TUG 2025

Separate Universe Approach : Jordan frame or Einstein frame ?

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Outline

Introduction and Motivations

- Conceptual problems

$\rightarrow$ No origin for the initial conditions

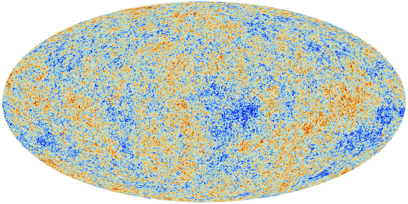
$\rightarrow$ Horizon problem

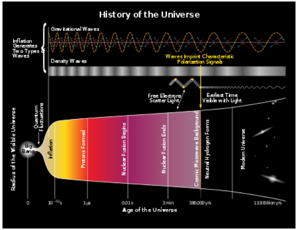
$\rightarrow$ Flatness problem

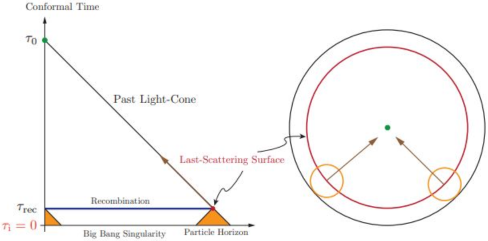
$\rightarrow$ Scale invariance origin

$\rightarrow$ origin of CMB and LSS

$\Rightarrow$ Accelerated expansion = **Inflation**







The Separate Universe approach

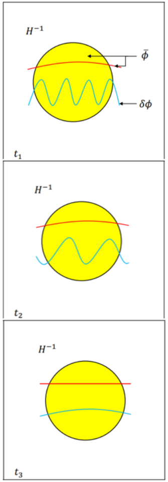


Figure: Quantum to classical transition of fluctuations Artigas Grav&Cosmo 2024

- In a Hubble patch the long wavelength modes eventually evolve as the background
- **GOAL :** evolve the superhorizon modes non linearly in homogeneous patches **and** evolve the subhorizon modes linearly in non homogeneous patches
- When and how should you switch from one description to another ?

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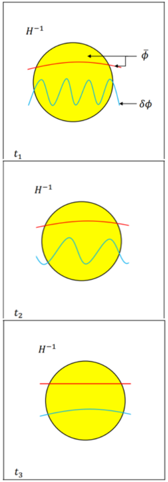

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$\Rightarrow$ Proven to work at leading order in perturbation theory for single field inflation Artigas+ 22 Pattison+ 19

$\Rightarrow$ Proof doesn't tell **how** to execute this matching correctly $\longrightarrow$ if you gauge fix in SU you can't guaranty a good gauge choice in CPT

Applications of the SUA

δN formalism

- Need to know the local initial conditions
- These initial conditions depend on where we coarse grain our universe *i.e.* the scale at which we define our separate universe approach

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Stochastic inflation

- Langevin equation : $\dot{\phi}_{IR} = -\frac{\partial V}{\partial \phi} + \xi_\phi$
- No gradients $\Rightarrow$ relies on the Separate Universe approach
- Noise defined by sub-horizon modes
- Diffusive effects are given by the expectation value of the noise

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$\Rightarrow$ depends on the scale at which we cut our modes

$\Rightarrow$ We **need** to coarse-grain properly at the right scale to define the Separate Universe approach properly

Outline

1. Stochastic inflation & Separate Universe
2. Separate Universe approach in multifield theories
3. Non minimal couplings to gravity

Stochastic inflation :

Stochastic inflation as I have presented it : suppose Slow Roll, de Sitter universe $\Rightarrow$ simple Langevin equation.

NOT always the case. No SR approximation $\Rightarrow$ no attractor solution $\Rightarrow$ need to keep the complete phase space, i.e. the associated momenta to our field(s) $\Rightarrow$ Hamiltonian framework to keep track of everything.

Hamiltonian approach & multifield inflation

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Multifield models :

Phenomenology

- Comological collider physics,
- Reheating : coupling inflaton to Standard model,
- PBH, exponential tails, large scale structures...

