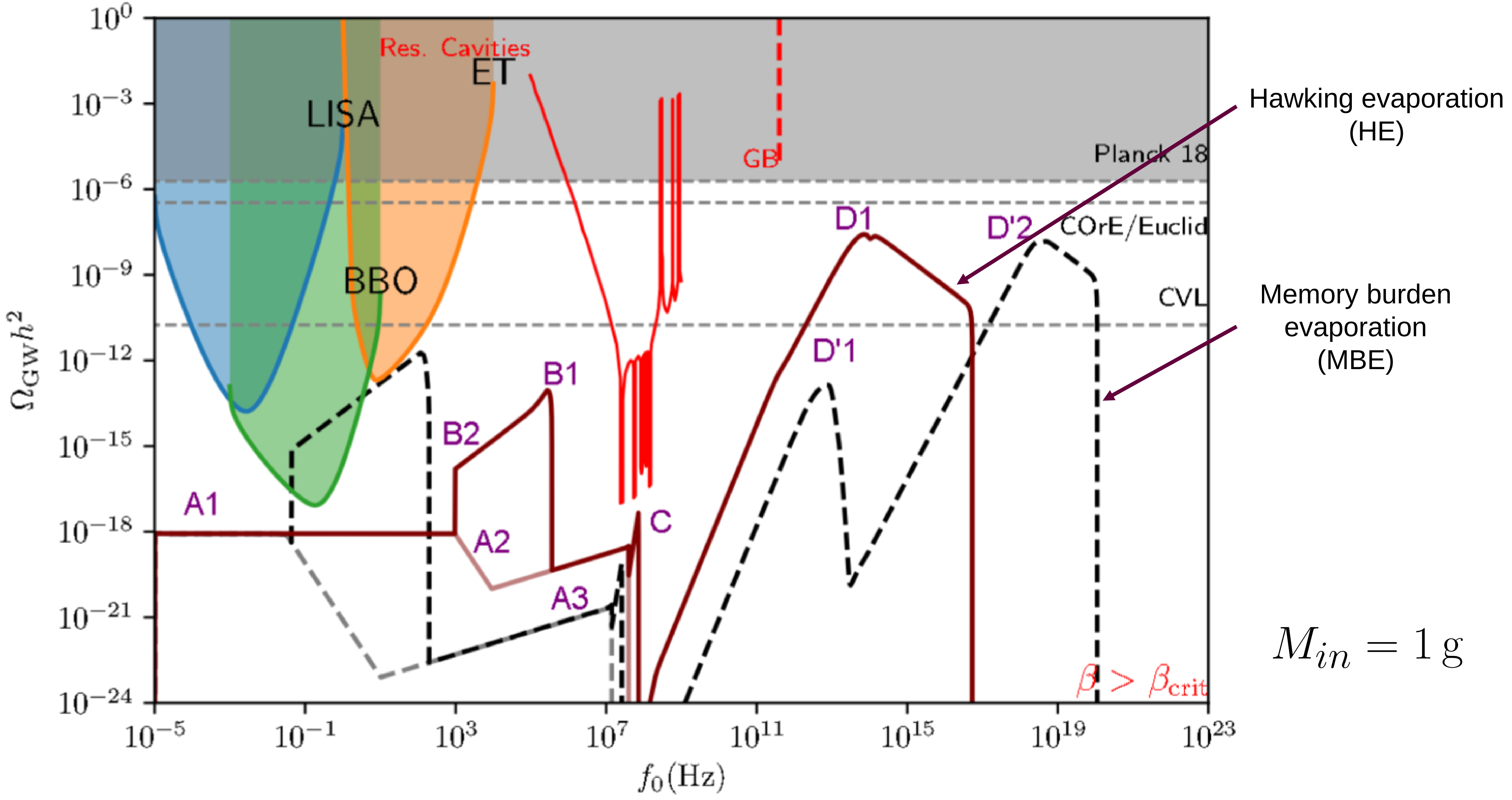
Burdening (or not) gravitational waves in the presence of primordial black holes

Mathieu Gross

TUG 2025

15/10/2025

Based on : arXiv:2411.04189 [hep-ph] and arXiv:2509.02701 [hep-ph]
In collaboration with: Riajul Haque , Essodjolo Kpatcha, Yann Mambrini, Maria Olalla Olea-Romacho, Rishav Roshan



Res. Cavities
ET
LISA
GB
Planck 18
BBO
D1
D'2
COrE/Euclid
CVL
D'1
B1
B2
A1
C
A2
A3
$\beta > \beta_{\mathrm{crit}}$
$\Omega_{\mathrm{GW}} h^2$
$f_0(\mathrm{Hz})$
10^{0}
10^{-3}
10^{-6}
10^{-9}
10^{-12}
10^{-15}
10^{-18}
10^{-21}
10^{-24}
10^{-5}
10^{-1}
10^{3}
10^{7}
10^{11}
10^{15}
10^{19}
10^{23}
Hawking evaporation (HE)
Memory burden evaporation (MBE)
$M_{in} = 1\,\mathrm{g}$

Outline

Introduction

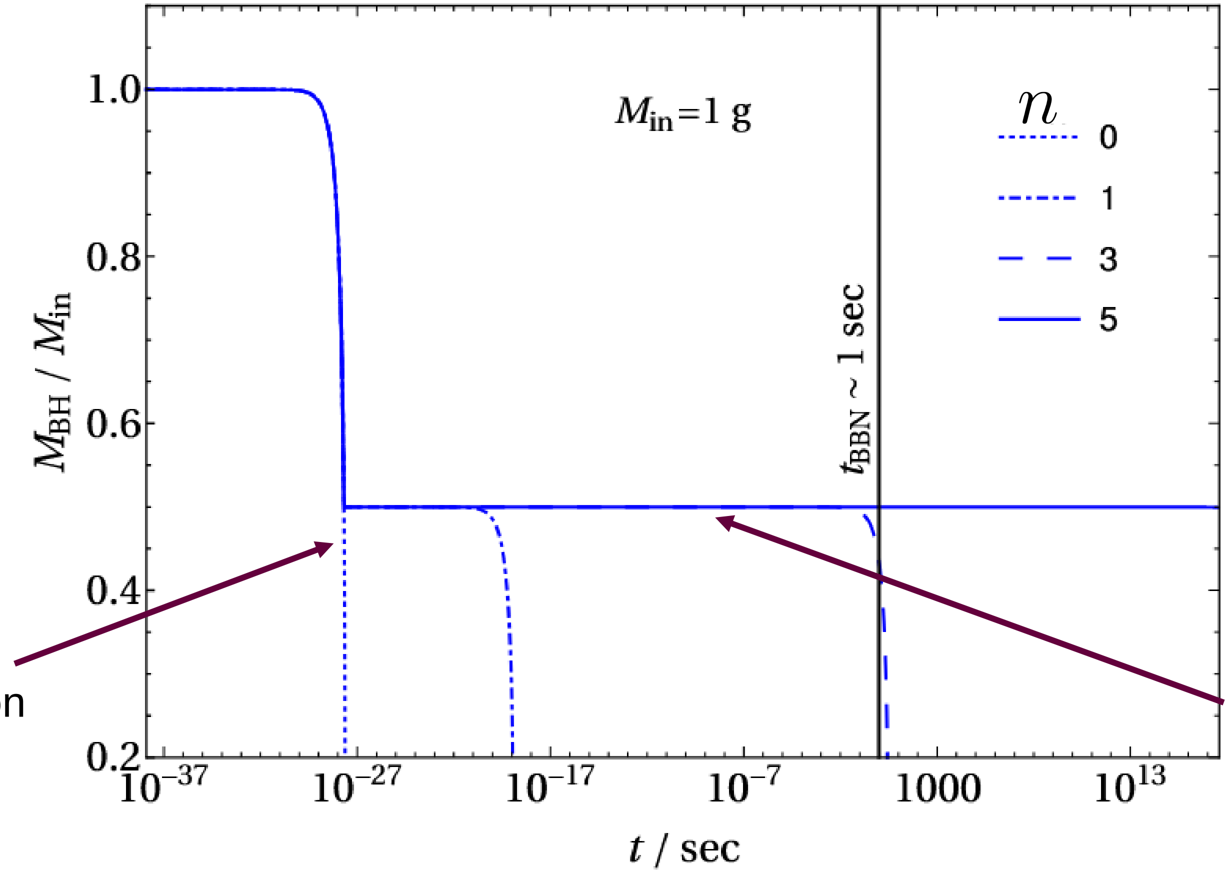
- Primordial Black Holes
- Reheating

PBH reheating

- Gravitationnal waves from inflation
- Inflaton scattering
- Scalar induced gravitationnal waves
- PBH evaporation

Memory burden evaporation

Memory burden effect: $$\frac{dM}{dt} = -\left(\theta(M - qM_{in}) + \frac{\theta(qM_{in} - M)}{S^n}\right)\frac{\epsilon M_P^4}{M^2}$$

