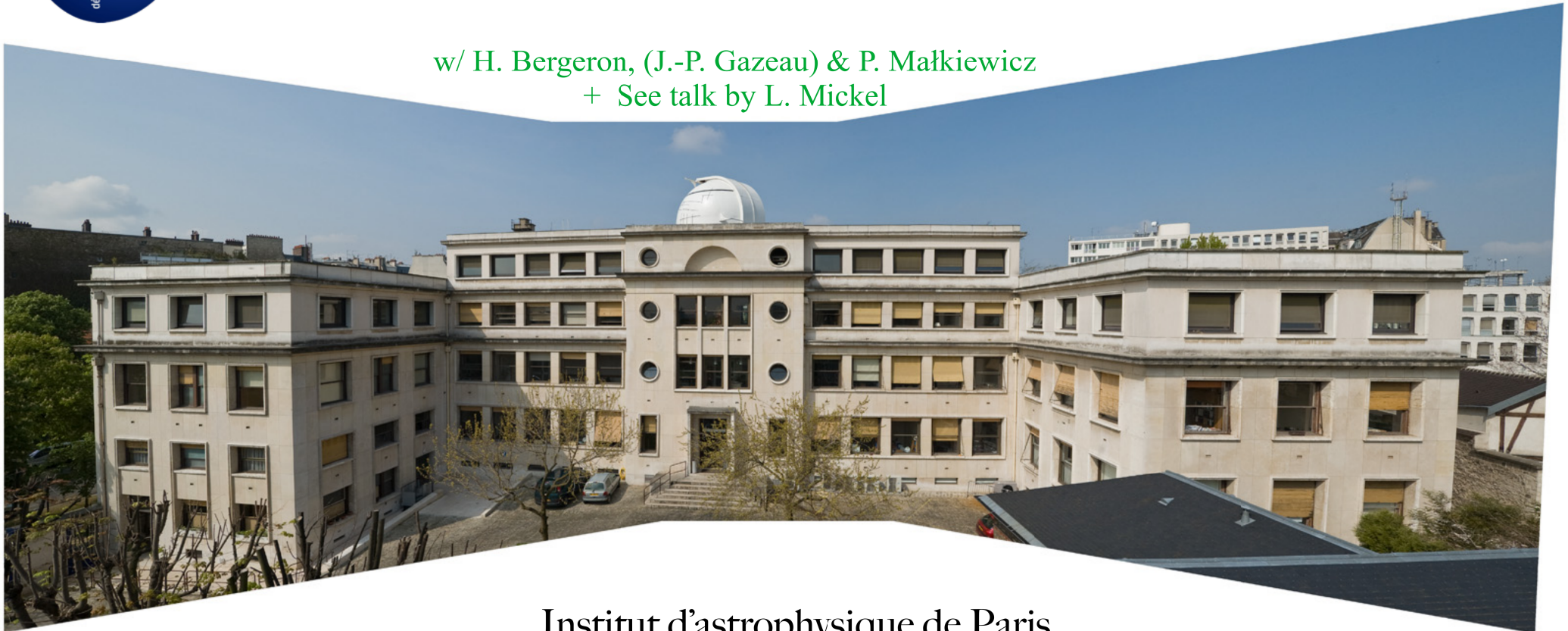


The Born–Oppenheimer approximation in quantum cosmology

w/ H. Bergeron, (J.-P. Gazeau) & P. Małkiewicz
+ See talk by L. Mickel



Institut d'astrophysique de Paris

TUG – IPhT Saclay – 14/10/2025

Overview

- Main hypothesis for quantum cosmology

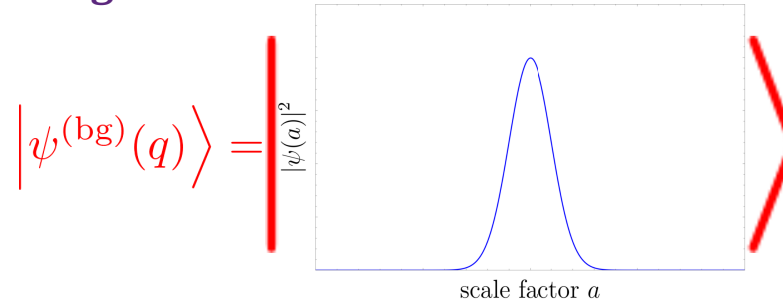
- Born-Oppenheimer approximation

H. Bergeron, P. Małkiewicz, PP – 2405.09307
Phys.Rev. **D110**, 043512 (2024)
+ 2503.14570

$$|\Psi^{(\text{BO})}\rangle = |\psi^{(\text{bg})}(q)\rangle \otimes |\psi^{(\text{pert})}[\{v_{\mathbf{k}}\}; q(\tau), p(\tau)]\rangle$$

- Simple background state

K. Mazde, L. Mickel, PP – 2508.06231



- Enhanced quantization on the half line

H. Bergeron, J.-P. Gazeau, P. Małkiewicz, PP – 2310.16868
Phys.Rev. **D109**, 023516 (2024)

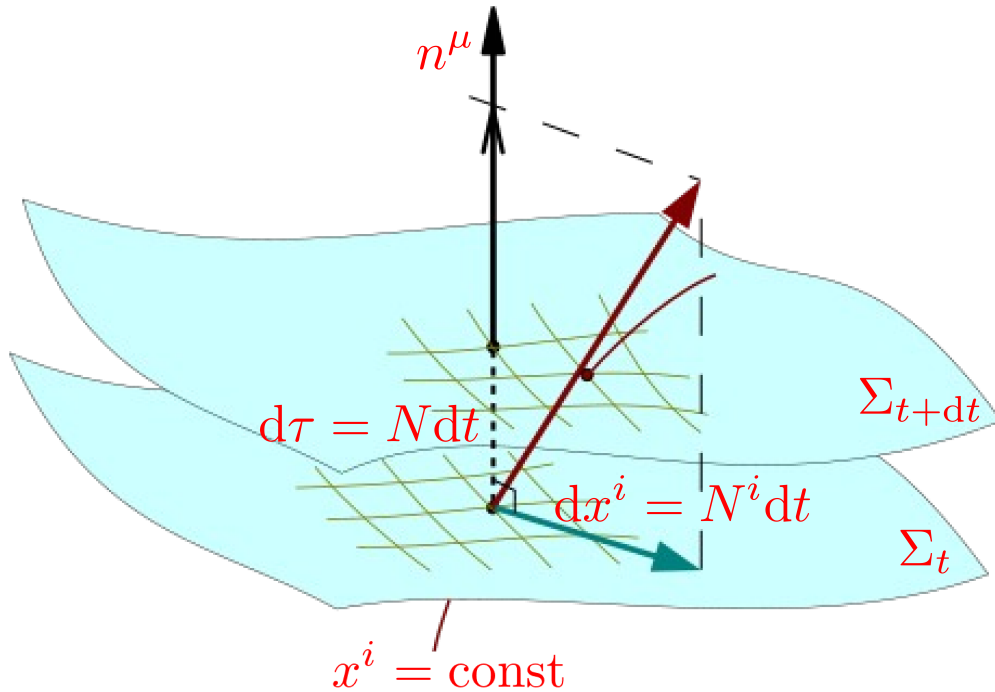
- A new class of exact coherent states

- Cosmological consequences

Basic model: 3+1 GR + perfect fluid

$$\mathcal{S}_{\text{EHS}} = \underbrace{\frac{1}{16\pi G_N} \int d^4x \sqrt{-g} R}_{\mathcal{S}_{\text{EH}}} + \underbrace{\int d^4x \sqrt{-g} P_w [(\partial\phi)^2]}_{\mathcal{S}_{\text{M}}}$$

Canonical quantization
Wheeler De Witt

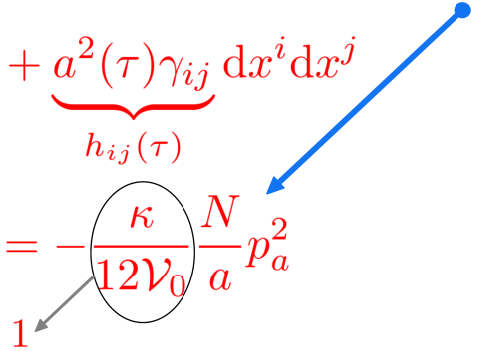


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FLRW metric

$$ds^2 = -N^2(\tau) d\tau^2 + \underbrace{a^2(\tau) \gamma_{ij}}_{h_{ij}(\tau)} dx^i dx^j$$

$$H_{\text{grav}} = - \underbrace{\left(\frac{\kappa}{12\mathcal{V}_0} \right)}_1 \frac{N}{a} p_a^2$$