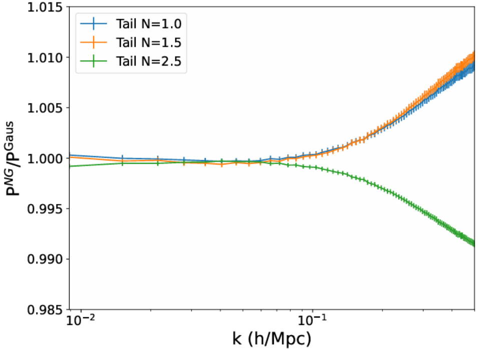
Theory

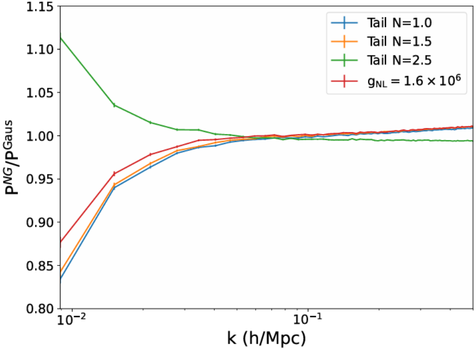
- EFT of inflation,
- non linear couplings,
- non minimally coupled to gravity,

$\Rightarrow$ coupling metric for a covariant theory.

Exponential tails from multiple scalar fields : a smoking gun ?

Multifield models lead to boosts in the power spectrum, which come with exponential tails and non linear effects, some of which can be observed :





Matter and Halo power spectrum for exponential tails, credit to Coulton+ 2024

Multifield hamiltonian constraints

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}M_{pl}^2\mathcal{R} - \frac{1}{2}g^{\mu\nu}G_{IJ}\partial_\mu\phi^I\partial_\nu\phi^J - V(\phi^I)\right]$$
$$= \int \mathrm{d}\tau \int \mathrm{d}^3x \left(\pi_I\dot{\phi}^I + \pi^{ij}\dot{\gamma}_{ij} - N\mathcal{C} - N^i\mathcal{D}_i\right)$$

No dependance on $\dot{N}$ or $\dot{N}^i$ $\Rightarrow$ they have no properly defined associated momenta $\Rightarrow$ We have constraints.
We get them by varying the action with respect with N and N^i :

$$\boxed{\mathcal{C} \equiv \frac{\delta S}{\delta N} = 0} \qquad \boxed{\mathcal{D}_i \equiv \frac{\delta S}{\delta N^i} = 0}$$

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Derive the second order action –so second order constraints –with and without the SUA

$\longrightarrow$ compare the results and make them match :

- Match the constraints themselves *i.e.* give conditions to neglect gradient terms
- Gauge fix and match the perturbed lapse in the two descriptions

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Validity conditions for SU

Compute equations of motion for $\{\mathcal{Q}_c, \mathcal{P}_b, \delta\gamma_1, \delta\pi_1, \delta\gamma_2, \delta\pi_2\}$ and $\{\bar{\mathcal{Q}}_c, \bar{\mathcal{P}}_b, \bar{\delta\gamma}_1, \bar{\delta\pi}_1\}$. Compare them at large scales and check when they match. Next step : check whether we have $\delta N = \bar{\delta N}$

The separate universe approach is valid if $\left(\frac{k}{aH}\right)^2 \ll 3\left(1-\epsilon_1\right)$ and if gradient terms can be neglected in :

$$\mathcal{M} = \begin{pmatrix} \frac{v}{4}\left(\frac{k^2}{v^{2/3}}\delta_{ab} + V_{;ab} - \mathcal{R}_{ab}\right) & \frac{\sqrt{3}v^{1/3}}{8}V_{;a} & 0 \\ \frac{\sqrt{3}v^{1/3}}{8}V_{;a}^{\mathrm{T}} & \frac{1}{v^{1/3}}\left(\frac{\pi_\sigma^2}{v^2} + \frac{V}{2} - \frac{M_{pl}^2 k^2}{4v^{2/3}}\right) & \frac{\sqrt{2}M_{pl}^2 k^2}{24v} \\ 0 & \frac{\sqrt{2}M_{pl}^2 k^2}{24v} & \frac{1}{v^{1/3}}\left(\frac{\pi_\sigma^2}{v^2} + \frac{V}{2} - \frac{M_{pl}^2 k^2}{8v^{2/3}}\right) \end{pmatrix}$$

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The separate universe approach is valid if :

$$\left(\frac{k}{aH}\right)^2 \ll 3\left(1-\epsilon_1\right), \quad \left(\frac{k}{aH}\right)^2 \ll 16\left(1-\epsilon_1\right), \quad \left(\frac{k}{aH}\right)^2 \ll m_a^2,$$

where a runs in $\{\sigma, 1, \ldots, n-1\}$. Grain, Holland 2025

Matching the constraint → 1

Matching the dynamics → { 2, 3 }

1 and 2 → We are working at large scales

3 → The effective mass has to be big enough

⚠ Dynamics play a role here

Outline

Multiple fields in the Jordan frame

In the Jordan frame, the action for n scalar fields coupled to gravity can be expressed as follows:

$$S_J = \int \mathrm{d}^4x\sqrt{-\tilde{g}}\left[f(\phi^I)\tilde{\mathcal{R}} - \frac{1}{2}\tilde{G}_{IJ}\partial_\mu\phi^I\partial^\mu\phi^J + \tilde{V}(\phi^I)\right].$$

On can perform a conformal transformation of the metric : $\tilde{g}_{\mu\nu} \to g_{\mu\nu} = \Omega^2(\phi^I)\tilde{g}_{\mu\nu}$, $\Omega(\phi^I) = \frac{2f(\phi^I)}{M_{pl}^2}$.

