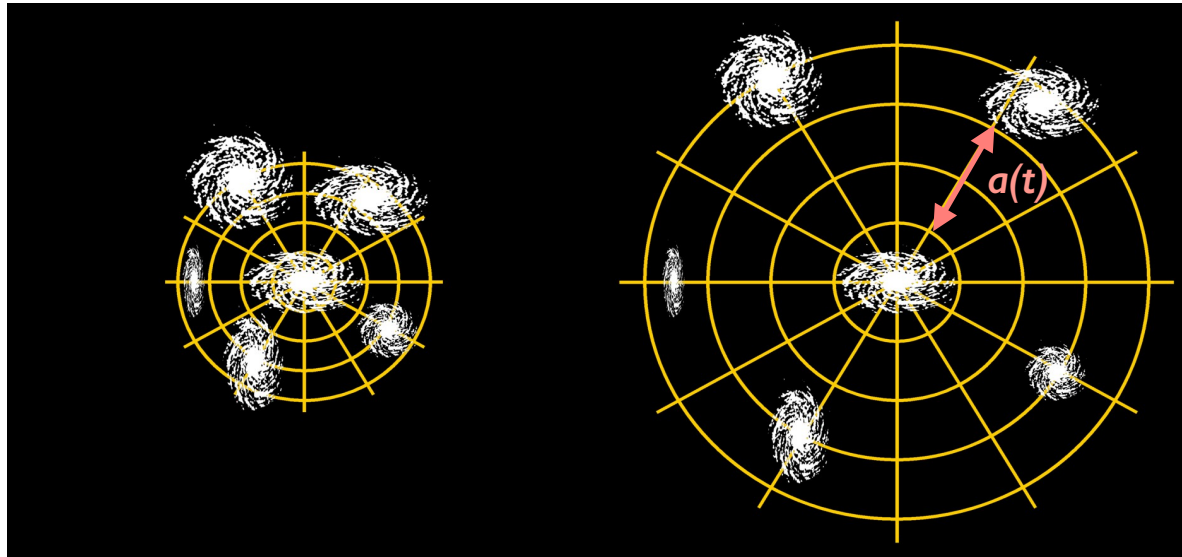


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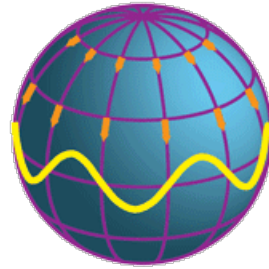


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# Second Pillar: “Hot Big Bang” & CMB

Photons freely expanding with the universe manifest the same ‘redshift’ phenomenon → becoming less energetic with time (remember,  $E=h\nu$ )

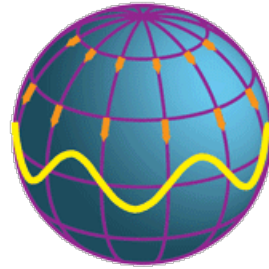


- Like a gas becoming hotter and denser when compressed, the **Universe should have been hotter and denser in the past** (Lemaître '34, Gamow '40)
- Sufficiently early on, one expects that “atoms” interact sufficiently frequently to allow for the instauration of a **thermal equilibrium. Blackbody spectrum** expected

*The colder and diluted relic of this photon bath should still exist today.*

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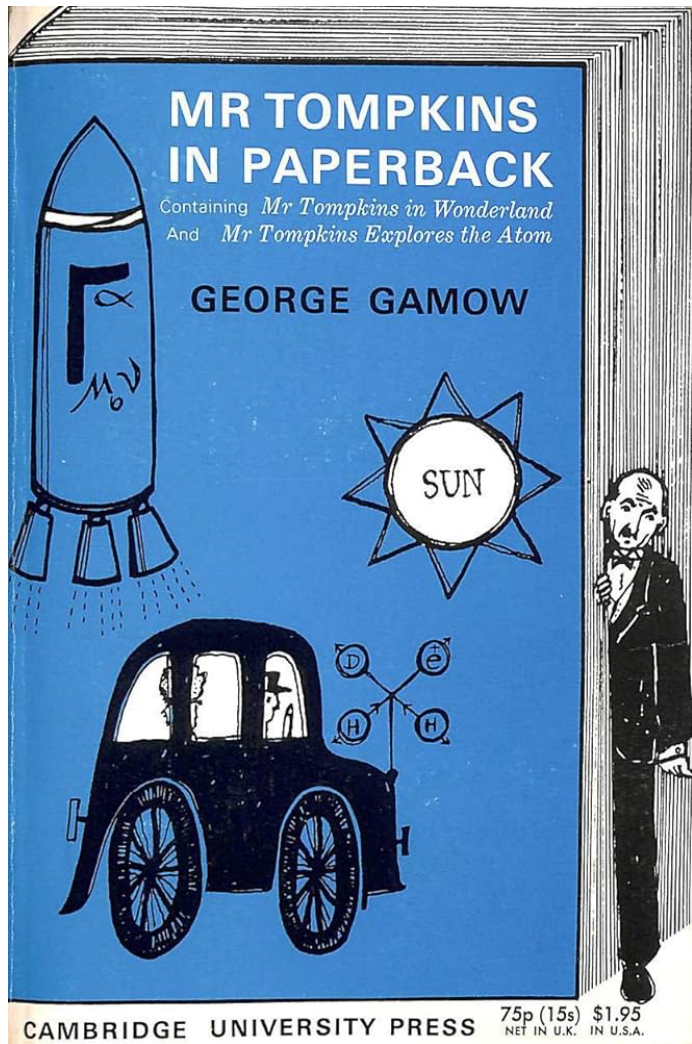
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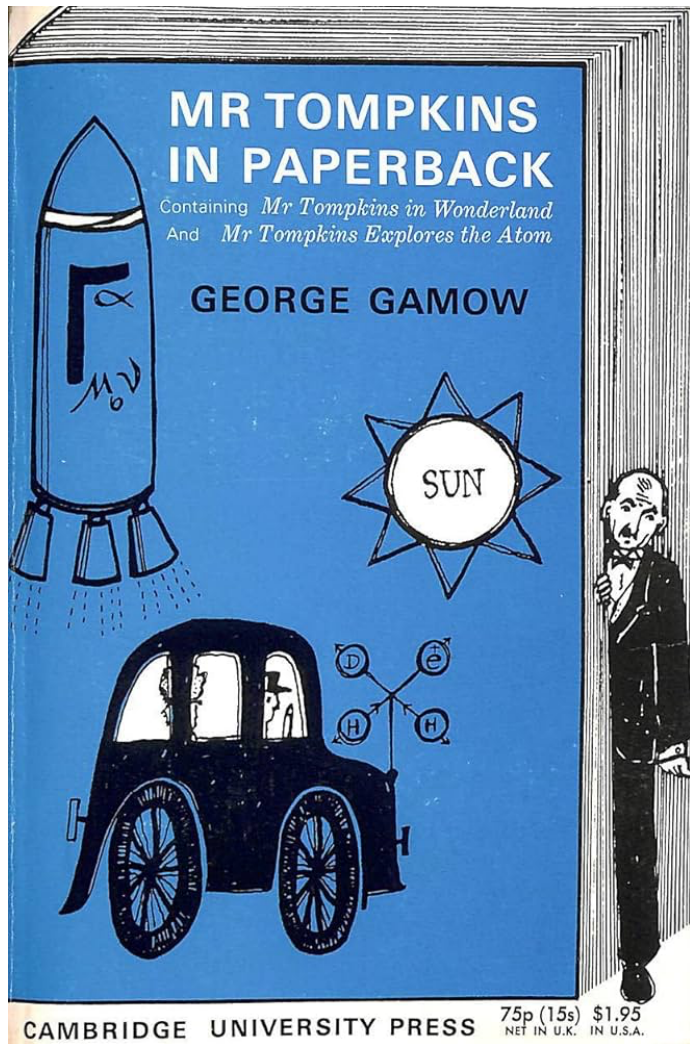
# Interlude: Mr. Tompkins, Gamow... and $\rho$



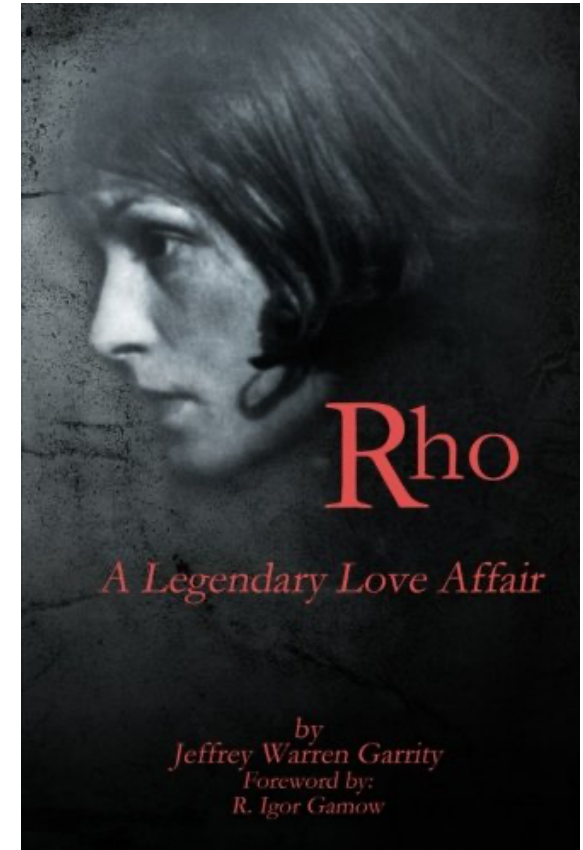
George Gamow  
(Georgiy Antonovich Gamov)  
(Odessa 1908 - Boulder 1968)

“What if... physics constants were different?”

# Interlude: Mr. Tompkins, Gamow... and *rho*



George Gamow  
(Georgiy Antonovich Gamov)  
(Odessa 1908 - Boulder 1968)



Lyubov "Rho" Vokhmintseva  
(1909-1985)

*From avant-garde Russian art world to  
"spy story" during early soviet period...*

"What if... physics constants were different?"

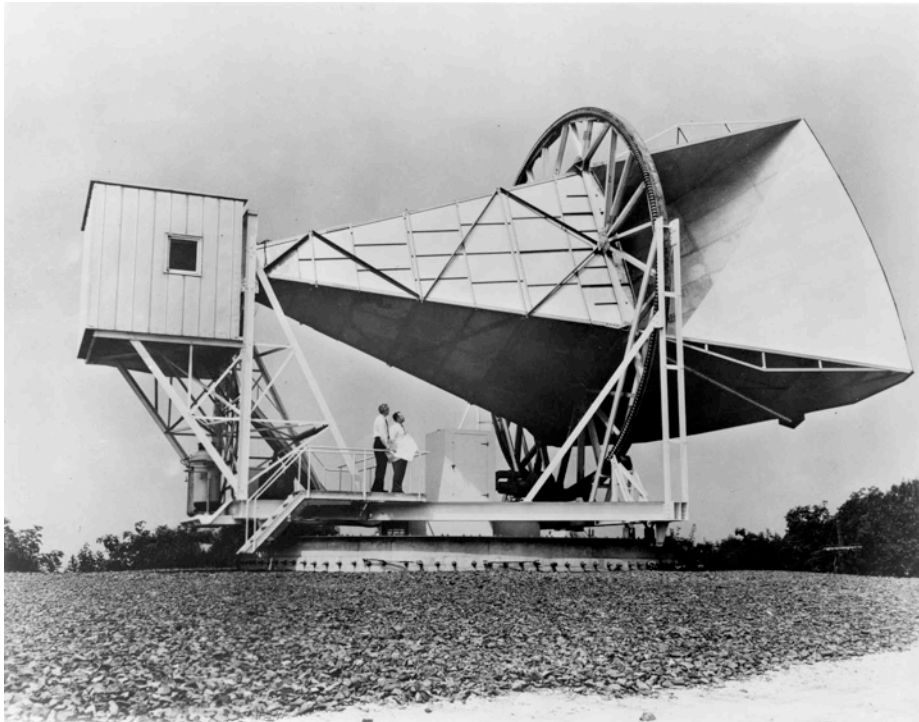


# Surprise!

Working on a new antenna at Bell Laboratories in 1965, Penzias et Wilson (Nobel 1978) found a strange radio noise...

...always present, even after cleaning up pigeon droppings...

Turned out to match Gamow's prediction of an isotropic thermal background ( $T \sim 3$  K). Can think of it as the **photons freed when  $e$  and  $p$  first combined into atoms**, freely streaming ever since.



## The Nobel Prize in Physics 1978

Pyotr Kapitsa, Arno Penzias, Robert Woodrow Wilson

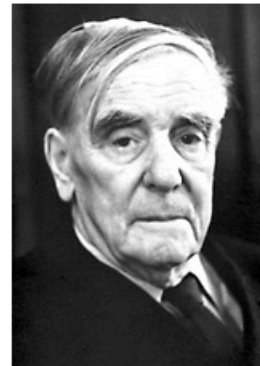
### The Nobel Prize in Physics 1978

Nobel Prize Award Ceremony

Pyotr Kapitsa

Arno Penzias

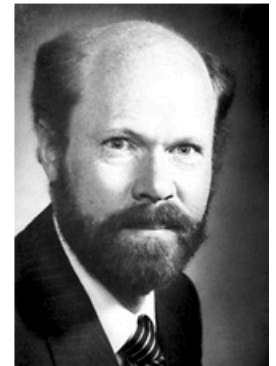
Robert Woodrow Wilson



Pyotr Leonidovich  
Kapitsa



Arno Allan Penzias

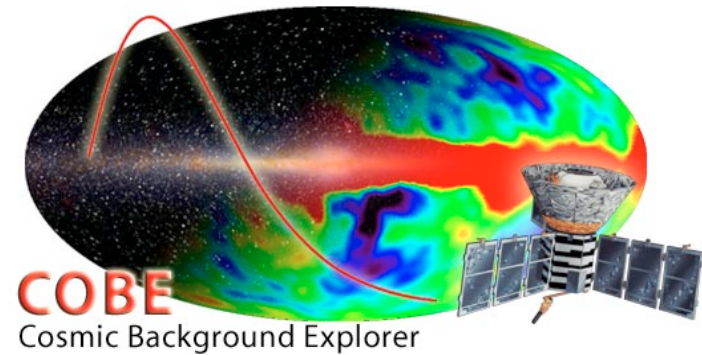


Robert Woodrow  
Wilson

The Nobel Prize in Physics 1978 was divided, one half awarded to Pyotr Leonidovich Kapitsa "for his basic inventions and discoveries in the area of low-temperature physics", the other half jointly to Arno Allan Penzias and Robert Woodrow Wilson "for their discovery of cosmic microwave background radiation".