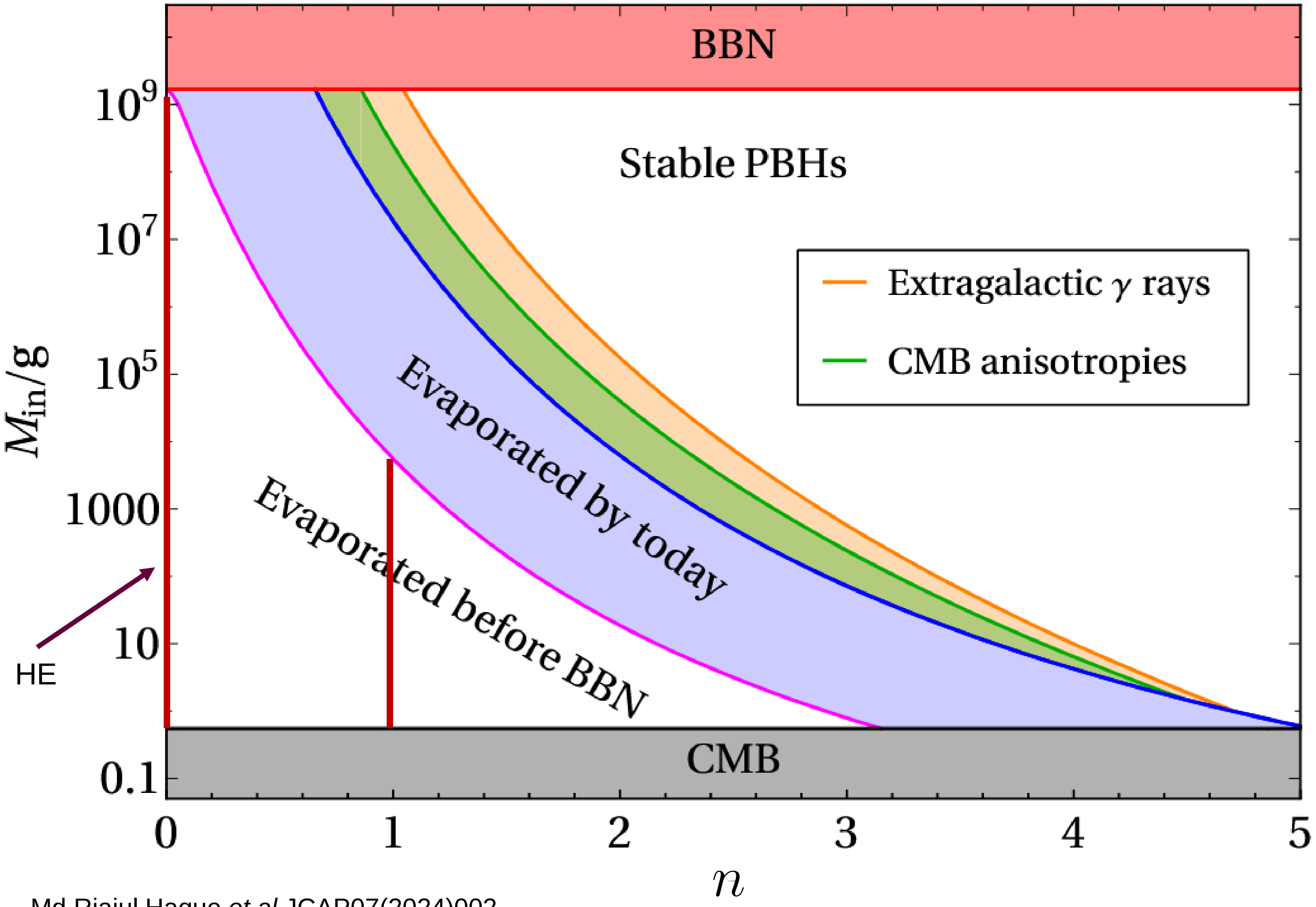


Hawking evaporation

Memory burden evaporation

G. Dvali, arXiv:1810.02336 [hep-th]
Gia Dvali *et al* JCAP03(2019)010
A. Alexandre et al Phys. Rev. D **110**, 036004
Gia Dvali *et al* Phys. Rev. D **110**, 056029
V. Thoss et al *Monthly Notices of the Royal Astronomical Society*

Memory burden evaporation



Md Riajul Haque *et al* JCAP07(2024)002

We will take a benchmark point

$$q = \frac{1}{2}\,, n = 1$$

$$t_{\rm in} \sim \frac{M_{\rm in}}{M_{\rm P}^2}$$

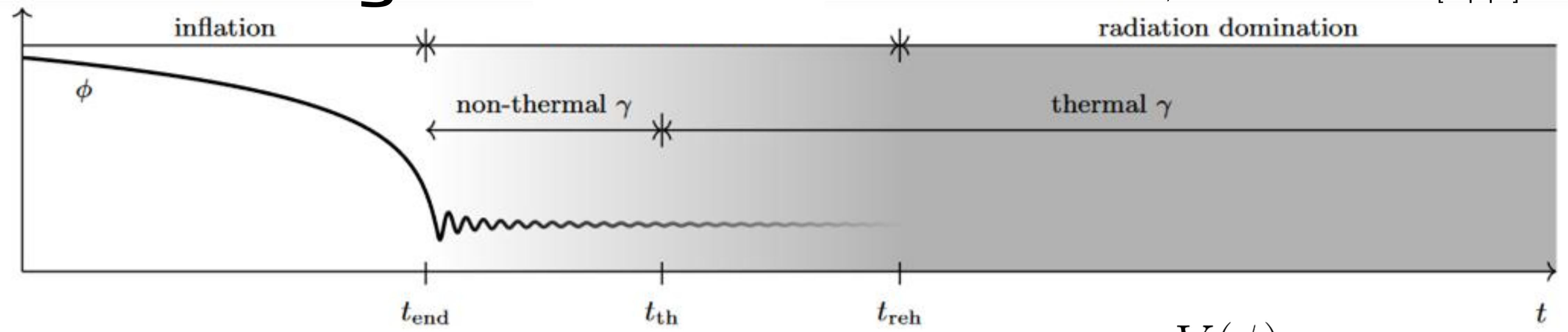
$$f_{\rm PBH} \propto \delta(M - M_{\rm PBH})$$

$$M_{PBH} \in \left[1{\rm g}, 10^4{\rm g}\right]$$

$$\beta = \left.\frac{\rho_{\rm PBH}}{\rho_\phi}\right|_{t_{\rm in}}$$

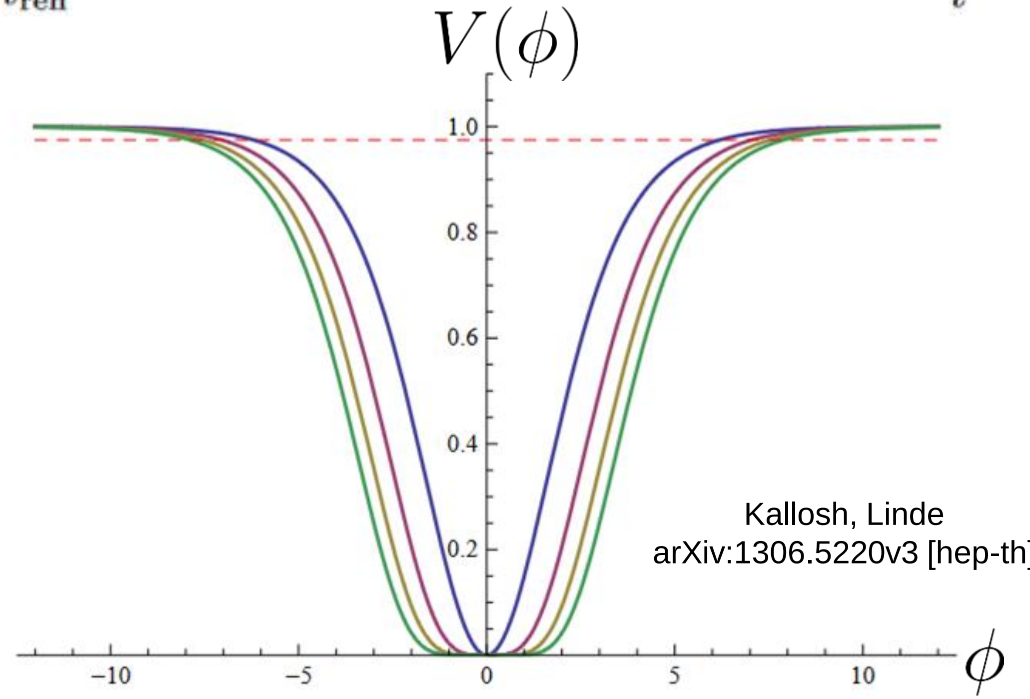
Reheating after inflation

Garcia, Amin arXiv:1806.01865v2 [hep-ph]



$$V(\phi) = \lambda M_P^4 \left[\sqrt{6}\tanh\left(\frac{\phi}{\sqrt{6}M_P}\right)\right]^k$$

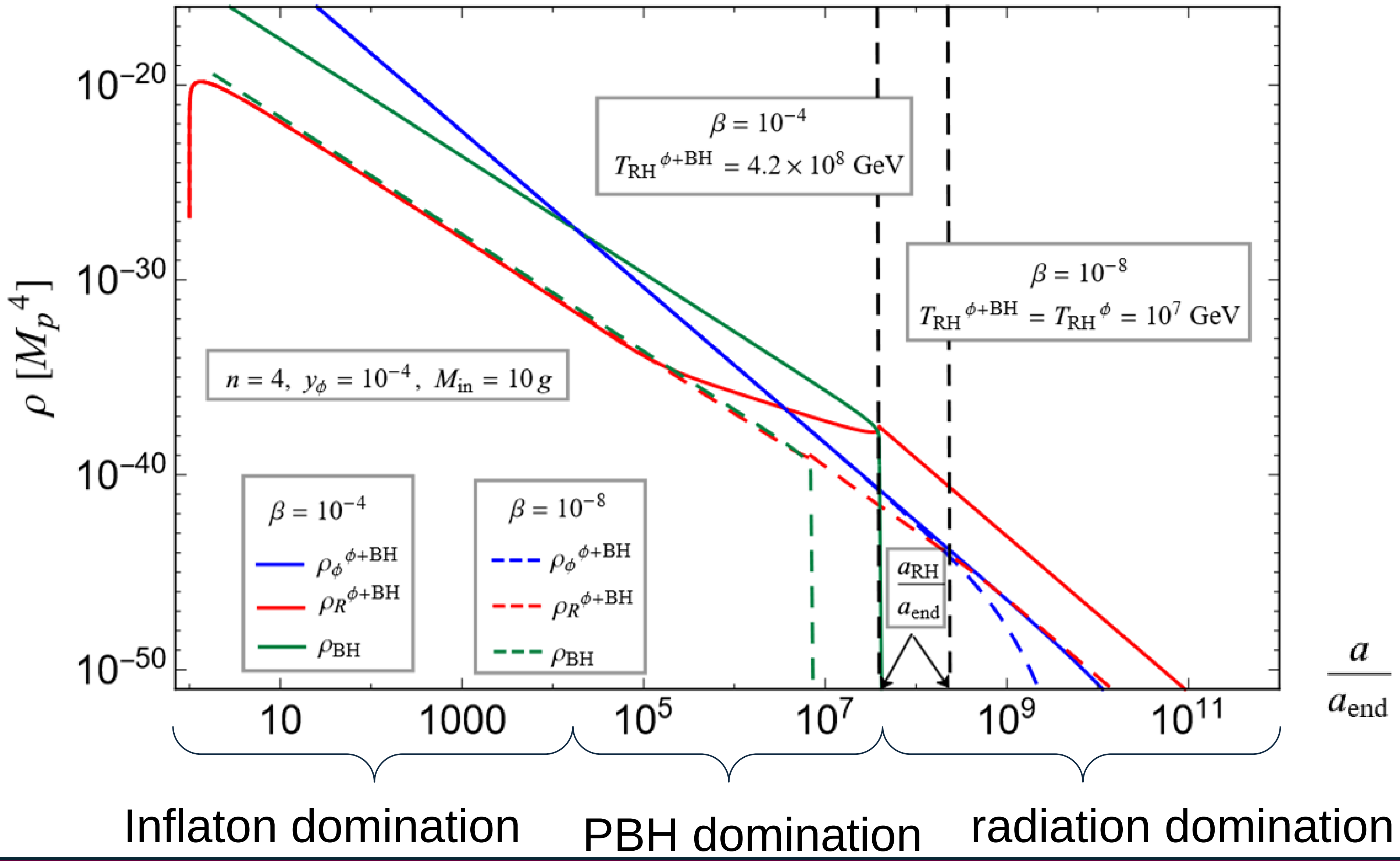
$$w_\phi = \frac{p_\phi}{\rho_\phi} = \frac{k-2}{k+2} = \begin{cases} 0 \quad \text{if} \quad k=2 \\ 1/3 \quad \text{if} \quad k=4 \\ 1/2 \quad \text{if} \quad k=6 \\ 1 \quad \text{if} \quad k \gg 1 \end{cases}$$

