

An Open System Approach to Gravity

Thomas Colas

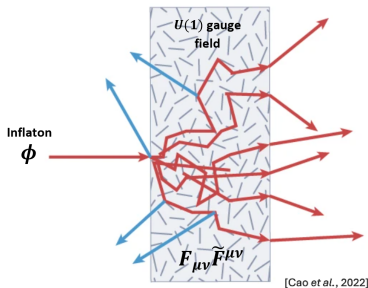
with S. Agui-Salcedo, L. Dufner & E. Pajer



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Cosmology in a medium

Cosmological components can get **sourced** or **damped**: [Anber & Sorbo, 0908.4089]



[Cao et al., 2022]

Model-by-model approach works but

- ① Models can be degenerate;
- ② Can miss components;
- ③ Can miss patterns.

Example: one-loop corrections to **gravitational waves spectrum** [Ema et al., 2506.15780]

$$\left[\text{1-loop diagram} \right] = \text{diagram 1} + \text{diagram 2} \Rightarrow \left[\text{diagram 3} + \text{diagram 4} \right]_{\text{soft}} = 0$$

Principle-based approach

Effective Field Theories (EFTs) leverage *symmetries and scale hierarchies*

EFT of Inflation [Cheung et al., 2008]: single-clock + slow-roll models:

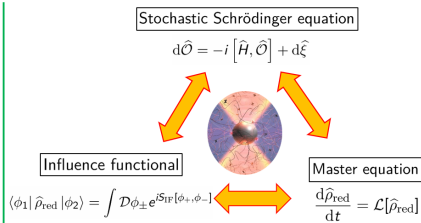
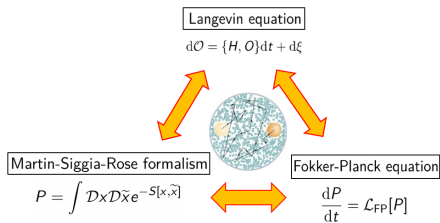
$$S_\pi = \int d^4x \sqrt{-g} \left\{ \frac{1}{2} M_{\text{Pl}}^2 R + \epsilon M_{\text{Pl}}^2 H^2 \left[\dot{\pi}^2 - \frac{(\partial_i \pi)^2}{a^2} \right] + 2M_2^4 \left[\dot{\pi}^2 + \dot{\pi}^3 - \dot{\pi} \frac{(\partial_i \pi)^2}{a^2} \right] - \frac{4}{3} M_3^4 \dot{\pi}^3 + \dots \right\}.$$



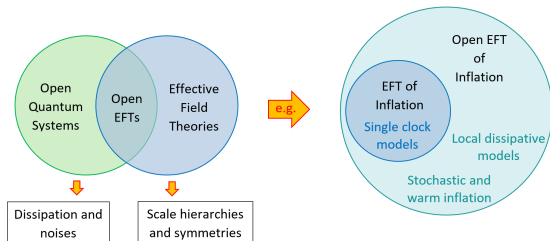
Shortcomings: **Conservative dynamics** only:
⇒ Cannot describe previous example.

*How do we account for **sourcing** and **damping**?*

Combining EFTs and Open Quantum Systems



Extend the **embedding power** of EFTs:



The Open EFT of Inflation [S.A. Agüí Salcedo, T.C. & E. Pajer, 2404.15416]

Early universe: one scalar degree of freedom $\pi(\mathbf{x}, t)$:

$$\text{Observed } \langle \text{Earth} \rangle \Leftrightarrow \langle \hat{\pi}^n \rangle(t) = \int d\pi \pi^n \text{Prob}_\pi(t).$$

- EFT of Inflation [Cheung *et al.*, 2008]: most generic **wavefunction**

$$\text{Prob}_\pi(t) = |\Psi_\pi(t)|^2 = \left| \int_\Omega \mathcal{D}\pi e^{iS_{\text{eff}}[\pi]} \right|^2 \quad \Rightarrow \quad \left| \longrightarrow \right|^2$$

- **Dissipation & noise:** most generic **density matrix**

$$\text{Prob}_\pi(t) = \rho_{\pi\pi}(t) = \int_\Omega \mathcal{D}\pi_+ \int_\Omega \mathcal{D}\pi_- e^{iS_{\text{eff}}[\pi_+, \pi_-]} \quad \Rightarrow \quad \longleftarrow \text{X} \longrightarrow$$

Physical principles restrict $S_{\text{eff}}[\pi_+, \pi_-]$:

- 1 **Unitarity:** $\{\text{Sys.} + \text{Env.}\}$ closed $\Rightarrow$ non-equilibrium constraints; [Liu & Glorioso, 2018]
- 2 **Symmetries:** in-in coset construction; [Akyuz, Goon & Penco, 2023]
- 3 **Locality:** truncatable power counting scheme.

From functionals to phenomenology

$$S_{\text{eff}} = \int d^4x \sqrt{-g} \left\{ \overbrace{\dot{\pi}_r \dot{\pi}_a - c_s^2 \partial_i \pi_r \partial^i \pi_a}^{\text{Kinetic term}} \right. \\ \left. - \underbrace{\gamma \dot{\pi}_r \pi_a}_{\text{Dissipation}} + i \underbrace{\left[\beta_1 \pi_a^2 - (\beta_2 - \beta_4) \dot{\pi}_a^2 + \beta_2 (\partial_i \pi_a)^2 \right]}_{\text{Noise}} + \dots \right\}$$

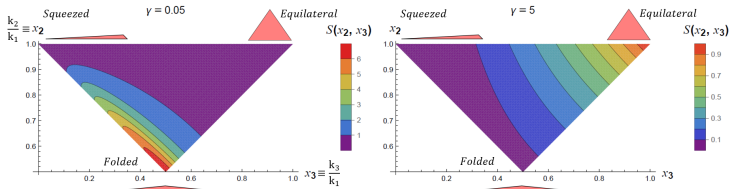
Keldysh basis:

$$\pi_r = \frac{\pi_+ + \pi_-}{2}$$

$$\pi_a = \pi_+ - \pi_-$$

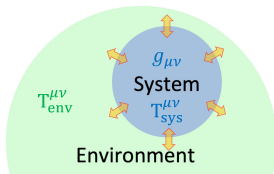
Recover and extend the EFT of Inflation

⇒ **Nearly-scale invariant power spectrum, primordial non-Gaussianities:**



Gravity in a medium [S.A. Agüf Salcedo, T.C. , L. Dufner & E. Pajer, 2507.03103]

Dissipative theory for a **massless spin 2 graviton** + matter sector.



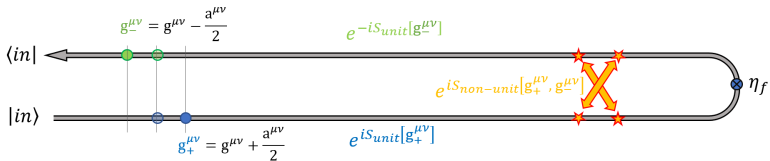
Diffeomorphisms invariance:

$$g_{\pm}^{\mu\nu}(x) \rightarrow \frac{\partial(x^{\mu} + \xi_{\pm}^{\mu})}{\partial x^{\alpha}} \frac{\partial(x^{\nu} + \xi_{\pm}^{\nu})}{\partial x^{\beta}} g_{\pm}^{\alpha\beta}(x)$$

for each branch of SK path integral contour.