# Detailed spectral check

Performed by COBE in  
1992, Nobel prize 2006



## The Nobel Prize in Physics 2006

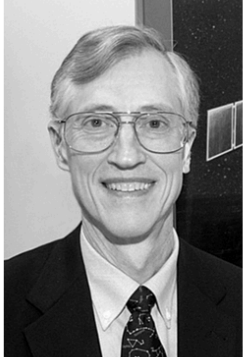


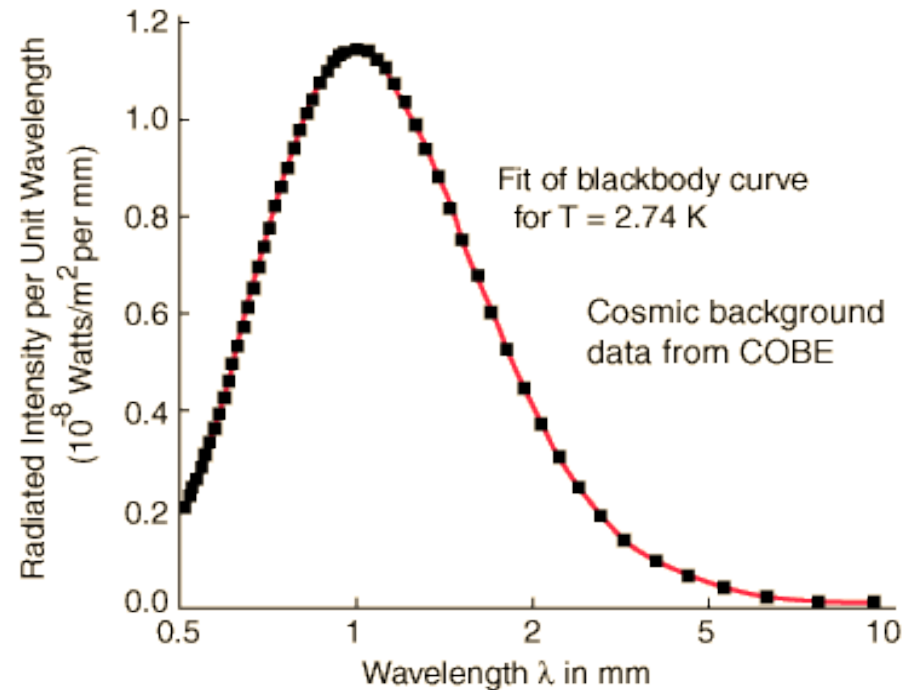
Photo: P. Izzo  
John C. Mather  
Prize share: 1/2



Photo: J. Bauer  
George F. Smoot  
Prize share: 1/2

The Nobel Prize in Physics 2006 was awarded jointly to John C. Mather and George F. Smoot "for their discovery of the blackbody form and anisotropy of the cosmic microwave background radiation."

Most precise blackbody spectrum known!

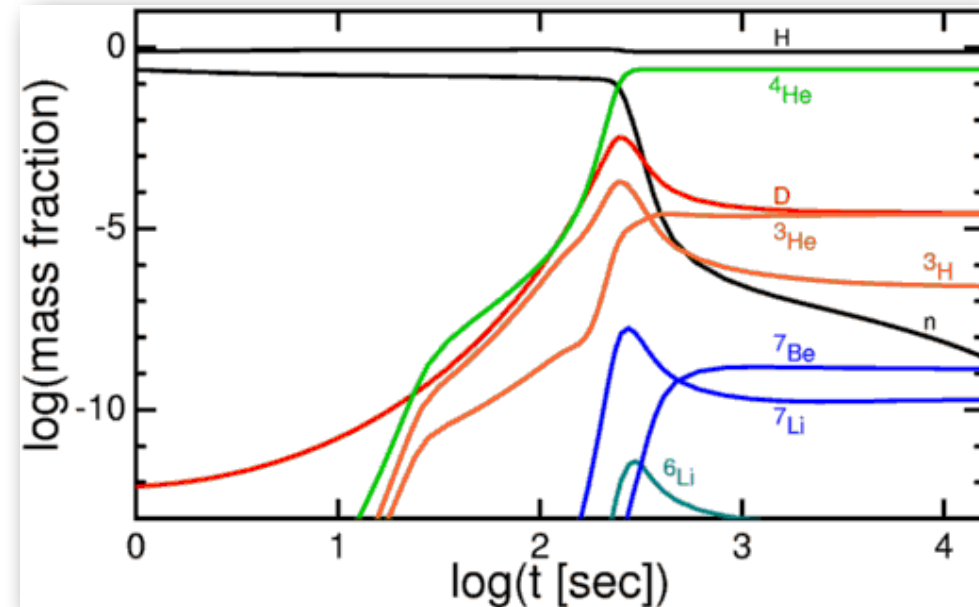




# Third pillar: Primordial (or big bang) nucleosynthesis

Yields of light nuclei (notably Deuterium & Helium)  $\gg$  stellar thermonuclear reprocessing matches expectations of the extrapolation of the pattern way backwards, the early universe was a hot enough place to host thermonuclear reactions!

Predictions can be obtained if evolving from initial hot universe with all nuclear species at equilibrium to a cooler phase due to the expansion of the universe



Alpher, Bethe, Gamow, "The origin of Chemical Elements", Phys. Rev. 73, (7), 803 (1948)

# The rise of the hot Big Bang

Evolving the expanding universe backwards in time → picture of hot Early Universe, “gas” which has been cooling while expanding. The CMB and light elements are the “atomic plasma” and “nuclear plasma” ashes of the early time

Basic (not unique!) task of cosmology: to understand what the universe is made of, now & in the past (the mixture does evolve with time!)

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Fred Hoyle (1915-2001) on BBC, 1949

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Natural units :  $c = \hbar = k_B = 1$

Will tend use them, but for quoting some astrophysical results

**If you're unfamiliar with them, exercise!**

- Compute your typical body temperature (assuming you are still alive) in eV.
- Check the working frequency of your mobile phone. Rephrase it into eV.
- Compute your height in  $\text{eV}^{-1}$
- Compute your age in  $\text{eV}^{-1}$
- Compute your density (estimate within  $\sim 10\%$  error) in  $\text{eV}^4$

## Lecture key objectives.

### You should be able:

To explain “classical” evidences for DM

To illustrate the modern evidences and their importance

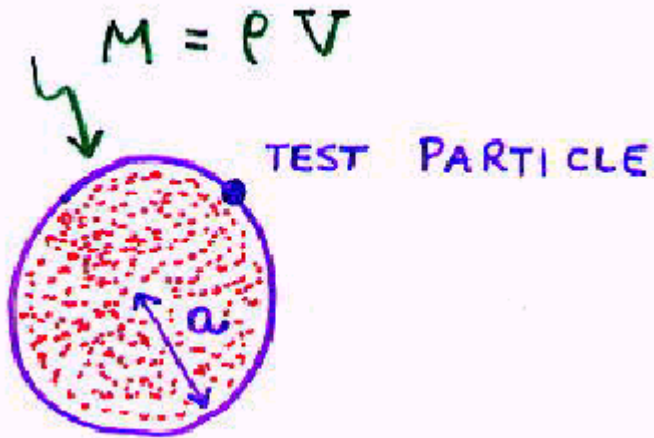
To explain three pillars of the hot big bang model

To derive smooth cosmology solutions in Newtonian toy model

To qualitatively explain what we learn from the pattern and the size of CMB anisotropies

To explain the motivation for the WIMP class of DM candidates, the key hypotheses underlying it, and the main search strategies.

# A trick: “Newtonian cosmology” I



Radially expanding sphere of pressureless dust.

$a$  = radius of the sphere (*scale factor*).

The **total energy** (conserved) writes

$$\frac{1}{2}m\dot{a}^2 - \frac{GMm}{a} = T + U = \text{const.} \equiv -\frac{1}{2}mk$$

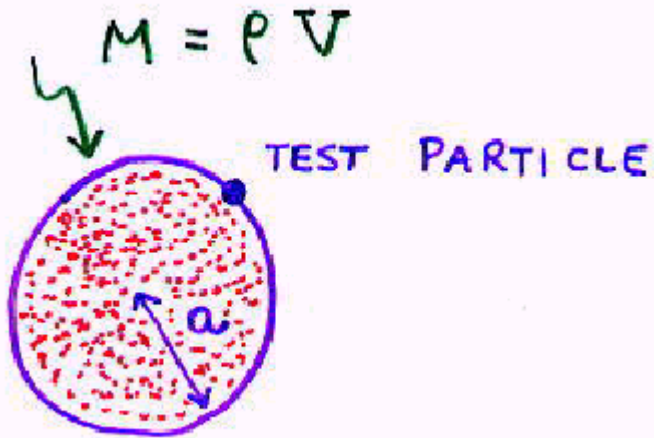
$$k > 0, k = 0, k < 0$$

$\leftrightarrow$

recollapsing, asymptotically still, forever expanding cloud



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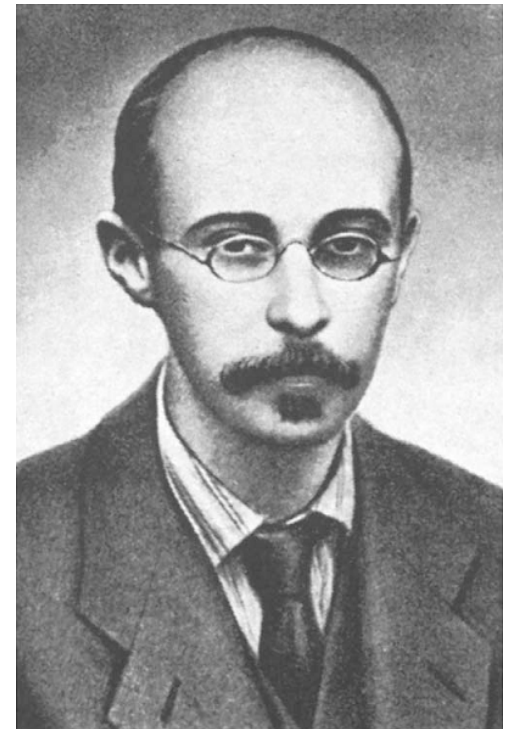
replace

$$M = \frac{4\pi}{3}\rho a^3$$

(First) Friedmann equation (1922)

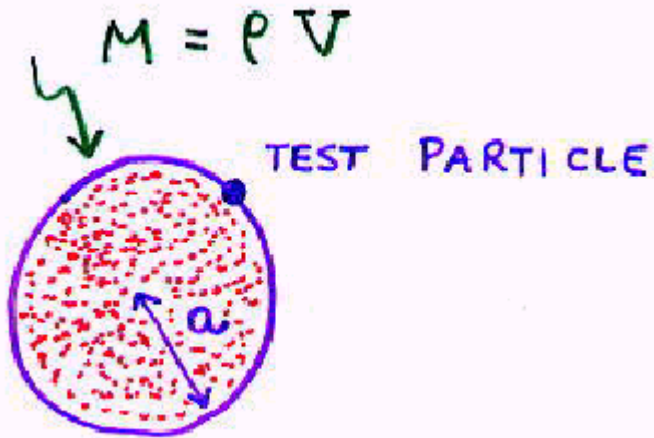
$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3}\rho - \frac{k}{a^2}$$

Equivalent to one of the 2 independent  
 GR equations in the FLRW metric



Alexander Friedmann (1888-1925)

# A trick: “Newtonian cosmology” II



Apply the first law of thermodynamics for perfect gas in the expanding volume  
(adiabatic expansion)

$$d(\rho a^3) = -P d(a^3) \Leftrightarrow dU = -P dV$$

This yields

$$\frac{d\rho}{dt} = -3 \frac{\dot{a}}{a} (\rho + P)$$

Equivalent to the **second independent** GR **equation** in the FLRW metric

With 1st eq., closed system for  $\rho, P, a$  if an **equation of state**  $P=P(\rho)$  is provided

# Solution for linear equations of state (EOS)

For an equation of state of type:  $P=w \rho$

$$\frac{d\rho}{dt} + 3\frac{1}{a} \frac{da}{dt} \rho(1+w) = 0$$

One obtains

$$\frac{\rho(t)}{\rho_0} = \left[ \frac{a(t)}{a_0} \right]^{-3(1+w)} = (1+z)^{3(1+w)}$$

plugging into the first Friedmann equation, can solve to obtain  $a(t)$ , hence  $\rho(t)$ ,  $P(t)$

# Some generic solutions (for $k=0$ )

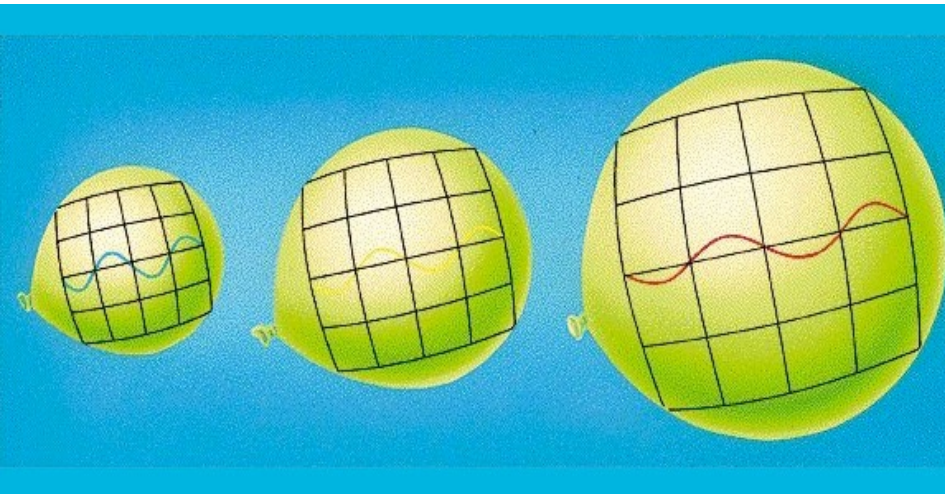
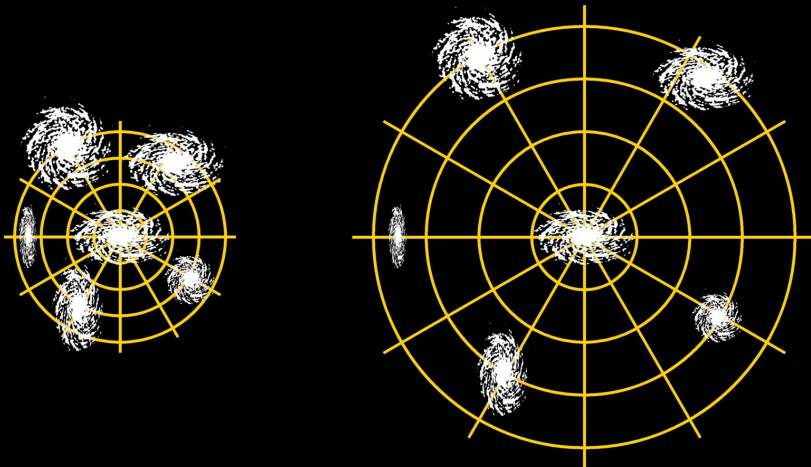
	Equation of State	Behaviour of $\rho$	Scale Factor
Matter	$P \simeq 0$ ( $T \ll m$ )	$\rho \propto a^{-3}$	$a \propto t^{2/3}$
Radiation	$P = \rho/3$	$\rho \propto a^{-4}$	$a \propto t^{1/2}$
Cosm. constant	$P = -\rho$	$\rho = \text{const.}$	$a \propto e^{H_0 t}$