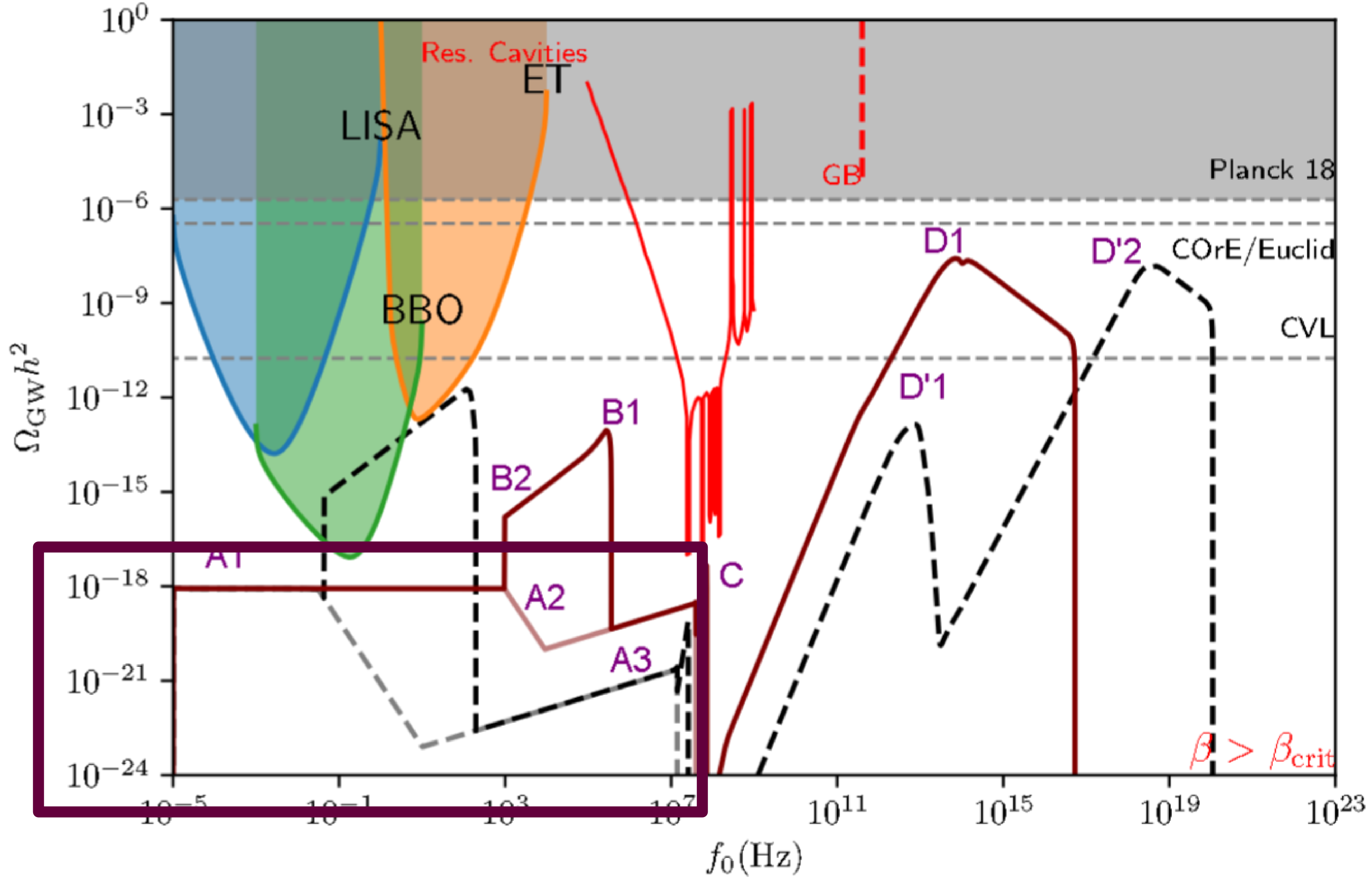


Kallosh, Linde
arXiv:1306.5220v3 [hep-th]

Haque, Kpatcha et al arXiv:2305.10518

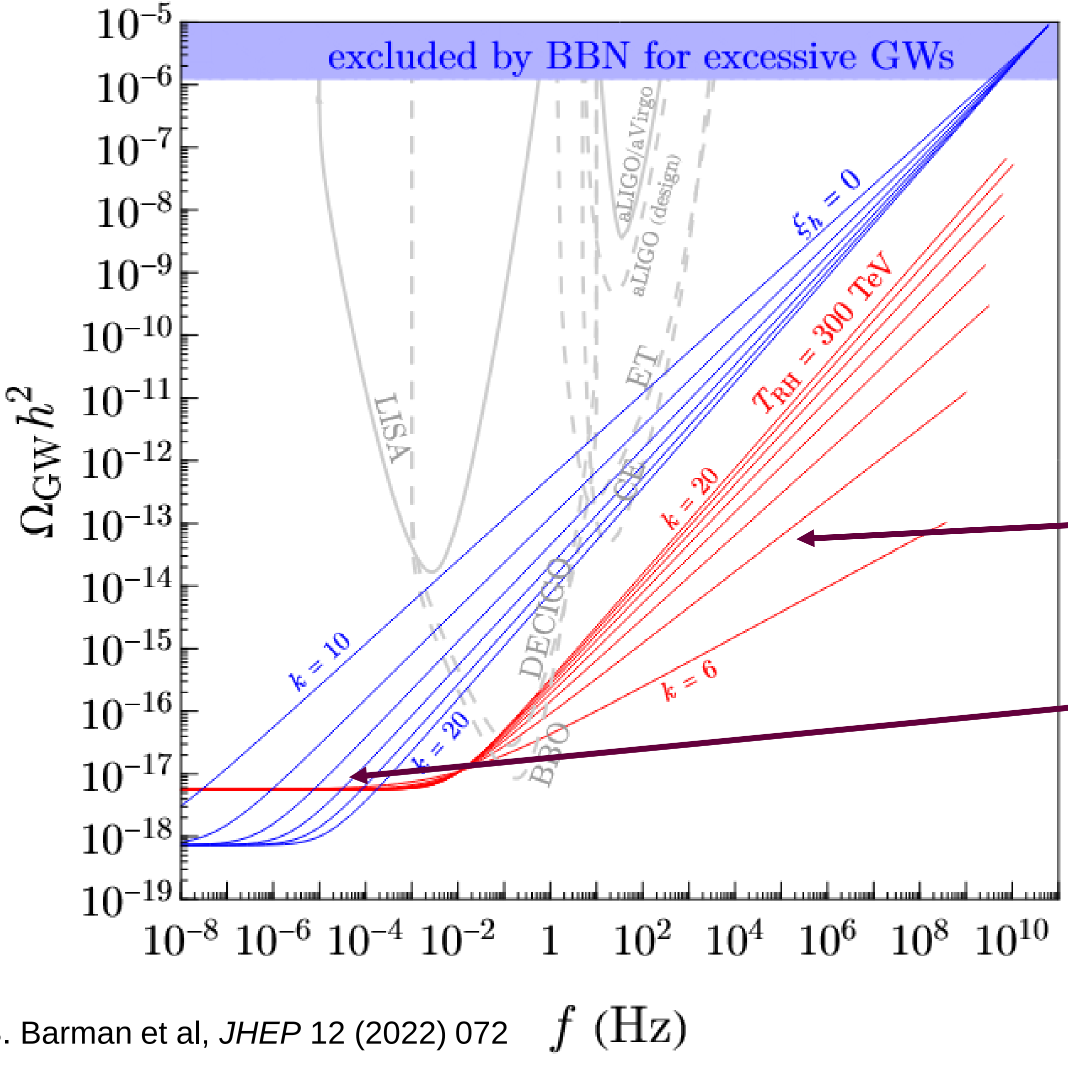
PBH domination/reheating





Res. Cavities
ET
LISA
BBO
GB
Planck 18
COrE/Euclid
CVL
D1
D'2
D'1
B1
B2
A1
A2
A3
C
$\beta > \beta_{\rm crit}$
$\Omega_{\rm GW}h^2$
$f_0\,(\rm Hz)$

