# Quantum (and classical) detection of gravitational waves: scope and limitations

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**Contributed talk**

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**Based on:** C. Beadle, PB, R.T. D'Agnolo, S.A.R. Ellis, arXiv:25XX.XXXXX

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**Can we analytically confirm this statement, starting from an *ab initio* computation in quantum mechanics and taking into account all form factors?**

# Motivation

- **Why high frequencies?**

**Characteristic wavelength** → 

$$\begin{cases} \lambda_* \leq H_*^{-1} & \text{(Process occurring at temperature } T_*\text{)} \\ \omega_0 = a(t_*)/a(t_0)\,\omega_* & \text{(Redshift of gravitons)} \end{cases}$$

Signal at GUT scale:

**# d.o.f.** → $g_*$

$$\omega_0 \gtrsim 100 \text{ MHz} \left(\frac{T_*}{10^{15} \text{ GeV}}\right) \left(\frac{g_*(T_*)}{100}\right)^{1/6}$$

- **What could have produced the GW stochastic background?**

Vacuum fluctuations, phase transitions, cosmic strings, domain walls,…
[M. Maggiore, *Gravitational waves* (Oxford University Press, 2007); Aggarwal *et al.*, *Living Rev. Rel.* **24**, 4 (2021)]

<u>*Other interesting signals*</u>: **Primordial black holes & superradiance**

# Main shortcoming: the minimal detectable strain

- The study of **cosmological stochastic backgrounds of GWs** can be performed by means of an ***energy density*** *per unit logarithmic interval of frequency:*

$$\Omega_g(\omega) \equiv \frac{8\pi G_N}{3H_0^2}\frac{d\rho_g(\omega)}{d\log\omega}$$

GW energy density

Signal-to-Noise ratio

$$\mathrm{SNR} = \left(t_{\mathrm{int}}\int\frac{\mathrm{d}\omega}{2\pi}\frac{\Omega_g(\omega)^2}{\Omega_n(\omega)^2}\right)^{\frac{1}{2}} \simeq 1$$

→ Minimal detectable (PSD) strain

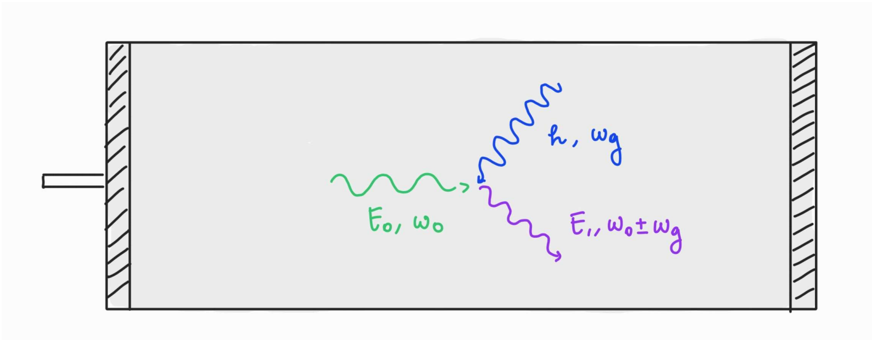
- **Bound on minimal signal strength for detection of GWs coming from primordial backgrounds** [M. Kawasaki *et al.*, Phys. Rev. Lett. 82, 4168 (1999) & Phys. Rev. D 62, 023506 (2000), M. Maggiore, *Physics Reports* 331 (2000), 283-367]

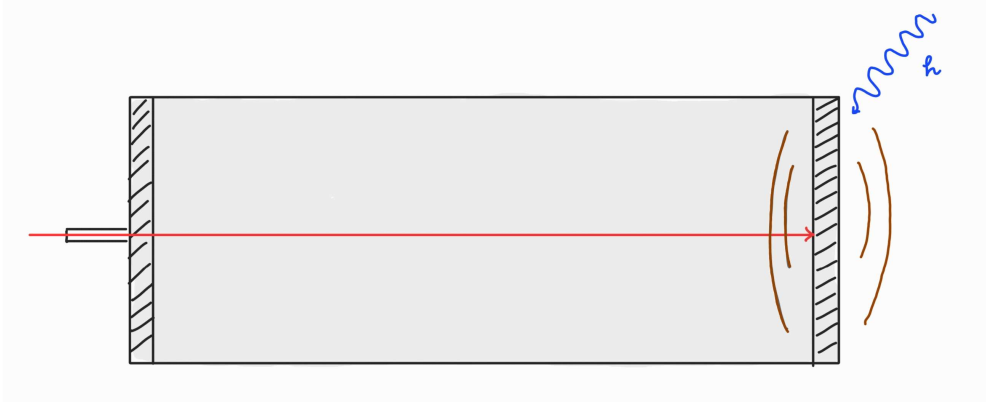
Reduced hubble parameter

$$\int d\log\omega \, h_{\mathrm{eff}}^2\Omega_g(\omega) \lesssim 5\times 10^{-6}\Delta N_{\mathrm{eff}}$$