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conservation of particles per comoving volume  
 For radiation, further a-factor due to wavelength stretching

$$1 + z = \frac{\lambda_{\text{today}}}{\lambda_{\text{then}}} = \frac{a_{\text{today}}}{a_{\text{then}}}$$





# CMB photon temperature as “clock”

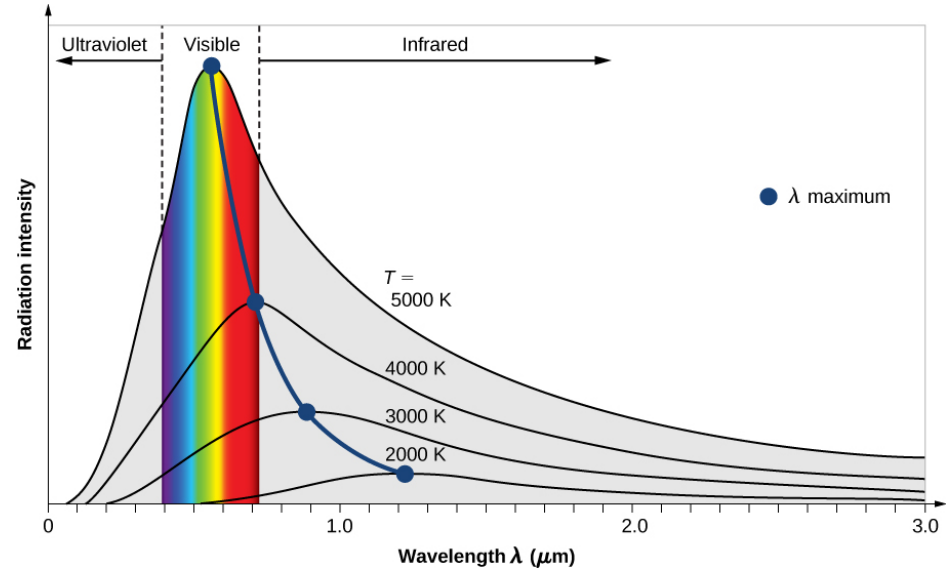
Stefan-Boltzmann law:  $\rho \propto T^4$

$$\langle E \rangle \propto E_{\text{peak}} \propto \nu_{\text{peak}} \propto \frac{1}{\lambda_{\text{peak}}} \propto T$$

(Wien's law)

also implies  $n = \frac{\rho}{\langle E \rangle} \propto T^3$

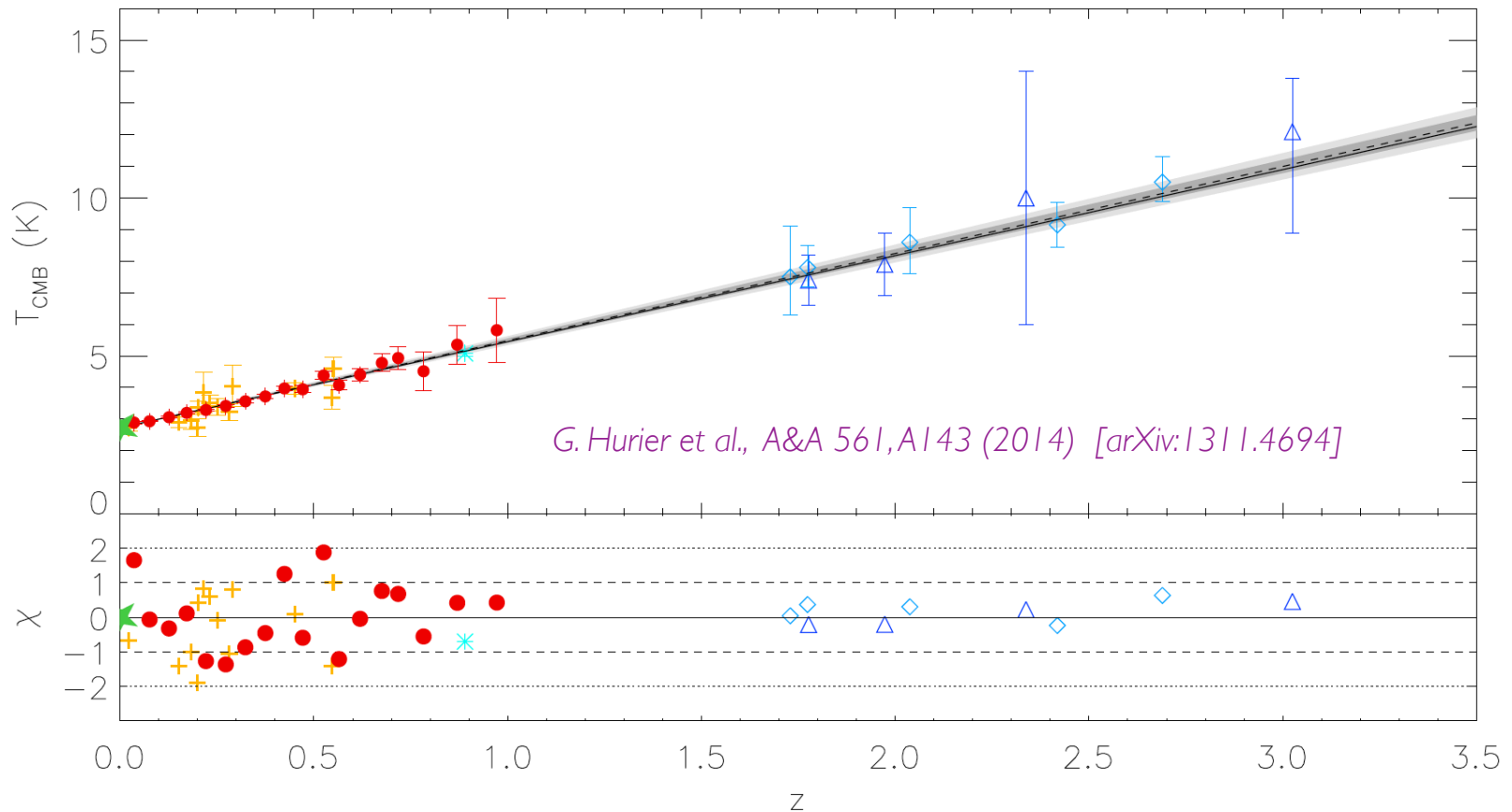
Since we also have  $\rho \propto a^{-4} \rightarrow T \propto a^{-1}$



**we can use CMB photon temperature as clock variable for the epoch of the universe, or their number density to rescale a volume expansion**

*(It turns out that these relations are also valid after the photons stop interacting, not trivial why!)*

# CMB Temperature evolution



Theory :  $T$  rises as  $T(z) = T_0(1+z)$

Observations  $T(z) = T_0(1+z)^{0.994 \pm 0.013}$

Measured via

- “Warming” of the CMB around galaxy clusters (remember, contain hot plasma);
- absorption in gas clouds (excited to fundamental state ratio depends on local  $T_{\text{CMB}}$ )

# Gaining familiarity with cosmo jargon & quantities...

Compositions usually expressed in  $\Omega_i$ 's, ratios of density of i-species to “critical density”

$$\rho_c = \frac{3}{8\pi G_N} H_0^2$$

**Ex:** compute  $\rho_c$  for  $H_0=70$  km/(s Mpc)

Reduced Hubble constant

$H_0 := h$  100 km/(s Mpc)

$h \sim 0.7$

For a flat case ( $k=0$ ), favoured by current data, we can simply write:

$$\frac{1}{H_0^2} \left( \frac{\dot{a}}{a} \right)^2 = \Omega_{m,0} \left( \frac{a_0}{a} \right)^3 + \Omega_{r,0} \left( \frac{a_0}{a} \right)^4 + \Omega_\Lambda$$

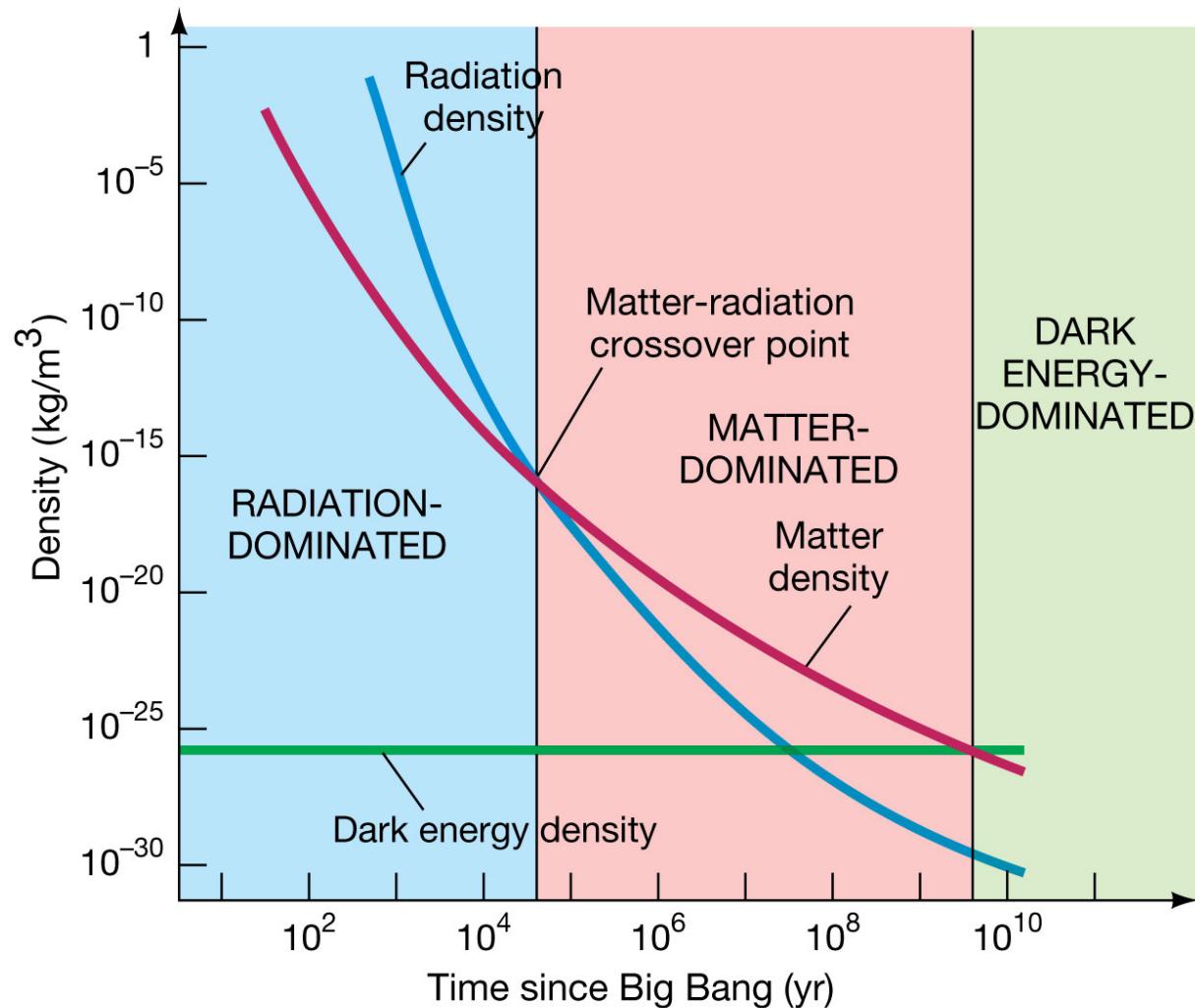
**Ex:** Knowing that today  $\Omega_m \sim 0.28$   $\Omega_\Lambda \sim 0.72$ , at which redshift  $z$  the matter and Cosmological constant contribution were equal?

$$1 + z = \frac{\lambda_{\text{today}}}{\lambda_{\text{then}}} = \frac{a_{\text{today}}}{a_{\text{then}}}$$

**Ex.:** Infer current value of  $\Omega_r$  from  $T_{\text{CMB}} \sim 2.73$  K. At which  $z$  there is matter-radiation equality?

**Ex.:** Plot the RHS of the above equation, expressed vs.  $1+z$ , in log-log scale. Also, plot the ratio of each term to the total RHS

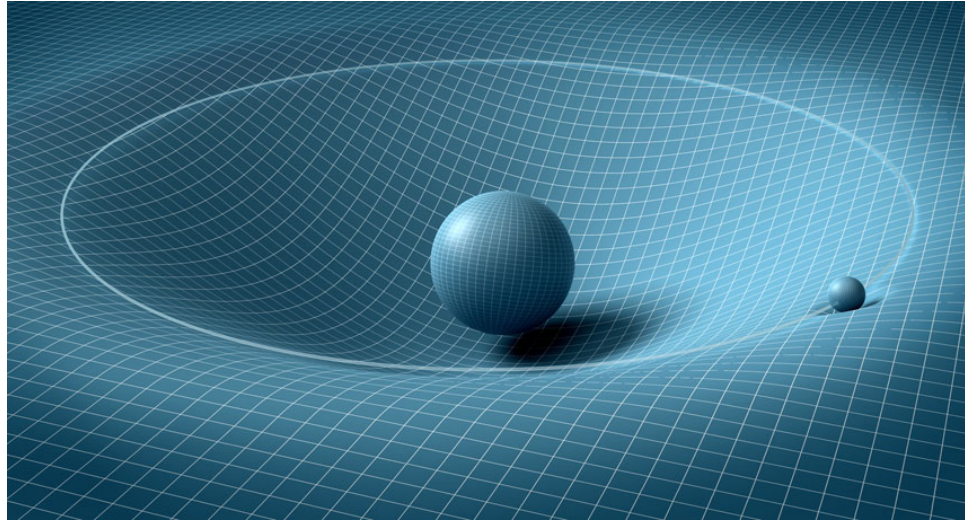
# Outcome



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time-evolution of density (and species dominance) in the universe  
(You have all the tools to quantitatively derive such a plot)

# Actual ingredient nr. 1: General relativity



- GR is a metric & relativistic theory of gravitation, i.e. the key function is the metric:

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu$$

- Metric determined by the matter-energy content via *Einstein equations* (+ - - -)

$$G_{\mu\nu}[g_{\mu\nu}] - \Lambda g_{\mu\nu} = \frac{8\pi G_N}{c^4} T_{\mu\nu}$$