Primodial gravitationnal waves



B. Barman et al, *JHEP* 12 (2022) 072

Inflation predicts above the horizon: $\mathcal{P}_T \approx \dfrac{H_I^2}{2\pi^2 M_P^2}$

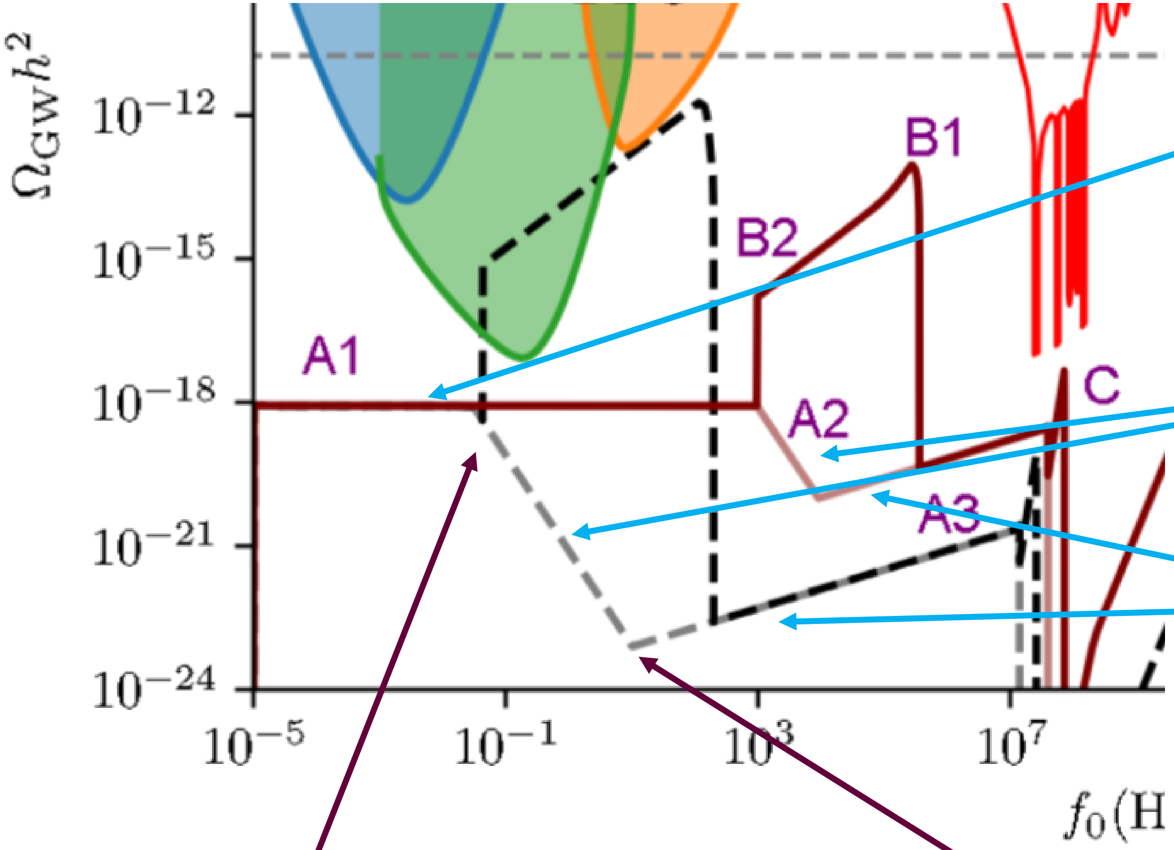
Horizon reentry modulation: $\Omega_{GW} h^2 \sim k^{-\frac{2(1-3w)}{1+3w}}$

Stiff equation of state domination

Radiation domination

$$T_{\rm RH} \geq \left(\frac{\Omega_{GW}^{\rm rad} h^2\, \zeta(w_\phi)}{5.61\times 10^{-6}\,\Delta N_{\rm eff}}\right)^{\frac{3(1+w_\phi)}{4(3w_\phi-1)}} \left(\frac{30\,\rho_{\rm end}}{\pi^2 g_{RH}}\right)^{1/4}.$$

Primodial gravitationnal waves



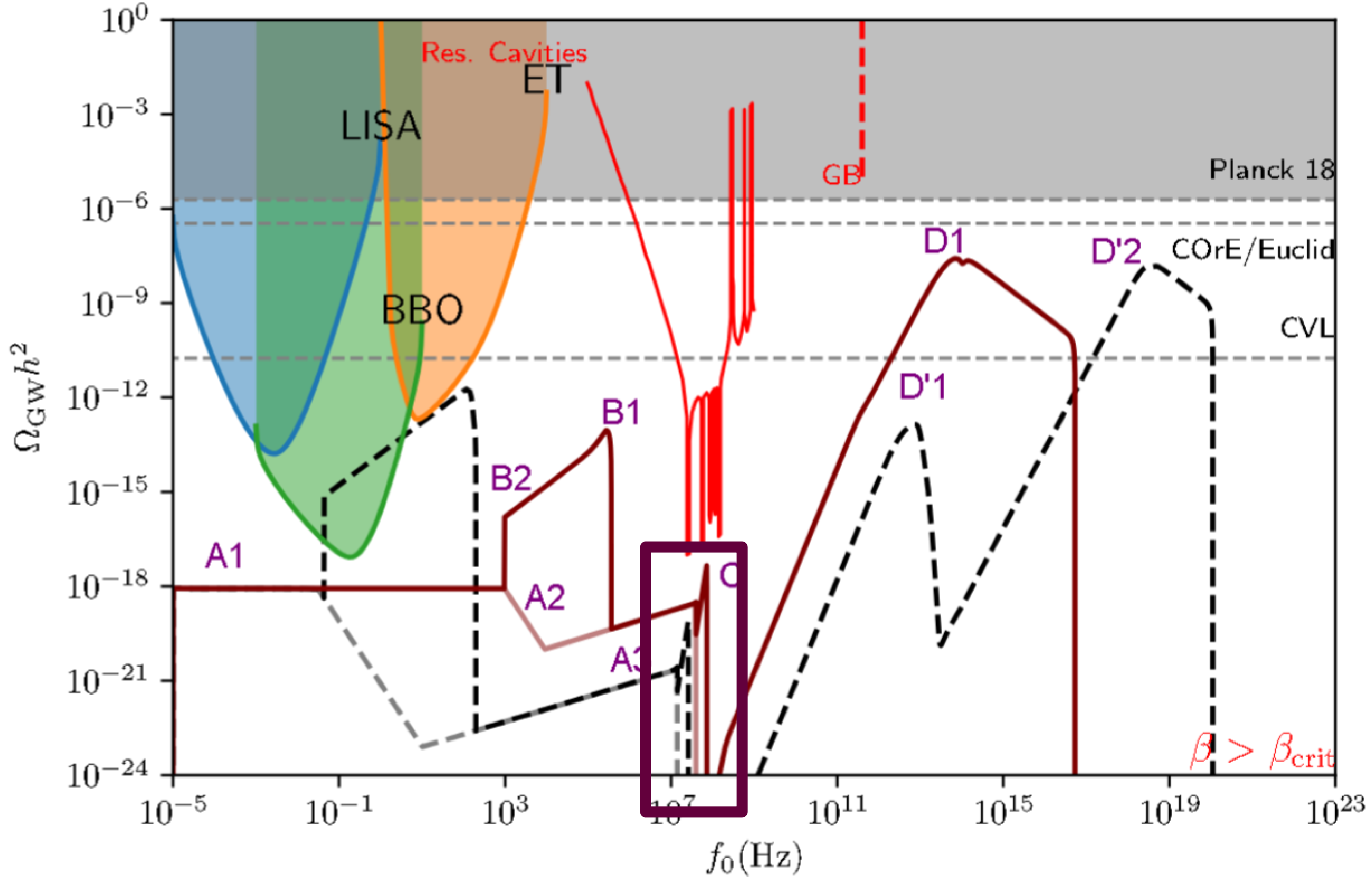
Radiation domination $\Omega_{GW}h^2 \propto k^0$

PBH domination $\Omega_{GW}h^2 \propto k^{-2}$

Inflaton domination $\Omega_{GW}h^2 \propto k^{-2\frac{1-3w_\phi}{1+3w_\phi}}$

$$f_{RH} = \frac{H_{RH}}{2\pi}\frac{a_{RH}}{a_0} \qquad f_{PBH} = \frac{H_{PBH}}{2\pi}\frac{a_{PBH}}{a_0}$$

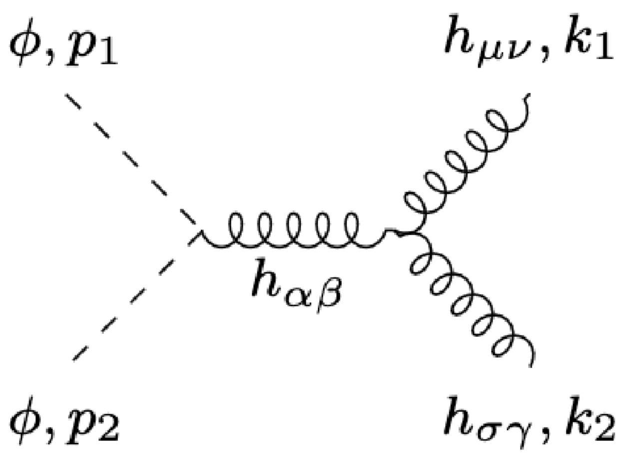
Memory burden evaporation makes it easier to have PBH domination!



