Testing theories of gravity with planetary ephemerides

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Living Reviews in Relativity

Aims and scope →

Agnès Fienga & Olivier Minazzoli

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Experimental Foundations of Gravitation

Abstract

We describe here how planetary ephemerides are built in the framework of General Relativity and how they can be used to test alternative theories. We focus on the definition of the reference frame (space and time) in which the planetary ephemeris is described, the equations of motion that govern the orbits of solar system bodies and electromagnetic waves. After a review on the existing planetary and lunar ephemerides, we summarize the results obtained considering full modifications of the ephemeris framework with direct comparisons with the observations of planetary systems, with a specific attention for the PPN formalism. We then discuss other formalisms such as Einstein-dilaton theories, the massless graviton and MOND. The paper finally concludes on some comments and recommendations regarding misinterpreted measurements of the advance of perihelia.

Sections | Figures | References

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Common issue in the literature

Example with a Yukawa suppression of gravity

Modified Newtonian potential:

$$U = \sum_a \frac{Gm_a}{|\boldsymbol{x} - \boldsymbol{x}_a|} \mathrm{e}^{-|\boldsymbol{x}-\boldsymbol{x}_a|/\lambda_\mathrm{g}}$$
$$= \sum_a \frac{Gm_a}{|\boldsymbol{x} - \boldsymbol{x}_a|} + \frac{1}{2} \sum_a \frac{Gm_a}{\lambda_\mathrm{g}^2} |\boldsymbol{x} - \boldsymbol{x}_a| + O(Gmr^2/\lambda_\mathrm{g}^3),$$

Modified equation of motion:

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = -\frac{Gm\boldsymbol{n}}{r^2}\left(1 - \frac{1}{2}\frac{r^2}{\lambda_\mathrm{g}^2}\right),$$

Perihelion advance per orbit (assuming 2 bodies only):

$$\Delta\varpi = \pi \left(\frac{a}{\lambda_\mathrm{g}}\right)^2 (1 - e^2)^{-1/2},$$

Common issue in the literature
Example with a Yukawa suppression of gravity

Perihelion advance per orbit (assuming 2 bodies only):

$$\Delta\varpi = \pi \left(\frac{a}{\lambda_g}\right)^2 (1 - e^2)^{-1/2},$$

Deduction of λ_g from uncertainties:

Table 1. Bounds on λ_g.

Planet	$\sigma(\dot{\varpi})$ (mas yr^{-1})	λ_g bound (10^{14} km)
Data from table 4 of [16]		
Mercury	0.03	0.18
Venus	0.016	0.28
Earth	0.0019	0.88
Mars	0.000 37	2.21
Jupiter	0.28	0.11
Saturn	0.0047	0.98
Data from [17]		
Mercury	0.02	0.22
Saturn	0.026	0.42

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Common issue in the literature

Example with a Yukawa suppression of gravity

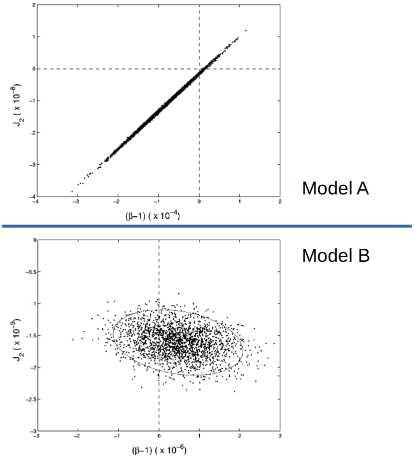
Correlations in INPOP17b:

	λ_g	a Mercury	a Mars	a Saturn	a Venus	a EMB	$GM_\odot$
λ_g	1	0.50	0.49	0.04	0.39	0.05	0.66
a Mercure	...	1	0.21	0.001	0.97	0.82	0.96
a Mars	...	...	1	0.03	0.29	0.53	0.06
a Saturn	...	...	...	1	0.003	0.02	0.01
a Venus	...	...	...	...	1	0.86	0.94
a EMB	...	...	...	...	...	1	0.73
$GM_\odot$	...	...	...	...	...	...	1

Bernus et al. [2019]

$\sim$ 400 parameters to adjust!

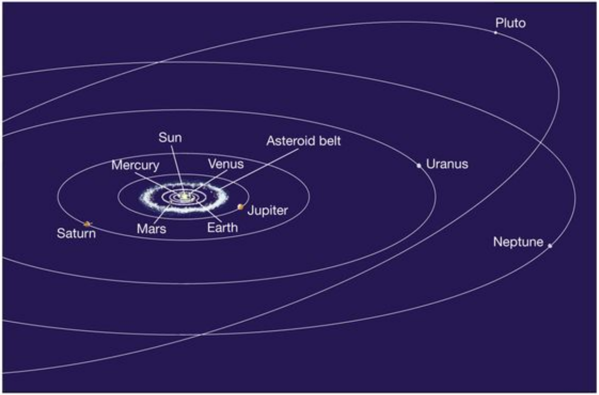
Another example of correlation: between J_2 and β



Milani et al. [2002]

Correlations: why?