Basic model: 3+1 GR + perfect fluid

$$\mathcal{S}_{\text{EHS}} = \underbrace{\frac{1}{16\pi G_N} \int d^4x \sqrt{-g} R}_{\mathcal{S}_{\text{EH}}} + \underbrace{\int d^4x \sqrt{-g} K \mu^{1+1/w}}_{\mathcal{S}_{\text{M}}}$$

$$\mu^2 = -g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi$$

$$\mu = \frac{\rho + P}{n}$$

$$P = w\rho$$

FLRW metric

$$ds^2 = -N^2(\tau) d\tau^2 + \underbrace{a^2(\tau) \gamma_{ij}}_{h_{ij}(\tau)} dx^i dx^j$$

$$H_{\text{grav}} = -\underbrace{\left(\frac{\kappa}{12\mathcal{V}_0} \right)}_1 \frac{N}{a} p_a^2$$

$$H_{\text{fluid}} = \gamma(K, w) \frac{N}{(\sqrt{h})^w} p_\phi^{1+w}$$

$$N = -a^{3w} \implies H_{\text{fluid}} = p_\tau$$

Use as clock

Basic model: 3+1 GR + perfect fluid

$$\mathcal{S}_{\text{EHS}} = \underbrace{\frac{1}{16\pi G_N} \int d^4x \sqrt{-g} R}_{\mathcal{S}_{\text{EH}}} + \underbrace{\int d^4x \sqrt{-g} K \mu^{1+1/w}}_{\mathcal{S}_{\text{M}}}$$

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1

$$H_{\text{fluid}} = \gamma(K, w) \frac{N}{(\sqrt{h})^w} p_\phi^{1+w}$$

$$N = -a^{3w} \implies H_{\text{fluid}} = p_\tau$$

Use as clock

$$q = \frac{2}{3(1-w)} a^{\frac{3}{2}(1-w)}$$

$$p = a^{\frac{1}{2}(3w-1)} p_a \propto a^{\frac{3}{2}(1+w)} (H)$$

New canonical variables

H
Hubble rate

$$H_{\text{T}} = H_{\text{grav}} + H_{\text{fluid}} = p^2 + p_\tau$$

Coherent state quantization

Background phase space $(q, p) \in \mathbb{R}^+ \times \mathbb{R} = \{(q, p) | q > 0, p \in \mathbb{R}\}$

Natural choice = 2 parameter affine group of the real line $\lambda \in \mathbb{R} \mapsto (q, p) \cdot \lambda = \frac{\lambda}{q} + p$

$$\{(q_0, p_0), (q, p)\} \mapsto (q', p') = (q_0, p_0) \circ (q, p) = \left(q_0 q, \frac{p}{q_0} + p_0\right)$$

“Usual” Coherent state quantization

Background phase space $(q, p) \in \mathbb{R}^+ \times \mathbb{R}$

Natural choice = 2 parameter affine group of the real line $\lambda \in \mathbb{R} \mapsto (q, p) \cdot \lambda = \frac{\lambda}{q} + p$

unitary, irreducible and square-integrable representation in the Hilbert space $\mathcal{H} = L^2(\mathbb{R}^+, dx)$

$$\langle x | U(q, p) | \psi \rangle = \langle x | q, p \rangle = \frac{e^{ipx}}{\sqrt{q}} \psi\left(\frac{x}{q}\right)$$

$\psi(x) = \langle x | \psi \rangle$

covariant integral quantization on (affine) coherent states

$$\mathbb{R}^+ \times \mathbb{R} \ni (q, p) \mapsto |q, p\rangle := U(q, p) |\xi\rangle \in \mathcal{H}$$

normalized fiducial state

$$U(q, p) = e^{ip\hat{x}} e^{-i(\ln q)\hat{d}}$$

algebra

$$[\hat{x}, \hat{d}] = i\hat{x}$$

dilation $\hat{d} := \frac{1}{2}(\hat{x}\hat{p} + \hat{p}\hat{x})$

“Enhanced” Coherent state quantization

Background phase space $(q, p) \in \mathbb{R}^+ \times \mathbb{R}$

Natural choice = 2 parameter affine group of the real line $\lambda \in \mathbb{R} \mapsto (q, p) \cdot \lambda = \frac{\lambda}{q} + p$

Equivalent representation $\implies$ canonical transformation $q \mapsto q^\alpha \quad \& \quad p \mapsto \frac{p}{\alpha q^{\alpha-1}}$

unitary, irreducible and square-integrable representation in the Hilbert space $\mathcal{H}_\alpha = L^2(\mathbb{R}^+, x^{-\alpha} dx)$

covariant integral quantization on (affine) coherent states

$$\mathbb{R}^+ \times \mathbb{R} \ni (q, p) \mapsto |q, p\rangle := \hat{V}_{q,p} |\xi_0\rangle \in \mathcal{H}$$

normalized fiducial state

$$\hat{V}_{q,p} = e^{i \frac{p}{2q} \hat{x}^2} e^{-i(\ln q)^2 \hat{d}/2} = e^{i \frac{p}{2q} \hat{x}^2} e^{-i(\ln q) \hat{d}}$$

algebra

$$\left[\hat{x}^\alpha, \frac{\hat{d}}{\alpha} \right] = i \hat{x}^\alpha$$

$$\alpha \rightarrow 2$$

$$\tilde{H} = p^2 + Cq^{-2}$$

$$\text{dilation } \hat{d} := \frac{1}{2}(\hat{x}\hat{p} + \hat{p}\hat{x})$$

$(\tilde{H}, q^2, d/2)$ = closed Lie algebra with structure constant independent of C

define $c_\gamma(\psi) = \int_0^{+\infty} \frac{dx}{x^{\gamma+2}} |\psi(x)|^2 \implies \int_{\Pi_+} \frac{dq dp}{2\pi c_0(\xi_0)} |q, p\rangle \langle q, p| = 1$ (resolution of unity)

affine coherent state quantization:

$$f \mapsto \hat{A}_f = \int_{\Pi_+} \frac{dq dp}{2\pi c_0(\xi_0)} |q, p\rangle f(q, p) \langle q, p|$$

$$\longrightarrow \hat{A}_1 = \mathbb{1}$$

$$\longrightarrow \forall \alpha \in \mathbb{R}, \quad \hat{A}_{q^\alpha} = \frac{c_\alpha(\xi_0)}{c_0(\xi_0)} \hat{x}^\alpha \implies \hat{A}_q = \frac{c_1(\xi_0)}{c_0(\xi_0)} \hat{x}$$

$$\longrightarrow \hat{A}_p = \frac{c_1(\xi_0)}{c_0(\xi_0)} \hat{p}$$

impose $c_1(\xi_0) = c_0(\xi_0)$ to ensure $[\hat{A}_q, \hat{A}_p] = i$

$$\longrightarrow \hat{H} = \hat{A}_{p^2} = \frac{c_2(\psi_0)}{c_0(\psi_0)} \left\{ p^2 + \underbrace{\left[\frac{K}{c_2(\psi_0)} - \frac{3}{2} \right] \frac{1}{\hat{x}^2}}_{> 0} \right\} \quad \text{Hamiltonian}$$

$$K = \int_0^\infty \frac{dy}{y^2} \xi'_0(y)^2 > 0$$

$$\hat{H}_\nu = \hat{p}^2 + \frac{\nu^2 - \frac{1}{4}}{\hat{x}^2}$$

generic (factor ordering...)

Coherent state semiclassical framework

new fiducial state $|\psi_0\rangle$

$$|q, p\rangle = \hat{V}_{q,p} |\psi_0\rangle$$

$$\forall \alpha \in \mathbb{R}, \langle q, p | \hat{x}^\alpha | q, p \rangle = c_{-\alpha-2}(\psi_0) q^\alpha$$

$$\left. \begin{aligned} \langle q, p | \hat{x} | q, p \rangle &= c_{-3}(\psi_0) q \\ \langle q, p | \hat{p} | q, p \rangle &= c_{-3}(\psi_0) p \end{aligned} \right\}$$

Rescale $\psi_0; c_{-3}(\psi_0) \rightarrow 1$

time-dependent functions $q(\tau)$ and $p(\tau)$

define time-dependent coherent states $|q(\tau), p(\tau)\rangle = \hat{V}_{q(\tau), p(\tau)} |\psi_0\rangle$

$$\langle \hat{x} \rangle = q(\tau)$$

$$\langle \hat{p} \rangle = p(\tau)$$

semiclassical hamiltonian: $\langle q, p | \hat{p}^2 | q, p \rangle = c_{-4}(\psi_0) p^2 + \frac{C}{q^2}$

$$C = \int_0^\infty \psi'_0(x)^2 dx > 0$$

Choosing the fiducial state $|\psi_0\rangle$ wisely...

$$\hat{H}_\nu = \hat{p}^2 + \frac{\nu^2 - \frac{1}{4}}{\hat{x}^2}$$

1) demand that the coherent state solves the Schrödinger equation

$$i\partial_\tau \left[e^{-i\phi(\tau)} |q_\tau, p_\tau\rangle_{\psi_0} \right] = e^{-i\phi(\tau)} \hat{H}_\nu |q_\tau, p_\tau\rangle_{\psi_0}$$

with extra time dependent phase

2) apply Ehrenfest theorem:

$$\frac{d}{d\tau} \langle \psi(\tau) | \hat{\mathcal{O}} | \psi(\tau) \rangle = i \langle \psi(\tau) | [\hat{H}_\nu, \hat{\mathcal{O}}] | \psi(\tau) \rangle$$

$$\longrightarrow e^{-i\phi(\tau)} |q_\tau, p_\tau\rangle_{\psi_0}$$

$$\frac{d}{d\tau} \langle \psi(\tau) | \hat{x} | \psi(\tau) \rangle = \frac{dq_\tau}{d\tau} = 2 \langle \psi(\tau) | \hat{p} | \psi(\tau) \rangle = 2p_\tau$$

$$\frac{d}{d\tau} \langle \psi(\tau) | \hat{p} | \psi(\tau) \rangle = \frac{dp_\tau}{d\tau} = 2 \left(\nu^2 - \frac{1}{4} \right) \langle \psi(\tau) | \hat{x}^{-3} | \psi(\tau) \rangle = 2 \left(\nu^2 - \frac{1}{4} \right) \frac{c_1(\psi_0)}{q_\tau^3}$$

$$\frac{d}{d\tau} \langle \psi(\tau) | \hat{H}_\nu | \psi(\tau) \rangle = \frac{d}{d\tau} \left[c_{-4}(\psi_0) p_\tau^2 + \frac{(\nu^2 - \frac{1}{4}) c_0(\psi_0) + C}{q_\tau^2} \right] = 0$$

effective
e.o.m

Choosing the fiducial state $|\psi_0\rangle$ wisely...