Res. Cavities
ET
LISA
BBO
GB
Planck 18
COrE/Euclid
CVL
D1
D'2
D'1
B1
B2
A1
A2
A3
C
$\beta > \beta_{\rm crit}$
$\Omega_{\rm GW}h^2$
$f_0\,(\rm Hz)$
10^0
10^{-3}
10^{-6}
10^{-9}
10^{-12}
10^{-15}
10^{-18}
10^{-21}
10^{-24}
10^{-5}
10^{-1}
10^3
10^7
10^{11}
10^{15}
10^{19}
10^{23}

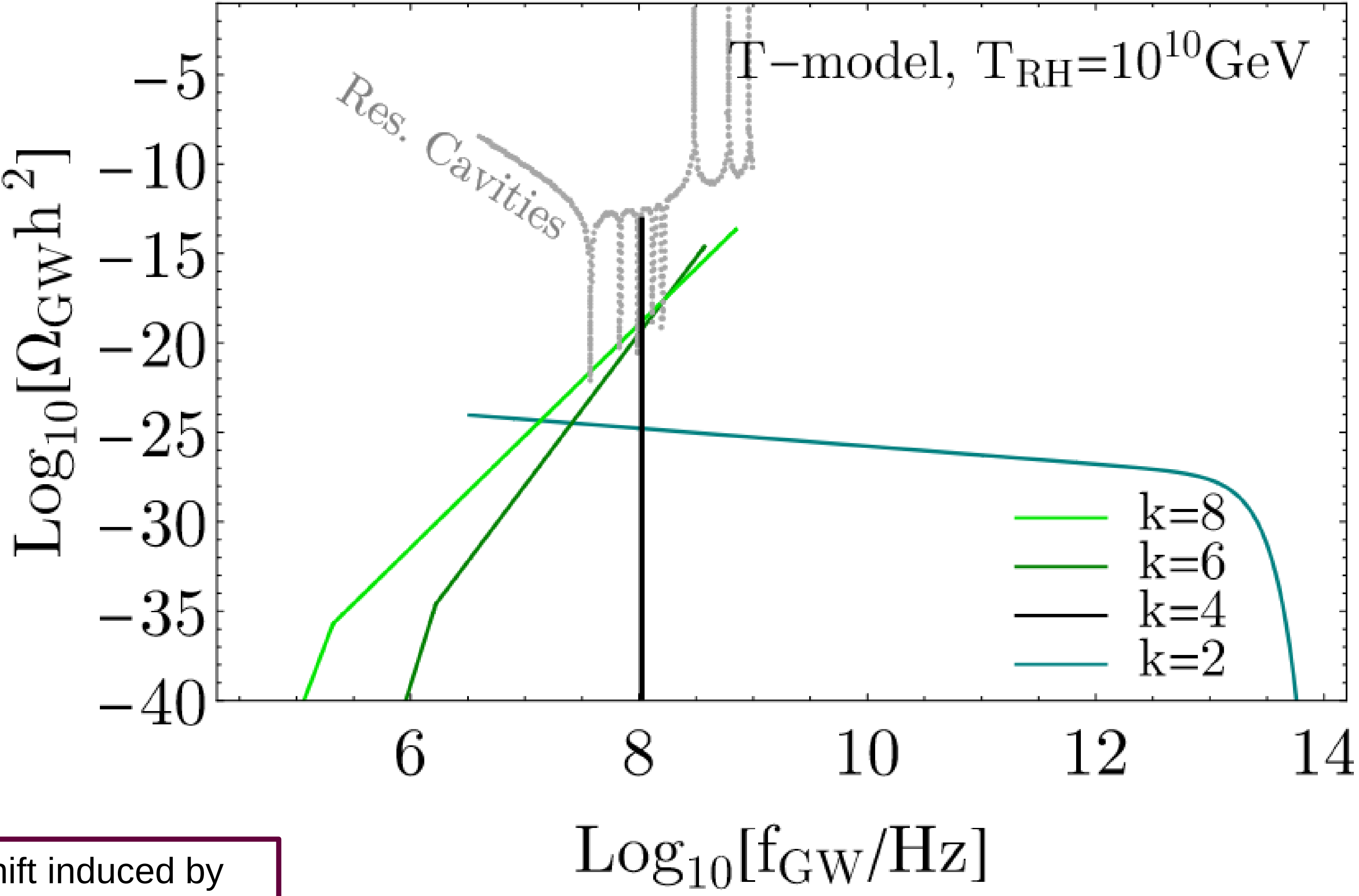
GW from inflaton scattering

Choi, Ke, Olive arXiv:2402.04310 [hep-ph]

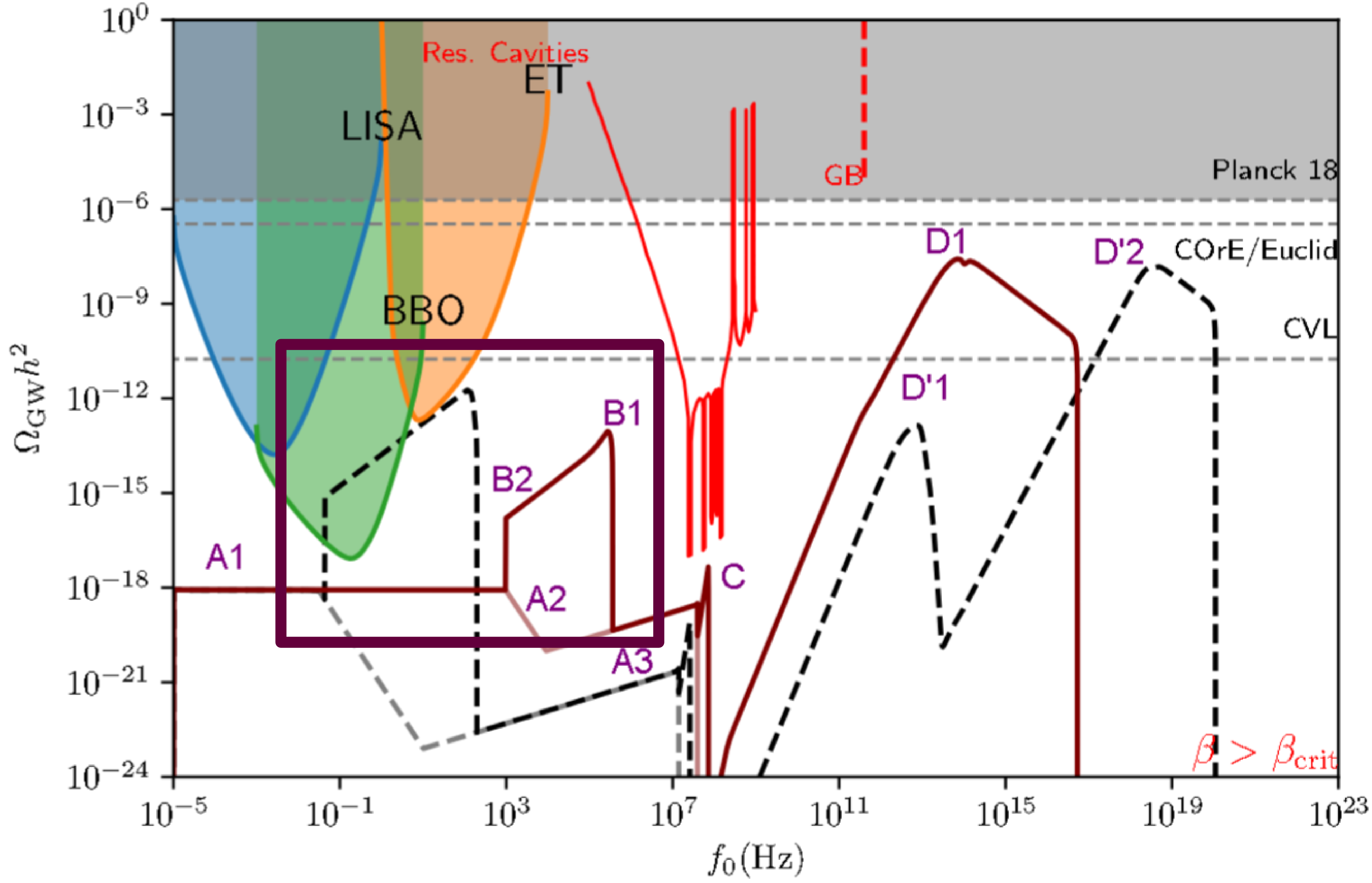


$$(1 + w_\phi)\Gamma_\phi \rho_\phi = \frac{\rho_\phi^2 \omega}{4\pi M_P^4} \Sigma^n$$

$$\omega \propto a^{-3w_\phi}$$



This source is only sensitive to the redshift induced by PBH domination!



Res. Cavities
ET
LISA
BBO
GB
Planck 18
COrE/Euclid
CVL
D1
D'2
D'1
B1
B2
A1
A2
A3
C
$\beta > \beta_{\rm crit}$
$\Omega_{\rm GW}h^2$
$f_0(\rm Hz)$
10^{0}
10^{-3}
10^{-6}
10^{-9}
10^{-12}
10^{-15}
10^{-18}
10^{-21}
10^{-24}
10^{-5}
10^{-1}
10^{3}
10^{7}
10^{11}
10^{15}
10^{19}
10^{23}

Scalar induced gravitationnal waves

We allow the metric to fluctuate:

$$ds^2 = (1-2\Phi)dt^2 - a^2(t)\left[\delta_{ij}(1+2\Phi) + \frac{2h_{ij}}{M_{\rm P}}\right]dx^i dx^j$$

At second order scalar fluctuations can source GW :

$$h''_{k,\lambda} + 2\mathcal{H}h'_{k,\lambda} + k^2 h_{k,\lambda} = \mathcal{S}_{k,\lambda},$$

PBH will intrinsically generate scalar fluctuations:

Theodoros Papanikolaou *et al* JCAP03(2021)053

$$\mathcal{P}_\Phi \approx \frac{2}{3\pi}\left(\frac{k}{k_{UV}}\right)^3$$

$$\frac{k_{UV}}{k_{in}} = \left(\frac{\beta}{\gamma}\right)^{\frac{1}{3}} \qquad \frac{k_{\rm BH}}{k_{in}} = \sqrt{2}\beta^{\frac{1+3w_\phi}{6w_\phi}}$$

$$\frac{k_{\rm RH}}{k_{\rm in}} = \left[\frac{((3+2k)2^n\epsilon)^2}{36\pi^2\gamma^2}\left(\frac{M_{\rm in}}{M_{\rm P}}\right)^2\left(\frac{M_{\rm P}}{qM_{\rm in}}\right)^{6+4n}\beta^2\right]^{\frac{1}{6}}$$

MBE modify the frequency of the GW peak!