All motions $\in\sim$ ecliptic plane & $\sim$ small eccentricities



Correlations: why?

Spread in methods, data types and accuracies

Planet/type [unit]	s/c or method	Period	Averaged accuracy
Mercury			
Direct range [m]	Surface	1971.29: 1997.60	900
Radio science range [m]	Messenger	2011.23: 2014.26	5
Navigation range [m]	Mariner	1974.24: 1976.21	100
Mercury			
VLBI [mas]	Magellan, Venus Express	1990.70: 2013.14	2.0
Direct range [m]	Surface	1965.96: 1990.07	1400
Navigation range [m]	Venus Express	2006.32: 2011.45	7.0
Mars			
VLBI [mas]	MGS, MRO	1989.13: 2013.86	0.3
Navigation range [m]	Mars Express	2005.17: 2019.37	2.0
Radio Science range [m]	MGS	1999.31: 2006.70	2.0
Radio Science range [m]	MRO/MO	2002.14: 2014.00	1.2
Navigation range [m]	Viking	1976.55: 1982.87	20.0
Jupiter			
VLBI [mas]	Galileo	1996.54: 1997.94	11
	Juno	2016:2020	0.5
Optical RA/Dec [arcsec]	Transit+CCD	1924.34: 2008.49	0.3
Flyby RA/Dec [mas]	U,C, P, V	1974.92: 2001.00	8.0
Flyby range [m]	U,C, P, V	1974.92: 2001.00	2000
Radio science range [m]	Juno	2016.65: 2020.56	20
Saturn			
Optical RA/Dec [arcsec]	Transit+CCD	1924.22: 2008.34	0.3
VLBI RA/Dec [mas]	Cassini	2004.69: 2017.9	0.6
JPL H14 [m]	Cassini	2004.41: 2014.38	25.0
Navigation [m]	Cassini	2006.01: 2009.83	6.0
Radio science: Titan flybys [m]	Cassini	2006.01: 2016.61	15.0
Radio science: grand finale) [m]	Cassini	2017.35: 2017.55	1.0

Correlations: why?

Temporal spread: accurate observations are rarely simultaneous

Planet/type [unit]	s/c or method	Period	Averaged accuracy
Mercury			
Direct range [m]	Surface	1971.29: 1997.60	900
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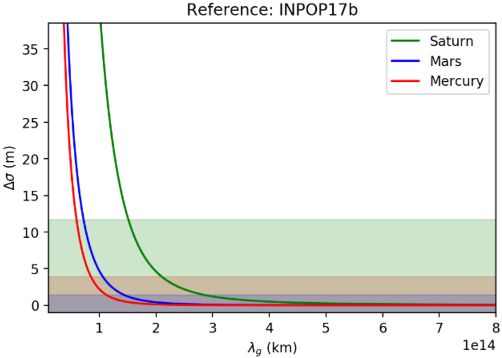
Effect of correlations

Example with a Yukawa suppression of gravity

Standard deviation without readjusting model parameters:

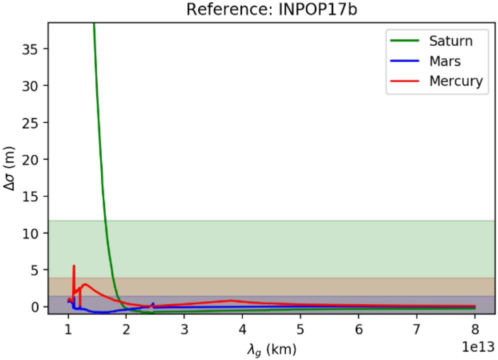


Bernus et al. [2019]

Effect of correlations

Example with a Yukawa suppression of gravity

Standard deviation after readjusting model parameters:



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a Saturn	⋯	⋯	⋯	1	0.003	0.02	0.01
a Venus	⋯	⋯	⋯	⋯	1	0.86	0.94
a EMB	⋯	⋯	⋯	⋯	⋯	1	0.73
$GM_\odot$	⋯	⋯	⋯	⋯	⋯	⋯	1

Bernus et al. [2019]