*10 second order,  
non-linearly coupled PDE*

- The matter-energy in turn moves according to metric 'gradients'

$$\Gamma_{\mu\nu}^\lambda = \frac{1}{2} g^{\lambda\rho} (\partial_\mu g_{\rho\nu} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu}) \quad (\text{Christoffel symbols})$$

➡ See Alba Romero-Rodriguez's lectures



# Actual ingredient nr. 2: Cosmological Principle

- Dynamical description of the universe can only be attempted in a 'statistical' sense.
- Key postulate: **Cosmological Principle**  
**At sufficiently large scales, the universe is isotropic and homogeneous**  
= generalisation of the **Copernican (?) Principle**: we occupy no special place in the Cosmos

In GR, mathematical consequence due to symmetries! A set of coordinates exists such that one can write ([Friedmann-Lemaître-Robertson-Walker metric](#), 1922-24, 1927-30, 1935)

$$g_{\mu\nu}(x)dx^\mu dx^\nu = dt^2 - a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 d\vartheta^2 + r^2 \sin^2 \vartheta d\varphi^2 \right] .$$

- **a(t)**: scale factor of the universe (t-dep. *the 4D curvature is non-vanishing*)
- **k** accounts for 3 possible constant *spatial curvatures* (i.e. of 3D slices t=const.); rescale of the 'ruler' can always be used to reduce to k=+1 or -1 or 0
- Also implies that the "matter-energy" content of the universe can be described simply by a density and pressure

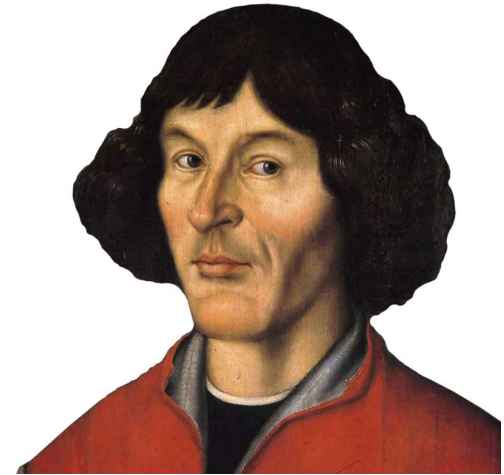
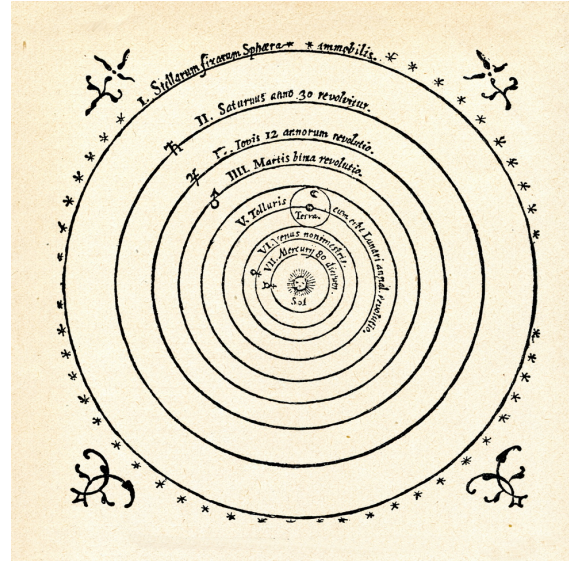
# History note on “Copernican” Principle

*we occupy no special place in the Universe*

Not true in Copernicus!

Sun is central,  
Solar system very different  
from “fixed stars”, etc.

Instead, fair representation of the thinking  
of **Giordano Bruno** (1548-1600)



Nicolaus Copernicus (1473–1543)

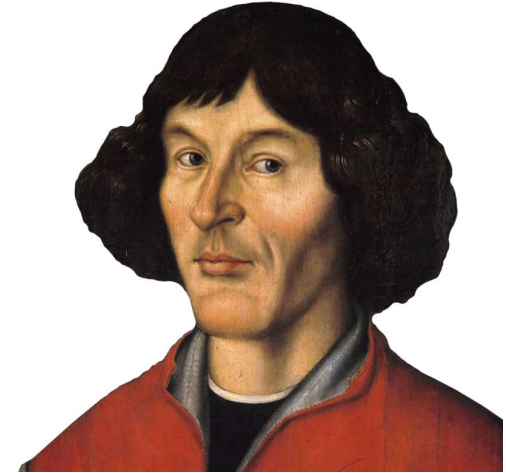
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“There are countless suns and countless Earths all rotating around their suns in exactly the same way as the seven planets of our system. We see only the suns because they are the largest bodies and are luminous, but their planets remain invisible to us because they are smaller and non-luminous”  
*(On the Infinite, Universe and Worlds, 1584)*

“The moon is no longer sky to us, than we are to the moon.”  
*(The Ash Wednesday Supper, 1584)*

# Anecdote on Bruno's character

Must be said that he was not a master of the politically correct:

“The fools of the world have been those who have established religions, ceremonies, laws, faith, rule of life. The greatest asses of the world are those who, lacking all understanding and instruction, and void of all civil life and custom, rot in perpetual pedantry.”

*“Cabal of the Cheval Pegasus with Appendix on the Cillenican Ass, Described by the Nolan” (1585)*



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Inspirational figure over the centuries, till now



Alexander Polzin  
Potsdamer Platz, Berlin



Campo de' Fiori, Rome



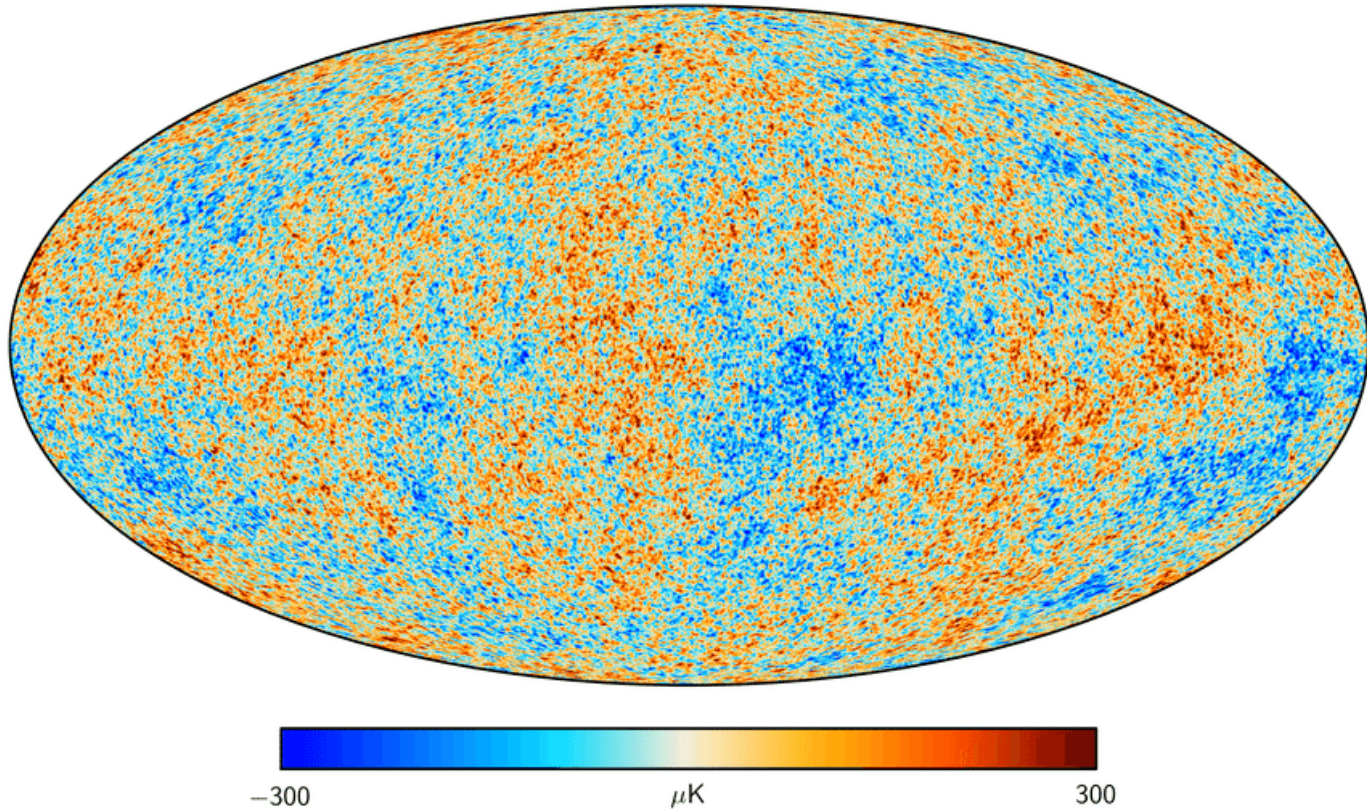


# Beyond smoothness

*(as you can imagine, in GR it is rather technical...)*

# The CMB is *not exactly* isotropic!

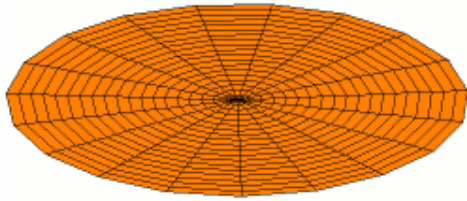
$T$ -differences with respect to the average



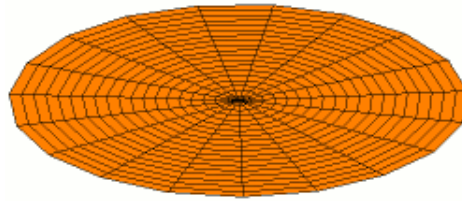
What are these perturbations?

What can we learn from them?

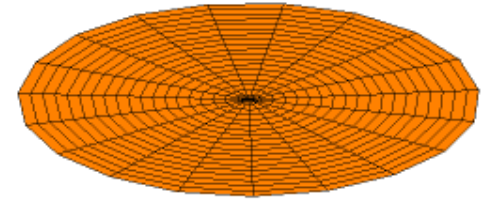
# Think of the oscillating modes of a membrane



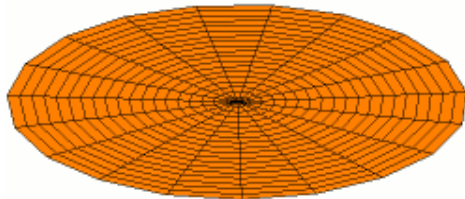
1s



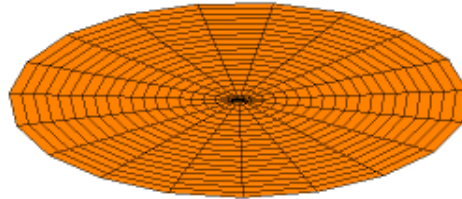
2s



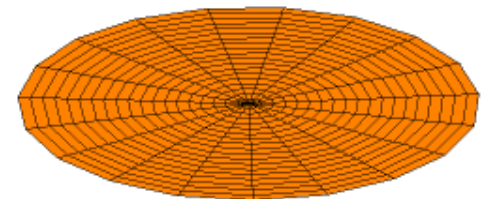
3s



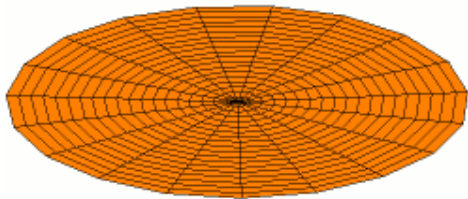
2p



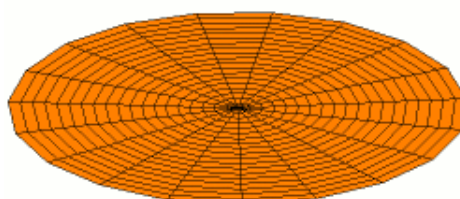
3p



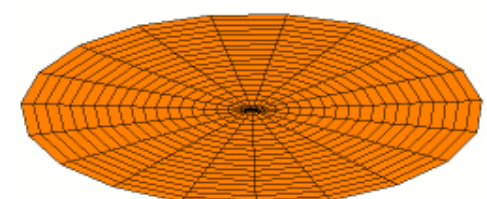
4p



3d

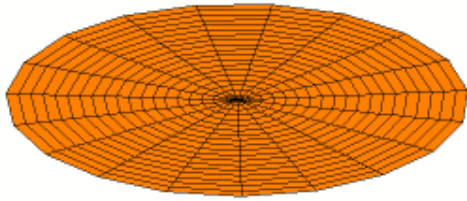


4d

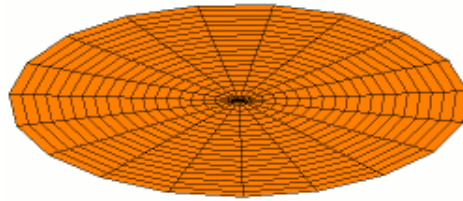


5d

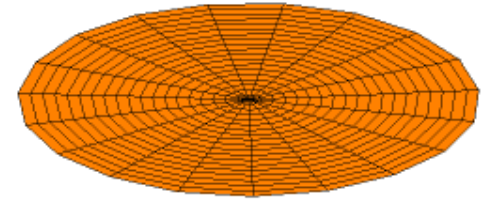
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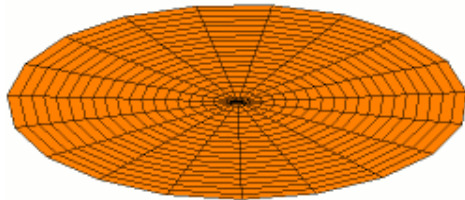
1s



2s

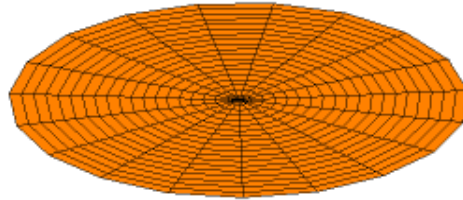


3s

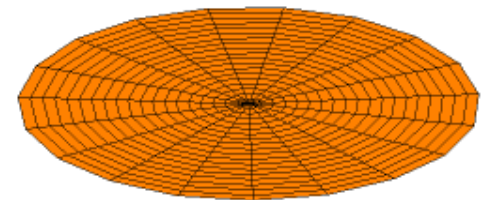


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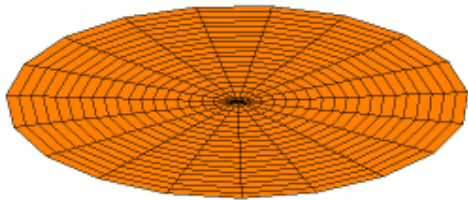
2p