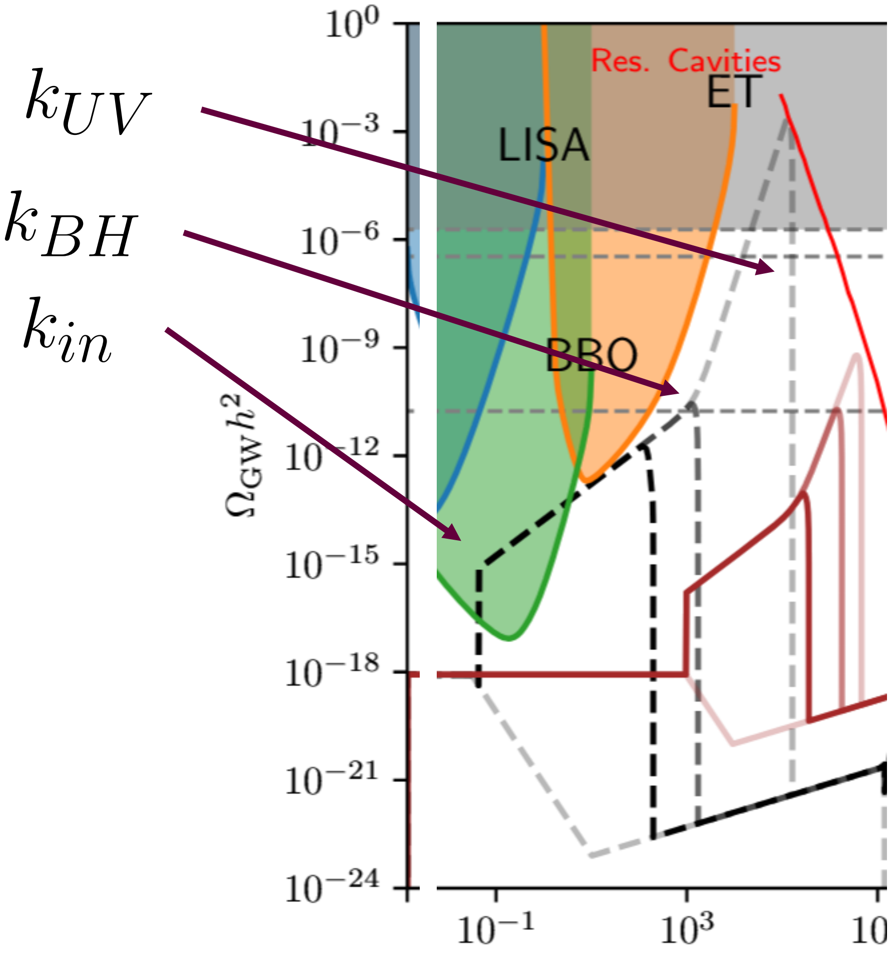
Scalar induced gravitationnal waves

$$\Omega_{\mathrm{GW,res}}(k) = \frac{(2c_{\mathrm{s}})^{2\frac{3\kappa+2}{3\kappa}} C^4(w) q^4 (c_{\mathrm{s}}^2 - 1)^2}{13824 c_{\mathrm{s}} \pi}$$

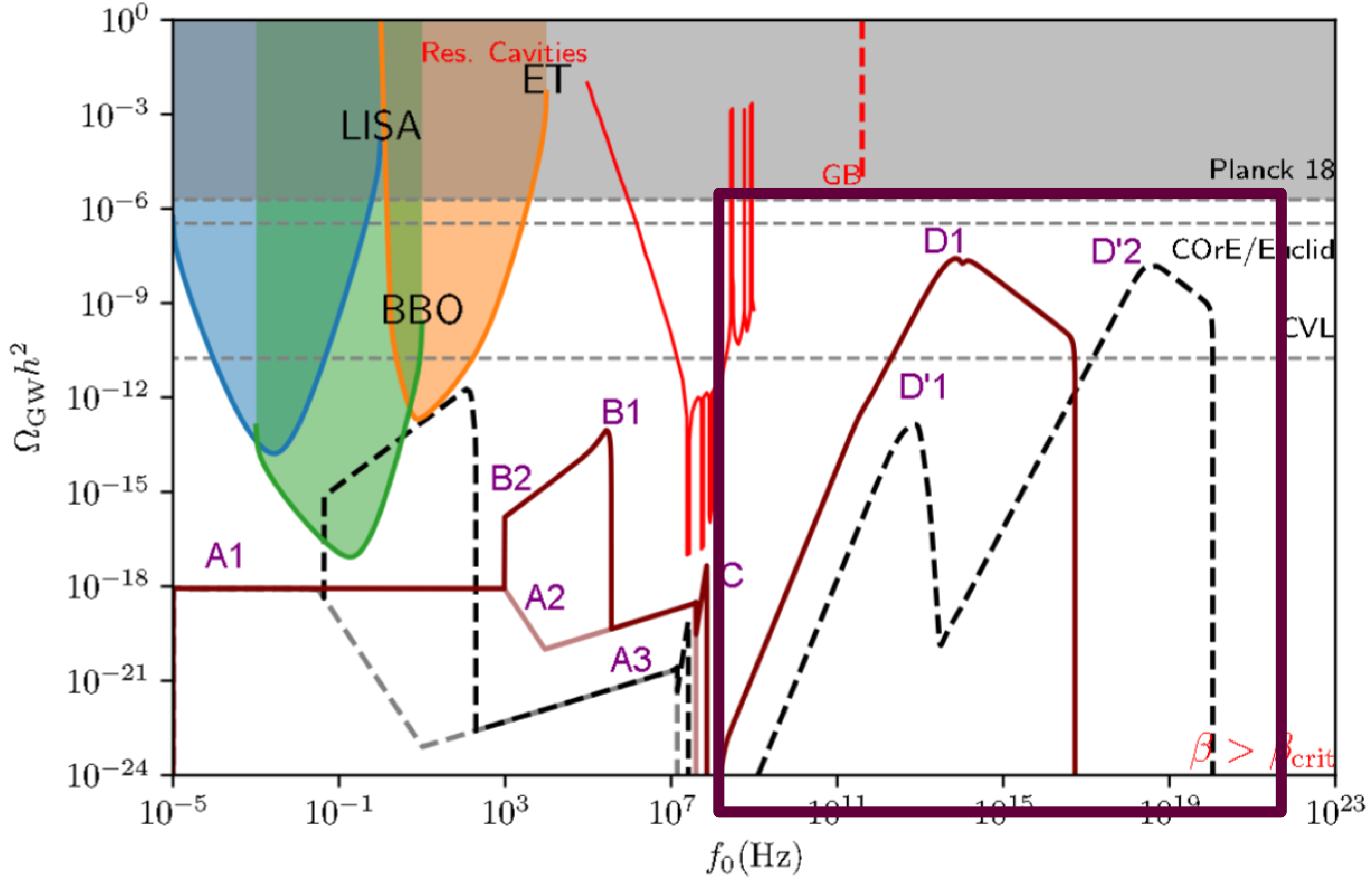
$$\left(\kappa\sqrt{\frac{2}{3}}\sqrt{\frac{2}{3\kappa-1}}\right)^{-\frac{4}{3\kappa}} \left(\frac{k}{k_{\mathrm{UV}}}\right)^{5-\frac{4}{3\kappa}} \left(\frac{k_{\mathrm{UV}}}{k_{\mathrm{RH}}}\right)^{7-\frac{4}{3\kappa}} \left(\frac{k_{\mathrm{BH}}}{k_{\mathrm{UV}}}\right)^{8}$$

$$\Omega_{\mathrm{GW,IR2}} = \frac{C_{\mathrm{s}}^4 C(w)^4 q^4}{36\pi^2} \frac{\kappa}{9\kappa-4} \left(\sqrt{\frac{2}{3}} \frac{\kappa\sqrt{2}}{\sqrt{3\kappa-1}}\right)^{-\frac{4}{3\kappa}}$$

$$\left(\frac{k_{\mathrm{BH}}}{k_{\mathrm{RH}}}\right)^{8} \left(\frac{k_{\mathrm{RH}}}{k_{\mathrm{UV}}}\right)^{\frac{6\kappa+4}{3\kappa}} \left(\frac{k}{k_{\mathrm{UV}}}\right)$$



G. Domenech, J. Tränkle, Phys. Rev. D **111**, 063528
Shyam Balaji *et al* JCAP11(2024)026
Theodoros Papanikolaou *et al* JCAP03(2021)053


Res. Cavities
ET
LISA
BBO
GB
Planck 18
COrE/Euclid
CVL
D1
D'2
D'1
B1
B2
A1
A2
A3
C
$\beta > \beta_{\rm crit}$
$\Omega_{\rm GW}h^2$
$f_0\,({\rm Hz})$
10^{0}
10^{-3}
10^{-6}
10^{-9}
10^{-12}
10^{-15}
10^{-18}
10^{-21}
10^{-24}
10^{-5}
10^{-1}
10^{3}
10^{7}
10^{11}
10^{15}
10^{19}
10^{23}

Hawking evaporation

$$\frac{dM}{dt} = -\left(\theta(M - qM_{in}) + \frac{\theta(qM_{in} - M)}{S^n}\right)\frac{\epsilon M_P^4}{M^2}$$

$$\frac{dN_{grav}}{dtdE} = \frac{1}{2\pi}\frac{\Gamma(M,E)}{e^{E/T_{BH}} - 1}\left(\theta(t_q - t) + \frac{\theta(t - t_q)}{S^n}\right)$$