Uncertainty on the number of neutrino species

# Two toy models to describe (almost) any detector




## EM resonators

- Large static magnetic field
- **Readout:** $\omega_1 \approx \omega_g$
- **MADMAX** [arXiv:2409.06462], **CAST** [arXiv:1705.02290], **IAXO** [*Eur. Phys. J. C* **79** (2019) 1032]

- Transition mode 0 (loaded) → 1 (readout)
- **Readout:** $\omega_1 = \omega_0 \pm \omega_g$
- **MAGO** [*Phys. Rev. D* **108** (2023) 084058]

**Resonant EM microwave cavities** [*Physical Review* D **105** 116011 (2022)], **Lumped LC resonators** [*Phys. Rev. Lett.* **129** (2022) 041101]

## Mechanical resonators

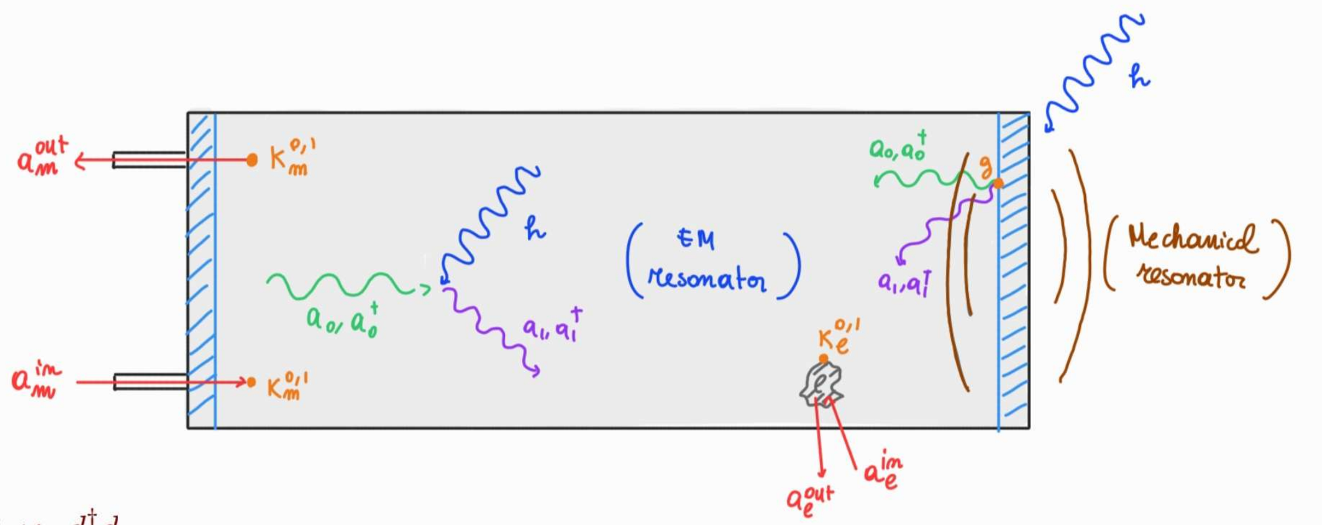
- Like test masses. Their position is measured through an **EM readout**

- **Interferometers** (**LVK**, **Holometer**), **Optomechanical sensors** (**levitating sphere** [A. Arvanitaki, A. A. Gercai, *Phys. Rev. Lett.* **110** (2013) 071105]), **Weber bars** (**AURIGA** [M. Cerdonio *et al.*, *Classical and Quantum Gravity* **14** (1997) 1491]), **Magnetic Weber bars** [V. Domcke, S.A.R. Ellis, N. L. Rodd, *Phys. Rev. Lett. 134, 231401*]