3p

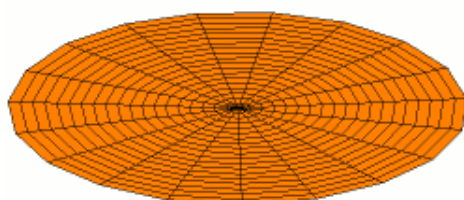


4p

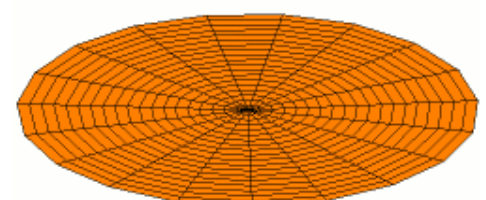


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3d



4d



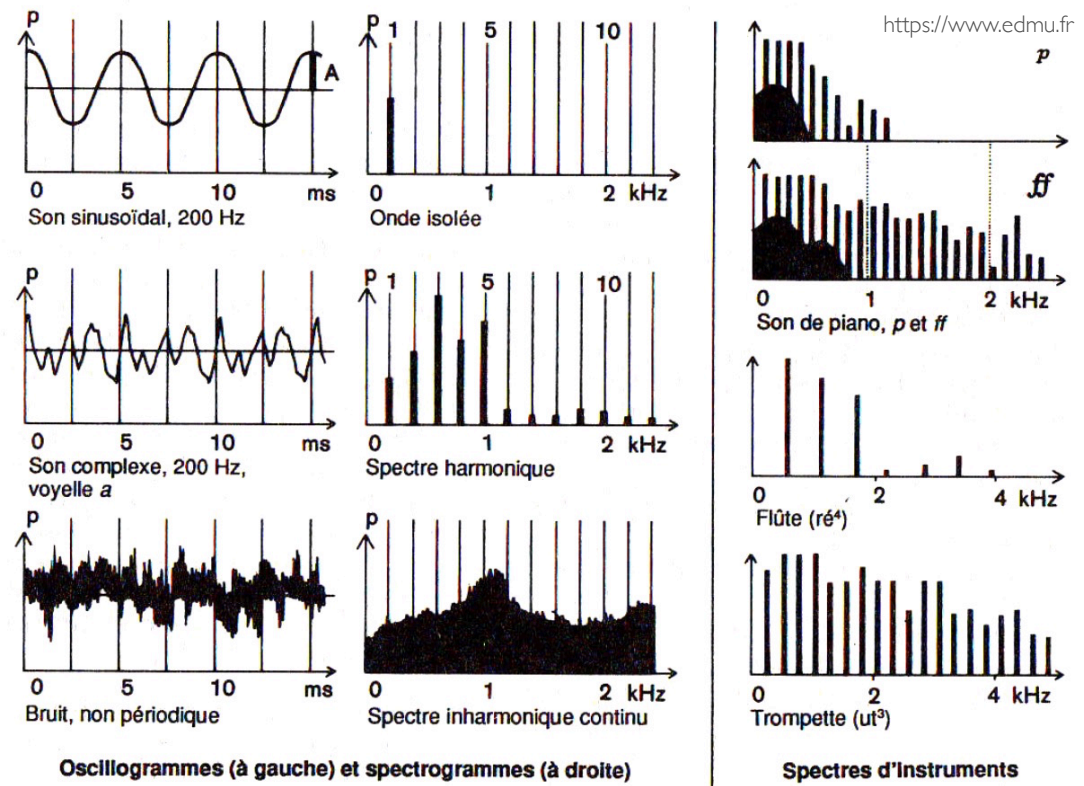
5d



# Acoustic analogy

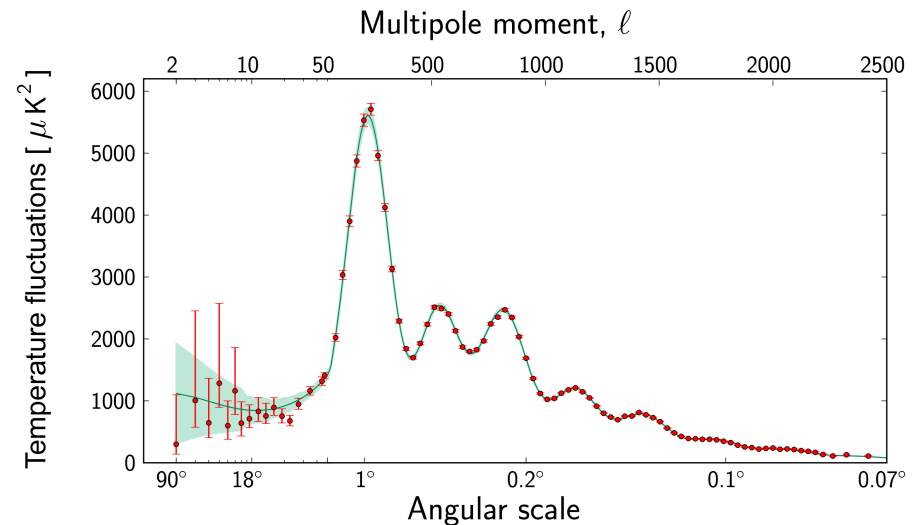
## Music

Can decompose pressure waves  
(sound) in frequencies



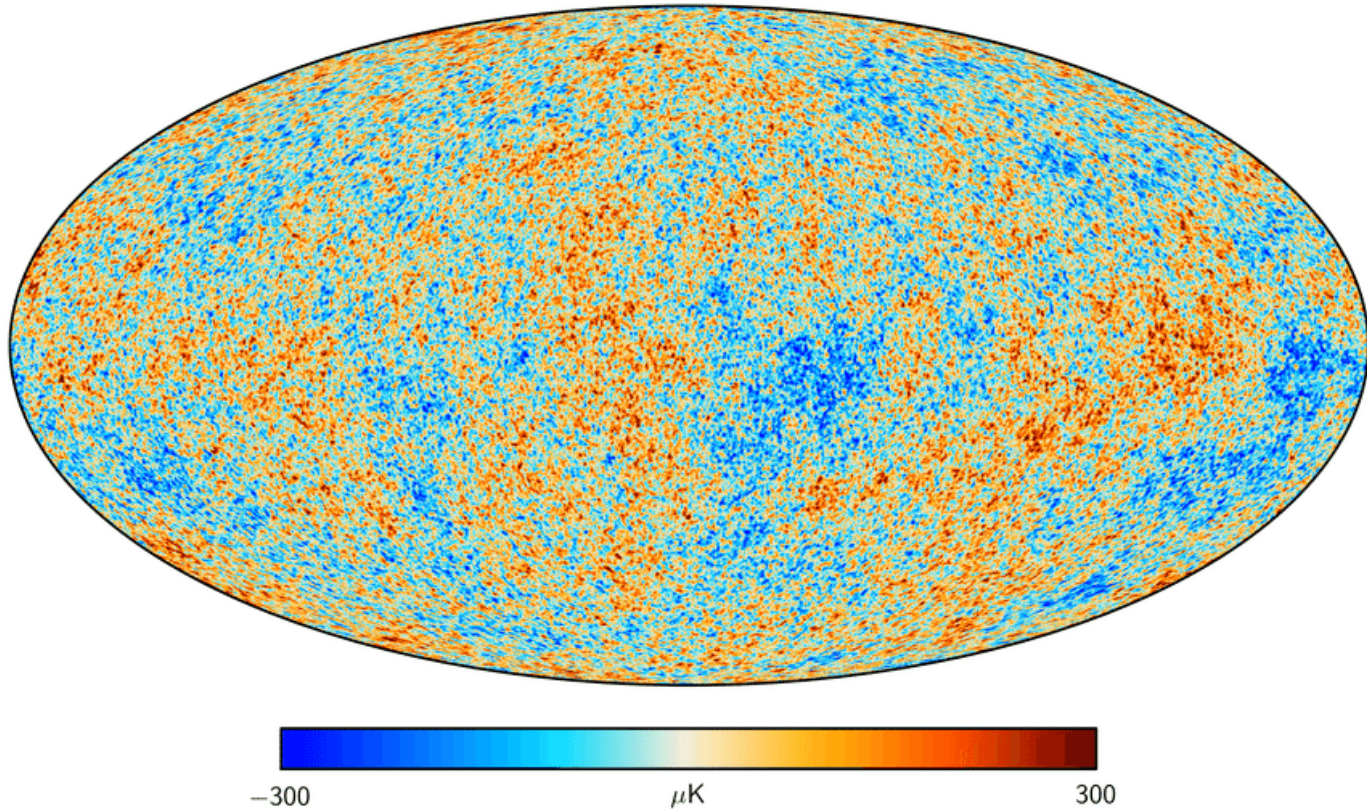
## Cosmology

Can decompose the T-difference fluctuations  
in angular wavelengths, deducing the excited  
frequencies & their intensities



# The CMB is *not exactly* isotropic!

$T$ -differences with respect to the average



**It's “sound”!**

Can be studied to infer

- i) The initial “sheet music”  $\rightarrow$  inflation (not covered)
- ii) The physical conditions & geometry of the medium

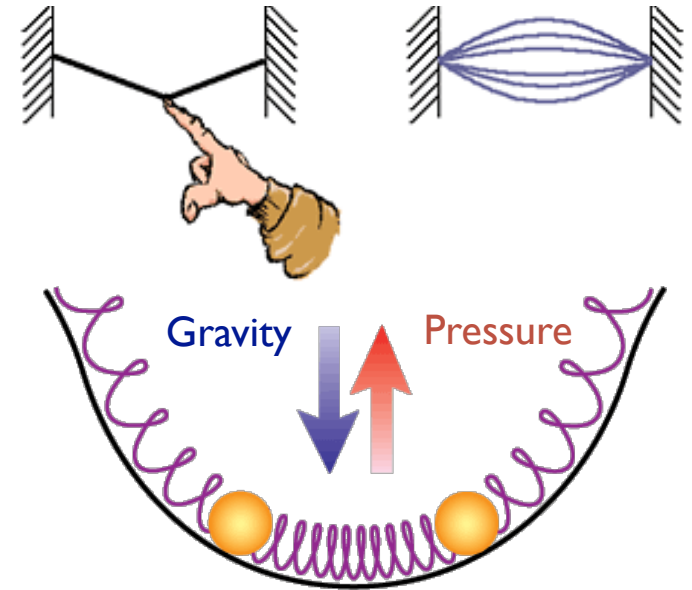


# Peaks are ‘the sound of the universe’

**Primary scale** = distance a wave can travel till release  
(~when electrons and protons combine into neutral atoms);  
**smaller scales: ‘harmonics’.**

Interplay between **plugging force** and **restoring tension**  
=

Interplay between **gravity** (caused by...everything!) and **gas pressure** (ordinary matter)



Just like for a membrane the characteristic frequencies write

$$\nu_n^2 \sim c_n^2 \text{ tension/density}$$

From angular analysis one can deduce:

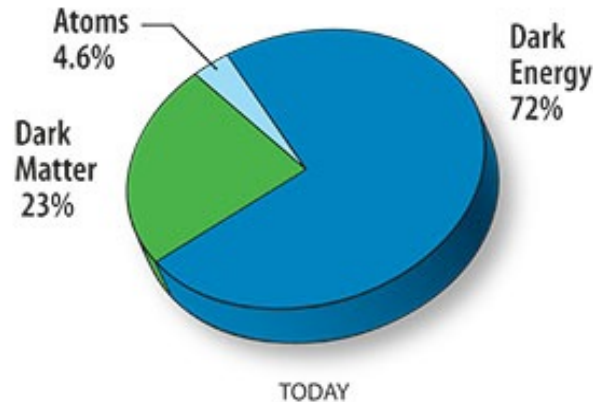
- Overall density (responsible for the gravitational force)
- Fraction of ‘ordinary’ gas that ‘resists to pull’ (via electric force involving e- and p+)
- Geometry (spatial curvature)



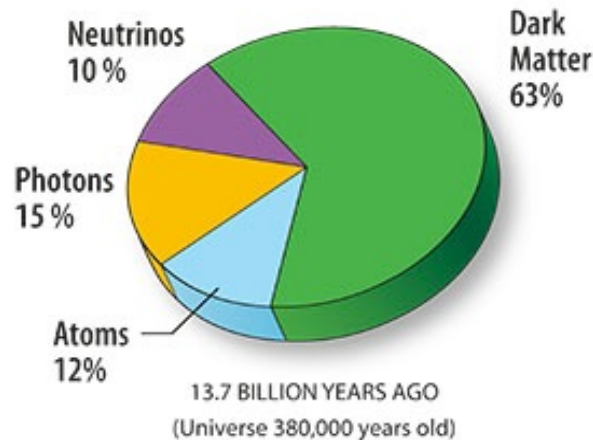
# An unfamiliar universe : $\Lambda$ CDM !

Both at CMB formation time and now, the microscopic nature of the dominant components of the energy budget of the universe are unknown → challenge for fundamental physics!

Today :  
**Dark energy**



at CMB formation:  
**Dark matter**



## James Peebles Facts



James Peebles  
The Nobel Prize in Physics 2019

Born: 25 April 1935, Winnipeg, Canada

Affiliation at the time of the award: Princeton University, Princeton, NJ, USA

Prize motivation: "for theoretical discoveries in physical cosmology."

Prize share: 1/2

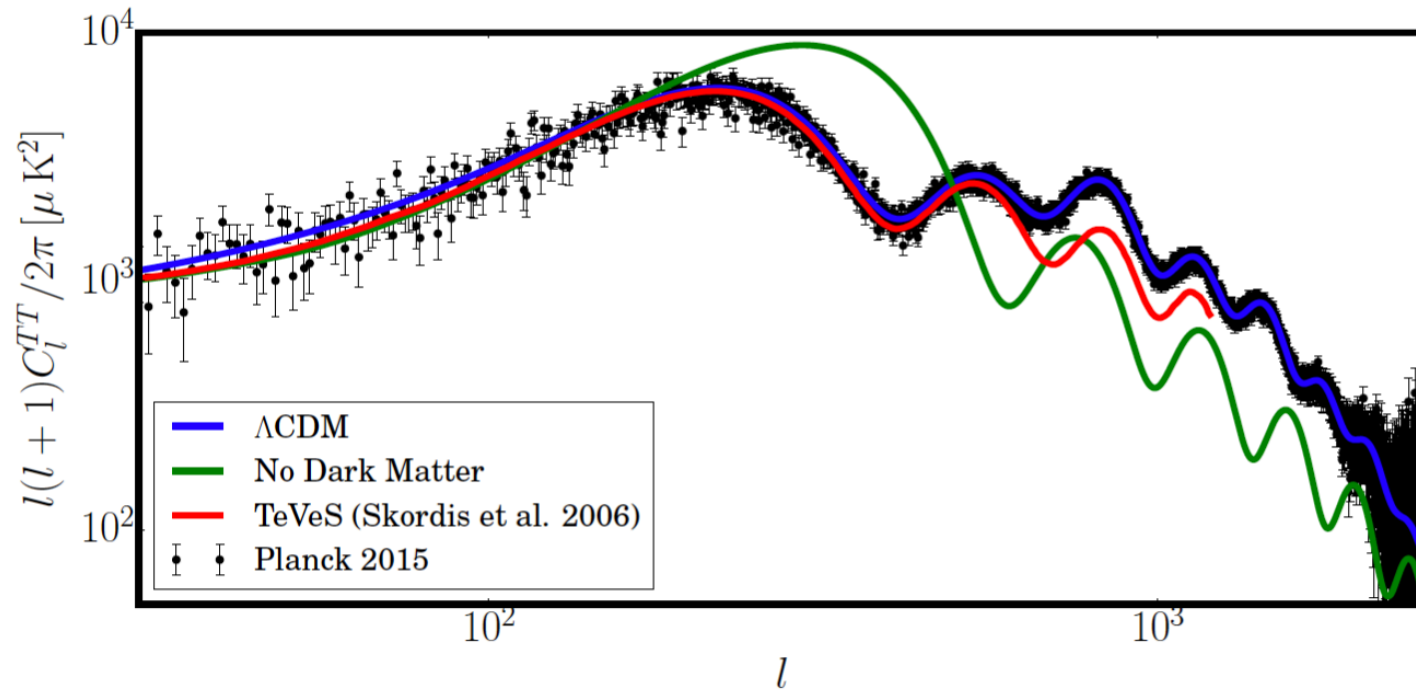
© Nobel Media. Photo: A. Mahmoud

Despite the uneasy feeling, this parametric model has passed a number of quantitative tests!