Keldysh basis: **retarded** $g_{\mu\nu}$ and **advanced** $a^{\mu\nu}$ metric:

$$g = \frac{g_+ + g_-}{2} = \bar{g} + \delta g, \quad \text{and} \quad a = g_+ - g_- = \delta a$$



Symmetry breaking pattern fix the theory

$$4\text{d-diff}_+ \times 4\text{d-diff}_- \xrightarrow{\text{open}} 4\text{d-diff}_r \xrightarrow{\text{clock}} 3\text{d-diff}_r$$

$$S_{\text{eff}} = \int d^4x \sqrt{-g} \sum_{\ell=0} (g^{00} + 1)^\ell \left[M_{\mu\nu,\ell} a^{\mu\nu} + i N_{\mu\nu\rho\sigma,\ell} a^{\mu\nu} a^{\rho\sigma} + \dots \right]$$

with $M_{\mu\nu,\ell}$ and $N_{\mu\nu\rho\sigma,\ell}$ rank-2 and 4 cotensors under *retarded spatial diffs.*

$$M_{00,\ell} = \gamma_{1,\ell}^{tt} + \gamma_{2,\ell}^{tt} K + \gamma_{3,\ell}^{tt} K^2 + \gamma_{4,\ell}^{tt} K_{\alpha\beta} K^{\alpha\beta} + \gamma_{5,\ell}^{tt} \nabla^0 K + \gamma_{6,\ell}^{tt} R + \gamma_{7,\ell}^{tt} R^{00};$$

$$M_{0\mu,\ell} = \gamma_{1,\ell}^{ts} R^0{}_\mu + \gamma_{2,\ell}^{ts} \nabla_\mu K + \gamma_{3,\ell}^{ts} \nabla_\beta K^\beta{}_\mu;$$

$$\begin{aligned} M_{\mu\nu,\ell} = & g_{\mu\nu} \left(\gamma_{1,\ell}^{ss} + \gamma_{2,\ell}^{ss} K + \gamma_{3,\ell}^{ss} K^2 + \gamma_{4,\ell}^{ss} K_{\alpha\beta} K^{\alpha\beta} + \gamma_{5,\ell}^{ss} \nabla^0 K + \gamma_{6,\ell}^{ss} R + \gamma_{7,\ell}^{ss} R^{00} \right) \\ & + \gamma_{8,\ell}^{ss} K_{\mu\nu} + \gamma_{9,\ell}^{ss} \nabla^0 K_{\mu\nu} + \gamma_{10,\ell}^{ss} K_{\mu\alpha} K^\alpha{}_\nu + \gamma_{11,\ell}^{ss} K K_{\mu\nu} + \gamma_{12,\ell}^{ss} R_{\mu\nu} + \gamma_{13,\ell}^{ss} R_\mu{}^0{}_\nu{}^0 \\ & + \gamma_{1,\ell}^{\text{P.O.}} \epsilon_\mu{}^{\alpha\beta 0} \nabla_\alpha K_{\beta\nu} + \gamma_{2,\ell}^{\text{P.O.}} \epsilon_\mu{}^{\alpha\beta 0} R_{\alpha\beta}{}^0{}_\nu. \end{aligned}$$

and similarly for $N_{\mu\nu\rho\sigma,\ell} \Rightarrow$ can be studied **systematically**.

Background evolution

Modified Friedmann equations with α_i functions of EFT coefficients:

$$\text{Open: } \begin{cases} 3M_{\text{Pl}}^2 H^2 = \alpha_1 + \alpha_2 H, \\ 2M_{\text{Pl}}^2 \dot{H} = \alpha_3 + \alpha_4 H, \end{cases} \quad \text{vs} \quad \text{Closed: } \begin{cases} 3M_{\text{Pl}}^2 H^2 = \rho_\phi, \\ 2M_{\text{Pl}}^2 \dot{H} = -(\rho_\phi + P_\phi), \end{cases}$$

① *Bulk viscosity* [Weinberg, 1971]: $\alpha_4 = 3\zeta$

$$2M_{\text{Pl}}^2 \dot{H} = -[\rho_\phi + (P_\phi - 3H\zeta)],$$

② *Brane-world gravity* [Dvali, Gabadadze & Porrati, 2000]: $\alpha_2 = \pm 3M_{\text{Pl}}^2/r_c$

$$3M_{\text{Pl}}^2(H^2 \pm H/r_c) = \rho_\phi,$$

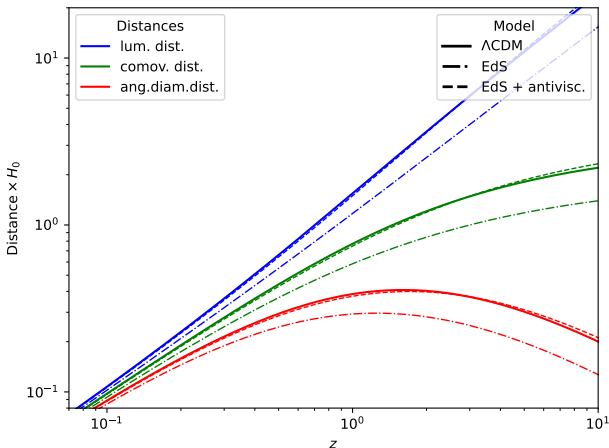
$\Rightarrow$ **Modified** continuity equation: $\dot{\alpha}_1 + (\dot{\alpha}_2 H + \alpha_2 \dot{H}) - 3H(\alpha_3 + \alpha_4 H) = 0$

③ *Interacting dark sector*: $Q = -(\dot{\alpha}_2 H + \alpha_2 \dot{H}) + 3\alpha_4 H^2$

$$\dot{\rho}_\phi + 3H(\rho_\phi + P_\phi) = Q.$$

CMB + BAO + SNe analysis

- CMB fix $\Omega_m h^2$ and angular scale $\theta_* = r_s(z_*)/D_A(z_*) \Rightarrow$ constrain α_i
- Compatible with low-redshift $H(z)$ from DESI DR2 + SNe?



Primordial gravitational waves

Transverse and traceless (TT) sector: $g_{ij} = a^2(t) (\delta_{ij} + h_{ij})$, $a^{ij} = a^{-2} h_{ij}^a$:

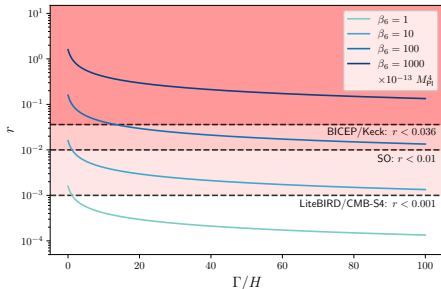
$$S^{(2)} = \int d^4x \sqrt{-g} \frac{M_{Pl}^2}{4c_T^2} h_{ij}^a \left\{ \ddot{h}_{ij} - c_T^2 \frac{\nabla^2}{a^2} h_{ij} + (\Gamma_T + 3H) \dot{h}_{ij} \right. \\ \left. + \frac{\chi_T}{a} \epsilon_{imn} (\partial_m \dot{h}_{nj} + 2H \partial_m h_{nj}) + i \frac{\beta_T}{M_{Pl}^2} h_{ij}^a \right\}.$$

① Speed of propagation c_T^2 ;

② Dissipation Γ_T ;

③ Birefringence χ_T ;

④ Noise β_T .



No birefringence: $\chi_T = 0$

⚠ In general, $\beta_T \propto f(\Gamma_T)$:

• Thermal equilibrium:
 $\beta_T \propto \Gamma_T T$ (FDR).