# Quantum mechanical set-up

**Prototypical system**

$$H(t) = H_0(t) + H_{G+OM}(t) + H_R(t)$$




**Free** $$H_0(t) = \sum_{n,r} \Delta_n a^\dagger_{n,r} a_{n,r} + \int_V d^3x |B_0|^2 + \omega_m d^\dagger d$$

**Int.** $$H_{G+OM}(t) = \int_V \mathrm{d}^3 x h_{\mu\nu} T^{\mu\nu} + gxX_1 = h(t) \left\{ \sum_{jj'} C_{jj'} a^\dagger_j(t) a_{j'}(t) + \sum_j \left[ D_j a_j(t) + D^*_j a^\dagger_j(t) \right] \right\} + gxX_1$$

Back-action

**Readout** $$H_R(t) = \sum_{j=0}^{1} \sum_l \int \mathrm{d}\omega \left\{ \omega\, b^\dagger_l(\omega) b_l(\omega) + i g^j_l \left[ b^\dagger_l(\omega) a_j(t) - b_l(\omega) a^\dagger_j(t) \right] \right\}$$

Thermal baths

# Procedure

- **Input-output formalism** [Beckey *et al.*, arXiv:2311.07270]

$$X_n = \frac{a_n + a_n^\dagger}{\sqrt{2}}, \quad Y_n = -\frac{i(a_n - a_n^\dagger)}{\sqrt{2}}$$

  - EoMs

$$\begin{cases} \dot{X}_n(t) = \Delta_n Y_n(t) + \sum_j h(t)\mathrm{Re}(C_{nj})Y_j(t) + \sum_j h(t)\mathrm{Im}(C_{nj})X_j(t) \mp \frac{1}{2}\sum_k \tilde{k}_j^n X_j(t) + \sum_\Lambda \sqrt{\kappa_\Lambda^n} X_\Lambda^{\substack{\mathrm{in}\\\mathrm{out}}}(t) + F_X^n(t) \\ \dot{Y}_n(t) = -\Delta_n X_n(t) - \sum_j h(t)\mathrm{Re}(C_{nj})X_j(t) + \sum_j h(t)\mathrm{Im}(C_{nj})Y_j(t) \mp \frac{1}{2}\sum_k \tilde{k}_j^n Y_j(t) + \sum_\Lambda \sqrt{\kappa_\Lambda^n} Y_\Lambda^{\substack{\mathrm{in}\\\mathrm{out}}}(t) + F_Y^n(t) \\ \dot{x}(t) = \frac{p(t)}{M} \\ \dot{p}(t) = -M\omega_m^2 x(t) - \gamma_m p(t) - gX_1(t) + F_m(t) \end{cases}$$

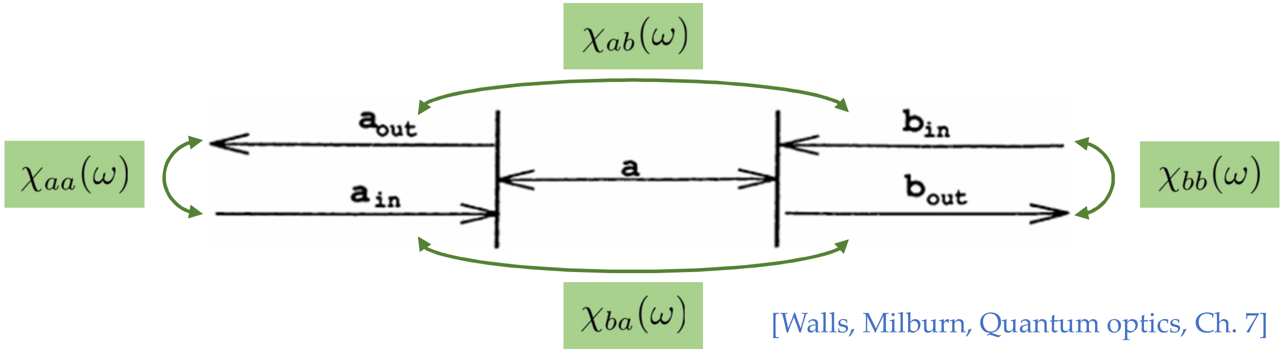
**EM**

**Mech.**

  - Input-output relations

$$\begin{cases} X_\Lambda^{\mathrm{out}} = X_m^{\mathrm{in}} - \sum_j \sqrt{\kappa_\Lambda^j} X_j \\ Y_\Lambda^{\mathrm{out}} = Y_m^{\mathrm{in}} - \sum_j \sqrt{\kappa_\Lambda^j} Y_j \end{cases}$$