# CMB acoustic peaks

The detailed study of the CMB anisotropies has allowed us to measure both the fraction of interacting matter & the total amount of matter, hence the DM amount.

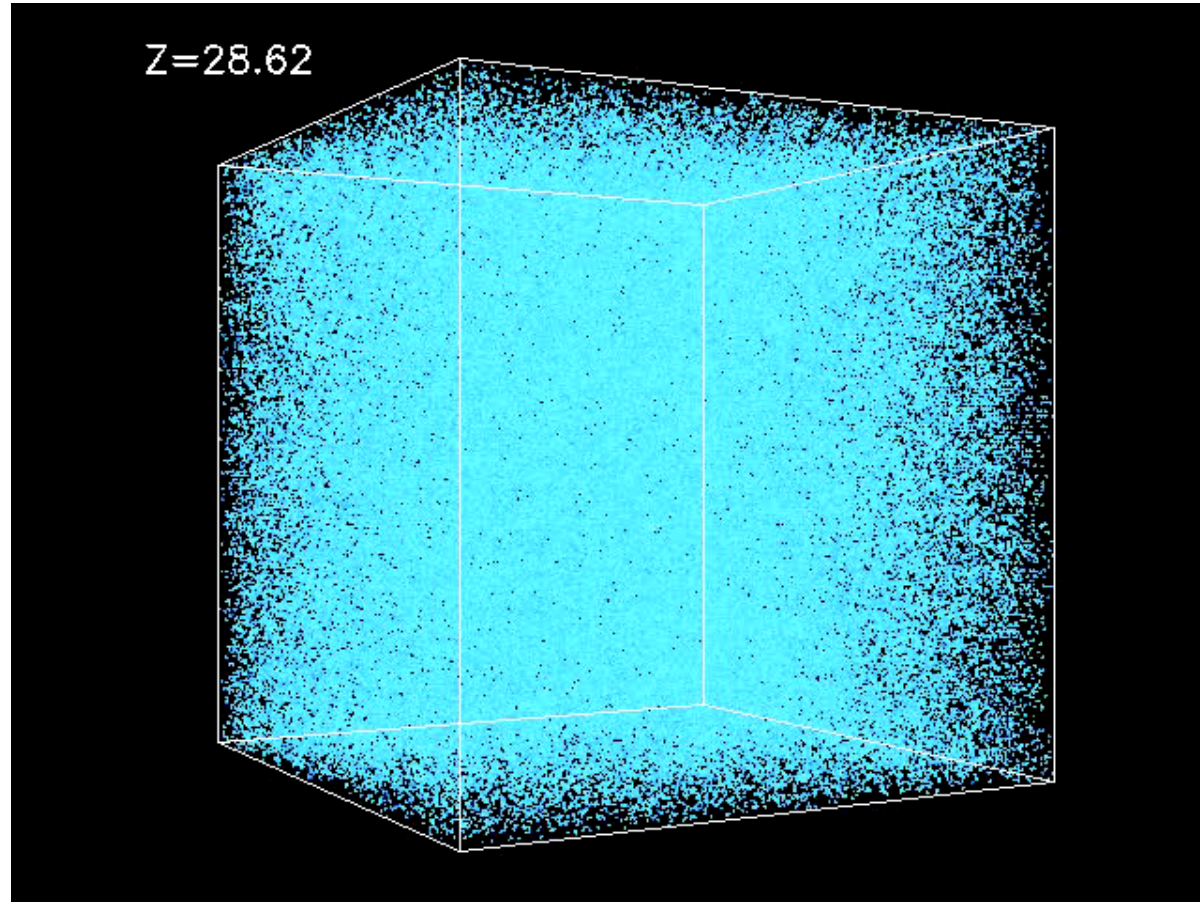
Amount of DM consistent with what inferred from previous considerations supports idea that we're observing the same phenomenon at different scales.



comparison of CMB with/without DM, and with a “Modified Gravity” model  
(with neutrino masses so large to be excluded by Lab measurements, by the way).

# Universe in a computer

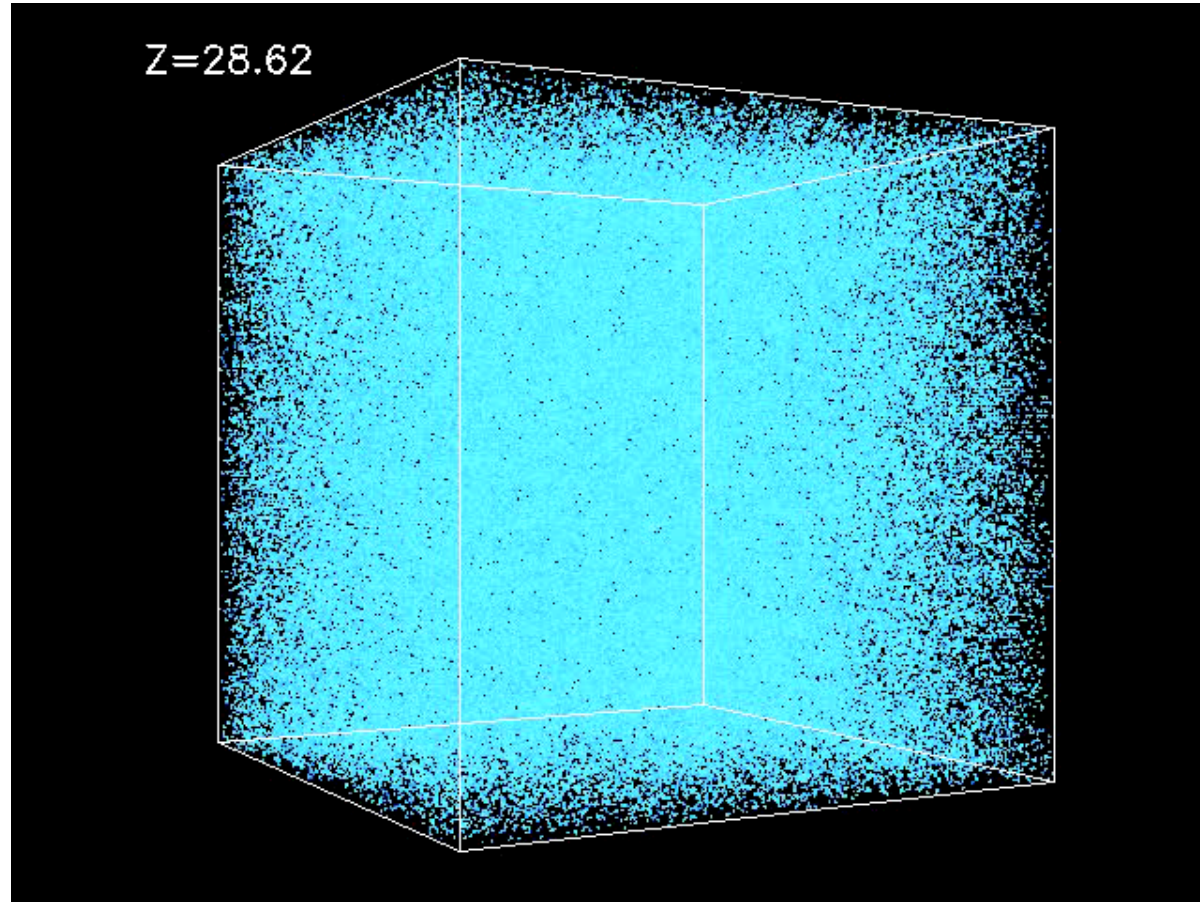
If starting from CMB 'initial conditions' and evolve in time under standard gravity (with DM)



<http://cosmicweb.uchicago.edu>

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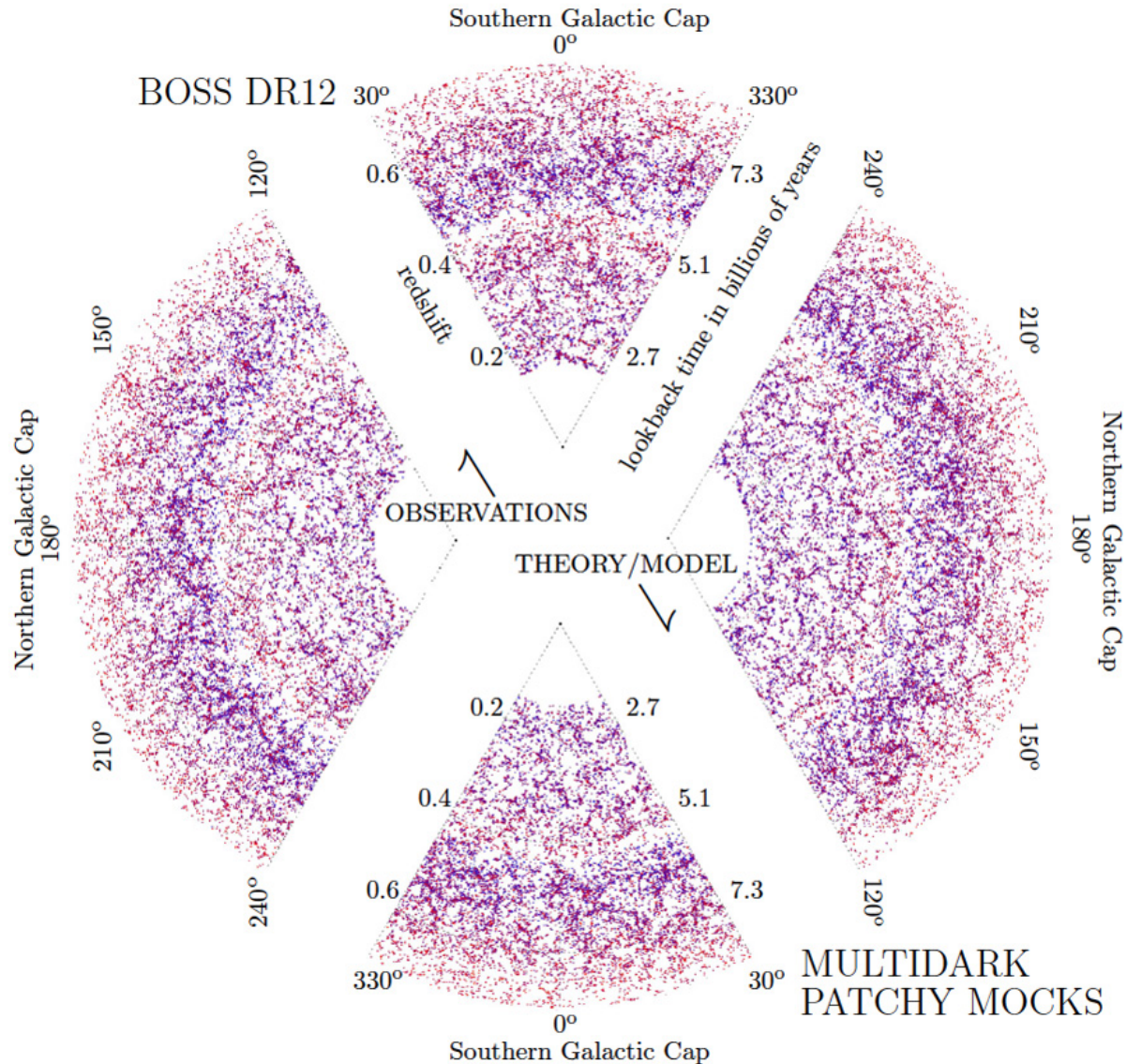


<http://cosmicweb.uchicago.edu>



# ...one finds the large scale structure of galaxies!

Same time evolution and spatial clustering

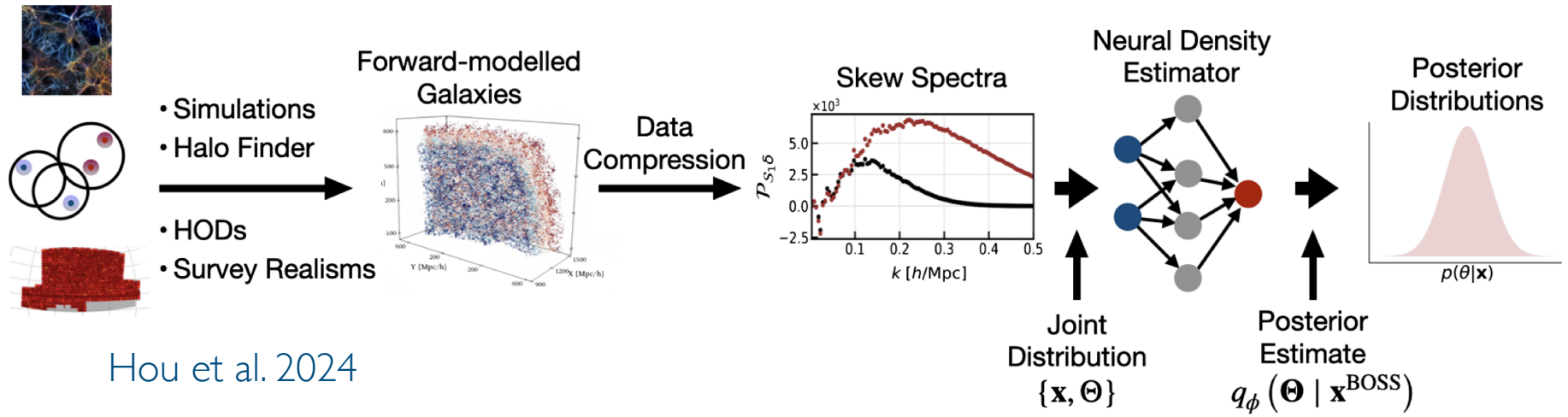




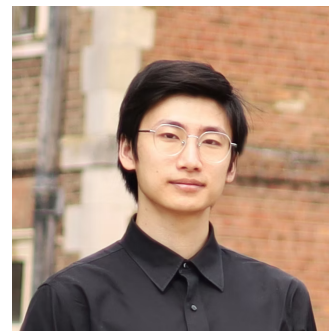
# Promotion

Highly non-trivial to extract cosmological parameters from these data, especially to beat CMB