BICEP/Keck bound:

$$r < 0.036 \Rightarrow \beta_T \lesssim (0.002 M_{Pl})^4$$

Conclusion

*Realistic environments are **noisy and dissipative**:
We need **systematic modelling** for **upcoming data**.*

Open inflation:

- 1 An **embedding** for **local dissipative models** of inflation;
- 2 Smoking gun near **folded triangles** in **primordial non-Gaussianities**.

CMB constraints & entropy bounds

Open gravity:

- 1 **Dissipation** and **noise** consistent with **diff invariance**;
- 2 Modified **Friedmann equations** & dissipative **gravitational waves**.

Dissipative dark sectors & LSS constraints

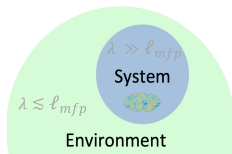
Backup

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Outline

- 1 More on Open EFTol
- 2 More on Open E&M
- 3 More on Open GR

Open systems, dissipation and noise

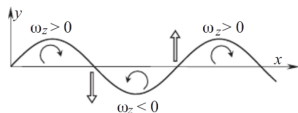


System interacts with environment:

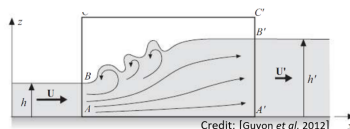
⇒ non-unitary evolution

Dissipation & noise = energy & information losses

Perfect fluid: Wilsonian EFT



Imperfect fluid: non-equilibrium EFT



What about cosmology?

• **Observable** universe $\neq$ whole;



• Continuously **evolving**;



• Always a **medium**;



• $\exists$ **fluxes** between sectors.



⇒ effective dynamics often **non-unitary** [Baumann, Nicolis, Senatore & Zaldarriaga, 2010]

Goal: EFTs accounting for **dissipation & noise** in cosmology

Dynamical gravity and a scalar field χ :

$$S = \frac{1}{2} \int d^{d-1}x d\tau f^2(\tau) \left[\left(\frac{d\chi}{d\tau} \right)^2 - c_s^2(\tau) [e^{-h}]_{ij} (\partial_i \chi)(\partial_j \chi) - m_\chi^2(\tau) \chi^2 \right]. \quad (3.2)$$

Schwinger-Keldysh expansion:

$$S_{\text{in-in}} = \int d^{d-1}x d\tau f^2(\tau) c_s^2(\tau) \left[\frac{1}{2} \hat{h}_a^{ij} (\partial_i \hat{\chi}_r) (\partial_j \hat{\chi}_r) + \hat{h}_r^{ij} (\partial_i \hat{\chi}_r) (\partial_j \hat{\chi}_a) - \frac{1}{2} \hat{h}_{ak}^i \hat{h}_r^{jk} (\partial_i \hat{\chi}_r) (\partial_j \hat{\chi}_r) \right] + \dots, \quad (3.3)$$

Effective vertex $h_{ij}^a h_r^{ij}$ breaks retarded diff!

Outline

- 1 More on Open EFTol
- 2 More on Open E&M
- 3 More on Open GR

Schwinger-Keldysh formalism

Consider some **observable**

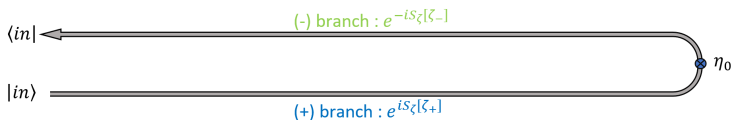
$$\hat{Q} \equiv \hat{\zeta}(\mathbf{x}_1) \hat{\zeta}(\mathbf{x}_2) \cdots \hat{\zeta}(\mathbf{x}_n)$$

and some unitary **evolution operator** $\hat{U}(\eta_0, \eta_i)$ so that

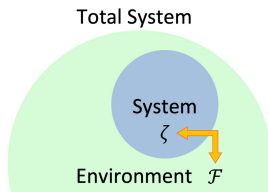
$$|\Psi(\eta_0)\rangle = \hat{U}(\eta_0, \eta_i) |\text{BD}\rangle \quad \text{with} \quad \langle \zeta | \hat{U}(\eta_0, \eta_i) | \zeta_1 \rangle = \int_{\zeta_1}^{\zeta} \mathcal{D}[\Phi] e^{iS[\Phi]}.$$

If $S[\Phi] = S_{\zeta}[\zeta]$, see [Weinberg, 2005]:

$$\begin{aligned} \langle \hat{Q}(\eta_0) \rangle &= \langle \Psi(\eta_0) | \hat{Q}(\eta_0) | \Psi(\eta_0) \rangle = \langle \text{BD} | \hat{U}^\dagger(\eta_0, \eta_i) \hat{Q}(\eta_0) \hat{U}(\eta_0, \eta_i) | \text{BD} \rangle \\ &= \int d\zeta d\zeta_1 d\zeta_2 [\zeta(\mathbf{x}_1) \cdots \zeta(\mathbf{x}_n)] \int_{\zeta_1}^{\zeta} \mathcal{D}[\zeta_+] \int_{\zeta_2}^{\zeta} \mathcal{D}[\zeta_-] e^{iS_{\zeta}[\zeta_+] - iS_{\zeta}[\zeta_-]} \langle \zeta_1 | \text{BD} \rangle \langle \text{BD} | \zeta_2 \rangle \end{aligned}$$



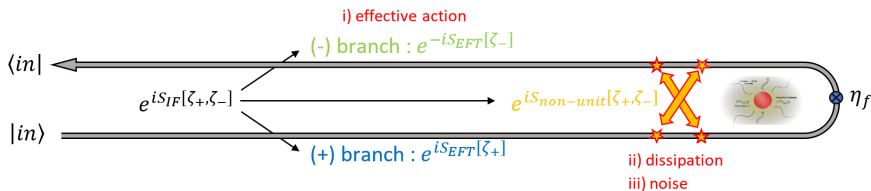
Integrating out an environment



- $S[\Phi] = S_{\zeta}[\zeta] + S_{\mathcal{F}}[\mathcal{F}] + S_{\text{int}}[\zeta; \mathcal{F}]$
with $\mathcal{F}$ a **hidden sector**.
- Goal: tracing out $\mathcal{F}$, the environment being **unobservable**.

Effects of the environment captured by the **Influence Functional (IF)**:

$$\langle \hat{Q}(\eta) \rangle = \int d\zeta d\zeta_1 d\zeta_2 [\zeta(\mathbf{x}_1) \cdots \zeta(\mathbf{x}_n)] \int_{\zeta_1}^{\zeta} \mathcal{D}[\zeta_+] \int_{\zeta_2}^{\zeta} \mathcal{D}[\zeta_-] e^{iS_{\zeta}[\zeta_+] - iS_{\zeta}[\zeta_-] + iS_{\text{IF}}[\zeta_+; \zeta_-]}$$



What are the rules obeyed by $S_{\text{IF}}[\zeta_+; \zeta_-]$?