$$\hat{H}_\nu = \hat{p}^2 + \frac{\nu^2 - \frac{1}{4}}{\hat{x}^2}$$

semi classical hamiltonian: $H_{\text{sc}}(q, p) = p^2 + \frac{(\nu^2 - \frac{1}{4})c_1(\psi_0)}{q^2}$


$$\left\{ \begin{array}{l} \frac{dq}{d\tau} = \frac{\partial H_{\text{sc}}}{\partial p} \\ \frac{dp}{d\tau} = -\frac{\partial H_{\text{sc}}}{\partial q} \end{array} \right.$$

+ **constraint:** $C = \int_0^\infty \psi'_0(x)^2 dx = \left(\nu^2 - \frac{1}{4} \right) [c_1(\psi_0)c_{-4}(\psi_0) - c_0(\psi_0)]$

POSSIBLE ???

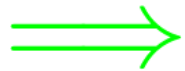
Choosing the fiducial state $|\psi_0\rangle$ wisely...

$$\hat{H}_\nu = \hat{p}^2 + \frac{\nu^2 - \frac{1}{4}}{\hat{x}^2}$$

3) Necessary condition: $i \hat{V}_{q_\tau p_\tau}^\dagger \partial_\tau \left[e^{-i\phi(\tau)} \hat{V}_{q_\tau p_\tau} \right] |\psi_0\rangle = e^{-i\phi(\tau)} \hat{V}_{q_\tau p_\tau}^\dagger \hat{H}_\nu \hat{V}_{q_\tau p_\tau} |\psi_0\rangle$

$$e^{i\phi(\tau)} i \hat{V}_{q_\tau p_\tau}^\dagger \partial_\tau \left[e^{-i\phi(\tau)} \hat{V}_{q_\tau p_\tau} \right] = \phi'(\tau) + p_\tau^2 \hat{x}^2 - \frac{(\nu^2 - \frac{1}{4}) c_1(\psi_0)}{q_\tau^2} \hat{x}^2 + \frac{2p_\tau}{q_\tau} \hat{d}$$

$$\hat{V}_{q_\tau p_\tau}^\dagger \hat{H}_\nu \hat{V}_{q_\tau p_\tau} = \frac{1}{q_\tau^2} \hat{p}^2 + p_\tau^2 \hat{x}^2 + \frac{2p_\tau}{q_\tau} \hat{d} + \frac{\nu^2 - \frac{1}{4}}{q_\tau^2 \hat{x}^2}$$



$$\left[\hat{p}^2 + \frac{\nu^2 - \frac{1}{4}}{\hat{x}^2} + \left(\nu^2 - \frac{1}{4} \right) c_1(\psi_0) \hat{x}^2 \right] |\psi_0\rangle = q_\tau^2 \phi'(\tau) |\psi_0\rangle$$

eigenvalue β

Choosing the fiducial state $|\psi_0\rangle$ wisely...

$$\hat{H}_\nu = \hat{p}^2 + \frac{\nu^2 - \frac{1}{4}}{\hat{x}^2}$$

4) Solve eigenvalue equation $\left[\hat{p}^2 + \frac{\nu^2 - \frac{1}{4}}{\hat{x}^2} + \left(\nu^2 - \frac{1}{4} \right) c_1(\psi_0) \hat{x}^2 \right] |\psi_0\rangle = \underbrace{q_\tau^2 \phi'(\tau)}_\beta |\psi_0\rangle$

$$e^{-i\phi(\tau)} = \left[\frac{\sqrt{(\nu^2 - \frac{1}{4})c_1(\psi_0) - i q_\tau p_\tau}}{\sqrt{(\nu^2 - \frac{1}{4})c_1(\psi_0) + i q_\tau p_\tau}} \right]^{\frac{1}{4}\beta[(\nu^2 - \frac{1}{4})c_1(\psi_0)]^{-1/2}}$$

using $\frac{d}{d\tau}(q_\tau p_\tau) = 2H_{\text{sc}}(q_\tau, p_\tau)$

5) Solve eigenfunction equation $\hat{H}_0 |\Phi_n\rangle = \left(\hat{p}^2 + \frac{\nu^2 - \frac{1}{4}}{\hat{x}^2} + \xi_\nu^2 \hat{x}^2 \right) |\Phi_n\rangle = \beta_n |\Phi_n\rangle$

$\nu \geq 1$ $\xi_\nu > 0$

$\longrightarrow \Phi_n(x) = \langle x | \Phi_n \rangle \in \mathbb{R}$

Choosing the fiducial state $|\psi_0\rangle$ wisely...

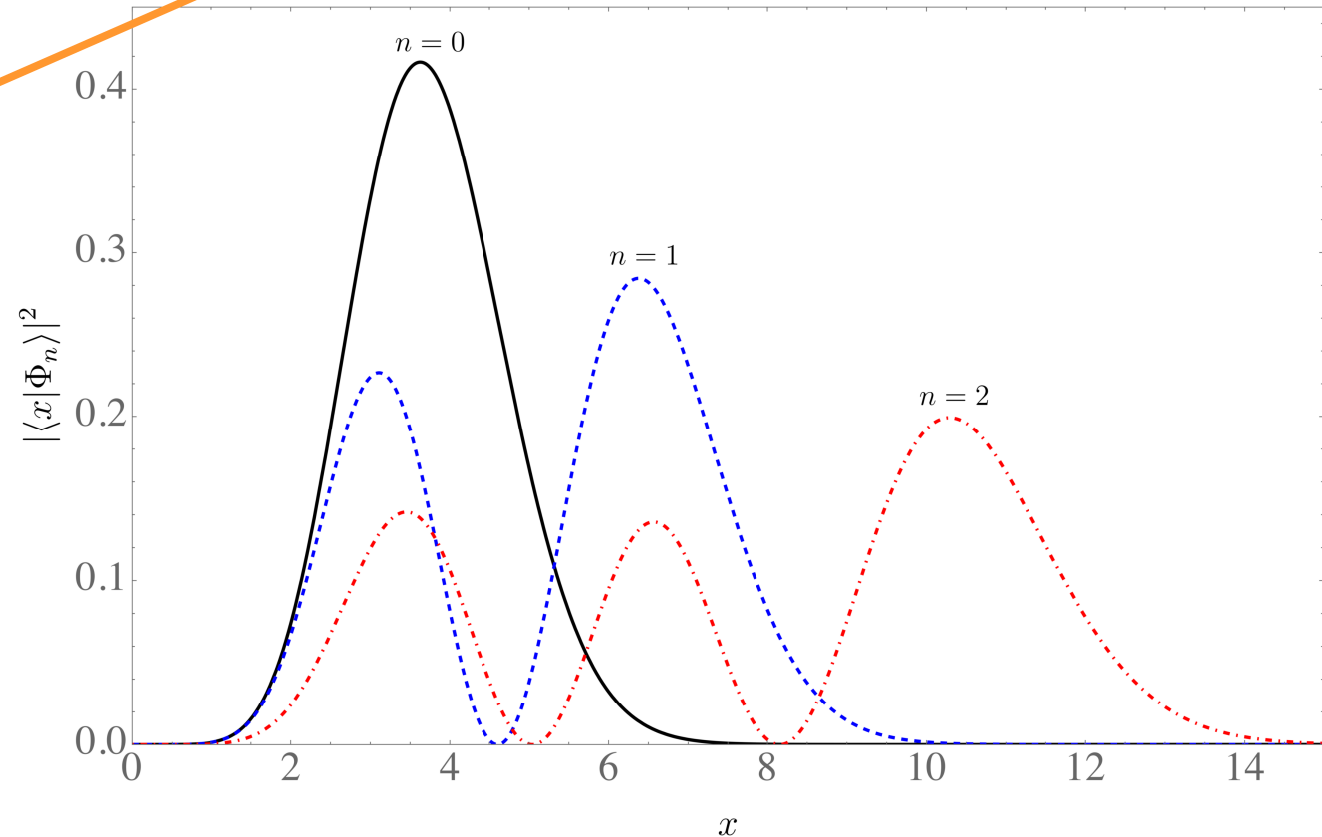
$$\hat{H}_\nu = \hat{p}^2 + \frac{\nu^2 - \frac{1}{4}}{\hat{x}^2}$$

6) Open book on known Hamiltonian eigenvalues and eigenvectors:

$$\rightarrow \beta_n = 2\xi_\nu(2n + \nu + 1)$$

$$\rightarrow \Phi_n(x) = \sqrt{\frac{2n!}{\Gamma(\nu + n + 1)}} \xi_\nu^{\frac{\nu+1}{2}} x^{\nu+1/2} L_n^\nu(\xi_\nu x^2) e^{-\frac{1}{2}\xi_\nu x^2}$$

