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Slowly rotating black holes in scalar-tensor theories of gravity

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In collaboration with Karim NOUI and David LANGLOIS (paper available soon)

Introduction

Motivations

More and more **tests of gravity** will come from black holes in the following years.

- LIGO-Virgo-KAGRA collaboration
- Event Horizon Telescopee
- In the future : LISA
- In the future : Einstein Telescope ?

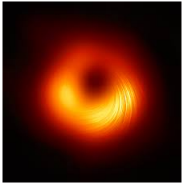

Figure: M87* by the Event Horizon Telescope

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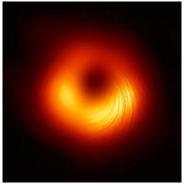

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What are the predictions of modified gravity theories on black holes ?

Modified gravity theories

There exists a large landscape of modifications of General Relativity.

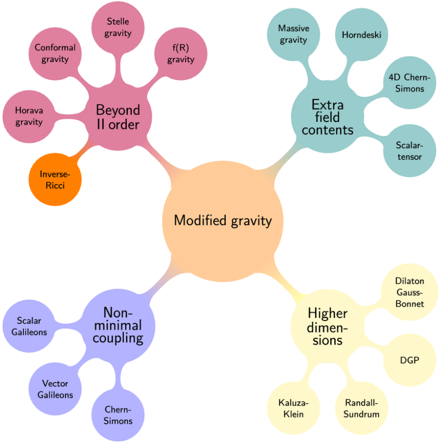

Figure: Landscape of modified gravity theories

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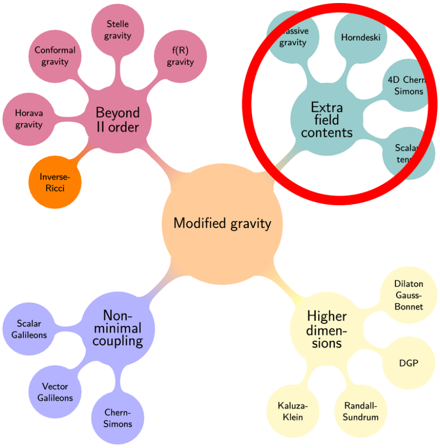

Figure: Landscape of modified gravity theories

Scalar-tensor theories

Definition (Scalar-tensor theories)

Theory of gravity described by both the **metric field** $g_{\mu\nu}$, and a **scalar field** ϕ.

$$S = S_g[g_{\mu\nu}, \phi] + S_m[g_{\mu\nu}, \psi_m] \tag{1}$$

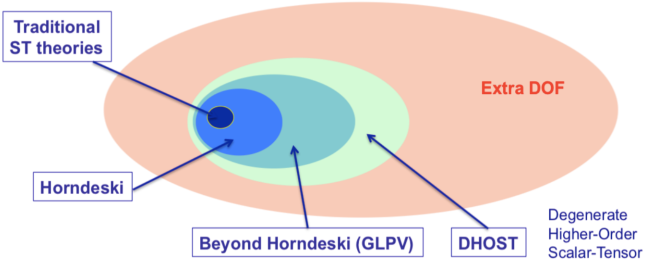

Figure: Landscape of scalar-tensor theories

DHOST theories

Definition (DHOST theories)

Degenerated Higher-Order Scalar-Tensor theories : most general scalar-tensor lagrangian that propagates 3 degrees of freedom : 2 tensorial modes + 1 scalar mode (no Ostrogadski ghost). [Langlois and Noui, 2016]

$$\mathcal{L} = F_2 R + P + Q\Box\phi + A_1 \phi_{\mu\nu}\phi^{\mu\nu} + A_2 \Box\phi^2 + \cdots + F_3 G^{\mu\nu}\phi_{\mu\nu} + B_1 \Box\phi^3 + \dots \quad (2)$$

With $\phi_\mu = \nabla_\mu \phi$, $\phi_{\mu\nu} = \nabla_\mu \nabla_\nu \phi$, and $X = -\frac{1}{2}\phi_\mu \phi^\mu$.
It depends on **functions** of ϕ and X : $F_2(\phi, X)$, $F_3(\phi, X)$, $A_1(\phi, X)$, $B_1(\phi, X)$, ...

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It depends on **functions** of ϕ and X : $F_2(\phi, X)$, $F_3(\phi, X)$, $A_1(\phi, X)$, $B_1(\phi, X)$, ...