Non-equilibrium constraints [Liu & Glorioso, 2018]

Requiring **Open QFT** originates from a **unitary** “closed” UV theory:

$$\text{i) } \text{Tr}[\hat{\rho}] = 1, \quad \text{ii) } \hat{\rho}^\dagger = \hat{\rho} \quad \text{and} \quad \text{iii) } \hat{\rho} \geq 0$$

implies constraints on $S_{\text{eff}}[\pi_+, \pi_-] \equiv S_{\text{unit}}[\pi_+] - S_{\text{unit}}[\pi_-] + S_{\text{non-unit}}[\pi_+, \pi_-]$:

$$\begin{aligned} \text{i) } S_{\text{eff}}[\pi_+, \pi_+] &= 0, & S_{\text{eff}}[\pi_r, \pi_a = 0] &= 0; \\ \text{ii) } S_{\text{eff}}[\pi_+, \pi_-] &= -S_{\text{eff}}^*[\pi_-, \pi_+], & S_{\text{eff}}[\pi_r, \pi_a] &= -S_{\text{eff}}^*[\pi_r, -\pi_a]; \\ \text{iii) } \Im S_{\text{eff}}[\pi_+, \pi_-] &\geq 0, & \Im S_{\text{eff}}[\pi_r, \pi_a] &\geq 0, \end{aligned}$$

for $\pi_r = (\pi_+ + \pi_-)/2$ and $\pi_a = \pi_+ - \pi_-$. *Consequences:*

- 1 $S_{\text{eff}}[\pi_r, \pi_a]$ starts **linear** in π_a ;
- 2 **Odd** powers of π_a are purely **real**; **even** powers of π_a purely **imaginary**;
- 3 **Bounds** on the **noise coefficients**.

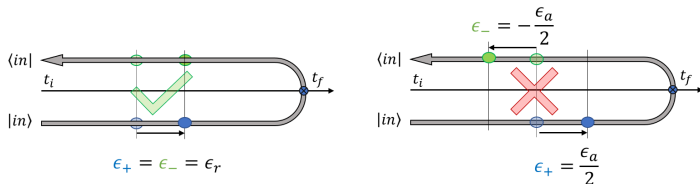
$\Rightarrow$ *Already reduce the scope of available Open EFTs*

Dissipative shift symmetric scalar [Hongo *et al.*, 2018], [Akyuz, Goon & Penco, 2023]

$S_{\text{unit}}[\pi_{\pm}]$ invariant under $\text{shift}_+ \times \text{shift}_-$:

$$\pi_{\pm}(t) \rightarrow \pi'_{\pm}(t) = \pi_{\pm}(t + \epsilon_{\pm}) + \epsilon_{\pm},$$

but $S_{\text{non-unit}}[\pi_+; \pi_-]$ **is not**: **SSB** $\text{shift}_+ \times \text{shift}_- \rightarrow \text{shift}_r$:



Retarded $\pi_r = (\pi_+ + \pi_-)/2$ and **advanced** $\pi_a = \pi_+ - \pi_-$ fields:

$$\pi_r(t) \rightarrow \pi'_r(t) = \pi_r(t + \epsilon_r) + \epsilon_r, \quad \pi_a(t) \rightarrow \pi'_a(t) = \pi_a(t + \epsilon_r).$$

Building blocks:

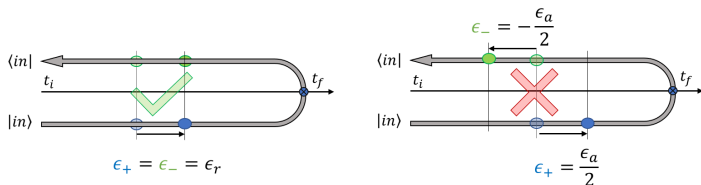
$$\pi_a, \quad t + \pi_r, \quad \partial_\mu \pi_a, \quad \partial_\mu(t + \pi_r).$$

In-in coset construction [Hongo et al., 2018], [Akyuz, Goon & Penco, 2023]

Two **simplifications** from [Cheung et al., 2008]: [regime of validity? $\Rightarrow$ later]

① *Decoupling limit*: Mixing $\pi/\delta g$ small as long as $E \sim H \gg E_{\text{mix}} \sim \epsilon^{1/2} H$

$\Rightarrow$ **enough** to construct theory of **dissipative shift symmetric scalar**:



$S_{\text{eff}}[\pi_r, \pi_a]$ invariant under *retarded time diffeomorphism*:

$$t \rightarrow t + \epsilon_r : \quad \pi_r \rightarrow \pi_r - \epsilon_r, \quad \pi_a \rightarrow \pi_a.$$

Building blocks: $\pi_a, t + \pi_r, \partial_\mu \pi_a, \partial_\mu(t + \pi_r)$.

② *Derivative expansion*: **locality** and truncatable power counting scheme.

Effective functional

Keldysh basis: $\pi_r = (\pi_+ + \pi_-)/2$, $\pi_a = \pi_+ - \pi_-$.

- *Quadratic order:* $1 \rightarrow 5$ EFT param (1 tadpole constraint):

$$S_{\text{eff}}^{(2)} = \int d^4x \sqrt{-g} \left\{ \overbrace{\dot{\pi}_r \dot{\pi}_a - c_s^2 \partial_i \pi_r \partial^i \pi_a}^{\text{Kinetic term}} - \underbrace{\gamma \dot{\pi}_r \pi_a}_{\text{Dissipation}} + i \underbrace{\left[\beta_1 \pi_a^2 - (\beta_2 - \beta_4) \dot{\pi}_a^2 + \beta_2 (\partial_i \pi_a)^2 \right]}_{\text{Noise}} \right\}$$

- *Cubic order:* $1 \rightarrow 13$ EFT param: EFTol famous for **relating operators at different orders** because of **non-linearly realised boosts** [López Nacir *et al.*, 2011].

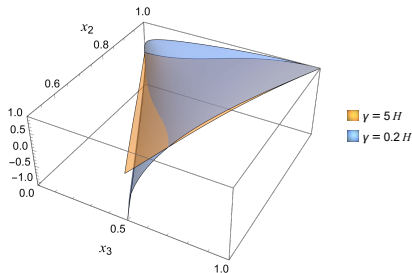
$$\text{EFTol:} \quad \mathcal{L} \supset (c_s^2 - 1) \left[-2\dot{\pi}_r + (\partial_\mu \pi_r)^2 \right] \dot{\pi}_a$$

$$\text{Dissipation:} \quad \mathcal{L} \supset \gamma \left[-2\dot{\pi}_r + (\partial_\mu \pi_r)^2 \right] \pi_a$$

$$\text{Noise:} \quad \mathcal{L} \supset i\beta_4 (-\dot{\pi}_a + \partial_\mu \pi_r \partial^\mu \pi_a)^2$$

$\Rightarrow$ *Recover and extend the EFT of Inflation.*

Primordial non-Gaussianities

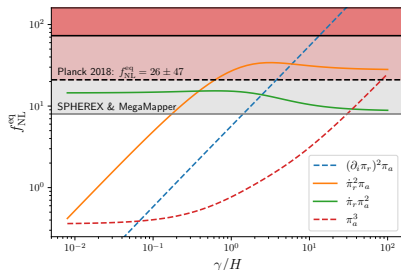


Main features:

- $\gamma \gg H$: **equilateral** shape;
- $\gamma \ll H$: **new folded** signal;
- Consistency relations hold.

Matching with [Creminelli et al., 2305.07695]

- inflaton ϕ
- massive scalar field χ
- softly-broken $U(1)$



New challenge: dynamical gravity

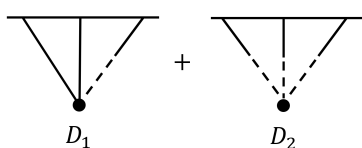
Standard observables

Symmetries ensure existence of **nearly scale invariant power spectrum**

$$\langle \zeta_{\mathbf{k}} \zeta_{\mathbf{k}'} \rangle = \frac{H^2}{f_\pi^4} \langle \pi_{\mathbf{k}}^c \pi_{\mathbf{k}'}^c \rangle \equiv (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}') \frac{2\pi^2}{k^3} \Delta_\zeta^2(k).$$

$\Rightarrow \Delta_\zeta^2 = 10^{-9}$ obtained by imposing **hierarchies of scales**.