Laguerre polynomials



Choosing the fiducial state $|\psi_0\rangle$ wisely...

$$\hat{H}_\nu = \hat{p}^2 + \frac{\nu^2 - \frac{1}{4}}{\hat{x}^2}$$

7) Wrap up with full solutions

$$\longrightarrow e^{-i\phi(\tau)} = \left(\frac{\xi_\nu - i q_\tau p_\tau}{\xi_\nu + i q_\tau p_\tau} \right)^{\frac{\beta_n}{4\xi_\nu}}$$

$$|q, p; \nu, n\rangle \stackrel{\text{def}}{=} e^{-i\phi(\tau)} |q_\tau, p_\tau\rangle_{\psi_0}$$

$$\langle x | q_\tau, p_\tau; \nu, n \rangle = \sqrt{\frac{2n!}{\Gamma(\nu+n+1)}} \left(\frac{\xi_{\nu,n} - i q_\tau p_\tau}{\xi_{\nu,n} + i q_\tau p_\tau} \right)^{\frac{1}{2}(2n+\nu+1)} \xi_{\nu,n}^{\frac{\nu+1}{2}} \frac{x^{\nu+1/2}}{q_\tau^{\nu+1}} L_n^\nu \left(\xi_{\nu,n} \frac{x^2}{q_\tau^2} \right) \exp \left[-\frac{1}{2} (\xi_{\nu,n} - i q_\tau p_\tau) \frac{x^2}{q_\tau^2} \right]$$

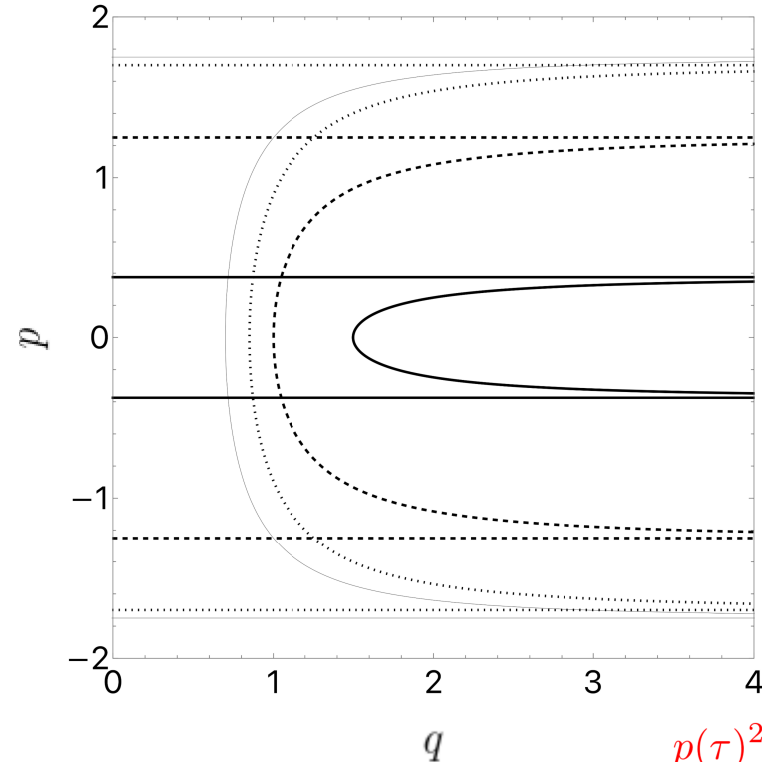
8) Plot nice figures for the phase space evolution...