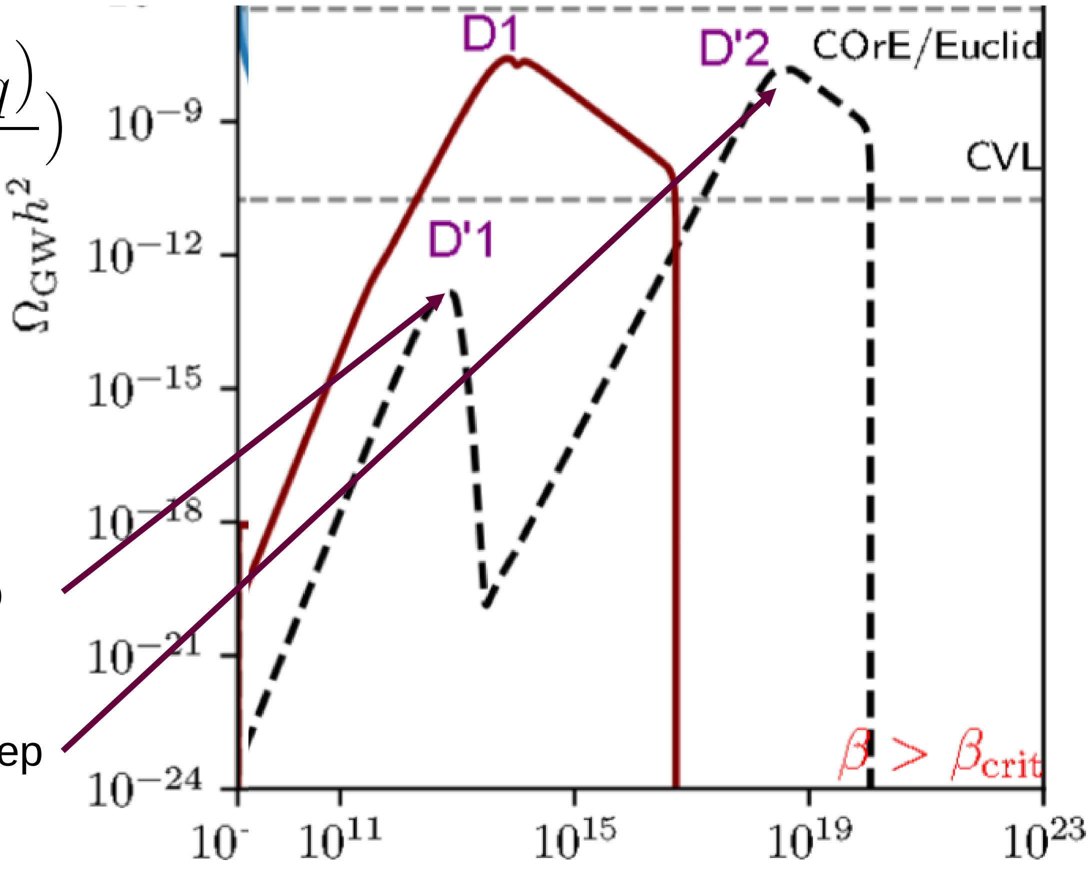
Memory burden slows down Hawking evaporation!

$$f_{D'1,2} \sim T_{BH}\frac{a_{ev1,2}}{a_0}$$

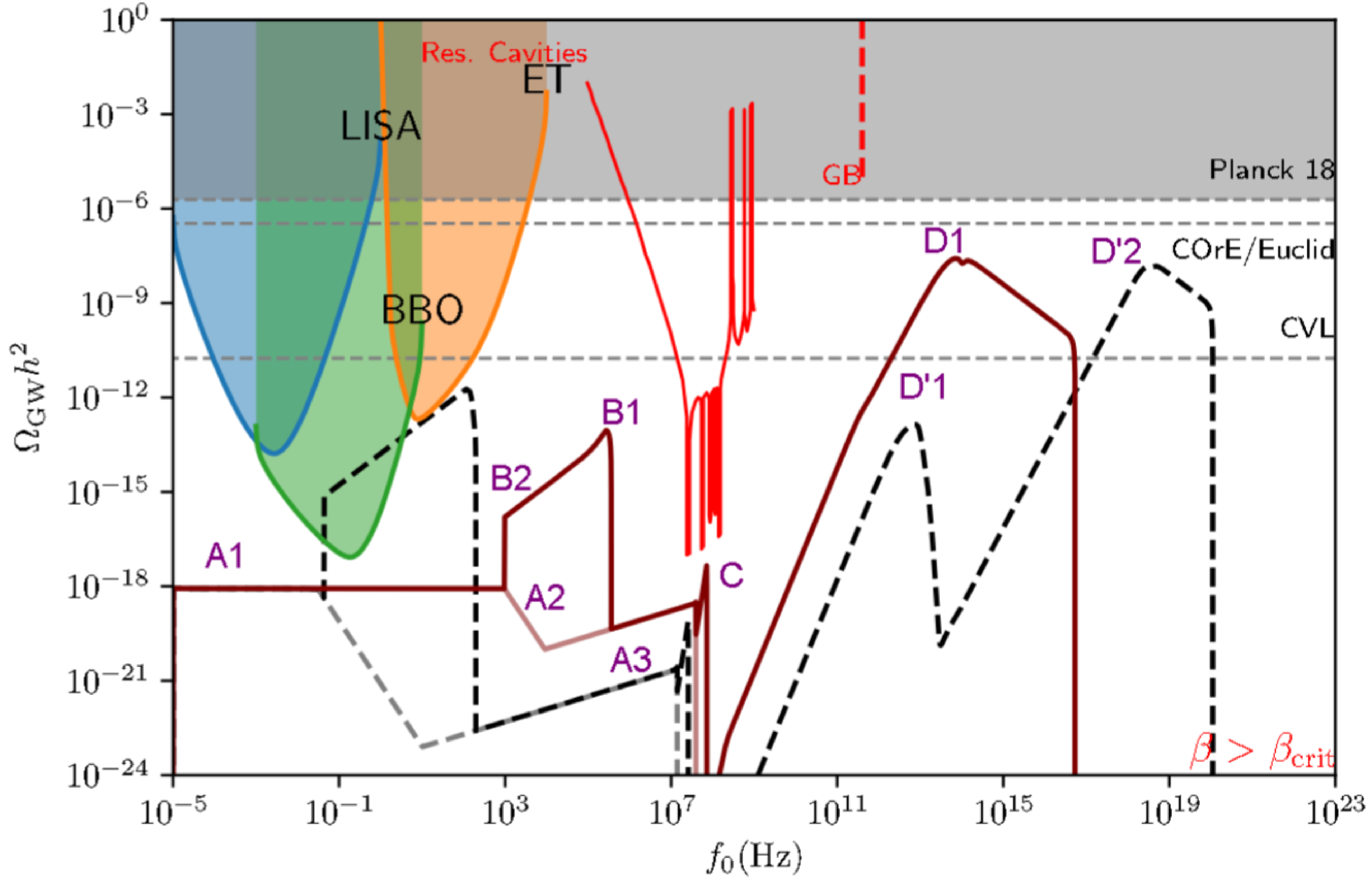
First evaporation step

$$\Omega_{GW}h^2\Big|_{D'2} \sim cst$$

Second evaporation step



The main evaporation of a burdened PBH is comparable to regular evaporation up to a different frequency



Res. Cavities
ET
LISA
GB
Planck 18
COrE/Euclid
D1
D'2
BBO
CVL
D'1
B1
B2
A1
C
A2
A3
$\beta > \beta_{\rm crit}$
$\Omega_{\rm GW} h^2$
$f_0(\rm Hz)$

Thank you !

Backup

Figure 3. GW spectra in the unburden case with $w_\phi = 1/2$ for $M = 1, 10^2$ and 10^4g respectivelly for black, red and green with $\beta = 10^{-5}, 10^{-6}$ and 10^{-7}