Bispectrum computed in **perturbation theory** using standard **in-in rules**.

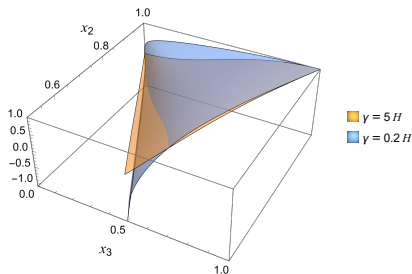


$$\begin{aligned} \eta \text{ --- } \eta' &\quad -iG^K(k; \eta, \eta') \\ \eta \text{ --- } \eta' &\quad -iG^R(k; \eta, \eta') \\ \eta' \text{ --- } \eta' &\quad ig \int_{-\infty(1 \pm i\epsilon)}^{\eta_0} \frac{d\eta'}{(H\eta')^4} \end{aligned}$$

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle = -\frac{H^3}{f_\pi^6} \langle \pi_{\mathbf{k}_1}^c \pi_{\mathbf{k}_2}^c \pi_{\mathbf{k}_3}^c \rangle \equiv (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B(k_1, k_2, k_3).$$

$$S(x_2, x_3) \equiv (x_2 x_3)^2 \frac{B(k_1, x_2 k_1, x_3 k_1)}{B(k_1, k_1, k_1)}, \quad f_{\text{NL}}(k_1, k_2, k_3) \equiv \frac{5}{6} \frac{B(k_1, k_2, k_3)}{P(k_1)P(k_2) + 2 \text{ perms.}}.$$

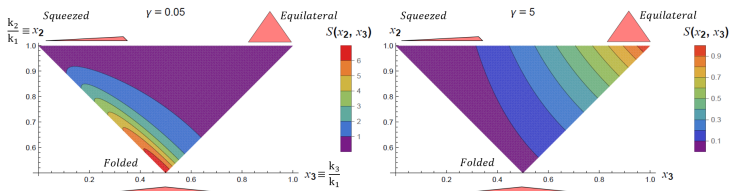
Bispectrum shapes



Main features:

- $\gamma \gg H$: equilateral;
- $\gamma \ll H$: folded;
- Consistency relations;
- Regularized divergence.

Consistent with **flat-space/sub-Hubble** analytic results:

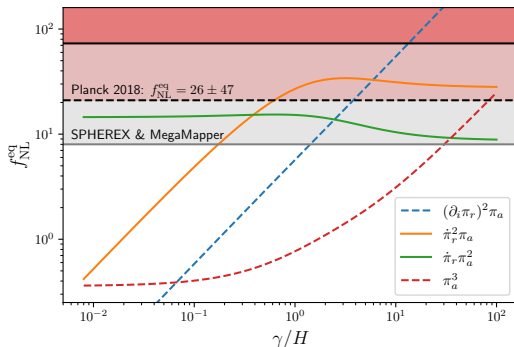


Matching and f_{NL} with [Creminelli *et al.*, 2305.07695]

UV completion: inflaton ϕ + massive scalar field χ with softly-broken $U(1)$:

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_{\text{Pl}}^2 R - \frac{1}{2} (\partial\phi)^2 - V(\phi) - |\partial\chi|^2 + M^2 |\chi|^2 - \frac{\partial_\mu \phi}{f} (\chi \partial^\mu \chi^* - \chi^* \partial^\mu \chi) - \frac{1}{2} m^2 (\chi^2 + \chi^{*2}) \right].$$

$\Rightarrow$ narrow **instability band** in sub-Hubble regime: *local* particle production.

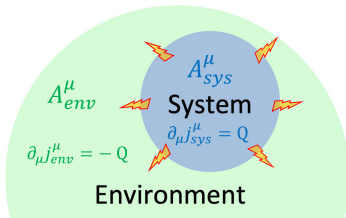


Outline

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- 2 More on Open E&M
- 3 More on Open GR

Open gauge theories

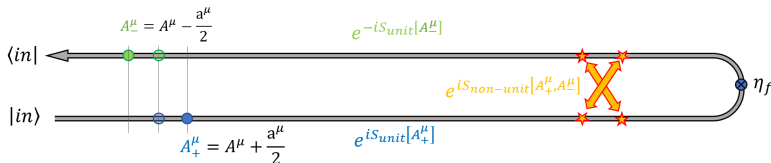
[S.A. Agüir Salcedo, T.C. & E. Pajer, 2412.12299]: *Dissipative theory* for of **light in a medium**.



- Gauge symmetry: $U(1)_r \times U(1)_a$
- Global symmetry:

$$U(1)_r \times U(1)_a \rightarrow U(1)_r$$

Keldysh basis: retarded $A^\mu = (A_+^\mu + A_-^\mu) / 2$; advanced $a^\mu = A_+^\mu - A_-^\mu$.



Retarded & advanced gauge transformation

Retarded gauge transformation $\epsilon_+ = \epsilon_- = \epsilon_r$:

$$A^\mu \rightarrow A^\mu + \partial^\mu \epsilon_r, \quad a^\mu \rightarrow a^\mu.$$

Advanced gauge transformation $\epsilon_+ = -\epsilon_- = \epsilon_a$:

$$A^\mu \rightarrow A^\mu, \quad a^\mu \rightarrow a^\mu + \partial^\mu \epsilon_a.$$

- Global $U(1)_a$ broken: $B_a^\mu = a^\mu + \partial^\mu \varphi_a$ with $\varphi_a \rightarrow \varphi_a - \epsilon_a$.
- **Unitary gauge**: $\varphi_a = 0$ and **retarded gauge invariance** only.

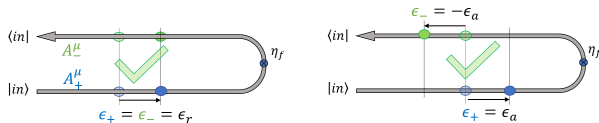
Effective functional constructed out of $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ and a^μ :

$$S_1 = \int_{\omega, k} \left[a^0 i k_i F^{0i} + a_i \left(\gamma_2 F^{0i} + \gamma_3 i k_j F^{ij} + \gamma_4 \epsilon_{jl}^i F^{jl} \right) + a^\mu j_\mu \right]$$

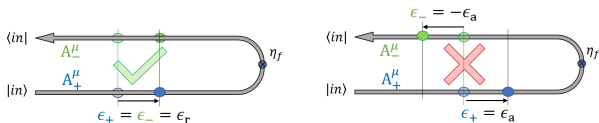
$$S_2 = \int_{\omega, k} i a^\mu N_{\mu\nu} a^\nu, \quad S_{n \geq 3} = \dots$$

Summary

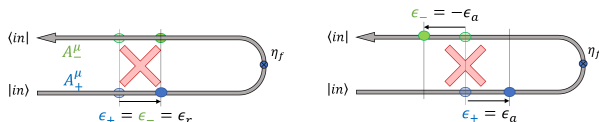
- ① **Unitary:** $\Delta S_{\text{eff}}^{\text{adv}} = 0 \Rightarrow \partial^\mu j_\mu = 0$: Maxwell in medium;



- ② **Non-unitary:** $\Delta S_{\text{eff}}^{\text{adv}} \neq 0 \Rightarrow \partial^\mu j_\mu \neq 0$: modified charge conservation;



- ③ **Conductor:** $\Delta S_{\text{eff}}^{\text{ret}} \neq 0 \Rightarrow$ new d.o.f.: dissipative Proca theory.