Transfer functions →




[Walls, Milburn, Quantum optics, Ch. 7]

# Power Spectral Densities

- **Power Spectral Density:** $\langle A(t)B^\dagger(t')\rangle = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} e^{i\omega(t-t')} S_{AB}(\omega)$

- Quadrature PSD:

$$
\begin{aligned}
S^{\text{out}}_{Y_m Y_m}(\omega) = &\sum_\Lambda \Big[ |\chi_{Y_m Y_\Lambda}(\omega)|^2 S^{\text{in}}_{Y_\Lambda Y_\Lambda}(\omega) + |\chi_{Y_m X_\Lambda}(\omega)|^2 S^{\text{in}}_{X_\Lambda X_\Lambda}(\omega) \Big] \\
&+ \sum_\Lambda \Big[ \chi_{Y_m X_\Lambda}(\omega)\chi_{Y_m Y_\Lambda}(\omega)^* S^{\text{in}}_{Y_\Lambda X_\Lambda}(\omega) + \chi_{Y_m Y_\Lambda}(\omega)\chi_{Y_m X_\Lambda}(\omega)^* S^{\text{in}}_{X_\Lambda Y_\Lambda}(\omega) \Big] \\
&+ \sum_I |\chi_{Y_m F_I}(\omega)|^2 S_{F_I F_I}(\omega) \supset S_{hh}(\omega)
\end{aligned}
$$

Noise

Signal

# Minimal detectable strain (PSD)

- **Signal-to-Noise-Ratio:**

$$\mathrm{SNR} = \left( t_{\mathrm{int}} \int \frac{\mathrm{d}\omega}{2\pi} \frac{S_{hh}(\omega)^2}{S_{nn}(\omega)^2} \right)^{\frac{1}{2}} \simeq 1$$

- **Ingredients:**

$$\begin{cases} S_{hh}(\omega) = S_{hh}(\omega_s)\,[\Theta(\omega - \omega_s + \Delta\omega) - \Theta(\omega - \omega_s - \Delta\omega)] \\ S_{nn}(\omega) = \dfrac{S^{\mathrm{out}}_{Y_m Y_m}|_{\mathrm{NOISE}}}{|\chi_{Y_m h}(\omega)|^2} \end{cases}$$

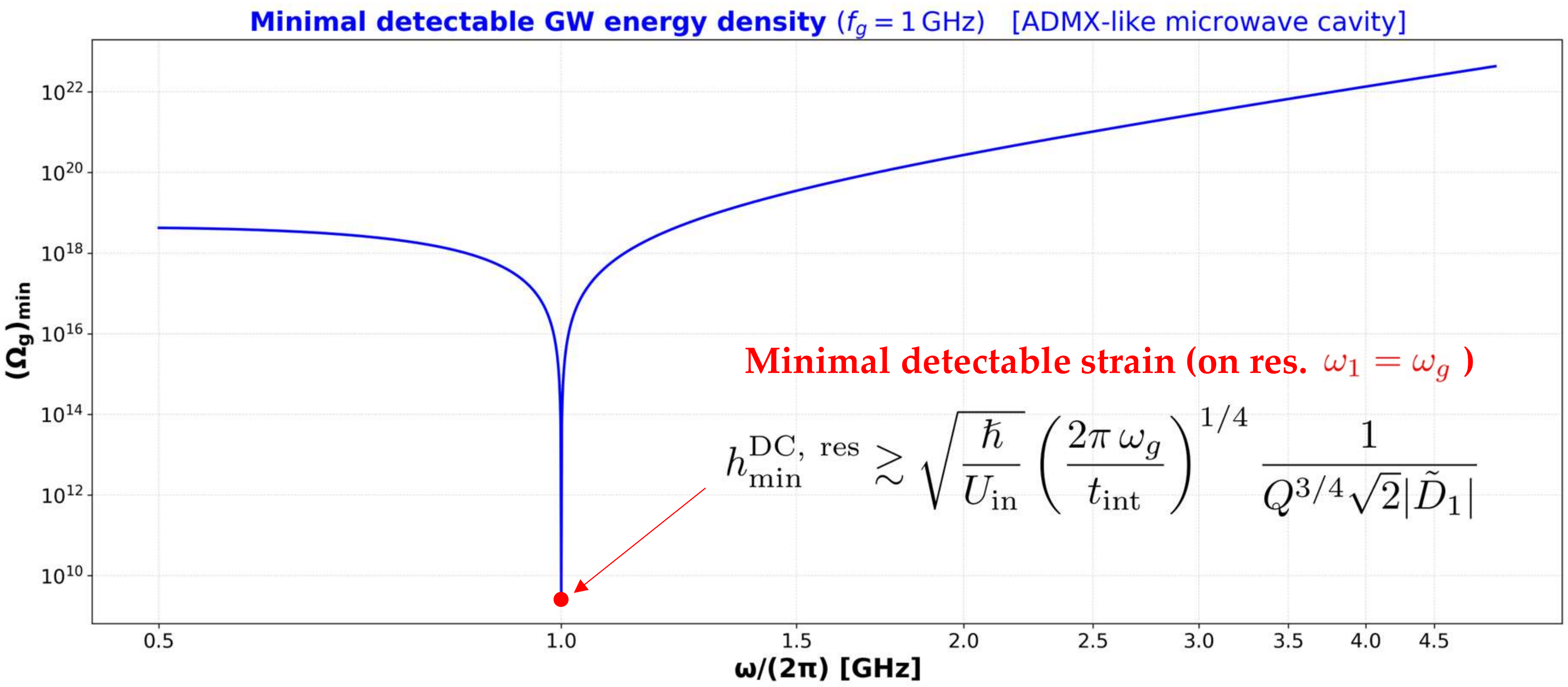
**Relation between the h-PSD and the GW energy density**