Beware

These functions are **not independant**, and must satisfy degeneracy conditions to avoid Ostrogradski instabilities. (Example : $A_1 = -A_2$)

Black holes in DHOST theories

Static black holes

Many **static spherically symetric black hole** solutions are known.

$$\mathrm{d}s^2 = -h(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2\left(\mathrm{d}\theta^2 + \sin^2(\theta)\mathrm{d}\varphi^2\right) \quad ; \quad \phi = qt + \psi(r) \tag{3}$$

Problem

Very few **rotating** black hole solutions are known.

- Stealth Kerr (Kerr metric + non trivial scalar field)
- Disformed Kerr (transformation of stealth Kerr)

It appears to be very difficult to solve the equations of motion in the general axisymmetric case for rotating black holes.

Hartle-Thorne formalism

Approach : find **approximate** rotating metrics for **slowly rotating** black holes. Expand at linear order in the angular momentum J.

Hartle-Thorne ansatz

Ansatz for a rotating metric at linear order in J. [Hartle, 1967][Hartle and Thorne, 1968]

$$\mathrm{d}s^2 = -h(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2\left(\mathrm{d}\theta^2 + \sin^2(\theta)\mathrm{d}\varphi^2\right) - 2\omega(r)r^2\sin^2(\theta)\mathrm{d}t\mathrm{d}\varphi \tag{4}$$

- The unkown is $\omega(r)$, **frame dragging** at radius r, proportional to J.
- $f(r)$ and $h(r)$ are unchanged compared to the static solution at linear order in J.
- The scalar field ϕ is also unchanged at linear order in J.

For Kerr spacetime, we have $\omega(r) = \frac{2J}{r^3}$.

Solving procedure

We want to solve for $\omega(r)$. Standard procedure is the following.

1. Inject the ansatz (4) in the equations of motion $\mathcal{E}_{\mu\nu} = \frac{1}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}}$.
2. Expand at linear order in $\omega(r)$.
3. Equations $\mathcal{E}_{tt}$, $\mathcal{E}_{rr}$, $\mathcal{E}_{\theta\theta}$, and $\mathcal{E}_{\varphi\varphi}$ don't depend on ω. It gives back the static solution for $f(r)$, $h(r)$, $\psi(r)$.
4. Equation $\mathcal{E}_{t\varphi}$ gives a **second order linear differential equation** in $\omega(r)$. Inject the static solution to solve for $\omega(r)$.

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4. Equation $\mathcal{E}_{t\varphi}$ gives a **second order linear differential equation** in $\omega(r)$. Inject the static solution to solve for $\omega(r)$.

However, we have found that a linear combination of those equations gives a simpler equation for $\omega(r)$.

Using symmetry to solve for $\omega(r)$

The idea is to exploit a shift symmetry of the frame dragging.

$$\omega(r) \to \omega(r) + \omega_0 \iff \varphi \to \varphi - \omega_0 t \tag{5}$$

From this symmetry, the following can be shown.

$$\mathcal{E}_{t\varphi} + \omega\mathcal{E}_{\varphi\varphi} = \frac{h}{2\sqrt{-g}}\sin^3(\theta)\frac{\mathrm{d}}{\mathrm{d}r}\left[Q(r)\omega'(r)\right] \tag{6}$$

$\omega(r)$ can be expressed as an integral.

$$\omega(r) = \int \frac{k}{Q(r)}\mathrm{d}r \tag{7}$$

$Q(r)$ depends on the functions of the theory F_2, F_3, A_1, ..., and on the static solution $f(r)$, $h(r)$, $\psi(r)$.

Expression of $Q(r)$

We can compute explicitly the expression of $Q(r)$ from the DHOST lagrangian.

$$Q = \sqrt{\frac{f}{h}}\left((F_2 + 2XA_1)r^4 + 2f\psi' XB_2r^3 - \frac{1}{2}f\psi' X'(F_{3X} + 4XB_6 - 2B_2)r^4\right) \tag{8}$$

Plug-in your theory functions F_2, A_1, ..., and the static solution $f(r)$, $h(r)$, $\psi(r)$, and you can compute the first order rotation $\omega(r)$.

$$\omega(r) = \int \frac{k}{Q(r)}\mathrm{d}r \tag{9}$$

Slowly rotating solutions in shift-symmetric theories

Shift-symmetric theories

We will consider a **subclass of DHOST theories**.

Definition (Shift-symmetric scalar-tensor theories)

Theories for which the lagrangian obeys the symmetry $\phi \to \phi + c$. The lagrangian must depend only on the **derivatives** of ϕ.

In these theories, a time dependance can be added to the scalar field, while preserving the stationnarity of the metric.

$$\phi = qt + \psi(r) \tag{10}$$

Many **static** black hole solutions were found in this framework.

