COLLÈGE DE FRANCE 1530

Review talk: Symmetries of asymptotically flat spacetimes

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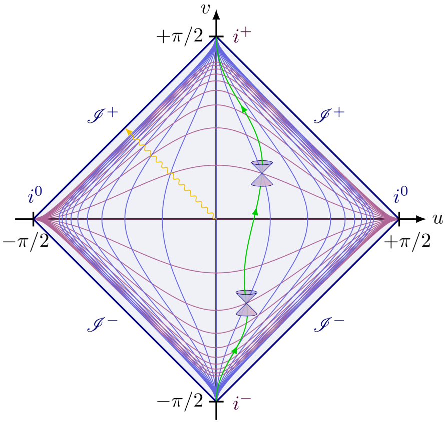
4d asymptotically flat spacetimes

- Spacetimes that are asymptotically Minkowski
- Why? Detectors are very far from the sources (BH mergers, etc)



[Figure Neutelings]

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Asymptotic symmetries as symmetries for gravity

Observables in gravity need a boundary

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- How to define energy in gravity?
- Locally, it is impossible because of equivalence principle.
- One needs a BOUNDARY to define a charge/generator associated to a symmetry via Noether procedure
- Same for electromagnetism: electric charge is measured using the Gauss law

Asymptotic symmetries

To get a charge:

1. Boundary
2. Symmetry

Asymptotic symmetries

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1. Boundary
2. Symmetry

- In general solutions of Einstein equations do not have a Killing vector

$$\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = 0$$

- Class of solutions to share the same asymptotic structure
- We will ask the symmetry to only preserve the asymptotic structure
  $=$ ASYMPTOTIC SYMMETRY

How to:

Theory:

1. Action and equations of motion
2. Choice of fall-offs and boundary conditions

Recipe for computing charges in gravity:

1. Asymptotic symmetries (preserving those fall-offs)
2. Compute the generators/charges (with some subtleties)
   Charges are the labels of a physical states
3. Symmetry algebra

Method: covariant phase space [review: Fiorucci '21] or Hamiltonian [review: Henneaux's lectures at College de France '21-'22 & '22-'23]

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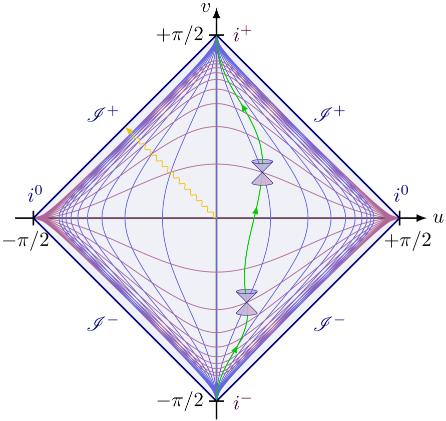
- Definition of observables for gravity (ex: energy)
- Bottom-up approach to new holographic dualities
- Quantum Gravity: quantum states form a representation of the asymptotic symmetry algebra $\rightarrow$ Non-perturbative handle on Quantum Gravity
- Connection with low energy physics (infrared red triangle)

Asymptotically flat spacetimes and BMS group

4d asymptotically flat spacetime

- Past and future timelike infinity ($i_\pm$)
- Past and future null infinity ($\mathscr{I}_\pm$)
- Spatial infinity (i_0)



Focus on future null infinity [Bondi et al. '61, Sachs '61]

- Bondi coordinates: u retarded time, r radial coordinates, x^A angular coordinates on a 2-sphere ($q_{AB} = d\theta^2 + \sin^2\theta d\phi^2$)

$$ds^2_{\text{Mink}} = -du^2 - 2dudr + r^2 q_{AB} dx^A dx^B$$

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- First correction in r

$$ds^2 = -\left(1 - \frac{1}{r}2M + ...\right) du^2 - 2dudr + r^2 \left(q_{AB} + \frac{1}{r}C_{AB} + ...\right) dx^A dx^B + ...$$

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- Trautman–Bondi mass-loss formula

$$\partial_u \oint M = -\frac{1}{8} \oint \dot{C}_B^A \dot{C}_A^B$$

Symmetry of future null infinity

- ξ s.t. $g_{\mu\nu} + \mathcal{L}_\xi g_{\mu\nu} = \eta_{\mu\nu} + \mathcal{O}(1/r)$

$$\xi^u = T(x^A) + \frac{1}{2} u\, D_A Y^A$$
$$\xi^r = -\frac{r}{2} D_A Y^A + \dots$$
$$\xi^A = Y^A(x^A) + \dots$$

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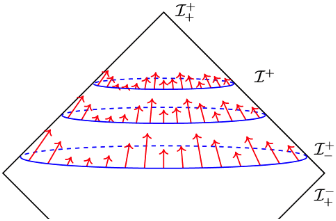
- Algebra

$$[\xi_{(T_1,Y_1)}, \xi_{(T_2,Y_2)}] = \xi_{(\hat{T},\hat{Y})}$$
$$\hat{T} = Y_1^A \partial_A T_2 - \frac{1}{2} D_A Y_1^A T_2 - (1 \leftrightarrow 2)\,,$$
$$\hat{Y} = \mathcal{L}_{Y_1} Y_2$$