$$\Omega_g(\omega) = \frac{\omega^3\, S_{hh}(\omega)}{24\pi H_0^2}$$

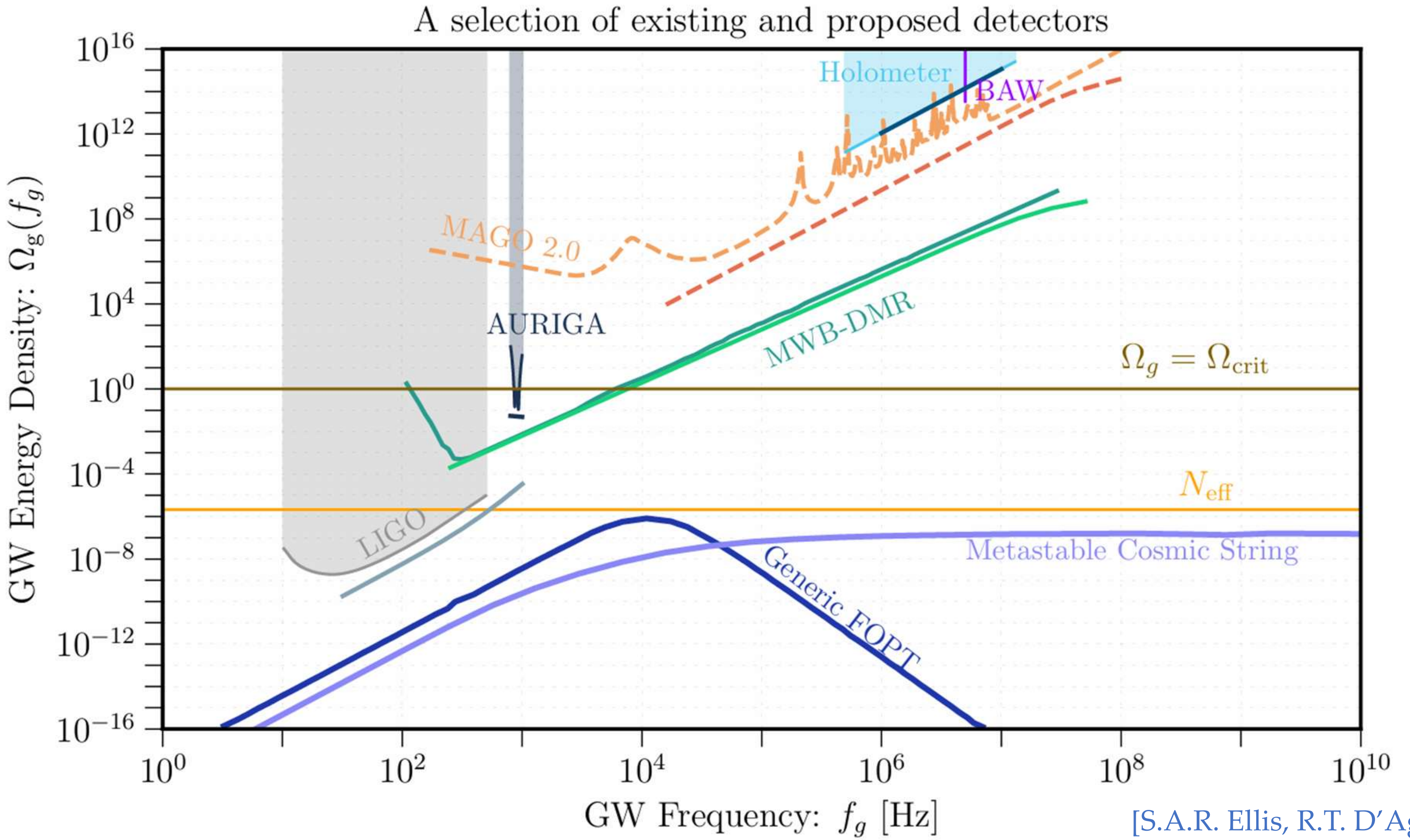
$$(\Omega_g(\omega))_{\mathrm{min}} = \frac{\omega^3}{24\pi H_0^2} \left[ t_{\mathrm{int}} \int_{\omega-\Delta\omega}^{\omega+\Delta\omega} \frac{\mathrm{d}\omega'}{2\pi} \left( \frac{|\chi_{Y_m h}(\omega')|^2}{S^{\mathrm{out}}_{Y_m Y_m}(\omega')|_{\mathrm{NOISE}}} \right)^2 \right]^{-\frac{1}{2}}$$