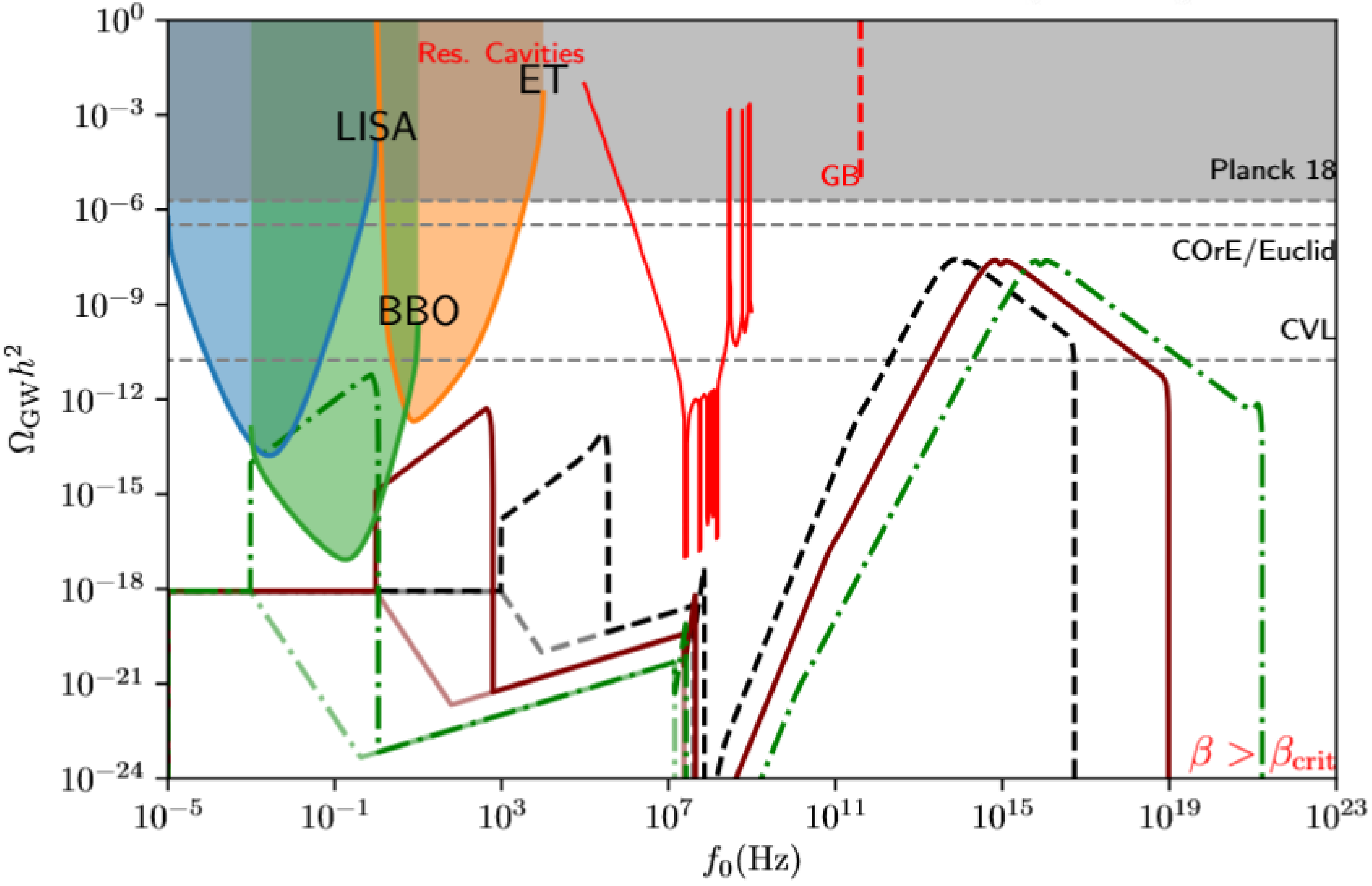
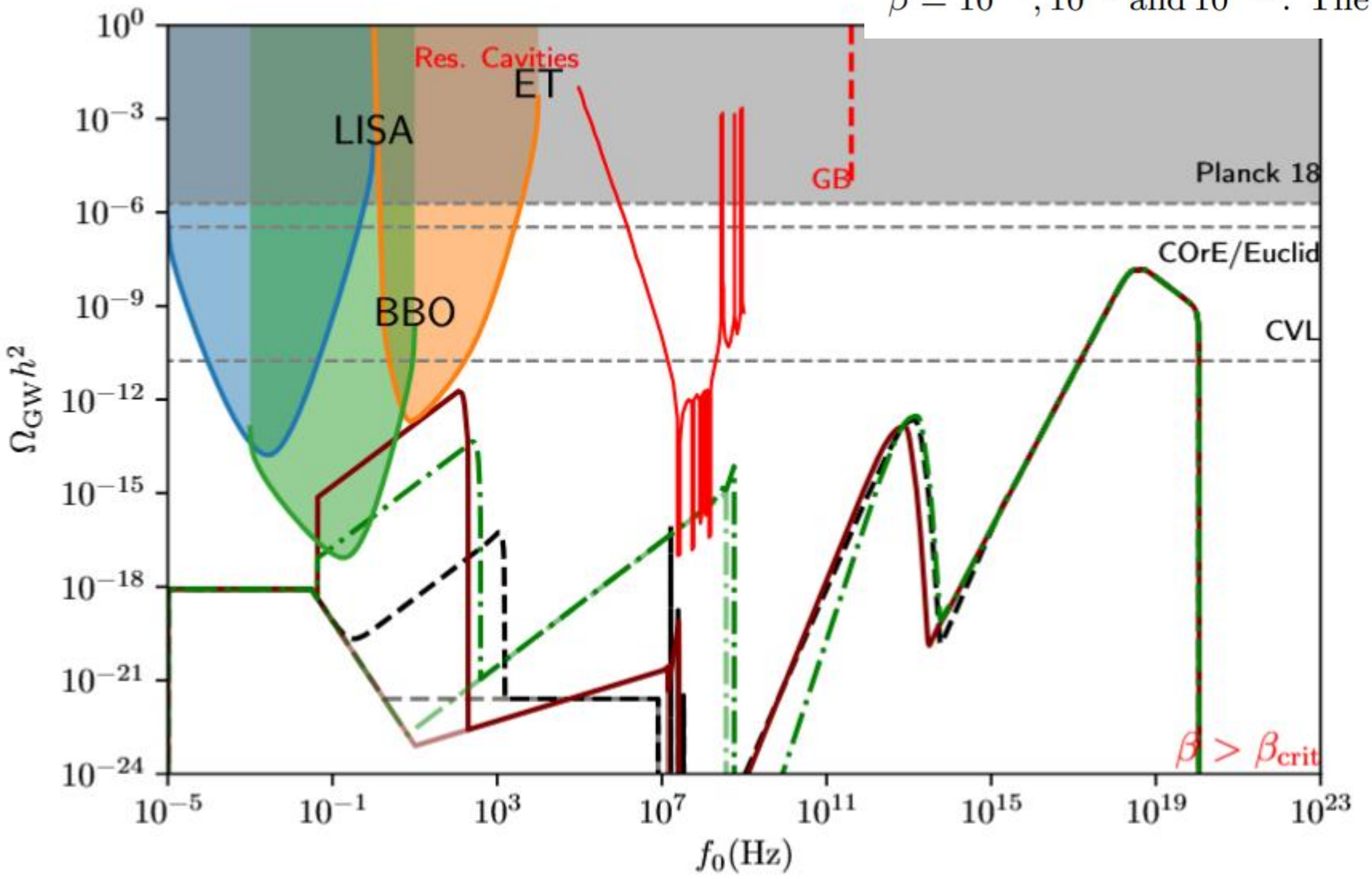
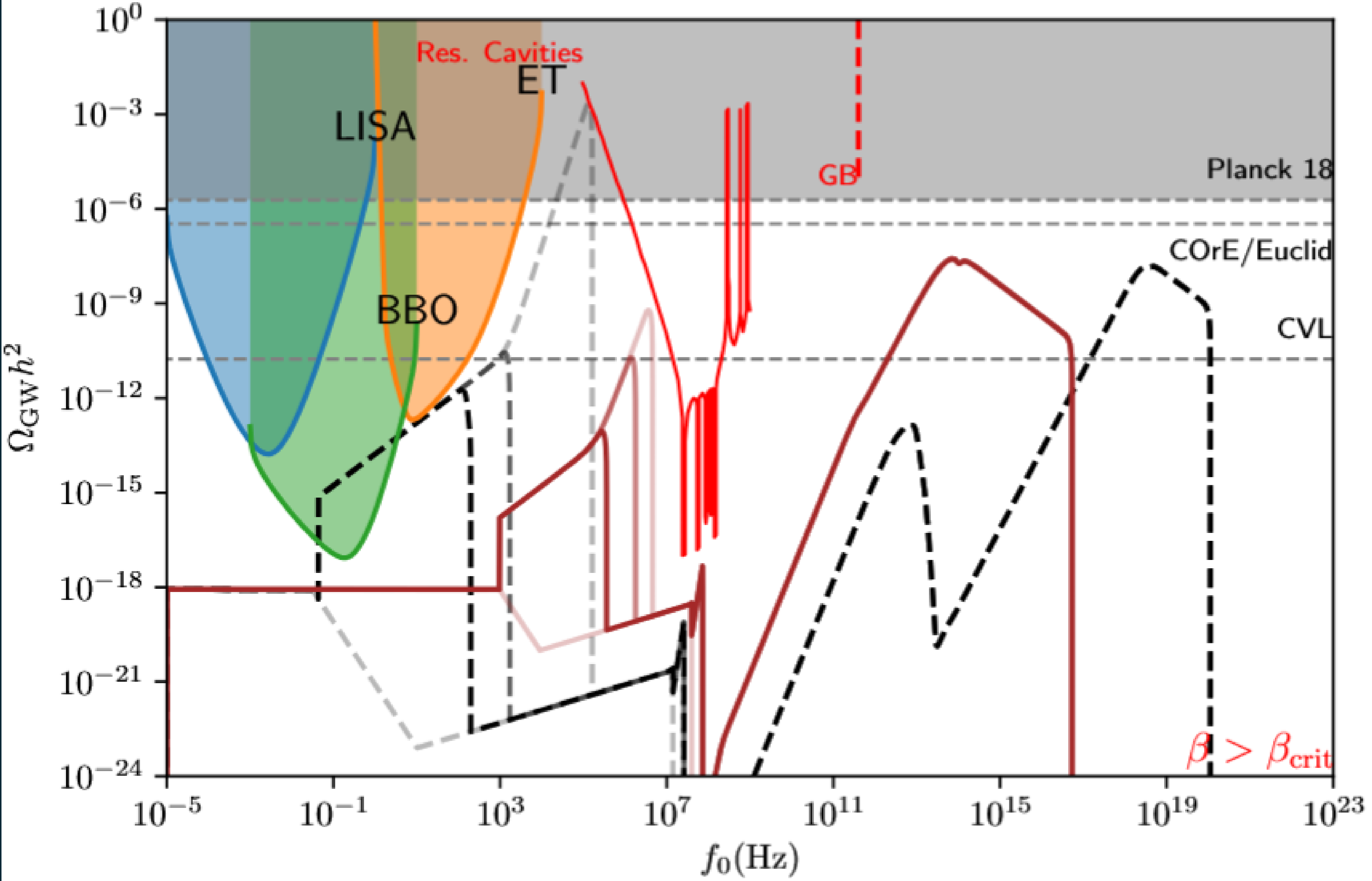


Figure 6. GW spectra in the burden case with $w_\phi = 1/3, 1/2$ and 1 respectivelly for black, red and green with $\beta = 10^{-7}, 10^{-8}$ and 10^{-13}. The PBH mass is fixed to $M = 1g$




Res. Cavities
ET
LISA
BBO
GB
Planck 18
COrE/Euclid
CVL
$\beta > \beta_{\rm crit}$
$\Omega_{\rm GW} h^2$
$f_0(\rm Hz)$
10^0
10^{-3}
10^{-6}
10^{-9}
10^{-12}
10^{-15}
10^{-18}
10^{-21}
10^{-24}
10^{-5}
10^{-1}
10^{3}
10^{7}
10^{11}
10^{15}
10^{19}
10^{23}