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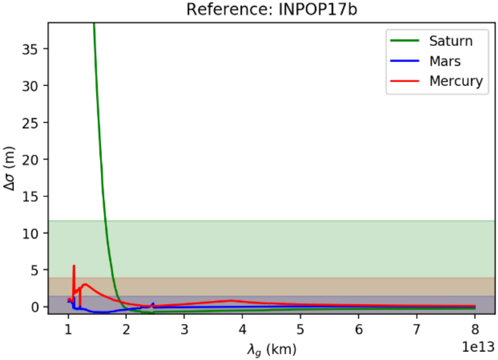
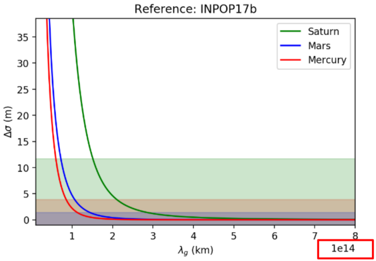


Bernus et al. [2019]

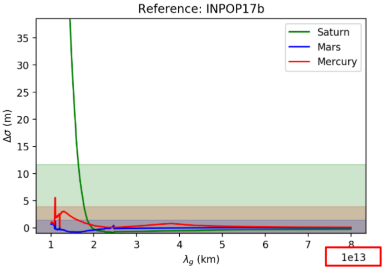
Effect of correlations

Example with a Yukawa suppression of gravity

Absorption of the effect of λ_g during the fit:



Without fit



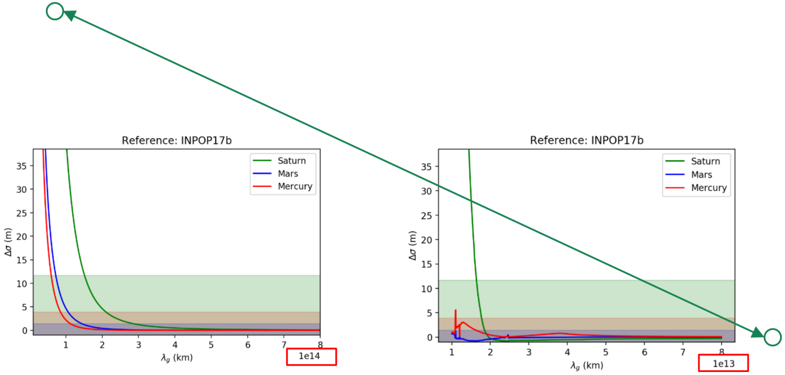
With fit

Bernus et al. [2019]

Effect of correlations

Example with a Yukawa suppression of gravity

Absorption of the effect of λ_g during the fit:



Without fit　　　　With fit

Bernus et al. [2019]

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Perihelion advances are not measured

Table 1. Bounds on λ_g.

Planet	$\sigma(\dot{\varpi})$ (mas yr^{-1})	λ_g bound (10^{14} km)
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The whole picture

The post-Newtonian framework is not only about the equations of motion!

- Dynamics of massive bodies
- Conserved quantities from Lagrangian or Hamiltonian of motion
- Shapiro effect on electromagnetic wave propagation
- Production of coordinate time for synchronisation between all clocks

Conclusion: The whole picture

Beware of correlations between parameters!

- Parameters must be adjusted in a novel theoretical framework

- Perihelion advances are deduced from the ephemerides (i.e., they are model-dependent).

The post-Newtonian framework is not only about the equations of motion!

- Dynamics of massive bodies
- Conserved quantities from Hamiltonian of motion
- Shapiro effect on electromagnetic wave propagation
- Production of coordinate time for synchronisation between all clocks

References I

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Accuracy of ephemerides

Ephemerides	Gaillot (Gaillot and Le Verrier 1913)		DE102 (Standish 1983)		DE440/INPOP19a (Park et al. 2021; Fienga et al. 2019)		GR
Data span	1800–1913		1913–1983		1924–2021		
	Angle ”	Distance Earth-km	Angle ”	Distance Earth-km	Angle ”	Distance Earth-km	$\Delta\dot{\varpi}$ ”.yr^{-1}
Mercury	1	450	0.050	5	0.002	0.004	0.43
Venus	0.5	100	0.050	2	0.002	0.006	0.14
Mars	0.5	150	0.050	0.050	0.001	0.0015	0.065
Jupiter	0.5	1400	0.1	10	0.010	0.020	0.019
Saturn	0.5	3000	0.1	600	0.001	0.020	0.010
Uranus	1	12,700	0.2	2540	0.050	10	0.005
Neptune	1	22,000	0.2	4400	0.050	50	0.0033
Pluto	1	24,000	0.2	4800	0.050	2400	0.0027