*Simulations, modelling, advanced statistical and inference techniques, assessing new probes...*



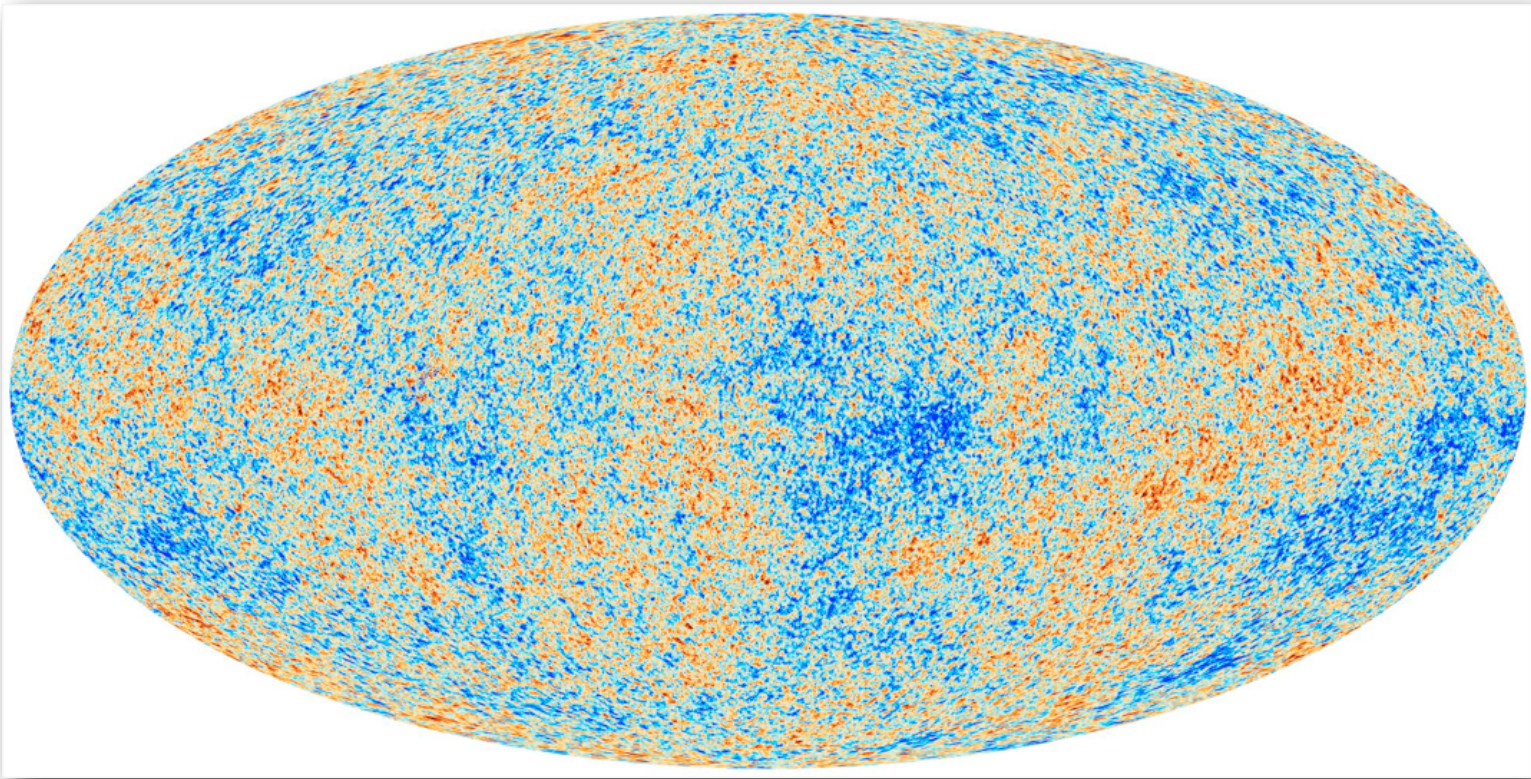
Hou et al. 2024



@ LAPTh Azadeh Moradinezhad (+Marina Cagliari, Zucheng Gao)

# Growth of structures

This picture, plus linear theory is a robust proof of the existence of DM!



## Key argument

- ▶ Before recombination: ordinary gas (plasma) & photons coupled, “share perturbations”
- ▶ We measure amplitude  $\sim \text{few} \times 10^{-5}$  at recombination (picture above)
- ▶ Evolving forward in time, **insufficient to achieve nonlinear structures** as we see nowadays, **unless lots of gravitating matter** (not coupled to photons) creates deep potential wells!

# Take-away optional project

Compute the growth rate of perturbations in linearly perturbed Newtonian cosmology and convince yourself of the previous statement!

# Back to DM & ‘particle cosmology’

## How can DM be produced in the early universe?

Need to specify what DM could be and how it is coupled to the rest.

At the basis of the (unavoidably) model-dependent searches!

Will only sketch one example...

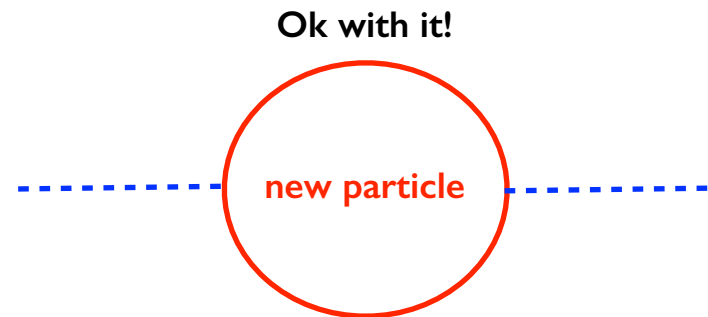
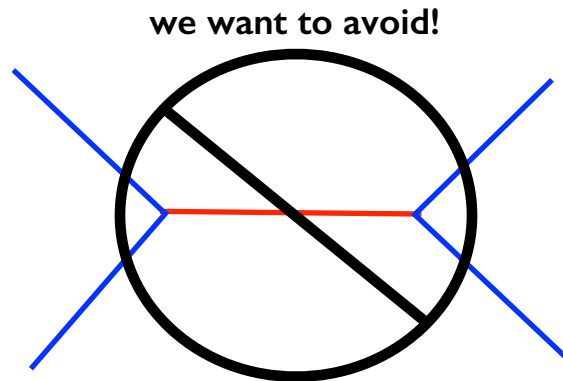
# Traditional link DM-particle physics

Strong prior for TeV-scale BSM (with SM-like couplings) to cure “the hierarchy problem”:

**why is the Higgs mass insensitive to quantum effects from physics at some much higher energy scale  $\Lambda_{UV}$  (e.g. gravity)?**

**Conjecture:** there is some symmetry (e.g. SUSY) @  $E \sim O(\text{TeV})$ , “shielding” low-E pheno from UV.

Precision data suggest that tree-level couplings SM-SM-BSM should be avoided!



**One** straightforward solution is to impose some **symmetry** (often “parity-like”, relic from some UV-sym): SUSY R-parity, K-parity in ED, T-parity in Little Higgs. New particles only appear in pairs!

- ➡ Automatically makes lightest new particle stable!
- ➡ It has other benefits, e.g. respect proton stability bounds!

# The Weakly Interacting Massive Particle paradigm

Cosmology tells us that the early universe was a hot plasma, with all “thermally allowed” species populated. Notion tested up to  $T \sim \mathcal{O}(\text{MeV})$  (BBN)

What if we extrapolate further backwards, adding to the SM just...

$$X \bar{X} \longleftrightarrow \ell \bar{\ell}$$

...a single **stable massive particle** in **equilibrium with SM** via **EW-strength binary interactions** in early universe down to  $T \ll m$ ?

What is left of it depends on the decoupling time, i.e. annihilation cross section: the weaker, the more abundant...

“Most popular” class of DM candidates: theoretical bias, “WIMP miracle”, independence of initial conditions, possibility to probe it...

**But by far not unique!!! Do not equate DM to WIMPs!**



# If particles are not created/destroyed ...

...like in the cold universe today, we recover “dust” evolution for the number density

$$\frac{dn}{dt} + 3\frac{\dot{a}}{a}n(t) = \frac{d}{dt}[n(t) a^3(t)] = 0$$

➤ i.e. when the universe expands (or contracts), the number of particles in a given comoving volume is constant

$$N = n(t) a^3(t) = \textit{const.}$$

# If particles kept at equilibrium by fast interactions

In the **relativistic limit**, behave as photons, for which we saw  $n_{\text{eq}} = \frac{\rho}{\langle E \rangle} \propto T^3$

In the **non-relativistic-limit**, can be obtained by integrating Boltzmann distribution  $e^{-E/T}$

$$n_{\text{eq}} = (2s_i + 1) \int \frac{d^3p}{(2\pi)^3} e^{-E/T} = (2s_i + 1) \left( \frac{mT}{2\pi} \right)^{3/2} e^{-m/T}$$

$$\rho = m n$$

$$P = n T \ll \rho$$

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Actual tool to prove these statements: Boltzmann equations for phase space distribution  $f$  in FLRW metric

$$\hat{L}[f] = \hat{C}[f] \quad \hat{L} \rightarrow p^\mu \frac{\partial}{\partial x^\mu} - p^\alpha p^\beta \Gamma_{\alpha\beta}^\mu \frac{\partial}{\partial p^\mu} \rightarrow E \left( \frac{\partial}{\partial t} - \frac{\dot{a}}{a} p \frac{\partial}{\partial p} \right)$$

$$\hat{C}[f_1] = \frac{1}{2} \int d\Pi_2 d\Pi_3 d\Pi_4 (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_3 - p_4) [|M_{34 \rightarrow 12}|^2 f_4(\vec{p}_4) f_3(\vec{p}_3) - |M_{12 \rightarrow 34}|^2 f_2(\vec{p}_2) f_1(\vec{p}_1)]$$

# Solution at generic time (& “WIMP miracle”)

Can check right limiting behaviours

$$\frac{dn}{dt} + 3H n = -\langle \sigma v \rangle [n^2 - n_{\text{eq}}^2]$$

“Annihilation cross section”

must be  
quadratic,  
for binary  
processes

Useful to factor out the expansion of the universe & put in dimensionless form:  $n \rightarrow n/T^3$  or  $n/n_{\text{eq}}$

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Asymptotic abundance  
(per coming volume)  $\propto \frac{1}{\langle\sigma v\rangle T_F M_P}$

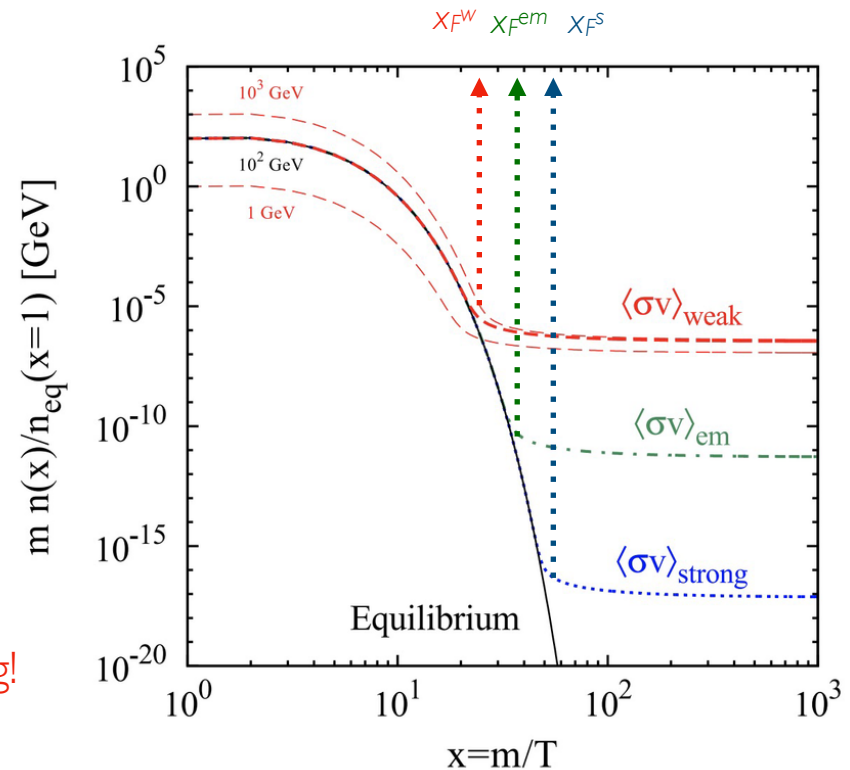
$T_F$  (**Freeze-out temperature**) defined by

$$\Gamma_{\text{eq}} \equiv \langle\sigma v\rangle(T_F) n_{\text{eq}}(T_F) = H(T_F)$$

Plugging numbers  $\Omega_X h^2 \simeq \frac{0.1 \text{ pb}}{\langle\sigma v\rangle}$

matches observations~for e.weak scale mass & coupling!

$$\langle\sigma v\rangle \sim \frac{\alpha^2}{m^2} \simeq 1 \text{ pb} \left( \frac{200 \text{ GeV}}{m} \right)^2$$