semiclassical trajectory in phase space

$$q_B = \xi_\nu / \sqrt{E}$$

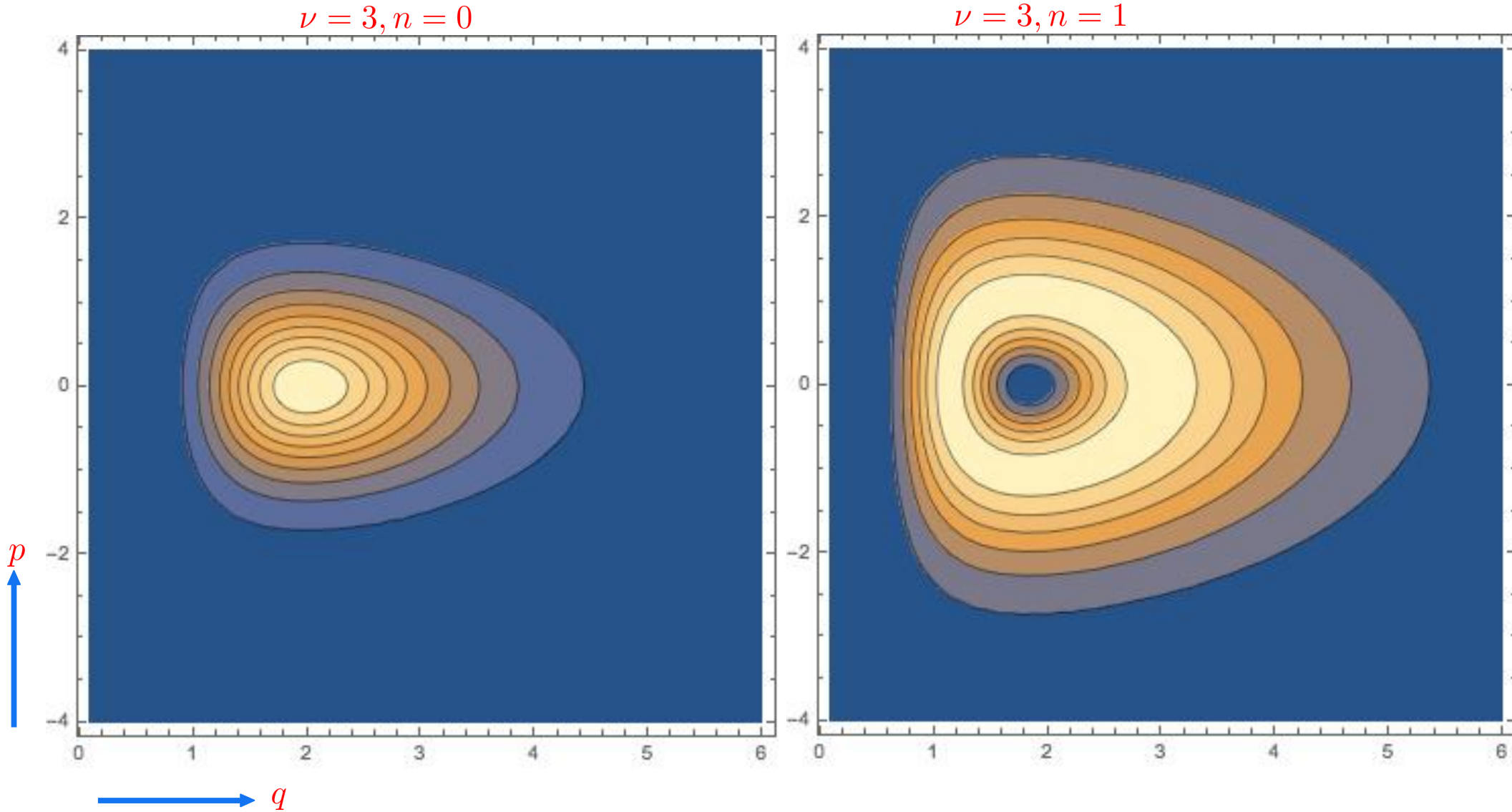
$$\omega = 2E/\xi_\nu$$

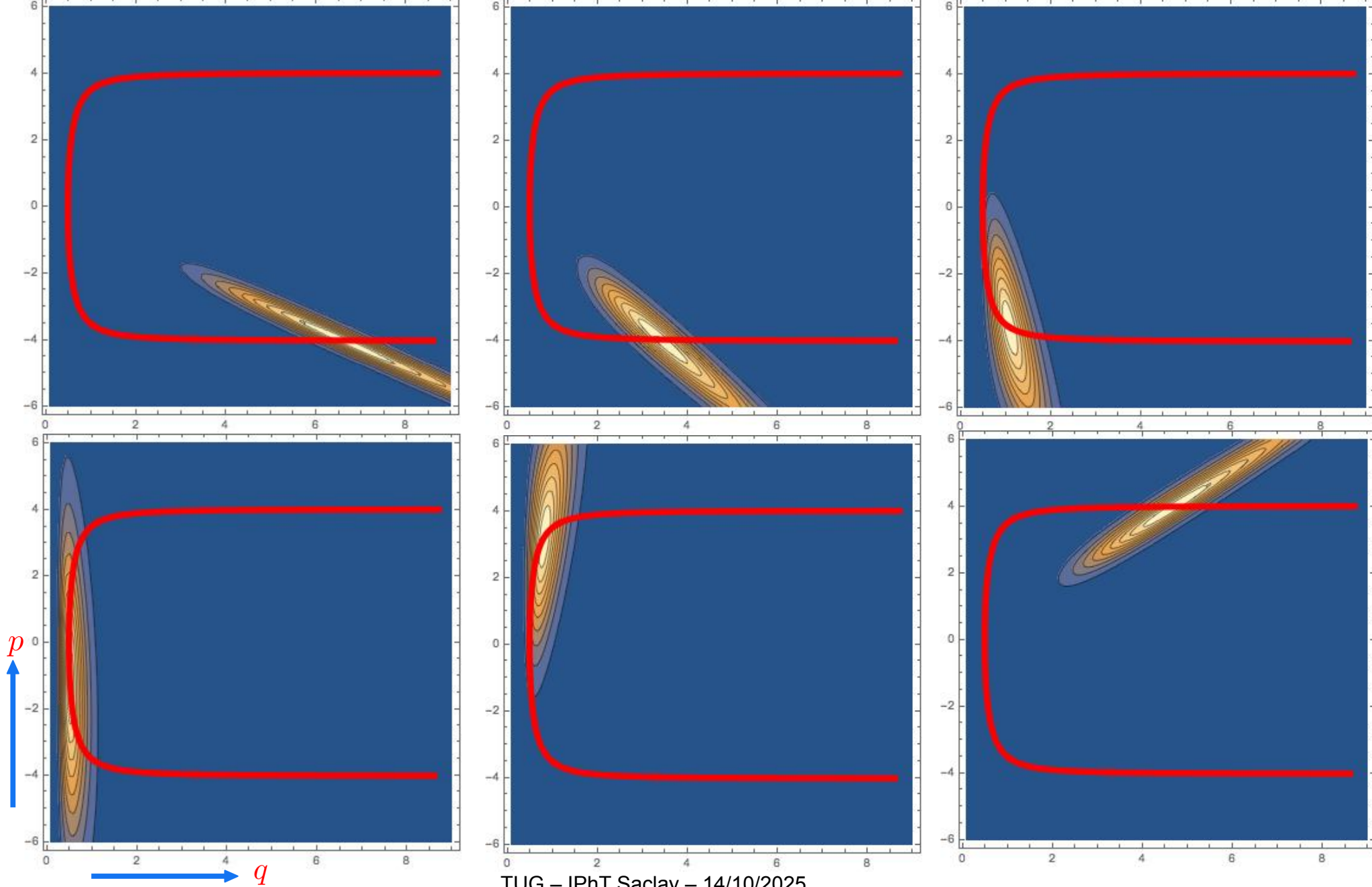
$$\begin{cases} q(\tau) = q_B \sqrt{1 + \omega^2 (\tau - \tau_B)^2} \\ p(\tau) = \frac{1}{2} \frac{dq}{d\tau} = \frac{\omega^2 q_B^2}{2q(\tau)} (\tau - \tau_B) \end{cases}$$



$$p(\tau)^2 + \frac{\nu^2 - \frac{1}{4}}{q(\tau)^2} \equiv E$$

semi classical probability density $\rho_\psi(q, p) = |\langle q, p; \nu, 0 | \psi \rangle|^2 / [2\pi c_0(\nu, 0)]$





Next step: include perturbations

$$H \rightarrow H_\nu + \sum_{\mathbf{k}} H_{\mathbf{k}}^{(2)} = H_\nu + \sum_{\mathbf{k}} \left[\frac{1}{2} |\pi_{v,\mathbf{k}}|^2 + \frac{1}{2} (wk^2 - \mathcal{V}) |v_{\mathbf{k}}|^2 \right]$$

$$\mathcal{V} = \frac{8(1-3w)}{(1+3w)^2} \frac{p^2}{q^2}$$

Mukhanov variable

$$\Rightarrow i \frac{d}{d\tau} |\Psi\rangle = \left(\hat{H}^{(0)} + \hat{H}^{(2)} \right) |\Psi\rangle$$

usual way to go

Born-Oppenheimer approximation

$$|\Psi^{(\text{BO})}\rangle = |\psi^{(\text{bg})}(q)\rangle \otimes |\psi^{(\text{pert})}[\{v_{\mathbf{k}}\}; q(\tau), p(\tau)]\rangle$$

$$+ \text{ Schrödinger } \quad i \frac{d}{d\tau} |\psi^{(\text{pert})}\rangle = \langle \psi^{(\text{bg})} | \hat{H}^{(2)} | \psi^{(\text{bg})} \rangle |\psi^{(\text{pert})}\rangle$$

but possible non diagonal terms... $\langle \psi_1^{(\text{bg})} | \hat{H}^{(2)} | \psi_2^{(\text{bg})} \rangle$

BO not stable $|\Psi\rangle = |\psi_0^{(\text{bg})}\rangle |\psi_0^{(\text{pert})}\rangle$ $\xrightarrow{\text{time evolution}}$ $|\Psi(\tau)\rangle = \sum_n |\psi_n^{(\text{bg})}(\tau)\rangle |\psi_n^{(\text{pert})}(\tau)\rangle$

BO initial condition interferences

Schrödinger $\Rightarrow i \frac{d}{d\tau} |\psi_n^{(\text{pert})}\rangle = \sum_{\ell m} M_{n\ell}^{-1} \hat{H}_{\ell m}^{(2)} |\psi_m^{(\text{pert})}\rangle$

$M_{nm} = \langle \psi_n^{(\text{bg})} | \psi_m^{(\text{bg})} \rangle$ $\langle \psi_\ell^{(\text{bg})} | \hat{H}^{(2)} | \psi_m^{(\text{bg})} \rangle$

overcomplete basis & $\langle \psi_n^{(\text{bg})} | \hat{H}^{(2)} | \psi_m^{(\text{bg})} \rangle \neq 0$

a simple 'semiclassical' state eventually evolves into a multibranch state

interferences / virtual universes / loop expansion

interpretation / meaning?



fixed basis

$$\left. \begin{aligned} |\psi_0^{(\text{bg})}(\tau)\rangle &= |q_0(\tau), p_0(\tau)\rangle \\ |\psi_1^{(\text{bg})}(\tau)\rangle &= |q_1(\tau), p_1(\tau)\rangle \end{aligned} \right\} \text{two background modes with energies } E_0 \text{ and } E_1$$

Born-Oppenheimer basis $|\phi_{E_0}(\tau)\rangle$ and $|\phi_{E_1}(\tau)\rangle \longrightarrow \begin{cases} i\partial_\tau |\phi_{E_0}\rangle = \langle q_0(\tau), p_0(\tau) | \hat{H}_{\mathbf{k}}^{(2)} | q_0(\tau), p_0(\tau) \rangle |\phi_{E_0}\rangle \\ i\partial_\tau |\phi_{E_1}\rangle = \langle q_1(\tau), p_1(\tau) | \hat{H}_{\mathbf{k}}^{(2)} | q_1(\tau), p_1(\tau) \rangle |\phi_{E_1}\rangle \end{cases}$

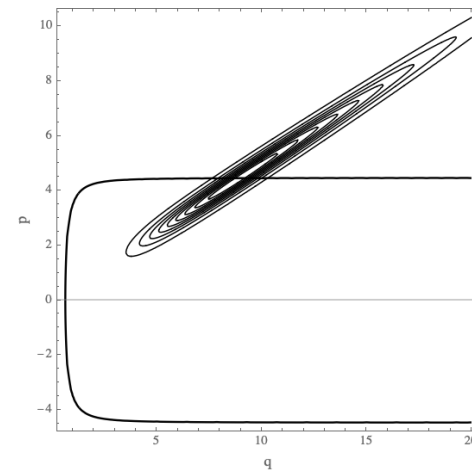
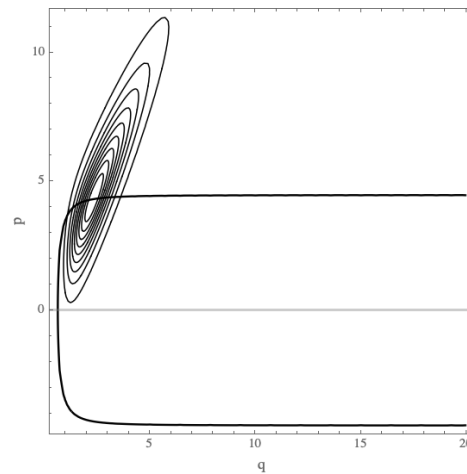
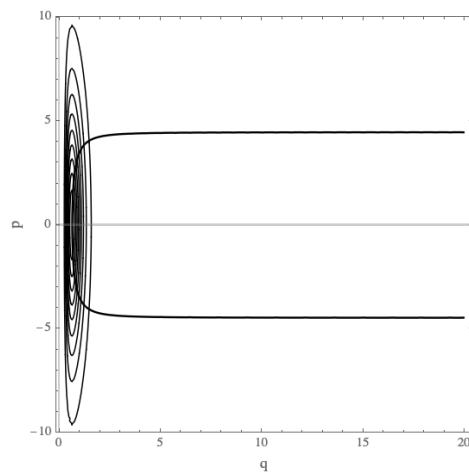
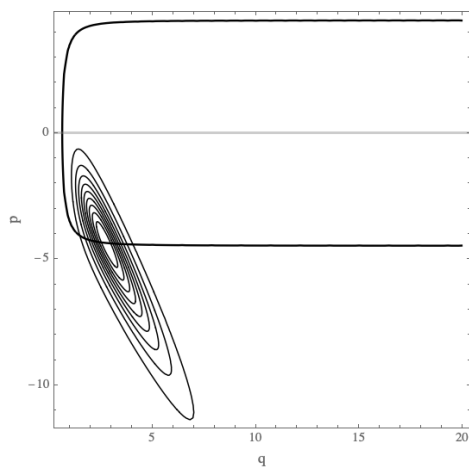
perturbation basis

$$|\psi_0^{(\text{pert})}(\tau)\rangle = \alpha_{00}(\tau) |\phi_{E_0}(\tau)\rangle + \alpha_{01}(\tau) |\phi_{E_1}(\tau)\rangle$$

$$|\psi_1^{(\text{pert})}(\tau)\rangle = \alpha_{10}(\tau) |\phi_{E_0}(\tau)\rangle + \alpha_{11}(\tau) |\phi_{E_1}(\tau)\rangle$$