Recovering electromagnetism in a medium

From S_{eff} , obtain modified **Gauss** and **Ampère** laws:

$$\frac{\delta S_{\text{eff}}}{\delta a^0} = 0 \quad \Rightarrow \quad \nabla \cdot \mathbf{E} = j_0 + \xi_0,$$

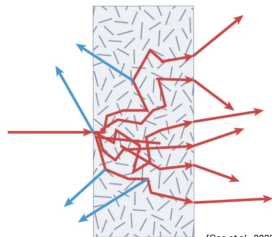
$$\frac{\delta S_{\text{eff}}}{\delta a^i} = 0 \quad \Rightarrow \quad \gamma_2 \mathbf{E} + \gamma_3 \nabla \times \mathbf{B} - 2\gamma_4 \mathbf{B} = \mathbf{j} + \xi,$$

and a **noise constraint**: *modified charge conservation in the system*

$$\partial^\mu (j_\mu + \xi_\mu) = \Gamma(j_0 + \xi_0).$$

Properties:

- Dispersive medium: $n = 1/\sqrt{-\gamma_3}$;
- Dissipative medium: $\gamma_2 = -i\omega + \Gamma$;
- Anisotropic medium: γ_4 ;
- Random medium: ξ^μ .

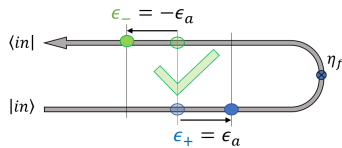
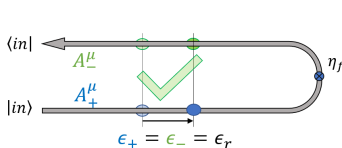


[Cao et al., 2022]

Unitary limit

$$\Delta S_1^{\text{ret}} = 0 \quad \text{but} \quad \Delta S_1^{\text{adv}} = \int_{\omega, \mathbf{k}} (i\omega + \gamma_2) \epsilon_a i k_i F^{0i}.$$

- For general γ_2 , $\Delta S_1^{\text{adv}} \neq 0$;
- When $\gamma_2 = -i\omega$, $\Delta S_1^{\text{adv}} = 0$.



Unitary theory with $S_1[A_+, A_-] = S[A_+] - S[A_-]$ and

$$S[A] = \frac{1}{4} \int d^4x \left[F^{\mu\nu} F_{\mu\nu} + (c_s^2 - 1) F^{ij} F_{ij} + \theta F^{\mu\nu} \tilde{F}^{\mu\nu} \right]$$

Recover *Maxwell in a medium*.

Deformed advanced gauge

When $\gamma_2 \neq -i\omega$, does it exist **physically equivalent** advanced configurations?

- ① Retarded gauge invariance: $M_{\mu\nu}k^\nu = 0$ where $k^\mu = (\omega, \mathbf{k})$.
- ② $\exists$ “right kernel” $\Rightarrow \exists$ “left kernel” such that $v^\mu M_{\mu\nu} = 0$.
- ③ M is non-Hermitian $\Rightarrow$ different left and right kernels: $v^\mu = (i\gamma_2, \mathbf{k})$.

Conclusion: retarded gauge invariance generates **advanced gauge invariance**.

S_1 remains unchanged under

$$A^\mu \rightarrow A^\mu + \epsilon_r k^\mu, \quad a^\mu \rightarrow a^\mu + \epsilon_a v^\mu.$$

Not broken but rather **deformed!** [Liu & Glorioso, 2018], [Akyuz, Goon & Penco, 2023].

Advanced gauge redundancy allows us to **reduce number of advanced components**.

Noise constraint

Add some noise: $N_{\mu\nu}$ positive semi-definite [Kamenev, 2011], [Breuer & Petruccione, 2007]

$$S = \int d^4x [a^\mu M_{\mu\nu} A^\nu + i a^\mu N_{\mu\nu} a^\nu],$$

Hubbard-Stratonovich trick:

$$\mathcal{Z} = \int [\mathcal{D}A^\mu] \int [\mathcal{D}a^\mu] \int [\mathcal{D}\xi_\mu] \exp \left[\int d^4x i a^\mu (M_{\mu\nu} A^\nu - j_\nu - \xi_\nu) - \frac{1}{4} \xi_\mu (N^{-1})^{\mu\nu} \xi_\nu \right]$$

Advanced gauge symmetry $a^\mu \rightarrow a^\mu + \epsilon_a v^\mu$ induces **noise constraint**:

$$v^\mu (j_\mu + \xi_\mu) = 0.$$

Meaning: $\gamma_2 = \Gamma - i\omega$ leads to

$$\partial^\mu (j_\mu + \xi_\mu) = \Gamma(j_0 + \xi_0).$$

Conservation of the total current $\Leftrightarrow$ **Non-conservation of the system's current**

Advanced Stueckelberg [Lau, Nishii & Noumi, 2412.21136]

Stueckelberg trick: $X_a \rightarrow X_a - \epsilon_a$ s.t. $\mathcal{A}_a^\mu = a^\mu + \partial^\mu X_a \rightarrow \mathcal{A}_a^\mu$ invariant

$$\begin{aligned} S_{\text{eff}} &= \int_{\omega, \mathbf{k}} \left[\mathcal{A}_{a0} i k_i F^{0i} + \mathcal{A}_{ai} (\gamma_2 F^{0i} + \gamma_3 i k_j F^{ij} + \gamma_4 \epsilon^i_{jl} F^{jl}) - \mathcal{A}_{a\mu} (j^\mu + \xi^\mu) \right], \\ &= S_{\text{eff}}^{\text{old}} + \int_{\omega, \mathbf{k}} X_a [(i\omega + \gamma_2) i k_i F^{0i} + \partial_\mu (j^\mu + \xi^\mu)]. \end{aligned}$$

X_a e.o.m. **recovers noise constraint:** for $\gamma_2 = \Gamma - i\omega$,

$$\frac{\delta S_{\text{eff}}}{\delta X_a} = 0 \quad \Rightarrow \quad \partial^\mu (j_\mu + \xi_\mu) = \Gamma(j_0 + \xi_0).$$

using on-shell relation $\delta S_{\text{eff}}/\delta a_0 = 0$ that is $i k_i F^{0i} = -(j^0 + \xi^0)$.

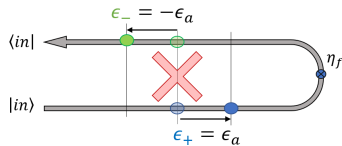
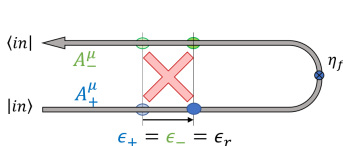
Lessons:

- **Manifest advanced gauge invariance** at the price of X_a ;
- If $\partial^\mu (j_\mu + \xi_\mu) = 0$, only solution is $\gamma_2 = -i\omega$, i.e. no dissipation $\Gamma = 0$.