This leads to the Einstein frame description :

$$S_J = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}\mathcal{R} - \frac{1}{2}G_{IJ}\partial_\mu\phi^I\partial^\mu\phi^J + V(\phi^I)\right].$$

$$V(\phi^I) \equiv \frac{\tilde{V}(\phi^I}{\Omega^4(\phi^I)}, \quad G_{IJ} \equiv \frac{M_{pl}^2}{2f}\left(\tilde{G}_{IJ} + \frac{3f_{;I}f_{;J}}{f}\right)$$

SUA validity conditions in the Jordan Frame

We get a more complicated mass matrix

$$\mathcal{M} = \begin{pmatrix} \frac{v}{4}\left(\frac{k^2}{v^{2/3}}\delta_{ab} + \tilde{V}_{;ab} - \tilde{\mathcal{R}}_{ab} + F_{;a}F_{;b} - \frac{1}{2}F_{;ab}\right) & \frac{\sqrt{3}v^{1/3}}{8}\tilde{V}_{;a} + \frac{\sqrt{3}v^{1/3}\theta^2}{8M_{pl}^2}\frac{F_{;a}}{F^2} - F_{;a}\frac{M_{pl}^2k^2}{3v^{1/3}} & F_{;a}\frac{M_{pl}^2k^2}{3\sqrt{2}v^{1/3}} \\ \frac{\sqrt{3}v^{1/3}}{8}\tilde{V}_{;a}^{\mathrm{T}} + \frac{\sqrt{3}v^{1/3}\theta^2}{8M_{pl}^2}\frac{F_{;a}^{\mathrm{T}}}{F^2} - F_{;a}^{\mathrm{T}}\frac{M_{pl}^2k^2}{3v^{1/3}} & \frac{1}{v^{1/3}}\left(\frac{\pi_\sigma^2}{v^2} + \frac{V}{2} - \frac{M_{pl}^2k^2}{4v^{2/3}}\right) & \frac{\sqrt{2}M_{pl}^2k^2}{24v} \\ F_{;a}^{\mathrm{T}}\frac{M_{pl}^2k^2}{3\sqrt{2}v^{1/3}} & \frac{\sqrt{2}M_{pl}^2k^2}{24v} & \frac{1}{v^{1/3}}\left(\frac{\pi_\sigma^2}{v^2} + \frac{V}{2} - \frac{M_{pl}^2k^2}{8v^{2/3}}\right) \end{pmatrix}$$

How can I neglect gradients here ? How many conditions do I get from doing this ?

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$$\left(\frac{k}{aH}\right)^2 \ll \alpha V_{;a} + \beta\frac{F_{;a}}{F^2}$$

Why is there a new set of conditions ?

The background quantities have also been changes, making any direct comparison inaccurate !

A two field case study - work in progress

We can't diagonalise the two matrix formally :

- n fields $\Rightarrow (n+2)$x$(n+2)$ matrices
- model dependance, in particular on $V(\phi^I)$ and $G_{IJ}(\phi^I)$.

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$$S = \int \sqrt{-\tilde{g}} \left[\frac{1+K(\phi_J)}{2} \tilde{\mathcal{R}} + \frac{1}{2} \left((\partial \phi_J)^2 + (\partial \chi_J)^2 \right) - \tilde{V}(\phi_J) \right]$$

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$$b = -\frac{1}{2} log(1 + K)$$

$\Rightarrow$ compute the two mass matrices and diagonalise them, see what the differences mean.

$\Rightarrow$ apply this to a complex scalar with a Higgs like potential, where ϕ_J is the modulus and χ_J the argument.

Conclusion and next steps

- We need the separate universe approach to hold for the δN and stochastic formalisms
- Multifield inflation models are interesting for these two formalisms : when does the SUA approach hold ?
- We derive the validity conditions in both the Einstein and the Jordan frame
- Quite easy to derive in the former, not so much in the latter
- *Next steps :* Study a two field toy model to see if the validity conditions can compare easily.