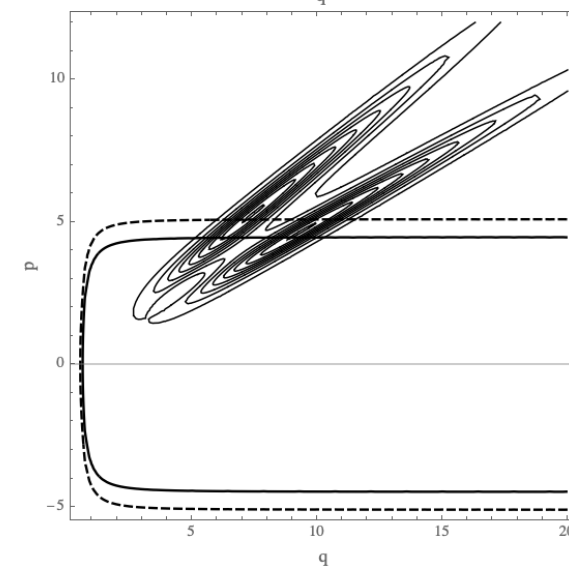
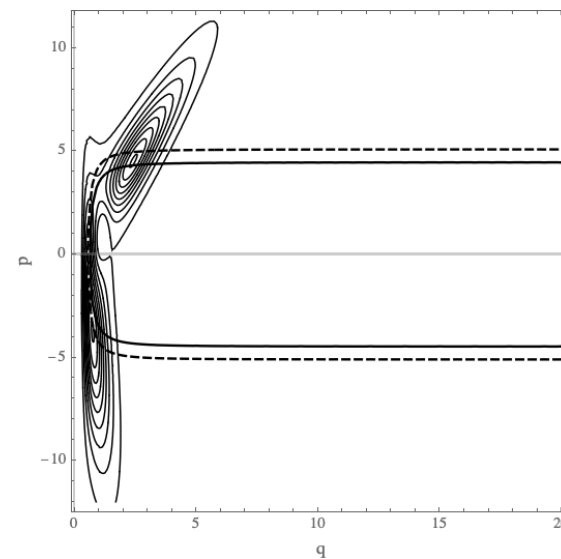
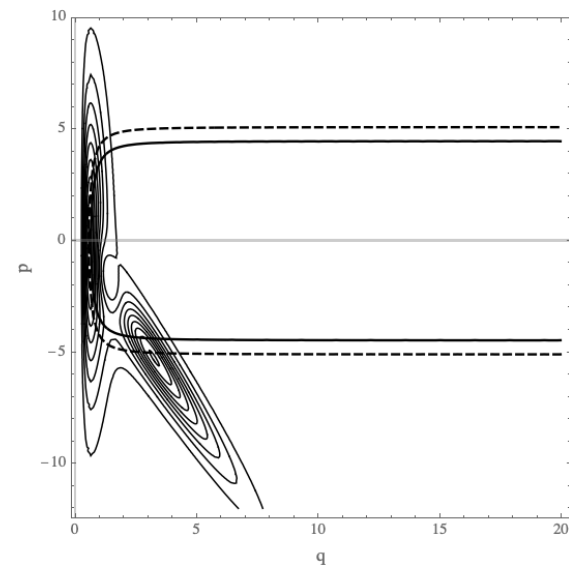
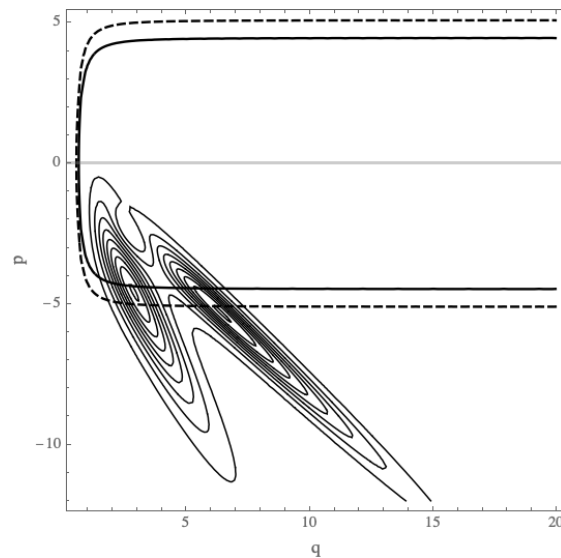
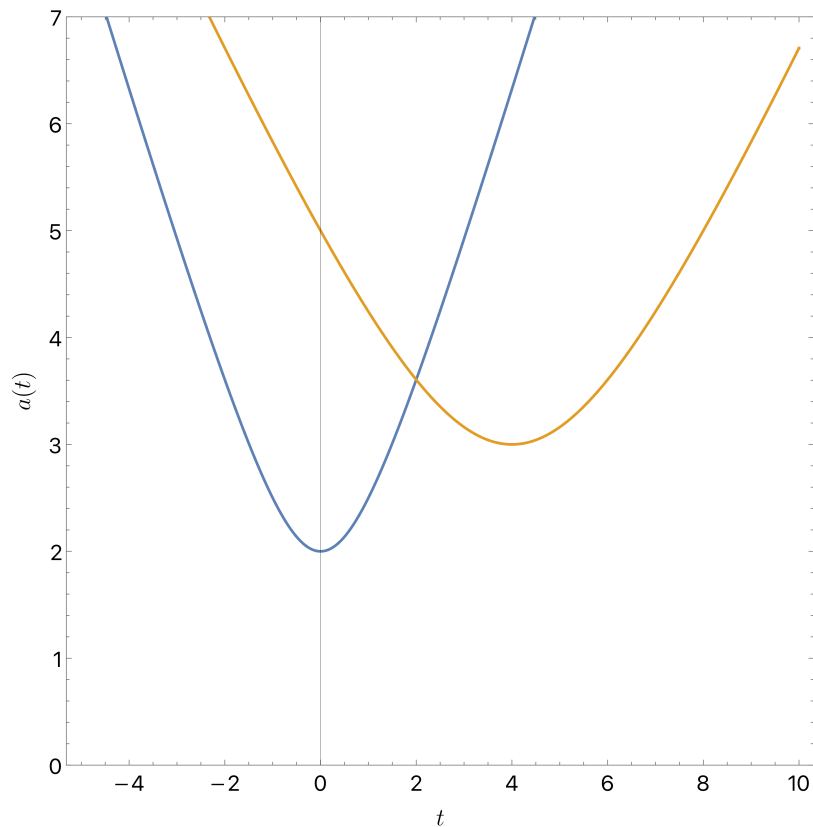
$$\Longrightarrow |\Psi(\tau)\rangle = |q_0, p_0\rangle [\alpha_{00}(\tau) |\phi_{E_0}(\tau)\rangle + \alpha_{01}(\tau) |\phi_{E_1}(\tau)\rangle] + |q_1, p_1\rangle [\alpha_{10}(\tau) |\phi_{E_0}(\tau)\rangle + \alpha_{11}(\tau) |\phi_{E_1}(\tau)\rangle]$$

$$\rho_E(q, p, \tau) = |\psi_E(q, p, \tau)|^2 = \frac{1}{2\pi c_0} \left| \int_0^\infty dx \langle q, p | x \rangle \langle x | \psi_E \rangle \right|^2$$



two modes

$$\rho_{E_0, E_1}(q, p, \tau) = \frac{|\psi_{E_0}(q, p, \tau - \tau_0) + \psi_{E_1}(q, p, \tau - \tau_1)|^2}{2(1 + \Re\langle q_0, p_0 | q_1, p_1 \rangle)}$$