Proca mass term

Adding a **mass term** breaks **both** retarded and advanced gauge symmetries:

$$S_{\text{eff}} = \int_{\omega, \mathbf{k}} ik_i a_0 F^{0i} - i\omega a_i F^{0i} - m^2 a_\mu A^\mu$$



Stueckelberg trick:

- $X_a \rightarrow X_a - \epsilon_a$ s.t. $\mathcal{A}_a^\mu = a^\mu + \partial^\mu X_a \rightarrow \mathcal{A}_a^\mu$ invariant;
- $X_r \rightarrow X_r - \epsilon_r$ s.t. $\mathcal{A}_r^\mu = A^\mu + \partial^\mu X_r \rightarrow \mathcal{A}_r^\mu$ invariant

$$S_{\text{eff}} = S_{\text{eff}}^{\text{old}} - m^2 \int d^4x [\partial_\mu X_a \partial^\mu X_r - a_\mu \partial^\mu X_r - \partial_\mu X_a A^\mu]$$

Decoupling: when $E \gg m$, mixing between X and A negligible.

Breaking retarded gauge invariance triggers new d.o.f.

Dispersion relations

Gauge fixing

- retarded **Coulomb gauge**: $\partial_i A^i = 0$

$$\exists \epsilon_r \quad \text{s.t.} \quad k_i A'^i = 0, \quad \text{where} \quad A'^\mu = A^\mu + \epsilon_r k^\mu.$$

- advanced **Coulomb gauge**: $\partial_i a^i = 0$

$$\exists \epsilon_a \quad \text{s.t.} \quad k_i a'^i = 0, \quad \text{where} \quad a'^\mu = a^\mu + \epsilon_a v^\mu.$$

Eigenvalues of the kinetic matrix: 1 **constrained** dof, 2 **propagating** dof

$$(k^2, i\gamma_2\omega + \gamma_3 k^2 + 2\gamma_4 k, i\gamma_2\omega + \gamma_3 k^2 - 2\gamma_4 k).$$

Introduce $\gamma_2 = \Gamma - i\omega$, $\gamma_3 = -c_s^2$:

$$\omega^2 + i\Gamma\omega - c_s^2 k^2 \pm 2\gamma_4 k = 0 \quad \Rightarrow \quad \omega = -i\frac{\Gamma}{2} \pm \sqrt{c_s^2 k^2 - (\Gamma/2)^2 \mp 2\gamma_4 k}.$$

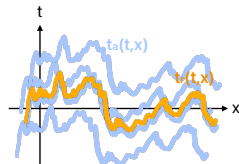
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A tale of two clocks

Fluctuating clocks: $\phi_{\pm}(t, \mathbf{x})$, i.e.

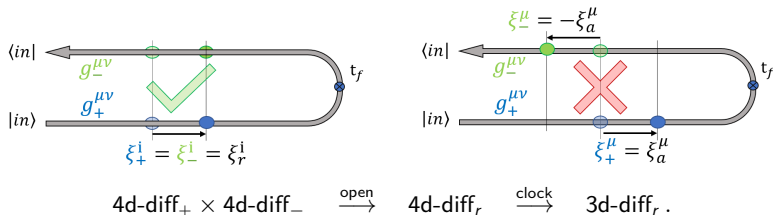
- $\phi_r(t, \mathbf{x}) \Leftrightarrow t_r(t, \mathbf{x})$: average clock;
- $\phi_a(t, \mathbf{x}) \Leftrightarrow t_a(t, \mathbf{x})$: stochasticity.



Gauge fixing. Work in *unitary gauges*: $\phi_+(t, \mathbf{x}) = \phi_-(t, \mathbf{x}) = \bar{\phi}(t)$:

$$\phi_r(t, \mathbf{x}) = \bar{\phi}(t) \quad \text{and} \quad \phi_a(t, \mathbf{x}) = 0 \quad \text{i.e.} \quad t_r = t \quad \text{and} \quad t_a = 0.$$

Following EFT of Inflation [Cheung *et al.*, 2008] and Dark Energy [Gubitosi, Piazza & Vernizzi, 2013]:



A glimpse on what to expect

- **Dissipative** and **stochastic** Einstein Equations: $G_{\mu\nu} + \Gamma \mathcal{D}_{\mu\nu} = T_{\mu\nu} + \xi_{\mu\nu}$
- **Non-conserved** stress-energy tensor: $\nabla_\mu T^{\mu\nu} \neq 0$

Phenomenology:

- **Background:** Viscous cosmology & Interacting dark sectors

$$\dot{\rho}_{DE} + 3H(\rho_{DE} + p_{DE}) = \Gamma \quad \text{and} \quad \dot{\rho}_m + 3H(\rho_m + p_m) = -\Gamma$$

- **Clustering** $\Rightarrow$ redshift space distortion (RSD) and weak lensing (WL)

$$k^2 \langle \psi \rangle = -4\pi G \mu(a, k) a^2 \rho_m \langle \delta \rangle, \quad k^2 \frac{\langle \psi + \phi \rangle}{2} = -4\pi G \Sigma(a, k) a^2 \rho_m \langle \delta \rangle$$

- **Gravitational waves** $\Rightarrow$ GW production, propagation and dissipation

$$\ddot{h}_{ij} + \Gamma \dot{h}_{ij} + c_T^2 h_{ij} + \chi \epsilon_{ilm} k_m h_{jl} = T_{ij} + \xi_{ij}$$

*Rich phenomenology to explore,
eventually **already constrained from data**.*

Reintroducing the scalar

Restore covariance by *retarded* and *advanced* time diffeomorphism:

$$S_{\text{eff}}[g_{\mu\nu}, a_{\mu\nu}] \rightarrow S_{\text{eff}}[g_{\mu\nu}, a_{\mu\nu}, \pi_r, \pi_a],$$

- **retarded Stückelberg trick:** $t_r \rightarrow t_r + \pi_r(x)$:

$$g^{00} \rightarrow g^{00} + 2g^{0\mu} \partial_\mu \pi_r + g^{\mu\nu} \partial_\mu \pi_r \partial_\nu \pi_r,$$

$$a^{00} \rightarrow a^{00} + 2a^{0\mu} \partial_\mu \pi_r + a^{\mu\nu} \partial_\mu \pi_r \partial_\nu \pi_r.$$

- **advanced Stückelberg trick:** $t_a \rightarrow t_a - \pi_a(x)$:

$$g_{\mu\nu} \rightarrow g_{\mu\nu},$$

$$a^{\mu\nu} \rightarrow a^{\mu\nu} + 2\nabla^{(\mu} \epsilon_a^{\nu)}.$$

Decoupling limit: slow-roll $\epsilon \ll 1$ (even with dissipation & noise):

$$-a^{00} + (g^{00}/2)g_{\mu\nu}a^{\mu\nu} \rightarrow 2\dot{\pi}_a - 2\partial^\mu \pi_r \partial_\mu \pi_a,$$

$$g_{\mu\nu}a^{\mu\nu} \rightarrow 2\dot{\pi}_a + 6H\pi_a,$$

$$1 + g^{00} \rightarrow -2\dot{\pi}_r + (\partial_\mu \pi_r)^2,$$

$\Rightarrow$ recover Open EFTol & more: *primordial GWs*

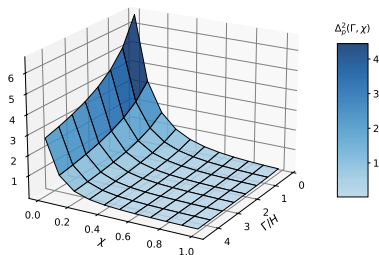
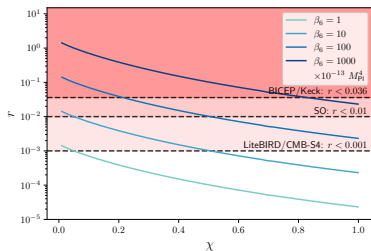
Including birefringence

$$\left[\partial_\eta^2 - \left(\frac{2 + \frac{\Gamma_T}{H}}{\eta} \mp \chi_T k \right) \partial_\eta + k^2 \right] G_{\times,+}^R(k; \eta, \eta') = H^2 \eta^2 \delta(\eta - \eta').$$

