Stability of circular orbits of compact binaries in general relativity
A post-Newtonian analysis

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Compact binaries

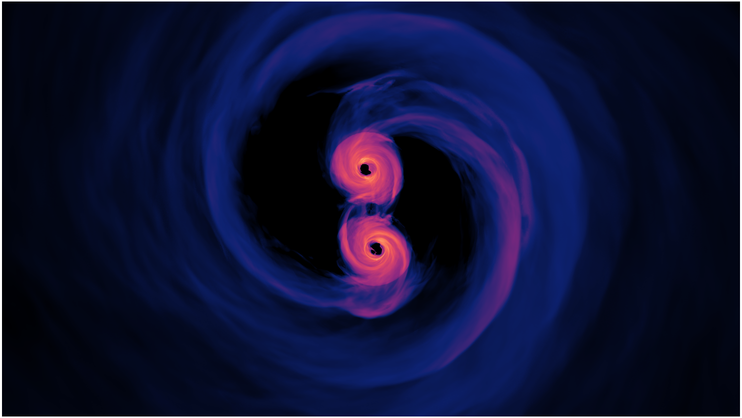

Figure: Binary system of black holes orbiting around each other. (source: https://svs.gsfc.nasa.gov/13086)

Stability in mechanics

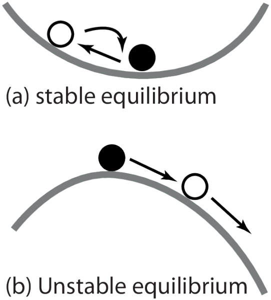

Figure: Stable (a) and unstable (b) equilibria.

Why look at the stability?

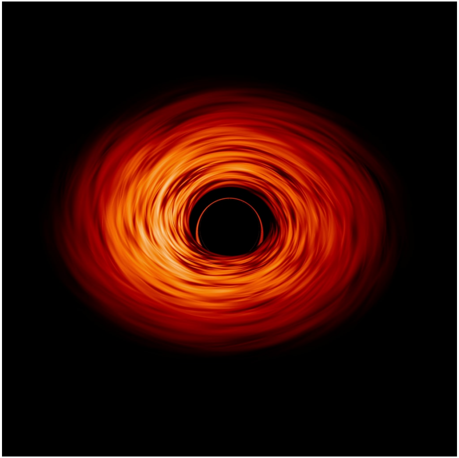

Figure: Accretion disk around a black hole. The disk stops at the *Innermost Stable Circular Orbit (ISCO)*. (source: `https://svs.gsfc.nasa.gov/13326`)

Problem at hand

Set-up

Two compact objects (black holes or neutron stars) orbiting one another around a circular orbit.

Question

Given the (conservative part of) the dynamics set by general relativity, what is the location of the ISCO?

Issue

The dynamics (*i.e.* Lagrangian or Hamiltonian) of the two-body problem has no exact realisation in general relativity: one must then resort to an approximation scheme.

Principle

$$
\begin{aligned}
(\text{General Relativity}) = {} & (\text{Newtonian theory}) + \left(\frac{v}{c}\right)^2 (\text{1PN correction}) \\
& + \left(\frac{v}{c}\right)^4 (\text{2PN correction}) \\
& + \ldots \\
& + \left(\frac{v}{c}\right)^{2n} (n\text{PN correction}) \\
& + \ldots
\end{aligned}
$$

Our work (with Luc Blanchet and David Langlois)

Literature

The stability analysis has been pursued up to the 3PN order (Blanchet et Iyer 2003).

Challenge

Extend this analysis to the 4PN order.

Non-local “tail” interaction at 4PN

Gist

The evolution of the system at time t depends on the state of the system at all other times $t' \neq t$.

Physical origin

Due to the non-linear nature of the gravitational interaction, the gravitational waves emitted by a body can influence it back at later times.

Results

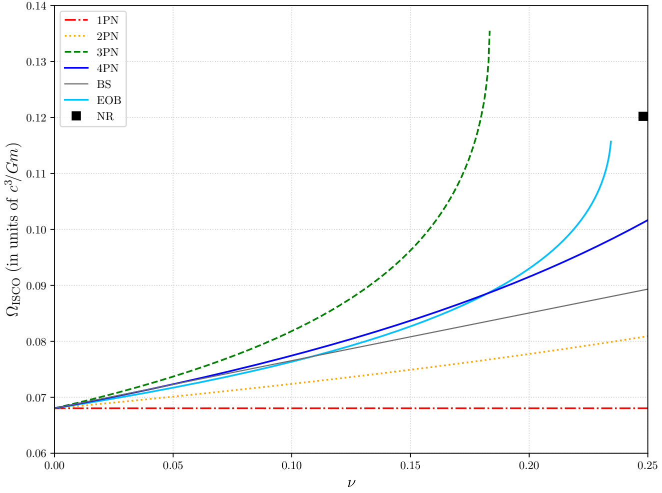

Figure: ISCO orbital frequency $\Omega_{\rm ISCO}$ in terms of the symmetric mass ratio $\nu = m_1 m_2/(m_1+m_2)^2$.

Thank You!

Questions?