# An example: EM resonator w/ an external static magnetic field

- **PSD:** $$S_{nn}(\omega) = \frac{(\kappa_1^2/4 + \omega_1^2 - \omega^2)^2 + \kappa_1^2\Omega^2}{2\kappa_m^1\left\{[\omega_1 \mathrm{Im}D_1 - \kappa_1/2\,\mathrm{Re}D_1]^2 + \omega^2 \mathrm{Re}{D_1}^2\right\}} S^{\mathrm{out}}_{Y_m Y_m}(\omega)$$

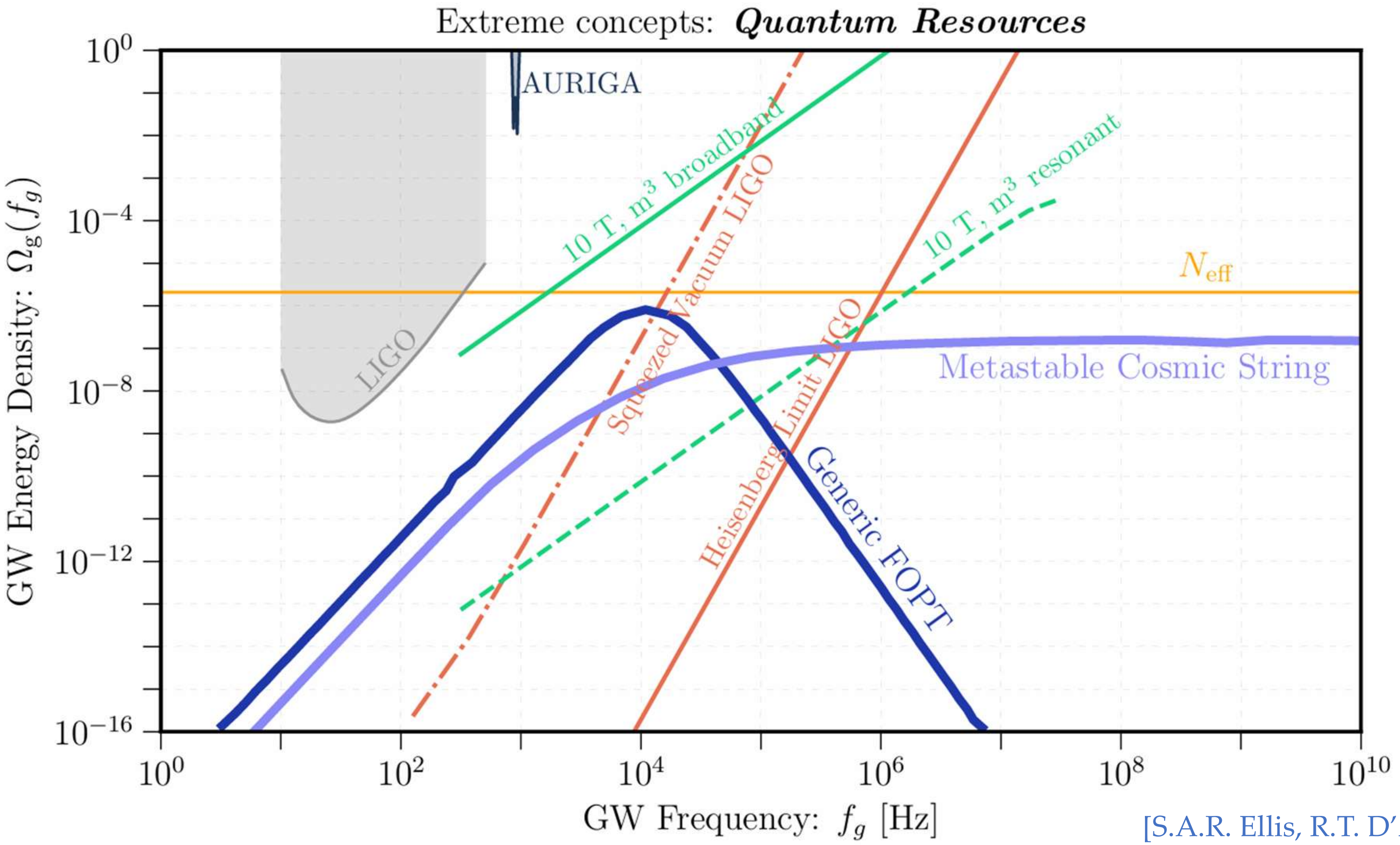


# Classical heuristics




[S.A.R. Ellis, R.T. D'Agnolo, arXiv:2412.17897]

# Quantum heuristics




[S.A.R. Ellis, R.T. D'Agnolo, arXiv:2412.17897]

# Conclusions (again!) and outlook

- The novel analysis based on the input-output formalism is going to give **analytic bounds (including form factors)** on GW detection and accurately tell us what we can expect from present and near-future GW detectors

- **However we expect the precise quantum computations to confirm the conclusions drawn from the heuristics**

*Upshot:* **High-frequency gravitational waves coming from primordial backgrounds might remain out of the experimental reach of current detectors**

# Backup slides

# Primordial black holes

**Motivations:**

1. **PBHs may constitute (at least) a fraction of DM**
2. **Their production requires new physics**