No-hair theorems for shift symmetry

No-hair theorems have been proven in the **Horndeski** framework, for shift-symmetric theories.

- For static black holes, with $q = 0$ [Hui and Nicolis, 2013]
- At first order in rotation, with $q \neq 0$ [Maselli et al., 2015]

Theorem

In shift-symmetric Horndeski theories, providing theory functions are ***analytic*** *in X,* ***asymptotically flat*** *black hole solutions are identical to Kerr at first order in J, with no scalar hair.*

$$f(r) = h(r) = 1 - \frac{2M}{r} \quad ; \quad \omega(r) = \frac{2J}{r^3} \quad ; \quad \psi'(r) = 0 \tag{11}$$

Evading the no-hair theorem

Hairy static solutions have been found, evading no-hair theorems hypotheses.

- Non-analytic functions : shift-symmetric Gauss-Bonnet $F_3 \propto \log(X)$ [Lu and Pang, 2020], BCL solution $F_2 = 1 + \lambda\sqrt{|X|}$ [Babichev et al., 2017].
- Hairy solutions in **beyond Horndeski** theories [Bakopoulos et al., 2024].

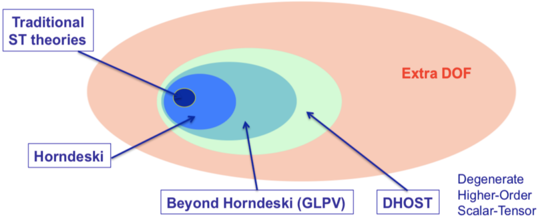

Figure: Landscape of scalar-tensor theories

Slowly rotating metric

Let's compute $\omega(r)$ for these solutions. We will restrict ourselves to shift-symmetric, **parity-symmetric** ($\phi \to -\phi$), beyond Horndeski theories.

$$Q = \sqrt{\frac{f}{h}}\left(F_2 + 2XA_1\right) r^4 \tag{12}$$

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For these theories, it can be shown from the equations of motion that we also have the following equation.

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Hence, $Q(r) \propto r^4$, and so we can deduce $\omega(r)$.

$$\omega(r) = \frac{2J}{r^3} \tag{14}$$

Summary of the solutions

In this class of theories, the first order correction in J is always identical to the one of Kerr, even for hairy static solutions.

[Babichev et al., 2017]	$f(r) = h(r) = 1 - \frac{2M}{r} - \frac{\beta^2}{2\zeta\eta r^2}$	$\omega(r) = \frac{2J}{r^3}$
[Bakopoulos et al., 2024]	$f(r) = h(r) = 1 - \frac{2M}{r} + \eta q^4 \left(\frac{\frac{\pi}{2} - \mathrm{Arctan}\left(\frac{r}{\lambda}\right)}{\frac{r}{\lambda}} + \frac{1}{1+\left(\frac{r}{\lambda}\right)^2} \right)$	$\omega(r) = \frac{2J}{r^3}$
[Bakopoulos et al., 2024]	$f(r) = h(r) = 1 + \eta q^2 - \frac{2M}{r} + \eta q^2 \frac{\frac{\pi}{2} - \mathrm{Arctan}\left(\frac{r}{\lambda}\right)}{\frac{r}{\lambda}}$	$\omega(r) = \frac{2J}{r^3}$

Scalar Gauss-Bonnet

This is not true for non-parity-symmetric theories, as scalar Gauss-Bonnet theories.

Static solution : [Lu and Pang, 2020]

$$f(r) = h(r) = 1 + \frac{r^2}{2\alpha}\left(1 - \sqrt{1 + \frac{8\alpha M}{r^3}}\right) \tag{15}$$

First order rotation : [Charmousis et al., 2022]

$$\omega(r) = \frac{J}{M}\frac{1 - f(r)}{r^2} = -\frac{J}{2\alpha M}\left(1 - \sqrt{1 + \frac{8\alpha M}{r^3}}\right) \tag{16}$$

Conclusion

Conclusion

- We have established a way to compute $\omega(r)$ in an integral form for any scalar-tensor theory, providing the static solution is known.

$$\omega(r) = \int \frac{k}{Q(r)} \mathrm{d}r \tag{17}$$

- For a large subclass of theories (shift and parity symmetric beyond Horndeski theories), $\omega(r)$ is always identical to the one of Kerr, independently of the static solution.

$$\omega(r) = \frac{2J}{r^3} \tag{18}$$

Thank you !

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