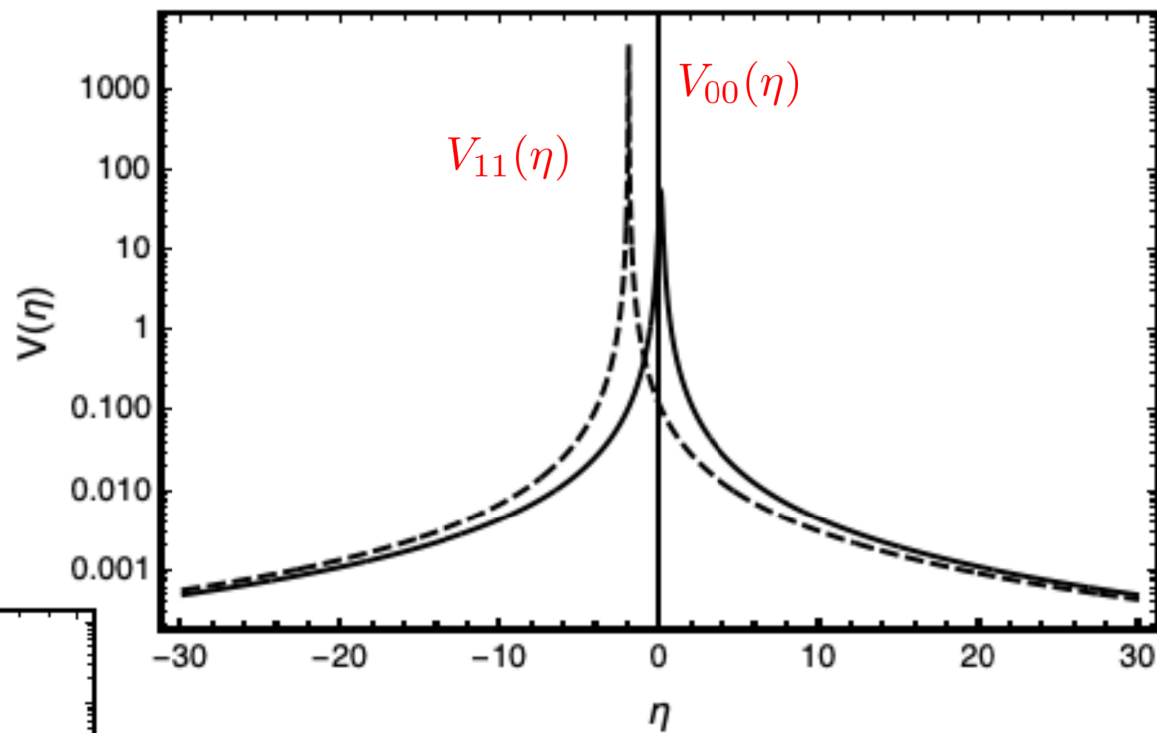
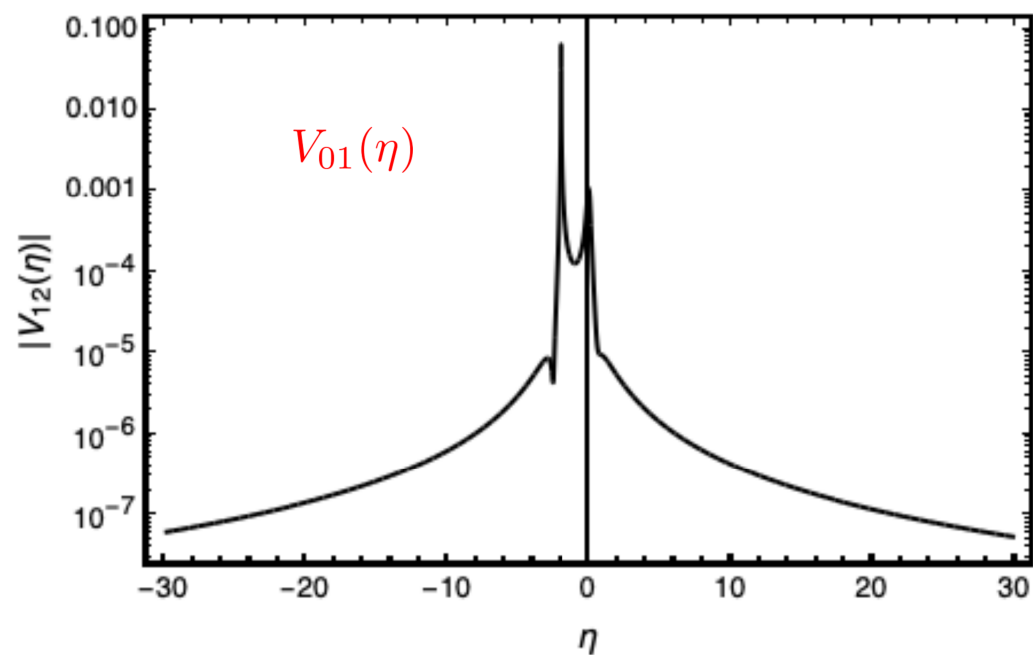


One k mode perturbation hamiltonian $\hat{H}_{ij}^{(2)} = \left(-\frac{1}{2} \frac{\partial^2}{\partial v_{\mathbf{k}}^2} + \frac{1}{2} w k^2 v_{\mathbf{k}}^2 \right) M_{ij} - \frac{1}{2} \mathcal{V}_{ij} v_{\mathbf{k}}^2$

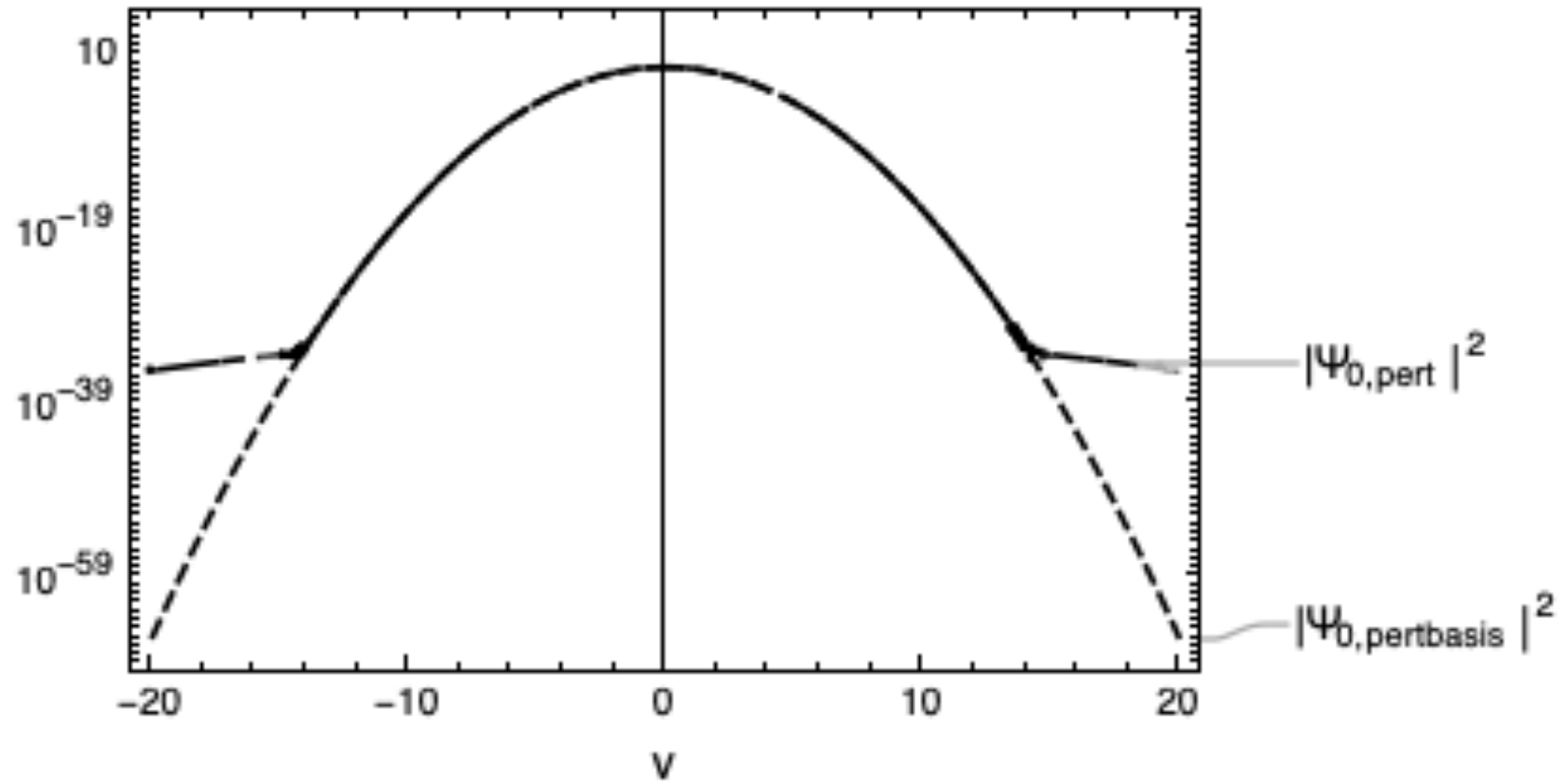
Gaussian perturbation representation $\langle v_{\mathbf{k}} | \phi_{E_i} \rangle = \left[\frac{2\Re(\Omega_i)}{\pi} \right]^{1/4} \exp \left[-\Omega_i(\eta) v_{\mathbf{k}}^2 \right]$

mode equation $f_i'' + (w k^2 - \mathcal{V}_{ii}) f_i = 0 \quad \Longrightarrow \quad \text{variance} \quad \Omega_i = -\frac{i}{2} \frac{f_i'}{f_i}$

potentials



projected distribution



Non gaussian

General framework

basis for background states $\mathcal{H}_B = \{|\alpha\rangle\}$ solving Hamiltonian $\hat{H}^{(0)}$

actual background state $\tau \mapsto |\alpha_B(\tau)\rangle \in \mathcal{H}_B$

general state $|\Psi(\tau)\rangle$ & BO initial conditions $|\Psi(0)\rangle = |\alpha_B\rangle \otimes |\psi_P(0)\rangle$ solves $\hat{H}^{(0)} + \hat{H}^{(2)}$