[Figure Lim Zheng Liang]

- The charge associated to T is the mass and to Y^A is the angular momentum

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- The algebra is BMS $= = SL(2,\mathbb{C})_Y \ltimes \mathbb{R}_T$

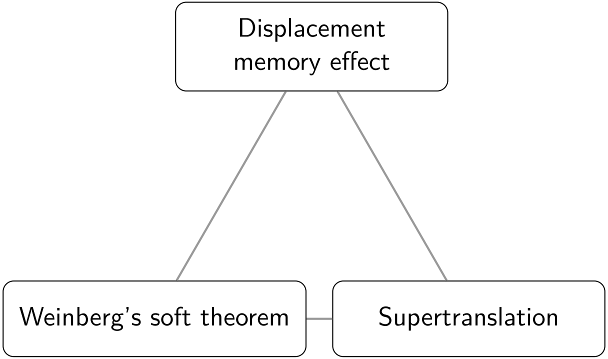
$$\{Q_{\xi_1}, Q_{\xi_2}\} = Q_{[\xi_1,\xi_2]}$$

- Remark: for an coordinate independent derivation of these results [Penrose '64]

Summary:

- BMS is realized at null infinity
- There is an antipodal matching between BMS generators defined at future and null infinity [Strominger '13, Troessaert 17' Prabhu et al. '19 '21, Capone et al. 22']
  This is relevant to studying scattering amplitudes
- All five asymptotic boundaries are compatible with BMS [Compère, Gralla, Wei '23]

Infrared triangle



[review: Strominger '17]

More symmetries

More symmetries at null infinity?

- Asymptotic symmetries are tied to fall-offs. Are there more general fall-offs such that the charge are still well-defined (finite, closure of the algebra, ...) ?

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- Asymptotic symmetries are tied to fall-offs. Are there more general fall-offs such that the charge are still well-defined (finite, closure of the algebra, ...) ?
- Yes!
- Consequence 1: the symmetry algebra is typically bigger
- Consequence 2: more inclusivity (more solutions consistently included in the same phase space)

Starting point: Bondi-Sachs gauge

$$g_{rr} = 0\,, \quad g_{rA} = 0\,, \quad \det g_{AB} = r^4 \det q\,, \quad q_{AB} \text{ round 2-sphere}$$

$$T(x^A), \quad Y^A(x^A) \text{ 6 conf. KV (global)} \quad \text{BMS } = SL(2,\mathbb{C})_Y \ltimes \mathbb{R}_T$$

$Y^A(x^A)$ conf. KV (local) [Barnich, Troessaert '11]	T, Y^A	eBMS= $(\text{VirxVir})_Y \ltimes \mathbb{R}_T$
$\delta q_{AB} \neq 0, \delta \det q = 0$ [Campiglia, Laddha '14, Compere et al. '18]	T, Y^A	gBMS= $(\text{Diff}(S_2)_Y \ltimes \mathbb{R}_T$
$\delta q_{AB} \neq 0$ [Barnich, Troessaert '11, Freidel et al. '21]	T, W, Y^A	WBMS= $((\text{Diff}(S_2)_Y \ltimes \mathbb{R}_T) \ltimes \mathbb{R}_W$
$\det g_{AB}$ free [Geiller, Zwikel '22, '24]	T, Y^A, W, k_1, k_2	$(((\text{Diff}(S_2)_Y \ltimes \mathbb{R}_T) \ltimes \mathbb{R}_W) \ltimes (\mathbb{R}_{k_1} \otimes \mathbb{R}_{k_2})$

Other generalizations include:

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- Polyhomogenous expansion (allowing log terms)
  Compatible with Einstein equations and generated by hyperbolic encounters [Winicour '85, Damour '86, Chrusciel et al. '93, Kroon '98, Christodoulou '02, Kehrberger '21→'24, ...]
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Subtleties in computing the charges: renormalization, Wald-Zoupas criteria, "correct" bracket for the algebra, slicing, ...

More symmetries at spatial infinity?

- gBMS [Fiorucci, Matulich, Ruzziconi '24]

More symmetries at spatial infinity?

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- Log BMS [Fuentealba, Henneaux, Troessaert '22] [Girelli, Langenscheidt, Neri, Pollack, Zwikel '25 (to appear)]

  Features

  - Related to allowing logarithmic terms in the radial expansion of the metric
  - Log BMS= BMS + Log supertranslations where log supertranslation have a central charge with supertranslations
  - Used for redefining the Lorentz generators such that Poincaré is an ideal of BMS

Now what?

What is the physics behind the new symmetries?

- How are they encoded in the different regions of asymptotically flat space
- New infrared triangles? (yes for certain symmetries)
- How are they encoded in flat holography proposals?

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  - other boundaries (ex: for instance: black hole horizon)
  - with matter and other gauge fields

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Merci!