The whole picture

Dynamics: coordinate acceleration of massive bodies

$$
\begin{aligned}
\boldsymbol{a}_T = &-\sum_{A\neq T}\frac{\mu_A}{r_{AT}^3}\boldsymbol{r}_{AT} \\
&-\sum_{A\neq T}\frac{\mu_A}{r_{AT}^3 c^2}\boldsymbol{r}_{AT}\left\{v_T^2 + 2v_A^2 - 4\boldsymbol{v}_A\cdot\boldsymbol{v}_T - \frac{3}{2}\left(\frac{\boldsymbol{r}_{AT}\cdot\boldsymbol{v}_A}{r_{AT}}\right)^2\right. \\
&\left. -\frac{1}{2}\boldsymbol{r}_{AT}\cdot\boldsymbol{a}_A - 4\sum_{B\neq T}\frac{\mu_B}{r_{TB}} - \sum_{B\neq A}\frac{\mu_B}{r_{AB}}\right\} \\
&+\sum_{A\neq T}\frac{\mu_A}{c^2 r_{AT}^3}\left[4\boldsymbol{r}_{AT}\cdot\boldsymbol{v}_T - 3\boldsymbol{r}_{AT}\cdot\boldsymbol{v}_A\right](\boldsymbol{v}_T-\boldsymbol{v}_A) \\
&+\frac{7}{2}\sum_{A\neq T}\frac{\mu_A}{c^2 r_{AT}}\boldsymbol{a}_A\,,
\end{aligned}
$$

The whole picture

Dynamics: Lagrangian, Hamiltonian and first integrals

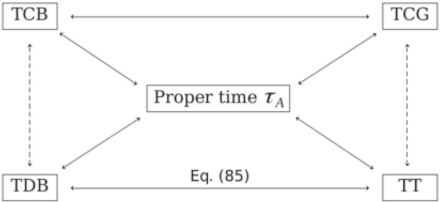
$$
\begin{aligned}
L_B = & - m_B c^2 + m_B \frac{v_B^2}{2} + \sum_{A \neq B} \frac{G m_A m_B}{r_{AB}} + \frac{m_B v_B^4}{8c^2} \\
& + \frac{1}{c^2} \sum_{A \neq B} \frac{G m_A m_B}{r_{AB}} \left[-4 \boldsymbol{v}_A \cdot \boldsymbol{v}_B + \frac{3}{2} v_B^2 + 2 v_A^2 - \frac{(\boldsymbol{v}_A \cdot \boldsymbol{r}_{AB})^2}{2 r_{AB}^2} - \frac{\boldsymbol{r}_{AB} \cdot \boldsymbol{a}_A}{2} \right] \\
& - \frac{1}{c^2} \sum_{A \neq B} \frac{G m_A m_B}{r_{AB}} \left[\sum_{D \neq A} \frac{G m_D}{r_{AD}} + \sum_{D \neq B} \frac{1}{2} \frac{G m_D}{r_{DB}} \right] + \mathcal{O}(c^{-4}).
\end{aligned}
$$

The whole picture

Dynamics: light rays and the Shapiro delay

$$c(t_r - t_e)_{GRT} = R + \sum_A 2\frac{\mu_A}{c^2} \ln \frac{\boldsymbol{n} \cdot \boldsymbol{r}_{rA} + r_{rA} + \frac{4\mu_A}{c^2}}{\boldsymbol{n} \cdot \boldsymbol{r}_{eA} + r_{eA} + \frac{4\mu_A}{c^2}} ,$$

The whole picture
Time



The whole picture

Time

The difference between TT and TDB is produced by planetary ephemerides, by integrating the following equation together with the equations of motion (Klioner 2008; Fienga et al. 2009)

$$\frac{d(TT - TDB)}{d(TDB)} = \left(L_B + \frac{1}{c_2} a\right)(1 + L_B - L_G) - L_G + \frac{1}{c_4} b, \qquad (85)$$

with L_B and L_G are defining constants for TDB relatively to TCB and TT relatively to TCG, respectively (see e.g. Klioner 2008, Petit and Luzum 2010 for the full definition) and where

$$a = -\frac{1}{2} v_T^2 - \sum_{A \neq T} \frac{\mu_A}{r_{AT}}$$

$$b = -\frac{1}{8} v_T^4 + \frac{1}{2}\left[\sum_{A \neq T} \frac{\mu_A}{r_{AT}}\right]^2 + \sum_{A \neq T} \frac{\mu_A}{r_{AT}} \left\{ 4 \boldsymbol{v}_T \cdot \boldsymbol{v}_A - \frac{3}{2} v_T^2 - 2 v_A^2 \right. \qquad (86)$$

$$\left. + \frac{1}{2} \boldsymbol{a}_A \cdot \boldsymbol{r}_{AT} + \frac{1}{2} \left(\frac{\boldsymbol{v}_A \cdot \boldsymbol{r}_{AT}}{r_{AT}} \right)^2 + \sum_{B \neq A} \frac{\mu_B}{r_{BA}} \right\},$$