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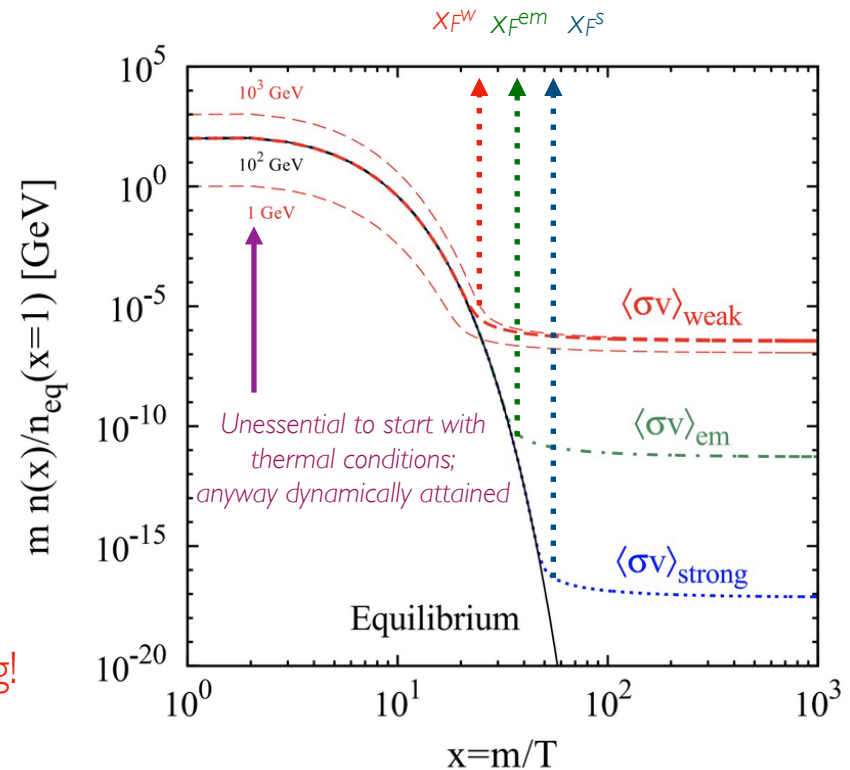
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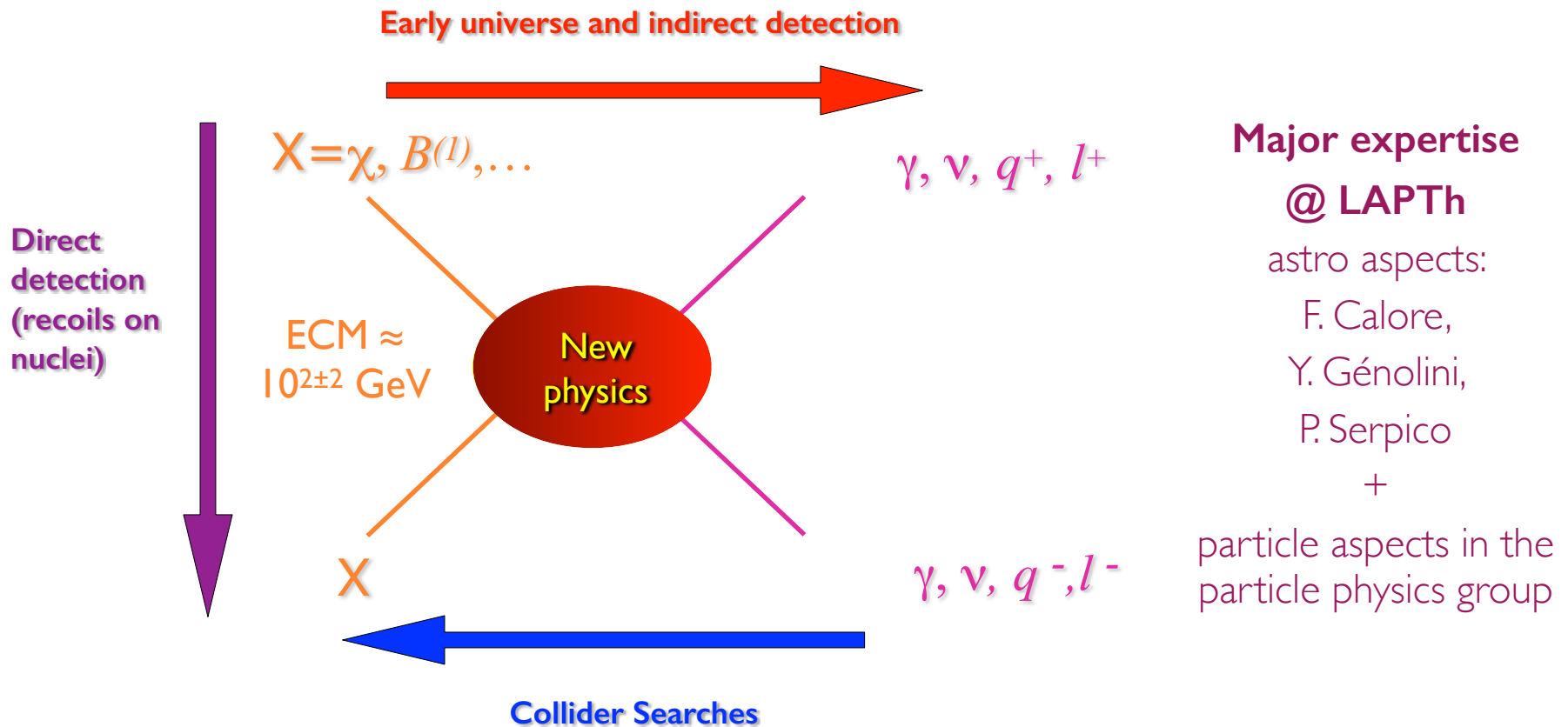
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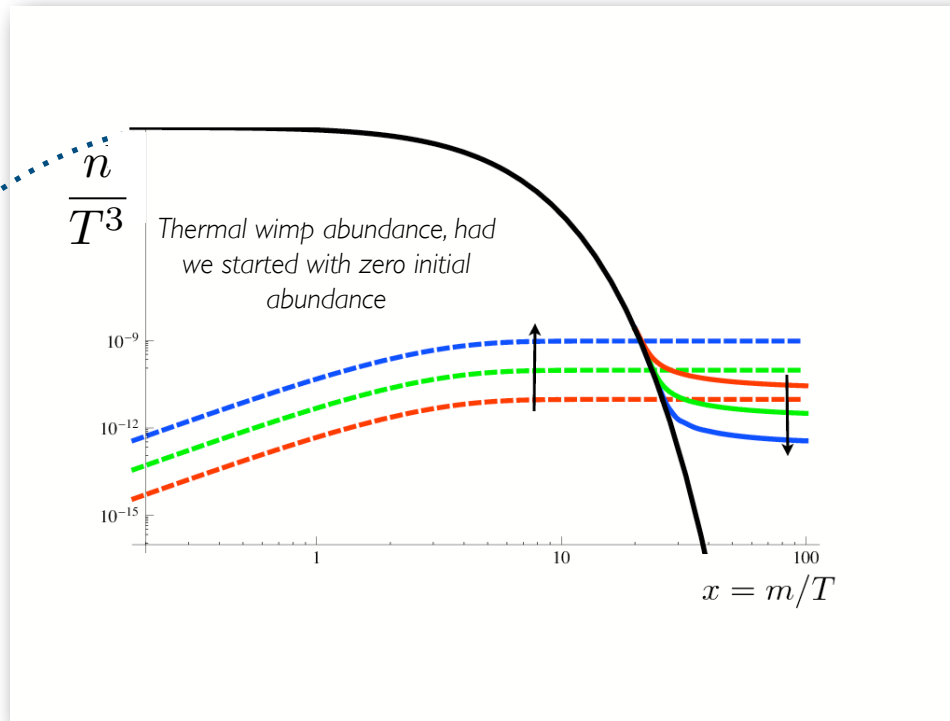


# WIMP (not generic!) DM search program



- ✓ demonstrate the “particle physics” nature of astrophysical DM (locally, via DD; remotely, via ID)
- ✓ Possibly, create DM candidates in the controlled environments of accelerators (but not enough! Neither stability nor relic density “directly tested”, for instance...)
- ✓ Find a consistency between properties of the two classes of particles. Ideally, we would like to calculate abundance and DD/ID signatures → link with cosmology/test of production

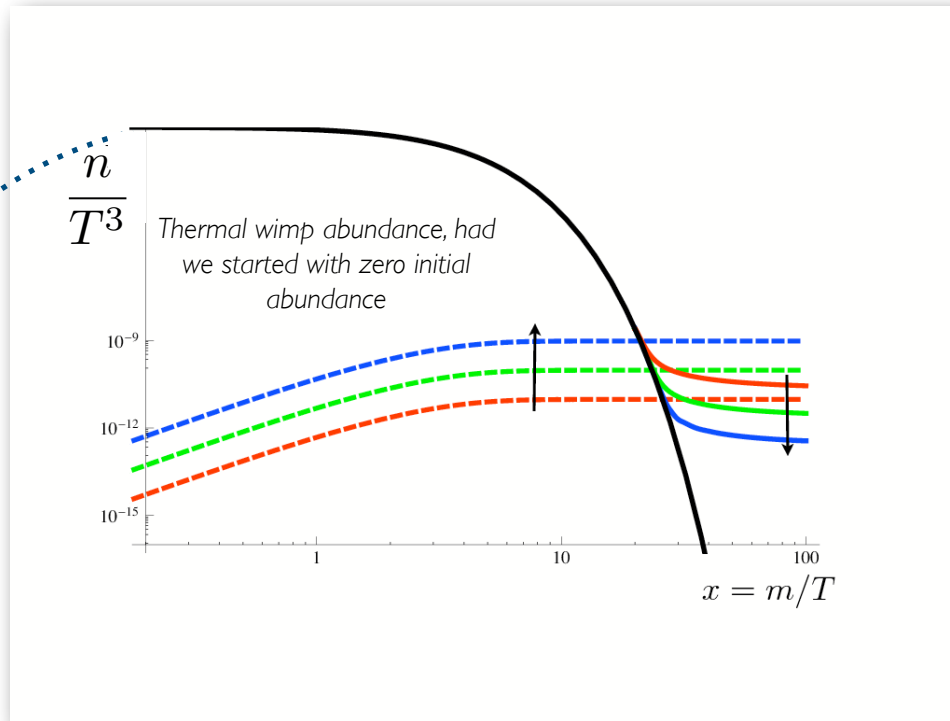
# Beware of model dependence!



That's called *Freeze In*, since it's the "reverse" of freeze out

Even in the same framework, another solution arises if dial the **x-section to such small values that DM never attains equilibrium**, and observations matched via the residual production from the plasma

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**Note:** now one finds asymptotic abundance (per coming volume)

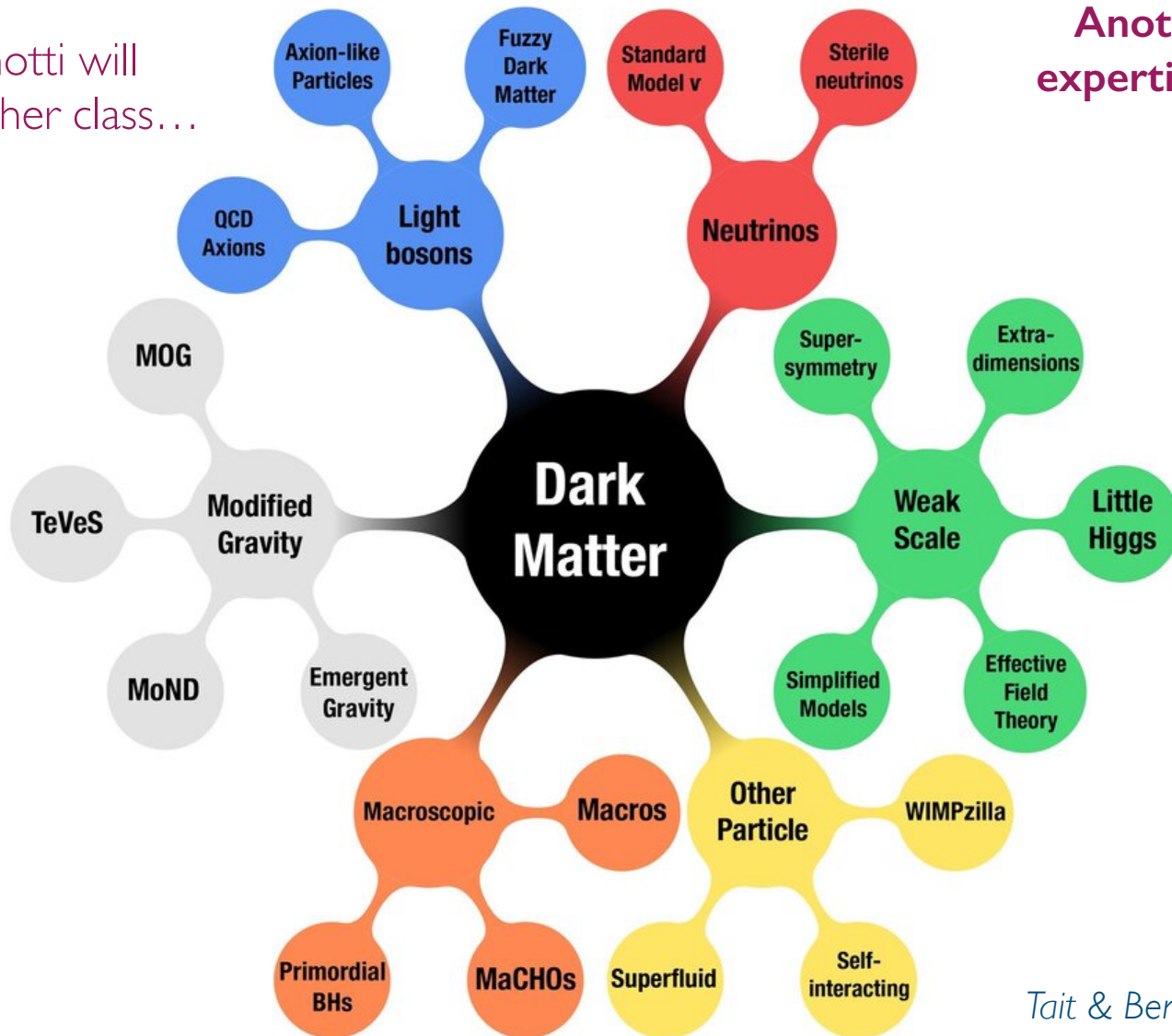
$$\propto \langle \sigma v \rangle$$

**Obvious pheno consequence:**  
much harder to search for it!

# Number of models is huge (some already excluded)

M. Giannotti will  
discuss another class...

Another area of  
expertise @ LAPTh



*Tait & Bertone*



# Summary

- ▶ Many observations collected over the past century show the need for “invisible stuff” contributing dominantly to the dynamics of bound objects from sub-Galactic to cluster scales.
- ▶ Also needed to explain the CMB anisotropy pattern and timely formation of non-linear scales via gravitational instabilities, starting from tiny fluctuations as inferred from CMB temperature perturbations.
- ▶ This DM cannot be made of “hidden ordinary matter” (e.g. dim stars, gas, planets...) because we can measure how much ordinary stuff was around when the universe was smooth (no stars, no planets...): CMB, LSS, clusters, galaxy-inferred DM all agree.
- ▶ A long-standing promising class of microscopic candidates are WIMPs, for which we sketched some motivation, production mechanism & search strategies. No discovery yet, but growing sensitivity... will we find them? Or something completely unexpected? The quest is on!

**Maybe current ideas are inadequate, but the robust feature is  
the need for new physics (likely new d.o.f.)  
This is at the root of the excitement!**

## Lecture key objectives.

### You should be able:

To explain “classical” evidences for DM

To illustrate the modern evidences and their importance

To explain three pillars of the hot big bang model

To derive smooth cosmology solutions in Newtonian toy model

To qualitatively explain what we learn from the pattern and the size of CMB anisotropies

To explain the motivation for the WIMP class of DM candidates, the key hypotheses underlying it, and the main search strategies.

Should be able to avoid incorrect shortcuts as (and find the mistakes in)  
*DM is a new particle they're looking for at LHC whose existence we know thanks to Galaxy rotation curves...*