The Toric Code in a Magnetic Field and at Finite Temperature

M1 Internship Defense

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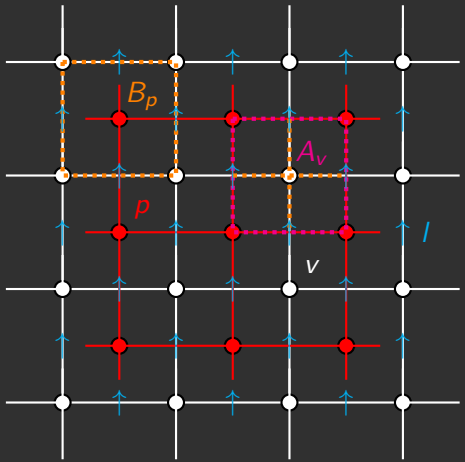
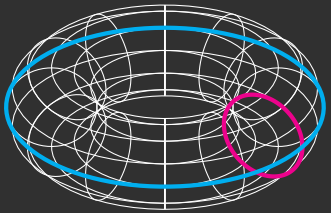
The Toric Code Model

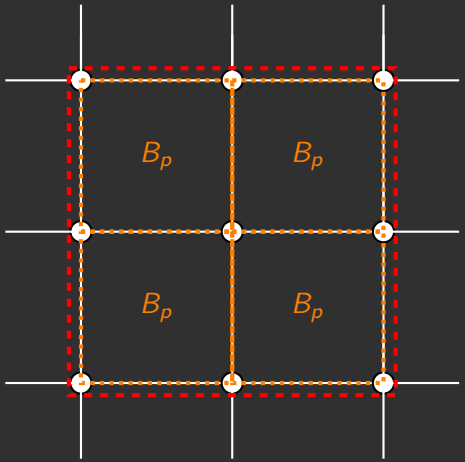


- $\mathcal{H}_{\mathrm{TC}} = -J_v \sum_v A_v - J_p \sum_p B_p$ (Kiteav, 2003)
- $B_p = \prod_{l \in p} \sigma_l^z,\ A_v = \prod_{l \in v} \sigma_l^x,\ [A_v, B_p] = 0$
- Topological constraints on a closed surface: $\prod_p B_p = \prod_v A_v = 1$
- "Logical" eigenbasis: $|\{a_v\}, \{b_p\}\rangle$
- Ground state: $|\{1\}, \{1\}\rangle$ with $E_0 = -J_p N_p - J_v N_v$

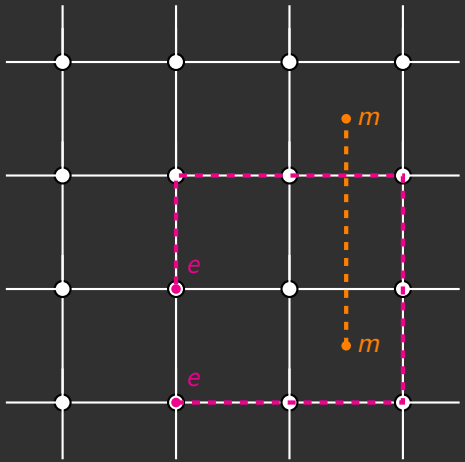
The Hidden Topological Order of the Ground State

- Link basis: $|\{s_l^x\}\rangle \implies \prod_p \left(\frac{1+B_p}{2}\right)|\Rightarrow\rangle$
- Loop Gas: Highly entangled states
- Bigger loops $\equiv$ product of B_p unless "hole" or open
- GSD in the states coming from the topology!





The Excitations as Particles



- Link flip increases energy by creating pairs of "particles": $E_{n_p,n_v} = E_0 + J_p n_p + J_v n_v$
- B_p like flux lines (m), A_v like charges (e)
- $W_e(\mathscr{C}) = \prod_{l \in \mathscr{C}} \sigma_l^z = \prod_{p \in \mathscr{C}} B_p$
- Closed loops don't cost energy

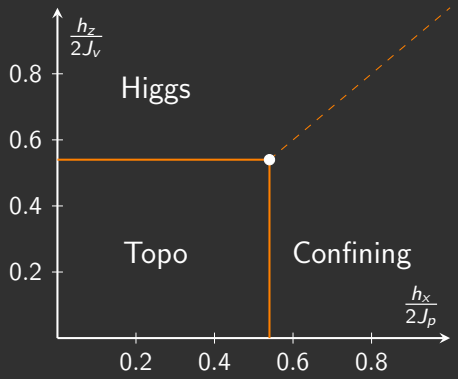
Motivation: Testing the Robustness of the Order

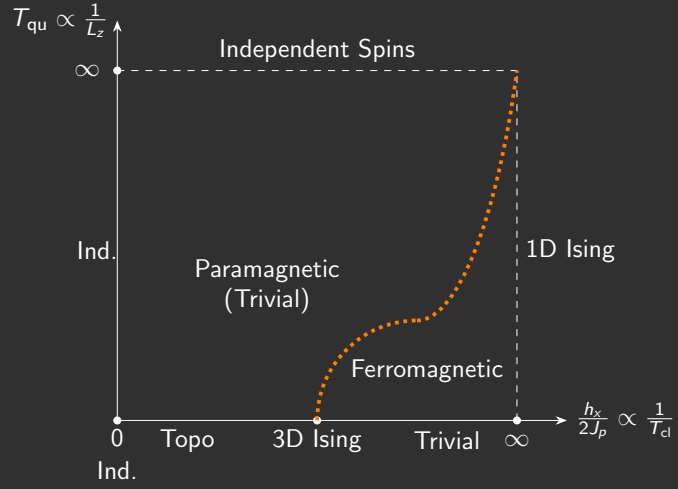
- Topological order $\implies$ robustness against weak local perturbations
- When does it exactly break down? $\implies$ test against quantum and thermal fluctuations
- Try to perturb using an external magnetic field (h):

$$\mathcal{H}_{\text{TCF}} = -J_v \sum_v A_v - J_p \sum_p B_p - h \sum_l (\sin(\theta)\sigma_l^x + \cos(\theta)\sigma_l^z)$$

- GSD on the Torus goes from 4 to 1 as h is increased $\implies$ Quantum Phase transition!

Exploring the Phase Diagrams through Perturbation Theory





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