Analytic expression in terms of Appell F_2 function: $\kappa_\pm \equiv \mp \frac{i\chi_T}{\sqrt{1-\chi_T^2}} (\nu_T - 1/2)$

$$\Delta_{\times,+}^2 = \frac{\beta_T / M_{\text{Pl}}^4}{2\pi^2 \nu_\Gamma^2 \chi_T^3} F_2 \left(3; \frac{1}{2} + \nu_\Gamma - \kappa_\pm, \frac{1}{2} + \nu_\Gamma + \kappa_\pm; 2\nu_\Gamma + 1, 2\nu_\Gamma + 1; \mp \sqrt{1 - \frac{1}{\chi_T^2}}, \pm \sqrt{1 - \frac{1}{\chi_T^2}} \right)$$

Recover **instability band** at low dissipation:



Primordial gravitational waves

Transverse and traceless (TT) sector: $g_{ij} = a^2(t) (\delta_{ij} + h_{ij})$, $a^{ij} = a^{-2} h_{ij}^a$:

$$S^{(2)} = \int d^4x \sqrt{-g} \frac{M_{Pl}^2}{4c_T^2} h_{ij}^a \left\{ \ddot{h}_{ij} - c_T^2 \frac{\nabla^2}{a^2} h_{ij} + (\Gamma_T + 3H) \dot{h}_{ij} \right. \\ \left. + \frac{\chi_T}{a} \epsilon_{imn} (\partial_m \dot{h}_{nj} + 2H \partial_m h_{nj}) + i \frac{\beta_T}{M_{Pl}^2} h_{ij}^a \right\}.$$

- ① Speed of propagation c_T^2 ;
- ② Dissipation Γ_T ;
- ③ Birefringence χ_T ;
- ④ Noise β_T .

Effects **correlated** with **background** and **scalar sector** [Lau, Nishii & Noumi, 2024]:

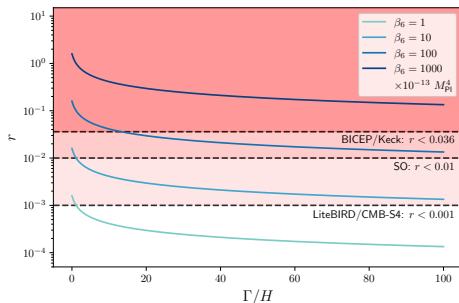
$$S_{\text{eff}} \supset \int d^4x \sqrt{-g} \gamma_{8,\ell}^{ss} K_{\mu\nu} a^{\mu\nu}.$$

- Background with bulk viscosity: $\zeta = 2\gamma_{8,\ell}^{ss}/3$.
- Dissipation in tensor sector: $\Gamma_T = 2\gamma_{8,\ell}^{ss}/M_{Pl}^2$.

Tensor-to-scalar ratio

Without birefringence ($\chi_T = 0$): define $\nu_T \equiv 3/2 + \Gamma_T/(2H)$

$$\Delta_h^2 = \frac{\beta_T}{M_{Pl}^4} 2^{2\nu_T} \frac{\Gamma(\nu_T - 1)\Gamma(\nu_T)^2}{\Gamma(\nu_T - \frac{1}{2})\Gamma(2\nu_T - \frac{1}{2})} \propto \begin{cases} \frac{\beta_T}{M_{Pl}^4}, & \text{for } \Gamma_T \ll H, \\ \frac{\beta_T}{M_{Pl}^4} \sqrt{\frac{H}{\Gamma_T}} \left[1 + \mathcal{O}\left(\frac{H}{\Gamma_T}\right) \right], & \text{for } \Gamma_T \gg H. \end{cases}$$



⚠ In general, $\beta_T \propto f(\Gamma_T)$:

- Thermal equilibrium:
 $\beta_T \propto \Gamma_T T$ (FDR);
- What about out-of-equilibrium?

BICEP/Keck bound: $r < 0.036 \Rightarrow \beta_T \lesssim (0.002 M_{Pl})^4$

Late-universe: clockless example

Subset that preserves retarded time-diff: **no dynamical scalar**

$$S_{\text{eff}} = \int d^4x \sqrt{-g} (f_1 R_{\mu\nu} + f_2 g_{\mu\nu} + f_3 T_{\mu\nu} + \text{h.d.}) a^{\mu\nu}.$$

$\Rightarrow$ *Very few ingredients!* To lowest order, e.o.m. takes the form:

$$G_{\mu\nu} + \tilde{f}_1 R g_{\mu\nu} = \tilde{f}_2 T_{\mu\nu} + \tilde{f}_3 T g_{\mu\nu}.$$

w.l.o.g., separating **Standard Model**, **dark matter** and **cosmological constant**:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = G_N [T_{\mu\nu}^{\text{SM}} + T_{\mu\nu}^{\text{DM}} + (\lambda^{\text{SM}} T^{\text{SM}} + \lambda^{\text{DM}} T^{\text{DM}}) g_{\mu\nu}].$$

If **dark matter** $\simeq$ **perfect fluid**, absorb $\lambda^{\text{DM}} T^{\text{DM}} g_{\mu\nu}$ into equation of state w^{DM} :

$\simeq$ *Gravitational Aether* [Afshordi, 2008] | *Massive Gravity* [Isham, Salam & Strathdee, 1971]

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = G_N [T_{\mu\nu}^{\text{SM}} + T_{\mu\nu}^{\text{DM}} + \lambda^{\text{SM}} T^{\text{SM}} g_{\mu\nu}].$$

Blueprint for cosmological analysis [with F. McCarthy, ACT & SO]

- Does not come from an action $\Rightarrow$ **evade** [Lovelock, 1971] **theorem**;
- Dissipative dark matter:

$$\nabla^\mu T_{\mu\nu}^{\text{SM}} = 0, \quad \nabla^\mu T_{\mu\nu}^{\text{DM}} + \lambda^{\text{SM}} \partial_\nu T^{\text{SM}} = 0.$$

- To **UV complete**, need to find sector s.t. $\langle T_{\mu\nu}^{\text{env}} \rangle = \lambda^{\text{SM}} T^{\text{SM}} g_{\mu\nu}$;
- One-parameter extension of Λ -CDM $\Rightarrow$ **data-analysis** CMB + BAO.

Background $\Rightarrow$ **Hubble diagram** function of redshift z :

$$\frac{H(z)}{H_0} = \sqrt{\Omega_\Lambda + \Omega_{\text{DM}}(1+z)^{3(1+w_{\text{DM}})} + \Omega_{\text{SM}}(1+z)^{3(1+w_{\text{SM}})} [1 + \lambda(1 - 3w_{\text{SM}})]}.$$

Clustering $\Rightarrow$ **redshift space distortion** (RSD) and **weak lensing** (WL):

$$k^2 \langle \psi \rangle = -4\pi G \mu(a, k) a^2 \rho_m \langle \delta \rangle, \quad k^2 \frac{\langle \psi + \phi \rangle}{2} = -4\pi G \Sigma(a, k) a^2 \rho_m \langle \delta \rangle$$