BO procedure $i \frac{d}{d\tau} |\psi_P^{(\text{BO})}(\tau)\rangle = \tilde{H}_{\alpha_B(\tau)}^{(2)} |\psi_P^{(\text{BO})}(\tau)\rangle$

$\tilde{H}^{(2)} = \langle \alpha_B(\tau) | \hat{H}^{(2)} | \alpha_B(\tau) \rangle$

$|\Phi(\tau)\rangle = e^{-i[\hat{H}^{(0)} + \tilde{H}^{(2)}](\tau - \tau_0)} |\Phi(\tau_0)\rangle$

$\Rightarrow |\Phi^{(\text{BO})}(\tau)\rangle = T \left(\exp \left\{ -i \int_{\tau_0}^{\tau} [\hat{H}^{(0)} + \tilde{H}_{\alpha_B(\tau')}^{(2)}] d\tau' \right\} \right) |\Phi^{(\text{BO})}(\tau_0)\rangle$

interaction picture $\hat{H}_I^{(2)}(\tau) = e^{i\hat{H}^{(0)}\tau} \hat{H}^{(2)} e^{-i\hat{H}^{(0)}\tau}$

$$\Rightarrow |\Phi(\tau)\rangle = e^{-i\hat{H}^{(0)}\tau} \underbrace{T \left[e^{-i \int_{\tau_0}^{\tau} \hat{H}_I^{(2)}(\tau') d\tau'} \right] T \left[e^{i \int_{\tau_0}^{\tau} \tilde{H}_{\alpha_B}^{(2)}(\tau') d\tau'} \right] e^{i\hat{H}^{(0)}\tau}}_{\text{expand the exponentials}} |\Phi^{(\text{BO})}(\tau)\rangle$$

→ expand the exponentials

→ only $\hat{V}(q, p)$ depends on time

⇒ Beyond Born-Oppenheimer

$$|\psi_P^{(\text{BBO})}(\tau)\rangle \simeq \left[1 - K_{\tau, \tau_0} \left(\sum_{\mathbf{k}} \hat{v}_{\mathbf{k}} \hat{v}_{-\mathbf{k}} \right)^2 \right] |\psi_P^{(\text{BO})}(\tau)\rangle$$

$$K_{\tau, \tau_0} = \int_{\tau_0}^{\tau} d\tau_1 \int_{\tau_0}^{\tau_1} d\tau_2 \langle \alpha_B(0) | \hat{V}_B^{(1)}(\tau_1) \hat{V}_B^{(1)}(\tau_2) | \alpha_B(0) \rangle - \frac{1}{2} \left[\int_{\tau_0}^{\tau} V_B(\tau_1) d\tau_1 \right]^2$$

$$S_R = \left\langle \sum_{\mathbf{p}} \hat{v}_{\mathbf{p}} \hat{v}_{-\mathbf{p}} \right\rangle_{\tau}^{(\text{BO})}$$

Cosmological consequences

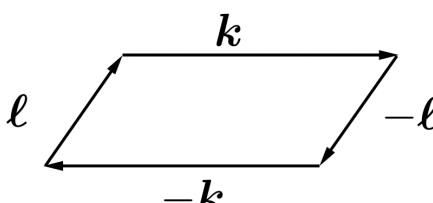
Power spectrum $\langle \hat{v}_{\mathbf{k}} \hat{v}_{\mathbf{\ell}} \rangle_{\tau}^{(\text{BBO})} = \frac{\left\langle \psi_{\text{P}}^{(\text{BBO})}(\tau) \left| \hat{v}_{\mathbf{k}} \hat{v}_{\mathbf{\ell}} \right| \psi_{\text{P}}^{(\text{BBO})}(\tau) \right\rangle}{\left\langle \psi_{\text{P}}^{(\text{BBO})}(\tau) \left| \psi_{\text{P}}^{(\text{BBO})}(\eta) \right\rangle}$

$$\Rightarrow P_{\tau}^{(\text{BBO})}(k) = P_{\tau}^{(\text{BO})}(k) \left[1 - 8 \Re(K_{\tau, \tau_0}) S_{\text{R}} P_{\tau}^{(\text{BO})}(k) \right]$$

non linear modification

Bispectrum $\langle \hat{v}_{\mathbf{k}} \hat{v}_{\mathbf{\ell}} \hat{v}_{\mathbf{m}} \rangle_{\tau}^{(\text{BBO})} = \frac{\left\langle \psi_{\text{P}}^{(\text{BBO})}(\tau) \left| \hat{v}_{\mathbf{k}} \hat{v}_{\mathbf{\ell}} \hat{v}_{\mathbf{m}} \right| \psi_{\text{P}}^{(\text{BBO})}(\tau) \right\rangle}{\left\langle \psi_{\text{P}}^{(\text{BBO})}(\tau) \left| \psi_{\text{P}}^{(\text{BBO})}(\tau) \right\rangle} \rightarrow 0$

Trispectrum $\langle \hat{v}_{\mathbf{k}} \hat{v}_{\mathbf{\ell}} \hat{v}_{\mathbf{m}} \hat{v}_{\mathbf{n}} \rangle_{\tau}^{(\text{BBO})} = \frac{\left\langle \psi_{\text{P}}^{(\text{BBO})}(\tau) \left| \hat{v}_{\mathbf{k}} \hat{v}_{\mathbf{\ell}} \hat{v}_{\mathbf{m}} \hat{v}_{\mathbf{n}} \right| \psi_{\text{P}}^{(\text{BBO})}(\tau) \right\rangle}{\left\langle \psi_{\text{P}}^{(\text{BBO})}(\tau) \left| \psi_{\text{P}}^{(\text{BBO})}(\tau) \right\rangle}$

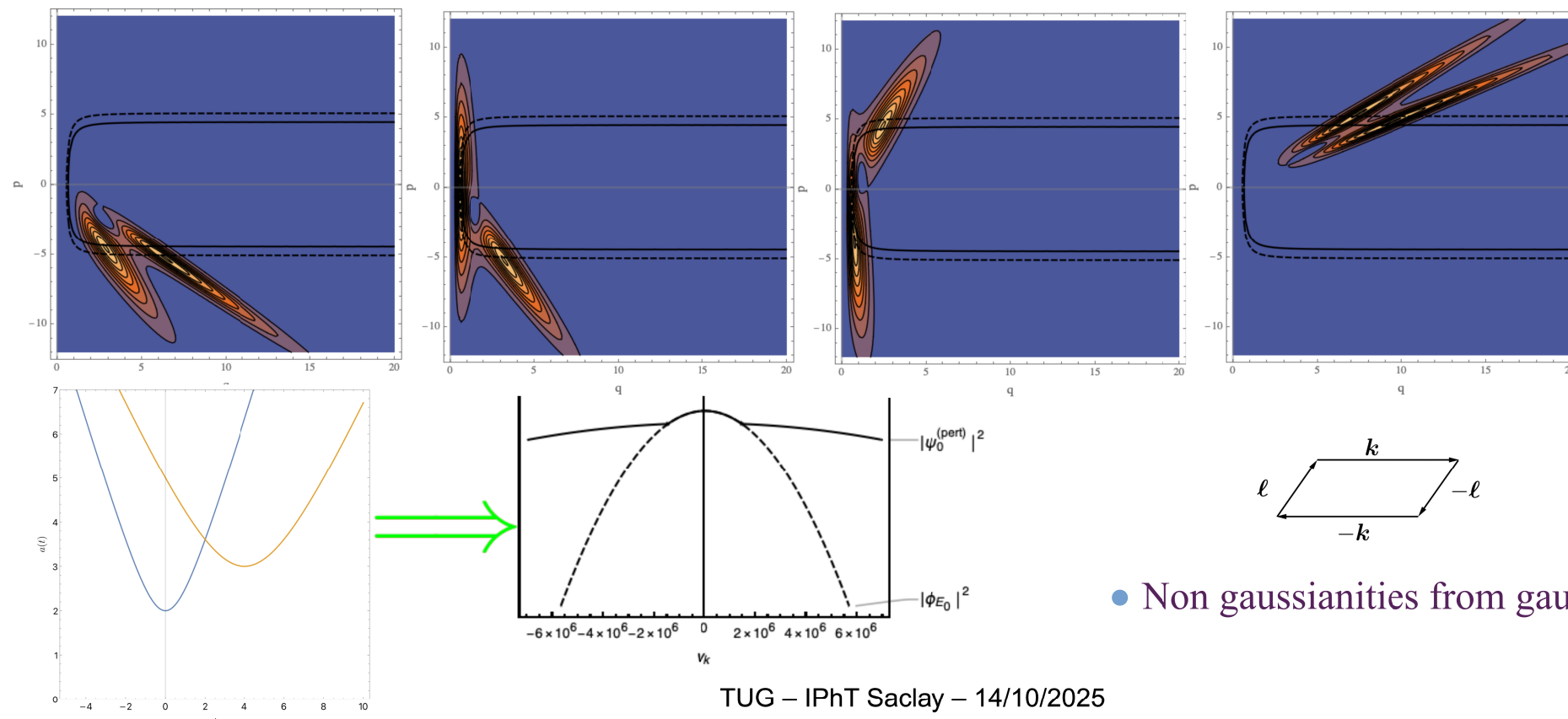


Conclusions

- A new class of coherent state basis for background quantum cosmology
- *quantum background + quantum fluctuations*



superposition principle $\rightarrow$ possible interferences



- Non gaussianities from gaussian states