   a. New mechanism enhancing curvature perturbations
   b. New long-range force

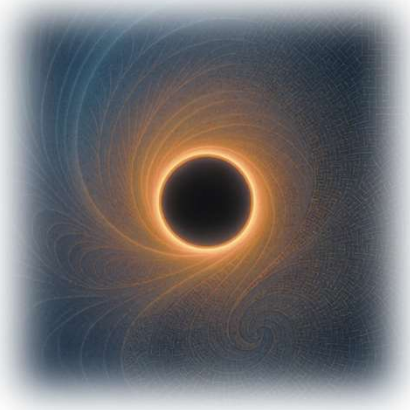
**Expected strain from inspirals**

$$h \simeq 10^{-29} \left(\frac{1\text{pc}}{d}\right) \left(\frac{M_{\text{PBH}}}{10^{-11} M_{\odot}}\right) \left(\frac{\omega_g}{\text{GHz}}\right)$$

**Challenges:**

1. **The expected strain is still out of reach**

2. **Signals coming from the most massive PBH binaries (the easiest to detect) are confined in the inspiral regime (very short coherence times)**



# Superradiance

**Motivation:**

**Given their simple waveform and long coherence time, superradiance singals are the easiest to detect**

→ Almost monochromatic signal around:

$$\omega_g = 2m_{\rm boson} \simeq 10^{-6}\ {\rm eV} \times \left(\frac{10^{-4} M_\odot}{M_{\rm PBH}}\right)$$

**Expected strain from annihilations**

$$h \simeq 10^{-27} \left(\frac{1{\rm kpc}}{d}\right) \left(\frac{10^{-4} M_\odot}{M_{\rm PBH}}\right)$$

[A. Arvanitaki and A. A. Geraci, Phys. Rev. Lett. **110**, 071105 (2013)]

**Challenge:**

**There is, however, a cut-off given by the large amount of energy that is required to be stored in the detector for detecting such small strains**